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A Novel Iterative System Identification and Modeling Scheme With Simultaneous Time-Delay and Rational Parameter Estimation

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ABSTRACT In this work, a new method is proposed to identify systems in the form of first and second-order continuous time-delayed (CTD) models where time-delay and rational parameters are estimated simultaneously. The time delay is explicitly brought into system parameters by discretizing the CTD models. The sampled input-output data from the system is used in an iterative prediction error minimization (PEM) algorithm to identify the discrete-time delayed (DTD) model from which the equivalent CTD model is extracted. The DTD model includes the fractional and integer part of time-delay, where the phase contribution due to the fractional part is used to correct the estimation of delay as well as the termination of the algorithm until the phase contribution lies within one sample time. The efficacy of the proposed method is demonstrated by the simulation study with noisy measurements for four different systems along with the experimental validation.

INDEX TERMS System identification, time-delay estimation, prediction error methods, iterative algorithms, continuous time-delayed systems, discrete time-delayed systems, robust parameter estimation, adaptive gradient optimization.

LIST OF SYMBOLS

δ	Relative tolerance
γ	Negative pure fraction
λ	Positive pure fraction
$\mathcal{L}$	Laplace transform
$\mathcal{Z}$	Z-transform
μ	A scalar quantity
ω_c	Cutoff frequency of state variable filter
τ	Time constant
θ	Parameter vector
ε	Prediction error variable
Ξ	Prediction error vector
D	Time delay
d	Positive integer
D_i	Initial time-delay
D_{max}	Upper time-delay bound
D_{min}	Lower time-delay bound
g	Gradient vector
H	Approximate Hessian matrix
I	Identity matrix
J	Jacobian matrix
K	System gain
N	Data length
T	Sample time
T_r	Transpose operator
u	Input variable
V	Objective function
w	Noise disturbance variable
x	Noise-free output variable
y	Noisy output variable

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I. INTRODUCTION

The system identification plays a vital role in estimating appropriate models from input/output data for behavior prediction, simulation modeling, monitoring and controller synthesis [1], [2]. The identification experiment on real systems is generally carried out using a digital instrument such as the computer with sampling and hold devices. It is demonstrated in [2] that the discretized models using zero-order hold (ZOH) and inputs, which are piecewise constant between sampling intervals, produce the same analytical solution as that of differential equations of the actual system. The inputs such as step, pulse, and pseudo-random binary sequence (PRBS) shows the piecewise constant property between sampling instants. A suitable choice of sample time is also essential to generate informative data for identification [2]–[5].

Time delay is an integral part of many industrial processes, which mainly occurs due to significant mass or volume transfer, energy or heat exchanges, and signal communication lags between sensors and actuators. The accurate knowledge of time delay is essential for effective controlling of time-delayed systems failing to, which can cause severe performance degradation and instability in the system. An excellent comparative survey on various time delay estimation techniques for linear systems has been reported in [6]. Another study on open problems and useful advances related to time-delayed systems can be found in [7]. In this work a technique is developed through which accurate time-delay estimation can be achieved.

In control theory, the lower order time-delayed system modeling of unknown processes having significant time delays or slower dynamics is preferred to design and tune suitable controllers based on these models [8]–[10]. It is observed that the time-delay introduces non-linearity in the cost function, which is required to be optimized for its estimation. Therefore, the gradient-based numerical optimization techniques such as steepest-descent, Newton-Raphson, Gauss-Newton, and Levenberg-Marquardt (LM) algorithms are generally being utilized in prediction error minimization (PEM) framework, especially for time-delay estimation. However, these techniques require suitable parameter initialization to avoid trapping into local optimum [1], [2], [11], [12]. The proposed work is also based on PEM approach, where it is demonstrated that by using explicitly computed time-delay one can avoid the problem of trapping to a local optimum.

The identification of both CTD [12]–[17] and DTD [19]–[22] systems are popular among researchers and have their advantages and disadvantages. The former one gives the benefit of direct physical connection with the actual system and ability to handle irregularly sampled data but has to be realized in the discrete form on digital hardware. The latter one has the advantage of direct digital implementation after estimation using the sampled data but lags the direct linking with the actual physical system in terms of its parameters. Another limitation in DTD system identification is that the time delay is generally limited to an integer multiple of sample time, which may cause some approximation errors in the estimation. Furthermore, in the last two decades, the interest has been shifted towards identifying continuous time models directly or indirectly using sampled data [23]–[27]. A novel approach is proposed here, in which all the parameters of a CTD model is identified indirectly using a DTD model, where the time delay is not restricted to an integer multiple of the sample time.

The existing literature involving time-delayed system identification uses different approaches such as: identification using relay experiments [28]–[30]; simultaneous identification of rational model parameters and time delay [15]–[17], [20]–[22]; identification of rational model parameters and time delay separately [12], [31], [32]. Although the step response based identification of time-delayed system is also popular in literature [33], [36], that is mainly due to its simplicity in performing the identification experiments. However, as pointed-out in [14] that the step inputs may not be suitable to excite all useful dynamics of the system. Therefore, inputs like PRBS, which are deterministic and show white noise like characteristics, are preferred to generate informative data for more accurate model estimation.

A recursive time delay estimation algorithm is developed in [19], where the time delay of a continuous time-delayed system is estimated based on the phase contribution of discrete-time zeros from its ZOH based sampled data system. The main limitation associated with this approach is that the delay estimation mismatch does not affect discrete-time zeros only but also affects the discrete-time poles as well. This problem can be resolved by computing the time-delay in terms of discrete-time zeros and poles, which is demonstrated in the proposed work while developing the discretization of CTD systems. Another interesting contribution, which uses step response data to identify time-delayed systems with least square estimation and ZOH discretization, is reported in [23]. Where the time-delay is identified by minimizing an error tolerance between two consecutive non-negative output samples; however, this type of approach is less useful as it often produces false estimates in the presence of noisy measurements. Moreover, most of the above CTD and DTD estimation techniques may not be robust with respect to parameter initialization and may not successfully converge for noisy measurements, whereas the proposed approach can give excellent convergence while being less sensitive towards parameter initialization and noisy measurements.

Recently, an impressive work is reported in [12] (also see [18]), where various issues related to separately identify rational parameters and time delay of a CTD system have been discussed in detail. The authors proposed a simplified refined instrument variable (SRIV) method for continuous transfer function estimation, which is referred to as the TFSRIVC algorithm, where the SRIV approach is used to estimate rational parameters and an adaptive gradient-based approach is used for time delay estimation, separately in each iteration. This method uses a state variable filter (SVF) with a suitable corner frequency to generate initial model parameters. The authors pointed out that the choice of SVF's corner frequency and initial time delay are very critical for the algorithm's global convergence and to achieve robustness against measurement noise. They also address an issue that the estimation of a CTD system with zero is somewhat more difficult and challenging as compared to all pole CTD systems. In-fact they demonstrated that the higher percentage of global convergence for CTD systems with zero could only be achieved through an exhaustive search for the time delay. The exhaustive search method is based on estimating several system models one by one for a given range of time delays and finally selecting the system model that produces a minimum error to initialize the estimation algorithm. In summary, they recommended choosing the SVF's cutoff frequency of less than one-tenth of the system bandwidth for rational parameter initialization of all pole systems and an exhaustive time delay search method for systems with zero. However, for an unknown system, it is difficult to know about its bandwidth in advance. Furthermore, the one by one time-delay search over a prespecified time delay range is computationally demanding and time-consuming.

In this work, iterative algorithms are proposed to identify the system dynamics in terms of lower-order CTD models. The proposed algorithms are based on the accurate discretized modeling of first order-CTD (FOCTD), second order-CTD (SOCTD), and SOCTD with zero (SOCTDZ) systems. These models are discretized using a ZOH circuit and a piecewise constant input, where the time delay is expressed in terms of integer and pure fractional multiple parts of sample time. The prime motivation behind using the discretization of CTD models is to explicitly bring the time-delay (more specifically, the fractional delay part) in the rational model parameters to enable simultaneous time-delay and rational parameters identification. In each iteration of the proposed algorithms, for a fixed integer delay part, rational parameters and fractional delay part are estimated and the anticipation of fractional delay part corresponding to integer multiples of sample time is added to the integer delay part for next iteration, and this process is continued until the phase due to fractional delay part lies within a sample time.

The proposed algorithm has the following major contributions and advantages:

1) Explicit expressions to compute fractional time-delay part and rational parameters are derived for FOCTD, SOCTD and, SOCTDZ systems in terms of their discrete equivalents. In fact, this is the first attempt where explicit expressions are derived to computes the fractional time-delay part for SOCTD and SOCTDZ systems.
2) Two new iterative algorithms are proposed, which enables simultaneous time-delay and rational parameter estimation.
3) Apart from algorithm termination, the identification of fractional delay part also helps the algorithms to escape local minima to reach global minima and hence makes the algorithm robust and accurate, especially in the cases of multi-model cost function and severe noise conditions.
4) It is demonstrated through simulations that the proposed approach is less sensitive for initial parameter choices, and excellent convergence can be achieved without using the exhaustive time-delay search for systems with a zero.
5) Most of the existing least square (LS) and instrument variable (IV) based methods for time-delayed system identification requires time-integral or time-derivatives of input-output data for parameter estimation whereas the proposed technique does not require such computations.

The systematic organization of the manuscript is as follows: The problem statement of the proposed work is defined in Section II. Section III is used for the detailed description of the proposed methodology and mathematical formulation for the discretization of CTD systems. The iterative algorithms for parameter estimation are developed in Section IV. Simulation results are presented in Section V, which is followed by a real process identification in Section VI. Finally, the work is concluded in Section VII.

II. PROBLEM DESCRIPTION

Consider the following general output error model representation:

$$\begin{aligned} X(s) &= G(s)U(s) \\ Y(s) &= X(s) + W(s) \end{aligned} \tag{1}$$

where $G(s)$ is the system model. The terms $U(s)$, $X(s)$, $Y(s)$ and $W(s)$ in (1) are the Laplace domain representations of the system's input $u(t)$, output $x(t)$, measured output $y(t)$, and noise disturbance $w(t)$ respectively.

The objective of the proposed work is to use the sampled data $[u(kT), y(kT); k = 1, 2, \ldots N]$ of length N, from the scheme of Fig. 1, with ZOH and at uniform sampling interval T, to estimate DTD system $G(z)$ in terms of parameters of original CTD system $G(s)$ with following assumptions:

(1) The $G(s)$ is a single-input single-output (SISO) stable system operating in open loop.
(2) The input $u(t)$ and output $x(t)$ have zero initial conditions.
(3) The disturbance signal $w(t)$ is white noise having zero mean and finite variance.
(4) The input $u(t)$ is piece-wise constant, persistently exciting signal and it is uncorrelated to $w(t)$.

Remark 1: Note that, apart from the above assumptions, one more consideration, which is typically assumed by most of the LS, IV, and PEM based approaches, that the dataset used for identification is complete. However, the presence of missing data samples in industrial datasets can not be denied, and suitable actions are required to be taken, which may require some added computation burden. The interested readers are referred to the excellent work in [34] and references therein to work in this direction.

III. FORMULATION OF DISCRETIZATION FOR PROPOSED METHOD

In this work, identification of following FOCTD, SOCTD, and SOCTDZ system model types are considered:

$$\left.\begin{aligned} G_1(s) &= \frac{K}{\tau s+1}e^{-DS}, \\ G_2(s) &= \frac{K}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ds}, \\ G_3(s) &= \frac{K(\tau_0 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ds}, \end{aligned}\right\} \begin{aligned} &\tau > 0, D \geq 0, \tau_1 \neq \tau_2, \\ &\tau_1\tau_2 > 0, (\tau_1+\tau_2) > 0, \\ &\tau_0 \neq \tau_1 \neq \tau_2 \end{aligned} \tag{2}$$

where K is the system gain, D is the time-delay, and τ's are the time constants. The CTD models in (2) are discretized according to the scheme of Fig. 1, where the corresponding mathematical relationships are developed in the following subsections.

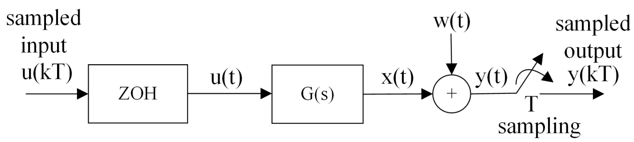

FIGURE 1. **Standard sampled-data system using sampling and ZOH devices.**

Remark 2: The proposed method can estimate all types of systems in (2) with unequal stable poles $\tau_1 \neq \tau_2$, which can also be complex-conjugate, as in the case of under-damped systems. Moreover, the proposed algorithm can also estimate systems with equal poles $\tau_1 = \tau_2$ using noisy data, which is always present in the data from real systems. However, equality does not hold for noise-free data.

A. DISCRETIZATION OF FOCTD SYSTEM

If the continuous time-delay system $G_1(s)$ in (2) is sampled using a ZOH device with transfer function:

$$H_0(s) = \frac{1-e^{-sT}}{s}, \tag{3}$$

then consider the following theorem as

Theorem 1: The modified $\mathcal{Z}$-transform to compute all the parameters of FOCTD system is:

$$\begin{aligned} &\mathcal{Z}\left[\mathcal{L}^{-1}\left\{H_0(s)\frac{K}{(\tau s+1)}e^{-Ds}\right\}\Big|_{t=kT}\right]; 0 \leq |\lambda| < 1 \\ &\quad = \frac{a_1 z^{-1} + a_2 z^{-2}}{1+b_1 z^{-1}} z^{-d}, \textit{ where} \\ &\quad K = (a_1+a_2)\,/\,(1+b_1)\,,\ \tau = -T\,/\log(-b_1), \\ &\quad \lambda = 1 + (\tau/T)\log\left(1 - a_1/K\right),\ D = dT + \lambda T. \end{aligned} \tag{4}$$

Proof: Consider a sampled transfer function $G_1(z)$, which can be computed by substituting (3) in the left-hand side of the Theorem 1, with $z = e^{sT}$ as

$$G_1(z) = (1-z^{-1})\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\frac{K}{s(\tau s+1)}e^{-Ds}\right\}\Big|_{t=kT}\right], \tag{5}$$

by applying the partial fraction in (5) we have

$$G_1(z) = (1-z^{-1})\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\left(\frac{K}{s} - \frac{K\tau}{\tau s+1}\right)e^{-Ds}\right\}\Big|_{t=kT}\right]. \tag{6}$$

If the time delay D in (6) is expressed in terms of integer and fractional multiples of sample time T as

$$D = \begin{cases} dT + \lambda T; 0 \leq \lambda < 1 \\ (d+1)T + \gamma T; -1 < \gamma \leq 0 \end{cases}, \tag{7}$$

where d is a positive integer and λ and γ are the pure positive and negative fractions respectively.

The results of following lemma and corollary are used to compute the modified $\mathcal{Z}$-transform ([2], [23], [37]) in (6), by considering $D = dT + \lambda T$, with λ being a pure positive fraction and then the results can be generalized for pure negative fractions as well.

Lemma 1: The modified Z-transform of following function type is

$$\begin{aligned} &z^{-d}\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\frac{1}{s+p}e^{-\lambda Ts}\right\}\Big|_{t=kT}\right]; 0 \leq \lambda < 1 \\ &\quad = \frac{e^{(\lambda-1)pT}z^{-1}}{1-e^{-pT}z^{-1}}z^{-d}. \end{aligned} \tag{8}$$

Proof: Rewriting the left side of (8) using the time-shifting property of Laplace-transform and assuming system to be causal then:

$$\begin{aligned} &z^{-d}\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\frac{1}{s+p}e^{-\lambda Ts}\right\}\Big|_{t=kT}\right] \\ &\quad = z^{-d}\mathcal{Z}\left[e^{-p(t-\lambda T)}1(t-\lambda T)\Big|_{t=kT}\right], \end{aligned} \tag{9}$$

where $1(t-\lambda T)$ is delayed unit step signal. Now, according to the definition of $\mathcal{Z}$-transform to the right side of (9) for $0 \leq \lambda < 1$ and by applying sum of infinite geometric progression with $|z| > \left|e^{-pT}\right|$, one can have

$$\begin{aligned} &z^{-d}\mathcal{Z}\left[e^{-p(t-\lambda T)}1(t-\lambda T)\Big|_{t=kT}\right] \\ &\quad = z^{-d}e^{p\lambda T}\sum_{k=1}^{\infty}e^{-pkT}z^{-k} = \frac{e^{(\lambda-1)pT}z^{-1}}{1-e^{-pT}z^{-1}}z^{-d}. \end{aligned} \tag{10}$$

This proves Lemma 2. □

Corollary 1: Consider the modified Z-transform of the following function

$$z^{-d}\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-\lambda Ts}\right\}\Big|_{t=kT}\right]; 0 \leq \lambda < 1 = \frac{z^{-1}}{1-z^{-1}}z^{-d}. \tag{11}$$

Proof: Consider the special case of Lemma 2, where by substituting $p = 0$, on both sides of the expressions in (8), one can have

$$z^{-d}\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-\lambda Ts}\right\}\Big|_{t=kT}\right] = \frac{z^{-1}}{1-z^{-1}}z^{-d}. \tag{12}$$

This proves Corollary 1. □

Remark 3: It can be easily proved that the expressions of Lemma 1 and Corollary 1 will remain the same if the time-delay $D = (d+1)T + \gamma T$ is expressed in terms of pure negative fraction $-1 < \gamma \leq 0$ with $\lambda = 1 + \gamma$. Therefore for subsequent computations the general representation of $0 \leq |\lambda| < 1$, for fractional delay part of time-delay will be considered.

Now by utilizing the results of Lemma 2 and Corollary 1, the complete discretization of (6) can be obtained as

$$\begin{aligned} G_1(z) &= (1-z^{-1})z^{-d}K\left[\frac{z^{-1}}{1-z^{-1}} - \frac{e^{\frac{(\lambda-1)T}{\tau}}z^{-1}}{1-e^{-\frac{T}{\tau}}z^{-1}}\right] \\ &= \frac{a_1z^{-1}+a_2z^{-2}}{1+b_1z^{-1}}z^{-d}; 0 \leq |\lambda| < 1, \end{aligned} \tag{13}$$

where the parameters of the discretized system are

$$\begin{aligned} &a_1 = K(1-e^{((\lambda-1)T/\tau)}), a_2 = K(e^{((\lambda-1)T/\tau)} - e^{-(T/\tau)}), \\ &b_1 = -e^{-(T/\tau)}, \end{aligned} \tag{14}$$

now all the parameters of the original FOCTD system can be recovered from (14) using direct algebraic manipulations as

$$\begin{aligned} &K = (a_1+a_2)/(1+b_1), \tau = -T/\log(-b_1), \\ &\lambda = 1 + (\tau/T)\log(1-(a_1/K)), D = dT + \lambda T. \end{aligned} \tag{15}$$

This proves the Theorem 1. □

B. DISCRETIZATION OF SOCTD SYSTEM

Theorem 2: The modified $\mathcal{Z}$-transform to recover all the parameters of SOCTD system is:

$$\begin{aligned} &\mathcal{Z}\left[\mathcal{L}^{-1}\left\{H_0(s)\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ds}\right\}\Bigg|_{t=kT}\right]; \\ &\times 0 \leq |\lambda| < 1 \\ &= \frac{a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}{1+b_1z^{-1}+b_2z^{-2}}z^{-d}, \textit{ where} \\ &\tau_1 = \frac{-T}{\log\left(\left(-b_1+\sqrt{b_1^2-4b_2}\right)/2\right)}, \\ &\tau_2 = \frac{-T}{\log\left(\left(-b_1-\sqrt{b_1^2-4b_2}\right)/2\right)}, \\ &\lambda = (\tau_2/T)\log\left(\frac{a_3/K+(1-a_1/K-e^{-T/\tau_1})e^{-T/\tau_2}}{(\tau_2/(\tau_1-\tau_2))\left(e^{-T/\tau_1}-e^{-T/\tau_2}\right)e^{-T/\tau_2}}\right), \\ &K = (a_1+a_2+a_3)/(1+b_1+b_2), \; D = dT + \lambda T. \end{aligned} \tag{16}$$

Proof: If $G_2(z)$ represents the sampled transfer function of (16) then we have

$$G_2(z) = \mathcal{Z}\left[\mathcal{L}^{-1}\left\{H_0(s)\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ds}\right\}\Bigg|_{t=kT}\right] \tag{17}$$

by using (3), (7) and applying partial fraction on (17) with Lemma 2 in (8) and Corollary 1 in (11), the following discretized time-delayed model is obtained

$$G_2(z) = \frac{a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}{1+b_1z^{-1}+b_2z^{-2}}z^{-d}; 0 \leq |\lambda| < 1, \tag{18}$$

where, the discretized model parameters of (18) are given by

$$\begin{aligned} a_1 &= K\left(1 - \frac{\tau_1}{(\tau_1-\tau_2)}e^{\frac{(\lambda-1)T}{\tau_1}} + \frac{\tau_2}{(\tau_1-\tau_2)}e^{\frac{(\lambda-1)T}{\tau_2}}\right) \\ a_2 &= -K\left(e^{-T/\tau_1}+e^{-T/\tau_2}\right) + \frac{K\tau_1 e^{(\lambda-1)T/\tau_1}\left(1+e^{-T/\tau_2}\right)}{(\tau_1-\tau_2)} \\ &\quad -\frac{K\tau_2 e^{(\lambda-1)T/\tau_2}\left(1+e^{-T/\tau_1}\right)}{(\tau_1-\tau_2)} \\ a_3 &= Ke^{-(\tau_1+\tau_2)T/\tau_1\tau_2} - \frac{K\tau_1 e^{(\lambda-1)T/\tau_1}e^{-T/\tau_2}}{(\tau_1-\tau_2)} \\ &\quad +\frac{K\tau_2 e^{(\lambda-1)T/\tau_2}e^{-T/\tau_1}}{(\tau_1-\tau_2)} \\ b_1 &= -e^{-T/\tau_1} - e^{-T/\tau_2}, b_2 = e^{-(\tau_1+\tau_2)T/\tau_1\tau_2}. \end{aligned} \tag{19}$$

Now, the parameters of SOCTD can be easily recovered through some algebraic manipulations in (19) as

$$\begin{aligned} \tau_1 &= -T/\log\left(\left(-b_1+\sqrt{b_1^2-4b_2}\right)/2\right), \\ \tau_2 &= -T/\log\left(\left(-b_1-\sqrt{b_1^2-4b_2}\right)/2\right), \\ \lambda &= \frac{\tau_2}{T}\log\left(\frac{\frac{a_3}{K}+(1-\frac{a_1}{K}-e^{-\frac{T}{\tau_1}})e^{-\frac{T}{\tau_2}}}{\left(\frac{\tau_2}{\tau_1-\tau_2}\right)\left(e^{-\frac{T}{\tau_1}}-e^{-\frac{T}{\tau_2}}\right)e^{-\frac{T}{\tau_2}}}\right), \\ K &= (a_1+a_2+a_3)/(1+b_1+b_2), D = dT + \lambda T. \end{aligned} \tag{20}$$

This proves the Theorem 3. □

Remark 4: Observe that, the numerator parameters (a_1, a_2, and a_3) of the DTD model in (19), uses both the CTD system's rational parameters and time-delay (in terms of the fractional delay part 'λ'), for their computation. Therefore, any error in estimating the CTD system's parameters and time-delay will directly affect these parameters, which then also affect the value of λ in (20). Hence, by correcting the estimates of λ until it lies within $0 \leq |\lambda| < 1$, one can correct the errors in estimating CTD system parameters.

C. DISCRETIZATION OF SOCTDZ SYSTEM

Theorem 3: The following modified $\mathcal{Z}$-transform is used to compute all the parameters of SOCTDZ system through its discretized system as

$$\begin{aligned} &\mathcal{Z}\left[\mathcal{L}^{-1}\left\{H_0(s)\frac{K(\tau_0 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}e^{-Ds}\right\}\Bigg|_{t=kT}\right]; \\ &\times 0 \leq |\lambda| < 1 \\ &= \frac{a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}{1+b_1z^{-1}+b_2z^{-2}}z^{-d}, \textit{ where} \\ &\tau_1 = \frac{-T}{\log\left(\left(-b_1+\sqrt{b_1^2-4b_2}\right)/2\right)}, \end{aligned}$$

$$\tau_2 = \frac{-T}{\log\left(\left(-b_1 - \sqrt{b_1^2 - 4b_2}\right)/2\right)},$$
$$K = (a_1 + a_2 + a_3)\,/\,(1 + b_1 + b_2)\,,\; D = dT + \lambda T,$$
$$\tau_0 = \frac{\tau_1\tau_2 a_1}{KT(1-\lambda)},$$

and λ is that root of $\alpha_1\lambda^2 + \alpha_2\lambda + \alpha_3 = 0$, that lies within -1 and 1, where α_1, α_2, and α_3 are defined as

$$\begin{aligned}
\alpha_1 &= KT^2(e^{-T/\tau_1} - e^{-T/\tau_2})/(\tau_2-\tau_1)a_1\\
\alpha_2 &= KT((\tau_2-2T)e^{-T/\tau_1} - (\tau_1-2T)e^{-T/\tau_2})/(\tau_2-\tau_1)a_1\\
&\quad -KT(e^{-T/\tau_1}e^{-T/\tau_2} - a_3/K)/a_1\\
&\quad -T(\tau_1 e^{-T/\tau_1} - \tau_2 e^{-T/\tau_2})/(\tau_2-\tau_1),\ \text{and}\\
\alpha_3 &= KT((T-\tau_2)e^{-T/\tau_1} - (T-\tau_1)e^{-T/\tau_2})/(\tau_2-\tau_1)a_1\\
&\quad +KT(e^{-T/\tau_1}e^{-T/\tau_2} - a_3/K)/a_1\\
&\quad +(\tau_1 e^{-T/\tau_1}(T+\tau_2) - \tau_2 e^{-T/\tau_2}(T+\tau_1))/(\tau_2-\tau_1).
\end{aligned} \tag{21}$$

Proof: Let the discretized model of (21) is $G_3(z)$ that can be computed as:

$$G_3(z) = z^{-d}(1-z^{-1})\mathcal{Z}\left[\mathcal{L}^{-1}\left\{\frac{K(\tau_0 s+1)e^{-\lambda Ts}}{s(\tau_1 s+1)(\tau_2 s+1)}\right\}\Bigg|_{t=kT}\right], \tag{22}$$

consider the partial fraction of the following term in (22) as

$$\frac{K(\tau_0 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)} = K_e\left(\frac{A}{s} + \frac{B}{s+p_1} + \frac{C}{s+p_2}\right) \tag{23}$$

where

$$\begin{aligned}
A &= \frac{q_0}{p_1p_2}, B = \frac{q_0-p_1}{p_1\,(p_1-p_2)}, C = \frac{p_2-q_0}{p_2\,(p_1-p_2)},\\
K_e &= \frac{K}{A}, q_0 = 1/\tau_0, p_1 = 1/\tau_1, p_2 = 1/\tau_2.
\end{aligned} \tag{24}$$

Now, according to Lemma 2 and Corollary 1, the modified Z-transform of the following terms can be computed as

$$\begin{aligned}
Z\left[\frac{1}{s}e^{-\lambda Ts}\right] &= \frac{z^{-1}}{1-z^{-1}},\\
Z\left[\frac{1}{s+p_1}e^{-\lambda Ts}\right] &= \frac{d_1^{1-\lambda}z^{-1}}{1-d_1z^{-1}},\\
Z\left[\frac{1}{s+p_2}e^{-\lambda Ts}\right] &= \frac{d_2^{1-\lambda}z^{-1}}{1-d_2z^{-1}},
\end{aligned} \tag{25}$$

where

$$d_1 = e^{-p_1T}, d_2 = e^{-p_2T}, \tag{26}$$

on substituting (23) into (22), and using (25) one can have the following expression as

$$G_3(z) = \frac{a_1z^{-1} + a_2z^{-2} + a_3z^{-3}}{1 + b_1z^{-1} + b_2z^{-2}}z^{-d};\, 0 \le |\lambda| < 1, \tag{27}$$

where

$$\begin{aligned}
b_1 &= -(d_1+d_2),\\
b_2 &= d_1d_2,\\
a_1 &= K_e\left(A + Bd_1^{1-\lambda} + Cd_2^{1-\lambda}\right),\\
a_2 &= -K_e\left(A(d_1+d_2) + Bd_1^{1-\lambda}(1+d_2) + Cd_2^{1-\lambda}(1+d_1)\right),\\
a_3 &= K_e\left(Ad_1d_2 + Bd_1^{1-\lambda}d_2 + Cd_2^{1-\lambda}d_1\right).
\end{aligned} \tag{28}$$

The values of K, τ_1 and, τ_2 of (21) can be directly computed in terms of expressions in (24), (26) and, (28) as

$$\begin{aligned}
K &= \frac{a_1+a_2+a_3}{1+b_1+b_2},\\
\tau_1 &= -T/\log\left(\frac{-b_1+\sqrt{b_1^2-4b_2}}{2}\right),\\
\tau_2 &= -T/\log\left(\frac{-b_1-\sqrt{b_1^2-4b_2}}{2}\right).
\end{aligned} \tag{29}$$

In order to estimate the parameter τ_0 of (21), rewrite expression of a_1 in (28) using (24) and (26) as

$$\left(\frac{a_1}{K} - 1\right) = \frac{(\tau_1-\tau_0)}{(\tau_2-\tau_1)}e^{-(1-\lambda)T/\tau_1} + \frac{(\tau_0-\tau_2)}{(\tau_2-\tau_1)}e^{-(1-\lambda)T/\tau_2}, \tag{30}$$

by using the first order approximation of exponential terms in (30) and on simplification we get

$$\tau_0 = a_1\tau_1\tau_2/KT(1-\lambda). \tag{31}$$

Remark 5: Note that, the first-order approximation of exponential terms $e^{-(1-\lambda)T/\tau_1}$ and $e^{-(1-\lambda)T/\tau_2}$ in (30) does not produce significant approximation error as the sample time T, is already considered to be sufficiently small as compared to the system time constants τ_1 and τ_2.

The following quadratic equation in λ can be obtained by using expressions of a_1 and a_3 in (28) and substitutions from (24), (26) and, (31) with first-order approximation of exponential terms involving λ as

$$\alpha_1\lambda^2 + \alpha_2\lambda + \alpha_3 = 0, \tag{32}$$

where

$$\begin{aligned}
\alpha_1 &= KT^2(e^{-T/\tau_1} - e^{-T/\tau_2})/(\tau_2-\tau_1)a_1\\
\alpha_2 &= KT((\tau_2-2T)e^{-T/\tau_1} - (\tau_1-2T)e^{-T/\tau_2})/(\tau_2-\tau_1)a_1\\
&\quad -KT(e^{-T/\tau_1}e^{-T/\tau_2} - a_3/K)/a_1\\
&\quad -T(\tau_1 e^{-T/\tau_1} - \tau_2 e^{-T/\tau_2})/(\tau_2-\tau_1),\ \text{and}\\
\alpha_3 &= KT((T-\tau_2)e^{-T/\tau_1} - (T-\tau_1)e^{-T/\tau_2})/(\tau_2-\tau_1)a_1\\
&\quad +KT(e^{-T/\tau_1}e^{-T/\tau_2} - a_3/K)/a_1\\
&\quad +(\tau_1 e^{-T/\tau_1}(T+\tau_2) - \tau_2 e^{-T/\tau_2}(T+\tau_1))/(\tau_2-\tau_1),
\end{aligned} \tag{33}$$

and the time delay is computed by $D = dT + \lambda T$. □

Remark 6: The quadratic equation in (32) does not return a pure fractional value of λ until the estimated time-delay is not converged to actual time-delay. Therefore an iterative procedure is adapted to update the time delay and other rational parameters until any one of the roots of (32) satisfies

$0 \leq |\lambda| < 1$. *The step by step procedure to select the suitable root until the convergence been achieved is described in the following section.*

IV. PROPOSED ALGORITHMS FOR PARAMETER ESTIMATION

In this section, algorithms are developed to estimate all the parameters of FOCTD, FOCTD, and SOCTDZ systems, including the fractional delay. As in the real situation, the identification data is corrupted by the measurement noise. Therefore, if we consider the generalized difference equation representation of discretized models as

$$\begin{aligned}\hat{y}(k) = \hat{a}_1 u\left(k-\hat{d}-1\right) + \ldots + \hat{a}_m u\left(k-\hat{d}-m\right)\\ -\hat{b}_1\hat{y}(k-1) - \ldots - \hat{b}_n\hat{y}(k-n),\end{aligned} \tag{34}$$

where $\hat{y}(k)$ represents the estimated output. For rest of the following representations the values of n & m are considered as 1 & 2 for FOCTD and 2 & 3 for SOCTD and SOCTDZ systems. Let the estimated parameter vector in (34) is given by

$$\hat{\theta} = \left[\hat{a}_1 \ldots \hat{a}_m \; \hat{b}_1 \ldots \hat{b}_n\right]^{T_r}, \tag{35}$$

where T_r is the transpose operator. If the prediction error at $\hat{\theta}$ between actual and predicted output is defined as

$$\varepsilon\left(k;\hat{\theta}\right) = y(k) - \hat{y}\left(k;\hat{\theta}\right), \tag{36}$$

then, the prediction error vector for data length N is denoted by

$$\Xi(\hat{\theta}) = \left[\varepsilon\left(1;\hat{\theta}\right)\; \varepsilon\left(2;\hat{\theta}\right) \ldots \varepsilon\left(N;\hat{\theta}\right)\right]^{T_r}. \tag{37}$$

The objective function for prediction error minimization is defined by

$$V(\hat{\theta}) = \frac{1}{2N}\Xi^{T_r}(\hat{\theta})\Xi(\hat{\theta}), \tag{38}$$

and the gradient vector can be computed as

$$g(\hat{\theta}) = \frac{\partial V(\hat{\theta})}{\partial\hat{\theta}} = \frac{1}{N}J^{T_r}(\hat{\theta})\Xi(\hat{\theta}), \tag{39}$$

where $J(\hat{\theta})$ is the Jacobian matrix defined as the partial derivative of residuals with respect to estimated parameters as

$$J\left(\hat{\theta}\right) = \left[\frac{\partial\Xi\left(\hat{\theta}\right)}{\partial\hat{a}_1} \ldots \frac{\partial\Xi\left(\hat{\theta}\right)}{\partial\hat{a}_m}\; \frac{\partial\Xi\left(\hat{\theta}\right)}{\partial\hat{b}_1} \ldots \frac{\partial\Xi\left(\hat{\theta}\right)}{\partial\hat{b}_n}\right], \tag{40}$$

where the Jacobian matrix for each parameter is calculated by

$$\begin{aligned}J_{\hat{a}_i}(\hat{\theta}) &= \frac{\partial\Xi\left(\hat{\theta}\right)}{\partial\hat{a}_i} = -\frac{\partial\hat{y}(k)}{\partial\hat{a}_i} = -u(k-d-i)\\ &\quad + \sum_{l=1}^{n}\hat{b}_l\frac{\partial\hat{y}(k-l)}{\partial\hat{a}_i},\\ J_{\hat{b}_j}(\hat{\theta}) &= \frac{\partial\Xi\left(\hat{\theta}\right)}{\partial\hat{b}_j} = -\frac{\partial\hat{y}(k)}{\partial\hat{b}_j} = \hat{y}(k-j) + \sum_{l=1}^{n}\hat{b}_l\frac{\partial\hat{y}(k-l)}{\partial\hat{b}_j},\end{aligned} \tag{41}$$

where, $\hat{a}_i$ is the i^{th} numerator and $\hat{b}_j$ is the j^{th} denominator parameter of the identified DTD model. Now, the approximate Hessian matrix is computed by the relation as

$$H(\hat{\theta}) = J^{T_r}\left(\hat{\theta}\right)J\left(\hat{\theta}\right), \tag{42}$$

finally, the parameters $\hat{\theta}$ can be updated using following LM algorithm update rule for $(r+1)^{th}$ iteration as

$$\hat{\theta}_{r+1} = \hat{\theta}_r - \Delta\hat{\theta}_{r+1}, \quad \Delta\hat{\theta}_{r+1} = \left[H(\hat{\theta}_r) + \mu I\right]^{-1} g(\hat{\theta}_r), \tag{43}$$

where I is an identity matrix, and μ is a scalar whose values are adaptively updated to switch between gradient descent and Gauss-Newton algorithm to achieve convergence. The initial model parameters of the proposed algorithm are computed similarly to the method referred in [12], which is based on least square estimates of filtered data through a filter $SVF(s) = 1/(s+\omega_c)^n$ of order n and cutoff frequency ω_c. The only difference is that the proposed algorithm uses the discretized version of SVF. The Algorithm 1 and Algorithm 2 are used to summarize the complete proposed methodology.

V. SIMULATION RESULTS

In this section three examples using following CTD systems with long time-delay (also see [11], [12]) are used to evaluate the performance of the proposed method

$$\begin{aligned}Sys1 &:= \frac{2}{s+1}e^{-8s}\\ Sys2 &:= \frac{1.5}{0.25s^2+0.25s+1}e^{-8s}\\ Sys3 &:= \frac{-4s+0.5}{s^2+s+4}e^{-8s}\\ Sys4 &:= \frac{-6400s+1600}{s^4+5s^3+408s^2+416s+1600}e^{-8s}\end{aligned}$$

Two state of the art methods are used for reference and comparative analysis with the proposed method. First is the PROCEST algorithm of MATLAB's system identification toolbox, and second is the TFSRIVC algorithm of [12], which is also available in the CONTSID toolbox [38]. All the simulations are performed using MATLAB (R2017b) software installed in a computer having Intel Core i7 2620M CPU with 8GB RAM.

A persistently exciting PRBS signal of clock period 20 samples and switching levels +1 and −1 is used in all examples to generate a total of 2540 input/output data samples with 0.09 s, sampling interval, which is not an integer multiple of actual time-delay of 8 s. For identification, a white noise of zero mean is added to the output, and its variance is adjusted to obtain a 10dB signal to noise ratio (SNR). The accuracy of the identified model is validated using normalized root mean square error (NRMSE) in terms of Fit value defined as

$$Fit = 100\left(1 - \frac{\|y(k)-\hat{y}(k)\|}{\|y(k)-mean(y(k))\|}\right) \tag{44}$$

Algorithm 1 Identification of FOCTD and SOCTD systems

1: Collect $[u(kT), y(kT)]_{k=1}^{k=N}$.
Consider: $n = 1 \& m = 2$ for FOCTD and $n = 2 \& m = 3$ for SOCTD systems.
Specify: max_it, $grad\_it$, $\hat{d}_0 = round(D_i/T)$, and ω_c.
Use SVF to compute $\hat{\theta}_0$.
2: **for** $r = 0$ **to** max_it **do**
3: Construct $\hat{G}(z)$ using $\hat{\theta}_r$ and $\hat{d}_r$.
If $\hat{G}(z)$ have unstable poles (outside the unit circle) then reflect them into stable region (inside the unit circle).
Set: $update = 1$ and $\mu = 10^{-3}$.
4: **for** $i = 1$ **to** $grad\_it$ **do**
5: If: $update == 1$.
Compute $V(\hat{\theta}_r^i)$, $g(\hat{\theta}_r^i)$, $J(\hat{\theta}_r^i)$, and $H(\hat{\theta}_r^i)$ using (38), (39), (40) and (42) respectively.
6: Compute $\Delta\hat{\theta}_{r+1}^i = \left[H(\hat{\theta}_r^i) + \mu I\right]^{-1} g(\hat{\theta}_r^i)$.
Stop If: $\left|\Delta\hat{\theta}_{r+1}^i\right| < \delta$ (relative tolerance).
7: Compute $\hat{\theta}_{r+1}^i = \hat{\theta}_r^i - \Delta\hat{\theta}_{r+1}^i$, and $V(\hat{\theta}_{r+1}^i)$.
If: $V(\hat{\theta}_{r+1}^i) < V(\hat{\theta}_r^i)$:
Set: $update = 1$, $\mu = \mu/10$, and $\hat{\theta}_r^i = \hat{\theta}_{r+1}^i$.
Else: Set $update = 0$, and $\mu = 10\mu$.
8: **end for**
9: Use $\hat{\theta}_{r+1}$ to compute $\hat{\lambda}_{r+1}$ (using (15) for FOCTD system or by (20) for SOCTD system) and $\hat{D}_{r+1} = \left|\hat{d}_r T + \hat{\lambda}_{r+1} T\right|$.
10: Stop If: $0 \leq \left|\hat{\lambda}_{r+1}\right| < 1$, and return $\hat{G}(s)$ using Theorem 1, for FOCTD system or using Theorem 3, for SOCTD system.
Else: Set $\hat{\theta}_r = \hat{\theta}_{r+1}$, and $\hat{d}_r = round(\hat{D}_{r+1}/T)$.
11: **end for**

where $\|.\|$ is the 2-norm, $y(k)$ is the measured output and $\hat{y}(k)$ is the estimated output. For the given noise conditions, the estimated model is considered to be successfully converged if $Fit > 68$.

Remark 7: It can be observed that the lower D_{min} and upper D_{max} time-delay bounds are not required in Algorithm 1, while it is needed in Algorithm 2 for convergence. This is due to the fact that the explicit expression of fractional delay part λ is used in Algorithm 1, whereas in Algorithm 2, it is computed by solving a quadratic expression. Furthermore, the value of D_{min} can be set to '0' and D_{max} can be computed by observing system's step response or by calculating the maximum cross-correlation between input $u(kT)$ and output $y(kT)$ data [12], or by performing some statistical tests [35].

A. EXAMPLE 1

In this example, two systems Sys1 and Sys2, are used to validate the proposed algorithm. The identification settings for PROCEST method are: the model structure is 'P1D'

Algorithm 2 Identification of SOCTDZ system

1: Collect $[u(kT), y(kT)]_{k=1}^{k=N}$.
Specify: $n = 2$, $m = 3$, D_{min}, D_{max}, max_it, $grad\_it$, $\hat{d}_0 = round(D_i/T)$, and ω_c.
Use SVF to compute $\hat{\theta}_0$.
2: **for** $r = 0$ **to** max_it **do**
3: Use Steps 3 to 8 of Algorithm 1.
4: Use $\hat{\theta}_{r+1}$ to compute λ_1 and λ_2 using (32), then calculate
$D_1 = \left|\hat{d}_r T + \lambda_1 T\right|$ and $D_2 = \left|\hat{d}_r T + \lambda_2 T\right|$.
5: If: Both D_1 and D_2 lies within $D_{\min}$ and $D_{\max}$, then choose the one with better Fit value in (44) and correspondingly assign $\hat{D}_{r+1}$ and $\hat{\lambda}_{r+1}$.
ElseIf: $D_{\min} < D_1 < D_{\max}$
Set: $\hat{D}_{r+1} = D_1$ and $\hat{\lambda}_{r+1} = \lambda_1$.
ElseIf: $D_{\min} < D_2 < D_{\max}$
Set: $\hat{D}_{r+1} = D_2$ and $\hat{\lambda}_{r+1} = \lambda_2$.
Else: Stop.
6: Stop If: $0 \leq \left|\hat{\lambda}_{r+1}\right| < 1$, and return $\hat{G}(s)$ using Theorem 4.
Else: Set $\hat{\theta}_r = \hat{\theta}_{r+1}$, and $\hat{d}_r = round(\hat{D}_{r+1}/T)$.
7: **end for**

for Sys1 and 'P2DU' for Sys2. The estimation options are defined by opt=procestOptions with fields 'Focus', 'InitialCondition', 'SearchMethod', 'SearchOption.MaxIter' and, 'SearchOption.Tolerance' being 'prediction', 'zero', 'lm', 50 and, 10^{-4}, respectively. An initial model is constructed with these settings where the initial time-delay 'Structure.Td.Value' and it's lower 'Structure.Td.Minimum' and upper 'Structure.Td.Maximum' delay bounds are set to D_i, 0 s, and 9 s, respectively. Rest options are set to their default values. The identification settings for TFSRIVC algorithm [12] are as follows: the name-value pairs in 'tfsrivc' routine of CONTSID toolbox is set to lower time delay bound 'TdMin=0 s', upper time delay bound 'TdMax=9 s', initial time-delay 'IODelay=D_i', tolerance in parameter change 'TolPar=10^{-4}' and tolerance in cost function change 'TolFun=10^{-4}'. The number of zeros 'nz' and the number of poles 'np' are set to '0' & '1' for Sys1 and '0' & '2' for Sys2. The identification settings for the proposed method is, according to Algorithm 1, where the initial time-delay, $grad\_it$ and the tolerance 'δ' in the relative change of estimated parameters $\Delta\hat{\theta}$ are set to D_i, 30 and 10^{-4}, respectively. The SVF's cutoff frequency for TFSRIVC and the proposed method is set to $\omega_c = 2$ rad/s, and a maximum of 50 iterations are allowed for convergence in all three methods.

The convergence performance of PROCEST, TFSRIVC and, proposed methods for different initial time delays of $[0, 3, 5, 7, 9]$ s, is presented in Table 1, and Table 2, for Sys1 and Sys2, respectively, where for every initial time delay total 100 Monte Carlo (MC) simulations are performed with different noise initialization. The results of Table 1 and Table 2 (see F_m: mean percentage of

TABLE 1. Simulation results of 100 MC tests for Sys1 with different initial time-delay. F_m is the mean percentage of successfully converged models, T_m is the mean time taken for each model estimation (including both successful and unsuccessful estimation) and, It_m is the mean number of iterations required by the successfully converged models.

Initial delay	PROCEST			TFSRIVC [12]			Proposed		
D_i	F_m	T_m	It_m	F_m	T_m	It_m	F_m	T_m	It_m
0s	98%	1.11s	16.7	65%	0.12s	8.5	99%	0.08s	11.1
3s	58%	0.86s	14.7	54%	0.12s	8.9	100%	0.05s	7.9
5s	54%	0.89s	9.9	55%	0.11s	8.5	100%	0.04s	5.9
7s	100%	0.49s	5.6	100%	0.09s	4.5	100%	0.02s	3.8
9s	98%	0.51s	6.1	69%	0.12s	8.4	100%	0.04s	5.8

TABLE 2. Simulation results of 100 MC tests for Sys2 with different initial time-delay.

Initial delay	PROCEST			TFSRIVC [12]			Proposed		
D_i	F_m	T_m	It_m	F_m	T_m	It_m	F_m	T_m	It_m
0s	56%	1.78s	16.7	55%	0.21s	5.9	99%	0.11s	8.1
3s	38%	1.84s	14.9	12%	0.29s	6.5	98%	0.08s	6.2
5s	58%	1.71s	15.1	82%	0.14s	5.8	100%	0.05s	4.1
7s	100%	0.73s	7.1	100%	0.11s	5.5	100%	0.03s	3.0
9s	96%	1.1s	11.9	46%	0.20s	5.7	99%	0.03s	3.0

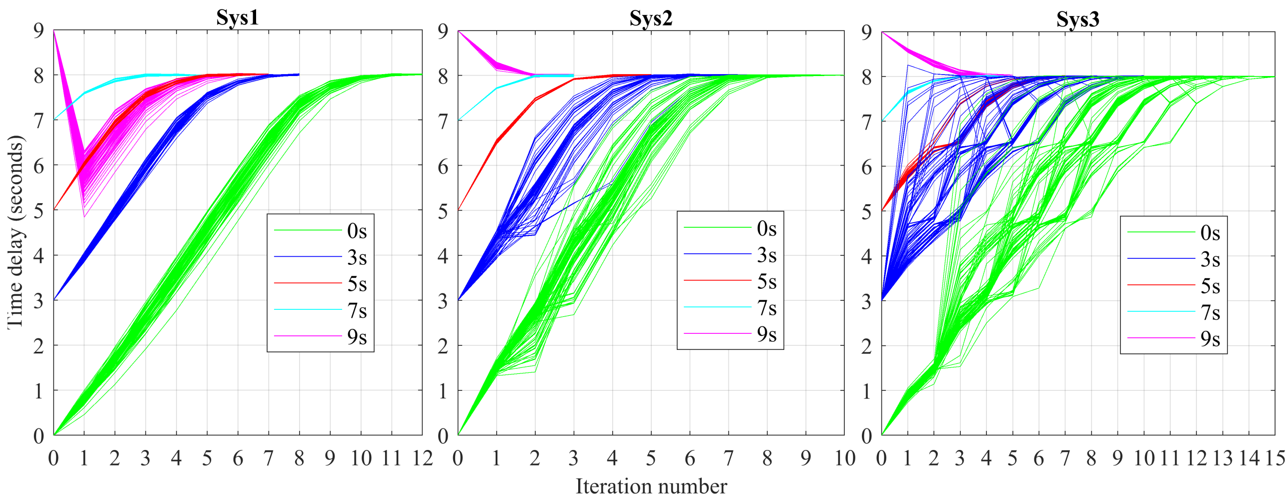

FIGURE 2. Evolution of estimated time delay in proposed method with iteration number using 100 MC tests with different noise and delay initialization for successfully converged models of Sys1, Sys2 and Sys3.

successfully converged models) indicate that the proposed method shows excellent convergence consistency with all initial time delays for both systems as compared to PROCEST and TFSRIVC methods. Furthermore, it can also be seen in Table 1 and Table 2 that the mean simulation time T_m required for each model estimation is minimum in proposed method for all initial delays and the mean number of iterations It_m required for successfully converged models are significantly less as compared to PROCEST and similar to TFSRIVC method. Fig. 2 is used to show the evolution of estimated time delay in the proposed method with iteration number using 100 MC tests with different noise and delay initialization for successfully converged models of Sys1 and Sys2.

A total number of 100 MC simulations are performed to compare the identification accuracy of all three methods for Sys1 and Sys2, with different noise seeds and a uniformly distributed initial delay between [0, 9]s. The results are presented in terms of estimated model's step responses in Fig. 3, and in terms of mean and standard deviation of estimated parameters in Table 3 and Table 4, where the mean and standard deviation values indicate that all three methods are accurate in parameter estimation. However, the values of simulation parameters (S_{par}) supports the superiority of the proposed method over PROCEST and TFSRIVC methods in terms of a high percentage of successful convergence (F_m) with less time required for identification (T_m).

Furthermore, to show the effect of the choice of ω_c on successful convergence 100 MC simulation with distinct noise seed and a uniform initial delay between $[0, 9]$ s, is performed. The results are presented in Table 5, where it can be observed that the proposed method is more robust for the choice of SVF's cutoff frequency ω_c as compared to the TFSRIVC method.

TABLE 3. Estimated parameters for 100 MC tests for Sys1.

True values	PROCEST			TFSRIVC [12]			Proposed		
2.0000	1.9997($\pm$0.0181)			2.0023($\pm$0.0272)			2.0015($\pm$0.0171)		
1.0000	1.0011($\pm$0.0194)			1.0018($\pm$0.0183)			1.0022($\pm$0.0162)		
8.0000	7.9990($\pm$0.0087)			8.0007($\pm$0.0065)			8.0001($\pm$0.0081)		
S_{par}	F_m	T_m	It_m	F_m	T_m	It_m	F_m	T_m	It_m
Value	78%	0.72s	8.7	77%	0.11s	6.6	100%	0.04s	6.4

TABLE 4. Estimated parameters for 100 MC tests for Sys2.

True values	PROCEST			TFSRIVC [12]			Proposed		
1.5000	1.4992($\pm$0.0228)			1.5003($\pm$0.0136)			1.5013($\pm$0.0175)		
0.2500	0.2453($\pm$0.0335)			0.2497($\pm$0.0022)			0.2501($\pm$0.0027)		
0.2500	0.2461($\pm$0.0336)			0.2504($\pm$0.0032)			0.2499($\pm$0.0034)		
8.0000	8.0072($\pm$0.0543)			8.0009($\pm$0.0069)			8.0002($\pm$0.0091)		
S_{par}	F_m	T_m	It_m	F_m	T_m	It_m	F_m	T_m	It_m
Value	63%	1.29s	9.3	62%	0.19s	5.4	99%	0.09s	6.6

TABLE 5. The effect of ω_c in SVF on mean successful convergence (F_m).

Method	System	F_m for ω_c(rad/s)			
		0.1	1.0	3.0	5.0
TFSRIVC [12]	Sys1	85%	80%	65%	54%
	Sys2	92%	68%	54%	52%
Proposed	Sys1	100%	100%	97%	96%
	Sys2	100%	99%	96%	92%

B. EXAMPLE 2

In this example, Sys3 is used, which is a second-order under-damped time-delayed system with a zero. The identification of such systems is challenging as the phase contribution due to the zero can cause false delay estimation, which further affects the accuracy of rational parameter estimation and may result in identification failure. The identification settings for the PROCEST, TFSRIVC methods are the same as that in Example 1, except the process model structure is set to 'P2DUZ' in PROCEST and 'nz=1' & 'np=2' are selected in TFSRIVC. The identification settings for the proposed method is according to Algorithm 2, with D_{min} and D_{max} are set to 0 and 9 s, respectively. The Initial rational model parameters are computed using SVF with $\omega_c = 2\, rad/s$ in all three approaches, and remaining settings are the same as that of Example 1.

The simulation results for 100 MC tests are shown in Table 6 for each initial time-delay D_i of 0, 1, 3, 5, 7, 8, and 9 s, respectively. It can be observed from the values of F_m: mean percentage of successfully converged models in Table 6, that the proposed method shows almost perfect convergence for all initial time-delays while the PROCEST and TFSRIVC methods do not converge well if the initial time-delay is not close enough to the actual time-delay. Furthermore, the values of mean simulation time for each model estimation (T_m) and the mean iterations required for successful convergence in the proposed method are competitive to the TFSRIVC method and much lesser as compared to PROCEST approach. Another interesting observation can also be made from Table 6, that the proposed method requires minimum time and only one iteration for convergence if the initial delay is equal to the actual delay of 8 s, whereas the PROCEST and TFSRIVC methods needed at least three iterations for convergence. Moreover, the evolution of the time-delay with each iteration in the proposed approach is shown in Fig. 2 for diffident initial delays D_i's of [0, 3, 5, 7, 9] s.

To compare the identification accuracy, Sys3 is used for 100 MC simulations with different noise seeds and uniformly distributed initial time-delays between $[0, 9]$ s. The results of PROCEST, TFSRIVC, and proposed methods are summarized in Fig. 3, as the step responses of estimated models and in Table 7, as the mean and standard deviation of estimated parameters. The values of estimated parameters in Table 7, shows that the TFSRIVC and proposed methods are more accurate than PROCEST method where the proposed method also gained the advantage of superior and fastest convergence with mean percentage of successful converged models F_m, which is nearly 100% and minimum mean time required for model estimation $T_m < 0.1$ s, as compared to the other two methods.

C. EXAMPLE 3

This example considers the system (Sys4), which is a fourth-order non-minimum phase time-delay system. The identification experiment with 10 dB noise is performed similarly to that of the previous examples to generate identification data. A SOCTDZ model is identified for Sys4 using all three approaches (PROCEST, TFSRIVC [12] and, the proposed methods) by keeping all identification settings same as that in Example 2. The identification results are presented (see: Table 8) in terms of mean and standard deviation of estimated parameters for 100 MC tests with an initial delay which is uniformly distributed in the interval [0 9] s. The simulation results in Table 8 show that the proposed approach is superior to the other two in achieving successful convergence and requires less computation time. Furthermore, a selected portion of measured and estimated model's response is plotted in Fig. 4, where it can be observed that the proposed

TABLE 6. Simulation results of 100 MC tests for Sys3 with different initial time-delay.

Initial delay	PROCEST			TFSRIVC [12]			Proposed		
D_i	F_m	T_m	It_m	F_m	T_m	It_m	F_m	T_m	It_m
0s	03%	5.5s	50	72%	0.12s	5.7	98%	0.22s	11.9
1s	04%	4.7s	50	19%	0.14s	5.1	99%	0.25s	12.6
3s	10%	4.6s	50	15%	0.21s	6.0	98%	0.18s	10.4
5s	40%	3.2s	50	82%	0.12s	5.5	100%	0.09s	5.1
7s	100%	1.9s	21	74%	0.11s	5.4	100%	0.06s	4.1
8s	100%	0.8s	3.1	100%	0.09s	3.0	100%	0.04s	1.0
9s	86%	2.2s	15.7	11%	0.16s	5.8	99%	0.06s	4.8

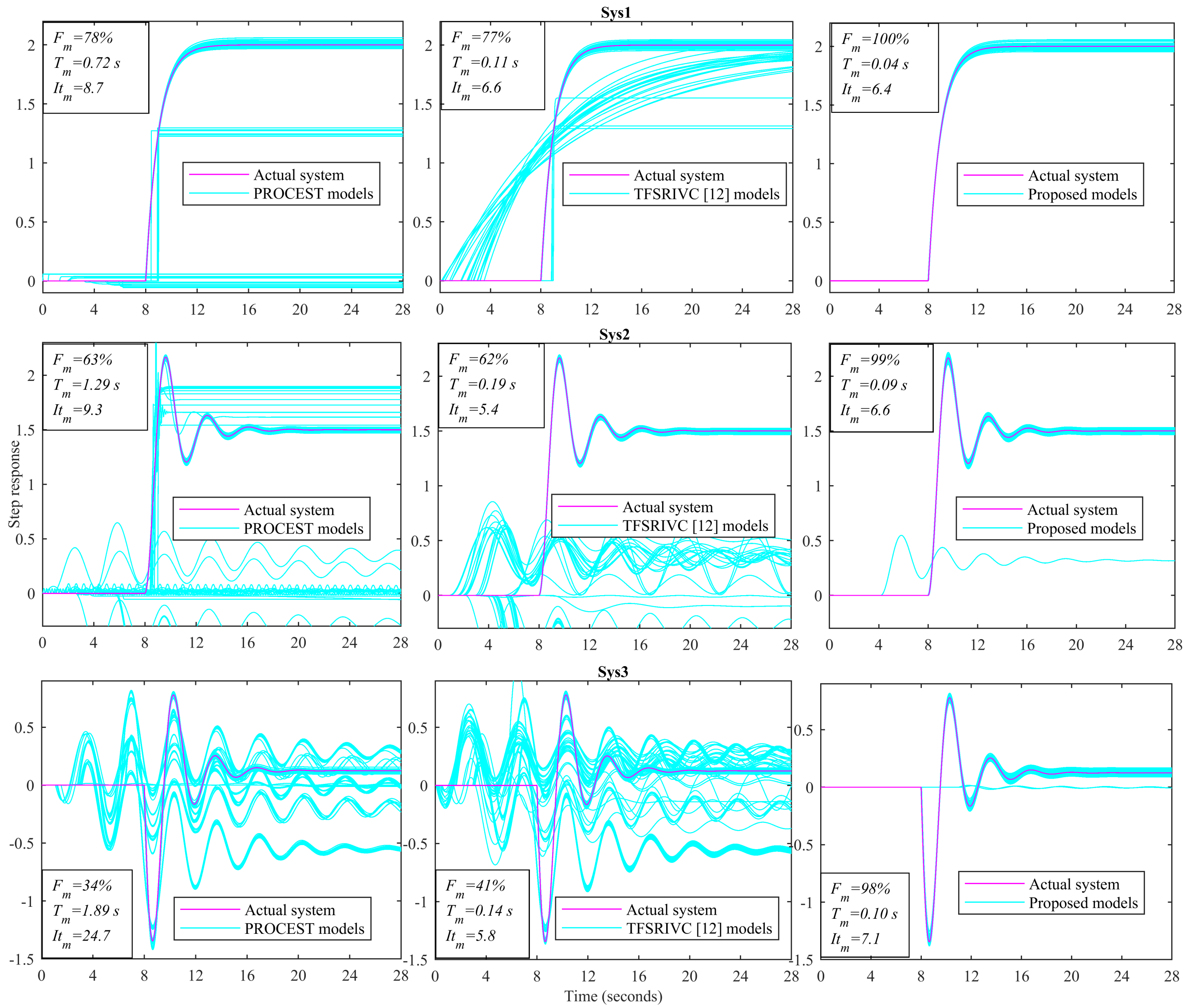

FIGURE 3. Identified models step responses using 100 MC tests with different noise and delay initialization for Sys1, Sys2 and Sys3. F_m is the mean percentage of successfully converged models, T_m is the mean time taken for each model estimation (including both successful and unsuccessful estimation) and, It_m is the mean number of iterations required by the successfully converged models.

method is capable in handling severe noise interferences to produce unbiased parameter estimates.

VI. REAL SYSTEM IDENTIFICATION APPLICATION

Let us now considered a real system to validate the proposed identification technique. The experimental setup is shown in Fig. 5, where the three identical conical tanks 1, 2, and 3 can be configured individually or together in a non-interactive or interactive manner. In this paper, a single conical tank 1 of height 70 cm, upper diameter 35 cm, and lower diameter 2.5 cm is considered. The water is fed to tank 1 through a dedicated pneumatic control valve (RK valve ltd), which

TABLE 7. Estimated parameters for 100 MC tests for Sys3.

True values	PROCEST			TFSRIVC [12]			Proposed		
-4.0000	$-3.9082(\pm 0.0810)$			$-4.0076(\pm 0.0492)$			$-3.9867(\pm 0.0526)$		
0.5000	$0.1294(\pm 0.2443)$			$0.5100(\pm 0.0645)$			$0.4952(\pm 0.0736)$		
1.0000	$0.9646(\pm 0.0269)$			$1.0031(\pm 0.0144)$			$1.0006(\pm 0.0163)$		
4.0000	$3.9917(\pm 0.0297)$			$4.0085(\pm 0.0283)$			$4.0027(\pm 0.0257)$		
8.0000	$7.9751(\pm 0.0182)$			$8.0024(\pm 0.0068)$			$8.0023(\pm 0.0055)$		
S_{par}	F_m	T_m	It_m	F_m	T_m	It_m	F_m	T_m	It_m
Value	34%	1.89s	24.7	41%	0.14s	5.8	98%	0.10s	7.1

TABLE 8. Identified SOCTDZ models for 100 MC tests in terms of mean and standard deviation of estimated parameters for successfully converged models and simulation parameters for Sys4.

Method	Estimated models	F_m	It_m	T_m
PROCEST	$\dfrac{\frac{-16.1540}{(\pm 0.3971)}s+\frac{3.3982}{(\pm 1.6425)}}{s^2+\frac{0.9937}{(\pm 0.0419)}s+\frac{4.0255}{(\pm 0.0318)}}e^{-\frac{8.0047}{(\pm 0.0255)}s}$	29%	15.6	2.6s
TFSRIVC [12]	$\dfrac{\frac{-16.2429}{(\pm 0.2139)}s+\frac{4.1652}{(\pm 0.2361)}}{s^2+\frac{1.0037}{(\pm 0.0149)}s+\frac{4.0167}{(\pm 0.0284)}}e^{-\frac{8.0153}{(\pm 0.0064)}s}$	34%	5.2	0.21s
Proposed	$\dfrac{\frac{-16.0201}{(\pm 0.2951)}s+\frac{4.1055}{(\pm 0.2479)}}{s^2+\frac{1.0062}{(\pm 0.0172)}s+\frac{4.0264}{(\pm 0.0296)}}e^{-\frac{8.0186}{(\pm 0.0074)}s}$	98%	6.9	0.12s

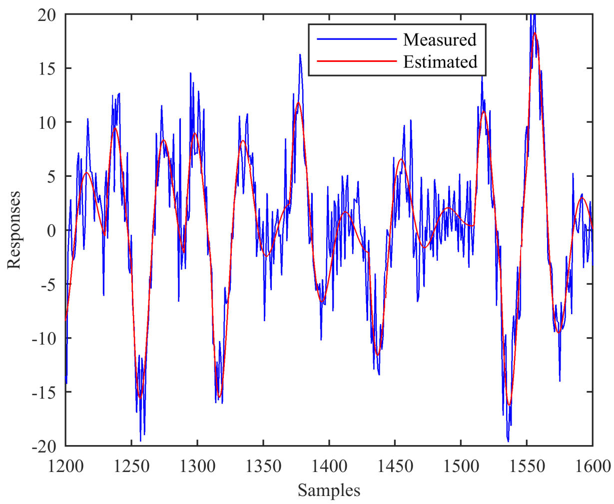

FIGURE 4. Measured and estimated model's response for Sys4 in Example 3.

FIGURE 5. Real conical tank system used for identification.

is coupled to an electric pump (Kirloskar) that works at a constant speed. The water returned to a reservoir by an outlet at the bottom of tank 1. The system's input is the flow rate command in percentage (%) of the maximum flow rate (500 liters per hour), and the output is the water level in the tank 1. The current water flow rate is indicated by a Rotameter (TELELINE). A differential pressure transducer 'DPT' (Rosemount) has been utilized to measure the liquid level in the tank. All the sensors and actuators of the conical tank system interact through MATLAB/Simulink in a computer with the help of a digital data exchange interface (VDPID-03) device, which converts the digital information from computer to an analog signal for the process and vice versa.

The data generation experiment uses two steps. In the first step, a fixed flow rate of 40% is supplied, and the manual outlet valve of tank 1 is adjusted to obtain a steady-state liquid level around 42.27 cm. Secondly, identification data is generated at $T = 0.11$ s sampling interval by applying a PRBS signal having flow levels 0 and 50% and clock period of 20 s. The static components in generated input/output data $u(k)|y(k)$ of 7500 samples are removed by subtracting the expected values as $\bar{u}(k) = u(k) - mean\{u(k)\}$ and $\bar{y}(k) = y(k) - mean\{y(k)\}$. Then the recovered data $\bar{u}(k)|\bar{y}(k)$ is divided into two parts first 3500 samples (from 0 to 385 s) are used for model identification, and the remaining 4000 samples (from 385 to 825 s) are used for model validation.The proposed algorithm uses this identification data with an initial delay $D_i = 0$ s and $\omega_c = 1$ rad/s in SVF to estimate a SOCTD model given by

$$\hat{G}(s) = \frac{34.29}{4.55 \times 10^4 s^2 + 5047s + 1} e^{-1.54s}. \tag{45}$$

The estimated model of the conical tank process in (45) is tested on the validation data set, and the comparison is shown in Fig. 6. The accuracy of the identified model is computed by the percentage fitting criterion as $Fit(\%) = 100\left(1 - \frac{\|\bar{y}(k) - \hat{y}(k)\|}{\|\bar{y}(k) - mean(\bar{y}(k))\|}\right)\%$, in terms of measured $\bar{y}(k)$ and estimated $\hat{y}(k)$ output. The obtained $Fit(\%)$ value for system in (45) for validation data is comes out to be 83.81%. The same identification experiment is also performed using PROCEST and TFSRIVC methods, where the identified models are $\hat{G}_{PROCEST}(s) = \frac{74.66}{8.36\times10^4 s^2 + 1.13\times10^4 s + 1} e^{-2.84s}$ and $\hat{G}_{TFSRIVC}(s) = \frac{70.85}{7.91\times10^4 s^2 + 1.05\times10^4 s + 1} e^{-2.85s}$ with $Fit(\%)$ values 78.65% and 78.96% respectively on validation data.

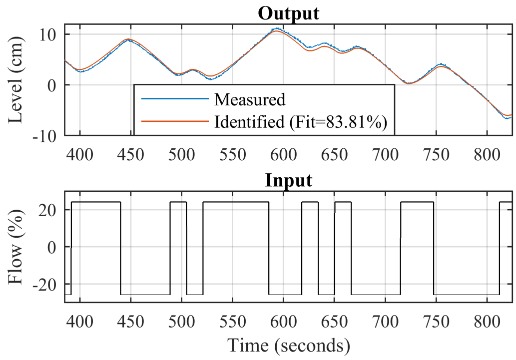

FIGURE 6. **Identification result of proposed method on validation data for conical tank process.**

VII. CONCLUSION

This paper presented a simple, fast, robust, and accurate method to simultaneously identify all the parameters, including the time-delay of FOCTD, SOCTD, and SOCTDZ models. It is mathematically formulated that the discretization of a CTD system brings time-delay into rational model parameters of the discretized system, where it can be effectively utilized in an iterative algorithm to simultaneously estimate all the parameters of original CTD system. The simulation outcomes of three numerical examples on four systems indicate that the proposed approach produces much better results as compared to the recent instrument variable based method TFSRIVC and the MATLAB's (R2017b) PROCEST routine in terms of identification speed, accuracy, and robustness. It is observed that the global convergence of the PROCEST and TFSRIVC algorithms for 100 MC simulations is less than 80%, 70% and 50% for FOCTD, SOCTD and, SOCTDZ systems respectively, while the proposed method gives excellent convergence which is more than 98% for all type of FOCTD, SOCTD, and SOCTDZ systems. Furthermore, the proposed method is also validated for the identification of a real conical tank system with good fitting accuracy. The future work directions could be the extensions of the proposed methodology for identifying the multi-variable and nonlinear time-delayed systems.

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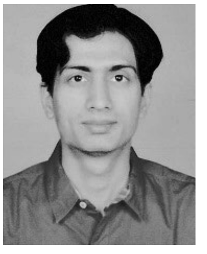

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