IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 2, 2020, accepted March 16, 2020, date of publication March 26, 2020, date of current version April 10, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2983566

Multivariate Stochastic Optimization Approach Applied in a Flux-Cored Arc Welding Process

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This work was supported in part by FAPEMIG, in part by CNPq, in part by CAPES, and in part by FAPEPE.

ABSTRACT One of the main goals in flux-cored arc welding processes is the optimization of bead geometry, in which multiple geometric characteristics of the welding bead are important; therefore, multiobjective optimization programming is often applied. However, several optimization problems that use stochastic programming do not consider the impact of the correlation between the output variables on their probabilistic constraints. In this context, this paper aims to present a multiobjective optimization method based on multivariate stochastic programming. To demonstrate the applicability of the proposal, we conducted a design of experiments to optimize a flux-cored arc welding process for stainless-steel claddings. The weighting-sums method was applied to formulate the multiobjective optimization problem. It was possible to formulate a multivariate probability distribution for the penetration and dilution. In addition, a 95% probability to meet the predefined specification limits of the geometric characteristics was achieved.

INDEX TERMS Bead geometry, bivariate normal distribution, flux-cored arc welding, multiobjective optimization programming, stainless steel claddings, stochastic programming.

I. INTRODUCTION

In practical optimization problems of industrial processes, the assumption that the input data are deterministic is rarely sustained. In fact, certain key inputs that are clearly random are instead represented by their expected values. Such an approach may be justified under special conditions; however, in several applications, it is possible to demonstrate that such a formulation is inadequate [1].

For example, in the general linear programming formulation, *Min* $\mathbf{f}(\mathbf{x}) = \mathbf{C}'\mathbf{x}$ is a vector of deterministic objective functions that needs to be minimized and $\mathbf{Ax} \leq \mathbf{b}$ is a set of constraints [2]. In most approaches reported in the literature, matrices $\mathbf{C}$ and $\mathbf{A}$ and vector $\mathbf{b}$ are composed of deterministic values. Nevertheless, these vectors and matrices may have random inputs in actual problems.

Therefore, it is important to model the stochastic nature of the inputs in optimization problems. To achieve this, stochastic programming (SP) can be used as a technique to measure and analyze the impact on the variability of the responses [3]–[6] and has been used in different sectors. Dai *et al.* [7] presented a literature review of all different SP methods that have been applied only in unit commitment (UC), including multi-stage SP and chance-constrained programming (CCP). Reddy *et al.* [8] provided a literature review on stochastic programming methods applied in the optimization of smart grids (SG). The recourse method and CCP were cited as widely used within the SG context.

In addition to the random aspect, most optimization problems present multiple and, often, conflicting responses of interest [9]. In such cases, it is necessary to consider the multivariate nature of the data [10]. For instance, correlated data present a significant variance–covariance structure; to properly compute the probabilities involved in the problem, multivariate techniques should be used. These strategies have been widely employed in various segments [11]–[14]. Considering a random distribution of vectors that contain correlated variables, there is a multivariate normal distribution (MND), each element of which has been assigned a univariate normal distribution [15].

Within this context, the present authors propose a multivariate stochastic optimization method, referred to as

The associate editor coordinating the review of this manuscript and approving it for publication was Utku Kose.

multivariate chance-constrained programming (MCCP). This method integrates multivariate statistics to the classic CCP method [16].

The MCCP will be applied to a flux-cored arc welding (FCAW) process for stainless-steel claddings based on weighted sums and the MND. Although several SP applications have already been published, the present authors could not find any studies in which SP had been applied to FCAW processes.

First, the response surface methodology (RSM) will be used, through which the geometric characteristics of the FCAW process will be analyzed. SP coupled with the MND will be used in the cases where the optimization problem would be defined as the maximization of the bead width (W) and reinforcement (R), subjected to the predefined minimum probability that both the penetration (P) and dilution (D) would be below their upper specification limits (USL).

This paper is organized as follows: in Section 2, the theoretical background of stainless-steel claddings, RSM, SP, MND, the weighted-sums method, and the generalized reduced gradient algorithm will be presented. In Section 3, the materials and the proposed method will be described. In Section 4, the results will be presented and the application of the method in the FCAW process will be discussed. Finally, the conclusions will be drawn in Section 5.

II. THEORETICAL BACKGROUND

A. STAINLESS-STEEL CLADDINGS

Stainless-steel cladding is an industrial process that has attracted significant attention from researchers. It refers to the surfacing of a low-carbon steel substrate with a corrosion-resistant material [17]. The advantages of this process include the improvement of surface properties, the recovery of elements affected by wear and corrosion, easier use of hardened steels and wear-resistant alloys, and cost reduction by using smaller amounts of expensive materials by minimizing the expenditures on maintenance and surfacing speed [18]. Consequently, by applying the aforementioned process, the surface presents better properties, the overall process cost decreases, and the manufacturability may increase. However, the challenge that remains is to control the dilution and to guarantee a satisfactory bead geometry [19].

A variety of methods have been used in the cladding of carbon steels with stainless steels, such as metal inert gas, metal active gas, tungsten inert gas (TIG), submerged arc welding, and FCAW.

FCAW comprises a metal fusion process that uses the heat of the electric arc between the wire and the workpiece [20]. This process, in particular, presents notable productivity results owing to the high electric current density, which guarantees a high fusion rate. Moreover, this welding method has been increasingly used because of advantages such as the quality of the deposited material and the excellent appearance of the weld beads. Owing to their high corrosion resistance, stainless-steel claddings are widely used in industrial equipment over carbon-steel claddings. In most cases, the materials suffer wear under severe conditions of use in harsh environments, often requiring restorations and maintenance using stainless weld metals.

Several FCAW applications already exist in the literature, most of which focus on bead microstructure and material properties. On the other hand, certain studies were aimed toward the analysis and optimization of other process outputs. Kumar *et al.* [21] investigated the interaction effects between the process parameters of FCAW on the hardness and bead geometry of cladded plates on super duplex stainless steel. The hardness values increased when the maximum wire feed rate and maximum welding speed were used owing to the dilution rate of the feed wire. In addition, the bead width and reinforcement were improved by using the minimum wire feed rate and minimum welding speed. Kannan and Yoganandh [22] followed a central composite design (CCD) to obtain mathematical models of bead geometry characteristics in austenitic stainless-steel claddings deposited by the gas metal arc welding process. Li *et al.* [23] applied the Taguchi experimental design to optimize the bead geometry by varying certain controlled machine parameters of an FCAW process. In their work, Balan *et al.* [24] optimized the bead geometry using a simulated annealing technique. Shao *et al.* [25] optimized the welding stress and deformation by varying the electric current, voltage, and the welding speed using the design of experiments (DOE), which is a strategy for modeling experiments using statistical and mathematical techniques [26]. Gomes *et al.* [19] studied a FCAW process for stainless-steel claddings. An experiment was conducted following a CCD to evaluate the influence of four controlled input variables on the geometry, productivity, and quality responses of interest. In addition, there are recently published studies in *IEEE Access* that address this type of methodology [27]–[29]. However, none of the aforementioned studies considered the natural variability of the process. Such consideration is important because the bead geometry is not uniform even in automatic welding. As a result, The optimization of only the expected values of the bead characteristics does not guarantee a robust process.

Considering the correlated nature of the response variables presented in the FCAW process, it is necessary to use multivariate strategies in this study.

B. RESPONSE SURFACE METHODOLOGY

In the industry, typically, the relationships between a set of outputs and the decision variables are unknown [30]. One of the manners to overcome this problem is to use the RSM [31]. In this method, a designed experimental array is followed to obtain real data on the process under analysis. Then, the results are used to define an analytical model that depends on the decision variables of each of the outputs. The CCD is a typical experimental array used to define the experiments. Then, second-order polynomial models are often used to build RS models $f(\mathbf{x})$, [32]–[34], and are expressed as in (1):

$$f(\mathbf{x}) \sim \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j}\sum \beta_{ij} x_i x_j + \varepsilon = \mathbf{z}'\boldsymbol{\beta} + \varepsilon, \tag{1}$$

where $f(\mathbf{x})$ is the response model, β_i, β_{ii}, and β_{ij} are the coefficients to be estimated; x_i represents the decision variables of vector $\mathbf{x}' = [x_1, x_2, \ldots, x_k]$, k is the number of input variables, and ε is the error observed in the response. Vector $\boldsymbol{\beta}' = [\beta_1, \ldots, \beta_p]$ of coefficients is estimated using the ordinary least-squares (OLS) regression method [35]. This methodology has been extensively used in [27], [36], [37].

If the models present adequate adjustments and residuals, a multiobjective optimization problem may be formulated. For this purpose, there are various techniques available in the literature. One of them is referred to as the weighted-sums (WS) method and it will be described in Section 2.2. After the problem has been formulated, an algorithm is used to search for the optimal solution. The generalized reduced gradient (GRG) algorithm is often used when polynomial analytical models represent the responses of interest [4].

C. STOCHASTIC PROGRAMMING

The input variables, responses of interest, and certain other parameters involved in real-world design problems have a random (or stochastic) character [3]. SP is a strategy used in the formulation of optimization problems to build objective functions and constraints whose coefficients or decision variables are described by random variables [38]. SP methods provide an important approach to linear programming under uncertainty which started in the 1950s and continues to be widely used up to present [39]. The stochastic problem can be remodeled as a deterministic one after considering the variability of the functions [40].

Different SP approaches have been published in the literature [9]. For instance, Liu *et al.* [41] developed a stochastic model, referred to as the inexact two-stage waste management (ITWM) model, for the planning of the long-term municipal solid waste management in the city of Changchun, China. In their approach, probability levels were used in the objective function in a two-stage stochastic programming model. Lindenschmidt and Rokaya [42] presented a method for the estimation of the probable maximum staging from the ice-jam floods. The idea was based on a deterministic hydraulic model with a stochastic framework.

Another widely used method is the CCP [16]. It refers to the formulation of an optimization problem, in which at least one constraint consists of a minimum value or an acceptable range for the probability function. Using the CCP and some other methods, Diaz-Garcia [6] proposed different strategies to formulate a multiobjective optimization problem using SP. One of them was the maximization of the probability that a response is within a predetermined level range. For instance, let Y be a response of interest. In each test, Y is measured k times. Therefore, $E[Y]$ is the Y' expected value of a run and s_Y is its standard deviation for the test. After all tests have been conducted, $E[Y(\mathbf{x})]$ can be expressed as (1); the standard deviation of $E[Y]$ can be estimated using $\sigma_{\bar{Y}} = \bar{s}_y / c_4$ as an alternative to the variance from the regression model. However, in multiobjective problems, this probability is multivariate; therefore, the possible correlations among the responses of interest must be considered. If the means of the response is, as per usual, normally distributed, the MND may be used.

D. MULTIVARIATE NORMAL DISTRIBUTION

The MND is an extension of the univariate distribution. For the case of a single variable with $\mu = E[x]$ and $\sigma^2 = Var(x)$, the probability density function is expressed as (2):

$$\mathrm{f}(\mathrm{x}) = \frac{1}{\sqrt{2\pi\sigma^2}} \mathrm{e}^{-\frac{1}{2}\mathrm{q}(\mathrm{x})}, \quad \mathrm{x} \in]-\infty, \infty[, \tag{2}$$

where $q(x) = (x - \mu)\left(\sigma^2\right)^{-1}(x - \mu)$. In (2), it may be noticed that $q(x)$ determines how far x is from μ in the scale of one standard deviation. According to Johnson [43], such a distance may be extended when $\mathbf{x}$ is a vector of the current values of the variables, i.e., when $\mathbf{x} = [x_1 x_2 \ldots x_n]'_{n\times 1}$, with $n \geq 2$. Then, $\boldsymbol{\mu} = [\mu_1 \mu_2 \ldots \mu_n]'_{nx1}$ is the vector of the expected values for each variable in $\mathbf{x}$, and $\boldsymbol{\Sigma}_{nn}$ is the corresponding definite positive variance–covariance matrix. Then, we have

$$Q_p(\mathbf{x}) = (\mathbf{x} - \boldsymbol{\mu})' \boldsymbol{\Sigma}_p^{-1} (\mathbf{x} - \boldsymbol{\mu}). \tag{3}$$

Equation (3) refers to the square distance between $\mathbf{x}$ and $\boldsymbol{\mu}$. Therefore, the multivariate normal probability density function is defined as

$$F_p(\mathbf{x}) = \frac{1}{(2\pi)^{p/2} |\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}Q_p(\mathbf{x})}, \tag{4}$$

where $x_i \in]-\infty, \infty[$ for $i = 1, 2, \ldots, n$.

E. WEIGHTED SUMS

The WS method has received attention owing to its implementation in multiobjective problems in different areas [2], [44]. WS consists of an agglutination technique that creates a global function as a linear combination—a weighted sum—of the individual objective functions in the problem. As shown in (5), a vector, $\mathbf{f}'(\mathbf{x}) = [f_1(\mathbf{x}) f_2(\mathbf{x}) \ldots f_m(\mathbf{x})]$, composed by m objective functions is multiplied by a vector of weights, $\mathbf{w}' = [w_1 w_2 \ldots w_m]$, where $0 \leq w_i \leq 1$, $i = 1, 2, \ldots, m$ and $\sum_{i=1}^{n} w_i = 1$.

$$\begin{aligned} \underset{\mathbf{x}}{\mathrm{Min}}\ \ & \mathbf{w}'\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{m} w_i f_i(\mathbf{x}), \\ s.t.:\ & h_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, l, \\ & g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \ldots, p, \end{aligned} \tag{5}$$

where $h_i(\mathbf{x})$ and $g_i(\mathbf{x})$ are the equality and inequality constraints, respectively. Because the outputs may have significantly different scales, they can be normalized according to

TABLE 1. Chemical composition of base metal and filler metal

Material	C	Mn	P	S	Si	Ni	Cr	Mo
AISI 1020	0.18 - 0.23	0.30 - 0.60	0.04	0.05	-	-	-	-
AWS E316LT1-1/4	0.03	1.58	-	-	1.00	12.4	18.5	2.46

the payoff matrix, $\mathbf{\Phi}$, which is defined in (6) [45]:

$$\mathbf{\Phi}_{(m\times m)} = \begin{bmatrix} f_1^*\left(\mathbf{x}_1^*\right)\ldots & f_1\left(\mathbf{x}_i^*\right)\ldots & f_1(\mathbf{x}_m^*) \\ \vdots & \vdots & \vdots \\ f_i\left(\mathbf{x}_1^*\right)\ldots & f_i^*\left(\mathbf{x}_i^*\right)\ldots & f_i\left(\mathbf{x}_m^*\right) \\ \vdots & \vdots & \vdots \\ f_m\left(\mathbf{x}_1^*\right)\ldots & f_m\left(\mathbf{x}_i^*\right)\ldots & f_m^*\left(\mathbf{x}_m^*\right) \end{bmatrix}, \quad (6)$$

where $\mathbf{x}_i^*$ represents the vector of input variables that optimizes $f_i(\mathbf{x})$; $f_i^*(\mathbf{x}_i^*)$ refers to the optimal value of the response, also known as the utopia value. The main diagonal of $\mathbf{\Phi}$ consists of utopian values. The remaining values are non-optimal; the worst of them regarding each response are referred to as Nadir values. Utopian and Nadir values are used to normalize the objective functions so that they all have the same scale. The normalized objective functions are obtained by (7):

$$\bar{f}_i(\mathbf{x}) = \frac{f_i(\mathbf{x}) - f_i^U}{f_i^N - f_i^U}, \quad i = 1, \ldots, m, \quad (7)$$

where f_i^U is the utopian value of response i and f_i^N is the Nadir value. After normalizing the objective functions, the multiobjective optimization problem can be written as (8). The problem is solved for different combinations of w and the solutions of the objective functions compose the Pareto boundary.

$$\begin{aligned} &\underset{\mathbf{x}}{\text{Min}}\ \mathbf{w}'\mathbf{F}(\mathbf{x}) = \sum_{i=1}^{m} w_i\bar{f}_i(\mathbf{x}) \\ &s.t.:\ h_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, l \\ &\qquad g_i(\mathbf{x}) \le 0, \quad i = 1, 2, \ldots, p \end{aligned} \quad (8)$$

F. GENERALIZED REDUCED GRADIENT (GRG)

Once the optimization problem has been formulated, the aim is to find the optimal solution. To achieve this, several search algorithms have been developed [46]. For instance, when the response surface models are second-order polynomials, the optimal solution corresponds to the stationary point of the fitted surface or to the point that is closest to the stationary one, depending on the experimental region.

Among the search algorithms, the GRG is one of the most efficient methods for constrained optimization. GRG reduces the number of variables by substituting the constraints of the objective functions. Therefore, the number of gradients is reduced [4].

The first stage of the GRG method is the classification of the original variables into basic ones, namely Z (or dependent) and non-basic Y (or independent). Let $\boldsymbol{F}(\mathbf{x}) = \boldsymbol{F}(\boldsymbol{Z}, \boldsymbol{Y})$ and $\boldsymbol{h}(\mathbf{x}) = \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{Y})$ be functions of variables Z and Y. To guarantee the optimality condition, it is necessary that $\boldsymbol{dhj}(\boldsymbol{x}) = \mathbf{0}$. Thus, if $\boldsymbol{A} = \boldsymbol{\nabla_Z h_j}(\mathbf{x})$ and $\boldsymbol{B} = \boldsymbol{\nabla_Y h_j}(\mathbf{x})$, then $\boldsymbol{dY} = -\boldsymbol{B}^{-1}\boldsymbol{AdZ}$. Finally, the gradient, G_R, is expressed as (9).

$$G_R = \frac{d}{dZ}F(\mathbf{x}) = \nabla_Z F(\mathbf{x}) - \left[B^{-1}A\right]' \nabla_Y F(\mathbf{x})' \quad (9)$$

The search direction is given by $\boldsymbol{S}_\mathbf{x} = [-\boldsymbol{G_R dY}]'$. To verify whether a potential solution $\mathbf{x}^{k+1}$ is, in fact, satisfactory and if $\boldsymbol{h}\left(\mathbf{x}^{k+1}\right) = \mathbf{0}$, the increment at each step, $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha\boldsymbol{S}^{k+1}$, can be used. Finally, using a one-dimensional algorithm of search, such as the Newton method, it is possible to obtain the solution of F($\mathbf{x}$).

III. MATERIALS AND METHOD

A. EXPERIMENTAL DESIGN

Experiments were conducted using the ESAB AristoPower 460 welding machine, the AristoFeed 30-4 W MA6 module (employed to feed the wire), and a mechanical system device to control the welding speed, torch distance, and torch angle, which was defined as 15° to "pushing". The base metal was AISI 1020 carbon steel cut into plates of 120 mm × 60 mm × 6.35 mm. The filler metal was a flux-cored stainless-steel wire of type AWS E316LT1-1/4, with a diameter of 1.2 mm and a linear density of 7.21 g/m. The chemical compositions of the materials are listed in Table 1.

A mixture of 75% Ar + 25% CO_2 was used as the shielding gas at a flow rate of 16 L/min. The welding technique used in the experiments was bead-on-plate, setting the input variables according to the selected DOE. The input variables were the wire feed rate (W_f), voltage (V), welding speed (S), and the distance from the contact tip to the work piece (N). Following a CCD, 31 experiments were conducted: 16 factorial points, 8 axial points, and 7 center points. The parameter levels were established according to Gomes *et al.* [47] based on previous tests and are summarized in Table 2.

The samples were cut at four different points along the specimens, as shown in Fig. 1a; their cross sections were attacked with nital solution (4%) and were then photographed. The Analysis Doc® software was used to measure (in mm) the bead width (W), penetration (P), and reinforcement (R). The penetration area (A_p) and the total area (A_t) of the weld were measured in mm^2 and are shown in Fig. 1b. Then, the dilution percentage (D) was obtained by calculating A_p/A_t, as shown in Fig. 1b. Table 3 lists the results of the experiments regarding the measured responses.

TABLE 2. Input variables and levels

Parameter	Unit	Notation	Levels				
			-2	-1	0	1	2
Wire feed rate	meter/minutes	W_f	5.5	7	8.5	10	11.5
Voltage	volt	V	24.5	27	29.5	32	34.5
Welding speed	centimeter/minute	S	20	30	40	50	60
Distance from contact tip to work piece	milimeter	N	10	15	20	25	30

ADAPTED FROM GOMES *ET AL.* [47]

TABLE 3. Experimental runs and results of the measured outputs

Run	Input variables				Output variables			
	Wf	*V*	*S*	*N*	*W*	*P*	*R*	*D*
1	7.0	27.0	30	15	11.19	1.37	2.63	26.4%
2	10.0	27.0	30	15	12.99	1.66	3.12	25.8%
3	7.0	32.0	30	15	12.70	1.69	2.50	31.5%
4	10.0	32.0	30	15	15.05	1.98	2.78	31.2%
5	7.0	27.0	50	15	9.21	1.65	2.17	36.2%
6	10.0	27.0	50	15	9.96	1.94	2.67	33.7%
7	7.0	32.0	50	15	9.75	1.54	2.06	37.1%
8	10.0	32.0	50	15	11.51	2.18	2.42	41.1%
9	7.0	27.0	30	25	10.32	1.25	2.87	22.5%
10	10.0	27.0	30	25	11.43	1.00	3.59	18.3%
11	7.0	32.0	30	25	11.27	1.32	2.85	23.7%
12	10.0	32.0	30	25	13.34	1.10	3.18	22.0%
13	7.0	27.0	50	25	7.99	1.11	2.55	25.0%
14	10.0	27.0	50	25	8.62	1.23	2.80	23.3%
15	7.0	32.0	50	25	8.48	1.37	2.36	28.8%
16	10.0	32.0	50	25	10.84	1.64	2.60	30.2%
17	5.5	29.5	40	20	9.07	1.38	2.21	31.6%
18	11.5	29.5	40	20	12.21	2.14	3.06	31.0%
19	8.5	24.5	40	20	9.42	1.20	3.03	22.8%
20	8.5	34.5	40	20	11.69	1.86	2.46	35.6%
21	8.5	29.5	20	20	14.93	0.95	3.45	18.6%
22	8.5	29.5	60	20	8.48	1.43	2.25	35.8%
23	8.5	29.5	40	10	11.73	2.18	2.61	40.4%
24	8.5	29.5	40	30	9.22	1.28	2.89	24.2%
25	8.5	29.5	40	20	10.82	1.71	2.60	31.0%
26	8.5	29.5	40	20	10.93	1.72	2.59	31.7%
27	8.5	29.5	40	20	10.74	1.62	2.65	30.9%
28	8.5	29.5	40	20	10.61	1.80	2.50	32.8%
29	8.5	29.5	40	20	10.64	1.49	2.62	30.0%
30	8.5	29.5	40	20	10.59	1.49	2.61	31.1%
31	8.5	29.5	40	20	10.57	1.50	2.56	31.0%

ADAPTED FROM GOMES *ET AL.* [47]

IV. RESULTS AND DISCUSSION

A. RESPONSE SURFACE MODELS

The five output variables listed in Table 3 were modeled using experimental data and the general second-order polynomial model presented in (1). The coefficients in vector $\boldsymbol{\beta}$ were estimated using the OLS algorithm. In this study, (10) through (13) present the obtained RMSs, for which the adjusted R-squared were 97.98%, 91.86%, 83.24%, and 93.43%, respectively.

$$\begin{aligned} E\left[W(\mathbf{x})\right] = {} & 10.7 + 0.8x_1 + 0.66x_2 - 1.45x_3 - 0.63x_4 \\ & - 0.0033x_1^2 - 0.02x_2^2 + 0.26x_3^2 - 0.04x_4^2 \\ & + 0.27x_1x_2 - 0.11x_1x_3 - 0.03x_1x_4 - 0.1x_2x_3 \\ & - 0.01x_2x_4 + 0.07x_3x_4 \end{aligned} \tag{10}$$

$$\begin{aligned} E\left[R(\mathbf{x})\right] = {} & 2.59 + 0.19x_1 - 0.11x_2 - 0.22x_3 + 0.12x_4 \\ & + 0.01x_1^2 + 0.04x_2^2 + 0.02x_3^2 + 0.04x_4^2 \end{aligned}$$

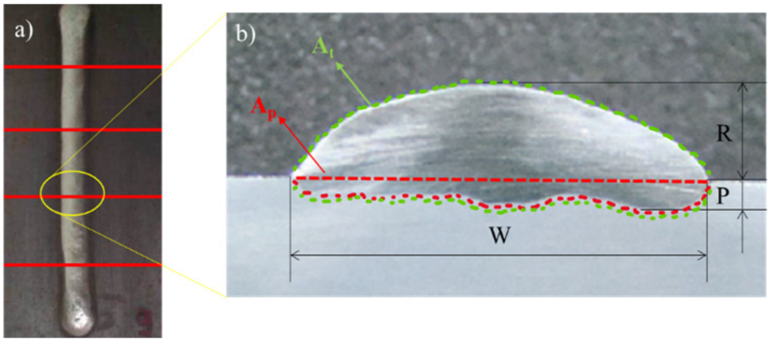

FIGURE 1. a) Welding bead cuts; b) bead geometry and parameters.

$$- 0.03x_1x_2 - 0.02x_1x_3 - 0.02x_1x_4 - 0.01x_2x_3 + 0.02x_2x_4 - 0.01x_3x_4 \quad (11)$$

$$E[P(\mathbf{x})] = 1.62 + 0.12x_1 + 0.12x_2 + 0.09x_3 - 0.24x_4 + 0.03x_1^2 - 0.03x_2^2 - 0.12x_3^2 + 0.02x_4^2 + 0.03x_1x_2 + 0.08x_1x_3 - 0.1x_1x_4 + 0.00019x_2x_3 + 0.01x_2x_4 + 0.01x_3x_4 \quad (12)$$

$$E[D(\mathbf{x})] = 31.22 - 0.28x_1 + 2.49x_2 + 3.68x_3 - 4.25x_4 - 0.23x_1^2 - 0.74x_2^2 - 1.25x_3^2 + 0.03x_4^2 + 0.77x_1x_2 + 0.5x_1x_3 - 0.42x_1x_4 + 0.23x_2x_3 - 0.2x_2x_4 - 0.77x_3x_4 \quad (13)$$

B. VARIANCE ESTIMATIONS

In each experimental run, the results of W, R, P, and D were measured four times. The standard deviations were considered deterministic and were obtained using $\sigma_{\bar{Y}} = \bar{s}_y/c_4$, as described in Section 2.3. For $n = 4$ observations, we obtained $c_4 = 0.921$. Table 4 and 5 list the standard deviations of the responses and the correlations between each pair of responses, respectively.

TABLE 4. Standard deviations of each response of interest

	W	R	P	D
$\bar{s}_y$	0.366	0.138	0.181	0.028
$\hat{\sigma}_y$	0.397	0.150	0.197	0.031

TABLE 5. Correlations between pairs of responses

	W	R	P	D
R	0,52			
P	0,22	-0,39		
D	-0,12	-0,77	0,82	

C. MULTIOBJECTIVE OPTIMIZATION PROBLEM

Equation (14) presents the optimization problem for this application.

$$\begin{aligned} &Max\ F(\mathbf{x}) = wE[W(\mathbf{x})] + (1-w)E[R(\mathbf{x})] \\ &s.t.:\ \mathbf{X}'\mathbf{X} \leq \rho^2 = 4 \\ &\quad \varphi[P(\mathbf{x}) \leq USL_P \cap D(\mathbf{x}) \leq USL_D] \geq 95\% \end{aligned} \quad (14)$$

The aim of this application was to maximize the responses of interest related to the process productivity, being subjected to maximum values of the critical quality (CTQ) characteristics.

As presented in Fig. 1a and b, all responses W, R, P, and D relate to the bead geometry. For the stainless-steel claddings, W is also related to the process productivity because higher values of the bead width would require less steps to cover the same surface area. The same productivity issue applies to the reinforcement (R), which is also linked to the protection of the surface under the stainless-steel layer. In addition, both R and W were measured in mm; therefore, their agglutination function, $\boldsymbol{F}(\mathbf{x})$, did not require their normalization.

The remaining results, namely P and D, are the CTQs of this process. A small penetration (P) and low dilution percentage are desired for the cladding process. According to the specialist of the present study, the penetration should remain below $USL_P = 1.5$ mm, with a dilution lower than $USL_D = 25\%$, to guarantee that the bead presents a satisfactory and functional geometry. Higher values of P and D would compromise the quality of the cladding.

In (14), $\varphi[P(\mathbf{x}) \leq USL_P \cap D(\mathbf{x}) \leq USL_D]$ is the probability of both P and D remaining below their USLs. This probability is obtained using (4), which was presented in Section 2. For $n = 2$, as is the case in this study, we have

$$\boldsymbol{\Sigma}_2 = \begin{bmatrix} \boldsymbol{\sigma}_1^2 & \boldsymbol{\sigma}_{12} \\ \boldsymbol{\sigma}_{12} & \boldsymbol{\sigma}_2^2 \end{bmatrix}. \quad (15)$$

Since the correlation between x_1 and x_2 is

$$\rho = \frac{\sigma_{12}}{\sigma_1\sigma_2} \leftrightarrow \sigma_{12} = \rho\sigma_1\sigma_2, \quad (16)$$

we have

$$\boldsymbol{\Sigma}_2^{-1} = \frac{1}{\sigma_1\sigma_2(1-\rho^2)} \begin{bmatrix} \sigma_2^2 & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^2 \end{bmatrix}; \quad (17)$$

therefore,

$$Q_2(x_1, x_2) = \frac{1}{1-\rho^2}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2 - 2\rho\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right)\right]. \quad (18)$$

TABLE 6. Results of the Optimization Problem in (14)

w	V_a	T	V_s	N	$E[W]$	$E[R]$	$E[P]$	$E[D]$	$\varphi[P \cap D]$
0.00	9.814	27.831	24.643	23.273	13.176	3.600	0.900	0.172	0,994
0.05	9.813	27.953	24.282	23.082	13.341	3.600	0.902	0.172	0,994
0.10	9.813	28.088	23.920	22.870	13.516	3.597	0.907	0.173	0,994
0.15	9.811	28.233	23.566	22.638	13.697	3.593	0.913	0.174	0,993
0.20	9.808	28.389	23.226	22.388	13.883	3.587	0.922	0.175	0,992
0.25	9.804	28.555	22.907	22.118	14.071	3.578	0.933	0.176	0,991
0.30	9.792	28.720	22.615	21.857	14.247	3.567	0.945	0.178	0,990
0.35	9.730	28.813	22.311	21.757	14.339	3.560	0.941	0.178	0,990
0.40	9.669	28.891	22.053	21.674	14.411	3.553	0.938	0.178	0,990
0.45	9.612	28.954	21.833	21.602	14.468	3.546	0.934	0.178	0,990
0.50	9.559	29.005	21.650	21.543	14.510	3.540	0.931	0.178	0,990
0.55	9.509	29.046	21.492	21.490	14.543	3.534	0.929	0.178	0,990
0.60	9.464	29.078	21.358	21.443	14.569	3.529	0.926	0.178	0,990
0.65	9.424	29.105	21.248	21.410	14.587	3.524	0.924	0.178	0,990
0.70	9.386	29.126	21.152	21.379	14.600	3.519	0.922	0.178	0,990
0.75	9.353	29.143	21.070	21.353	14.609	3.515	0.921	0.178	0,990
0.80	9.322	29.156	21.001	21.331	14.616	3.511	0.919	0.178	0,990
0.85	9.294	29.166	20.940	21.312	14.620	3.508	0.918	0.178	0,990
0.90	9.268	29.175	20.886	21.296	14.623	3.505	0.917	0.178	0,990
0.95	9.247	29.181	20.842	21.282	14.624	3.502	0.916	0.178	0,990
1.00	9.225	29.186	20.800	21.270	14.625	3.500	0.915	0.178	0,990

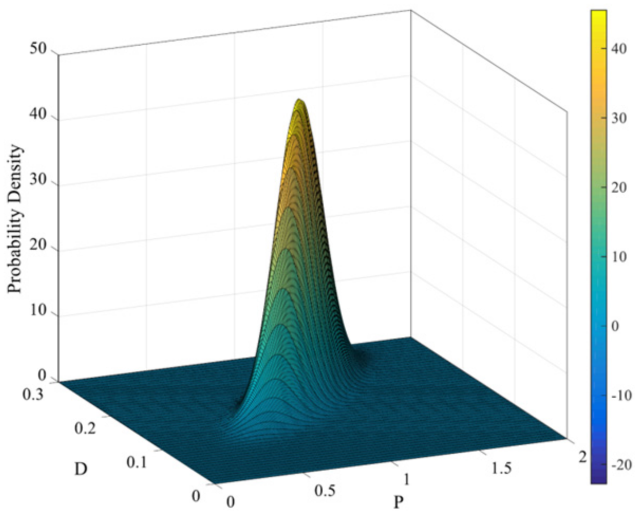

FIGURE 2. Bivariate normal distribution of P and D.

Consequently, the bivariate normal distribution is given by (19).

$$F_2(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} e^{-\frac{1}{2}Q_2(x_1,x_2)} \tag{19}$$

Fig. 2 shows the bivariate normal distribution and the probability density of P and D.

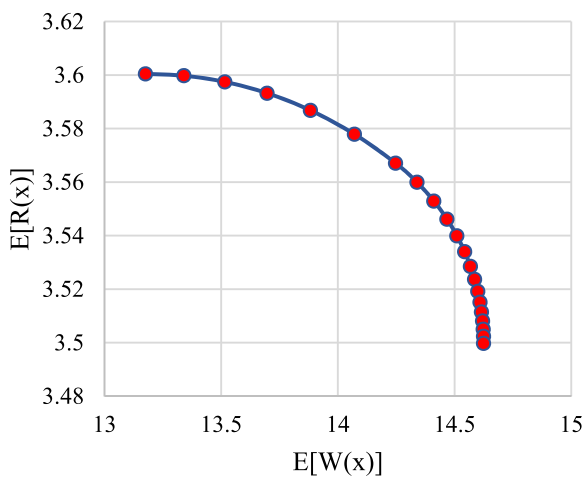

FIGURE 3. Pareto boundary of the problem in (14).

D. MULTIOBJECTIVE OPTIMIZATION PROBLEM

Equation (14) was solved in different weighting sets. Table 6 lists all the results. Fig. 3 presents the Pareto boundary for 21 different weighting sets from 0 to 1 with 0.05 increments. As may be observed, in Fig. 3, the points at the bottom right corner are very close to each other. They were obtained by solving (14) for w $\geq$ 0.6. The stochastic constraint becomes active for the aforementioned weighting sets; therefore, the algorithm becomes more limited in finding the optimal solutions.

The results obtained in the present study were compared with those of Gomes *et al.* [19]. These authors carried out a deterministic multivariate optimization using the same data set summarized in Table 3. In their approach, they used principal component analysis (PCA), which is a method that reduces the dimensionality of extense and correlated data [14]. The PCA approach was integrated to the mean square error (MSE) optimization method.

Based on individual optimizations, Gomes *et al.* [19] defined that the targets for the geometrical responses of interests were the following: $W = 15.57$ mm, $R = 3.34$ mm, $P = 0.83$ mm, and $D = 16.27\%$. The optimal solution achieved by using the PCA–MSE strategy was: $W = 8.99$ mm, $R = 2.87$ mm, $P = 1.45$ mm, and $D = 25.88\%$.

After analyzing the behavior of the Pareto-optimal solutions obtained in the present study, it is possible to observe that the results of the proposed multivariate stochastic optimization method are slightly closer to the targets established in the work of Gomes *et al.* [19]. It is important to state that in their work, additional responses of interest were included to the optimization problem, not only the geometric characteristics of this welding process.

V. CONCLUSION

In this work, we aimed to optimize a FCAW process used in the manufacturing of stainless-steel claddings. The authors proposed the use of stochastic programming and multivariate statistics to include the variability of the geometry characteristics to the optimization problem. The main results were the following.

- The standard deviations of the expected values of the responses were estimated from the measurements of different sessions of the bead of each experimental run. After conducting Anderson–Darling goodness-of-fit tests, the normal distribution was considered an appropriate representation of the data.
- The WS method was efficient in determining a Pareto boundary for the multiobjective optimization problem. The normal boundary intersection (NBI) method was also tested; however, it could not define a Pareto boundary for $w \geq 0.6$. The NBI could not converge after the multivariate stochastic constraint became active.
- The multivariate stochastic constraint of the optimization problem was active for certain weighting sets present in the Pareto boundary, which approximated the solutions when $w \geq 0.6$.
- To obtain the desired results, it was important to consider the correlation between the penetration (P) and the dilution (D). If P and D were considered independent, the error in the multivariate probability would be significant.

As future research, the authors suggest that the comparison between the variance present in the RSMs and the variance of the real measured data be included in the solution of the optimization problem.

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