Received February 23, 2020, accepted March 30, 2020, date of publication April 3, 2020, date of current version April 20, 2020.
Digital Object Identifier 10.1109/ACCESS.2020.2985411

Using a Random Coefficient Regression Model to Jointly Determine the Optimal Critical Level and Lot Sizing

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This work was supported in part by the National Natural Science Foundation of China under Grant 71601019, Grant 71701038, Grant 71729001, and Grant 71871018, in part by the Fundamental Research Funds for the Central Universities under Grant FRF-TP-18-006A2, in part by the China Ministry of Education Humanities and Social Sciences Research Youth Fund Project under Grant 16YJC630174, and in part by the Natural Science Foundation of Hebei Province under Grant G2019501074.

ABSTRACT This paper proposes an integrated preventive maintenance and economic production quantity model. A condition-based maintenance policy is described by a random coefficient regression model, based on which the monitored condition is divided into two parts: the actual condition and random error. Products are produced in batches and the system is monitored at the end of each batch. If the observed system condition either reaches or exceeds the critical level, the system should be renewed by preventive maintenance. However, if the actual system condition reaches the failure level during the production process, the system fails and should be renewed immediately. Based on these two renewal situations, we construct a model of expected cost per unit time using the renewal reward theory. The critical level and production lot size are decision variables, which can be obtained by minimizing the cost model. We also develop a simulation process to obtain the optimal results in another way and validate our proposed cost model. Finally, a real case study is given to demonstrate the model and the simulation process.

INDEX TERMS Economic production quantity, condition-based maintenance, renewal reward theory, inventory, preventive maintenance.

I. INTRODUCTION

The joint optimization of maintenance and production has two main research streams. The first research stream is the joint optimization of the capacitated lot-sizing and scheduling problem (CLSP) and maintenance. This considers several periods in a finite planning horizon, where more than one type of product demand needs to be satisfied in each period and preventive maintenance (PM) is assumed to be carried out at the end of some periods to minimize production interruptions. The optimal production planning and PM schedule can be obtained by simultaneously minimizing the production and maintenance cost in the planning horizon [1]–[3]. The second research stream is the joint optimization of the economic production quantity (EPQ) and maintenance. Most research in this area considers an infinite planning horizon, where the product is repeatedly produced in batches and PM is assumed to be carried out at the end of some batches. The optimal lot size and PM schedule can be calculated by minimizing the maintenance and production cost per unit time [4], [5]. In this paper, we focus only on the second research stream, i.e., the integrated EPQ and PM problem.

Most existing research on this problem considers regular PM, i.e. time-based maintenance (TBM), where 1) the expected number of system failures is always calculated to obtain the expected maintenance cost and 2) the PM schedule and lot size are optimized by minimizing the maintenance and production cost per unit time. Here, we only review some representative studies. Giri and Yun [6] proposed an integrated EPQ and PM model for an unreliable production system subject to two types of random failure. Shortages are considered in their model because of the longer repair time, and, as the PM is regular, the optimal lot sizing policies can be obtained. Giri and Dohi [7] considered corrective maintenance (CM) and PM times, and the time to failure in their model is random. They constructed an integrated EPQ and maintenance

The associate editor coordinating the review of this manuscript and approving it for publication was Luca Cassano.

model based on the net present value method, demonstrating the advantages of this method over a traditional model. El-Ferik [8] used random failure and imperfect maintenance in an integrated EPQ and maintenance model, where PM is carried out either upon system failure or when the usage of the system reaches a predetermined level. Chakraborty *et al.* [9] assumed that a production facility has two states – the 'in-control' state and the 'out-of-control' state – and that the transfer epoch of the system state is random and can occur during a production run. They proposed an integrated model of the expected cost per unit time, where CM is carried out when a machine breakdown occurs and PM is performed at the end of a lot size. Similarly, Chakraborty *et al.* [10] considered the in-control state and out-of-control state of a system with two different inspection policies. Jin *et al.* [11] considered uncertain demand and proposed an option-based and analytical cost model for scheduling joint production and PM, where the 'option' is a financial derivative tool used to tackle the optimization problem under an uncertain environment. Bouslah *et al.* [4] considered the joint determination of optimal lot sizing and production planning for an unreliable and imperfect production system, where the quality control of the produced lots was performed using a sampling plan, and the lot sizing and production rate were the decision variables. Wee and Widyadana [12] developed a production model for deteriorating items with stochastic PM time and rework using the first-in first-out rule. Liu *et al.* [13] proposed an integrated production, inventory and PM model for a multi-product production system. They studied multiple products and used the delay-time concept to describe maintenance activity.

All these studies used a TBM policy in their models. However, in recent years, because of the development of sensor technologies, some research on integrated EPQ and PM models has started to consider condition-based maintenance (CBM). Based on CBM, the degradation process of a system can be monitored to predict the residual life of the system and reduce unnecessary maintenance activities [14]. Many probability models have been proposed for modeling the degradation process. Almost all of these models can be divided into two categories, one is stochastic process models, such as a Wiener process, gamma process and inverse Gaussian (IG) process, and the other is the random coefficient regression model (RCRM), which is also called the general path model [15]. Here, we only review some representative works about the degradation models. Regarding the first category, a Wiener process is widely used in modeling the degradation process. Zhang *et al.* [16] proposed a prognostic model for degradation systems, where performance degrades with usage and recovers in storage. A nonlinear Wiener process was used to model the degradation process, and a case study of Li-ion batteries was carried out to illustrate the model. Zhai and Ye [17] proposed an adaptive Wiener process model, where the time-varying drift rate follows a Wiener process. A maximum likelihood estimation procedure was developed to estimate the model parameters, and the degradation-based remaining useful life was also addressed based on the proposed model. Another important stochastic model is a gamma process model, which is used to model a strictly monotonic degradation process. Lawless and Crowder [18] constructed a tractable gamma process model considering the random effect and covariates, and the model was applied to some data on crack growth. Lu *et al.* [19] presented a maximum likelihood estimation (MLE) method for estimating the parameters of a gamma process model, and the Genz transform and a quasi-Monte Carlo method were applied in the estimation. An IG process model is also an attractive model with monotonic paths. Compared with a gamma process, an IG process has many superb properties when considering covariates and random effects [20]. The MLE method is always used to estimate the parameters in IG process models for degradation data, and the estimation-maximization (EM) method is always applied to obtain the unknown parameters of the maximum likelihood estimators [21]. The second category concerns the RCRM, which has been widely used in the fields of social science, economics, pharmacokinetics and quality management, among others [22], [23]. It has also been used in the field of maintenance and reliability, especially regarding the degradation process (e.g., wear and fatigue and crack length) of a system. Lu and Meeker [22] modeled fatigue crack growth progress on the basis of the RCRM, defining a particular time-to-failure distribution. Wang [23] not only applied the RCRM to model the time-to-failure distribution but also to determine the optimal PM threshold and the monitoring interval. Tang *et al.* [24] developed a random coefficient autoregressive model with a time effect to describe system degradation. Sun *et al.* [25] used the RCRM as a general degradation model, proposing a two-stage degradation model to derive the degradation path for reliability prediction.

In addition to the aforementioned works, some other methods are available for modeling a degradation process, although they have not yet received as much attention, for example, shock models, continuous-time Markov models, and delay-time models. Although many models have been studied for modeling a degradation process based on a CBM policy, few studies exist on the joint optimization of a CBM policy and EPQ planning. To the best of our knowledge, Jafari and Makis [26] proposed a proportional hazards model, that considers both the CM data and the age of a production facility. Here, the deterioration process of the system is determined by the age and covariate values, and the covariate process is modeled by a continuous-time Markov process. The optimal EPQ and PM levels can be calculated by minimizing the long-run expected average cost per unit time. Jafari and Makis [27] extended the work in [26] by modeling the covariate process with a continuous-time hidden Markov process. Bouslah *et al.* [28] also constructed an integrated EPQ and CBM model. They obtained the qualification rate of the products through continuous sampling monitoring, and then analyzed the qualification rate to determine the deterioration process of the system. Peng and Houtum [29] considered the continuous-time and continuous-state degradation processes in an integrated EPQ and CBM model, where the production lot size is optimized by taking the CBM activities into account. Their model of the average long-run cost was developed based on the renewal theory. Cheng *et al.* [30] proposed a joint EPQ and CBM optimization model for a multicomponent production system, where the components deteriorate gradually with use and age. Cheng *et al.* [31] proposed an integrated EPQ and CBM optimization model for an imperfect production system, where the quality control is studied and the degradation process is modeled by a gamma process. They used the structural importance measure to make maintenance decisions, and they set the production lot size and PM threshold as the decision variables. All these works used a stochastic process or Markov process to describe the system deterioration processes. They also set some form of critical level as the PM threshold (e.g. the system condition threshold, the reliability threshold, or the hazard rate threshold) and used the PM threshold and lot size as decision variables. The optimal results were all calculated by minimizing the average long-run maintenance and production costs. However, despite the many studies about the degradation process using the RCRM, the RCRM has not been used for the integrated EPQ and CBM problem. In fact, the RCRM is very easy to use, and the theory has been well studied; it is very flexible in incorporating random effects, and it is more robust than stochastic process models and other process-based models.

In this paper, we propose an integrated EPQ and CBM model using the renewal reward theory. We use the RCRM to describe the system deterioration process, and the coefficients in the model include random variables, that follow a multivariate distribution. Most degradation models used for this particular problem are restricted to a single characteristic of the system (i.e., the fixed effects, which give the inherent characteristics of the system). However, as the RCRM is a more general model, it can be used to describe not only the fixed effects but also the random effects of the system (i.e., those effects that are relevant to the individual characteristics of the system). Furthermore, in the RCRM, the observed system condition has two parts: the actual system condition, which cannot be observed accurately, and the random error, which is relate to the measurement errors and some unknown factors. Previous research has treated the observed condition as the actual condition, and the random error has not been considered. The RCRM can fill this research gap. In addition, unlike previous studies that considered only the PM action triggered by the system condition threshold, we also consider the product quality problem caused by system deterioration, i.e. the qualification rate of products could gradually decrease in line with the system deterioration. Unqualified products should therefore be reworked to improve their quality back to normal or be sold at a lower price. As such, we add the handling cost of unqualified products to our model.

We consider an infinite planning horizon, where the product is repeatedly produced in batches and the system is monitored at the end of each lot size. Once the observed system deterioration condition meets or exceeds the critical level (which is a decision variable) the system should be renewed by PM. If the system deterioration condition meets the failure level, the system is likely to break down and a repair or replacement should be carried out immediately to renew the system. The same monitoring process is then resumed. Our objective is to find the optimal lot size and critical level by minimizing the expected cost per unit time, where the costs include the inventory cost, setup cost, maintenance cost, shortage cost and cost of unqualified products. For model verification, we also develop a simulation process to describe the maintenance and production strategy in detail.

Our integrated EPQ and CBM model provides innovation in the following areas: 1) for this type of model, we are the first study to use the RCRM to describe a system degradation process; 2) we consider the product quality problem in the model; and 3) we develop a simulation process to verify our model and provide a new method for finding the optimal results.

The rest of this paper is organized as follows. Notations and the problem description are given in Section II. Section III introduces the cost models. Section IV describes the simulation process. Numerical examples are presented in Section V, and Section VI concludes the paper.

II. NOTATIONS AND PROBLEM DESCRIPTION

A. NOTATIONS

p	production rate
d	demand rate
Q	production lot size – a decision variable
τ	production time of a lot size
CI	unit inventory cost
C_S	unit setup cost
C_M	unit cost of condition monitoring check
C_P	unit PM cost
C_F	unit cost of failure repair or replacement
CL	unit shortage cost
CU	unit handling cost of unqualified products
t_j	the time of the j^{th} monitoring check since new, $j = 1, 2, \ldots\ldots$
$y(t_j)$	the observed system condition at time t_j
η_j	the actual system condition at time t_j
C	the first condition level, which is called the critical level. Once the observed condition meets or exceeds this level, the system should be renewed by PM – a decision variable
D	the second condition level, which is called the failure level. Once the system condition meets this level, the system is likely to break down
$F_T(t)$	the distribution of the time to failure
$P(Y)$	the distribution of $y(t_j)$
d_P	unit time of PM
d_f	unit time of failure repair or replacement
$\Phi(\cdot)$	the standard normal distribution function

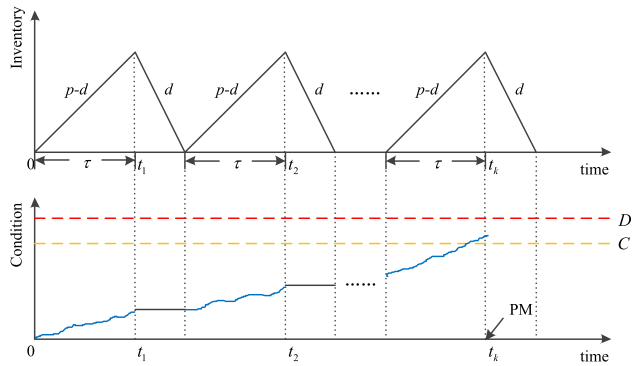

FIGURE 1. Maintenance and lot sizing strategy of PM renewal.

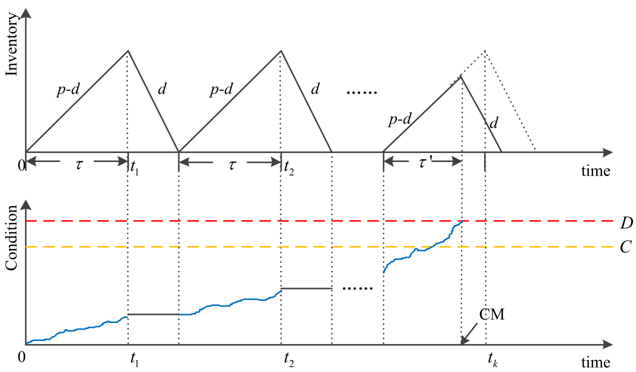

FIGURE 2. Maintenance and lot-sizing strategy of failure renewal.

B. PROBLEM DESCRIPTION

1) MAINTENANCE AND LOT-SIZING STRATEGY

The system has two renewal scenarios: PM renewal and failure renewal. We first describe the PM renewal scenario, as shown in Fig. 1. The product is produced in batches at production rate p, and the product is consumed at demand rate d, where $p > d$. The product accumulates at rate p-d until a lot size is finished. The production time of a lot size is τ, where $\tau = Q/p$; τ can also be treated as a decision variable because lot size Q is a decision variable and p is constant. When a lot size is finished, the system is shut down. Here, the product inventory is $(p-d)\,\tau$, the inventory is still consumed at demand rate d, and the length of the idle time is $[(p-d)\,\tau]\big/d$. The degradation process is monitored when a lot size is finished. If the observed system condition, $y(t_j)$, is less than the critical level, C, then no maintenance action is needed. However, if the observed system condition meets or exceeds C but is less than D, PM should be carried out to renew the system. We assume that the PM time d_P satisfies $d_P \le [(p-d)\,\tau]\big/d$ because the system is in an 'in-control' scenario, not a sudden failure scenario. In practice, the shortage cost is very high. It is therefore logical to disallow shortages at the planning stage, and this is a common setting in deterministic demand environments [3]. A new batch and a new renewal cycle will be started when the inventory decreases to zero.

Fig. 2 shows the failure renewal scenario, which is identified by the failure level, D. D is not a decision variable; it can be obtained by data analysis or system specification. For example, Lu and Meeker [22] defined a critical crack length of 1.6 inches to be a failure level according to the data of 21 test units, and Wang [23] collected the measured wear data of 8 identical components over their life cycles. When the actual deterioration, η_j, reaches D, the system is likely to break down. This will be observed immediately, at which point CM should be carried out. As shown in Fig. 2, in the batch where system failure occurs, $\tau' < \tau$, and the idle time from system breakdown to when the inventory decreases to zero is $\left[(p-d)\,\tau'\right]\big/d$. We cannot set $d_f \le \left[(p-d)\,\tau'\right]\big/d$ directly since the failure time is random; therefore, if $d_f > \left[(p-d)\,\tau'\right]\big/d$, shortage occurs. A new cycle starts when the inventory decreases to zero under the situation $d_f \le \left[(p-d)\,\tau'\right]\big/d$ or when CM is finished under the situation $d_f > \left[(p-d)\,\tau'\right]\big/d$.

2) THE RANDOM COEFFICIENT REGRESSION MODEL

The condition of the system deteriorates over time, and the observed condition of the system can be modeled by an RCRM, where the mean and variance of the observed condition can be increasing functions with time. At time t_j, the observed system condition is $y(t_j)$, which is composed of two parts: the actual system condition, η_j, and the random error, ε. We use an RCRM to describe the system deterioration process as follows:

$$y(t_j) = \eta_j + \varepsilon = \eta(t_j; \theta, \xi) + \varepsilon. \tag{1}$$

In (1), ε denotes the random error, which is assumed to be normally distributed with constant variance and mean zero, that is, $\varepsilon \sim N(0, \sigma)$. The normal distribution has been commonly used in existing research because it has a good fitting effect for the random error. Additionally in (1), θ is the fixed-effect parameter, representing the inherent characteristics for a system; ξ is the random-effect parameter with probability distribution function $g(\xi)$, representing the individual characteristics of a system; ξ and ε are assumed to be mutually independent; η_j can be a linear or nonlinear function of t_j with θ and ξ. For example, $\eta_j = \theta + \xi t_j$, $\eta_j = \theta_1 + \xi \exp(\theta_2 t_j)$ [22].

For simplification, we use t_j in the model. This denotes the actual running time of the system at the j^{th} monitoring point. However, please note that the actual running time, such as that in interval $[0, t_j]$ in Fig. 1, is composed of the actual running time, $j\tau$, and the down time, $j\,[(p-d)\,\tau]\big/d$. Therefore, Eq. (1) in this paper should be calculated using the actual running time, $j\tau$.

III. THE INTEGRATED MODEL

A. THE DISTRIBUTION FUNCTIONS OF THE CONDITION AND TIME TO FAILURE

We use the model proposed in [22] and [23] to describe the distribution functions of $y(t_j)$ and the time to failure. In (1), ξ is a random variable. For a given ξ, the distribution function of $y(t_j)$ is

$$P(Y|\xi) = P\left(y(t_j) \le Y|\xi\right) = P\left(\eta(t_j;\theta,\xi) + \varepsilon \le Y\right) = \Phi\left(\frac{Y - \eta(t_j;\theta,\xi)}{\sigma}\right). \quad (2)$$

Based on (2), we have

$$P(Y) = P\left(y(t_j) \le Y\right) = \int_A g(\xi) P\left(y(t_j) \le Y|\xi\right) \mathrm{d}\xi, \quad (3)$$

where $g(\xi)$ is the probability distribution function of ξ, A is the sample space of ξ, and $P(Y)$ is only influenced by the actual system running time.

If the actual condition meets failure threshold D, the system fails and should be renewed. We use T to denote the failure time. For a given ξ, the distribution function of T is $P(T \le t|\xi) = F_T(t|\xi)$. Therefore, we have

$$P(T \le t) = F_T(t) = \int_A g(\xi) F_T(t|\xi)\, \mathrm{d}\xi. \quad (4)$$

In (4), if the time at which η meets D is less than or equal to t, the system has failed before or at time t: $F_T(t|\xi) = 1$. However, if the time at which η meets D is greater than t, the system has not failed at time t: $F_T(t|\xi) = 0$. For example, assuming that $\eta = \xi t + \theta$, and ξ follows a Weibull distribution, we have $D = \xi T + \theta$. We therefore have

$$P(T \le t) = P\left(\frac{D-\theta}{\xi} \le t\right) = P\left(\xi \ge \frac{D-\theta}{t}\right) = \int_0^{\frac{D-\theta}{t}} g(\xi) F_T(t|\xi)\, \mathrm{d}\xi + \int_{\frac{D-\theta}{t}}^{\infty} g(\xi) F_T(t|\xi)\, \mathrm{d}\xi = \int_{\frac{D-\theta}{t}}^{\infty} g(\xi) \mathrm{d}\xi. \quad (5)$$

In (5), if $0 \le \xi < \frac{D-\theta}{t}$, then $T > t$ and $F_T(t|\xi) = 0$. If $\xi \ge \frac{D-\theta}{t}$, then $T \le t$ and $F_T(t|\xi) = 1$.

B. THE INTEGRATED COST MODEL

Based on the problem description, there are two types of system renewal: PM renewal and failure renewal. We first discuss PM renewal. When the system condition $y(t_k)$ is equal to or higher than the critical level C at time t_k, the system should be renewed by preventive replacement. After preventive replacement, a renewal cycle is generated. The probability of this situation is denoted by $P_p(t_k; C)$. The system must not have failed or been preventively replaced before t_k; therefore, for a given ξ, we have

$$P_p(t_k; C|\xi) = [1 - F_T(t_k|\xi)] \left[\prod_{j=1}^{k-1} P(y(t_j) < C|\xi)\right] P(y(t_k) \ge C|\xi). \quad (6)$$

In (6), $1 - F_T(t_k|\xi)$ is the probability that the system has not failed before t_k, $\prod_{j=1}^{k-1} P(y(t_j) < C|\xi)$ is the probability that the system has not been preventively replaced before t_k, and $P(y(t_k) \ge C|\xi)$ is the probability that the system condition meets or exceeds critical level C at t_k. We therefore have

$$P_p(t_k; C) = \int_A g(\xi) P_p(t_k; C|\xi) d\xi. \quad (7)$$

Within a PM renewal cycle, the total monitored cost is kC_M, the total setup cost is kC_S, the PM cost is C_P, and the total product inventory cost is $C_I\left(\frac{kp(p-d)\tau^2}{2d}\right)$. We also consider the total handling cost of unqualified products, which can be obtained through the unqualified product rate $R(t)$. We denote the unqualified product rate $R(t|\xi)$ for a given ξ, where t is the actual system running time and $R(t|\xi)$ is an increasing function of t. We therefore have $R(t) = \int_A g(\xi) R(t|\xi) d\xi$. The production quantity within a PM renewal cycle is $kQ = kp\tau$; therefore, the expected total handing cost of unqualified products is $C_U \cdot R(k\tau) \cdot kp\tau$. The expected total cost during a PM renewal cycle is

$$EC_p(t_k) = \left[\begin{array}{l} kC_M + C_I\left(\dfrac{kp(p-d)\tau^2}{2d}\right) \\ + C_U \cdot R(k\tau) \cdot kp\tau + kC_S + C_P \end{array}\right] P_p(t_k; C). \quad (8)$$

As previously described, PM should be carried out to renew the system at t_k. We assume that $d_P \le [(p-d)\tau]/d$. A new production and renewal cycle starts when the inventory decreases to zero, so the expected length of a PM renewal cycle is

$$EL_p(t_k) = \left(k\frac{p\tau}{d}\right) P_p(t_k; C). \quad (9)$$

We now similarly discuss the failure renewal situation. Assume that the actual system condition reaches failure level D during the production process of the k^{th} batch. Here, the system fails and is renewed, and a failure renewal cycle is generated. The probability of this situation is denoted by $P_f(t_k; C)$. For a given ξ, the probability of a failure with the k^{th} batch production process is

$$P_f(t_k; C|\xi) = [F_T(t_k|\xi) - F_T(t_{k-1}|\xi)] \left[\prod_{j=1}^{k-1} P(y(t_j) < C|\xi)\right], \quad (10)$$

where $F_T(t_k|\xi) - F_T(t_{k-1}|\xi) = P(t_{k-1} < T \le t_k|\xi)$ is the probability that the system has failed between t_{k-1} and t_k, and $\prod_{j=1}^{k-1} P(y(t_j) < C|\xi)$ is the probability that the system has not been preventively replaced before t_{k-1}. Based on (10), we have

$$P_f(t_k; C) = \int_A g(\xi) P_f(t_k; C|\xi) d\xi. \quad (11)$$

From Fig. 2, we know that the k^{th} lot size might not have finished when the system fails. If we suppose that the running time of the k^{th} batch is τ_k, the consumption time of inventory accumulated within τ_k is $(p-d)\tau_k/d$. Therefore, if $d_f \le (p-d)\tau_k/d$, i.e. $\tau_k \ge \frac{d_f \cdot d}{p-d}$, there is no shortage in the renewal cycle; otherwise, $\tau_k < \frac{d_f \cdot d}{p-d}$, a shortage occurs, and the shortage time is $d_f - (p-d)\tau_k/d$. If we let $\Delta = \frac{d_f \cdot d}{(p-d)}$, the expected shortage cost within the renewal cycle is

$$EC_L = C_L \left[\begin{array}{l} 0 \cdot \displaystyle\int_{(k-1)\tau+\Delta}^{k\tau} \frac{\partial P_f(t;C)}{\partial t} dt \\ + \displaystyle\int_{(k-1)\tau}^{(k-1)\tau+\Delta} \left(d_f - \frac{(p-d)}{d}(t-(k-1)\tau) \right) \\ \dfrac{\partial P_f(t;C)}{\partial t} dt \end{array} \right].$$

Then, the expected inventory cost within the renewal cycle is

$$EC_I = C_I \left(\begin{array}{l} \dfrac{(k-1)p(p-d)\tau^2}{2d} P_f(t_k;C) \\ + \displaystyle\int_{(k-1)\tau}^{k\tau} \frac{p(p-d)\,(t-(k-1)\tau)^2}{2d} \frac{\partial P_f(t;C)}{\partial t} dt \end{array} \right),$$

the total monitored cost is $(k-1)C_M$, the total setup cost is kC_s, the failure repair or replacement cost is C_F, and the expected handling cost of unqualified products is $EC_U = \int_{(k-1)\tau}^{k\tau} (C_U R(t)pt) \frac{\partial P_f(t;C)}{\partial t} dt$.

According to the abovementioned derivation, the expected cost in a failure renewal cycle is

$$EC_f(t_k) = EC_L + EC_I + EC_U + [(k-1)C_M + kC_s + C_F]\,P_f(t_k;C). \quad (12)$$

The expected length of the failure renewal cycle is

$$\begin{aligned} EL_f(t_k) &= (k-1)\frac{p\tau}{d} P_f(t_k;C) \\ &+ \int_{(k-1)\tau+\Delta}^{k\tau} \left((t-(k-1)\,\tau)\,\frac{p}{d} \right) \frac{\partial P_f(t;C)}{\partial t} dt \\ &+ \int_{(k-1)\tau}^{(k-1)\tau+\Delta} \left((t-(k-1)\tau) + d_f \right) \frac{\partial P_f(t;C)}{\partial t} dt. \end{aligned} \quad (13)$$

Based on the renewal reward theory, which has been widely applied in existing research [32], [33], the expected cost per unit time is

$$EC(\tau;C) = \frac{\sum_{k=1}^{\infty} \left(EC_p(t_k) + EC_f(t_k) \right)}{\sum_{k=1}^{\infty} \left(EL_p(t_k) + EL_f(t_k) \right)}, \quad (14)$$

where Q and critical level C are the decision variables, which can be obtained by minimizing $EC(\tau;C)$. However, we cannot obtain the explicit form of the solutions because the integral, differential coefficient and unknown distribution of ξ exist in the model.

Many effective methods can be used to solve this problem because there are only two decision variables. For example, a sequential quadratic programming algorithm [34] can be used to search for the optimal results, or an enumeration algorithm can be used to solve the problem according to the reasonable scope and precision of the decision variables, as described before. Where D is predetermined, $0 < C < D$. Where p is predetermined, $Q = p\tau$, in which τ is the batch time length that, in reality, cannot be too long and has a scope according to experience and specification, so an enumeration algorithm is an effective method for solving the problem in this paper.

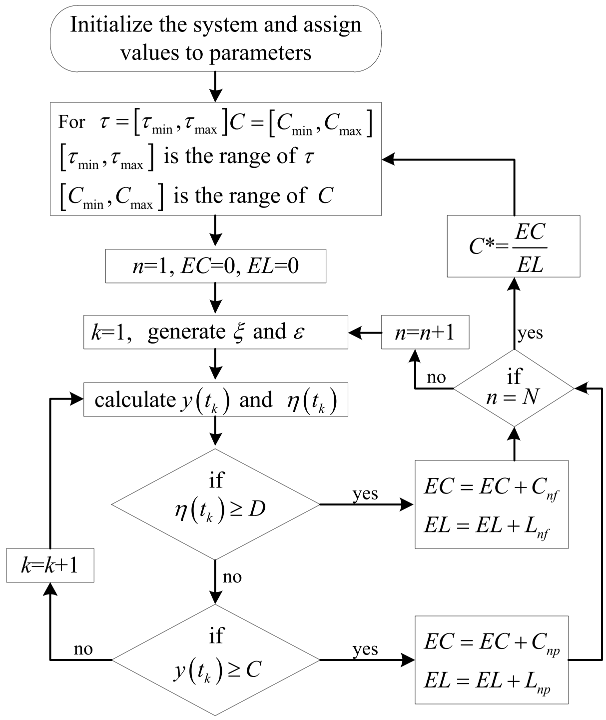

FIGURE 3. Simulation process.

IV. SIMULATION

In this section, we develop a simulation process to obtain the optimal Q and critical level C. In addition to using a secondary method to obtain the results, this also validates the models proposed in Section III. We set the reasonable value ranges of τ and C, all combination values of τ and C can be enumerated based on the given precision, although τ and C are both continuous variables. The cost of unit time can be calculated based on each combination value of τ and C, the optimal result of τ and C can be obtained according to the minimal cost of unit time, then the optimal Q can be obtained. Fig. 3 shows the simulation process. It contains 6 steps:

Step 1: Initialization

1) Initializing the system and assigning values to the cost and time parameters;
2) Setting the ranges of τ and C to $[\tau_{\min}, \tau_{\max}]$ and $[C_{\min}, C_{\max}]$, respectively;
3) N is the simulation time, and n denotes the n^{th} simulation;

4) EC denotes the total cost of all renewal cycles, EL denotes the total length of all renewal cycles, and k denotes the k^{th} monitoring point of the k^{th} batch.

Step 2: Generating random values of ξ and ε

In each renewal cycle, based on the parameter values in step 1, ξ and ε are generated according to (1).

Step 3: Calculating $y(t_k)$ and $\eta(t_k)$

$y(t_k)$ and $\eta(t_k)$ are calculated based on the values of ξ and ε in step 2.

Step 4: Calculating the cost and length of the renewal cycle

1) If $\eta(t_k) \geq D$, the system fails during the production process of the k^{th} lot size and should be renewed. Assuming that the length of the k^{th} lot size is τ_k, we have $\eta[(k-1)\tau + \tau_k] = D$. τ_k can be calculated, so the total cost in the failure renewal cycle is

$$\begin{aligned} C_{nf} &= (k-1)C_M + kC_s + C_F \\ &\quad + C_L \max\left\{0, d_f - \frac{\tau_k(p-d)}{d}\right\} \\ &\quad + C_I\left(\frac{(k-1)p(p-d)\tau^2}{2d} + \frac{p(p-d)\tau_k^2}{2d}\right) \\ &\quad + C_U R\left[(k-1)\tau + \tau_k \,|\xi\,\right] p\left[(k-1)\tau + \tau_k\right]. \end{aligned} \tag{15}$$

The length of the failure renewal cycle is

$$L_{nf} = \left((k-1)\frac{p\tau}{d}\right) + \tau_k + \max\left\{d_f, \frac{\tau_k(p-d)}{d}\right\}. \tag{16}$$

2) If $y(t_k) \geq C$, the system should be renewed by PM. The total cost in the PM renewal cycle is

$$\begin{aligned} C_{nP} &= kC_M + kC_s + C_P + C_I\left(\frac{kp(p-d)\tau^2}{2d}\right) \\ &\quad + C_U R\left(k\tau \,|\xi\,\right) pk\tau. \end{aligned} \tag{17}$$

The length of the PM renewal cycle is

$$L_{np} = k\frac{p\tau}{d}. \tag{18}$$

3) If $\eta(t_k) < D$ and $y(t_k) < C$, the system does not need to be renewed: $k = k+1$. Go back to step 3.

Step 5: The calculation of $C*$

If the numbers of simulation $n = N$, the cost of unit time is $C* = \frac{EC}{EL}$.

Step 6: Obtain the optimal results

All values of $C*$ are calculated according to the ranges of τ and C. By selecting the best results of τ and C based on the minimum $C*$, the optimal Q and C can be obtained.

V. NUMERICAL EXAMPLE

In this section, we use data collected from a steel factory that we previously visited. The system produces steel pipes. Several methods exist to assist with parameter estimation in the RCRM, such as the maximum likelihood estimation method [35], the expectation maximization algorithm [36], and the two-stage estimation method, which is simpler than that above [22], [23]. We give the parameters directly since the parameter estimation is not the innovation in this paper. We use (19) to model the system condition, in which we assume that random variable ξ follows a Weibull distribution because of its applicability and the fact that it is well studied [22], [23]. Equation (20) is the probability density function of ξ, where, for simplicity, the fixed parameter $\theta = 0$ and $\varepsilon \sim N(0, 0.0312)$.

$$y(t_j) = \left(\xi t_j + \theta\right) + \varepsilon. \tag{19}$$

$$f(x;\alpha,\beta) = \begin{cases} \alpha\beta(\alpha x)^{(\beta-1)}e^{-(\alpha x)^{\beta}}, & x \geq 0; \\ 0, & x < 0. \end{cases} \tag{20}$$

TABLE 1. The values of the model parameters.

p	d	d_f	d_p	α	β	D
10	6	0.2	0.15	0.4	2.42	5
C_I	C_S	C_M	C_P	C_F	C_L	C_U
5	50	50	200	500	50	10

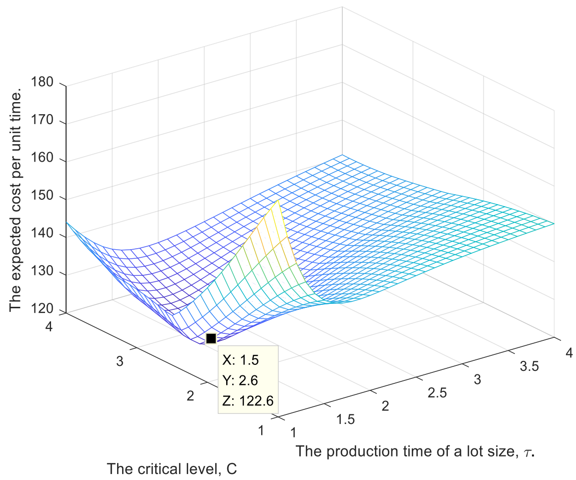

FIGURE 4. The results of the integrated cost model.

The relevant parameter values are presented in TABLE 1. The unit of weight is 'ton', the unit of time is 'day', and the failure level of the system condition is $D = 5$ (10 μm), which is the vibration that is collected by the sensor and used to assess the condition of the system. Regarding the rate of unqualified products, the data collected had an unqualified rate of 2–4% of the production quantity. Therefore, in this example, we use the mean value of 3% as the unqualified rate.

A. THE OPTIMAL RESULTS

Based on the above content, we use the models in Section III to calculate the optimal result, as shown in Fig. 4. Note that we set the interval of critical level C to 1∼4 and the production time of lot size τ to 1∼4. These are not the same unit of measurement, and the allowable ranges of C and τ are

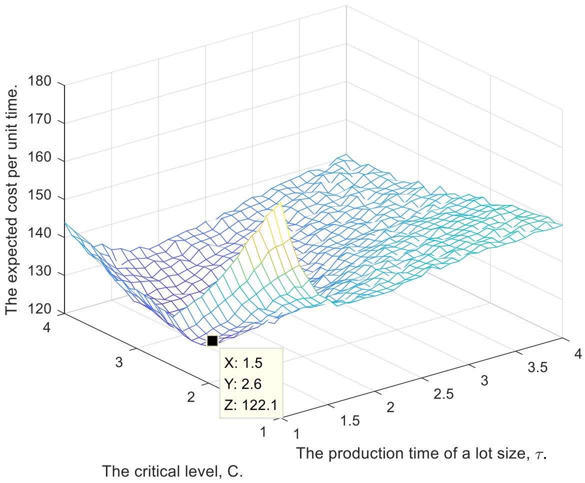

FIGURE 5. The simulation results.

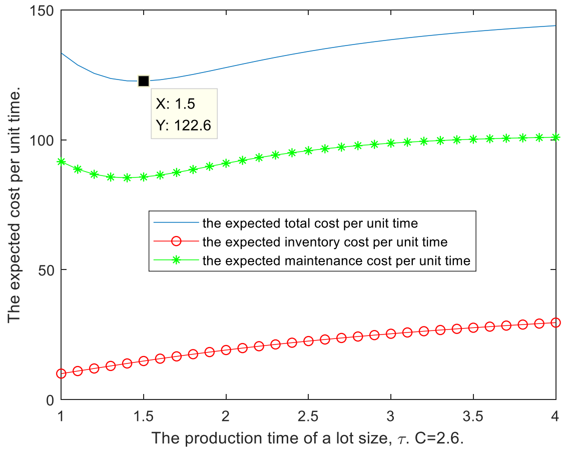

FIGURE 6. The expected cost per unit time when C=2.6.

greater than those in Fig. 4. To make Fig. 4 clear, we select only the part of the range that contains the optimal result.

Fig. 4 shows that the optimal result is $EC(\tau*; C*) = EC(1.5; 2.6) = 122.6$. Therefore, the optimal EPQ is $Q* = p\tau* = 15$, and the optimal critical level is $C* = 2.6$. When the monitored system condition reaches or exceeds this critical level, the system should be renewed by PM. Here we can see that the optimal lot size and the critical level can be obtained simultaneously, that is, the optimal production planning and maintenance policy can be arranged simultaneously, which is the main objective of this paper.

Fig. 5 shows the results obtained using the simulation process from Section IV. The number of simulations is set to 20000 and the optimal result is $EC(\tau*; C*) = EC(1.5; 2.6) = 122.1$. This result is very close to the result that was calculated through the models. This shows that the simulation method is another way to obtain the optimal results and also further validates the model results from Section III, which is also an important objective of this paper.

B. ANALYSIS OF THE RESULTS

We use the model to analyze the results separately. Fig. 6 shows three types of expected cost per unit time for C=2.6: total cost, inventory cost and maintenance cost. Each of these costs changes along with different τ. The inventory cost per unit time increases when τ increases (i.e., when the EPQ increases), which is a practical result. The trend of the expected maintenance cost per unit time is similar to the trend of the expected total cost per unit time, but the gap between the two gradually increases as the inventory cost increases.

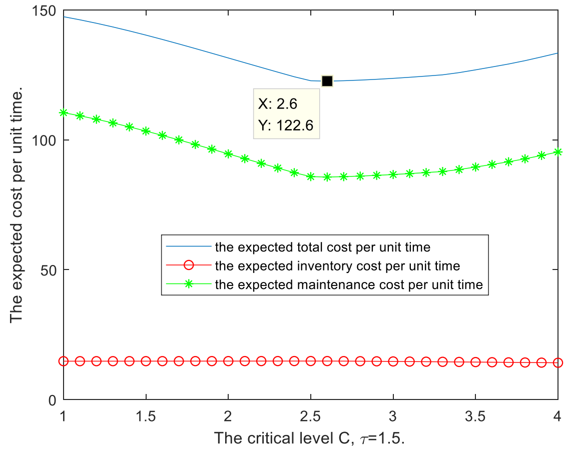

FIGURE 7. The expected cost per unit time when $\tau = 1.5$.

TABLE 2. The results of the sensitivity analysis of C_F.

C_F	300	400	500	600	700	800
$(\tau^*; C^*)$	(2.8;4.1)	(1.9;3.3)	(1.5;2.6)	(1.3;2.5)	(1.3;2.5)	(1.2;2.5)
$EC(\tau^*; C^*)$	102	115.4	122.6	125.3	127.1	128.5

Fig. 7 shows three types of expected costs per unit time for $\tau = 1.5$: total cost, inventory cost and maintenance cost. The trend of the maintenance cost is similar to the trend of the total cost. In contrast, the inventory cost is nearly constant. This occurs because τ is fixed, which makes the EPQ fixed. Fig. 6 and Fig. 7 show that the maintenance cost is influenced by both C and τ, but the inventory cost is only influenced by τ, which is a practical result.

C. SENSITIVITY ANALYSIS

Sensitivity analysis shows the influence of the parameters in a model. This paper uses several model parameters; however, because the failure state is always catastrophic and C_F is always larger than the other unit costs, we only select C_F as an example for sensitivity analysis here. TABLE 2 shows the results of the sensitivity analysis with different values of C_F.

TABLE 2 shows that $\tau*$ gradually decreases as C_F increases. This is because the monitoring interval should be decreased to prevent system failure. $C*$ also gradually decreases as C_F increases but becomes stable when $C_F \geq 600$. This is because more frequent PM actions not only reduce system failure but also cause an increase in PM and setup costs; therefore, the critical level cannot always decrease. The results of the sensitivity analysis match the real-world situation. Sensitivity analysis of the other parameters can be similarly studied, but we do not analyze them here.

VI. CONCLUSION

This paper studied the integrated economic production quantity (EPQ) and condition-based maintenance (CBM) problem. We used the random coefficient regression model (RCRM) to describe the degradation processes of the system, where both the actual condition and the random error were considered. Two renewal scenarios were studied according to a pre-set failure level and the critical level, which is the decision variable of the system condition. Based on the distributions of the system condition and time to failure, we proposed an integrated EPQ and CBM cost model. We applied the renewal reward theory to find the optimal critical level and lot size. A simulation process was also proposed to verify our model. A real case study was presented to demonstrate the applicability of both our model and the simulation method. The results show that the optimal lot size and critical level can be calculated simultaneously. This also proves the validity of our model because the results calculated through the model are very close to those calculated through simulation. Through sensitivity analysis, we found that the optimal production time of a lot size gradually decreases as the failure cost increases. Further, the optimal results of the critical level also decrease as the failure cost increases, but this becomes stable when the failure cost exceeds a high value, which is reasonable in reality.

As we introduced before, the RCRM had advantages in describing the degradation process, such as its flexibility and being more robust than stochastic models; however, the original RCRM is simplified in reality and may not be able to be applied to more complex situations. Based on this paper, there are many possible research extensions as follows: 1) To reduce the randomness in the observed degradation, the time-varying covariates in the degradation process should be considered in the RCRM, such as temperature, humidity, and voltage, and these environmental stress factors may affect the degradation process. 2) Additionally, in this paper, PM was assumed to be perfect (i.e., where PM renews the system), but this assumption could be relaxed. If the PM is imperfect, the usage of the system might decrease when PM is carried out, but it will not decrease to zero. The random demand rate and production rate could also be studied further. 3) Only one type of product is studied in this paper, and more than one type of product can be produced alternately in batches by a system in reality, so multiple products can be studied in the integrated RCRM and EPQ model. 4) An RCRM could be used to model economic and social science problems, and the model proposed in this paper could be applied to quality control problems.

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