IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received February 17, 2020, accepted March 23, 2020, date of publication April 6, 2020, date of current version April 29, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2985763

Fractional Neuro-Sequential ARFIMA-LSTM for Financial Market Forecasting

AYAZ HUSSAIN BUKHARI[1], MUHAMMAD ASIF ZAHOOR RAJA[2,3], MUHAMMAD SULAIMAN[1], SAEED ISLAM[1], MUHAMMAD SHOAIB[4], AND POOM KUMAM[5,6], (Member, IEEE)

[1]Department of Mathematics, Abdul Wali Khan University Mardan, Mardan 23200, Pakistan
[2]Future Technology Research Center, National Yunlin University of Science and Technology, Yunlin 64002, Taiwan
[3]Department of Electrical and Computer Engineering, COMSATS University Islamabad–Attock, Attock 43600, Pakistan
[4]Department of Mathematics, COMSATS University Islamabad–Attock, Attock 43600, Pakistan
[5]Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand
[6]Department of Medical Research, China Medical University Hospital, China Medical University, Taichung City 40402, Taiwan

Corresponding authors: Muhammad Sulaiman (msulaiman@awkum.edu.pk) and Poom Kumam (poom.kum@kmutt.ac.th)

This work was supported by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT.

ABSTRACT Forecasting of fast fluctuated and high-frequency financial data is always a challenging problem in the field of economics and modelling. In this study, a novel hybrid model with the strength of fractional order derivative is presented with their dynamical features of deep learning, long-short term memory (LSTM) networks, to predict the abrupt stochastic variation of the financial market. Stock market prices are dynamic, highly sensitive, nonlinear and chaotic. There are different techniques for forecast prices in the time-variant domain and due to variability and uncertain behavior in stock prices, traditional methods, such as data mining, statistical approaches, and non-deep neural networks models are not suited for prediction and generalized forecasting stock prices. While autoregressive fractional integrated moving average (ARFIMA) model provides a flexible tool for classes of long-memory models. The advancement of machine learning-based deep non-linear modelling confirms that the hybrid model efficiently extracts profound features and model non-linear functions. LSTM networks are a special kind of recurrent neural network (RNN) that map sequences of input observations to output observations with capabilities of long-term dependencies. A novel ARFIMA-LSTM hybrid recurrent network is presented in which ARFIMA model-based filters having the linear tendencies better than ARIMA model in the data and passes the residual to the LSTM model that captures nonlinearity in the residual values with the help of exogenous dependent variables. The model not only minimizes the volatility problem but also overcome the over fitting problem of neural networks. The model is evaluated using PSX company data of the stock market based on RMSE, MSE and MAPE along with a comparison of ARIMA, LSTM model and generalized regression radial basis neural network (GRNN) ensemble method independently. The forecasting performance indicates the effectiveness of the proposed AFRIMA-LSTM hybrid model to improve around 80% accuracy on RMSE as compared to traditional forecasting counterparts.

INDEX TERMS ARIMA model, ARFIMA model, GARCH model, RNN, LSTM model, RMSE, MSE, MAPE.

I. INTRODUCTION

The fast emergence of digital economics is one of the most innovative contributions in the modern global economy. With the development of globalization trades and business contact on financial activities among nations are increasing. The international trades and financial business are closely connected with stock rates [1]. The rapid development of digital currencies in the financial market abrupt impact on the movement of stock price [2]. The forecasting of financial data depends on the collection frequency of financial market. The modeling of high-frequency dynamic data of finance has to become a research focus in the research community.

Forecasting future values of time series has been a major research area since ago. The application of time series modeling finds its significance in business, stock exchange,

The associate editor coordinating the review of this manuscript and approving it for publication was Avishek Guha.

weather, electricity demand and many other fields [3]. The accurate forecasting of stock prices can help investors as a guideline to minimize risk and reducing investment losses [4]. The scientific way of modeling time series emerged when Box-Jenkins [5] introduced the methodology for time series in 1970 in which ARIMA model was introduced to forecast future behavior. The traditional time series forecasting methods depend mainly on exponential smoothing, Auto Regression and on Moving Average parameters, including ARMA model, ARIMA model [6], GARCH model [7]. Peters [8] noted that the dynamic nature of the stock market which is mostly non-Gaussian in nature with sharper peaks and fat tails [9]. In the presence of such evidence, the traditional methods have their limitation to provide accurate forecasts based on non-Gaussian data [10]. Sheng and Chen [11] proposed a new Autoregressive Fractional Integrated Moving Average (ARFIMA) model to analyze the GSL data to predict future levels and compared accuracy with previously published results [12]. The ARFIMA class model presented by Diebold and Rudebusch [13] provided flexible techniques to capture a long memory process. A neural network was used by Gao *et al.* [14] to predict daily closing prices of S&P 500 stocks exchange. The ARIMA and neural network hybrid models were discussed in Peter Zhang's literature [15]. Chen *et al.* [16] predicted stock exchange data of China stock market using sequential features of LSTM, He used 30 days long sequential step with 10 learning features in the model. A comparison of the traditional ARIMA model with deep learning features of LSTM for economics and financial data was carried out by Siami and Namin [17]. Stock prediction using LSTM and MLP model was estimated by Khare *et al.* [18]. Short Hybrid ARIMA-LSTM model was presented by Choi [19] in which the stock price correlation coefficient was analyzed by applying LSTM recurrent neural networks. The effect of currency and foreign exchange on stock market volatility was studied by Fang [20]. Fractional-order derivative is a generalized form of integer derivative which is extensively applied for modeling of different real phenomena in finance, psychology, bioengineering. mechanics and control theory. The concept fractional-order derivative emerged back in 1695 with famous correspondence between L'Hopital and Leibniz about the possibility of fractional-order derivatives. The first application of fractional order mathematics contributed by Abel [21] in 1823 who solved the autochrome integral order problem with the fractional derivative of half order. The application of fractional order differential equations has introduced new concepts and techniques in financial market forecasting. Modeling with fractional order with the Adomian decomposition method was introduced by Song [22] in an approximation of European price modeling and China's financial market. Biologists deducted that biological organisms also have fractional-order electric conductivity in their cell membranes [23], which is classified as non-integer group models. Kumar and Rawat [24] proposed techniques to estimate coefficients of fractional order differential equations.

Objective of the study

There are two main objectives of the study:

• To analyze the time-series data and identify the nature of phenomenon in the sequence of observation and study the pattern based on fractional differences.

• Forecasting nonlinear time series and predict future values on the bases of pattern identified.

The innovative contributions of designed hybrid neurocomputing approach with the exploration of different capabilities are presented in terms of following salient features:

• Provision of flexible tool for classes of long-memory model.

• The ARFIMA model filters linear tendencies better than ARIMA model in the hybrid scenarios.

• The proposed model is capable to overcome the over fitting problem of neural networks besides minimizing volatility problems.

• Dynamical features capturing the ability of the desired model ARFIMA-LSTM by the addition of exogenous dependent variables.

The rest of the paper is organized as follows. Section II describes different definitions of fractional order. section III describes the statistical analysis of data. Section IV presents the component model of ARIMA, ARFIMA, and LSTM and generalized regression radial basis neural network. The construction of the proposed nonlinear combination model is described in Section V. Section VI presents the experimental results and summary of implication based on the real Hybrid ARFIMA-LSTM time series. Finally, Section VII. is the conclusion.

For the convenience of readers, the notations used in this paper are summarized in Table 1.

II. PRELIMINARIES

Few preliminaries regarding fractional-order derivatives are presented here along with fractional time series and GRNN.

Definition 1 Grunwald-Letnikov:

Grunwald-Letnikov [25] presented a generalized form of fractional order using binomial expansion.

$$ {}_aD_t^{\alpha} f(\mathrm{t}) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum\nolimits_{j=0}^{[(x-2)/h]} (-1)^j \binom{a}{j} f(\mathrm{t}-\mathrm{jh}), \quad (1) $$

where $\binom{a}{j}$ is binomial coefficient and a is constant order, which can express by Euler's Gamma function defined as follows:

$$ \binom{a}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}. \quad (2) $$

Definition 2 Michele Caputo:

Michele Caputo [26] defined fractional order by applying the integral equation as follows:

$$ {}_a^cD_t^{\alpha} f(\mathrm{t}) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(\mathrm{n})}(\xi)}{(\mathrm{t}-\xi)^{\alpha+1-n}} d\xi, \quad (3) $$

where α is a real number and n is an integer. Grunwald-Letnikov definition is identical to Caputo's definition for

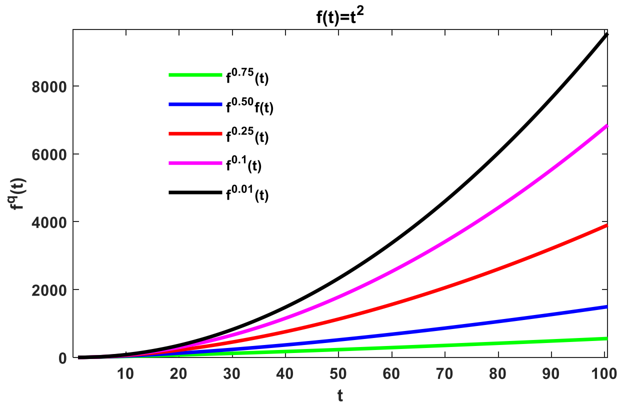

FIGURE 1. Fractional order representation of function f(x)=x2.

fractional derivative except in case of constant function for which Caputo derivative is zero, while Riemann-Liouville derivative of constant is a non-zero value.

Definition 3 Atangana-Baleanu:

The left Atangana-Baleanu [27] definition in term of fractional derivate for the interval $0 < \alpha < 1$ in Sobolev space is defined by:

$$T_{\alpha}(\mathrm{h})(\mathrm{x}) = \frac{B(\alpha)}{1-\alpha}\int_{0}^{x} h'(\mathrm{s})E_{\alpha}\left[-\frac{\alpha}{1-\alpha}(\mathrm{x}-s)^{\alpha}\right]\mathrm{ds}, \quad (4)$$

where $h \in H^{1}(0,1)$ in Sobolev space, $B(\alpha) > 0$ is a function in normalized form satisfying the condition: $B(0) = B(1) = 1$ and E_{α} is Mittag-Leffler function of a single variable.

Definition 4 Riemann-Liouville:

Riemann-Liouville [28] used derivatives instead of integral order to defined fractional-order derivatives defined as:

$$ {}_{a}^{c}D_{t}^{\alpha}f(\mathrm{t}) = \frac{1}{\Gamma(\mathrm{n}-\alpha)}\frac{d^{n}}{dx^{n}}\left[\int_{a}^{t}(\mathrm{t}-\xi)^{n-\alpha-1}f(\xi)\,d\xi\right]. \quad (5)$$

Fractional derivative by using the definition of Riemann-Liouville in term of gamma function is defined as

$$\frac{d^{q}}{dx^{q}}x^{m} = \frac{\Gamma(\mathrm{m}+1)}{\Gamma(m-q+1)}x^{m-q}, \quad (6)$$

for $m = 2$ the equation become

$$\frac{d^{q}}{dx^{q}}x^{2} = \frac{\Gamma(2+1)}{\Gamma(2-q+1)}x^{2-q}, \quad (7)$$

by taking fractional derivatives of order 0.75, 0.50, 0.25, 0.1 and 0.01 the geometrical representation of fractional derivative is shown in Figure 1.

A. FRACTIONAL TIME SERIES

Fractional Time series was developed by Harold Hurst [29] while calculating optimal dam size for the River Nile, which was directly linked with a fractional dimension of the dam. Consider d as periodic time duration over the range R, which was calculated by differencing of largest and smallest deviation encounter during d time interval which can be represented as:

$$R\alpha dH,$$

TABLE 1. Notations.

Abbreviation	Description
AE	Absolute Error
ARFIMA	Auto Regressive Fractional Integrated Moving Average
D	Degree of differencing
FFC	Fauji Fertilizer Company
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GRNN	Generalized regression radial basis neural network
H	Hurst Parameter
L	Backward-shift operator
LSTM	Long-short term memory
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MSE	Mean Squared Error
RBFs	Radial basis function neural network
RMSE	Root Mean Square Error
RNN	Recurrent Neural Network
p	Autoregressive order
q	Moving average order
r	Differencing in decimal form
Γ(·)	Gamma Function
ψ1	Autoregressive coefficients
ψ2	Moving average coefficients
σ	Sigmoid function

where H is the Hurst exponent varying from zero to one and the higher value of the Hurst component was represented with a smaller size of the curve.

B. GENERALIZED REGRESSION RADIAL BASIS NEURAL NETWORK (GRNN)

Mathematically, GRNN [30] can be represented by the equation.

$$Y(\mathrm{x}) = \frac{\sum_{k=1}^{N} w_{k}K(\mathrm{x},\mathrm{x}_{k})}{\sum_{k=1}^{N} K(\mathrm{x},\mathrm{x}_{k})}, \quad (8)$$

where Y(x) is a prediction for input variable x, w_k is activation weight for the pattern layer and $K(\mathrm{x},\mathrm{x}_k)$ is Gaussian radial basis function formulated as:

$$K(\mathrm{x},\mathrm{x}_{k}) = e^{-d_{k}/2\sigma^{2}}, \quad (9)$$

where d is Euclidean distance defined as:

$$d_{k} = (x-x_{k})^{T}(x-x_{k}).$$

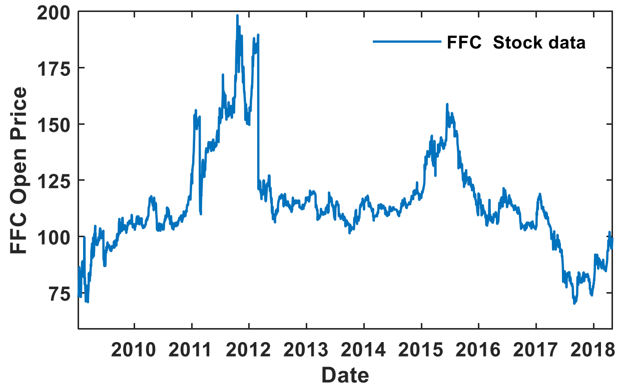

FIGURE 2. Graphical representation of FFC daily data 2009-2018.

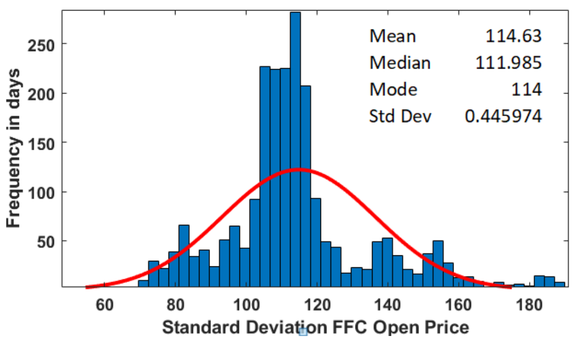

FIGURE 3. The probability distribution of FFC open Price.

C. STATISTICAL DESCRIPTION OF DATA

In this section, the statistical description of Fauji Fertilizer Company (FFC) open price data [31] is presented. We have used daily open price pf FFC data from 01 January 2009 to 30 May 2018 with $n = 3437$ observation. However, for modeling purposes, we have considered the daily data until 30 April 2018. The remaining data of one month is used to analyze the forecasting behavior of the proposed model. From the graphical analysis, it is easy to identify a most expressive increasing trend from 01 January 2009 up to 19 October 2012, then, a sudden declining trend in open price can be noticed until 05 January 2015 followed by another jump in open prices till 19 December 2017 after which last descending trend was noticed till 30 May 2018 as shown in Figure 2.

The highest non-Gaussian variation was noticed in the interval of 120-300 can be noticed from 2009 to 2018 as shown in Figure.3.

FFC open price data has shown sharper peaks which represent high-frequency data of the non-Gaussian distribution curve as shown in Figure 3.

The probability distribution of data with percentile Gaussian distribution is shown in Figure 4.

The Probability value of the fit is calculated with P value lesser then 0.005 displaying the non-Gaussian distribution and making kurtosis in vertical spread.

The seasonal plot of FFC open price with a strong upward periodic trend with a high degree of automation trading was noticed in the data as shown in Figure 5. The yearly variation in June, July and at the end of each year remain high as compared to the remaining months.

The statistical description of dependent variables used as exogenous input for the prediction of FFC open price is presented in Table 2 and Figure 6.

The correlation between oil prices and FFC open price remains high as compared to other dependent variables. The relationship between foreign reserves and FFC open price

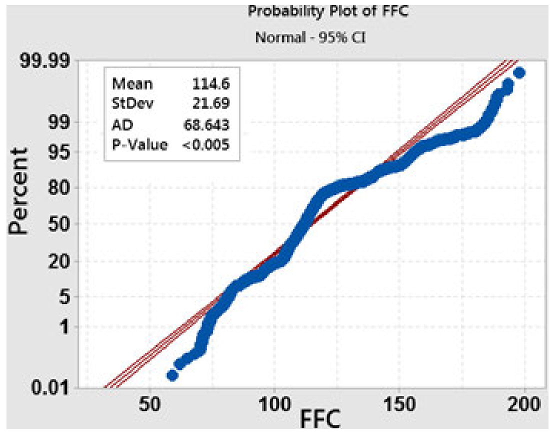

FIGURE 4. Percentile Gaussian fit of FFC open price.

FIGURE 5. The seasonal plot of FFC company from 2009-2018.

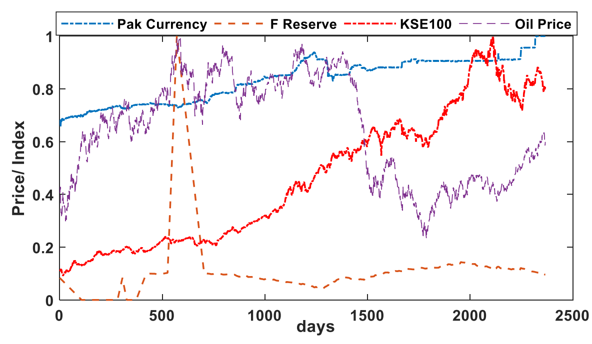

FIGURE 6. Graph of dependent variables used in the modeling.

TABLE 2. The statistical description of FFC open price & dependent variable.

Statistics	FFC open	PK Currency	KSE100 index	Oil Prices	Foreign Reserves
Mean	114.67	96.52	24359.08	73.47	22126.75
Median	112.02	98.65	22930.06	76.01	16432.42
Mode	114.00	104.80	10519.02	44.66	13248.56
St Dev	21.66	9.49	13431.98	22.46	24086.01
Kurtosis	1.48	-1.19	-1.26	-1.37	18.05
Skewness	0.93	-0.26	0.32	-0.12	4.23
Minimum	58.73	68.21	4815.34	26.21	7589.60
Maximum	198.35	115.64	52876.46	113.93	170454.00

perfectly remained very close in the highest variation years of 2012 and 2016.

III. ARIMA AND ARFIMA MODELS

In this section, we will discuss some basic concepts and background of both models, i.e., ARIMA and ARFIMA, and hybridization with LSTM.

A. ARIMA MODEL

The mathematical representation of ARIMA Model was first introduced by Box and Jekin in his book in 1970 [5], to forecast the future trend representing by the equations as:

$$\begin{aligned} x_t &= c + \psi_1 x_{t-1} + \psi_2 x_{t-2} + \ldots + \psi_p x_{t-p} \\ &\quad - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q}, \\ x_t &= c + \sum\nolimits_{k=1}^{P} \psi_1 x_{t-k} + \sum\nolimits_{l=1}^{q} \theta_l \varepsilon_{t-l}, \end{aligned} \tag{10}$$

where $\psi(B) = 1 - \psi_1 B - \ldots - \psi_p B^p$, and $\theta(B) = 1 - \theta_1 B - \ldots - \theta_p B^p$, are polynomials in B and $\psi_i (i = 1, 2, \ldots, p)$ and $\theta_i \; (i = 1, 2, \ldots, q)$ are autoregressive and moving average parameters ε_i is representing white noise with mean zero and variance σ^2 and such a time series with white noise not depending on their previous terms but also depend on other phenomena and other variables [32].

A process $\{X_t\}$ with value of $t = 1, 2, \ldots, T$ satisfying

$$y_t = (1 - B)^d (X_t - \mu),$$

become a long memory process [33] after satisfying the following condition.

(a) $\lim_{n \to \infty} (\sum_{k=-n}^{n} |\rho_k|)$ is not finite i.e. ACF process diverges.

(b) The series $\{X_t\}$ is fractional differenced series.

ARIMA(p, d, q) model can only capture short-range dependency with d as integer order, where the Autoregressive Fractional Integrated Moving Average model (ARFIMA) was introduced by Granger and Joyeux [34], applied in long-range dependent time series.

We have used R software to fit the data with ARIMA and ARFIMA models. The residual of ARIMA fitted model of FFC open price with ACF lag and Residual plot is shown in Figure 7.

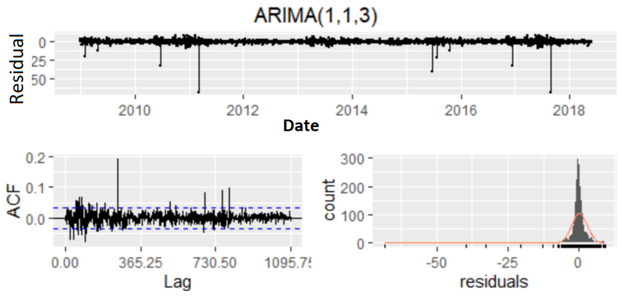

FIGURE 7. ARIMA Residual plot and its ACF and PACF Lag plot of FFC open price.

B. ARFIMA MODEL

ARFIMA (p, d, q) model define d for any real number using binomial expansion and Gamma function as

$$(1 - B)^d \sum_{j=0}^{\infty} \binom{d}{j} (-B)^j = \sum_{j=0}^{\infty} \frac{\Gamma(d+1)}{\Gamma(j+1)(d+1-j)} (-B)^j, \tag{11}$$

where $-1/2 < d < 1/2$.

Shaofei et al [35] and many other authors [36] suggest the use of Fractional ARIMA instead of an integer one can improve forecasting. The general form of ARFIMA (p, d, q) process defined as:

$$\varphi(B)(1 - B)^d X_t = \psi(B)\varepsilon_t, \tag{12}$$

where $-1/2 < d < 1/2$.

The above model widely used for LRD and SRD time series [37]. In ARFIMA (p, d, q), p is autoregressive order, q is moving average order and d is differencing in decimal form. The ARFIMA (p, d, q) process is a generalized form of ARIMA process, where d form integer value shift in decimal form in the ARFIMA modeling. Many non-stationary time series contain nonlinear trend and removing the trend is the first step of modeling of such time series. Box-Jekins theory served as a filter point to separate signals from the noise. In the residual of ARIMA model in Figure 7, we may notice a pattern of fractional correlation that commences with the first lag. In such a condition, fractional differences are useful to capture non-linearity by applying binomial expression to estimate ARFIMA(p, d, q) parameters. By applying a fractional order difference filter, the residual obtain is uncorrelated with lags of its variables. Mandelbrot [38] suggested the use of range over standard deviation R/S statistics called "rescaled range", which used by hydrologist Harold Hurst [39] in the Hurst exponent. The main concept of R/S analysis is to analyze rescaled cumulative deviation from the mean. The first estimation of Range R is given by:

$$R_n = max_{m=1,2,\ldots n} \sum_{i=1}^{n} (Y_j - \overline{Y}) - min_{m=1,2,\ldots n} \sum_{i=1}^{n} (Y_j - \overline{Y}),$$

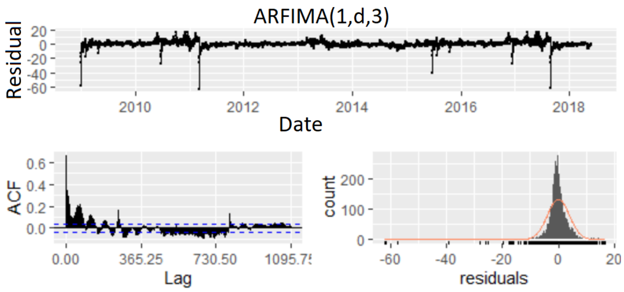

FIGURE 8. ARFIMA residual of open price FFC Company from 2009-2018 open price.

TABLE 3. Parameter estimation result ARFIMA(1,d,3) for FFC company.

Parameter	Coefficient	Std err	t-Ratio	p-Value
d	0.499914	0.00123	14.32	0.003
Ψ1	-0.60693	0.0188	18.65	0.021
Ψ2	-0.44672	0.02083	-3.2	0.01
constant	2.547838	3.01795	4.06	0.014

(13)

where R_n is the range of Accumulated deviation defined over period n of Y. The standard deviation S_n is defined as

$$S_n = [\sum_{i=1}^{n} (Y_j - \overline{Y})^2]^{1/2}, \tag{14}$$

with the increase in n it holds the equation,

$$\log[R_n/S_n] = \log \alpha + H \log n. \tag{15}$$

The above equation reflects linearity in the estimation of Hurst slope H. In the ARFIMA model the intensity d of fractional Gaussian noise of the data is estimated with the maximum likelihood of Hurst Parameter defined as:

$$d = h - 1/2. \tag{16}$$

The relationship permits researchers to define certain boundaries to some limit as follow:

(a) if d=0 the process does not contain long term memory and is stationary.
(b) if 0<d<1 the process is persistent with long term memory.
(c) If d=0.5 the process represents a random walk and unpredictable.

Estimation of d in financial data series is different from 0 and 0.5. Caporale [40] pointed out the presence of long-term memory in the US Stock Exchange. The parametric estimation of ARFIMA process for the FFC company is shown in Table 3. ARFIMA Residual plot and its ACF and PACF Lag plot for the FFC Company are shown in Figure 8. The best fitted fractional difference is calculated as d=0.499914.

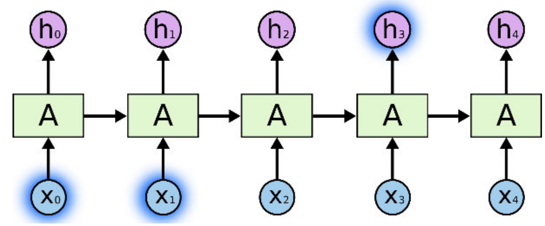

FIGURE 9. Structure of RNN Neural Network.

C. LSTM MODEL

Neural networks are efficient to extract nonlinear features for long memory data because of its versatility and use of non-linear activation functions in each layer. Kumarasinghe et al [41] designed Long Short-Term Memory (LSTM) network for intelligent prediction of the Colombo Stock Exchange. To understand the working of LSTM model, consider the RNN mechanism which is a sequential model that performs effectively by sequencing time series data as an input vector and provides vector output by neural network structure in the model's cell as shown in Figure 9. The time-series data passed through a cell in sequential vector, at each step the cell output value is concatenated with next time step data and the output value of cell serve as input for the next time step. The process is repeated until the last time step data, see Figure 10.

Standard LSTM is selected with forget gates in the research to model exogenous variables as an additional input for FFC open price forecasting. LSTM was introduced by F. Gers [42] consisting of interactive neural networks, each representing forget gate, input gate, input candidate gate, and output gate as shown in Figure 11. The output value of the forget gate varies between zero and one. The function representing forget gate which forgets the cell state from a previous time step that is not needed and keep the necessary information cell state for prediction represented as

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right), \tag{17}$$

the σ function representing activation function often called sigmoid which enables nonlinear capabilities of the model

$$\sigma(X) = \frac{1}{1 - e^{-x}}. \tag{18}$$

In the next step, the input gate and input candidate gate activate together to make a new cell state C_t which shifts to the next time step as a renewal cell state. Sigmoid activation function and hyperbolic tangent function are used as activation function at input gates and input candidate gate respectively providing output ii select and new cell state C_t' represented by the equations.

$$\begin{aligned} i_t &= \sigma(W_i \cdot [h_{t-1}, x_t] + b_t), \\ C_t' &= \tanh(W_c[h_{t-1}, x_t] + b_c). \end{aligned} \tag{19}$$

The *tanh* function is a hyperbolic tangent function that renders between -1 and 1.

$$\tanh(X) = \frac{e^x - e^{-x}}{e^x + e^{-x}}. \tag{20}$$

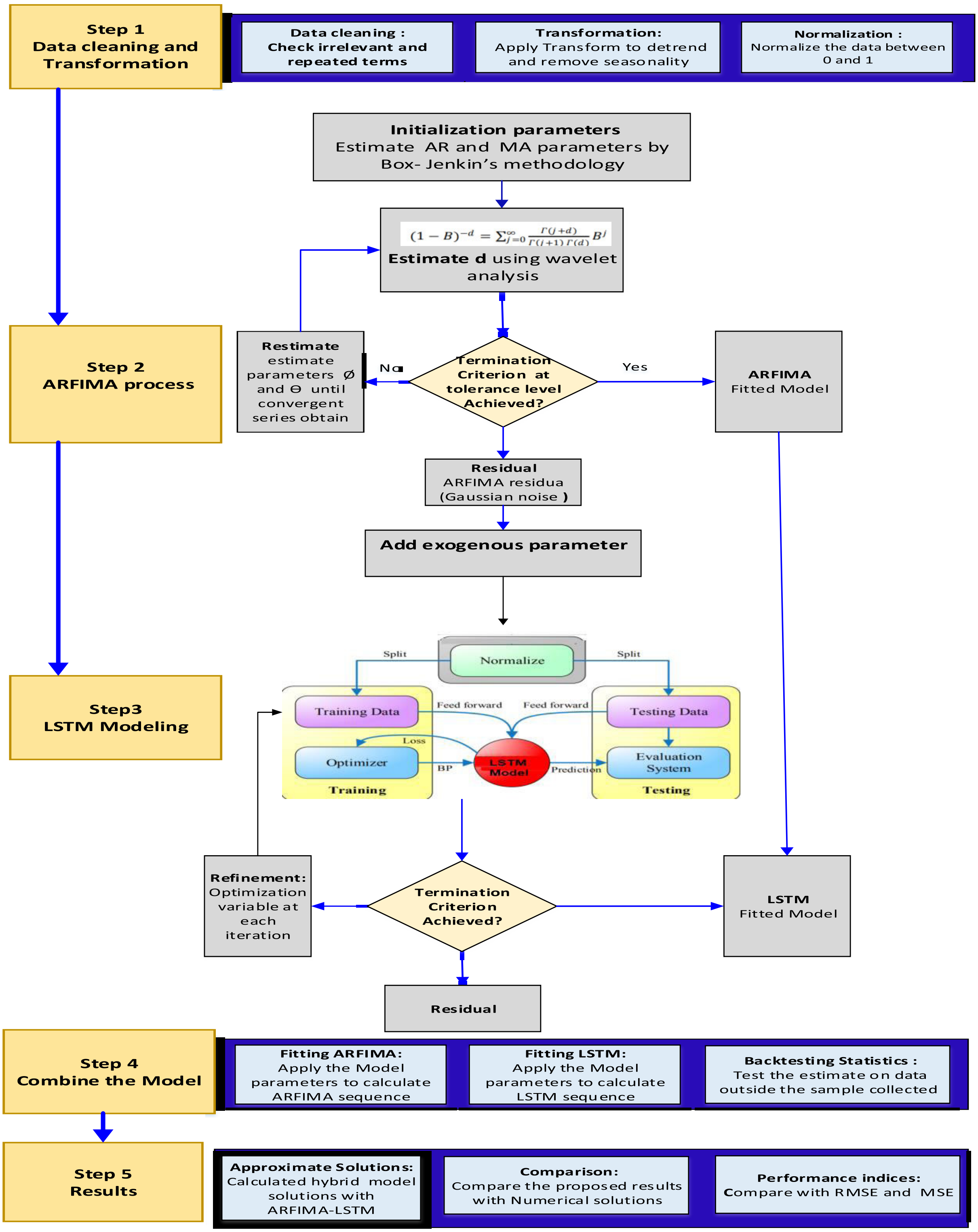

FIGURE 10. Graphical abstract of the proposed technique, ARFIMA-LSTEM for modeling of FFC open price.

Augmented Dickey–Fuller (ADF) test is used to transform non-stationary time series to stationary time series. The LSTM input is residual of open price FFC historical data modeled by ARFIMA model. We have also used the dependent variables to model the residual values of FFC data after filtered by ARFIMA Model.

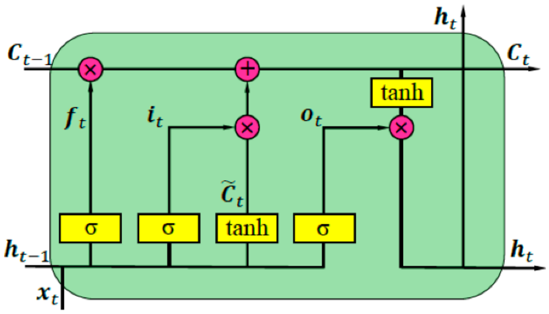

FIGURE 11. Hybrid LSTM Neural Network Structure.

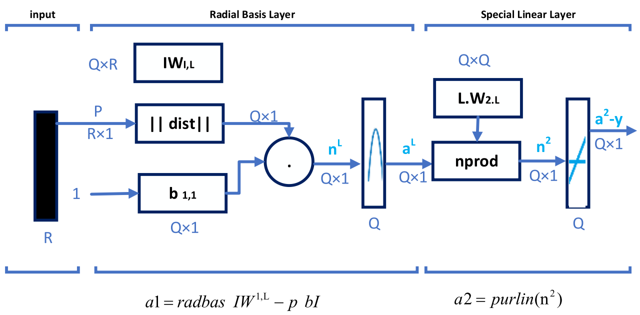

FIGURE 12. The architecture of Generalized regression radial basis neural network.

D. GENERALIZED REGRESSION RADIAL BASIS NEURAL NETWORK (GRNN)

Generalized regression neural network (GRNN) is used for approximation of function [43]. It consists of two layers in which its first layer comprises of a radial basis layer and the second layer consist of a special linear layer. The architecture for the GRNN is shown in Figure 12. It is similar to RBF neural network; the only difference is addition of second layer. The input vector is represented by P. and bias vector b1 is set to a column vector. Each neuron in the radial basis function computes weighted input with bias value which passes through the second input layer to produce generalized regression output.

where

R = no of elements in the input vector

Q = no of neurons in each Layer

E. PROPOSED HYBRID ARFIMA-LSTM MODEL

The residual white noise of ARFIMA model is processed in hybrid LSTM model to detect the pattern with the exogenous variables as an input. The overall graphical abstract of the proposed technique, ARFIMA-LSTM for modeling of FFC open price is shown in Figure 10. The noise has passed through LSTM neural network to model leftover signals with the help of external variables. Time series data decomposes into linear and nonlinear components expression as follow:

$$x_t = L_t + N_t, \tag{21}$$

here L_t represent linearity modeling of data with ARFIMA model which works decently on linear problems.

$$\varepsilon_t = x_t - L_t, \tag{22}$$

where ε_t is the residual left by the ARFIMA Model. The LSTM model calculated by the equation defined as:

$$N_t = f(\varepsilon_t) = f(x_t - L_t), \tag{23}$$

while N_t representing nonlinearity modeling for the period t of the time series ARFIMA residual and dependent variables with the hybrid LSTM neural network. The two models are combined to comprehend both linear and non-linear tendencies of the data. In the predictive model selection, we have used 30 steps forecast to evaluate the performance of the model as shown in Figure 13.

LSTM model for training, testing and prediction phases are depicted in the form of the algorithm as follows:

IV. EVALUATION CRITERIA

To evaluate the performance of the proposed nonlinear combination model, mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE) are used defined as follows:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|y_t - \hat{y}_t\right|,$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2},$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_t - \hat{y}_t}{y_t}\right| \times 100\%. \tag{24}$$

V. EXPERIMENTAL RESULT OF ARFIMA-LSTM

Training and cross-validation in LSTM Model is carried out using Adam algorithm. 75:25 proportion of data is set as a training and testing process respectively. Model performance is measured using mean absolute error (MAE), root mean square error (RMSE) and mean absolute percentage error (MAPE) as formulated in Eq 12-. The performance accuracy of each model is summarized in Table 4, and their forecasting results are described in Table 5. LSTM model fitting of the ARFIMA residual model is shown in Figure 14.

Training and Testing error of LSTM Model open price of FFC data found minimum at 150 epochs as shown in Figure 15. The hybrid ARFIMA-LSTM achieved the lowest RMSE of 0.0539 as compared to LSTM, ARFIMA, and ARIMA models individually. The comparison of results for different models is shown in Table 5. Graphical comparison of FFC forecast results using ARIMA, ARFIMA, GRNN, and hybrid ARFIMA-LSTM is shown in Figure 16 and Error comparison for the proposed model with its comparison is shown in Figures 17 and 18.

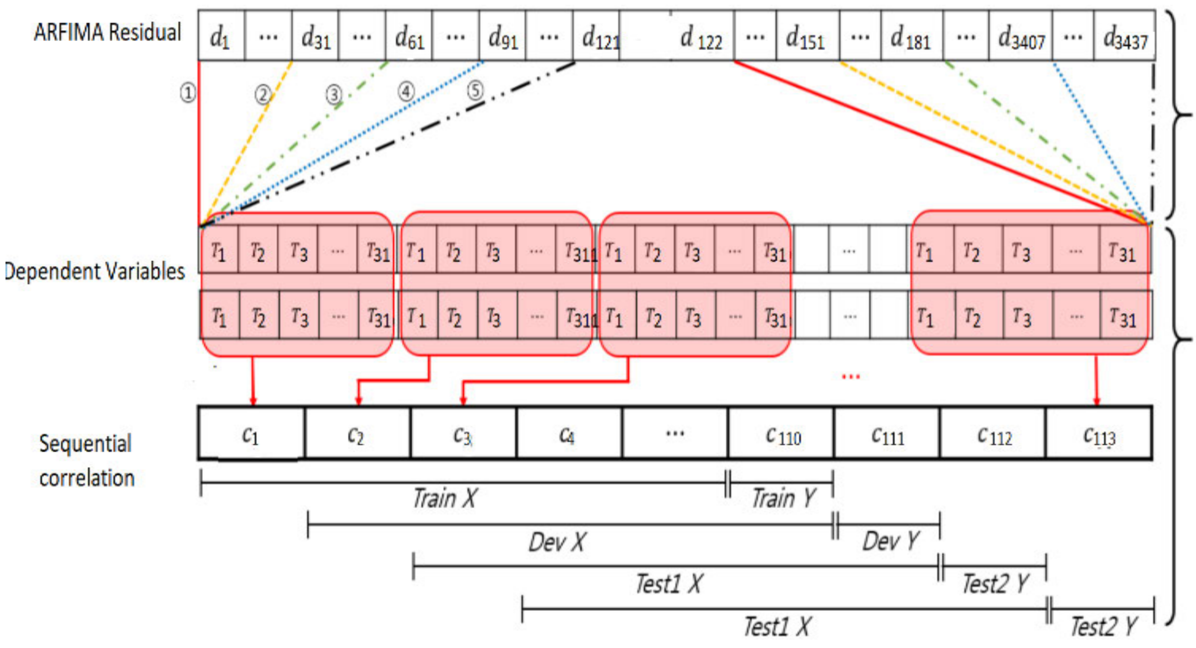

FIGURE 13. Hybrid LSTM model of FFC data open price with sequential correlation.

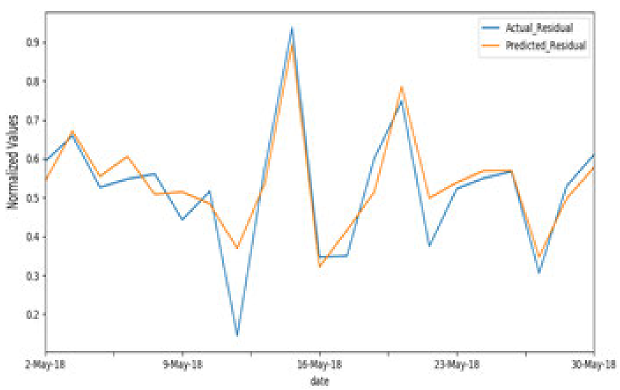

FIGURE 14. LSTM model fitting for ARFIMA residual model FFC open price,

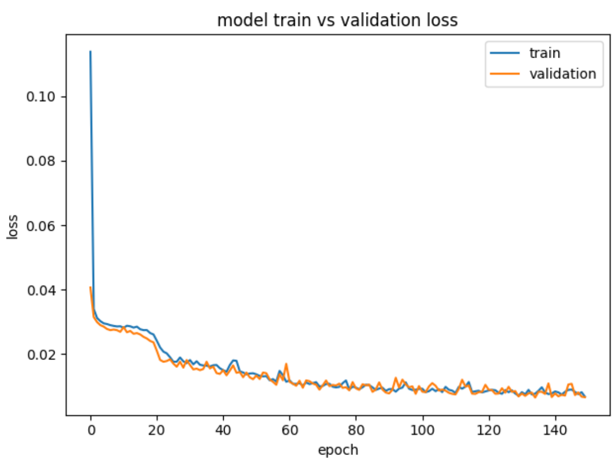

FIGURE 15. Training and Testing error LSTM Model residual of FFC open price.

TABLE 4. The FFC forecast statistics using ARIMA, ARFIMA and hybrid ARFIMA-LSTM.

MODEL	MAE	RMSE	MAPE (%)
ARIMA	0.1566	0.3132	0.1896
ARFIMA	0.1352	0.2704	0.1633
ARFIMA-LSTM	0.02694	0.0539	0.002
GRNN	0.03150	0.0629	0.0114

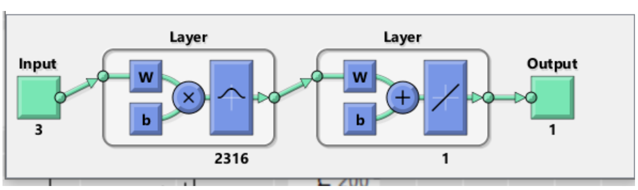

FIGURE 16. GRNN architecture for prediction of FFC open Price.

In the GRNN modeling, we used two layers, in the first layer total of 2316 neurons were used to fit regression with RBFs neural network as shown in Figure 16. The 3317 observation of daily FFC stock open price data from 01 January 2009 to 30 April 2018 was used for training output in Generalized regression radial basis neural network while the three modeled variable ARIMA, ARFIMA and ARFIMA-LSTM were used as input training purpose The remaining 30 values of 03 modeled variables for the month May 2019 was used to predict FFC open price in Generalized RBFs neural network.

Algorithm 1 LSTM Model Training Algorithm

```
The LSTM prediction algorithm works in the following four main phases:
 (a) Preprocessing requirement of Data
 (a) Fixation of parameters for the model
 (c) Fitting and estimation of the model
 (d) Prediction of the Model
Input: Five dependent variable based residual ARFIMA model. N lag steps between
all input and output of the dataset.
Output: train/test prediction data
Phase1: Preprocessing of the data
 (e) Normalization of the Residual dataset
 (f) Conversion of input/output as 75:25
 g) 'Train.LSTM and Test.LSTM' = divide (Residual, 0.75)
 (h) 'X.train,y.train' = 'split(train.LSTM, N.step)'
 (i) 'X.test,y.test' = split(test.LSTM, N.step)
 (j) Reshape 'train' and 'test' data
Phase 2: Determination of parameter for the model parameters
 (k) Model definition
 (l) Add 'LSTM(units=30,activation).activation'='relu',
 (m) 'Input.shape=(N.steps,n.features)'
 (n) Add 'LSTM(units=30,activation).activation'='relu')
 (o) Add 'Dense (n.features=2)'
Phase 3: Fitting of Model along with estimation
 (p) Repeatition
 (q) Forward.propagate model with 'X.train'
 (r) Backward.propagate model with 'y.train'
 (s) Adjust model parameters
 (t) MSE, MAE = evaluate.model ('X.train', 'y.train')
 (u) If convergence is observed on MSE:
 (v) End else Repeat
Phase 4: Prediction
 (w) 'Train.Pred' = 'predict (X.train)'
 (x) 'Test.Pred' = 'predict (X.test)'
 (y) Return 'train.Pred', 'test.Pred'
```

TABLE 5. The FFC forecast results using ARIMA, ARFIMA and hybrid ARFIMA-LSTM.

DATE	FFC	ARFIMA-LSTM	GRNN	ARIMA	ARFIMA	DATE	FFC	ARFIMA-LSTM	GRNN	ARIMA	ARFIMA
2-May-18	99.76	103.89	103.96	58.74	119.60	16-May-18	96.61	95.51	96.55	58.74	119.60
3-May-18	99.58	105.06	103.96	67.88	119.49	17-May-18	96.59	94.11	96.55	67.88	119.49
4-May-18	98.3	103.81	103.96	69.51	119.40	18-May-18	96.51	95.10	96.55	69.51	119.40
7-May-18	98.05	103.65	98.24	71.23	118.73	21-May-18	94.7	97.36	97	71.23	118.73
8-May-18	99.39	102.42	98.24	74.80	118.47	22-May-18	94.22	94.03	87.38	74.80	118.47
9-May-18	97.21	99.85	98.24	78.45	115.84	23-May-18	98.42	94.30	87.38	78.45	115.84
10-May-18	97.44	98.99	97.37	81.60	115.29	24-May-18	98.75	94.47	96.55	81.60	115.29
11-May-18	98.05	97.93	96.55	77.78	115.36	25-May-18	98.33	94.36	96.55	77.78	115.36
14-May-18	98.14	98.29	96.55	73.94	114.07	28-May-18	97.95	92.54	86.27	73.94	114.07
15-May-18	96.49	101.29	105.96	74.53	113.50	29-May-18	98.56	94.10	86.27	74.53	113.50

VI. CONCLUSION

In this paper, hybrid ARFIMA-LSTM is presented based on a combination of the ARFIMA model, and LSTM model of ARFIMA residual. The hybrid model extracts potential information from the residual with the help of exogenous dependent variables and achieves better performance in terms of prediction accuracy by joining both models. The addition of exogenous input of dependent variables in hybrid ARFIMA-LSTM improves prediction accuracy as compared to ARIMA, ARFIMA, and GRNN independently. Error analysis for all the models is presented in Table 3, which reflects the proposed model acquires the lowest MAPE

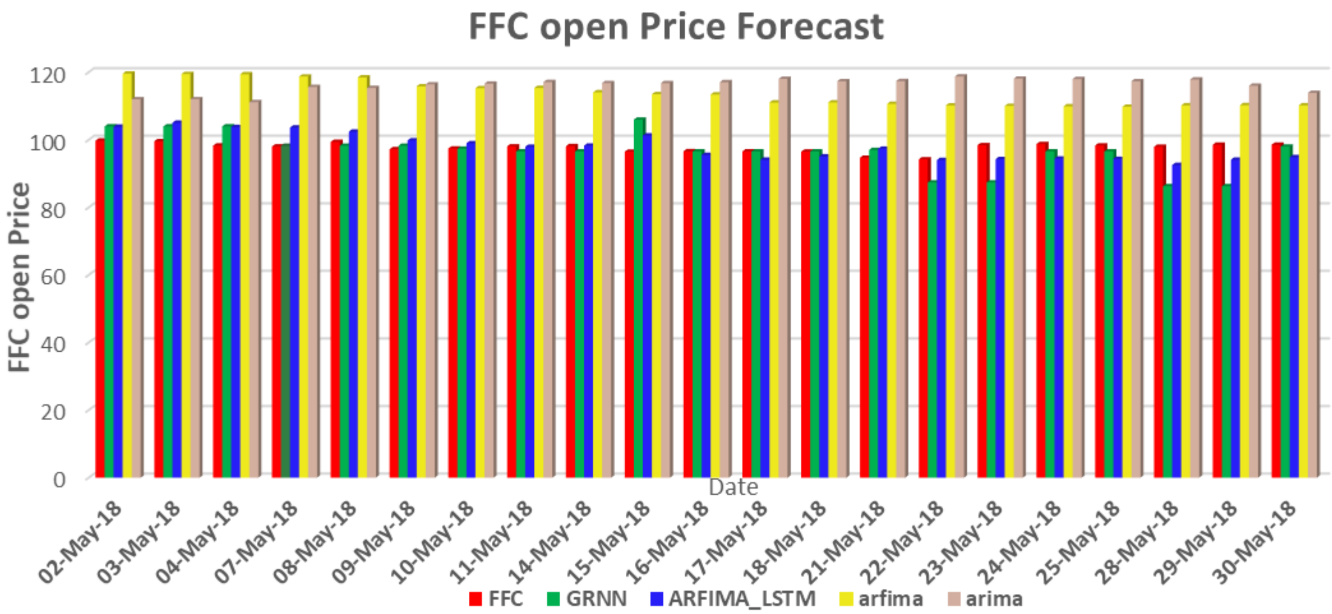

FIGURE 17. Graphical comparison of FFC forecast results using ARIMA, ARFIMA, GRNN and hybrid ARFIMA-LSTM.

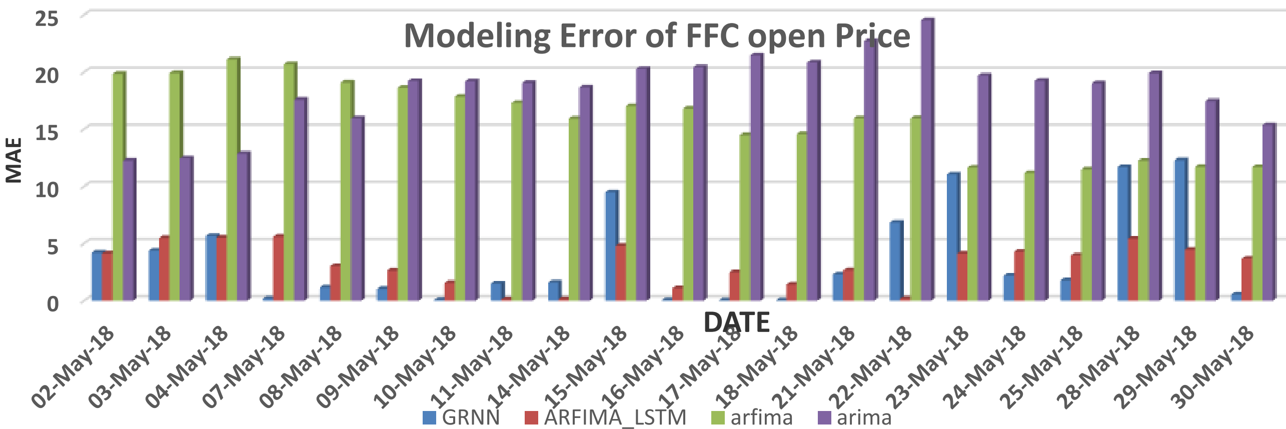

FIGURE 18. Graphical comparison of MAE Error FFC open price forecast.

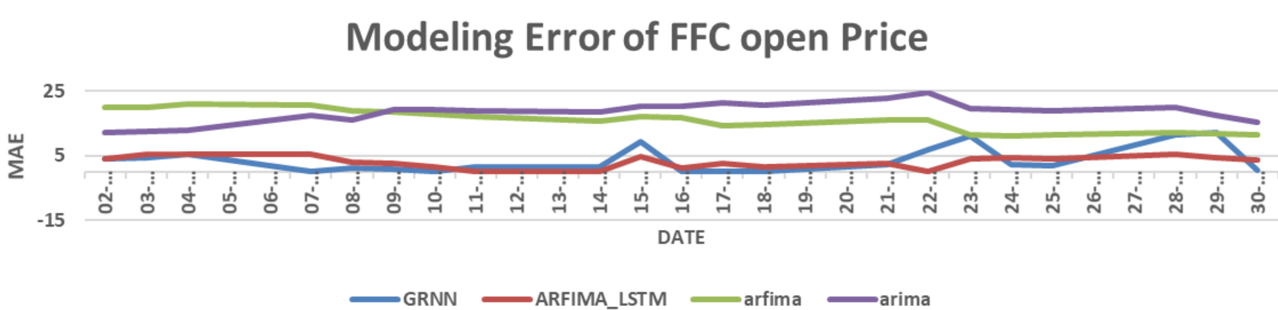

FIGURE 19. Parametric comparison of MAE Error FFC open price forecast.

of 0.002%. Therefore, it can be concluded that the proposed hybrid ARFIMA-LSTM model outperforms as compare to individual models independently. The superior performance of the proposed hybrid model significantly proved the best-parameterized model to enhance the financial series prediction by increasing the accuracy rate of 80% as compared to traditional models.

The ARFIMA-LSTM looks promising to be investigated solving the nonlinear stiff mathematical models representing diversified applications in applied sciences [44]–[50].

ACKNOWLEDGMENT

The authors would like to thank Syed Asghar Abbas Naqvi Regional Head, Islamabad Pakistan Stock Exchange for providing us with the PSX dataset that has been used in the research. The authors acknowledge the financial support provided by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT. They are obliged to respectable referees for their important and fruitful comments to enhance the quality of the current article.

DATA AVAILABILITY

All datasets generated during the current study are available from the corresponding author upon request.

REFERENCES

[1] K. Phylaktis and F. Ravazzolo, "Stock prices and exchange rate dynamics," *J. Int. Money Finance*, vol. 24, no. 7, pp. 1031–1053, Nov. 2005.
[2] P. R. Lane and J. C. Shambaugh, "Financial exchange rates and international currency exposures," *Amer. Econ. Rev.*, vol. 100, no. 1, pp. 518–540, Mar. 2010.
[3] G. Mahalakshmi, S. Sridevi, and S. Rajaram, "A survey on forecasting of time series data," in *Proc. Int. Conf. Comput. Technol. Intell. Data Eng. (ICCTIDE)*, Jan. 2016, pp. 1–8.
[4] K. Pawar, R. S. Jalem, and V. Tiwari, "Stock market price prediction using LSTM RNN," in *Emerging Trends in Expert Applications and Security*. Singapore: Springer, 2019, pp. 493–503.
[5] M. S. Ahmed and A. R. Cook, "Analysis of freeway traffic time-series data by using box-Jenkins techniques," *Transp. Res. Rec.*, no. 722, pp. 1–9, 1979.
[6] G. S. Atsalakis and K. P. Valavanis, "Surveying stock market forecasting techniques—Part II: Soft computing methods," *Expert Syst. Appl.*, vol. 36, no. 3, pp. 5932–5941, Apr. 2009.
[7] P. H. Franses and D. Van Dijk, "Forecasting stock market volatility using (non-linear) Garch models," *J. Forecasting*, vol. 15, no. 3, pp. 229–235, 1996.
[8] M. P. Deisenroth, C. E. Rasmussen, and J. Peters, "Gaussian process dynamic programming," *Neurocomputing*, vol. 72, nos. 7–9, pp. 1508–1524, Mar. 2009.
[9] A. Gupta, P. Bhatia, K. Dave, and P. Jain, "Stock market prediction using data mining techniques," in *Proc. 2nd Int. Conf. Adv. Sci. Technol. (ICAST)*, Apr. 2019, p. 5.
[10] Q. Dong, R. Kontar, M. Li, G. Xu, and J. Xu, "A simple approach to multivariate monitoring of production processes with non-Gaussian data," *J. Manuf. Syst.*, vol. 53, pp. 291–304, Oct. 2019.
[11] K. Liu, Y. Chen, and X. Zhang, "An evaluation of ARFIMA (autoregressive fractional integral moving average) programs," *Axioms*, vol. 6, no. 4, p. 16, 2017.
[12] H. Sheng and Y. Chen, "FARIMA with stable innovations model of great salt lake elevation time series," *Signal Process.*, vol. 91, no. 3, pp. 553–561, Mar. 2011.
[13] F. X. Diebold and G. D. Rudebusch, "Long memory and persistence in aggregate output," *J. Monetary Econ.*, vol. 24, no. 2, pp. 189–209, Sep. 1989.
[14] T. Gao, Y. Chai, and Y. Liu, "Applying long short term momory neural networks for predicting stock closing price," in *Proc. 8th IEEE Int. Conf. Softw. Eng. Service Sci. (ICSESS)*, Nov. 2017, pp. 575–578.
[15] O. Lieberman and P. C. B. Phillips, "Refined inference on long memory in realized volatility," *Econ. Rev.*, vol. 27, nos. 1–3, pp. 254–267, Feb. 2008.
[16] K. Chen, Y. Zhou, and F. Dai, "A LSTM-based method for stock returns prediction: A case study of China stock market," in *Proc. IEEE Int. Conf. Big Data (Big Data)*, Oct. 2015, pp. 2823–3024.
[17] S. Siami-Namini and A. Siami Namin, "Forecasting economics and financial time series: ARIMA vs. LSTM," 2018, *arXiv:1803.06386*. [Online]. Available: http://arxiv.org/abs/1803.06386
[18] K. Khare, O. Darekar, P. Gupta, and V. Z. Attar, "Short term stock price prediction using deep learning," in *Proc. 2nd IEEE Int. Conf. Recent Trends Electron., Inf. Commun. Technol. (RTEICT)*, May 2017, pp. 482–486.
[19] H. Kyu Choi, "Stock price correlation coefficient prediction with ARIMA-LSTM hybrid model," 2018, *arXiv:1808.01560*. [Online]. Available: http://arxiv.org/abs/1808.01560
[20] W. Fang, "The effects of currency depreciation on stock returns: Evidence from five east Asian economies," *Appl. Econ. Lett.*, vol. 9, no. 3, pp. 195–199, Feb. 2002.
[21] L. Debnath, "A brief historical introduction to fractional calculus," *Int. J. Math. Edu. Sci. Technol.*, vol. 35, no. 4, pp. 487–501, Jul. 2004.
[22] L. Song, "A semianalytical solution of the fractional derivative model and its application in financial market," *Complexity*, vol. 2018, pp. 1–10, Apr. 2018.
[23] A. AboBakr, L. A. Said, A. H. Madian, A. S. Elwakil, and A. G. Radwan, "Experimental comparison of integer/fractional-order electrical models of plant," *AEU-Int. J. Electron. Commun.*, vol. 80, pp. 1–9, Oct. 2017.
[24] M. Kumar and T. K. Rawat, "Fractional order digital differentiator design based on power function and least squares," *Int. J. Electron.*, vol. 103, no. 10, pp. 1639–1653, Oct. 2016.
[25] A. A. Kilbas, O. Marichev, and S. G. Samko, *Fractional integrals and derivatives*. Yverdon-les-Bains, Switzerland: Gordon and Breach Science, 1993.
[26] M. Caputo and M. Fabrizio, "A new definition of fractional derivative without singular kernel," *Prog. Fractional Differ. Appl.*, vol. 1, no. 2, pp. 1–13, 2015.
[27] A. Atangana and I. Koca, "Chaos in a simple nonlinear system with Atangana–Baleanu derivatives with fractional order," *Chaos, Solitons Fractals*, vol. 89, pp. 447–454, Aug. 2016.
[28] T. Sandev and A. Iomin, "Finite-velocity diffusion on a comb," *Europhys. Lett.*, vol. 124, no. 2, p.20005, 2018.
[29] E. Cadenas, R. Campos-Amezcua, W. Rivera, M. A. Espinosa-Medina, A. R. Méndez-Gordillo, E. Rangel, and J. Tena, "Wind speed variability study based on the hurst coefficient and fractal dimensional analysis," *Energy Sci. Eng.*, vol. 7, no. 2, pp. 361–378, Apr. 2019.
[30] M. Rout and K. M. Koudjonou, "An evolutionary algorithm based hybrid parallel framework for Asia foreign exchange rate prediction," in *Nature Inspired Computing for Data Science*. Cham, Switzerland: Springer, 2020, pp. 279–295.
[31] P. Klimek, S. Poledna, and S. Thurner, "Quantifying economic resilience from input–output susceptibility to improve predictions of economic growth and recovery," *Nature Commun.*, vol. 10, no. 1, p. 1677, Dec. 2019.
[32] J. Runge *et al.*, "Inferring causation from time series in Earth system sciences," *Nature Commun.*, vol. 10, no. 1, p. 2553, Dec. 2019.
[33] S. Sivarajasingham and A. M. M. Mustafa, "Does LKR/AUD exchange rate exhibit long memory? A fractional integration approach," *J. Bus. Econ.*, vol. 1, no. 1, pp. 21–30, 2019.
[34] A. M. M. S. Quoreshi, R. Uddin, and V. Jienwatcharamongkhol, "Equity market contagion in return volatility during euro zone and global financial crises: Evidence from FIMACH model," *J. Risk Financial Manage.*, vol. 12, no. 2, p. 94, 2019.
[35] S. Wu, "Nonlinear information data mining based on time series for fractional differential operators," *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 29, no. 1, Jan. 2019, Art. no. 013114.
[36] M. Isoardi and L. A. Gil-Alana, "Inflation in argentina: Analysis of persistence using fractional integration," *Eastern Econ. J.*, vol. 45, no. 2, pp. 204–223, Apr. 2019.
[37] P. D. Domański, "CPA industrial applications," in *Control Performance Assessment: Theoretical Analyses and Industrial Practice*, vol. 245. Cham, Switzerland: Springer, 2020, pp. 345–357.
[38] N. W. Watkins, "Mandelbrot's stochastic time series models," *Earth Space Sci.*, vol. 6, no. 11, pp. 2044–2056, 2019.
[39] S. A. Nazarychev, A. R. Zagretdinov, S. G. Ziganshin, and Y. V. Vankov, "Classification of time series using the hurst exponent," *J. Phys., Conf. Ser.*, vol. 1328, Oct. 2019, Art. no. 012056.
[40] G. M. Caporale, M. Karanasos, S. Yfanti, and A. Kartsaklas, "Investors' trading behaviour and stock market volatility during crisis periods: A dual long-memory model for the korean stock exchange," CESifo Group Munich, Munich, Germany, Tech. Rep. 7984, 2019.
[41] H. N. Kumarasinghe, D. M. A. B. Moneravilla, I. B. M. R. K. P. Muwanwella, and J. B. Ekanayake, "An intelligent predicting approach based long short-term memory model using numerical and textual data: The case of colombo stock exchange," 2019.
[42] N. C. Petersen, F. Rodrigues, and F. C. Pereira, "Multi-output bus travel time prediction with convolutional LSTM neural network," *Expert Syst. Appl.*, vol. 120, pp. 426–435, Apr. 2019.

[43] H.-Z. Li, S. Guo, C.-J. Li, and J.-Q. Sun, "A hybrid annual power load forecasting model based on generalized regression neural network with fruit fly optimization algorithm," *Knowl.-Based Syst.*, vol. 37, pp. 378–387, Jan. 2013.

[44] C. K. Ho, E. Hawkins, L. Shaffrey, J. Bröcker, L. Hermanson, J. M. Murphy, D. M. Smith, and R. Eade, "Examining reliability of seasonal to decadal sea surface," *Geophys. Res. Lett.*, vol. 40, no. 21, pp. 5770–5775, 2013.

[45] C. K. Ho, E. Hawkins, L. Shaffrey, J. Broecker, L. Hermanson, J. M. Murphy, D. M. Smith, and R. Eade, "Examining reliability of seasonal to decadal sea surface temperature forecasts: The role of ensemble dispersion," *Geophys. Res. Lett.*, vol. 40, no. 21, pp. 5770–5775, 2013, doi: 10.1002/2013GL057630.

[46] *MissUrooj Fatima|AssistantManagerMarketing&BusinessDevelopment Dept Program Manager|PSX Stock*, RP Data, Sydney, NSW, Australia, 2018.

[47] I. Ahmad, S. Ahmad, M. Awais, S. U. I. Ahmad, and M. A. Z. Raja, "Neuro-evolutionary computing paradigm for Painlevé equation-II in nonlinear optics," *Eur. Phys. J. Plus*, vol. 133, no. 5, p. 184, May 2018.

[48] I. Ahmad, H. Ilyas, A. Urooj, M. S. Aslam, M. Shoaib, and M. A. Z. Raja, "Novel applications of intelligent computing paradigms for the analysis of nonlinear reactive transport model of the fluid in soft tissues and microvessels," *Neural Comput. Appl.*, vol. 31, no. 12, pp. 9041–9059, Dec. 2019.

[49] S. U. I. Ahmad, F. Faisal, M. Shoaib, and M. A. Z. Raja, "A new heuristic computational solver for nonlinear singular Thomas–Fermi system using evolutionary optimized cubic splines," *Eur. Phys. J. Plus*, vol. 135, no. 1, pp. 1–29, Jan. 2020.

[50] A. Zameer, M. Muneeb, S. M. Mirza, and M. A. Z. Raja, "Fractional-order particle swarm based multi-objective PWR core loading pattern optimization," *Ann. Nucl. Energy*, vol. 135, Jan. 2020, Art. no. 106982.

AYAZ HUSSAIN BUKHARI was born in Rajan Shah, Layyah, Punjab, Pakistan. He received the M.Sc. degree in mathematics from the University of Engineering and Technology Lahore, Pakistan, and the M.Phil. degree in applied mathematics from Federal Urdu University, Karachi, Pakistan. He is currently a Ph.D. Scholar of Mathematics Department, Abdul Wali Khan University Mardan, Pakistan. He has 18 years of teaching experience at different universities in Pakistan, including UET Lahore, NED UET Karachi, National University of Science and Technology Islamabad, and Air University Islamabad. His areas of interest are machine learning, artificial intelligence, metaheuristic optimization, data science, statistical analysis, and fractional calculus.

MUHAMMAD ASIF ZAHOOR RAJA was born in muhallah Sadiqabad, Rawalpindi, Pakistan. He received the M.Sc. degree in mathematics from Forman Christen College Lahore, Pakistan, in 1996, the M.Sc. degree in nuclear engineering from Quaid-e-Azam University, Islamabad, Pakistan, in 1999, and the Ph.D. degree in electronic engineering from International Islamic University, Islamabad, in 2011. He was involved in the research and development assignment of Engineering and Scientific Commission of Pakistan, from 1999 to 2012. He is currently working as an Assistant Professor with the Department of Electrical Engineering, COMSATS Institute of Information Technology–Attock, Attock, Pakistan. He has developed the Fractional least mean square algorithm and the computational platform is formulated for the first time for solving fractional differential equations using artificial intelligence techniques during his Ph.D. studies. He has been the author of more than 176 publications, out of which more than 165 are reputed journal publications with impact factor more than 500. He acts as a resource person and gives invited talks on many workshops and conferences held at the national level. His areas of interest are solving linear and nonlinear differential equations of arbitrary order, active noise control system, fractional adaptive signal processing, nonlinear system identification, the direction of arrival estimation, and bioinformatics problems.

MUHAMMAD SULAIMAN received the B.Sc., M.Sc., and M.Phil. degrees in mathematics from the University of Peshawar, and Quaid-e-Azam University, Islamabad, Pakistan, in 2004, 2007, and 2009, respectively, and the Ph.D. degree in mathematics from the University of Essex, U.K., in 2015. From 2009 to 2016, he was a Lecturer of mathematics with the Abdul Wali Khan University Mardan, Pakistan. Since February 2016, he has been an Assistant Professor with the Department of Mathematics, Abdul Wali Khan University Mardan. He is the author of two book chapters, more than 17 articles. His research interests include optimization, including mathematical optimization techniques, global optimization, evolutionary algorithms, heuristics, metaheuristics, multi-objective optimization, design engineering optimization problems, structural engineering optimization problems, linear programming, linear and non-linear least squares optimization problems, evolutionary algorithms, and nature-inspired metaheuristics. He is an Associate Editor of the journal *COJ Reviews and Research*, and the *SCIREA Journal of Mathematics*.

SAEED ISLAM received the M.Sc. degree from Quaid-e-Azam University, Islamabad, Pakistan, and the Ph.D. degree from the Harbin Institute of Technology Shenzhen Graduate School, China. He is currently a Professor/Chairman with the Department of Mathematics, Abdul Wali Khan University Mardan, Mardan, Pakistan. He has written several articles and books in various field of mechanical engineering. His research interests include nanofluid, CFD, simulation, heat transfer, MHD, biomathematics, mesoscopic modeling, nonlinear science, magnetohydrodynamic, ferrohydrodynamic, electrohydrodynamic, and heat exchangers.

MUHAMMAD SHOAIB was born in Basal, Attock, Punjab, Pakistan. He received the master's degree in mathematics from Punjab University Lahore, Pakistan, in 2012, and the Ph.D. degree in mathematics from Riphah International University, Islamabad, Pakistan, in 2016. He has been working as an Assistant Professor with COMSATS University Islamabad–Attock, since 2017. He has about 30 research publications in international Journals. His areas of interest include mathematical modeling, neural networks, artificial intelligence and implementation of computational techniques based on traditional as well as the heuristic paradigms.

POOM KUMAM (Member, IEEE) received the Ph.D. degree in mathematics from Naresuan University, Thailand. He is currently a Full Professor with the Department of Mathematics, King Mongkut's University of Technology Thonburi (KMUTT). He is also the Head of the Fixed Point Theory and Applications Research Group, KMUTT, and also with the Theoretical and Computational Science Center (TaCS-Center), KMUTT. He is also the Director of the Computational and Applied Science for Smart Innovation Cluster (CLASSIC Research Cluster), KMUTT. He has authored or coauthored more than 400 international peer-reviewed journals. His main research interests include fixed point theory and applications, computational fixed point algorithms, nonlinear optimization, and control theory, and optimization algorithms.