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# Combining Cooperative With Non-Cooperative Game Theory to Model Wi-Fi Congestion in Apartment Blocks

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**ABSTRACT** The unlicensed spectrum utilized by Wi-Fi can be regarded as an economic commons in many deployments. Operators of Wi-Fi-enabled devices are usually non-cooperative, vying for spectral resources when in close range to each other, typically adopting a strategy of maximizing their transmission power. With an ever-growing number of wireless devices, this will ultimately lead to depletion of the spectrum, unless players collaborate. Previous studies used cooperative game theory to explore various collaboration strategies, enabled by the presence of some central authority or controller that executes an agreed-upon interference mitigation policy. However, the regulatory nature of unlicensed spectrum dictates that players cannot be forced into such collaboration. Most deployments therefore involve a mix of cooperative and non-cooperative players. In this paper, we propose a new way of modeling use cases involving a central authority or controller by combining non-cooperative and cooperative game theory. Our model uses the non-cooperative concept of Nash equilibriums as well as the cooperative concept of Nash bargaining. To the best of our knowledge, this paper is the first to propose a hybrid non-cooperative and cooperative game theoretic model for communication networks that offers the players the opportunity to strategize between non-cooperative and cooperative natures. It is successfully applied to the case of a densely-populated apartment block. The results show that, if only a subset of players joins the collaboration, most of the remaining non-joining players may obtain an SINR that is worse than what they would have obtained in the fully non-cooperative scenario; they are punished for their uncooperative behavior.

**INDEX TERMS** Computer network management, game theory, interference suppression, utility theory, wireless communication.

## I. INTRODUCTION

The vast popularity of smart devices is one of the main contributors to the high density of Wi-Fi Access Points (APs) in today's homes, offices and public spaces with an Internet connection. The drawback of this dense deployment is the potential for co-channel and adjacent channel interference with nearby devices. With an ever-growing number of wireless devices in urban areas, driven by developments such as the Internet of Things, this type of spectral congestion will lead to a significant loss of performance [1].

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Current channel selection procedures have only little effect, and the deployment of additional repeaters and APs also fails [2]. This is because Wi-Fi uses unlicensed spectrum, which in use cases such as deployments in densely populated apartment blocks can be regarded as an economic commons [3]. As Wi-Fi AP operators (often the residents themselves) are usually non-cooperative, they typically adopt a strategy in which they maximize their transmission power for a given transmission frequency (channel). The channel may be chosen to be a relatively non-congested one but, nowadays, all available channels are often equally badly congested. Most APs deployed in today's homes operate at maximum power by default. Therefore, users can only achieve

a higher Signal-to-Interference-and-Noise Ratio (SINR) for their consumer devices by adding more APs and repeaters to the home Wi-Fi network. However, this strategy will ultimately lead to the depletion of the spectral resources [1], [2]. AP operators thus need to collaborate to prevent depletion from happening [4], [5].

Previous studies used cooperative game theory to explore various collaboration strategies, and assumed the presence of some central authority or controller that executes an agreed-upon interference mitigation policy. In the Wi-5 project [6], we developed such a controller. It implements an integrated set of algorithms and mechanisms to improve the utilization of the wireless spectrum. More specifically, it continuously monitors the radio properties and traffic characteristics of the associated APs, and subsequently performs transmit power control, dynamic channel selection, device hand-overs, and frame aggregation. The controller thus executes the resource distribution policy as agreed upon by the AP operators.

However, the regulatory nature of unlicensed spectrum dictates that Wi-Fi users cannot be forced into such collaboration [3]. Realistically speaking, most use cases will thus consist of a mix of both cooperative and non-cooperative players (users). In this paper, we propose a new way of modeling use cases involving a controller by combining non-cooperative and cooperative game theory. This model is then used to investigate whether users have an incentive to collaborate with each other via the controller when their objective is to maximize the SINR.

In the following section, we first provide an overview of the relevant literature. In sections III and IV, we then develop a mixed cooperative/non-cooperative model, using the non-cooperative concept of Nash equilibriums as well as the cooperative concept of Nash bargaining. It facilitates the option of considering both Wi-Fi users who join and users who do not join the controller's regime simultaneously, while enabling them to take joint strategic decisions. The model is subsequently applied to a typical dense apartment block, as described in [2] and [7].

## II. RELATED WORK

### A. MODELING SPECTRUM ALLOCATION WITH GAME THEORY

In the last two decades, extensive research has been performed on Spectrum Allocation (SA) in wireless networks and transmission power control using game theory. Game theoretic models describe the interaction among rational, mutually aware players, where the decisions of a player may impact the payoff of others. A game is described by its players, the players' strategies, and the resulting payoffs from each outcome. Payoff is often defined by utility, which represents the satisfaction experienced by the consumer of a good. For a given player, the utility function then expresses utility as a function of the amounts of the various goods consumed.

An overview of the most relevant literature on modeling communication networks using game theory can be found in [8] and [9]. In this paper we focus on the research using non-cooperative and cooperative game theory. Non-cooperative games are games in which players selfishly try to maximize their utility, conditional on the rational choices made by the other players who try to do the same. A stable distribution of utilities can only be enforced through self-enforcing agreements such as a Nash Equilibrium (NE). Cooperative games, in contrast, assume that players adhere to a jointly chosen strategy, which can be externally enforced.

### B. NON-COOPERATIVE GAMES

Potential games are a type of non-cooperative strategic game in which the existence of a NE is guaranteed. They are popular building blocks to model wireless networks [10]. Potential games are first used in the context of wireless radio networks in [11], and a framework for transmission Power Allocation (PA) using this technique is proposed in [12]. The authors of [12] define the radios as the players, who are allowed to choose any power level. The utility function of the radios depends on the SINR and the costs associated with choosing a certain power level. In [13], the effect of adding a pricing element to the utility of selfish players is investigated. In addition to transmission power control, the authors of [14] take channel allocation into account in their model. They show that combined management of channel allocation and transmission power leads to improved normalized achievable throughput, compared to optimizing channel allocation and transmit power independently. Potential games are also used in [15]–[17] for SA. The authors of [15] show how this technique can optimize the use of spectrum and converge to NE in a short period of time, while the authors of [16] apply potential games to mitigate the interference that is caused by massive deployment of unmanned aerial vehicles. In [17] internal interference among users and external interference caused by jammers is considered to propose a distributed anti-jamming channel selection algorithm.

A recent literature study on resource allocation in wireless networks is presented in [18]. The authors of [18] consider a densely deployed AP scenario to propose an adaptive and distributed spectrum management algorithm. They take each AP as a rational player, define one AP as the row player, and group all other interfering APs as the column player of the game. They eliminate the need for a central resource manager by using a distributed approach, and their algorithm can handle changes in the wireless channel and in the number of APs joining to network. The authors of [19] use non-cooperative game theory to model use cases with Cognitive Radios (CR) to balance the conflicting requirements of spectrum assignment to Primary Users (PU, i.e. users of licensed spectrum) and Secondary Users (SU, i.e. users of unlicensed spectrum). In CR, PUs try to gain financial profit by leasing their unused spectrum, and SUs try to meet their Quality of Service (QoS) requirements by using the licensed spectrum as cost-effective as possible. In [19], a model is

developed for power trading, and Stackelberg games are used to reach NE where the PUs gain the optimal profit and the SUs use their optimal transmit power. The problem of their proposed model lies in the high chance of monopoly behavior from PUs, because of the freedom they have in adjusting the price dynamically.

Other types of non-cooperative game theory have also been used for spectrum allocation. A strategic bargaining game is used in [20] to find the Nash bargaining solution for spectrum allocation. Without knowledge about competitors, users of licensed and unlicensed spectrum (the latter being wireless sensors, for instance) can choose their best strategy to share the spectrum, including the possibility to share licensed spectrum with unlicensed users. The non-cooperative distributed model applied in [20] is similar to what is used in [18], and removes the need for a central operator. In [21], a game-theoretic solution is investigated for spectrum allocation in highly mobile vehicular networks. An incentive mechanism is designed to motivate the macro cell to share its unused bandwidth with Road Side Units (RSU). While considering transmission power and interference of RSUs, the authors of [21] then formulate the resource allocation problem for moving vehicles as an  $n$ -person game and calculate the equilibrium.

Power control is the main metric in [22] to decrease the interference and improve the throughput. By using transmission power as the cost function, non-cooperative game theory is leveraged to achieve NE.

The major limitation in all the previous work mentioned is the lack of differentiation between wireless users. In most realistic use cases, users have different Quality of Service (QoS) needs and pay higher fees to the service provider for higher needs. Different bandwidth requirements have been considered in [23], but only non-cooperative and selfish behavior of wireless users has been taken into account, which is mitigated by assigning a pre-defined channel to each user and a heavy fine for defiance and occupying another channel.

### C. COOPERATIVE GAMES

Two classes of cooperative games are used in modeling wireless networks: coalition games and bargaining games. Coalition games can be used to decide on optimal collaboration strategies [24], and are applied to unlicensed spectrum interference modeling for Wi-Fi and Long Term Evolution-Unlicensed (LTE-U) coexistence in [25]. In this paper, we do not consider coalition games because they do not scale: as the non-transferable QoS of individual players must be considered, and the QoS of a coalition also depends on all other players in the system, these games require  $O(n^n)$  computations in a system with  $n$  players.

Bargaining games are cooperative games in which all players may decide to cooperate while observing a disagreement point. The disagreement point represents the utility the players can expect to receive if negotiations break down and they decide not to cooperate. It can be defined in various ways. For example, in [26], a bargaining model is proposed for SA

and PA problems in cooperative communication networks. The authors divide the game into two sub-games, one for spectrum allocation and another one for power allocation, to decrease the complexity. The game is solved using Nash bargaining in which the disagreement point is the capacity that the selfish node can gain by direct transmission. In [27], a bargaining model is proposed for throughput sharing or channel access time sharing. The players are licensed mobile users who lease their unused spectrum and unlicensed Wi-Fi users who bargain to occupy it. The disagreement point is defined as the sum of achievable throughput when there is no cooperation among players.

In [28], a bargaining model for channel selection is proposed in which the disagreement point is chosen as the threat made by individual players to other players. The threats are defined as the chosen policy of a player, with the other player not acting in compliance with the former player's will. The utility set is based on the SINR. In [29], bargaining is used to deal with the case in which players only know the frequency channel and transmission power of other players that are physically close, i.e., only local information is available. In [30], a centralized platform is described to which state-of-the-art interest-independent and self-configured Wi-Fi APs connect, but which does not control the APs directly. It only provides the APs with information about how they mutually interfere and entices them to reduce their own power levels based on the cooperative Nash bargaining rule.

Of course, many papers exist in which bargaining games are applied to mobile networks such as 5G, as these networks are centrally controlled by definition. For instance, the authors of [31] use cooperative Nash bargaining to investigate the joint uplink subchannel and power allocation problem in cognitive small cells.

### D. COMBINING COOPERATIVE AND NON-COOPERATIVE GAMES

The Wi-5 project allows cooperative players to join a collaborative, overarching system-wide configuration optimization mechanism, i.e., the APs of cooperative players can be configured by a central controller that globally optimizes the settings. This allows for coalition formation between the cooperative players, execution of joint actions, and collective utility optimization, and should be modelled with cooperative game theory.

However, Wi-Fi operates in an unlicensed part of the radio spectrum. Current regulations regarding access to unlicensed spectrum and anti-trust make it impossible to force Wi-Fi AP operators to cooperate or collaborate in an overarching Wi-Fi interference optimization scheme. Participation should thus be voluntarily, and AP operators should be enticed to cooperate by means of a positive business case [3]. It can therefore be realistically expected that many apartment blocks will be home to both players who will join and players who will not join an interference optimization scheme. Consequently, our case should be modelled using a combination of non-cooperative and cooperative game theory.

Models that embed both non-cooperative and cooperative game theory are relatively scarce in the literature on communication networks. The authors of [32] investigate the efficiency of radio spectrum utilization when users join in a cooperative as well as non-cooperative manner, they do not propose a combined game theoretical model. The authors just observe that more efficiency is obtained when applying a cooperative joining strategy, but as there is a lack of incentive for the users to act cooperatively, they propose to define an admission fee in the non-cooperative scenario, and show that a near optimal utilization of spectrum usage can then be achieved. The main limitation of their approach is the assumption that all players act selfishly and make strategic decisions independently. In our case, however, players who join the interference optimization scheme connect their AP to the controller and employ strategies that are in the best interest of the group rather than the individual.

In [33] and [34], the authors apply hierarchical game theoretic models that consist of a nested combination of non-cooperative and cooperative games. These games are solved sequentially for different levels of the system architecture (e.g., operators and users), where negotiation between the players of different levels affects the utilities that can be obtained. In hierarchical games, players do not have the ability to convert from non-cooperative to cooperative strategies and vice versa. This is different from our set-up, where users can be assigned a cooperative strategy if joining the interference optimization scheme yields a higher utility. To the best of our knowledge, this paper is the first to propose a combined non-cooperative and cooperative game theoretic model for a communication network that offers the players to strategize between non-cooperative and cooperative natures.

### III. PRELIMINARIES

#### A. JOINING AND NON-JOINING PLAYERS

In this paper we consider non-cooperative strategic games with the one-shot solution concept of the mixed Nash equilibrium, and cooperative bargaining games with the weighted Nash bargaining solution. We assume that every player has perfect foresight, that is, each player decides unilaterally on a strategy conditional on the expected (future) decisions of all other players. We use the idea of transmission power control as in [14], but consider a different power control criterion. We also implement the concept of a pricing element [13] to prioritize the players that join a collaborative Wi-Fi interference optimization scheme. In addition, we adopt the approach of [28] that sets the disagreement point of a cooperative player equal to the utility received if one would act selfishly and non-cooperatively.

In a mixed cooperative/non-cooperative game there are two types of players who are not participating in a joined interference optimization scheme: non-cooperative players, and cooperative players who do not benefit from joining. To model this properly, we need to make a distinction between cooperative and non-cooperative players, and joining and non-joining players:

- *Cooperative players* are capable of joining the optimization scheme, but will not do so if it does not benefit them;
- *Non-cooperative players* are not capable of joining the optimization scheme, and will not do so even if it would benefit them;
- *Joining players* are cooperative players joining the optimization scheme;
- *Non-joining players* are not joining the optimization scheme.

A cooperative player will only join a collaborative interference optimization scheme if it is beneficial to that player. Therefore, the set of joining players may constitute only a subset of the total number of cooperative players, the remaining cooperative players identifying themselves as non-joining players. Fig. 1 depicts these different sets of players. The model we developed considers both joining and non-joining players simultaneously, with only joining players jointly agreeing on their AP settings.

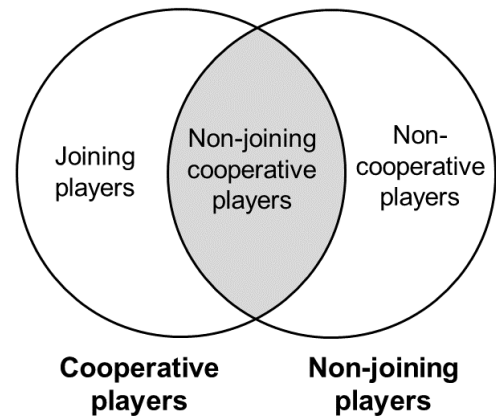


FIGURE 1. Sets of players.

#### B. THE MIXED NASH EQUILIBRIUM

Strategic games are non-cooperative games in which all players make their strategic decisions simultaneously. Strategic games are of the form

$$(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N}), \tag{1}$$

where  $N = \{1, \dots, n\}$  defines the finite set of players. The set of pure strategies of player  $i \in N$  is given by  $S_i$ , and  $S = \times_{i \in N} S_i$  is the set of pure strategy profiles. A pure strategy provides a complete definition of how a player acts in the game. The set of probability distributions over  $S_i$  is defined by  $\Delta(S_i)$ . An element  $s_i \in \Delta(S_i)$  is called a mixed strategy of player  $i \in N$ , and  $\times_{i \in N} \Delta(S_i)$  is the set of mixed strategy profiles. A strategic game is called finite if  $S_i$  is finite for all  $i \in N$ . The utility function  $u_i : S \rightarrow \mathbb{R}$  of player  $i \in N$  imposes a preference relation on the set of pure strategy profiles. The expected utility function of player  $i \in N$ ,  $U_i : \times_{i \in N} \Delta(S_i) \rightarrow \mathbb{R}$ , defines the expected utility of player  $i$ .

The mixed Nash equilibrium, first introduced in [24], is a one-shot solution concept of finite strategic games in which no player has the incentive to deviate.

*Definition 1:* Consider the strategic game  $\langle N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$ . A mixed Nash equilibrium of the strategic game is a profile  $s^* \in \times_{i \in N} \Delta(S_i)$  such that for every player  $i \in N$  it holds that

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*) \quad (2)$$

for all  $s_i \in \Delta(S_i)$ , where  $s_{-i}^*$  is the equilibrium strategy profile of all players in  $N \setminus \{i\}$ .

*Theorem 2 (Nash, [35]):* Every finite strategic game has at least one mixed Nash equilibrium.

The proof of Theorem 2 can be found in the Appendix.

### C. THE WEIGHTED NASH BARGAINING SOLUTION

Bargaining games are cooperative games in which either all players agree on an alternative or no agreement is reached at all. In the latter case, all players agree to disagree and a disagreement point is the chosen alternative. An  $n$ -person bargaining game  $B^n$  is of the form

$$B^n = (A, d), \quad (3)$$

where  $A \subseteq \mathbb{R}^n$  is the convex set of alternatives that can be achieved by the players and the vector  $d \in A$  is their disagreement point.

A well-known solution concept for bargaining games is the Nash Bargaining rule, proposed in [36]. In this paper we consider the weighted Nash bargaining rule proposed in [37].

*Definition 3:* The weighted Nash bargaining rule  $\mathcal{W}$  assigns to each bargaining game  $B^n = (A, d)$  an alternative  $\mathcal{W}(A, d) \in A$  such that

$$\mathcal{W}(A, d) = \arg \max_{a \in A} \prod_{i=1}^n (a_i - d_i)^{w_i}, \quad (4)$$

such that  $a_i \geq d_i \geq 0$  for  $i \in \{1, \dots, n\}$  and  $\sum_{i=1}^n w_i = 1$ . Note that the Nash bargaining rule is obtained for  $w_i = 1/n$  for all  $i \in \{1, \dots, n\}$ .

## IV. THE MIXED-COOPERATIVE MODEL

### A. ASSUMPTIONS

We consider three different scenarios to research whether Wi-Fi users benefit from joining a collaborative interference optimization scheme: the non-cooperative scenario, the cooperative scenario, and the mixed scenario. The mixed-cooperative model provides a one-shot solution for each of the three scenarios, i.e. the solution is found without prior iterative learning procedures.

First we introduce the model's players, parameters, and functions. Then, we describe the three scenarios. We consider a fixed set  $N = \{1, \dots, n\}$  of Wi-Fi users who are the players in our model. Each player owns a Wi-Fi receiver (station) and a transmitter (AP) node which are interconnected. The location of the nodes is given by  $(x_i, y_i, z_i)$  coordinates in the Euclidean space  $\mathbb{R}^3$ ,  $i \in N$ , where two nodes cannot have

the same coordinates. All receivers are equal, and the same holds for the transmitters.

To be able to prioritize between various players, we consider the players' individual monthly broadband contract fees  $m_i \geq 0$  for  $i \in N$ . In other words, we assume that the controller can prioritize the access of some players to the Wi-Fi network, for instance because they run a business from home (and pay a higher broadband subscription fee in return for prioritized access). Such a model may be controversial but is not illegal [3].

Each player has to make two strategic choices, namely (i) on which channel to transmit and (ii) the transmission power level. There is a fixed finite set of non-overlapping frequency channels  $C$ , which are the same for each player, from which a player can select a channel. It is realistic to assume that non-overlapping channels are selected whenever possible. So, for instance, if 3 non-overlapping channels are available (as is the case in the 2.45 GHz frequency band), and there are 3 players, they will all choose a different channel. If there are 4 players, one player has to select an already occupied channel. In a collaborative scheme, players may then, e.g., agree upon a rotation schedule regarding which players have to share a channel at any given time. Furthermore, the transmission power of the players has an upper bound  $p_{max}$ , imposed by regulations [38].

We assume that we know in advance which of the cooperative players are willing to join the collaborative Wi-Fi interference optimization scheme (joining players) and which players are not willing or able to join (non-joining players). Furthermore, we also have perfect information about the nature of the cooperative players in the current scenario: at any given time we know a priori who has (already) joined the interference optimization scheme (is a joining player), and who has not (yet) joined the scheme (is a non-joining player).

### B. THE UTILITY FUNCTION

Determining the utility of a Wi-Fi network is not a trivial task. Ideally, the utility is the Quality-of-Experience (QoE) that players observe when consuming a service that is supported by a Wi-Fi network, taking external factors into account, such as display quality, processing capabilities of the end device, and various subjective parameters. It is close to impossible to mathematically relate QoE to low-level Wi-Fi configuration settings in a practical way. Taking network throughput as a proxy for QoE seems to make more sense, but also here a simple mathematical expression is hard to obtain, as throughput also depends on switching capacity, link-layer protocol details, and the packet sizes being transported (which depends on the service consumed). In this paper we take the SINR observed by each player's receiver as utility. The SINR is expressed as follows.

Let  $p_i$  be the transmission power of player  $i$ , and  $c_i \in C$  the frequency channel selected by player  $i$ ,  $i \in N$ . The path loss between the transmitter of player  $j$  and the receiver of player

$i$  is given by [39], [40]:

$$G_{ij} = \left( 4\pi \left( \frac{300}{f \cdot 10^3} \right)^{-1} \right)^2 D_{ij}^{-\alpha_k},$$

where  $D_{ij}$  is the Euclidean distance between the transmitter and receiver under consideration, and  $f$  is the frequency in GHz at which the signal is transmitted.

The path loss exponent  $\alpha_k$  depends on whether the received signal originates from a player’s own transmitter ( $k = 1$  if  $i = j$ ) or not ( $k = 2$  if  $i \neq j$ ). This way, the effect of indoor propagation is taken into account, albeit in a simplified manner: when the receiver and the transmitter are in the same apartment ( $i = j$ ), the attenuation is smaller than when the receiver and transmitter are in different apartments ( $i \neq j$ ). Here we assume that the attenuation is equal for every  $i, j$  with  $i \neq j$ .

The utility function  $u_i$  is the SINR (in decibels) as a function of all transmission powers  $p_1, \dots, p_N$  and all channel selections  $c_1, \dots, c_N$ :

$$u_i(p_1, \dots, p_N, c_1, \dots, c_N) = 10 \log_{10} \left( \frac{G_{ii} p_i}{n_0 + \sum_{j \in N, j \neq i} G_{ij} p_j \mathbb{I}(c_i, c_j)} \right), \quad (5)$$

where  $n_0$  is a constant representing background noise from other sources, and the indicator  $\mathbb{I}(c_i, c_j)$  equals 1 if players  $i$  and  $j$  transmit on the same channel (i.e.,  $c_i = c_j$ ) and 0 otherwise. So, as stated previously, we only take co-channel interference into account, and assume that players actively avoid adjacent channel interference by selecting non-overlapping channels.

### C. THE NON-COOPERATIVE SCENARIO

In the non-cooperative scenario, we assume that all players are non-cooperative, and therefore will never join the interference optimization scheme. To determine an equilibrium strategy profile we model this scenario as a strategic game. We assume that it is in a player’s best (self-)interest to transmit with maximal transmission power. This is a direct result of the fact that, for many use cases, the Wi-Fi spectrum can be regarded as an economic commons to which the so-called Tragedy of the Commons applies [3], [41]. The authors of [41] and [42] argue that the Tragedy of the Commons is equivalent to a multi-player prisoner’s dilemma, in which the single stable dominant strategy for all players is to defect (or free-ride). Since transmitting with less than maximal transmission power is the equivalent cooperative strategy of the multi-player prisoner’s dilemma, non-cooperative players do not have an incentive to deviate from their maximal transmission power strategy.

This implies that there are  $|C|^n$  pure strategy profiles in total, since the only strategic decision that has to be made is the channel on which to transmit, and because the strategic game is finite. An equilibrium strategy profile is then a mixed Nash equilibrium as given by Definition 1, and always exists (Theorem 2). For every pure strategy profile, the equilibriums

can be computed using the utility profiles of the players provided by the utility function (5).

### D. THE COOPERATIVE SCENARIO

In the cooperative scenario, we assume that all players are cooperative players. We model this scenario as an  $n$ -person bargaining game (3). In each of the  $|C|^n$  channel allocations, a central controller determines the transmission power of the joining players. We assume that it tries to find a “fair” utility profile, i.e. a stable state in which all players obtain the maximum “fair” utility level. It does so by tuning the power settings such that whenever a particular player’s utility level is smaller than the others’, it is maximized:

$$\begin{aligned} & \max_{p_1, \dots, p_N} \min_{i \in N} u_i(p_1, \dots, p_N), \\ & \text{with } 0 < p_i \leq p_{max}. \end{aligned} \quad (6)$$

An example of an algorithm that provides such a stable state can be found in [43].

We define the alternative set  $A$  as the convex hull of these utility levels, and define the disagreement point  $d \in \mathbb{R}^n$  as the minimal utility a player requires to join the collaborative Wi-Fi interference optimization scheme. It can be obtained by calculating the player’s utility as if he would decide not to join the collaborative optimization scheme, which follows from the fully non-cooperative scenario described in the previous section. It is therefore possible that  $d \notin A$ , as  $d$  corresponds to a solution in which all players transmit with maximal transmission power, and  $A$  follows from optimized transmission powers which could all be less than the maximal powers.

If  $\{a \in A \mid a \geq d\} \neq \emptyset$ , individually rational alternatives exist in  $A$  in which all players obtain a higher (or equal) expected utility compared to the disagreement point. In such a case, all players have an incentive to continue their participation in the collaborative optimization scheme. If  $\{a \in A \mid a \geq d\} = \emptyset$ , there is no individually rational alternative, and one or more players have an incentive to discontinue the collaboration.

We use the weighted Nash bargaining rule (Definition 3, (4)) to find a solution for the previously described bargaining game. The weights  $w_i, i \in \{1, \dots, n\}$  are used to prioritize players, and relate to the  $m_i$  of the cooperative players as follows:

$$w_i = \frac{m_i}{\sum_{i=1}^n m_i}.$$

If the solution that optimizes (6) is an alternative in  $A$  other than  $d$ , the cooperative players have an incentive to join the interference optimization scheme.

### E. THE MIXED SCENARIO

In the mixed scenario there are both joining and non-joining players. The joining players are all cooperative players. The non-joining players comprise all non-cooperative players plus the subset of cooperative players for whom cooperation would not be beneficial.

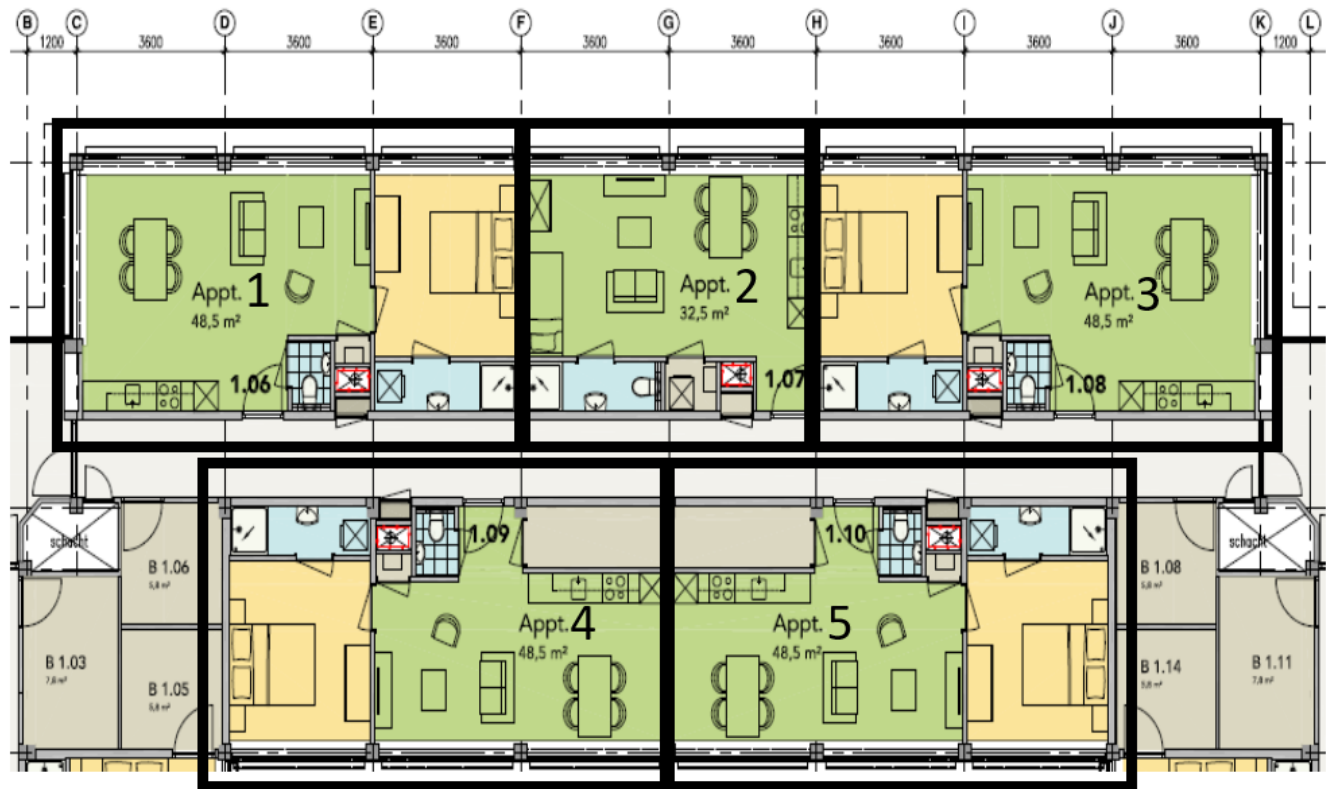


FIGURE 2. Layout of the apartment building.

Let  $N_c$  be the set of cooperative players,  $|N_c| = n_c$ , and  $N_{nc} = N \setminus N_c$  the set of non-cooperative players. The players in  $N_c$  only become joining players if their individual expected utility does not deteriorate. We model this scenario using  $n_c$ -person bargaining games,  $B^{n_c} = (A_c, d_c)$ , and strategic games for the players in  $N_{nc}$ . Let the disagreement point  $d_c \subseteq d$ ,  $d_c \in \mathbb{R}^{n_c}$ , be the sub-vector of  $d \in \mathbb{R}^n$  with indices in  $N_c$ , where  $d$  represents the utilities obtained from a mixed scenario with fewer joining players (which may be a fully non-cooperative scenario). This is a feasible choice for the disagreement point, since the non-joining cooperative players act as if they were non-cooperative players.

We compute the expected utilities (SINRs) of both the cooperative and non-cooperative players as follows:

- Step 1** Determine the optimized transmission power and the SINRs for each of the channel allocations of the cooperative players for all possible channel allocations of the non-cooperative players. Determine the SINRs for the non-cooperative players in each of their channel allocations for all possible channel allocations and for the previously determined optimal transmission powers of the cooperative players.
- Step 2** Determine the mixed Nash equilibriums of the non-cooperative players for each of the possible channel allocations of the cooperative players.
- Step 3** Determine the probability that the non-cooperative players select a particular channel, given the mixed Nash

equilibriums computed in Step 2 and the imperfect information regarding the channel selection of the cooperative players.

- Step 4** Determine the weighted Nash bargaining solutions for the cooperative players for each of the channel selections of the non-cooperative players.
- Step 5** Determine the SINRs of the cooperative players, combining the results of Step 3 and Step 4.
- Step 6** Determine the SINRs of the non-cooperative players, combining results from Step 2 and Step 4. (From Step 4 we obtain the probabilities that the cooperative players use a certain channel allocation.)

## V. RESULTS

### A. USE CASE

To illustrate the effectiveness of the mixed-cooperative model, we consider an apartment building with Wi-Fi users. The layout of a typical floor, with five apartments, is illustrated in Fig.2. For a more elaborate description of the building considered we refer to [2].

The location and dimensions of apartment  $i$  are determined by four  $(x_i, y_i)$  coordinates in the Euclidean space  $\mathbb{R}^2$ , each coordinate specifying the position of a corner of the apartment. The coordinates are given in Table 1, with respect to origin (0,0) in the bottom left corner of Fig. 2.

We assume that each apartment owner is a Wi-Fi user, and therefore a player in the context of our simulations,

TABLE 1.  $(x_i, y_i)$  coordinates of the corners of the five apartments.

| Apartment $i$ | Upper left (m) | Upper right (m) | Bottom left (m) | Bottom right (m) |
|---------------|----------------|-----------------|-----------------|------------------|
| 1             | (0.0,12.0)     | (10.8,12.0)     | (0.0, 7.2)      | (10.8, 7.2)      |
| 2             | (10.8,12.0)    | (18.0,12.0)     | (10.8, 7.2)     | (18.0, 7.2)      |
| 3             | (18.0,12.0)    | (28.8,12.0)     | (18.0, 7.2)     | (28.8, 7.2)      |
| 4             | (3.6, 4.8)     | (14.4, 4.8)     | (3.6, 0.0)      | (14.4, 0.0)      |
| 5             | (14.4, 4.8)    | (25.2, 4.8)     | (14.4, 0.0)     | (25.2, 0.0)      |

and operates one Wi-Fi AP  $i$ , for  $i \in \{1, \dots, 5\}$ . The APs are placed in the little grey hallways between the red-white cupboard and the apartments' front doors (opening toward the long hallway running through the center of the plan), and the receivers are placed in the middle of the apartments at a height of 2.0 m. The middle of the apartments is the Euclidean average of the corners of each apartment using the coordinates in Table 1.

Table 2 shows the coordinates  $(x_i, y_i, z_i)$  of the APs and the receivers. By default, the monthly contract fee of the individual players is set to be equal, so no player is prioritized.

TABLE 2. Location of the transmitters (APs) and the receivers (users' devices).

|          | Transmitter (m)     | Receiver (m)        |
|----------|---------------------|---------------------|
| player 1 | (5.40, 9.60, 1.50)  | (6.82, 7.42, 2.00)  |
| player 2 | (14.40, 9.60, 1.50) | (16.20, 7.42, 2.00) |
| player 3 | (21.98, 7.42, 1.50) | (23.40, 9.60, 2.00) |
| player 4 | (9.00, 2.40, 1.50)  | (7.58, 4.58, 2.00)  |
| player 5 | (19.80, 2.40, 1.50) | (21.22, 4.58, 2.00) |

**B. SIMULATION SET-UP**

We used MATLAB R2016a on a computer with an Intel i5-4310M 2.70GHz processor and 8GB of Random Access Memory (RAM). We used the functions *fminimax* and *fmincon* from MATLAB's optimization toolbox to compute the transmission powers (6) and the weighted Nash Bargaining solution respectively. To compute a mixed Nash equilibrium we used the method proposed in [44] and the corresponding implementation [45].

We set  $n_0 = 10^{-9}$  mW and  $\alpha_1 = 2$  as suggested in [46], and  $\alpha_2 = 4$  for inter-apartment propagation loss. The frequency  $f$  is chosen to be either 2.45 GHz or 5.21 GHz, and  $|C|$  is chosen between 1 and 3. (Although the 5.21 GHz band allows for many more channels, it was recently found that most of the current implementations do not use these additional channels [3].) The maximum transmission power  $p_{max} = 100$  mW and the distances  $D_{ij}$  are expressed in meters.

All simulations start with the non-cooperative scenario. The disagreement point thus obtained serves as input for the cooperative scenario and the mixed scenarios.

**C. THE FULLY COOPERATIVE SCENARIO**

We first investigate if the scenario where all five players join the interference optimization scheme is beneficial to all

of them, given the placement of their equipment as shown in Table 2. The players are considered to have access to channels 1, 2 or 3 in the 2.45 GHz frequency band. Table 3 and Fig. 3 show per player the SINR obtained for different numbers of available channels in the non-cooperative (NC) and cooperative (C) scenarios. The results in the NC column of the table represent the mixed Nash equilibrium, and provide the disagreement point used as input to obtain the values in the C column. The highest of the two values (C or NC) is printed bold, and the  $\Delta$  column represents the difference between the values (C minus NC).

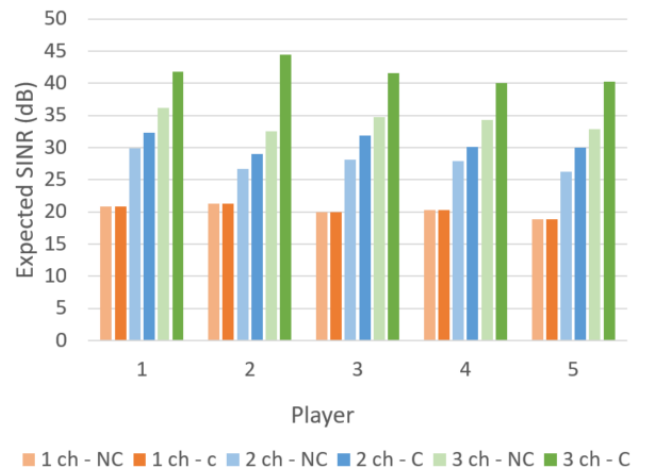


FIGURE 3. SINR per player in the non-cooperative (NC) and cooperative (C) scenario, for different numbers of available channels.

$\Delta$  is always equal to or greater than zero, indicating that joining the optimization scheme is always advantageous to the players, or at least not disadvantageous. In this case, the only reason for a player to defect cooperation is (within the scope of this paper) a non-rational one. The effect of such a situation is studied in the mixed scenario (see section V-E).

The channel allocation algorithm applied by the controller is bound from below by the disagreement points of each player (see (4), with the disagreement point equal to the SINRs of the fully non-cooperative scenario). It therefore aims to include as many players as possible first, before improving the expected individual SINRs. This provides players the incentive to join the interference optimization scheme rather than to act on their own. In other words, the controller applies an include-first, maximize-last principle.

If only one channel is available, the controller simply copies the settings from the non-cooperative players: consequently, the expected SINR of the cooperative players in Table 3 is identical to the SINR of the non-cooperative players. In this case, the choice between joining or not joining appears to be irrelevant. This can be understood by observing that, with only one channel available, the controller has only one parameter left to vary to improve the SINR of a player, namely the transmission power. But once all players have



**TABLE 3.** SINR (dB) in the non-cooperative (NC) and cooperative (C) scenario with five players.

| Channels | player 1    |             |          | player 2    |             |          | player 3    |             |          | player 4    |             |          | player 5    |             |          |
|----------|-------------|-------------|----------|-------------|-------------|----------|-------------|-------------|----------|-------------|-------------|----------|-------------|-------------|----------|
|          | NC          | C           | $\Delta$ | NC          | C           | $\Delta$ | NC          | C           | $\Delta$ | NC          | C           | $\Delta$ | NC          | C           | $\Delta$ |
| 1        | <b>20.8</b> | <b>20.8</b> | 0.0      | <b>21.3</b> | <b>21.3</b> | 0.0      | <b>20.0</b> | <b>20.0</b> | 0.0      | <b>20.3</b> | <b>20.3</b> | 0.0      | <b>18.8</b> | <b>18.8</b> | 0.0      |
| 2        | 29.8        | <b>32.3</b> | 2.5      | 26.7        | <b>29.0</b> | 2.3      | 28.1        | <b>31.9</b> | 3.8      | 27.9        | <b>30.1</b> | 2.2      | 26.2        | <b>30.0</b> | 3.8      |
| 3        | 36.2        | <b>41.8</b> | 5.6      | 32.5        | <b>44.4</b> | 11.9     | 34.7        | <b>41.6</b> | 6.9      | 34.3        | <b>40.0</b> | 5.7      | 32.8        | <b>40.2</b> | 7.4      |

joined, the controller can only increase the SINR of a player by lowering the transmission power of another player. Since this will effectively deteriorate that other player's SINR, any such action will lead to a solution that is no longer Pareto optimal.

If multiple non-overlapping channels are available, the controller can improve the SINR levels for all joining players relative to the disagreement points following from the non-cooperative scenario. Therefore, all players have an incentive to join the interference optimization scheme. Even higher SINRs can be achieved with more channels available. With 3 non-overlapping channels available, as is the case in normal 2.45 GHz Wi-Fi operation, cooperative players can achieve a total increase in SINR of more than 100% compared to a situation with only non-cooperative players using only 1 channel. This is similar to what is found by the authors of [47] by means of applying a coalition game and taking airtime as utility.

Obviously, the controller has more spectral capacity available in such case and can allocate a lower number of players to each channel, leading to a higher SINR per player. This observation holds true for both cooperative and non-cooperative players. But in the cooperative scenario, the effect is larger than in the non-cooperative scenario. In the cooperative scenario, the controller can apply settings to the APs that would not represent a stable state of the system if the players would act selfishly and non-cooperative. That is, non-equilibrium strategies that would normally lead players to deviate (defect) from the global optimum owing to selfish and non-cooperative behavior (e.g. choosing a transmission lower than the maximum), can now be maintained in stable equilibrium conditions by a central authority. Players voluntarily give up control over their AP settings but obtain a higher SINR in return.

#### D. PRIORITIZING PLAYERS

We now consider the scenario with only three players active (using their Wi-Fi), namely players 1, 2, and 4, who share 2 available channels. Player 2 has a varying monthly contract fee  $m_2$ , and we assume that a higher contract fee gives this player the right on higher prioritization and *vice versa*. Both player 1 and player 3 have a fixed contract fee  $m_1 = m_3 = 40$ .

The results are illustrated in Fig. 4. The dotted lines indicate each player's individual indifference levels, determined by the disagreement values of fully non-cooperative behavior. The solid lines indicate each player's expected SINR as a function of the monthly fee paid by player 2. Values below the dotted lines correspond to preferred non-cooperative

behavior, i.e. the player will defect, whereas values above the dotted lines correspond to preferred cooperative behavior.

With  $m_2 = 0$  we find that player 2 receives his disagreement value, while the SINR of the other two players is relatively high. With  $m_2 = 40$  the SINRs of all players are of the same order. At higher  $m_2$ , the SINRs of both player 1 and 3 converge to their respective disagreement values. We never found that a player had an incentive to defect the cooperation.

Fig. 4 shows that it is indeed possible to prioritize players effectively, i.e. spectral resources can be redistributed among players depending on an inter-player negotiation outcome. This can be understood in the same way as the result for cooperative behavior with multiple available channels (Fig. 3) without prioritization: the controller can apply settings to the APs that would not represent a stable state of the system if the players would act selfishly and non-cooperative. The idea that cooperative players can negotiate amongst each other how much unlicensed spectrum each gets is a completely new way of looking at the use and valuation of unlicensed spectrum.

#### E. THE INFLUENCE OF NON-JOINING PLAYERS

As stated previously, our results so far indicate that the only reason for a player to defect cooperation is (within the scope of this paper) an irrational one. But since players may cherish values that are not captured by our game theoretic model, players defecting or not joining the cooperation in the first place is a realistic scenario that needs to be studied. To study the influence of non-joining players on the SINR of the joining players, we gradually decrease the number of cooperative players in our model by considering the following six scenarios:

**Scenario 1.** The fully cooperative scenario: players 1–5 are cooperative players;

**Scenario 2.** A mixed scenario: players 1–4 are cooperative players, player 5 is a non-cooperative player;

**Scenario 3.** A mixed scenario: players 1–3 are cooperative players, players 4 and 5 are non-cooperative players;

**Scenario 4.** A mixed scenario: players 1 and 2 are cooperative players, players 3–5 are non-cooperative players;

**Scenario 5.** A mixed scenario: player 1 is a cooperative player, players 2–5 are non-cooperative players;

**Scenario 6.** The fully non-cooperative scenario: players 1–5 are non-cooperative players.

Fig. 4 shows the results of the expected SINR per player for every scenario. A player's bar is green if for that scenario the player is a cooperative player. The bar is orange if the player is non-cooperative. The results are obtained while considering the (sub)vector of the solution of the fully non-cooperative

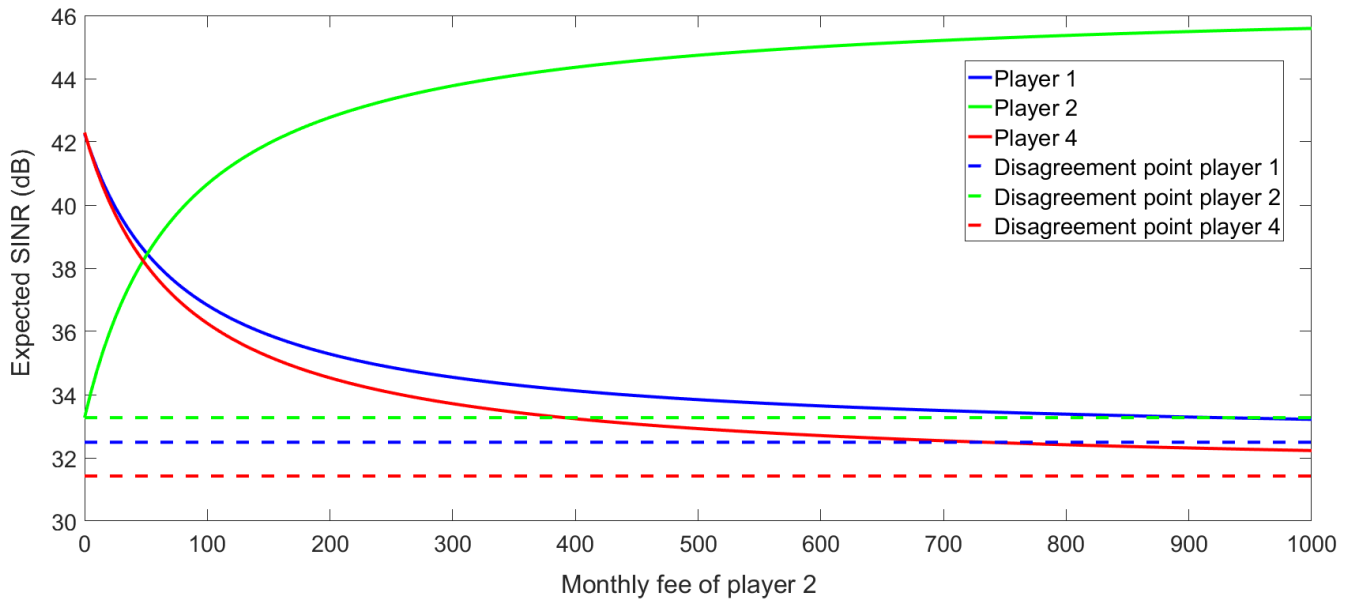


FIGURE 4. Varying  $m$  for player AP2.

scenario as disagreement setting. The disagreement point is the outcome of Scenario 6, and is represented with a horizontal red line. We have taken this as a given, but in reality, when a player decides to join or leave the collaboration, the controller may opt to reevaluate its disagreement settings to reflect the new mix of joining and non-joining players. The players are as described in Table 2.

In Fig. 5a, we assumed that there are only 2 channels available in the 2.45 GHz frequency band. These results suggest that it is always beneficial for all non-cooperative players except player 4 to join the interference optimization scheme, as their SINR will increase compared with what they obtain in Scenario 6. We found this to be independent of the subset of players who decide to cooperate.

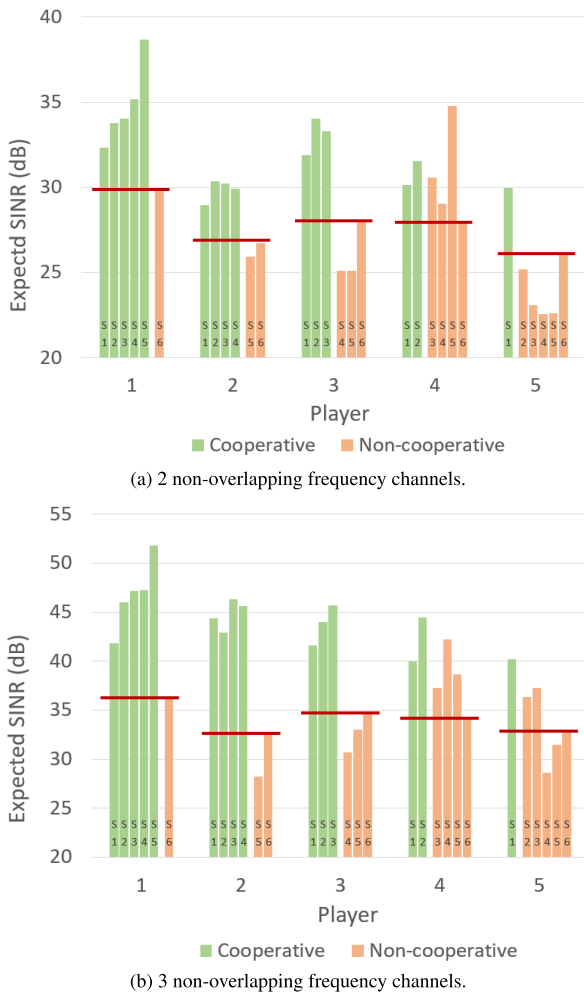
The results for player 4 show that he obtains the highest SINR in scenario 5, in which the only joining player is player 1. This can be understood by looking at the relative AP positions of players 1 and 4 in Fig. 2. Since the AP of player 1 is relatively close to the AP of player 4, these APs experience most interference from each other. Therefore, in the scenario that only player 1 is joining, the controller ensures that player 1 always transmits on a different channel than player 4. Both player 1 and player 4 therefore take advantage of Scenario 5. If also player 2 is joining (Scenario 4), the controller assigns the two available channels to the two cooperating players, and player 4's SINR will decrease to a level that is not particularly beneficial anymore.

There does not appear to be a correlation between the expected SINR of any individual player and the number of non-cooperative players. The exception seems to be player 1, for whom it seems beneficial that as few as possible other players cooperate. This suggests that any correlation largely depends on player-specific attributes, such as location.

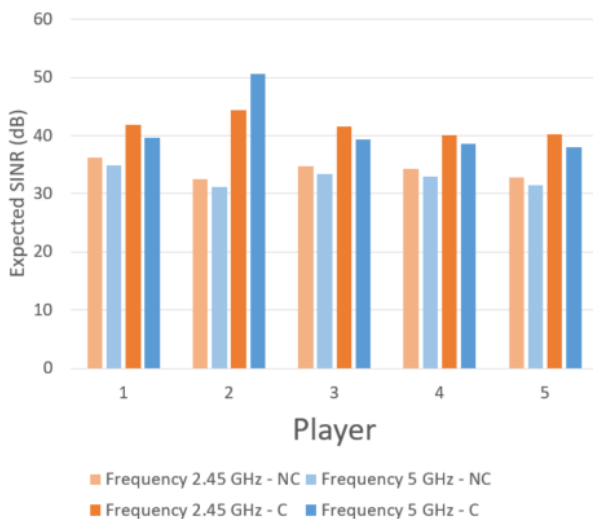
Fig. 5b presents the results for 3 available channels in the 2.45 GHz frequency band. They are similar to what we found for 2 available channels. We conclude that our combination of cooperative and non-cooperative game theory can be effectively used to model scenarios with a mixed group of joining and non-joining players. We find that joining a collaborative interference optimization scheme is always beneficial to players, compared to the fully non-cooperative scenario. If only a subset of players joins the collaboration, the remaining non-joining players may obtain an SINR that is even worse than the disagreement point: they are punished for their uncooperative behavior. But they may also be lucky and benefit anyway, though generally not as much as when they would have joined. That is, the outcomes achieved in scenarios 2–6 are all unstable if all players would be rational.

F. APPLYING OUR MODEL TO THE 5.21 GHz BAND

It is a popular belief that migrating Wi-Fi systems to the 5.21 GHz frequency band will solve all congestion problems, as that band has, in theory, tens of channels available. However, as [3] shows, most current implementations only operate on the lowest four channels, and the first indications of congestion in this band are already observed. Therefore, we also applied our model to the 5.21 GHz band, considering the fully non-cooperative and the fully cooperative scenario, in the same apartment block and with all five players. As with the 2.45 GHz band we assume that there are only three channels available. This could be a scenario in which two of the four channels in the 5.21 GHz band are occupied by a system that applies channel bonding. Fig. 6 presents the results, together with the results we presented earlier concerning the 3 available non-overlapping channels in the 2.45 GHz band.



**FIGURE 5.** Expected SINR for 6 different scenarios (S1...S6) with cooperative and non-cooperative players. The red lines are the SINRs expected in a fully non-cooperative scenario.



**FIGURE 6.** The non-cooperative and cooperative scenario for five players and three channels,  $f = 2.45$  GHz and  $f = 5.21$  GHz.

The results for the 5.21 GHz band appear to be very similar to those of the 2.45 GHz band. This contrasts with the common belief that systems in the 5.21 GHz band are

thought to cause less interference between players because of the reduced range that generally comes with higher carrier frequencies, especially indoors. Indeed, the expected SINRs in the non-cooperative case are similar to the ones found in 2.45 GHz: although the 5.21 GHz devices receive a lower signal, they also receive less interference and noise. Nevertheless, the increase in SINR when players decide to cooperate is still significant, evidencing that also in this case collaboration can be highly beneficial to all players.

## VI. CONCLUSIONS AND FUTURE WORK

Many private wireless networks involving networking technologies such as Wi-Fi use unlicensed spectrum in a self-destructive way. With an ever-growing number of wireless devices being actively used in densely populated areas, driven by developments such as the Internet of Things, the spectral resources are being depleted, leading to a significant loss of network performance. Wi-Fi users need to collaborate to avoid this from happening. They need to share the resources in a mutually accepted and enforced way.

Various current technological developments facilitate the making and execution of intelligent spectrum sharing policies. However, players cannot be forced to enter interference optimization schemes. We modeled these Wi-Fi use cases with game theory involving a mixed inclusion of cooperative and non-cooperative players which, to our knowledge, has not been done before. The model allows us to research whether neighboring Wi-Fi users benefit from cooperation or not, taking into account their free choice. Put differently, it facilitates users to opt for non-cooperation regardless of whether it could be beneficial.

Furthermore, our model incorporates a way to prioritize different Wi-Fi users, based on their mutual agreement, e.g., given their differences in monthly Internet subscription fees. The concept of cooperative players negotiating on the amount of spectral resources that each player gets, possibly involving monetary transactions, is a novel way of looking at the use of unlicensed spectrum, and opens a new field of systems engineering research.

The unique properties of our model made it possible to create realistic results for the use case of a typical floor in an apartment block, consisting of 5 apartments. For such a configuration, we demonstrated that joining a collaborative AP channel allocation and transmission power optimization scheme leads to improved Wi-Fi performance for all individual users in terms of SINR, compared to a fully non-cooperative setting. Our results indicate that cooperation is also beneficial even if some users choose not to join the collaboration. Besides, most of these defectors are punished for their behavior as they obtain an SINR that is even worse than the disagreement point. As such, they are enticed to join the collaboration anyway. Our results are largely independent of the chosen frequency band (2.45 GHz or 5.21 GHz).

Future work includes the extension of the simulations to different and larger apartment blocks (and more floors) with multiple APs per apartment, and to different networks

possibly using different frequency bands (such as Zigbee, Bluetooth, LTE-U, LoRa, and some varieties of 5G). We are also considering to use secure multi-party computation to address potential privacy concerns regarding the exchange of personal data (e.g., monetary information and location) that may prevent Wi-Fi users from joining the interference optimization scheme. Furthermore, we are considering the design of a brokering platform that contains agents representing the demand and offer of the individual players' resources, and then automates the negotiation. As the players are untrusted parties, the automated negotiation may be facilitated using blockchain and smart contracts. The platform then generates an agreed-upon policy, to be executed by the controller and the APs [30], [48]. We can subsequently validate our simulation results by measurements in real apartment blocks.

## APPENDIX

### PROOF OF THEOREM 2

The proof of Theorem 2 is analogous to the proof in [35]. Let us first introduce the following preliminaries.

*Definition 4 (Fixed Point):* Let  $f : X \rightarrow X$  be a mapping.  $x \in X$  is a fixed point of  $f$  if and only if  $f(x) = x$ .

*Theorem 5 (Brouwer, [49]):* For any continuous function  $f$  mapping a convex set to itself there is a point  $x$  such that  $f(x) = x$ .

*Proof (Theorem 2, [35]):* Let  $s = (s_1, \dots, s_n) \in \times_{i \in N} \Delta(S_i)$  be an  $n$ -tuple and let  $p_i(s)$  be the pay-off of  $s$  to player  $i \in N$ . We have that  $s_i$  is an affine combination of  $\pi_{i,\alpha}$ 's,  $\pi_{i,\alpha} \in S_i$ , and let  $p_{i,\alpha}(s)$  be the pay-off of player  $i$  if he selects  $\pi_{i,\alpha}$  instead of  $s_i$  when all other players do not deviate from  $s$ . The set of functions  $\phi_{i,\alpha}$  is defined as

$$\phi_{i,\alpha}(s) = \max(0, p_{i,\alpha}(s) - p_i(s)).$$

To improve pay-off, player  $i$  can deviate from  $s_i$  to  $s'_i$  by increasing the selection probabilities of pure strategies with a pay-off higher than  $p_i(s)$ ,

$$s'_i = \frac{s_i + \sum_{\alpha} \phi_{i,\alpha}(s) \pi_{i,\alpha}}{1 + \sum_{\alpha} \phi_{i,\alpha}(s)} \text{ for } \alpha \in \{1, |S_i|\},$$

where  $s' = (s'_1, \dots, s'_n)$ . Let  $T : \times_{i \in N} \Delta(S_i) \rightarrow \times_{i \in N} \Delta(S_i)$  be the mapping  $T(s) = s'$ . Note that  $\times_{i \in N} \Delta(S_i)$  is a convex set. It follows from Theorem A5 that  $T$  has a fixed point.

Let  $s^*$  be a mixed equilibrium point. It follows from Definition 1 that  $\phi_{i,\alpha}(s^*) = 0$  for all  $\alpha \in \{1, |S_i|\}$ , hence  $T(s^*) = s^*$ .

Let  $s^*$  now be a fixed point of  $T$  and let  $\pi_{i,\beta} \in S_i$  be the least profitable pure strategy considered in  $s^*$  of player  $i \in N$ . We have that  $p_{i,\beta}(s) \leq p_i(s)$  and therefore  $\phi_{i,\beta} = 0$ . Since  $s^*$  is fixed,  $T$  must not decrease the proportion of  $\pi_{i,\beta} \in S_i$  in  $T(s^*)$ . It follows that all  $\phi_{i,\alpha}(s^*) = 0$  for  $\alpha \in \{1, |S_i|\}$  to ensure that the denominator of  $T$  equals 1, hence  $s^*$  is a mixed equilibrium point.

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