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Stabilization of Chaotic Systems With Both Uncertainty and Disturbance by the UDE-Based Control Method

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ABSTRACT This paper investigates the stabilization of a class of chaotic systems with both model uncertainty and external disturbance. By combining the dynamic feedback control method, and the uncertainty and disturbance estimator (UDE)-based control method, a new UDE-based control method is developed. By using this method, the system stabilization can be achieved by three steps. Illustrative examples using numerical simulations verify the soundness and effectiveness of the proposed method.

INDEX TERMS Chaotic system, stabilization, dynamic feedback control, uncertainty, disturbance, UDE

I. INTRODUCTION

Lorenz firstly found the classical chaotic attractor [1] in 1963. Chaotic systems possess trajectories that have embedded within a large number of unstable periodic orbits. A chaotic system has complex dynamical behaviors that depend sensitively on tiny variations of initial conditions and has bounded trajectories in the phase space. As one of the most fascinating phenomenon in nonlinear dynamical system, chaos has been intensively studied over the past few decades. Since Ott, Grebogi and York first observed chaos control [2] in 1990, and Pecora and Carroll first presented chaos synchronization [3] in 1990, respectively, chaos becomes an interesting phenomenon and has been widely applied in engineering, science, and communications [4]–[7]. Up to today, many works have been done to deal with all kinds of control problems of the chaotic systems, such as stabilization, complete synchronization, anti- synchronization, coexistence of synchronization and anti-synchronization, simultaneous anti-synchronization and synchronization, projective synchronization, and so on, please see [8]–[15]. Among the above mentioned control problems, the stabilization of the system needs to be addressed at first. It is because only when the system is stabilized, we can further discuss and solve all kinds of control problems of the chaotic system. Therefore, it is very important to achieve the stabilization of the system at first.

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It should be pointed out that among the above mentioned chaotic systems, system uncertainty and external disturbance are not considered. Unfortunately, there are still many deficiencies in the current research, and in practice this is not the case. For the chaotic systems, a lot of methods have been proposed to investigate the robust stabilization problems, e.g., [16], [17]. However, the disturbance is bounded in some forms is a basic assumption in the most of those methods, such as $d(t)$ belongs to $L_2^n[0, +\infty)$, or $d(t)$ is assumed to be L_2-norm bounded. More importantly, the robust control and disturbance rejection problems in the above mentioned works are mainly solved by the linear matrix inequalities (LMIs). However, the stability conditions which are derived by the LMIs are only sufficient conditions and often result in conservative conclusions. The robust control problem of chaotic systems with both model uncertainty and external disturbance has remained very challenging.

The UDE-based control method [18] is a good method to deal with the model uncertainty and external disturbance, and it has the following two advantages:

1. The system model or a disturbance model is not known completely;
2. both structured (or unstructured) uncertainties and external disturbances can be suppressed.

Being an effective robust control strategy, UDE-based control has been widely applied in various systems. Thus, it is of interest to apply the UDE-based control to chaotic systems with both model uncertainty and external disturbance. However, by reviewing the literature, it was found that this problem has not been well addressed. Therefore, it is very important to solve the uncertainty of the model and the stability of the system with external disturbance. The goal of this paper is to develop a new UDE-based control method for the chaotic systems, by which the problem of model uncertainty and external disturbance can be solved. The rest of this paper is organized as follows. Section 2 introduces the preliminary knowledge and the problem formation, Section 3 presents the main results of this paper, Section 4 provides the illustrative examples with numerical simulation, and Section 5 gives the conclusions.

II. PRELIMINARIES AND PROBLEM FORMATION

A. PRELIMINARIES

Consider the following chaotic system

$$\dot{x} = f(x) \tag{1}$$

where $x \in R^n$ is the state, $f(x) = (f_1(x), \cdots, f_n(x))^T$ is a smooth vector function with $f(0) = 0$.

Definition 1: Consider the following controlled system

$$\dot{x} = f(x) + Bu \tag{2}$$

where $x \in R^n$ is the state, $B \in R^{n\times r}$, $r \geq 1$, $(f(x), B)$ is controllable, u is the designed controller. If $\lim_{t\to\infty} \|x(t)\| = 0$, we call the chaotic system (1) is stabilized by the controller u.

There are many results about the stabilization of chaotic systems. Among them, the dynamic feedback control method [11] has a wide range of applications because it is simply designed and can be easily applied. The introduction of the dynamic feedback control method are provided at first.

Lemma 1 [11]: Consider the chaotic system (2). If $B = (b_{ij})_{n\times r}$, and $b_{ij} = 0$ or $b_{ij} = 1$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, r$, and $(f(x), B)$ is controllable, then a dynamic feedback controller u is as follows

$$u = K(t)x \tag{3}$$

where $K(t) = k(t)B^T$, and

$$\dot{k}(t) = -\gamma x^T x = -\gamma \|x(t)\|^2 \tag{4}$$

where $\gamma > 0$ is a constant.

After that, the UDE-based control method is introduced.

Lemma 2 [18]: Considering the following system

$$\dot{x} = f(x) + Bu + \Delta f(x) + d(t) \tag{5}$$

where $x \in R^n$ is the state, $(f(x), B)$ is controllable, $B \in R^{n\times r}$, $r \geq 1$, $\Delta f(x)$ is the model uncertainty, $d(t)$ is the external disturbance.

The stable linear reference model is described as:

$$\dot{x}_m = A_m x_m + B_m C \tag{6}$$

where $x_m \in R^n$ is the reference state, $A_m \in R^{n\times n}$ is Hurwitz, $B_m \in R^{n\times r}$, $C \in R^{r\times 1}$ is a piecewise continuous and uniformly bounded command.

If a designed filter $g_f(t)$ satisfies

$$\tilde{u}_d = \hat{u}_d - u_d \to 0, \quad t \to \infty$$

where $\hat{u}_d = (\dot{x} - f(x) - Bu) * g_f(t)$, $u_d = \Delta f(x) + d(t)$, then the UDE-based controller u is designed as

$$\begin{aligned} u = B^+ &\left\{ \ell^{-1}\left[\frac{1}{1-G_f(s)}\right] * (A_m x + B_m C - Ke)\right\} \\ &- B^+ \left\{ f(x) + \ell^{-1}\left[\frac{sG_f(s)}{1-G_f(s)}\right] * x(t)\right\} \end{aligned} \tag{7}$$

where $B^+ = (B^TB)^{-1}B^T$, $G_f(s) = \ell[g_f(t)]$, $e = x_m - x$, ℓ denotes Laplace transformation and the matrix K is Hurwitz, ℓ^{-1} represents the Laplace inverse transformation, $*$ stands for the convolution.

B. PROBLEM FORMATION

Consider the following system

$$\dot{x} = f(x) + Bu + \Delta f(x) + d(t) \tag{8}$$

where $x \in R^n$ is the state, $(f(x), B)$ is controllable, $B \in R^{n\times r}$, $r \geq 1$, $\Delta f(x)$ is the model uncertainty, $d(t)$ is the external disturbance.

The main goal of this paper is to design a controller u to make the system (8) reach stabilization, i.e., $\lim_{t\to\infty} \|x(t)\| = 0$.

III. MAIN RESULTS

Considering the advantages of dynamic feedback control method and the UDE-based control method, we combine these two methods and obtain the following results.

Theorem 1: Consider the system given in Equation (8). If a filter $g_f(x)$ is designed to satisfy the following condition:

$$\tilde{u}_d = \hat{u}_d - u_d \to 0, \quad t \to \infty \tag{9}$$

where $\hat{u}_d = (\dot{x} - F(x) - Bu_{ude}) * g_f(t)$ and $u_d = \Delta f(x) + d(t)$, then the UDE-based controller u is

$$u = u_s + u_{ude} \tag{10}$$

where

$$u_s = K(t)x(t) = k(t)B^T x(t), \tag{11}$$

$k(t)$ is updated by the update law (4),

$$\begin{aligned} u_{ude} = B^+ &\left\{ \ell^{-1}\left[\frac{G_f(s)}{1-G_f(s)}\right] * F(x)\right\} \\ &- B^+ \left\{ \ell^{-1}\left[\frac{sG_f(s)}{1-G_f(s)}\right] * x(t)\right\} \end{aligned} \tag{12}$$

$B^+ = (B^TB)^{-1}B^T$, $F(x) = f(x) + Bu_s$, $G_f(s) = \ell[g_f(t)]$, ℓ represents the Laplace transformation, ℓ^{-1} represents the Laplace inverse transformation, $*$ stands for the convolution.

Proof: Substituting the controller u in (10) into the system (8), we get

$$\dot{x} = f(x) + Bu_s + Bu_{ude} + u_d = F(x) + Bu_{ude} + u_d \tag{13}$$

According to Lemma 1, the system $\dot{x} = F(x)$ is globally asymptotically stable. Noting that the condition (9), i.e.,

$$Bu_{ude} = -\hat{u}_d$$

Thus, the system (13) is rewritten as

$$\dot{x} = F(x) + \tilde{u}_d$$

and this system is globally asymptotically stable.

Remark 1: According to the existing results in [18], the following two filters can be applied to the most of cases in applications.

One is the low-pass filter:

$$G_f(s) = \frac{1}{1 + \tau s}, \tag{14}$$

where $\tau > 0$, which is suitable for $d(t)$ is a constant vector.

The other is presented as follows:

$$G_f(s) = \frac{a_1 s + (a_2 - \omega_0^2)}{s^2 + a_1 s + a_2}, \tag{15}$$

where $\omega_0 = 4\pi$, $a_1 = 10\omega_0$,$a_2 = 100\omega_0^2$.

The development of adaptive UDE-based control method consists of following three steps:

1. Design an dynamic feedback stabilization controller u_s for the chaotic systems without model uncertainty and external disturbance;
2. Design a filter to estimate model uncertainty and external disturbance, and derive the UDE-based controller u_{ude};
3. By combining the two controllers obtained in previous two steps, the adaptive UDE-based control controller u can be derived as $u = u_s + u_{ude}$.

IV. ILLUSTRATIVE EXAMPLES WITH NUMERICAL SIMULATION

In this section, using Lorenz system and the complex Lorenz system as examples, the numerical simulation were conducted for verifying the soundness and effectiveness of the proposed method.

Example 1: The controlled Lorenz system with both model uncertainty and external disturbance:

$$\dot{x} = f(x) + Bu + \Delta f(x) + d(t) \tag{16}$$

where $x = (x_1, x_2, x_3)^T$, and

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} = \begin{pmatrix} 10(x_2 - x_1) \\ 28x_1 - x_2 - x_1x_3 \\ -\frac{8}{3}x_3 + x_1x_2 \end{pmatrix}, \tag{17}$$

$$B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \tag{18}$$

$$\Delta f(x) = \begin{pmatrix} \Delta f_1(x) \\ \Delta f_2(x) \\ \Delta f_3(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0.1x_2x_3 \\ 0 \end{pmatrix}, \tag{19}$$

$$d(t) = \begin{pmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 1000 \\ 0 \end{pmatrix} \tag{20}$$

The controlled Lorenz system is presented as

$$\dot{x} = f(x) + Bu_s \tag{21}$$

where $x \in R^3$, $f(x)$ and B are given as (17).

According to [1], the Lorenz system is $\dot{x} = f(x)$, where $f(x)$ is given in (17).

Our goal is to design a controller $u = u_s + u_{ude}$ to make the system given in Equation (16) reach stabilization, i.e., $\lim_{t\to\infty} \|x(t)\| = 0$.

According to the results in Section II, the first step is to design u_s.

For the Lorenz system: $\dot{x} = f(x)$, if $x_2 = 0$, then the following two-dimensional system

$$\begin{aligned} \dot{x}_1 &= -10x_1 \\ \dot{x}_3 &= -\frac{8}{3}x_3 \end{aligned}$$

is globally asymptotically stable.

Therefore, according to Lemma 1, the controller u_s is designed as follows:

$$u_s = k(t)B^T x(t) = k(t)(0\ 1\ 0)x(t) \tag{22}$$

and the feedback gain $k(t)$ is updated by (4).

The numerical simulation is designed as follows: the initial values of the controlled Lorenz chaotic system given in Equation (21) and the system given in Equation (4) are given as follows: $x_0 = [1, 2, 3]^T$, $k(0) = -1$. Fig. 1 shows that the Lorenz system is stabilized, while Fig.2 shows that the feedback gain $k(t)$ converges to a negative constant.

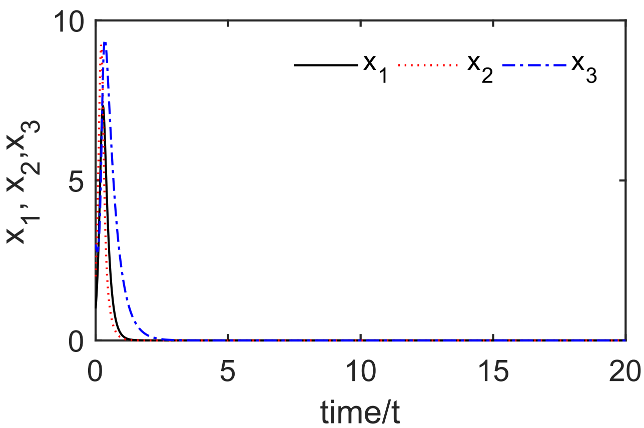

FIGURE 1. **The Lorenz system is stabilized.**

The second step is to design the UDE controller u_{ude}.

Let $u_d = \Delta f(x) + d(t)$, $F(x) = f(x) + Bu_s$, the system (16) is rewritten as

$$\dot{x} = F(x) + Bu_{ude} + u_d \tag{23}$$

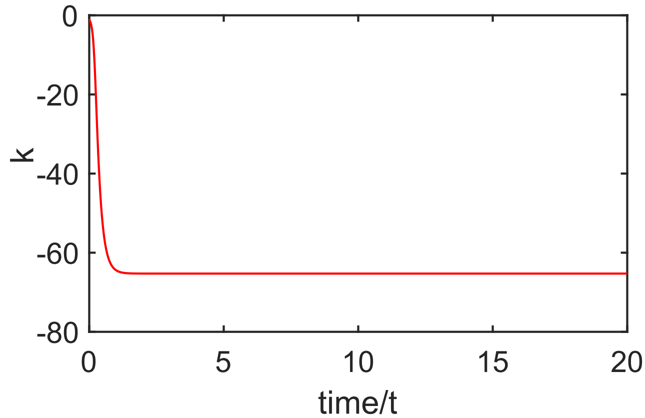

FIGURE 2. ***k*(*t*) converges to a negative constant.**

According to Theorem 1, the controller u_{ude} is presented as

$$u_{ude} = B^{+}\left\{\ell^{-1}\left[\frac{G_f(s)}{1-G_f(s)}\right] * F(x)\right\} - B^{+}\left\{\ell^{-1}\left[\frac{sG_f(s)}{1-G_f(s)}\right] * x(t)\right\} \tag{24}$$

where $B^{+} = (B^T B)^{-1}B^T$, $G_f(s) = \ell[g_f(t)]$, ℓ represents the Laplace transformation, ℓ^{-1} represents the Laplace inverse transformation, $*$ stands for the convolution.

The third step is to combine the two controllers u_s and u_{ude}, i.e., $u = u_s + u_{ude}$. Thus, the system given in Equation (16) can be stabilized by the above obtained controllers.

The numerical simulation is designed as follows: the initial values of the controlled Lorenz chaotic system given in Equation (23) and the system given in Equation (4) are given as follows: $x_0 = [1, 2, 3]^T$, $k(0) = -1$. Fig.3 shows that the Lorenz system is also stabilized, Fig.4 shows that $\hat{u}_{d2}$ tends to u_{d2} as $t \to \infty$, while Fig.5 shows that the feedback gain $k(t)$ converges to a negative constant.

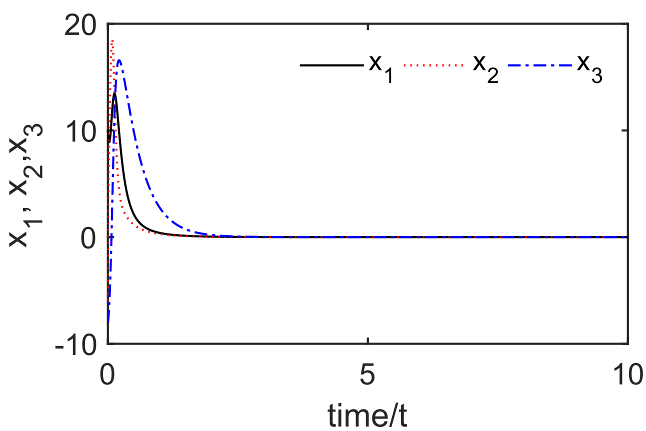

FIGURE 3. The Lorenz system is also stabilized.

Remark 2: For Example 1, according to the controller design given in Equation (7), the controller u can be

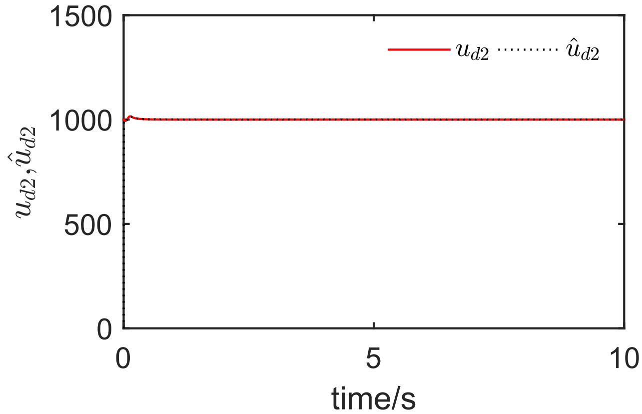

FIGURE 4. $\hat{u}_{d2}$ **tends to** u_{d2} **as** $t \to \infty$.

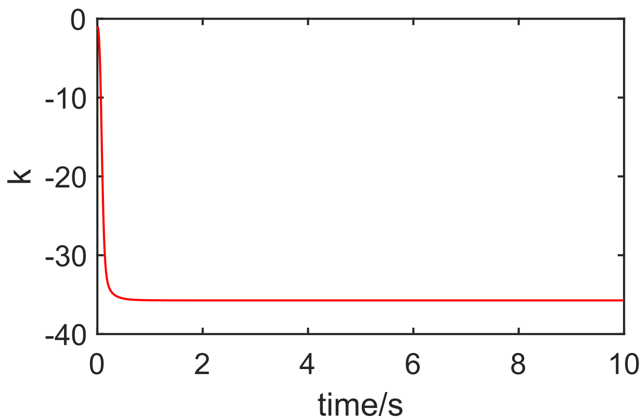

FIGURE 5. ***k*(*t*) converges to a negative constant.**

derived as

$$u = B^{+}\left\{\ell^{-1}\left[\frac{1}{1-G_f(s)}\right] * (A_m x + B_m C - Ke)\right\} - B^{+}\left\{f(x) + \ell^{-1}\left[\frac{sG_f(s)}{1-G_f(s)}\right] * x(t)\right\} \tag{25}$$

The second term in Equation (25)

$$B^{+}(-f(x)) = -f_2(x) \tag{26}$$

thus,

$$BB^{+}(-f(x)) = \begin{pmatrix} 0 \\ -f_2(x) \\ 0 \end{pmatrix} \neq -f(x) \tag{27}$$

That is to say, $Bu = -f(x) + u_d + A_m x$ is impossible to satisfied whatever u is. Therefore, the existing UDE-based control method [18] is not utilized directly for this example.

Example 2: the following complex Lorenz system [19]:

$$\begin{aligned} \dot{u}_1 &= a_1(u_2 - u_1) \\ \dot{u}_2 &= a_2 u_1 - u_2 - u_1 u_3 \\ \dot{u}_3 &= -a_3 u_3 + \frac{1}{2}(\bar{u}_1 u_2 - u_1 \bar{u}_2) \end{aligned} \tag{28}$$

where $u_1 = x_1 + jx_2$, $u_2 = x_3 + jx_4$, are the complex state variables of the system, u_3 is the state variable of the system and a_i, $i = 1, 2, 3$, are the real parameters.

According to the results in [19], the system (28) is chaotic when $a_1 = 10$, $a_2 = 110$, $a_3 = 2$.

Separating the real and imaginary parts of each variable in the system (28), the following real system is presented:

$$\begin{aligned} \dot{x}_1 &= 10(x_3 - x_1) \\ \dot{x}_2 &= 10(x_4 - x_2) \\ \dot{x}_3 &= 110x_1 - x_1x_5 - x_3 \\ \dot{x}_4 &= 110x_2 - x_2x_5 - x_4 \\ \dot{x}_5 &= -2x_5 + x_1x_3 + x_2x_4 \end{aligned} \tag{29}$$

The controlled complex Lorenz system with both model uncertainty and external disturbance is given as

$$\dot{x} = f(x) + Bu + \Delta f(x) + d(t) \tag{30}$$

where $x = (x_1, x_2, \cdots, x_5)^T$, and

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \end{pmatrix} = \begin{pmatrix} 10(x_3 - x_1) \\ 10(x_4 - x_2) \\ 110x_1 - x_1x_5 - x_3 \\ 110x_2 - x_2x_5 - x_4 \\ -2x_5 + x_1x_3 + x_2x_4 \end{pmatrix}, \tag{31}$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{32}$$

$$\Delta f(x) = \begin{pmatrix} \Delta f_1(x) \\ \Delta f_2(x) \\ \Delta f_3(x) \\ \Delta f_4(x) \\ \Delta f_5(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ x_2^2 x_3 \\ -x_3^2 \\ 0 \end{pmatrix}, \tag{33}$$

$$d(t) = \begin{pmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \\ d_4(t) \\ d_5(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 20\sin(2t) \\ 0 \end{pmatrix}. \tag{34}$$

The controlled complex Lorenz system is presented as

$$\dot{x} = f(x) + Bu_s \tag{35}$$

where $x \in R^5, f(x)$ and B are given as in Equation (31).

Our goal is to design a controller $u = u_s + u_{ude}$ to make the system in Equation (30) reach stabilization, i.e., $\lim_{t\to\infty} \|x(t)\| = 0$.

According to the results in Section II, the first step is to design u_s.

For the uncontrolled system (35), i.e., $\dot{x} = f(x)$, it is easy to note that: if $x_3 = x_4 = 0$, then the following three dimensional system

$$\begin{aligned} \dot{x}_1 &= -10x_1 \\ \dot{x}_2 &= -110x_2 \\ \dot{x}_5 &= -2x_5 \end{aligned} \tag{36}$$

is globally asymptotically stable.

Therefore, according to Lemma 1, the controller u_s is designed as follows:

$$u_s = k(t)B^T x(t) = k(t)\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} x(t) \tag{37}$$

and the feedback gain $k(t)$ is updated by (4).

The numerical simulation is designed as follows: the initial values of the controlled Lorenz chaotic system given in Equation (35) and the system given in Equation (4) are given as follows: $x_0 = [1, 2, 3, -4, -5]^T$, $k(0) = -1$. Fig.6 shows that the complex Lorenz system is stabilized, while Fig.7 shows that the feedback gain $k(t)$ converges to a negative constant.

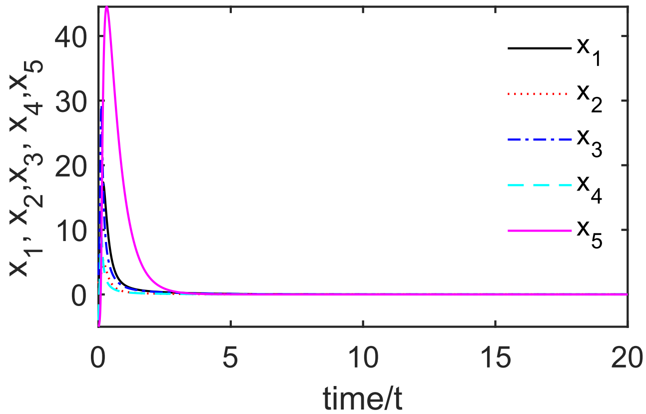

FIGURE 6. **The complex Lorenz system is stabilized.**

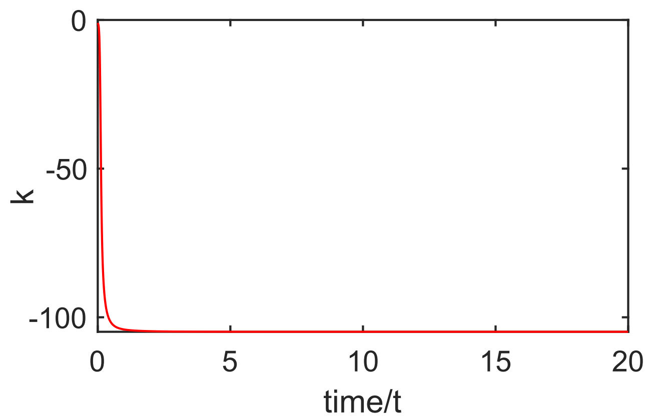

FIGURE 7. ***k*(*t*) converges to a negative constant.**

The second step is to design the UDE controller u_{ude}.

Let $u_d = \Delta f(x) + d(t)$, $F(x) = f(x) + Bu_s$, the system given in Equation (30) can be rewritten as

$$\dot{x} = F(x) + Bu_{ude} + u_d \tag{38}$$

According to Theorem 1, the controller u_{ude} is presented as

$$u_{ude} = B^+ \left\{ \ell^{-1}\left[\frac{G_f(s)}{1 - G_f(s)} \right] * F(x) \right\} - B^+ \left\{ \ell^{-1}\left[\frac{sG_f(s)}{1 - G_f(s)} \right] * x(t) \right\} \tag{39}$$

where $B^+ = (B^T B)^{-1} B^T$, $G_f(s) = \ell[g_f(t)]$, ℓ represents the Laplace transformation, ℓ^{-1} represents the Laplace inverse transformation, $*$ stands for the convolution.

The third step is to combine the two controllers u_s and u_{ude}, i.e., $u = u_s + u_{ude}$. Thus, the system given in Equation (30) can be stabilized by the above obtained controller.

The numerical simulation is designed as follows: the initial values of the controlled Lorenz chaotic system given in Equation (35) and the system given in Equation (4) are given as follows: $x_0 = [1, 2, 3, -4, -5]^T$, $k(0) = -1$. Fig.8 shows that the complex Lorenz system is also stabilized, Fig.9 shows that $\hat{u}_{d3}$ tends to u_{d3} as $t \to \infty$, Fig.10 shows that $\hat{u}_{d4}$ tends to u_{d4} as $t \to \infty$,while Fig.11 shows that the feedback gain $k(t)$ converges to a negative constant.

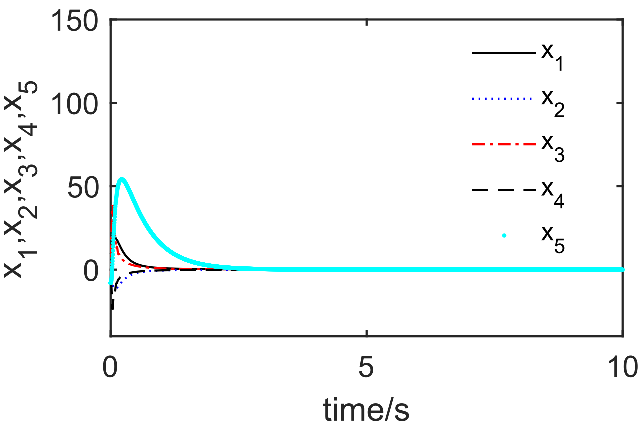

FIGURE 8. The complex Lorenz system is also stabilized.

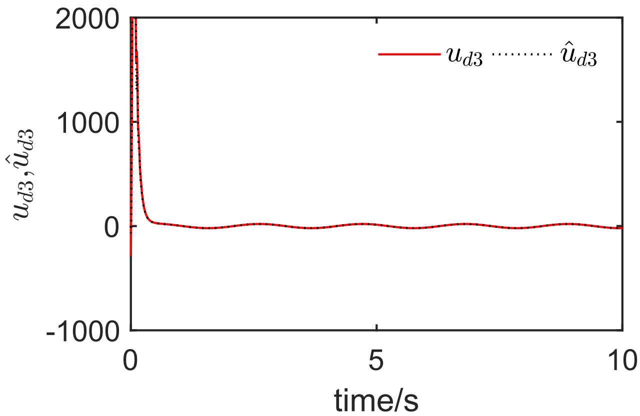

FIGURE 9. $\hat{u}_{d3}$ tends to u_{d3} as $t \to \infty$.

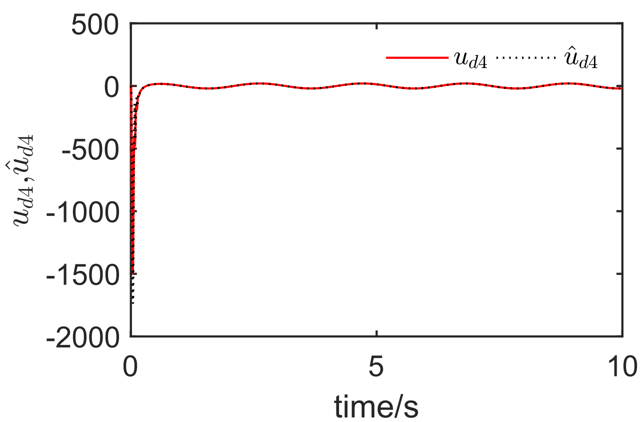

FIGURE 10. $\hat{u}_{d4}$ tends to u_{d4} as $t \to \infty$.

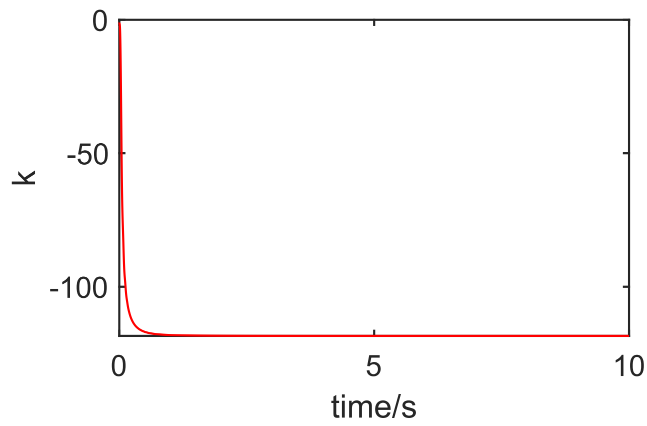

FIGURE 11. $k(t)$ converges to a negative constant.

V. CONCLUSION

This paper has investigated the stabilization of a class of chaotic systems with both model uncertainty and external disturbance. By combining the dynamic feedback control method and the UDE-based control method, a new UDE-based control method has been developed, by which the stabilization problem of chaotic systems has been solved by three steps. Theoretically, the developed method has some advantages over the existing methods. In this paper, the soundness and the effectiveness of the proposed results have been verified by the illustrative examples.

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