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Effect of Measurement Errors on the Performance of Coefficient of Variation Chart With Short Production Runs

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ABSTRACT This paper investigates the performance of the coefficient of variation chart in the presence of measurement errors for finite production horizon. We study a two-sided Shewhart coefficient of variation chart with measurement errors for detecting both increase and decrease in the coefficient of variation for short run processes using an error model with linear covariate. The performance of the coefficient of variation chart is evaluated by the truncated average run length and the expected value of the truncated average run length. The numerical results indicate that the precision error and the accuracy error have negative effect of the measurement errors on the performance of the coefficient of variation chart. In addition, the constant coefficient *B* in the linear covariate error model reduces the negative effect of the measurement errors on the performance of the coefficient of variation chart. However, taking multiple measurements per item in each sample is not an effective method to enhance the performance of the coefficient of variation chart. An example is provided to illustrate the implementation of the coefficient of variation chart. In addition, the economic criterion is also added to study the effect of measurement errors on the expected inspection cost. The result shows that an increase in the precision error reduces the expected inspection cost.

INDEX TERMS Coefficient of variation, measurement errors, short production runs.

I. INTRODUCTION

Control chart for monitoring the coefficient of variation (CV) is a useful tool for statistical process control when the mean is proportional to the standard deviation so that the ratio of the standard deviation to the mean is a constant. Kang *et al.* [1] proposed Shewhart-type control chart for monitoring the CV. Castagliola *et al.* [2] proposed a Shewhart CV control chart with variable sample size (VSS) and they designed the CV chart with VSS to obtain the optimal design parameters by minimizing the out-of-control average run length or average sample size. Tran and Tran [3] presented a method to monitor the squared CV by using the cumulative sum chart and they studied the run length properties of the proposed chart using a Markov chain approach. You *et al.* [4] presented a side sensitive group runs chart for monitoring the CV and the proposed chart surpasses the Shewhart CV, runs rules CV, synthetic CV and exponentially weighted moving average (EWMA)

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CV charts by means of average run length and standard deviation of run length. Khaw *et al.* [5] used the variable sample size and sampling interval (VSSI) feature to improve the performance of the CV chart and they measured the performance of the proposed chart, in terms of the average time to signal and expected average time to signal criteria. Teoh *et al.* [6] developed a rum sum chart for monitoring the CV and their results showed that the proposed chart outperforms the Shewhart CV, run rules CV and EWMA CV charts. Yeong *et al.* [7] proposed a VSS scheme directly monitoring the CV, instead of monitoring the transformed statistics. The advantage over the VSS chart based on the transformed statistics is the proposed chart provides an easier alternative as no transformation is involved. Yeong *et al.* [8] proposed an EWMA chart with variable sampling interval (VSI) to monitor the CV. The comparative studies showed that the VSI EWMA CV chart outperforms other competing charts such as Shewhart CV, synthetic CV, VSI CV and EWMA CV charts. Muhammad *et al.* [9] proposed a VSS EWMA CV chart and the performance comparison showed that the proposed chart

outperforms the Shewhart CV, EWMA CV, synthetic CV, run rules CV and VSS CV charts in almost all scenarios. Yeong *et al.* [10] proposed a variable parameters chart to monitor the CV. The variable parameters CV chart consistently outperforms the five alternative CV charts, which are the VSSI CV, VSI CV, VSS CV, synthetic CV and Shewhart CV charts for all the given shift sizes. Zhang *et al.* [11] proposed a new EWMA chart for monitoring the CV. They presented the implementation and optimization procedures for the proposed chart. Chen *et al.* [12] proposed a generally weighted moving average control chart with adjusted time-varying control limits for monitoring the CV. As demonstrated by extensive simulation results, the proposed chart is clearly more sensitive than other competing procedures in the literature.

The presence of measurement errors affects the performance of control charts. Since the measurement errors do exist in practice, many researchers studied the measurement errors on the performance of various control charts. Daryabari *et al.* [13] investigated the effect of measurement errors on the performance of the maximum exponentially weighted moving average and mean squared deviation control chart for jointly monitoring the process mean and variance. Maleki *et al.* [14] presented an overview for the effect of measurement errors on different aspects of statistical process monitoring. They reported an extensive survey of the research on control charts with measurement errors and they also provided some directions to motivate the future studies. Tran *et al.* [15] examined the performance of Shewhart median control chart in the presence of measurement errors by assuming the measurement error model as in Linna and Woodall [16]. Based on their results, it is obvious that measurement errors greatly affect the performance of Shewhart median chart. Yeong *et al.* [17] proposed the CV chart with a linear covariate error model and they found that using the control limits computed by ignoring the presence of measurement errors leads to erroneous conclusions regarding the average run length. Amiri *et al.* [18] incorporated the measurement errors into a hybrid method based on the generalized likelihood ratio and EWMA control charts for simultaneously monitoring the multivariate process mean and variability. They also suggested four remedial approaches to decrease the effect of measurement errors on the performance of the proposed control chart. Cheng and Wang [19] investigated the effect of measurement errors on the EWMA median and cumulative sum median charts. Their results indicated that the presence of measurement errors significantly affect the performance of the monitoring procedure. Cheng and Wang [20] presented a VSSI median control chart with estimated parameters in the presence of measurement errors. Their results showed that the VSSI median chart performs better than the Shewhart median, VSS median and VSI median control charts in terms of the average time to signal. Tang *et al.* [21] investigated the performance of an adaptive EWMA chart when measurement errors exist using the linear covariate error model. The comparisons with the

classical EWMA scheme confirmed the superiority of the adaptive EWMA scheme in detecting a wide range of shifts in the presence of measurement errors. Nguyen *et al.* [22] proposed a VSI Shewhart chart for monitoring the squared CV, denoted by VSI CV chart. Their numerical simulations showed that the precision error and the accuracy error do have negative influences on the VSI CV chart. Sabahno *et al.* [23] evaluated the effect of measurement errors on the overall performance of the VSS Hotelling's *T* 2 control chart and they used an optimization algorithm to find the minimum overall values of the performance measure for two different models. Tran *et al.* [24] studied the performance of the CV Shewhart-type control chart and the one-sided CV EWMA-type control chart using a model with linear covariate. Their results showed that the precision and accuracy errors significantly affect the performance of both control charts. Tran *et al.* [25] examined the performance of synthetic median chart in the presence of measurement errors by assuming the measurement error model as in Linna and Woodall [16]. Their results showed that the performance of the synthetic median chart deteriorates when the measurement errors increase. Tran *et al.* [26] investigated the effect of the measurement errors on the performance of cumulative sum chart for monitoring the CV. Their results showed that the precision error and the accuracy error have negative impact on the chart's performance.

Ladany [27] first proposed the control chart for short production runs where he introduced the economic optimization of a *p*-chart for short runs. Since then, the design of various control charts in a short run context is receiving increased attention in the literature. Crowder [28] derived an algorithm that allows the implementation of a short-production-run version of an economic-process-control model of Bather and Box and Jenkins. Castillo and Montgomery [29] presented some modifications that enhance the average run length properties of the *Q* chart. Castillo *et al.* [30] reviewed and commented on the statistical control methods for short production runs. Nenes and Tagaras [31] proposed the performance measure for the statistical performance of control charts with short production runs, where the production run ends after *H* hours. Trovato *et al.* [32] compared different strategies for monitoring the dispersion of a quality characteristic within a stage of a manufacturing process producing a short run of parts. Celano *et al.* [33] presented the theoretical background underlying the *t*-chart implementation and some statistical measures of performance have been computed to evaluate the chart statistical sensitivity during the short run. Celano *et al.* [34] proposed the economic design of a CUSUM *t* chart for monitoring short production runs without the need of Phase I samples. Celano *et al.* [35] also investigated the economic design of an SPC inspection procedure in short production runs based on a Shewhart control chart for monitoring the *t* statistic. Castagliola *et al.* [36] presented the theoretical background that allows the statistical performance measures of the variable sample size *t* Shewhart chart to be computed.

Celano *et al.* [37] investigated the statistical performance of the Shewhart, EWMA and CUSUM *t* charts for short production runs when the shift size is unknown and modeled by means of a statistical distribution. Castagliola *et al.* [38] investigated the CV chart for finite production horizon and they investigated the statistical properties of the proposed chart when the shift size is deterministic. Amdouni *et al.* [39] proposed an adaptive Shewhart chart implementing a VSS strategy in order to monitor the CV in a short run context and they obtained the optimal chart parameters by minimizing the out-of-control truncated average run length. Amdouni *et al.* [40] proposed a method to monitor the CV in a short run context by means of one-sided run rules type chart. A comparison analysis has been performed to show that implementing one-sided run rules type charts is the best decision most of the time. Celano *et al.* [41] compared the performance of several control charts for observations with a location-scale distribution in a finite horizon process for jointly monitoring the location and scale. Celano *et al.* [42] investigated the statistical performance of a nonparametric Shewhart sign control chart for monitoring the location of quality characteristic in a production process with a finite horizon. Amdouni *et al.* [43] investigated the design and implementation of a VSI chart to monitor the CV in a short run context. They derived the formulas for the truncated average time to signal. Performance comparison with the Shewhart CV chart demonstrated the outperformance of the VSI CV chart over the fixed sampling rate chart. Nikolaidis and Tagaras [44] evaluated the statistical performance of various one-sided Bayesian \bar{X} charts for monitoring the process mean in finite production runs. They found out that very often the Bayesian X chart with adaptive sample sizes performs better than other Bayesian \bar{X} charts from a statistical point of view. Celano and Castagliola [45] investigated the implementation of the EWMA sign control chart for finite production horizon process. They evaluated the statistical properties of the EWMA sign control chart with varying control limits using a nonhomogeneous Markov chain model. Zhou *et al.* [46] evaluated the control chart's performance for four different monitoring schemes based on fractional nonconformance for short run productions. Their results showed that the choice of the monitoring scheme does not heavily depend on the distribution of the quality characteristic. Chong *et al.* [47] proposed the multivariate fixed sample size and VSS Hotelling's T^2 short-run charts to monitor the mean shift in short production runs. Based on the comparison between the control charts, the VSS Hotelling's *T* 2 short-run chart surpasses its fixed sample size counterpart for most shift sizes. Khatun *et al.* [48] investigated the statistical performance of one-sided chart for monitoring the multivariate CV in short production runs. They found out that the upward chart is faster in detecting the same shift size of the multivariate CV compared to its downward counterpart.

In most practical applications, production processes are set up to produce quantities over a short time period. In such cases, research work has to consider the design of control charts with a short run context since the duration of the production runs is limited. Furthermore, the performance of control charts in the presence of measurement errors significantly differs from their counterparts without measurement errors. Therefore, the goal of this study is to investigate the statistical performance of the CV chart for short production runs in the presence of measurement errors. The numerical results show that the measurement errors influence the performance of the CV chart for short production runs.

II. COEFFICIENT OF VARIATION CHART FOR SHORT PROUCTION RUNS

When monitoring the CV in a process, samples $\{X_1, X_2, \ldots\}$ $..., X_n$ of size *n* are selected, then the sample CV, that is $\hat{\gamma} = \hat{S}/\bar{X}$ is plotted on the CV chart, where $\vec{X} = \sum_{j=1}^{n} X_j/n$ is the sample mean and $S = \sqrt{\sum_{j=1}^{n} (X_j - \bar{X})^2 / (n-1)}$ is the sample standard deviation. Let F_t^{-1} (*i*|*n* − 1, $\sqrt{}$ \overline{n}/γ_0) be the inverse of the cumulative function of the non-central *t* distribution with (*n*−1) degrees of freedom and noncentrality α as a model of $(n-1)$ degrees of freedom and noncentrality parameter \sqrt{n}/γ_0 , where γ_0 is the in-control target CV value. We investigate a two-sided CV chart in this study, where the detection of increasing and decreasing shifts is of equal importance. Let *p* be the desired false alarm probability of the CV chart, then the lower control limit and the upper control limit of the CV chart are calculated as

$$
LCL = F_{\gamma}^{-1} \left(\frac{p}{2} \mid n, \gamma_0 \right) \tag{1}
$$

and

UCL =
$$
F_{\gamma}^{-1} \left(1 - \frac{p}{2} \mid n, \gamma_0 \right)
$$
, (2)

respectively, where $F_{\gamma}^{-1}(\omega | n, \gamma_0) = \sqrt{\frac{\gamma_0^2}{n}}$ ively, where $F_{\gamma}^{-1}(\omega \mid n, \gamma_0) = \sqrt{n}/(F_t^{-1}(1 - \omega))$ $n-1, \sqrt{n}/\gamma_0$).

We assume that a small lot of *N* parts has been produced during a production having finite length *H* and there are *I* number of scheduled inspections within the production horizon *H*. Let $F_\gamma(x|n, \gamma) = 1 - F(\sqrt{n}/x|n-1, \sqrt{n}/\gamma)$ and $\gamma =$ $\tau \gamma_0$, where τ is the shift in the CV and $F(\sqrt{n}/x|n-1, \sqrt{n}/\gamma)$ is the cumulative function of the non-central *t* distribution with $(n - 1)$ degrees of freedom and noncentrality parameter \sqrt{n}/γ , then the truncated average run length of the CV chart for short production runs is TARL = $(1 - \beta^{I+1})/(1 - \beta)$, where $\beta = F_{\nu}(\text{UCL}|n, \gamma) - F_{\nu}(\text{LCL}|n, \gamma)$. Note that $\tau = 1$ when the process is in-control, while $\tau \neq 1$ when the process is out-of-control. Values of $0 < \tau < 1$ correspond to the decrease in the CV, while values of $\tau > 1$ correspond to the increase in the CV.

III. STATISTICAL DESIGN OF COEFFICIENT OF VARIATION CHART WITH MEASUREMENT ERRORS FOR SHORT PRODUCTION RUNS

Recently, Tran *et al.* [24] investigated the effect of measurement errors on a two-sided CV chart in which the production horizon is considered as infinite. However, the production horizon in many situations is very short, that is few hours or

few days and this is considered as finite. Consequently, this paper extends the work in Tran *et al.* [24] to study the effect of measurement errors on the CV chart in a short run context.

In this study, we investigate a two-sided CV chart for short production runs with measurement errors in detecting both increasing and decreasing shifts. We use the linear covariate error model in Tran *et al.* [24] to evaluate the performance of the two-sided CV chart for short production runs. A set of samples $\{X_{i,1}, X_{i,2}, \ldots, X_{i,n}\}\$ is selected at the *i*th sampling point, for $i = 1, 2, \ldots$, where $X_{i,j}$ follows a normal distribution with mean $\mu_0 + \delta_1 \sigma_0$ and standard deviation $\delta_2 \sigma_0$. Here, μ_0 is the in-control mean, σ_0 is the in-control standard deviation, δ_1 is the shift size of the mean and δ_2 is the shift size of the standard deviation.

To reduce the effect of measurement errors, many researchers suggested taking multiple measurements on each item. Therefore, the true value of *Xi*,*^j* in the linear covariate error can be accessible via $\{X_{i,j,1}^{\#}, X_{i,j,2}^{\#}, \ldots, X_{i,j,m}^{\#}\}$, where *m* is the number of multiple measurements per item in each sample. As suggested by Tran *et al.* [24], the linear covariate error model is given as

$$
X_{i,j,k}^{\#} = G + H X_{i,j} + E_{i,j,k},
$$
\n(3)

where $X_{i,j,k}^{\#}$ is the value observed for the *k*th measurement of the *j*th item at the *i*th sampling point; *G* and *H* are the known constant coefficients of the error model; $E_{i,j,k}$ is the random error following a normal distribution with mean 0 and standard deviation σ*EM* .

The mean of *m* measurements for the *j*th item at the *i*th sampling point is

$$
\bar{X}_{i,j}^{\#} = \frac{1}{m} \sum_{k=1}^{m} X_{i,j,k}^{\#} = G + H X_{i,j} + \frac{1}{m} \sum_{k=1}^{m} E_{i,j,k}, \qquad (4)
$$

where $\bar{X}_{i,j}^{\#}$ follows a normal distribution with mean $\mu_E = G +$ *H*($\mu_0 + \delta_1 \sigma_0$) and variance $\sigma_E^2 = H_{\frac{1}{2} \pi}^2 \delta_0^2 + \sigma_{EM}^2 / m$ (Tran *et al.* [24]). Consequently, the CV of $\bar{X}_{i,j}^{\#}$ is defined as

$$
\gamma_E = \frac{\sigma_E}{\mu_E} = \frac{\sqrt{H^2 \delta_2^2 \sigma_0^2 + \sigma_{EM}^2/m}}{G + H(\mu_0 + \delta_1 \sigma_0)}.
$$
 (5)

Let $\zeta = \frac{\sigma_{EM}}{\sigma_0}$ be the precision error, $\gamma_0 = \frac{\sigma_0}{\mu_0}$ be the in-control target CV value and $\rho = G/\mu_0$ be the accuracy error, then (5) becomes

$$
\gamma_E = \frac{\sqrt{H^2 \delta_2^2 + \varsigma^2 / m}}{\rho + H(1 + \delta_1 \gamma_0)} \times \gamma_0.
$$
 (6)

Based on this equation, the in-control CV value with measurement errors is computed as

$$
\gamma_{E0} = \frac{\sqrt{H^2 + \varsigma^2/m}}{H + \rho} \times \gamma_0 \tag{7}
$$

corresponding to $\delta_1 = 0$ and $\delta_2 = 1$, while the out-of-control CV value with measurement errors is computed as

$$
\gamma_{E1} = \frac{\sqrt{H^2 \delta_2^2 + \varsigma^2 / m}}{\frac{H \delta_2}{\tau} + \rho} \times \gamma_0 \tag{8}
$$

corresponding to $\delta_1 \neq 0$ and $\delta_2 \neq 1$ [24]. Note that $H = 1$, $\zeta = 0$ and $\rho = 0$ if the measurement errors are not taken into consideration. According to Tran *et al.* [24], the sample CV with measurement errors at the *i*th sampling point is defined as

$$
\hat{\gamma}_{E i} = \frac{S_i^{\#}}{\bar{X}_i^{\#}},\tag{9}
$$

where $\bar{\bar{X}}_i^{\#} = \frac{1}{n} \sum_{i=1}^n$ *j*=1 $\bar{X}_{i,j}^{\#}$ is the sample mean of $\bar{X}_{i,j}^{\#}$ and $S_i^{\#}$ = $\sqrt{\sum_{i=1}^n S_i^{\#}}$ *j*=1 $(\bar{X}_{i,j}^{\#} - \bar{\bar{X}}_i^{\#})^2/(n-1)$ is the sample standard

deviation of $\bar{X}_{i,j}^{\#}$.

Let $F_t^{-1}(\cdot|n-1, \sqrt{n})$ \overline{n}/γ_{E0}) be the inverse of the cumulative function of the noncentral *t* distribution with $(n - 1)$ degrees function of the noncentral *t* distribution with $(n - 1)$ degrees of freedom and noncentrality parameter $\sqrt{n}/γ_{E0}$ and let α be the type I error probability of the CV chart with measurement errors, then the lower control limit and the upper control limit of the CV chart with measurement errors are calculated as

$$
LCL = F_{\gamma_E}^{-1} \left(\frac{\alpha}{2} \middle| n, \gamma_{E0} \right)
$$
 (10)

and

$$
\text{UCL} = F_{\gamma_E}^{-1} \left(1 - \frac{\alpha}{2} \middle| n, \gamma_{E0} \right),\tag{11}
$$

respectively, where $F_{\gamma_E}^{-1}(c|n, \gamma_{E0}) = \sqrt{\frac{c_1^2}{c_1^2}}$ spectively, where $F_{\gamma_E}^{-1}(c|n, \gamma_{E0}) = \sqrt{n}/F_t^{-1}(1 - c|n - \gamma_{E0})$ $1, \sqrt{n}/\gamma_{E0}$).

Let $F_{\gamma_E}(x|n, \gamma_E) = 1 - F_t(\sqrt{n}/x|n-1, \sqrt{n/2})$ $x|n, \gamma_E$) = 1 – $F_t(\sqrt{n}/x|n-1, \sqrt{n}/\gamma_E)$, where $F_t(\cdot|n-1, \sqrt{n}/\gamma_E)$ is the cumulative function of the noncentral *t* distribution with (*n*− 1) degrees of freedom and non-The *t* distribution with $(n-1)$ degrees of freedom and non-
centrality parameter \sqrt{n}/γ_E . In general, the truncated average run length of control charts for short production runs [31] is computed as

$$
\text{TARL} = \frac{1 - \beta^{I+1}}{1 - \beta},\tag{12}
$$

where $\beta = F_{\gamma_E}(\text{UCL}|n, \gamma_E) - F_{\gamma_E}(\text{LCL}|n, \gamma_E)$ for the CV chart with measurement errors. Here, TARL $=$ TARL₀ for γ_E = γ_{E0} when the process is in-control, while TARL = TARL₁ for $\gamma_E = \gamma_{E1}$ when the process is out-of-control (i.e. $0 < \tau < 1$ for the decreasing shift or $\tau > 1$ for the increasing shift), where $TARL_0$ and $TARL_1$ are the in-control and outof-control TARLs, respectively. Once the UCL and LCL (i.e. chart parameters) are defined based on [\(10\)](#page-3-0) and (11), respectively for the given values of H , m , ρ , ζ , γ_0 and n such that TARL₀ = *I*, then the TARL₁ value can be numerically evaluated for a given shift size τ .

When the shift size is not deterministic, especially when a specific shift size cannot be determined a priori, then the performance of the CV chart for a short run context can be measured by the out-of-control expected value of the TARL as follows:

$$
\text{ETARL}_1 = \int_{\Omega} \text{TARL}_1 \times f(\tau) d\tau,\tag{13}
$$

TABLE 1. LCL and UCL of the CV Chart for Short Production Runs when H = 1, m = 1, ρ = 0.05 and I = 50 for Different Values of ς .

TABLE 2. LCL and UCL of the CV Chart for Short Production Runs when H = 1, m = 1, ς **= 0.28 and I = 50 for Different Values of** ρ **.**

where $f(\tau) = 1/(b - a)$ is a uniform distribution giving equal weight to each shift size within the interval Ω \in [*a*, *b*] and the in-control ETARL (ETARL₀) is equal to TARL₀. When designing the CV chart based on ETARL, the UCL and LCL (i.e. chart parameters) are determined using [\(10\)](#page-3-0) and (11), respectively such that $ETARL₀ = TARL₀ = I$. Then, the ETARL₁ value can be numerically evaluated for a range of shift sizes between *a* and *b*.

Note that the sensitivity of the CV chart in shift detection increases with a decreasing value of $TARL₁$ (or $ETARL₁$) for a given value of $TARL₀$ (or $ETARL₀$). Therefore, the CV chart performs better with a smaller value of $TARL₁$ (or ETARL₁), indicating a smaller value of TARL₁ (or ETARL₁) decreases the negative effect of the measurement errors on the TARL₁ (or ETARL₁) performance of the CV chart for short production runs.

IV. EFFECT OF MEASUREMENT ERRORS ON THE STATISTICAL PERFORMANCE OF COEFFICIENT OF VARIATION CHART FOR SHORT PRODUCTION RUNS

In this section, we investigate the effect of measurement errors on the performance of the CV chart for short production runs using the linear covariate error model discussed in the previous section. Tables 1-4 present the values of LCL and UCL, in which the CV chart is designed such that $TARL₀ =$ $I = 50, n \in \{5, 10, 15\}$ and $\gamma_0 \in \{0.05, 0.1, 0.2\}$, for different parameter combinations of measurement errors (i.e. ζ , ρ , *H* and *m*). According to Tran *et al.* [24], it is assumed that $\delta_2 = 1$ without loss of generality.

Tables 5-8 provide the values of $TARL₁$ of the CV chart with short production runs when $\gamma_0 \in \{0.05, 0.1, 0.2\}$, *n* \in {5, 10, 15} and $\tau \in$ {0.5, 0.7, 0.8, 1.3, 1.5, 2.0} for different parameter combinations of measurement errors

γ_{0}	\boldsymbol{n}		$H=1$	$H=2$	$H = 3$	$H = 4$	$H = 5$
0.05	5	LCL	0.005894	0.005871	0.005888	0.005901	0.005910
		UCL	0.112438	0.111992	0.112308	0.112560	0.112741
	10	LCL	0.015788	0.015726	0.015770	0.015804	0.015830
		UCL	0.090911	0.090552	0.090807	0.091009	0.091155
	15	LCL	0.021264	0.021181	0.021240	0.021287	0.021321
		UCL	0.082453	0.082128	0.082359	0.082542	0.082674
0.1	5	LCL	0.011763	0.011717	0.011750	0.011776	0.011794
		UCL	0.227834	0.226908	0.227565	0.228087	0.228464
	10	LCL	0.031494	0.031371	0.031458	0.031527	0.031577
		UCL	0.183326	0.182590	0.183112	0.183527	0.183826
	15	LCL	0.042421	0.042255	0.042373	0.042466	0.042533
		UCL	0.165954	0.165291	0.165762	0.166135	0.166405
0.2	5	LCL	0.023329	0.023239	0.023303	0.023353	0.023390
		UCL.	0.481540	0.479359	0.480906	0.482136	0.483024
	10	LCL	0.062358	0.062119	0.062288	0.062423	0.062520
		UCL	0.379251	0.377625	0.378779	0.379696	0.380357
	15	LCL	0.084012	0.082509	0.082002	0.081747	0.084230
		UCL	0.340550	0.333883	0.331638	0.330511	0.341523

TABLE 3. LCL and UCL of the CV Chart for Short Production Runs when m = 1, ς **= 0.28,** ρ **= 0.05 and I = 50 for Different Values of H.**

TABLE 4. LCL and UCL of the CV Chart for Short Production Runs when H = 1, ς **= 0.28,** ρ **= 0.05 and I = 50 for Different Values of m.**

γ_{0}	\boldsymbol{n}		$m=1$	$m = 3$	$m = 5$	$m = 7$	$m=10$
0.05	5	LCL	0.005894	0.005750	0.005721	0.005708	0.005698
		UCL	0.112438	0.109656	0.109092	0.108849	0.108667
	10	LCL	0.015788	0.015401	0.015322	0.015289	0.015263
		UCL	0.090911	0.088669	0.088214	0.088018	0.087871
	15	LCL	0.021264	0.020743	0.020637	0.020592	0.020558
		UCL	0.082453	0.080422	0.080010	0.079833	0.079699
0.1	5	LCL	0.011763	0.011476	0.011418	0.011393	0.011374
		UCL	0.227834	0.222055	0.220883	0.220379	0.220001
	10	LCL	0.031494	0.030726	0.030570	0.030503	0.030453
		UCL	0.183326	0.178732	0.177801	0.177400	0.177099
	15	LCL	0.042421	0.041387	0.041177	0.041086	0.041018
		UCL	0.165954	0.161816	0.160977	0.160616	0.160344
0.2	5	LCL	0.023329	0.022769	0.022655	0.022606	0.022569
		UCL	0.481540	0.467981	0.465247	0.464072	0.463190
	10	LCL	0.062358	0.060867	0.060564	0.060433	0.060335
		UCL	0.379251	0.369124	0.367076	0.366196	0.365535
	15	LCL	0.084012	0.082002	0.081593	0.081417	0.081285
		UCL	0.340550	0.331638	0.329833	0.329058	0.328475

(i.e. ς , ρ , *H* and *m*) based on the values of LCL and UCL in Tables 1-4, where $TARL₀ = I = 50$. From the numerical results in Tables 5-8, it can be noticed that when the shift size τ decreases (when $0 < \tau < 1$) or increases (when τ > 1) for the fixed values of *m*, *n*, *ς*, *ρ*, *H*, γ_0 and I , the value of TARL₁ decreases. The tables also show that the value of $TARL₁$ decreases when the sample size *n* increases for the fixed values of *m*, *n*, ζ , ρ , *H*, γ_0 and *I*.

Table 5 presents the $TARL₁$ values for different combinations of the precision error $\zeta \in \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$ when $H = 1$, $m = 1$, $\rho = 0.05$ and $I = 50$ with $\gamma_0 \in \{0.05,$ $0.1, 0.2$, $n \in \{5, 10, 15\}$ and $\tau \in \{0.5, 0.7, 0.8, 1.3, 1.5, 2.0\}$. It can be noticed from Table 5 that as the value of ζ increases for the fixed values of ς , ρ , H , I , γ_0 , n and τ , the negative effect of the measurement errors on the $TARL₁$ performance of the CV chart for short production runs increase. This is

to say, the increase in ζ value reduces the sensitivity of the CV chart. For instance, when $H = 1$, $m = 1$, $\rho = 0.05$, $I =$ 50, $n = 5$, $\gamma_0 = 0.1$ and $\tau = 1.5$, we have TARL₁ = 19.26, 19.28, 19.32, 19.40, 19.50 and 19.64, respectively for $\zeta = 0$, 0.2, 0.4, 0.6, 0.8 and 1.0.

Table 6 presents the $TARL₁$ values for different combinations of the accuracy error $\rho \in \{0, 0.01, 0.02, 0.03, 0.04,$ 0.05} when $H = 1$, $m = 1$, $\zeta = 0.28$ and $I = 50$ with $\gamma_0 \in$ $\{0.05, 0.1, 0.2\}, n \in \{5, 10, 15\}$ and $\tau \in \{0.5, 0.7, 0.8, 1.3,$ 1.5, 2.0}. According to Tran *et al.* [24], an acceptable value for the signal-to-noise ratio is at $\zeta = 0.28$. Table 6 shows that the ρ value gives negative influence on the TARL₁ performance of the CV chart for short production runs. In other words, the larger the value of ρ in the linear covariate error model, the larger the value of $TARL₁$. For example, for $m = 1, H = 1, \varsigma = 0.28, I = 50, n = 10, \gamma_0 = 0.2$ and $\tau = 0.5$, the TARL₁ values of the CV chart with linear

TABLE 5. TARL₁ of the CV Chart for Short Production Runs when H = 1, m = 1, ρ = 0.05 and I = 50 for Different Values of ς.

TABLE 6. TARL¹ of the CV Chart for Short Production Runs when H = 1, m = 1, ς = 0.28 and I = 50 for Different Values of ρ.

	$\Omega =$	$\overline{0}$	0.01	0.02	0.03	0.04	0.05	θ	0.01	0.02	0.03	0.04	0.05	Ω	0.01	0.02	0.03	0.04	0.05	
\boldsymbol{n}		$\chi = 0.05$					$\chi_0 = 0.1$							$\chi_0 = 0.2$						
\mathcal{D}	0.5	44.06	44.18	44.29	44.40	44.51	44.62	44.10	44.22	44.33	44.44	44.55	44.65	44.27	44.38	44.49	44.59	44.69	44.79	
	0.7	48.99	49.01	49.03	49.05	49.08	49.10	49.00	49.02	49.04	49.06	49.08	49.10	49.03	49.05	49.08	49.10	49.11	49.13	
	0.8	49.79	49.80	49.81	49.82	50.19	49.84	49.80	49.81	49.82	49.82	49.83	49.84	49.81	49.82	49.83	49.84	49.85	49.85	
	1.3	34.71	35.08	35.45	35.80	36.14	36.47	34.92	35.29	35.65	35.99	36.32	36.65	35.80	36.13	36.45	36.76	37.06	37.35	
	1.5	16.42	16.94	17.45	17.97	18.48	19.00	16.74	17.25	17.76	18.28	18.78	19.29	18.09	18.58	19.07	19.56	20.05	20.53	
	2.0	3.67	3.81	3.95	4.10	4.25	4.40	3.77	3.91	4.06	4.20	4.36	4.51	4.22	4.36	4.51	4.67	4.82	4.99	
10	0.5	14.26	14.65	15.05	15.45	15.85	16.25	14.45	14.84	15.24	15.64	16.04	16.44	15.20	15.59	15.99	16.38	16.78	17.17	
	0.7	43.34	43.49	43.64	43.78	43.92	44.05	43.42	43.57	43.71	43.85	43.99	44.12	43.74	43.87	44.00	44.13	44.25	44.37	
	0.8	48.06	48.10	48.14	48.19	48.23	48.26	48.08	48.13	48.17	48.21	48.25	48.28	48.18	48.22	48.26	48.29	48.33	48.36	
	1.3	23.47	24.00	24.53	25.05	25.55	26.06	23.87	24.39	24.91	25.42	25.91	26.40	25.47	25.94	26.42	26.88	27.34	27.78	
	1.5	7.04	7.31	7.59	7.87	8.16	8.46	7.26	7.53	7.81	8.10	8.40	8.70	8.20	8.48	8.77	9.07	9.38	9.69	
	2.0	1.74	1.79	1.84	1.89	1.95	2.00	1.79	1.84	1.89	1.94	2.00	2.05	1.99	2.04	2.10	2.15	2.21	2.27	
15	0.5	3.79	3.90	4.01	4.13	4.26	4.38	3.84	3.96	4.08	4.20	4.32	4.44	4.08	4.20	4.32	4.45	4.57	4.70	
	0.7	33.15	33.52	33.88	34.23	34.57	34.91	33.37	33.73	34.09	34.43	34.77	35.09	34.20	34.54	34.86	35.18	35.49	35.79	
	0.8	45.30	45.41	45.51	45.62	45.71	45.81	45.37	45.47	45.58	45.68	45.77	45.86	45.62	45.72	45.81	45.90	45.99	49.44	
	1.3	15.79	16.29	16.79	17.29	17.80	18.30	16.19	16.69	17.19	17.69	18.19	18.68)	17.84	18.31	18.79	19.27	19.75	20.22	
	1.5	4.14	4.29	4.45	4.61	4.77	4.94	4.27	4.42	4.58	4.75	4.92	5.09	4.84	5.00	5.17	5.34	5.52	5.71	
	2.0	.28	1.31	1.33	1.35	1.38	1.41	1.31	1.33	1.36	1.38	1.41	1.44	1.41	1.44	1.47	1.49	1.52	1.55	

TABLE 7. TARL₁ of the CV Chart for Short Production Runs when m = 1, ς = 0.28, ρ = 0.05 and I = 50 for Different Values of H.

covariate error model are obtained as 15.20, 15.59, 15.99, 16.38, 16.78 and 17.17 when $\rho = 0, 0.01, 0.02, 0.03, 0.04$ and 0.05, respectively.

Table 7 presents the $TARL₁$ values for different combinations of the constant coefficient $H \in \{1, 2, 3, 4, 5\}$ when $\zeta = 0.28$, $m = 1$, $\rho = 0.05$ and $I = 50$ with $\gamma_0 \in \{0.05,$

	$m =$		3	5		10		3	5		10		3	5		10	
n	τ	$\chi = 0.05$							$\chi = 0.1$			$\chi = 0.2$					
5	0.5	44.62	44.62	44.62	44.62	44.62	44.65	44.65	44.65	44.65	44.65	44.79	44.79	44.78	44.78	44.78	
	0.7	49.10	49.10	49.10	49.10	49.10	49.10	49.10	49.10	49.10	49.10	49.13	49.13	49.13	49.13	49.13	
	0.8	49.84	49.84	49.84	49.84	49.84	49.84	49.84	49.84	49.84	49.84	49.85	49.85	49.85	49.85	49.85	
	1.3	36.47	36.47	36.47	36.47	36.47	36.65	36.64	36.63	36.63	36.63	37.35	37.31	37.30	37.29	37.29	
	1.5	19.00	18.99	18.99	18.99	18.99	19.29	19.27	19.27	19.27	19.27	20.53	20.45	20.43	20.43	20.42	
	2.0	4.40	4.40	4.40	4.40	4.40	4.51	4.50	4.50	4.50	4.50	4.99	4.95	4.95	4.94	4.94	
10	0.5	16.25	16.25	16.25	16.25	16.24	16.44	16.42	16.42	16.42	16.42	17.17	17.12	17.11	17.11	17.11	
	0.7	44.05	44.05	44.05	44.05	44.05	44.12	44.11	44.11	44.11	44.11	44.37	44.36	44.35	44.35	44.35	
	0.8	48.26	48.26	48.26	48.26	48.26	48.28	48.28	48.28	48.28	48.28	48.36	48.36	48.36	48.36	48.36	
	1.3	26.06	26.05	26.05	26.05	26.05	26.40	26.38	26.38	26.38	26.37	27.78	27.70	27.68	27.67	27.66	
	1.5	8.46	8.46	8.46	8.46	8.46	8.70	8.68	8.68	8.68	8.68	9.69	9.62	9.61	9.60	9.60	
	2.0	2.00	2.00	2.00	2.00	2.00	2.05	2.05	2.05	2.05	2.05	2.27	2.26	2.25	2.25	2.25	
15	0.5	4.38	4.38	4.38	4.38	4.38	4.44	4.44	4.44	4.44	4.44	4.70	4.69	4.68	4.68	4.68	
	0.7	34.91	34.91	34.90	34.90	34.90	35.09	35.08	35.08	35.08	35.08	35.79	35.75	35.74	35.74	35.74	
	0.8	45.81	45.81	45.81	45.81	45.81	45.86	45.86	45.86	45.86	45.86	46.07	46.06	46.06	46.06	46.06	
	1.3	18.30	18.29	18.29	18.29	18.29	18.68	18.66	18.65	18.65	18.65	20.22	20.12	20.10	20.09	20.09	
	1.5	4.94	4.94	4.94	4.94	4.94	5.09	5.08	5.08	5.08	5.08	5.71	5.66	5.66	5.65	5.65	
	2.0	1.41	1.41	1.41	1.41	1.41	1.44	1.43	1.43	1.43	1.43	1.55	1.55	1.55	1.54	1.54	

TABLE 8. TARL₁ of the CV Chart for Short Production Runs when H = 1, ζ = 0.28, ρ = 0.05 and I = 50 for Different Values of m.

0.1, 0.2}, $n \in \{5, 10, 15\}$ and $\tau \in \{0.5, 0.7, 0.8, 1.3, 1.5,$ 2.0}. The larger *H* value leads to the decrease of TARL₁ value. This indicates that the increase of *H* value enhances the performance of the CV chart. For example, $TARL_1 = 19.29$, 18.00, 17.57, 17.35 and 17.22 for *B* = 1, 2, 3, 4 and 5, respectively at $\tau = 1.5$ when $\gamma_0 = 0.1$, $n = 5$, $\zeta = 0.28$, $m = 1$, $\rho = 0.05$ and $I = 50$.

Table 8 presents the $TARL₁$ values for different combinations of the number of multiple measurements per item in each sample $m \in \{1, 3, 5, 7, 10\}$ when $\zeta = 0.28$, *H* = 1, ρ = 0.05 and *I* = 50 with $\gamma_0 \in \{0.05, 0.1, 0.2\}$, $n \in \{5, 10, 15\}$ and $\tau \in \{0.5, 0.7, 0.8, 1.3, 1.5, 2\}$. The results in Table 8 indicates that the decrease of $TARL₁$ is insignificant as the *m* value increase, in which the value of TARL₁ reduces negligibly or remains unchanged as the value of *m* increases. For instance, if we consider the case $n = 15$, $\gamma_0 = 0.1, \tau = 1.3, \zeta = 0.28, H = 1, \rho = 0.05 \text{ and } I = 50,$ the TARL₁ values are 18.68, 18.66, 18.65, 18.65 and 18.65 for $m = 1, 3, 5, 7$ and 10, respectively.

Fig. 1, 2, 3 and 4 provide the impact of parameters for measurement errors ς , ρ , *H* and *m*, respectively on the overall performance of the CV chart for short production runs when $n \in \{5, 10, 15\}$ for $\gamma_0 = 0.1$ and ETARL₀ = TARL₀ = $I = 50$. Fig. 1 and 2 show that the parameters ζ and ρ significantly affect the performance of the CV chart, in which larger values of ς and ρ increase the value of ETARL₁. Fig. 3 shows that increasing the value of parameter *H* improves the efficiency of the CV chart, while Fig. 4 shows that the *m* value does not significantly affect the performance of the CV chart.

V. COEFFICIENT OF VARIAION CHART WITH MEASUREMENT ERRORS FOR SHORT PRODUCTION RUNS BASED ON ECONOMIC CRITERION

During a finite horizon process with length of I, N parts are scheduled to be produced. The parts will be loaded and

FIGURE 1. ETARL₁ of the CV chart for short production runs when $\gamma_0 = 0.1$, $H = 1$, $m = 1$, $\rho = 0.05$ and $I = 50$ for different values of ς .

worked within a workstation at one stage of the process and then released to an adjacent inspection area individually or in small groups of pallet size B. Let T be the number of

FIGURE 2. ETARL₁ of the CV chart for short production runs when $\gamma_0 = 0.1$, $H = 1$, $m = 1$, $\varsigma = 0.28$ and $I = 50$ for different values of ρ .

scheduled inspection within the production horizon I, then the last inspection is scheduled at the end of the production run and the sampling interval between two consecutive inspections is $h = I/(T + 1)$ (Celano *et al.* [35]).

The occurrence of an assignable cause when the process is shifted to an out-of-control state for the short production runs is assumed to be Poisson distributed with an exponentially distributed inter-arrival time having the mean of 1/v, where ν is the failure rate. In general, the parameters for the economic design of a control chart are the inspection interval (*h*), the rate of production (*rPR*), the rate of inspection (*R*), the failure rate (*v*), the production horizon (*I*), the total number of inspection during production horizon (*T*), the demand of parts during production horizon (*N*), the pallet size (B) , the fixed inspection cost (f) , the hourly cost of the inspection resource (c_{LR}) , the out-of-control loss rate (W) , the fixed set-up cost (c_F) , the cost per false alarm (L_0) and the cost of search and restoration (L_1) . In this study, $\alpha_0 =$ $1 - F_{\gamma_E}(\text{UCL}|n, \gamma_{E0}) + F_{\gamma_E}(\text{LCL}|n, \gamma_{E0})$ is the type I error

FIGURE 3. ETARL₁ of the CV chart for short production runs when $\gamma_0 = 0.1$, $\varsigma = 0.28$, $m = 1$, $\rho = 0.01$ and $I = 50$ for different values of H.

probability of the CV chart and $\alpha_1 = 1 - F_{\gamma_E}(\text{UCL}|n, \gamma_{E1}) +$ F_{γ_E} (LCL|*n*, γ_{E1}) is the power of the CV chart. The expected inspection cost $E(TC)$ of the CV control chart for short production runs is given as follows:

$$
E(TC) = C_1 + C_2 + C_3 + C_4 + C_5, \tag{14}
$$

where $C_1 = c_F [N/B]$ is the work-holding set-up cost with $\lceil x \rceil$ denotes the smallest integer greater than or equal to x; $C_2 = (f + c_{LR}n/R)$ *T* is the sampling cost; $C_3 = C_{31} + C_{32}$ is the expected out-of-control production cost, where $C_{31} =$ *Wh*(1 – e^{-vh}) $\sum_{n=1}^{T-2}$ *i*=0 $F(i) \left\{ \frac{1-\alpha_1}{\alpha_1} \left[1 - (1 - \alpha_1)^{T-1-i} \right] \right\}$ and $C_{32} = W\left(\frac{vh-1+e^{-vh}}{v}\right)$ $\frac{1+e^{-\nu h}}{\nu}\bigg\}\sum_{n=0}^{T-1}$ $\sum_{i=0}^{T-1} F(i); \ C_4 = L_0 \alpha_0 e^{-\nu h} \sum_{i=0}^{T-1}$ *i*=0 *F*(*i*) is the expected cost of false alarms; and $C_5 = \overline{L_1}(1$ $e^{-\nu h}$) $\sum_{i=1}^{T-1} F(i)[1 - (1 - \alpha_1)^{T-i}]$ is the expected search and restoration cost. Here, $F(0) = 1$ and $F(i) = F(i -$ 1)[$e^{-\nu h}$ + (1 − $e^{-\nu h}$)α₁] + [1 − *F*(*i* − 1)]α₁. The detailed

FIGURE 4. ETARL₁ of the CV chart for short production runs when $\gamma_0 = 0.1$, $H = 1$, $\varsigma = 0.28$, $\rho = 0.01$ and $I = 50$ for different values of m.

derivations of C_1 , C_2 , C_3 , C_4 and C_5 are presented in Celano *et al.* [35].

In the study, when designing the CV chart with short production runs based on the economic criterion, the values of *B*, *T* , *n*, UCL and LCL should be selected by minimizing the expected inspection cost in (14), subject to the following constraints: (i) $\alpha_0 < 0.01$ is the statistical constraint for the economic optimization, in order to limit the expected number of false alarms issued by the CV chart; (ii) $0 < T \leq \lfloor N/B \rfloor$ is related to the maximum allowable number of inspections to be scheduled during the rolling horizon; (iii) $0 < n < |N/T|$ is related to the maximum allowable sample size; and (iv) $0 < B < 20$ is related to the maximum pallet size, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

In this section, the effect of measurement errors on the parameters for the economic design of the CV chart with short production runs is investigated. The values of the parameters for this design based on the economic criterion are $v = 0.01$, $L_0 = 10$, $r_{PR} = 1$, $R = 30$, $c_F = 5$, $f = 0$, $c_{LR} = 20$ and $W = 100$, where $N = r_{PR}I$ and $L_1 = 1.5L_0$.

Fig. 5, 6, 7 and 8 provide the impact of ς , ρ , *H* and *m*, respectively on the economic performance of the CV chart for short production runs when $I = 50$, $\gamma_0 = 0.1$ and $\tau = 2.0$. Fig. 5 shows that the parameter ζ affects the economic performance of the CV chart, in which a larger value of ς slightly reduces the value of the expected inspection cost *E*(*TC*). Fig. 6, 7 and 8 show that the values of ρ , *H* and *m*, respectively, do not significantly affect the *E*(*TC*) value.

VI. ILLUSTRATIVE EXAMPLE

Castagliola *et al.* [38] provided an example for the implementation of the CV chart in short production runs. This example

FIGURE 5. $E(TC)$ of the CV chart for short production runs when $y_0 = 0.1$, $H = 1$, $m = 1$ and $\rho = 0.05$ for different values of ς .

FIGURE 6. $E(TC)$ of the CV chart for short production runs when $y_0 = 0.1$, $H = 1$, $m = 1$ and $\varsigma = 0.28$ for different values of ρ .

FIGURE 7. $E(TC)$ of the CV chart for short production runs when $y_0 = 0.1$, $\varsigma = 0.28$, $m = 1$ and $\rho = 0.01$ for different values of H.

FIGURE 8. $E(TC)$ of the CV chart for short production runs when $y_0 = 0.1$, $H = 1$, $\varsigma = 0.28$ and $\rho = 0.01$ for different values of m.

considers actual data from a die casting hot chamber process provided by a Tunisian company manufacturing zinc alloy parts for the sanitary section. The quality characteristic of

TABLE 9. Illustrative Example From A Die Casting Hot Chamber Process (Adopted from Nguyen et al. [22]).

interest is the weight (*X* in grams) of scrap zinc alloy material to be removed between the molding process and the continuous plating surface treatment. Then, Nguyen *et al.* [22] considered the example in Castagliola *et al.* [38] for the CV chart with variable sampling interval in the presence of measurement errors. This section illustrates the use of the CV chart for short production runs under the presence of measurement errors by considering the example in Nguyen *et al.* [22], where $\zeta = 0.28$, $\rho = 0$, $H = 1$ and $m = 1$ for the parameters of the linear covariate error model. From the Phase I data, the in-control CV is estimated as $\gamma_0 = 0.01$. The dataset with sample size of $n = 5$ in Phase II is listed in Table 9. The first 17 samples are supposed to be in-control while the last 23 subgroups are supposed to be out-of-control with an increase of 20% (i.e. $\tau = 1.2$) of the CV when designing the CV chart for a short run production of 31 hours calling for $I = 30$ inspections, that is an inspection every hour. Consequently, the out-of-control CV is determined as $\gamma_1 = \tau \gamma_0 = 1.2(0.01) = 0.012$. For detecting a shift from $\gamma_0 = 0.01$ to $\gamma_1 = 0.12$, the LCL and UCL of the CV chart are found to be 0.002947 and 0.018666 using [\(10\)](#page-3-0) and (11),

respectively. Then, the TARL₁ and $ETARL₁$ are computed as 14.38 and 13.71 using (12) and (13), respectively.

FIGURE 9. CV chart for the illustrative example.

The sample CV in Phase II are plotted on the CV chart in Fig. 9. At the 18th, 19th and 23rd sampling points, the samples are plotted above UCL, indicating the CV chart detects an out-of-control situation at each of these sampling points. This confirming the occurrence of an assignable cause for each of these sampling points and corrective actions will be taken to bring the process back to the in-control situation.

VII. CONCLUSION

This study evaluates the statistical performance of the CV chart with measurement errors for short production runs. The performance measures used to investigate the performance of the CV chart are TARL and ETARL. The effect of measurement errors on the performance of the CV chart for short production runs is studied by assuming a linear covariate error model.

From the numerical results, it can be noticed that measurement errors affect the performance of the CV chart in detecting the out-of-control situation for short production runs. The performance of the CV chart for short production runs deteriorates when both the precision and accuracy errors increase. In addition, increasing the constant coefficient *B* in the linear covariate error model can reduce the negative effect of measurement errors on the CV chart for short production runs. Although using multiple measures per item is a common approach to compensate the effect of measurement errors but the results in this study show that the efficiency of the CV chart for short production runs is not reduced significantly by increasing the number of multiple measurements per item in each sample. Furthermore, a lower expected inspection cost is expected when the precision error value increases based on the economic criterion.

The property of the CV chart for short production runs with linear covariate error model developed in this study is under the assumption that the observations are normally distributed. Thus, this study can be extended to the CV chart under non-normality for future work. Furthermore, advanced strategies such as synthetic-type chart and adaptive-type chart can also be considered under the presence of measurement errors.

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