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Multi-Attribute Group Decision-Making Methods Based on Pythagorean Fuzzy *N*-Soft Sets

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ABSTRACT In this paper, by integrating Pythagorean fuzzy set with N-soft set, we propose a generalization of N-soft set called the theory of Pythagorean fuzzy N-soft set and explore some of related operations concerning this theory including the weak complement, extended union and intersection, restricted union and intersection. Then two algorithms are introduced to Pythagorean fuzzy N-soft sets for dealing with multi-attribute group decision-making problems. Finally, a practical example is provided to illustrate the validity and practicality of Pythagorean fuzzy N-soft sets in multi-attribute group decision-making problems. Compared with the existing models, we also elaborate the advantages of this model.

INDEX TERMS Algorithm, multi-attribute group decision-making problem, N-soft set, Pythagorean fuzzy set, Pythagorean fuzzy N-soft set.

I. INTRODUCTION

With the continuous development of society, there exist a large number of fuzzy phenomena in all fields of life. For example, in linguistics, economics, engineering, aesthetics, medical sciences and many other fields, fuzziness is a common phenomenon. For this reason, fuzzy set is introduced by Zadeh [48] in order to deal with these uncertainties. Since its appearance, fuzzy set has produced many generalizations including interval-valued fuzzy sets [34], multi-valued fuzzy sets [31], intuitionistic fuzzy set [2] and Pythagorean fuzzy set [39], [40], etc. As one of generalizations of fuzzy set, Pythagorean fuzzy sets have drawn extensive attention from many researchers, since it came into being. In recent years, the researchers have carried on an extensive research about Pythagorean fuzzy sets and achieved plenteous theoretical results [12]–[14], [16], [18], [24], [26], [28], [30], [36]–[38], [49], [50]. For example, Zhang and Xu [49] discussed some basic operations and properties of Pythagorean fuzzy sets. Peng *et al.* [26] studied the system conversion of distance measure, similar measure and other measures, and proposed a new information measure formula. Wei *et al.* [36] proposed several Pythagorean fuzzy similarities based on cosine function method and applied it to handle the decision-making problems. Zhang [50] introduced an interval-valued Pythagorean fuzzy set and discussed its application in decision problems. Liang *et al.* [18] introduced the Pythagorean fuzzy number into decision-theoretic rough sets and constructed a new model of Pythagorean fuzzy decision-theoretic rough sets. Wan *et al.* [37] developed a Pythagorean fuzzy mathematical programming method to solve multi-attribute group decision-making problems under Pythagorean fuzzy environment. Literatures [16], [38] established a method of Pythagorean fuzzy analytic hierarchy process, and applied it to risk assessment.

Although fuzzy sets, Pythagorean fuzzy sets and other theories can deal with uncertain problems independently and each of them has its own unique characteristics, there exists the limitation of inadequacy of the parameterization tool. For this reason, Molodtsov developed the theory of soft sets [19] as a parameterization tool for dealing with the uncertainties which other mathematical tools can not handle. With the establishment of soft set theory, research works on soft sets are very active in these years, especially on the merging study of soft sets and other uncertainty theories. For example, in [20], Maji *et al.* observed that the soft set model can be integrated with other uncertainty mathematical models, resulting in a new soft set model. As a result, Maji *et al.* made the study

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of soft sets by initiating the concept of fuzzy soft sets [21] and intuitionistic fuzzy soft set [22], and proceeded to solve various decision making problems by using these theories. Following the research ideas, many new fusion models have been produced [6], [8], [17], [23], [27], [29], [32], [33], [35], [41], [43]–[47]. For example, Majumdar and Samantha [23] proposed a generalized fuzzy soft set on the basis of fuzzy soft set and applied it to decision-making problems. Yang *et al.* [41] presented the concept of interval-valued fuzzy soft sets by combining interval-valued fuzzy sets with soft sets. Based on [41], Jiang *et al.* [17] constructed interval-valued intuitionistic fuzzy soft sets model. In addition, Yang *et al.* [42] proposed the concept of multi-fuzzy soft sets and applied it to decision making problems. Alkhazaleh *et al.* [8] defined the concept of possibility fuzzy soft sets. Recently, Peng *et al.* [27] has extended fuzzy soft sets into Pythagorean fuzzy environment to handle decision making problems more effectively, and developed the concept of Pythagorean fuzzy soft sets. Later, the researchers successively came up with many fusion models for soft sets including generalized intuitionistic fuzzy soft sets [6], hesitant fuzzy soft sets [35], interval-value hesitant fuzzy soft sets [43], dual hesitation fuzzy soft sets [44], interval-value intuition hesitation fuzzy soft sets [32], soft rough sets [46], soft fuzzy rough sets [45] and soft rough fuzzy sets [47], etc.

Based on the models mentioned above, it is observed that whether or not it is fuzzy set and its generalizations or it is soft set theory and its generalizations, most of the works focus on binary estimation (either 0 or 1), or else real numbers between 0 and 1 [25]. However, there exist a large number of non-binary evaluation problems in practical life, such as voting situations [1] and ranking or evaluation systems [9]. In ranking or evaluation systems, it is observed that the ranking of the objects such as hotels, excellent students and scenic spots can be presented in the form of the number of mark symbols (like 'one big dot', 'two stars' or 'three hearts'). Take the ranking of hotels as an example. The number of stars represents the ranking of the hotel. In fact, the more stars the hotel has, the higher the ranking of the hotel. 'One star' can present an ordinary hotel; 'two stars' can present a relatively good hotel; 'three stars' can present a very good hotel; 'four stars' can present a luxury hotel; 'five stars' can present a very luxury hotel. Further, researchers have investigated the rating system in soft sets [7], [10], [15]. Very recently, Fatimah *et al.* [11] extended soft set under non-binary evaluation environment, proposed a new model called N-soft set (*NSS*), and explained the importance of ordered grades in practical problems. Later, in order to portray the hesitancy of decision makers, Akram *et al.* [3] introduced a novel hybrid model called hesitant N-soft sets by combining hesitancy and N-soft sets (*HNSS*). Meanwhile, in [4] they also proposed the concept of fuzzy N-soft set (*FNSS*) by integrating fuzzy set with N-soft set. The proposed model provides a more flexible decision-making method for dealing with uncertainties concerning which specific level is assigned to objects in the parameterizations by attributes. However, the fuzzy N-soft set model proposed by Akram *et al.* only involves the possibility of membership degree to the parameterized characterization of objects without considering the possibility of non-membership degree. Therefore, Akram *et al.* [5] introduced the N-soft set into the intuitionistic fuzzy environment and proposed an intuitionistic fuzzy N-soft set (*IFNSS*). However, in realistic problems, the sum of membership degree and non-membership degree in the process of decision making may be greater than 1, so it is difficult to solve the problem by using *IFNSS* model. In the light of this limitation, we intend to introduce a novel model to deal with the decision-making problem. In this paper, by introducing N-soft set into Pythagorean fuzzy environment, a new hybrid model called Pythagorean fuzzy N-soft set (*PFNSS*) model is constructed in which the square sum of membership degree and non-membership degree is allowed to be less than to 1. We attempt to explain the circumstances in which the decision-makers evaluate the objects to determine their rankings based on the same multiple fuzzy characteristics from the perspective of the membership degree and non-membership degree, and the practitioners provide the overall evaluation by combining multiple decision makers under different multiple fuzzy characteristics environment. So in order to model the group decision problem, the novel model can offer more possibility and flexibility by combining two different models, which makes the decision result more scientific and reasonable.

The paper is organized as follows. Section 2 briefly reviews some backgrounds on soft set, Pythagorean fuzzy set and N-soft set. In Section 3, we propose the concept of Pythagorean fuzzy N-soft set and further explore some of its properties. Section 4 investigates the relationships between the model and the existing models including Pythagorean fuzzy soft sets, N-soft sets and soft sets. In Section 5, two algorithms are established to demonstrate the effectiveness of *PFNSSs*. In Section 6, we describe the application of *PFNSS*s in practical problems, and use examples of grade estimation to illustrate the effectiveness and practicality of our model. Section 7 concludes the paper.

II. PRELIMINARIES

The following definitions are required in the sequel of our work and hence presented in brief.

A. PYTHAGOREAN FUZZY SETS

The concept of Pythagorean fuzzy sets has been introduced by Yager [39] to extend intuitionistic fuzzy sets.

Definition 1 [39]: Let U be a universe of discourse. A Pythagorean fuzzy set (*PFS*) is an object having the following form:

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in U\},$$

where $\mu_A : U \to [0, 1]$ represents the membership degree and $\nu_A : U \to [0, 1]$ represents the non-membership degree of the element $x \in U$ to the set A, respectively, and for any $x \in U$, it holds that $0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$. The degree of indeterminacy is given as:

$\pi_A(x) = \sqrt{1-(\mu_A(x))^2-(\nu_A(x))^2}$, where $\mu_A \in [0,1]$ and $\nu_A \in [0,1]$.

For convenience, we call $\alpha = (\mu_\alpha, \nu_\alpha)$ a Pythagorean fuzzy number (PFN).

Definition 2 [40], [49]: Let $\alpha = (\mu_\alpha, \nu_\alpha)$, $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ be any three $PFNs$ over U, then the following holds:

(1) $\alpha^c = (\nu_\alpha, \mu_\alpha)$;
(2) $\alpha_1 \cup \alpha_2 = (max(\mu_{\alpha_1}, \mu_{\alpha_2}), min(\nu_{\alpha_1}, \nu_{\alpha_2}))$;
(3) $\alpha_1 \cap \alpha_2 = (min(\mu_{\alpha_1}, \mu_{\alpha_2}), max(\nu_{\alpha_1}, \nu_{\alpha_2}))$;
(4) $\alpha_1 \geq \alpha_2$ *iff* $\mu_{\alpha_1} \geq \mu_{\alpha_2}$ *and* $\nu_{\alpha_1} \leq \nu_{\alpha_2}$;
(5) $\alpha_1 = \alpha_2$ *iff* $\mu_{\alpha_1} = \mu_{\alpha_2}$ *and* $\nu_{\alpha_1} = \nu_{\alpha_2}$;

Definition 3 [49]: Let $\alpha = (\mu_\alpha, \nu_\alpha)$ be any a PFN over U. The score function and accuracy function of α are defined as follows:

$$S(\alpha) = \mu_\alpha^2 - \nu_\alpha^2 \ \ and \ \ Q(\alpha) = \mu_\alpha^2 + \nu_\alpha^2,$$

where $S(\alpha) \in [-1,1]$ and $Q(\alpha) \in [0,1]$. For any two PFNs α_1, α_2, if $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;

If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 > \alpha_2$;
If $S(\alpha_1) = S(\alpha_2)$, then
(a) If $Q(\alpha_1) > Q(\alpha_2)$, then $\alpha_1 > \alpha_2$;
(b) If $Q(\alpha_1) = Q(\alpha_2)$, then $\alpha_1 \sim \alpha_2$.

Definition 4 [40]: Let $\alpha = (\mu_\alpha, \nu_\alpha)$ be any a PFN over U. The ranking function of α is defined as follows:

$$R(\alpha) = \frac{1}{2} + r_\alpha(\frac{1}{2} - \frac{2\theta_\alpha}{\pi}),$$

where $r_\alpha = \sqrt{\mu_\alpha^2 + \nu_\alpha^2}$ is called commitment strength. The angle between commitment strength r_α and membership degree μ_α is θ_α. The direction of commitment $d_\alpha = 1 - \frac{2\theta_\alpha}{\pi}$, $\mu_\alpha = r_\alpha cos\theta_\alpha$, $\nu_\alpha = r_\alpha sin\theta_\alpha$.

For any two PFNs α_1, α_2, if $R(\alpha_1) < R(\alpha_2)$, then $\alpha_1 < \alpha_2$;
If $R(\alpha_1) > R(\alpha_2)$, then $\alpha_1 > \alpha_2$;
If $R(\alpha_1) = R(\alpha_2)$, then $\alpha_1 = \alpha_2$.

B. N-SOFT SETS

Molodtsov developed the theory of soft sets [19] as a parameterization tool for dealing with the uncertainties which the classical mathematical tools can not handle.

Definition 5 [19]: Let U be an initial universe set and E be a universe set of parameters. A pair (F, A) is called a soft set over U if $A \subseteq E$ and $F : A \to P(U)$, where $P(U)$ is the set of all subsets of U.

Definition 6 [11]: Let C be a non-empty universal set of objects, A be a set of attributes, and $Z \subseteq A$. Let $D = \{0, 1, 2, \cdots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \cdots\}$. The triple (F, Z, N) is called a N-soft set on C, if F is a mapping $F : Z \to 2^{C \times D}$, with the property that for each $z \in Z$ there exists a unique $(c, d_z) \in C \times D$ such that $(c, d_z) \in F(z)$, $c \in C$, $d_z \in D$.

Definition 7 [11]: Let (F, Z, N) and (K, L, M) be two N-soft set on C. We say $(F, Z, N) = (K, L, M)$ if and only if $F = K$, $Z = L$ and $N = M$.

Definition 8 [11]: A weak complement of the N-soft set (F, Z, N), denoted by (F', Z, N), is any N-soft set, where $F'(z) \cap F(z) = \varnothing$, for each $z \in Z$.

III. PYTHAGOREAN FUZZY N-SOFT SET

A. CONCEPT OF PYTHAGOREAN FUZZY N-SOFT SETS

In this subsection, along the idea of Refs. [3], [4] we further extend the N-soft sets into Pythagorean fuzzy environment and introduce a Pythagorean fuzzy N-soft set model by combining the N-soft set and Pythagorean fuzzy set. Then, we explain its intuitive interpretation and suggest to use tabular form to illustrate the application of its simplification in practical problems.

Definition 9: Let C be a non-empty universal set of objects, A be a set of attributes, and $Z \subseteq A$. Let $D = \{0, 1, 2, \cdots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \cdots\}$. The triple (F_P, Z, N) is called a $PFNSS$ on C, if F_P is a mapping $F_P : Z \to 2^{C \times D} \times PFN$, in which $F : Z \to 2^{C \times D}$, and $P : Z \to PFN$, that PFN denote a Pythagorean fuzzy number, i.e. $\mu : Z \to [0,1]$ and $\nu : Z \to [0,1]$ with for all $z \in Z$, $0 \leq \mu_z^2(c) + \nu_z^2(c) \leq 1$.

For each $z \in Z$ and $c \in C$ there exists a unique$(c, d_z) \in C \times D$ such that $d_z \in D$ and $PFN = (\mu_z(c), \nu_z(c))$. Hence, (F_P, Z, N) can be written as

$$(F_P, Z, N) = ((c, d_z), (\mu_z(c), \nu_z(c))),$$

where d_z denotes the level of the element attribute, $\mu_z(c)$ denotes the membership degree, and $\nu_z(c)$ denotes the non-membership degree of the element $c \in C$ to the attribute z.

Example 10: Suppose that a class wants to select a student as the most outstanding student of the year, and only one can be selected. To solve this problem, it is necessary to seek the advice of the teachers. Assume that there are five students who meet the requirements. Given that $C = \{c_1, c_2, c_3, c_4, c_5\}$ is a family of five qualified students under consideration and A is a set of attributes ‘‘evaluations of students by the teacher’’. For the subset $Z \subseteq A$ with $Z = \{z_1, z_2, z_3, z_4, z_5\}$, a 5-soft set can be defined as Table 1, where

- Four hearts represent ‘highly recommended’,
- Three hearts represent ‘recommended’,
- Two hearts represent ‘more recommended’,
- One heart represents ‘not recommended ’,
- Hollow circle represents ‘disagree’.

This level assessment by hearts can be represented by numbers as $D = \{0, 1, 2, 3, 4\}$, where

- 0 means as ‘○’,
- 1 means as ‘♡’,
- 2 means as ‘♡♡’,
- 3 means as ‘♡♡♡’,
- 4 means as ‘♡♡♡♡’.

Based on the overall quality of the students, the teachers give evaluate scores of the students which is shown as Table 1 in which the tabular representation of its associated 5-soft set is given in Table 2.

Although it is easy to extract the grade data in actual information, the data possess the fuzzy anti uncertainty

TABLE 1. Evaluation data provided by teachers.

C/Z	z_1	z_2	z_3	z_4	z_5
c_1	♡	♡♡	♡	♡♡♡	♡
c_2	♡♡	♡	♡♡	♡♡	♡♡
c_3	♡	♡	◦	♡♡	♡♡♡♡
c_4	◦	♡♡	♡	♡	♡♡
c_5	♡♡	♡	◦	♡♡	♡

TABLE 2. Tabular representation of the corresponding 5-soft set.

C/Z	z_1	z_2	z_3	z_4	z_5
c_1	1	2	1	3	1
c_2	2	1	2	2	2
c_3	1	1	0	2	4
c_4	0	2	1	1	2
c_5	2	1	0	2	1

TABLE 3. Corresponding criteria.

d_z/P	$\mu_z(C)$	$\nu_z(C)$
$d_z = 0$	$[0, 0.15)$	$(0.85, 1]$
$d_z = 1$	$[0.15, 0.35)$	$[0.65, 0.85)$
$d_z = 2$	$[0.35, 0.65)$	$[0.35, 0.65)$
$d_z = 3$	$[0.65, 0.85)$	$[0.15, 0.35)$
$d_z = 4$	$[0.85, 1)$	$(0, 0.15]$

characteristics. When facing the problem, we attempt to explain the situation in which the teachers evaluate the students to determine their rankings based on the same multiple fuzzy characteristics from the perspective of the membership degree and non-membership degree. So by integrating Pythagorean fuzzy sets with N-soft sets we introduce Pythagorean fuzzy N-soft sets to give the grade division. This assessment of students by teachers complies with the guidelines as follows:

$$\begin{aligned}
-1 \leq S_z(C) < -0.7 \quad & \text{when } d_z = 0,\\
-0.7 \leq S_z(C) < -0.3 \quad & \text{when } d_z = 1,\\
-0.3 \leq S_z(C) < 0.3 \quad & \text{when } d_z = 2,\\
0.3 \leq S_z(C) < 0.7 \quad & \text{when } d_z = 3,\\
0.7 \leq S_z(C) \leq 1 \quad & \text{when } d_z = 4.
\end{aligned}$$

According to the above criteria, we can obtain Table 3. Therefore, by Definition 9, the Pythagorean fuzzy 5-soft set can be defined as follows:

$$\begin{aligned}
(\mu(z_1), \nu(z_1)) = \{&((c_1, 1), (0.32, 0.76)), ((c_2, 2), (0.58, 0.63)),\\
&((c_3, 1), (0.34, 0.77)), ((c_4, 0), (0.12, 0.88)),\\
&((c_5, 2), (0.63, 0.62))\} \in PF(F(z_1)),\\
(\mu(z_2), \nu(z_2)) = \{&((c_1, 2), (0.62, 0.64)), ((c_2, 1), (0.26, 0.75)),\\
&((c_3, 1), (0.33, 0.81)), ((c_4, 2), (0.62, 0.58)),\\
&((c_5, 1), (0.29, 0.84))\} \in PF(F(z_2)),\\
(\mu(z_3), \nu(z_3)) = \{&((c_1, 1), (0.32, 0.74)), ((c_2, 2), (0.61, 0.46)),\\
&((c_3, 0), (0.11, 0.97)), ((c_4, 1), (0.26, 0.79)),\\
&((c_5, 0), (0.11, 0.94))\} \in PF(F(z_3)),\\
(\mu(z_4), \nu(z_4)) = \{&((c_1, 3), (0.78, 0.32)), ((c_2, 2), (0.62, 0.57)),\\
&((c_3, 2), (0.60, 0.56)), ((c_4, 1), (0.24, 0.82)),\\
&((c_5, 2), (0.61, 0.64))\} \in PF(F(z_4)),\\
(\mu(z_5), \nu(z_5)) = \{&((c_1, 1), (0.18, 0.81)), ((c_2, 2), (0.58, 0.61)),\\
&((c_3, 4), (0.92, 0.14)), ((c_4, 2), (0.64, 0.37)),\\
&((c_5, 1), (0.34, 0.83))\} \in PF(F(z_5)).
\end{aligned}$$

Here we can express it more clearly in the tabular form shown as in Table 4.

It is worth noting that a tabular form can reveal the information concerning general Pythagorean fuzzy N-soft sets shown in Table 5 when the set of alternatives and the set of attributes are both finite.

Note 11: (1) In Example 10, we consider five assessment grades, but the assessment grades in practical problems do not necessarily the five grades, which can be arbitrary. Generally speaking, the range concerning the score function of Pythagorean fuzzy numbers PFNs can vary with actual grade requirements.

(2) Grade 0 represents the lowest score and does not indicate that the information is incomplete and can not be evaluated.

Definition 12: Suppose that (F_P, Z, N) and (K_Q, L, M) are two $PFNSS$s on a universe C. Now (F_P, Z, N) and (K_Q, L, M) are said to be equal if and only if

(1) $F_P = K_Q$, *where* $F = K, P = Q$;

(2) $Z = L$;

(3) $N = M$,

which can be denoted by $(F_P, Z, N) = (K_Q, L, M)$.

B. OPERATIONS ON PYTHAGOREAN FUZZY N-SOFT SETS

In this subsection, we shall explore the basic operations of $PFNSS$, such as "weak complement", "union operation" and "intersection operation".

Definition 13: Let (F_P, Z, N) be a $PFNSS$ on C. It is a weak complement of (F'_P, Z, N), if and only if (F', Z, N) is a weak complement of (F, Z, N).

Consider (F_P, Z, N) in Example 10, then its weak complement (F'_P, Z, N) is represented as Table 6.

Definition 14: Let (F_P, Z, N) be a $PFNSS$ on C. $(F_{P'}, Z, N)$ is said to be a Pythagorean fuzzy complement if and only if P' is the complement of Pythagorean fuzzy number P in F_P.

Consider (F_P, Z, N) in Example 10, then its Pythagorean fuzzy complement $(F_{P'}, Z, N)$ is shown in Table 7.

Definition 15: Let (F_P, Z, N) be a $PFNSS$ on C. $(F'_{P'}, Z, N)$ is referred to as a weak Pythagorean fuzzy complement if and only if (F'_P, Z, N) is the weak complement and $(F_{P'}, Z, N)$ is the Pythagorean fuzzy complement of (F_P, Z, N).

Reconsider (F_p, Z, N) in Example 10, then its weak Pythagorean fuzzy complement $(F'_{P'}, Z, N)$ is show in Table 8.

In order to further explore the application of Pythagorean fuzzy N-soft set, in what follows we shall initiate the concepts

TABLE 4. Tabular representation of the $(F_P, Z, 5)$-soft set in Example 10.

$(F_P, Z, 5)$	z_1	z_2	z_3	z_4	z_5
c_1	(1, (0.32, 0.76))	(2, (0.62, 0.64))	(1, (0.32, 0.74))	(3, (0.78, 0.32))	(1, (0.18, 0.81))
c_2	(2, (0.58, 0.63))	(1, (0.26, 0.75))	(2, (0.61, 0.46))	(2, (0.62, 0.57))	(2, (0.58, 0.61))
c_3	(1, (0.34, 0.77))	(1, (0.33, 0.81))	(0, (0.11, 0.97))	(2, (0.60, 0.56))	(4, (0.92, 0.14))
c_4	(0, (0.12, 0.88))	(2, (0.62, 0.58))	(1, (0.26, 0.79))	(1, (0.24, 0.82))	(2, (0.64, 0.37))
c_5	(2, (0.63, 0.62))	(1, (0.29, 0.84))	(0, (0.11, 0.94))	(2, (0.61, 0.64))	(1, (0.34, 0.83))

TABLE 5. Tabular representation of a general (F_P, Z, N)-soft set.

(F_P, Z, N)	z_1	z_2	$\cdots$	z_m
c_1	$(d_{11}, (\mu_{11}, \nu_{11}))$	$(d_{12}, (\mu_{12}, \nu_{12}))$	$\cdots$	$(d_{1m}, (\mu_{1m}, \nu_{1m}))$
c_2	$(d_{21}, (\mu_{21}, \nu_{21}))$	$(d_{22}, (\mu_{22}, \nu_{22}))$	$\cdots$	$(d_{2m}, (\mu_{2m}, \nu_{2m}))$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
c_n	$(d_{n1}, (\mu_{n1}, \nu_{n1}))$	$(d_{n2}, (\mu_{n2}, \nu_{n2}))$	$\cdots$	$(d_{nm}, (\mu_{nm}, \nu_{nm}))$

TABLE 6. A weak complement of the $(F_P, Z, 5)$-soft set in Example 10.

$(F'_P, Z, 5)$	z_1	z_2	z_3	z_4	z_5
c_1	(2, (0.32, 0.76))	(3, (0.62, 0.64))	(2, (0.32, 0.74))	(4, (0.78, 0.32))	(2, (0.18, 0.81))
c_2	(3, (0.58, 0.63))	(2, (0.26, 0.75))	(3, (0.61, 0.46))	(3, (0.62, 0.57))	(3, (0.58, 0.61))
c_3	(2, (0.34, 0.77))	(2, (0.33, 0.81))	(1, (0.11, 0.97))	(3, (0.60, 0.56))	(0, (0.92, 0.14))
c_4	(1, (0.12, 0.88))	(3, (0.62, 0.58))	(2, (0.26, 0.79))	(2, (0.24, 0.82))	(3, (0.64, 0.37))
c_5	(3, (0.63, 0.62))	(2, (0.29, 0.84))	(1, (0.11, 0.94))	(3, (0.61, 0.64))	(2, (0.34, 0.83))

of extended intersections, extended unions, restricted intersections and restricted unions.

Definition 16: Let C be a non-empty universal set of objects. Given that (F_P, Z, N) and (K_Q, L, M) are two *PFNSS*s on C, their restricted intersection is defined as $(R_T, J, X) = (F_P, Z, N) \cap_\xi (K_Q, L, M)$, where $R_T = F_P \cap_\xi K_Q, J = Z \cap L$ and $X = min(N, M)$, i.e. $\forall z_j \in J$ and $c_i \in C, (d_{ij}, (\mu_{ij}, \nu_{ij})) \in R_T(z_j), d_{ij} = min(d^1_{ij}, d^2_{ij}), \mu_{ij}(z_j) = min(\mu^1_{ij}(z^1_j), \mu^2_{ij}(z^2_j))$ and $\nu_{ij}(z_j) = max\ (\nu^1_{ij}(z^1_j), \nu^2_{ij}(z^2_j))$ where $(d^1_{ij}, (\mu^1_{ij}(z^1_j), \nu^1_{ij}(z^1_j))) \in (\mu_{F_P}(z^1_j), \nu_{F_P}(z^1_j))$ and $(d^2_{ij}, (\mu^2_{ij}(z^2_j), \nu^2_{ij}(z^2_j))) \in (\mu_{K_Q}(z^2_j), \nu_{K_Q}(z^2_j))$ with $z^1_j \in Z$ and $z^2_j \in L$.

Definition 17: Let C be a non-empty universal set of objects. Given that (F_P, Z, N) and (K_Q, L, M) are two *PFNSS*s on C, their extended intersection is defined as $(G_T, B, Y) = (F_P, Z, N) \cap_\zeta (K_Q, L, M)$, where $G_T = F_P \cap_\zeta K_Q, B = Z \cup L$ and $Y = max(N, M)$, i.e. $\forall z_j \in B$ and $c_i \in C$, $(d_{ij}, (\mu_{ij}, \nu_{ij})) \in G_T(z_j)$ with

$$G_T(z_j) = \begin{cases} (\mu_P(z^1_j), \nu_P(z^1_j)), & if\ z_j \in Z - L, \\ (\mu_Q(z^2_j), \nu_Q(z^2_j)), & if\ z_j \in L - Z, \\ (d_{ij}, (\mu_{ij}(z_j), \nu_{ij}(z_j))), & such\ that\ d_{ij} = min(d^1_{ij}, d^2_{ij}), \\ & \mu_{ij}(z_j) = min(\mu^1_{ij}(z^1_j), \mu^2_{ij} \\ & (z^2_j))\ and\ \nu_{ij}(z_j) = max(\nu^1_{ij} \\ & (z^1_j), \nu^2_{ij}(z^2_j)), where\ (d^1_{ij}, \\ & (\mu^1_{ij}(z^1_j), \nu^1_{ij}(z^1_j))) \in (\mu_P(z^1_j), \\ & \nu_P(z^1_j))\ and\ (d^2_{ij}, (\mu^2_{ij}(z^2_j), \\ & \nu^2_{ij}(z^2_j))) \in (\mu_Q(z^2_j), \nu_Q(z^2_j)), \\ & with\ z^1_j \in Z\ and\ z^2_j \in L. \end{cases}$$

Definition 18: Let C be a non-empty universal set of objects. Given that (F_P, Z, N) and (K_Q, L, M) are two *PFNSS*s on C, their restricted union is defined as :$(R_S, J, Y) = (F_P, Z, N) \cup_\xi (K_Q, L, M)$, where $R_S = F_P \cup_\xi K_Q, J = Z \cap L$ and $Y = max(N, M)$, i.e. $\forall z_j \in J$ and $c_i \in C, (d_{ij}, (\mu_{ij}, \nu_{ij})) \in R_S(z_j), d_{ij} = max(d^1_{ij}, d^2_{ij}), \mu_{ij}(z_j) = max(\mu^1_{ij}(z^1_j), \mu^2_{ij}(z^2_j))$ and $\nu_{ij}(z_j) = min(\nu^1_{ij}(z^1_j), \nu^2_{ij}(z^2_j))$, where $(d^1_{ij}, (\mu^1_{ij}(z^1_j), \nu^1_{ij}(z^1_j))) \in (\mu_P(z^1_j), \nu_P(z^1_j))$ and $(d^2_{ij}, (\mu^2_{ij}(z^2_j), \nu^2_{ij}(z^2_j))) \in (\mu_Q(z^2_j), \nu_Q(z^2_j))$ with $z^1_j \in Z$ and $z^2_j \in L$.

Definition 19: Let C be a non-empty universal set of objects. Given that (F_P, Z, N) and (K_Q, L, M) are two *PFNSS*s on C, their extended union is defined as $(G_S, B, Y) = (F_P, Z, N) \cup_\zeta (K_Q, L, M)$, where $G_S = F_P \cup_\zeta K_Q, B = Z \cup L$ and $Y = max(N, M)$, i.e. $\forall z_j \in B$ and $c_i \in C$, $(d_{ij}, (\mu_{ij}, \nu_{ij})) \in G_S(z_j)$, where

$$G_S(z_j) = \begin{cases} (\mu_P(z^1_j), \nu_P(z^1_j)), & if z_j \in Z - L, \\ (\mu_Q(z^2_j), \nu_Q(z^2_j)), & if\ z_j \in L - Z, \\ (d_{ij}, (\mu_{ij}(z_j), \nu_{ij}(z_j))), & such\ that\ d_{ij} = max(d^1_{ij}, d^2_{ij}), \\ & \mu_{ij}(z_j) = max(\mu^1_{ij}(z^1_j), \mu^2_{ij} \\ & (z^2_j))\ and\ \nu_{ij}(z_j) = min(\nu^1_{ij} \\ & (z^1_j), \nu^2_{ij}(z^2_j)), where\ (d^1_{ij}, \\ & (\mu^1_{ij}(z^1_j), \nu^1_{ij}(z^1_j))) \in (\mu_P(z^1_j), \\ & \nu_P(z^1_j))\ and\ (d^2_{ij}, (\mu^2_{ij}(z^2_j), \\ & \nu^2_{ij}(z^2_j))) \in (\mu_Q(z^2_j), \nu_Q(z^2_j)), \\ & with\ z^1_j \in Z\ and\ z^2_j \in L. \end{cases}$$

Example 20: Consider $(F_P, Z, 5)$ in Example 10 and $(K_Q, L, 6)$ given in Table 9. Their restricted intersection $(R_T, J, X) = (F_P, Z, 5) \cap_\xi (K_Q, L, 6)$, extended intersection $(G_T, B, Y) = (F_P, Z, 5) \cap_\zeta (K_Q, L, 6)$, restricted union $(R_S, J, Y) = (F_P, Z, 5) \cup_\xi (K_Q, L, 6)$ and extended union $(G_S, B, Y) = (F_P, Z, 5) \cup_\zeta (K_Q, L, 6)$ are, respectively, shown in Tables 10-13.

TABLE 7. Pythagorean fuzzy complement of the $(F_P, Z, 5)$-soft set in Example 10.

$(F_{P'}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
c_1	(1, (0.76, 0.32))	(2, (0.64, 0.62))	(1, (0.74, 0.32))	(3, (0.32, 0.78))	(1, (0.81, 0.18))
c_2	(2, (0.63, 0.58))	(1, (0.75, 0.26))	(2, (0.46, 0.61))	(2, (0.57, 0.62))	(2, (0.61, 0.58))
c_3	(1, (0.77, 0.34))	(1, (0.81, 0.33))	(0, (0.97, 0.11))	(2, (0.56, 0.60))	(4, (0.14, 0.92))
c_4	(0, (0.88, 0.12))	(2, (0.58, 0.62))	(1, (0.79, 0.26))	(1, (0.82, 0.24))	(2, (0.37, 0.64))
c_5	(2, (0.62, 0.63))	(1, (0.84, 0.29))	(0, (0.94, 0.11))	(2, (0.64, 0.61))	(1, (0.83, 0.34))

TABLE 8. A weak Pythagorean fuzzy complement of the $(F_P, Z, 5)$-soft set in Example 10.

$(F'_{P'}, Z, 5)$	z_1	z_2	z_3	z_4	z_5
c_1	(2, (0.76, 0.32))	(1, (0.64, 0.62))	(2, (0.74, 0.32))	(4, (0.32, 0.78))	(2, (0.81, 0.18))
c_2	(3, (0.63, 0.58))	(2, (0.75, 0.26))	(3, (0.46, 0.61))	(3, (0.57, 0.62))	(3, (0.61, 0.58))
c_3	(2, (0.77, 0.34))	(2, (0.81, 0.33))	(1, (0.97, 0.11))	(3, (0.56, 0.60))	(0, (0.14, 0.92))
c_4	(1, (0.88, 0.12))	(3, (0.58, 0.62))	(2, (0.79, 0.26))	(2, (0.82, 0.24))	(3, (0.37, 0.64))
c_5	(3, (0.62, 0.63))	(2, (0.84, 0.29))	(1, (0.94, 0.11))	(3, (0.64, 0.61))	(2, (0.83, 0.34))

TABLE 9. Tabular representation of $(K_Q, L, 6)$-soft set.

$(K_Q, L, 6)$	z_1	z_2	z_5	a
c_1	(2, (0.45, 0.62))	(3, (0.63, 0.34))	(2, (0.42, 0.67))	(4, (0.81, 0.24))
c_2	(3, (0.69, 0.46))	(2, (0.48, 0.61))	(3, (0.68, 0.49))	(5, (0.92, 0.09))
c_3	(2, (0.36, 0.73))	(3, (0.62, 0.44))	(0, (0.08, 0.94))	(3, (0.64, 0.45))
c_4	(3, (0.67, 0.46))	(2, (0.42, 0.66))	(4, (0.85, 0.28))	(1, (0.26, 0.87))
c_5	(3, (0.69, 0.45))	(0, (0.07, 0.96))	(1, (0.24, 0.82))	(4, (0.89, 0.28))

TABLE 10. Tabular representation of restricted intersection (R_T, J, X).

(R_T, J, X)	z_1	z_2	z_5
c_1	(1, (0.32, 0.76))	(2, (0.62, 0.64))	(1, (0.18, 0.81))
c_2	(2, (0.58, 0.63))	(1, (0.26, 0.75))	(2, (0.58, 0.61))
c_3	(1, (0.34, 0.77))	(1, (0.33, 0.81))	(0, (0.08, 0.94))
c_4	(0, (0.12, 0.88))	(2, (0.42, 0.66))	(2, (0.64, 0.37))
c_5	(2, (0.63, 0.62))	(0, (0.07, 0.96))	(1, (0.24, 0.83))

Based on the definitions mentioned above, the following properties concerning *PFNSS*s are straightforward.

Theorem 21: Given that (F_P, Z, N), (K_Q, L, M) and (G_T, B, Y) are any three *PFNSS*s on C, then the commutative and associative properties hold:

(1) $(F_P, Z, N) \cap_\zeta (F_P, Z, N) = (F_P, Z, N) \cup_\zeta (F_P, Z, N) = (F_P, Z, N)$;

(2) $(F_P, Z, N) \cap_\xi (F_P, Z, N) = (F_P, Z, N) \cup_\xi (F_P, Z, N) = (F_P, Z, N)$;

(3) $(F_P, Z, N) \cap_\zeta (K_Q, L, M) = (K_Q, L, M) \cap_\zeta (F_P, Z, N)$;

(4) $(F_P, Z, N) \cap_\xi (K_Q, L, M) = (K_Q, L, M) \cap_\xi (F_P, Z, N)$;

(5) $(F_P, Z, N) \cup_\zeta (K_Q, L, M) = (K_Q, L, M) \cup_\zeta (F_P, Z, N)$;

(6) $(F_P, Z, N) \cup_\xi (K_Q, L, M) = (K_Q, L, M) \cup_\xi (F_P, Z, N)$;

(7) $(F_P, Z, N) \cap_\zeta ((K_Q, L, M) \cap_\zeta (G_T, B, Y)) = ((F_P, Z, N) \cap_\zeta (K_Q, L, M)) \cap_\zeta (G_T, B, Y)$;

(8) $(F_P, Z, N) \cap_\xi ((K_Q, L, M) \cap_\xi (G_T, B, Y)) = ((F_P, Z, N) \cap_\xi (K_Q, L, M)) \cap_\xi (G_T, B, Y)$;

(9) $(F_P, Z, N) \cup_\zeta ((K_Q, L, M) \cup_\zeta (G_T, B, Y)) = ((F_P, Z, N) \cup_\zeta (K_Q, L, M)) \cup_\zeta (G_T, B, Y)$;

(10) $(F_P, Z, N) \cup_\xi ((K_Q, L, M) \cup_\xi (G_T, B, Y)) = ((F_P, Z, N) \cup_\xi (K_Q, L, M)) \cup_\xi (G_T, B, Y)$.

Theorem 22: Given that (F_P, Z, N) and (K_Q, L, M) are any two *PFNSS*s on C, then the following results hold:

(1) $(F_{(P')'}, Z, N) = (F_P, Z, N)$;

(2) $((F \cap_\zeta K)_{(P \cup_\zeta Q)'}, Z \cup L, max(N, M)) = (F_{P'}, Z, N) \cap_\zeta (K_{Q'}, L, M)$;

(3) $((F \cup_\zeta K)_{(P \cap_\zeta Q)'}, Z \cup L, max(N, M)) = (F_{P'}, Z, N) \cup_\zeta (K_{Q'}, L, M)$.

Theorem 22 reveals that the complementary law is only satisfied with extended intersection and extended union operations in the case of Pythagorean fuzzy complement, but not with restricted intersection and restricted union operations. Meanwhile, both extended intersection and union operations and restricted intersection and union operations do not satisfy the complementary law under weak complement and weak Pythagorean fuzzy complement environment.

In what follows, we can easily obtain the following distributive law of *PFNSS*s.

Theorem 23: Given that (F_P, Z, N), (K_Q, L, M) and (G_T, B, Y) are any three *PFNSS*s on C, then the following results hold:

(1) $((F_P, Z, N) \cup_\zeta (K_Q, L, M)) \cap_\xi (F_P, Z, N) = (F_P, Z, N)$;

(2) $((F_P, Z, N) \cap_\xi (K_Q, L, M)) \cup_\zeta (F_P, Z, N) = (F_P, Z, N)$;

(3) $((F_P, Z, N) \cup_\xi (K_Q, L, M)) \cap_\zeta (F_P, Z, N) = (F_P, Z, N)$;

(4) $((F_P, Z, N) \cap_\zeta (K_Q, L, M)) \cup_\xi (F_P, Z, N) = (F_P, Z, N)$;

TABLE 11. Tabular representation of extended intersection (G_T, B, Y).

(G_T, B, Y)	z_1	z_2	z_3	z_4	z_5	a
c_1	$(1, (0.32, 0.76))$	$(2, (0.62, 0.64))$	$(1, (0.32, 0.74))$	$(3, (0.78, 0.32))$	$(1, (0.18, 0.81))$	$(4, (0.81, 0.24))$
c_2	$(2, (0.58, 0.63))$	$(1, (0.26, 0.75))$	$(2, (0.61, 0.46))$	$(2, (0.62, 0.57))$	$(2, (0.58, 0.61))$	$(5, (0.92, 0.09))$
c_3	$(1, (0.34, 0.77))$	$(1, (0.33, 0.81))$	$(0, (0.11, 0.97))$	$(2, (0.60, 0.56))$	$(0, (0.08, 0.94))$	$(3, (0.64, 0.45))$
c_4	$(0, (0.12, 0.88))$	$(2, (0.42, 0.66))$	$(1, (0.26, 0.79))$	$(1, (0.24, 0.82))$	$(2, (0.64, 0.37))$	$(1, (0.26, 0.87))$
c_5	$(2, (0.63, 0.62))$	$(0, (0.07, 0.96))$	$(0, (0.11, 0.94))$	$(2, (0.61, 0.64))$	$(1, (0.24, 0.83))$	$(4, (0.89, 0.28))$

TABLE 12. Tabular representation of restricted union (R_S, J, Y).

(R_S, J, Y)	z_1	z_2	z_5
c_1	$(2, (0.45, 0.62))$	$(3, (0.63, 0.34))$	$(2, (0.42, 0.67))$
c_2	$(3, (0.69, 0.46))$	$(2, (0.48, 0.61))$	$(3, (0.68, 0.49))$
c_3	$(2, (0.36, 0.68))$	$(3, (0.62, 0.44))$	$(4, (0.92, 0.14))$
c_4	$(3, (0.67, 0.46))$	$(2, (0.62, 0.58))$	$(4, (0.85, 0.28))$
c_5	$(3, (0.69, 0.45))$	$(1, (0.29, 0.84))$	$(1, (0.34, 0.82))$

TABLE 13. Tabular representation of extended union (G_S, B, Y).

(G_S, B, Y)	z_1	z_2	z_3	z_4	z_5	a
c_1	$(2, (0.45, 0.62))$	$(3, (0.63, 0.34))$	$(1, (0.32, 0.74))$	$(3, (0.78, 0.32))$	$(2, (0.42, 0.67))$	$(4, (0.81, 0.24))$
c_2	$(3, (0.69, 0.46))$	$(2, (0.48, 0.61))$	$(2, (0.61, 0.46))$	$(2, (0.62, 0.57))$	$(3, (0.68, 0.49))$	$(5, (0.92, 0.09))$
c_3	$(2, (0.36, 0.73))$	$(3, (0.62, 0.44))$	$(0, (0.11, 0.97))$	$(2, (0.60, 0.56))$	$(4, (0.92, 0.14))$	$(3, (0.64, 0.45))$
c_4	$(3, (0.67, 0.46))$	$(2, (0.62, 0.58))$	$(1, (0.26, 0.79))$	$(1, (0.24, 0.82))$	$(4, (0.85, 0.28))$	$(1, (0.26, 0.87))$
c_5	$(3, (0.69, 0.45))$	$(1, (0.29, 0.84))$	$(0, (0.11, 0.94))$	$(2, (0.61, 0.64))$	$(1, (0.34, 0.82))$	$(4, (0.89, 0.28))$

(5) $(F_P, Z, N) \cup_\zeta ((K_Q, L, M) \cap_\xi (G_T, B, Y)) = ((F_P, Z, N) \cup_\zeta (K_Q, L, M)) \cap_\xi ((F_P, Z, N) \cup_\zeta (G_T, B, Y))$;

(6) $(F_P, Z, N) \cap_\zeta ((K_Q, L, M) \cup_\xi (G_T, B, Y)) = ((F_P, Z, N) \cap_\zeta (K_Q, L, M)) \cup_\xi ((F_P, Z, N) \cap_\zeta (G_T, B, Y))$;

(7) $(F_P, Z, N) \cup_\xi ((K_Q, L, M) \cap_\zeta (G_T, B, Y)) = ((F_P, Z, N) \cup_\xi (K_Q, L, M)) \cap_\zeta ((F_P, Z, N) \cup_\xi (G_T, B, Y))$;

(8) $(F_P, Z, N) \cap_\xi ((K_Q, L, M) \cup_\zeta (G_T, B, Y)) = ((F_P, Z, N) \cap_\xi (K_Q, L, M)) \cup_\zeta ((F_P, Z, N) \cap_\xi (G_T, B, Y))$.

IV. SOME RELATIONSHIPS

In the above sections, we introduce N-soft sets into Pythagorean fuzzy environment, and construct a *PFNSS* model. In this section, we shall establish the relationships between *PFNSS* and the extant theories including Pythagorean fuzzy soft set, N-soft set and soft set. Let C be a non-empty universal set of objects and A be a set of attributes, $Z \subseteq A$. *PFN* denotes a Pythagorean fuzzy number of Z. Let $D = \{0, 1, 2, \cdots, N-1\}$ be a set of ordered grades where $N \in \{2, 3, \cdots\}$.

In order to extract Pythagorean fuzzy soft sets and N-soft sets from (F_P, Z, N), the following definition can be defined.

Definition 24: Given the threshold $0 < D < N$ for the level, the Pythagorean fuzzy soft sets over C associated with (F_P, Z, N) and D is, denoted by (F_P^D, Z), defined as: $\forall z \in Z$,

$$F_P^D(z) = \begin{cases} (c, d_z)(\mu(z), \nu(z)), & \begin{array}{l} if\ (c, d_z) \in F(z) \\ and\ d_z \geq D, \end{array} \\ \begin{cases} (0, 0.5), & if\ \dfrac{d_z}{N} \geq 0.5, \\ (0, 1), & if\ \dfrac{d_z}{N} < 0.5, \end{cases} & otherwise. \end{cases}$$

Next, we can extract N-soft sets from (F_P, Z, N) by using a threshold concerning the score function of *PFN*s.

Definition 25: Given the threshold $\rho \in [-1, 1]$ for the score function of *PFN*s, the N-soft set over C associated with (F_P, Z, N) and ρ is, denoted by (F_{P^ρ}, Z, N), defined as: $\forall z \in Z$,

$$F_{P^\rho}(z) = \begin{cases} (c, d_z), & if\,(c, d_z) \in F(z)\ and\ S_z(c) > \rho, \\ \begin{cases} 1, & if S_z(c) > 0, \\ 0, & if S_z(c) \leq 0, \end{cases} & otherwise. \end{cases}$$

Definition 26: Given the threshold $0 < D < N$ for the level and the threshold $\rho \in [-1, 1]$ for the score function of *PFN*s, the soft set over C associated with (F_P, Z, N) and (D, ρ) is, denoted by $(F_P^{(D,\rho)}, Z)$, defined as: $\forall z \in Z$, $F_P^{(D,\rho)}(z) = \{c \in C : S_z^{F_P^D}(c) > \rho\}$.

Example 27: Consider $(F_P, Z, 5)$ in Example 10. Take $D = 2$ and $\rho = 0.2$, then Pythagorean fuzzy soft sets and N-soft sets can respectively be obtained from the corresponding thresholds which are shown in Tables 14-15. Meanwhile, when taking $(D, \rho) = (2, 0.2)$, we can get the soft set $F_P^{(2,0.2)}(z) = \{c_1, c_3, c_4\}$.

In view of the foregoing analysis, it is observed that *PFNSS* in a certain condition can convert into Pythagorean fuzzy soft set, N-soft set and soft set, respectively. In other words, *PFNSS* is a generalization of Pythagorean fuzzy soft set, N-soft set and soft set.

V. RELATED ALGORITHMS

In order to demonstrate the effectiveness of *PFNSSs*, this section will establish two decision making methods on *PFNSS* model.

A. ALGORITHM 1 THE ALGORITHM OF CHOICE VALUES OF PFNSSs

1. Input $C = \{c_1, c_2, \cdots, c_n\}$ as a universe of objects.
2. Input $Z = \{z_1, z_2, \cdots, z_m\}$ as a set of attributes.

TABLE 14. Pythagorean fuzzy soft set associated with $(F_P, Z, 5)$-soft set in Example 10 and threshold $D = 2$.

$(F_P^2, O, 5)$	z_1	z_2	z_3	z_4	z_5
c_1	$(0, 1)$	$(0.62, 0.64)$	$(0, 1)$	$(0.78, 0.32)$	$(0, 1)$
c_2	$(0.58, 0.63)$	$(0, 1)$	$(0.61, 0.46)$	$(0.62, 0.57)$	$(0.58, 0.61)$
c_3	$(0, 1)$	$(0, 1)$	$(0, 1)$	$(0.60, 0.56)$	$(0.92, 0.14)$
c_4	$(0, 1)$	$(0.62, 0.58)$	$(0, 1)$	$(0, 1)$	$(0.64, 0.37)$
c_5	$(0.63, 0.62)$	$(0, 1)$	$(0, 1)$	$(0.61, 0.64)$	$(0, 1)$

TABLE 15. N-soft set associated with $(F_P, Z, 5)$-soft set in Example 10 and threshold $\rho = 0.2$.

$(F_{P^{0.2}}, O, 5)$	z_1	z_2	z_3	z_4	z_5
c_1	0	0	0	3	0
c_2	0	0	1	1	0
c_3	0	0	0	1	4
c_4	0	1	0	0	2
c_5	1	0	0	0	0

3. Input (F, Z, N) with $D = \{0, 1, 2, \cdots, N-1\}$, $N \in \{2, 3, \cdots\}$, for each $c_i \in C$, $z_j \in Z$, there exists $d_{ij} \in D$.
4. Input *PFNSS* (F_P, Z, N).
5. Calculate $H_i = (\sum_{j=1}^{m} d_{ij}, \sum_{j=1}^{m} R_{z_{ij}})$ when $(c_i, d_{ij}) \in F(z)$ and $R_z = \frac{1}{2} + r_z(\frac{1}{2} - \frac{2\theta_z}{\pi})$.
6. Calculate all the indices k for which $H_k = maxH_i$ with $i = 1, 2, \cdots, n$.
7. Any of the alternatives for which $H_k = maxH_i$ can be chosen.

B. ALGORITHM 2 THE ALGORITHM OF D-CHOICE VALUES OF PFNSSs

1. Input $C = \{c_1, c_2, \cdots, c_n\}$ as a universe of objects.
2. Input $Z = \{z_1, z_2, \cdots, z_m\}$ as a set of attributes.
3. Input (F, Z, N) with $D = \{0, 1, 2, \cdots, N-1\}$, $N \in \{2, 3, \cdots\}$, for each $c_i \in C$, $z_j \in Z$, there exists $d_{ij} \in D$.
4. Input *PFNSS* (F_P, Z, N) and D threshold.
5. Calculate

$$F_P^D(z) = \begin{cases} (c, d_z)(\mu(z), \nu(z)), & if\,(c, d_z) \in F(z)\ and\ d_z \geq D, \\ \begin{cases} (0, 0.5), & if\,\frac{d_z}{N} \geq 0.5, \\ (0, 1), & if\,\frac{d_z}{N} < 0.5 \end{cases} & otherwise. \end{cases}$$

6. Calculate all the indices k for which $H_k^D = maxH_i^D$, where $H_i^D = \frac{\sum_{j=1}^{m} S_{z_{ij}}(c_i)}{m}$, $S_z(c) = \mu_z^2(c) - \nu_z^2(c)$ with $i = 1, 2, \cdots, n$.
7. Any of the alternatives for which $H_k^D = maxH_i^D$ can be chosen.

VI. APPLICATION IN PRACTICAL DECISION MAKING PROBLEMS

Multi-attribute decision making is a decision-making problem that selects the best alternative or sorts the scheme when considering multiple attributes or indicators. Group decision-making is a decision-making method in which multiple decision makers participate in negotiation to solve problems. And thus the multi-attribute group decision-making is a new interdisciplinary research area across multi-attribute decision making and group decision-making. In other words, the essence of multi-attribute group decision-making problem is to integrate the opinions and preferences of multiple decision makers to make the objective, fair, scientific and democratic evaluation of each alternative under multiple attributes or indicators environment. It has a wide range of applications and practical backgrounds in economics, management, engineering and military. However, in many practical multi-attribute group decision-making problems, due to the complexity of decision-making problems and the indeterminacy or fuzziness of human thinking, when they evaluate the objects, decision-makers often give their subjective preference information for uncertainty including interval number, linguistic interval, intuitionistic fuzzy number, hesitant fuzzy number and Pythagorean fuzzy number, etc. Although the research on decision-making problems with uncertain preference information draws more and more attention from domestic and foreign scholars, it is deficient both in theoretical analysis and practical application, which needs to be studied deeply and systematically. So far, no matter home or abroad the researches on multiple attribute group decision-making problems with uncertain preference information concerning the combination of Pythagorean fuzzy number and N-soft set are still blank both in theoretical analysis and practical application. And for that, this section studies the hybrid multiple attribute decision-making problem with uncertain preference information, in which the information about attribute can be divided into two categories: one is the information provided by practitioners with the same multiple fuzzy characteristics; the other is the information provided by practitioners with different multiple fuzzy characteristics.

In what follows we use the *PFNSS* model to study the problem relating to scenic spot selection from multi-attribute group decision-making perspective.

A. SCENIC SPOT SELECTION BASED ON PFNSSs

With the continuous development of society, no matter where you live worldwide, no matter what kind of power and wealth you have and no matter what field you have been in, there will always lead to fluctuations in psychology due to various factors. Some of these factors may cause such

TABLE 16. Evaluation information of tourist attractions on different web-sites.

A/W	w_1	w_2	w_3	w_4	w_5
a_1	♡♡♡	♡♡♡♡♡	♡♡♡	♡♡♡♡♡	♡♡♡♡
a_2	♡♡	♡♡♡♡♡	♡♡♡♡	♡♡♡♡♡	♡♡♡♡
a_3	♡	♡♡♡	♡♡	♡♡♡	♡
a_4	♡	♡♡♡♡	♡♡♡	♡♡♡	♡
a_5	∘	♡♡♡♡	♡♡♡	♡♡♡♡♡	♡♡♡♡
a_6	♡	♡♡♡♡	♡♡♡	♡♡♡	♡♡♡

phenomena as uneasiness, impulsiveness, anxiety and gloom, etc. When facing these psychological problems, people will always choose to put aside what they are doing and look for opportunities to solve this uncomfortable and impetuous psychological problem. Meanwhile, there are variations in the choice of relaxation to alleviate their psychological pressure among individuals. Different people will have the different engagement way. Some choose sports to relax; some choose to drink; some choose to talk with others, even many girls choose cry to relieve pressure of depression. Now, travel is one of the best ways of relaxing for many people among the ways of relaxing. Tourism can improve our impetuous and uneasy heart not only, also broaden our horizons and train our emotional experiences, but also expand our social circles in the process of tourism. Given the above, tourism is one activity performed by the combination of reducing stress, mental relaxation, vision cleared and nurturing spirituality. Today, with the development of economy and the improvement of people's living standard, tourism has become an essential part of our life and tourism industry will definitely turn into an industry with long-term growing, which may lead to a dramatic increase in the number of tourism companies. For the moment, with the prosperous development of information technology and tourism industry, the amount of tourism web-sites also increases sharply and these different tourism web-sites will provide many different tourist information. Yet even so, it is especially difficult for most of us to travel due to some factors such as time, energy and financial resources. In fact, even if we have decided to travel, it is also a very difficult question to decide where to go and choose a scenic spot. In this subsection, we shall use our model to solve the decision making problem concerning the selection of tourist attractions.

Now, we know that when a person decides to have a trip, it is very important to decide where to go and choose a scenic spot because of the influence of factors such as time, money and other factors. Generally speaking, the same tourist attraction provided by different web-sites has different star ratings and word of mouth. As a result, when choosing a scenic spot we are eager to query the star ratings of tourist attractions by different web-sites to determine our destination.

Assume that the traveling people have conducted a rough screening of scenic spots. There are six scenic spots $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$, where $a_1 = Potala\ Palace$, $a_2 = Jiuzhaigou$, $a_3 = Yarlung\ Zangbo\ Grand\ Canyon$, $a_4 = Qinghai\ Lake$, $a_5 = Shangri - La\ Grand\ Canyon$

TABLE 17. Tabular representation of 6-soft sets on tourist information.

A/W	w_1	w_2	w_3	w_4	w_5
a_1	3	5	3	5	4
a_2	2	5	4	5	4
a_3	1	3	2	3	1
a_4	1	4	3	3	1
a_5	0	4	3	5	4
a_6	1	4	3	3	3

Balague zong and $a_6 = Jade\ Dragon\ Snow\ Mountain$ represent six scenic areas as a universe, respectively. Meanwhile, the travellers inquire about the star ratings of these scenic spots through the web-sites. Given that $W = \{w_1, w_2, w_3, w_4, w_5\}$ is a set of attributes, where $w_1 =$ *www.ilvping.com/*, $w_2 =$ *www.qunar.com/*, $w_3 =$ *www.tripadvisor.cn/*, $w_4 =$ *lvyou.baidu.com/* and $w_5 =$ *www.ctrip.com/* represent respectively five different tourism web-sites in which these scenic spots mentioned above are classified into six grades based on the overall satisfaction related to the same five factors of scenic spots including beauty, characteristics, humanities, leisure and comprehensive management. As a result, we can obtain a 6-soft set shown in Table 16, where

- Five hearts represent 'Very satisfied',
- Four hearts represent 'Satisfaction',
- Three hearts represent 'More satisfied',
- Two hearts represent 'General',
- One heart represents 'Difference',
- Hollow circle represents 'Very bad'.

The grade evaluation by hearts can be represented by numbers as $D = \{0, 1, 2, 3, 4, 5\}$, where

- 0 means as '∘',
- 1 means as '♡',
- 2 means as '♡♡',
- 3 means as '♡♡♡',
- 4 means as '♡♡♡♡',
- 5 means as '♡♡♡♡♡'.

The ranking information extracted from different tourism web-sites is shown in Table 16, and thus tabular representation of 6-soft set is given by Table 17. Therefore, by Definition 9, $(F_P, W, 6)$-soft set is shown in Table 18. In Table 18, it is observed that the evaluation grade for the scenic spots provided by different tourism web-sites on the five parameters (like beauty, characteristics, humanities, leisure and comprehensive management) is known while the precise

TABLE 18. Tabular representation of $(F_P, W, 6)$-soft sets on tourist information.

$(F_P, W, 6)$	w_1	w_2	w_3	w_4	w_5
a_1	(3, (0.67, 0.42))	(5, (0.91, 0.12))	(3, (0.55, 0.48))	(5, (0.93, 0.07))	(4, (0.86, 0.27))
a_2	(2, (0.42, 0.59))	(5, (0.9, 0.12))	(4, (0.85, 0.48))	(5, (0.93, 0.09))	(4, (0.72, 0.46))
a_3	(1, (0.22, 0.84))	(3, (0.61, 0.42))	(2, (0.37, 0.68))	(3, (0.65, 0.32))	(1, (0.14, 0.83))
a_4	(1, (0.21, 0.75))	(4, (0.84, 0.23))	(3, (0.56, 0.47))	(3, (0.51, 0.48))	(1, (0.26, 0.72))
a_5	(0, (0.06, 0.92))	(4, (0.86, 0.25))	(3, (0.57, 0.49))	(5, (0.93, 0.04))	(4, (0.87, 0.29))
a_6	(1, (0.13, 0.88))	(4, (0.89, 0.19))	(3, (0.56, 0.42))	(3, (0.63, 0.47))	(3, (0.59, 0.42))

TABLE 19. Tabular representation of choice value of $(F_P, W, 6)$-soft sets.

$(F_P, W, 6)$	w_1	w_2	w_3	w_4	w_5	H_i
a_1	(3, (0.67, 0.42))	(5, (0.91, 0.09))	(3, (0.55, 0.48))	(5, (0.93, 0.07))	(4, (0.86, 0.27))	(20, 3.742249)
a_2	(2, (0.42, 0.59))	(5, (0.9, 0.12))	(4, (0.85, 0.48))	(5, (0.93, 0.09))	(4, (0.72, 0.46))	(20, 3.496243)
a_3	(1, (0.22, 0.84))	(3, (0.61, 0.42))	(2, (0.37, 0.68))	(3, (0.65, 0.32))	(1, (0.14, 0.83))	(10, 1.970615)
a_4	(1, (0.21, 0.75))	(4, (0.84, 0.23))	(3, (0.56, 0.47))	(3, (0.51, 0.48))	(1, (0.26, 0.72))	(12, 2.372383)
a_5	(0, (0.06, 0.92))	(4, (0.86, 0.25))	(3, (0.57, 0.49))	(5, (0.92, 0.04))	(4, (0.87, 0.29))	(16, 3.104633)
a_6	(1, (0.13, 0.88))	(4, (0.89, 0.19))	(3, (0.56, 0.42))	(3, (0.63, 0.47))	(3, (0.59, 0.42))	(14, 2.682926)

TABLE 20. Tabular representation of 3-choice value of $(F_P, W, 6)$-soft sets.

$(F_P^3, W, 6)$	w_1	w_2	w_3	w_4	w_5	H_i^3
a_1	(0.67, 0.42)	(0.91, 0.09)	(0.55, 0.48)	(0.93, 0.07)	(0.86, 0.27)	0.5383
a_2	(0, 1)	(0.9, 0.12)	(0.85, 0.48)	(0.93, 0.09)	(0.79, 0.46)	0.29026
a_3	(0, 1)	(0.61, 0.42)	(0, 1)	(0.65, 0.32)	(0, 1)	−0.4968
a_4	(0, 1)	(0.84, 0.23)	(0.56, 0.47)	(0.51, 0.48)	(0, 1)	−0.2450
a_5	(0, 1)	(0.86, 0.25)	(0.57, 0.49)	(0.92, 0.04)	(0.87, 0.29)	0.2559
a_6	(0, 1)	(0.89, 0.19)	(0.56, 0.42)	(0.63, 0.47)	(0.59, 0.42)	0.0482

evaluation for the scenic spots provided by different tourism web-sites on the five parameters is unknown. For example, under the overall evaluation index system for the five parameters, the tourism web-site w_1 = *www.ilvping.com/* deems the Potala Palace a_1 as 3 stars which indicate 'more satisfied'. Meanwhile, the tourism web-site w_1 = *www.ilvping.com/* can provide the membership degree of 0.67 to describe the degree to which the Potala Palace a_1 is satisfying; they can also provide the non-membership degree of 0.42 to describe the degree to which the Potala Palace a_1 is satisfying.

1) CHOICE VALUES OF *PFNSS*s

Based on Table 18, each $H_i (1 \leq i \leq 6)$ can be calculated by using Algorithm 1 and thus Table 19 can be obtained. According to the choice values $H_i (1 \leq i \leq 6)$ provided by Table 19, the ranking of the above mentioned tourist attractions is listed as follows: $a_1 > a_2 > a_5 > a_6 > a_4 > a_3$. As you can see, the tourist attraction *Potala Palace* is selected as the most perfect one.

2) *D*-CHOICE VALUES OF *PFNSS*s

Now, suppose that travelers only consider the scenic spots with star ratings no less than 3-star, i.e., we take $D = 3$ in Algorithm 2. Based on Table 18, F_P^D can be calculated by using Algorithm 2 and thus Table 20 can be obtained. According to *D*-choice values $H_i^D (1 \leq i \leq 6)$ with $D = 3$ provided by Table 20, the ranking of tourist attractions mentioned above is listed as follows: $a_1 > a_2 > a_5 > a_6 > a_4 > a_3$. As a result, the tourist attraction *Potala Palace* is still selected as the most perfect one.

B. COMPARATIVE ANALYSIS

As mentioned above, Akram *et al.* have successively introduced the concepts of *NSS*, *FNSS* and *IFNSS* and applied them to decision processing problems. In this section, we shall compare the proposed *PFNSS* model with the existing models.

Applying the methods of *NSS*, *FNSS*, *IFNSS* and *PFNSS*, we reconsider the practical decision making problem in subsection 6.1. Comparison results are shown in Table 21.

When applying the method of *NSS*, we find that the scenic spots a_1 and a_2 have the same score, so it is difficult to determine the priority of the two scenic spots. The reason is that *NSS* is a method to deal with the evaluation information without considering the credibility or other aspects, so the decision result may not perfect and scientific. As an extension model of *NSS*, *FNSS* considers the membership degree of evaluation parameters. Applying the method of *FNSS*, it can be seen from Table 21 that there is no difference seen between the scenic spots a_1 and a_2. The reason is that *FNSS* only considers the membership degree of the evaluation parameters, and doesn't take into account the effects of other factors, so the decision result may also imperfect and unreasonable. On the other hand, although *IFNSS* extends *FNSS* and considers the degrees of membership and non-membership, it can be seen from Table 21 that the distinction between the scenic spots a_1 and a_2 is still quite narrow. As a result, in order to significantly distinguish the differences between the scenic spots we should take into account more aspects including commitment strength and commitment direction as the ranking function. In this context, we can apply *PFNSS* to deal with the problem. From Table 21, we observe that the difference between the scenic spots a_1 and a_2 is significant based on the method of *PFNSS*. The reason is that when dealing with the ranking of evaluation information, *PFNSS* involves more aspects including the degrees of membership and non-membership, commitment strength and commitment direction of evaluation parameters. Therefore, the decision results based on the method of *PFNSS* has more reliability and strongly persuasion.

TABLE 21. Comparison table of *PFNSS* with existing models.

$(F_P, W, 6)$	$NSS(\sigma_i)$	$FNSS(Q_i)$	$IFNSS(S_i)$	$PFNSS(H_i)$
a_1	20	$(20, 3.92)$	$(20, 3.16)$	$(20, 3.742249)$
a_2	20	$(20, 3.82)$	$(20, 3.01)$	$(20, 3.496243)$
a_3	10	$(10, 1.99)$	$(10, -0.75)$	$(10, 1.970615)$
a_4	12	$(12, 2.38)$	$(12, -0.19)$	$(12, 2.372383)$
a_5	16	$(16, 3.28)$	$(16, 1.64)$	$(16, 3.104633)$
a_6	14	$(14, 2.8)$	$(14, 0.72)$	$(14, 2.682926)$

All in all, *PFNSS* allows the sum of membership and non-membership of parameters to be greater than 1 when dealing with uncertainty, and also takes into account the influence of commitment strength and commitment direction. Compared with the existing models, the discriminatory power of *PFNSS* models in the process of decision making can be significantly improved. Therefore, *PFNSS* is more reasonable and practical method under different complex environments.

VII. CONCLUSION

The main purpose of this study is to introduce a Pythagorean fuzzy N-soft set by integrating Pythagorean fuzzy set with N-soft set, and to quantify the uncertainty of the overall rank evaluation under Pythagorean fuzzy environment where the sum of the membership degree and non-membership degree is greater than 1. We have also examined some of interesting operational properties, such as weak complement, extended intersection, extended union, restricted intersection and restricted union. In addition, we have established two algorithms to deal with multi-attribute group decision making problems and verified the effectiveness of the algorithms by using practical problems. In the future, the research on algebraic structure of Pythagorean fuzzy N-soft sets is a very important and interesting question for us.

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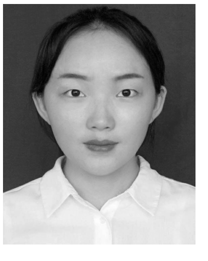

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