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# Small Sample-Based Fatigue Reliability Analysis Using Non-Intrusive Polynomial Chaos

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**ABSTRACT** Based on small sample of fatigue test data, a new method to obtain *p*-*S*-*N* curve for fatigue reliability analysis using non-intrusive polynomial chaos (NIPC) is proposed to lower test cost. Parameter *C* in Basquin *S*-*N* model is regarded as random variable. Samples of *C* are calculated through inverse analysis based on small sample of fatigue test life. Then non-intrusive polynomial chaos expansions of *C* with respect to fatigue life are constituted under different stress levels. Statistics of *C* can be calculated directly by polynomial coefficients. A fast large-sample of *C* can be obtained based on NIPC and probability distribution type can be determined through EDF test. Then samples of *C* under the stress levels can be obtained and substituted into *S*-*N* model to calculate corresponding fatigue life samples. The fatigue life under different reliabilities are calculated for fitting *p*-*S*-*N* curve. Fatigue test of Al 2024-T3 plate with hole is performed. *p*-*S*-*N* curves are obtained by proposed method and compared with that obtained by linear regression based on least square method. Almost all relative errors are less than 5%, which show that the proposed method can predict *p*-*S*-*N* curve effectively.

**INDEX TERMS** Fatigue reliability analysis, non-intrusive polynomial chaos, p-S-N curve, small sample, Basquin S-N model, aluminium alloy 2024-T3.

## I. INTRODUCTION

Metal are widely used as primary load-carrying components in engineering structure [1]. Fatigue damage caused by cyclic loading in long-term working period is one main failure mode of metal components [2], which is still the focus of safety and reliability study [3]–[6]. Researchers have proposed various fatigue damage and life prediction models [7]. The studies were focus on deterministic life prediction before 1990s. The most classic and still widely used models are *S*-*N* model and Palmgren-Miner rule [8], [9].

Numerous fatigue test data show fatigue life has great dispersion, which is caused by randomly distributed microscopic defects and uncertainty of workpiece process. Fatigue reliability design becomes more significant in engineering structure design. *p*-*S*-*N* curve is the most straightforward way, which can predict fatigue life for given reliability under constant amplitude loading. In general, *p*-*S*-*N* curve is fitted by linear regression and maximum likelihood method based on groups of fatigue test data [10], which is time-consuming and costly. In many pratical situations, only fewer samples of fatigue test life are avaiable. In such circumstances, many studies that focus on small sample based technique to lower time and test cost are carried out [11]–[14]. Gao *et al.* [15] proposed a method for obtaining *P*-*S*-*N* curves under the condition of small sample from a perspective of geometry. Based on the failure trajectory concept and backwards statistical inference technique, Xie *et al.* [16] designed experimental programs to fit *P*-*S*-*N* curve with a small number of test data. Xiong and Shenoi [17] proposed two new randomized models of time-dependent processes for estimating the *P*-*a*-*t* and *P*-*S*-*N* curves.

In recent decades, approximate response surface method, based on spectral representation of the uncertainty, developed rapidly. Non-intrusive polynomial chaos method is the representative one. It has been widely applied in computional fluid mechanics and reliability analysis [18]–[20], because probability properties of output response can be obtained accurately with smaller smaple size of input random variables in comparison to Monte Carlo [21].

The contribution of this paper distinguishes from publications in previous paragraph. The study is based on Basquin

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*S-N* model, which is randomized by regarding model parameter $C$ as random variable. To lower test cost, non-intrusive polynomial chaos method is used to calculate statistical characteristics of $C$.

The paper is organized as follows: basic formulas for non-intrusive polynomial chaos are briefly introduced in section 2. Algorithm framework to randomize Basquin *S-N* model and predict *p-S-N* curve are given in section 3. Validation of proposed method through fatigue test data of Al 2024-T3 plate with hole is provided in section 4. We summarize the paper and draw conclusions in section 5.

## II. NON-INTRUSIVE POLYNOMIAL CHAOS METHOD FOR STOCHASTIC ANALYSIS

The polynomial chaos method [21], [22], based on the spectral representation of the uncertainty, provides a random function with an approximate response surface with respect to its control random variables, the advantage of which is that statistic characteristics of random function can be obtained with smaller sample of input random variables.

Define $\xi = \{\xi_1, \ldots, \xi_n\}$ is a set of standard orthogonal random variables. Random function $\varphi$ can be expanded as a truncated series of polynomials,

$$\varphi \approx \sum_{k=0}^{N_{pc}-1} C_k \psi_k(\xi) \tag{1}$$

where $N_{pc}$ is number of terms in polynomial chaos expansion, which is also the minimum number of samples required to determine coefficient $C_k$. Eq. (1) indicates that random function $\varphi$ can be decomposed into deterministic and stochastic components. The expansion coefficients $C_k$ are deterministic components, and the basis functions $\psi_k(\xi)$ denote stochastic components. Number of terms $N_{pc}$ can be given by [19],

$$N_{pc} = \frac{(p+n)!}{p!n!} \tag{2}$$

where $p$ is the order of polynomial chaos and $n$ denotes the number of random variables in $\xi$.

**TABLE 1. Some continuous probability distributions and corresponding polynomial basis.**

| Distribution | Basis | Weight function | Support |
|---|---|---|---|
| Gaussian | Hermite | $e^{-\frac{x^2}{2}}$ | $[-\infty,+\infty]$ |
| Gamma | Laguerre | $x^a e^{-x}$ | $[0,+\infty]$ |
| Uniform | Legendre | 1 | [-1,1] |
| Poisson | Charlier | $\frac{\lambda^x}{x!}$ | $x=0,1,\ldots$ |
| Exponential | Laguerre | $e^{-x}$ | $[0,+\infty]$ |

To improve convergence of polynomial chaos expansion, basis function $\psi_k(\xi)$ should be chosen according to Askey rule [23] such that their orthogonal weight function matches the probability density function of random variables $\xi$. The polynomial basis for some commonly used continuous probability distributions are listed in Table 1.

In engineering application, each set of samples of $\xi = \{\xi_1, \ldots, \xi_n\}$ can lead to a corresponding response $\varphi$. Enough accurate statistics of $\varphi$ can be achieved while the order of polynomial chaos is no less than 3 [19]. With the left side of Eq. (1) obtained from experiment or the solutions of deterministic evaluations at the chosen random points, a linear system of equations can be obtained:

$$\begin{pmatrix} \psi_0(\xi_0) & \psi_1(\xi_0) & \cdots & \psi_{N_{pc}-1}(\xi_0) \\ \psi_0(\xi_1) & \psi_1(\xi_1) & \cdots & \psi_{N_{pc}-1}(\xi_1) \\ \vdots & \vdots & & \vdots \\ \psi_0(\xi_M) & \psi_1(\xi_M) & \cdots & \psi_{N_{pc}-1}(\xi_M) \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N_{pc}-1} \end{pmatrix} = \begin{pmatrix} \varphi(\xi_0) \\ \varphi(\xi_1) \\ \vdots \\ \varphi(\xi_M) \end{pmatrix} \tag{3}$$

where $M$ is no less than $N_{pc}$. The above linear equations are calculated using a single linear least squares solution. Based on the polynomial coefficients, the statistic characteristics, such as the mean, variance, skewness and kurtosis, of the random response $\varphi$ can be achieved. The mean of the random response $\varphi$ is given by,

$$\begin{aligned} \bar{\varphi} &= \int_{D_\varphi} \left( \sum_{k=0}^{N_{pc}-1} C_k \psi_k(\xi) \right) f_\xi(\xi) \mathrm{d}\xi \\ &= C_0 \int_{D_\varphi} \psi_0(\xi) f_\xi(\xi) \mathrm{d}\xi + \sum_{k=1}^{N_{pc}-1} C_k \int_{D_\varphi} \psi_k(\xi) f_\xi(\xi) \mathrm{d}\xi \\ &= C_0 + \sum_{k=1}^{N_p-1} C_k \int_{D_\varphi} \psi_0(\xi)\psi_k(\xi) f_\xi(\xi) \mathrm{d}\xi = C_0 \end{aligned} \tag{4}$$

The variance of the random response $\varphi$ is given by,

$$\sigma_\varphi^2 = \sum_{k=1}^{N_{pc}-1} C_k^2 \left\langle \psi_k^2(\xi) \right\rangle \tag{5}$$

Similarly, the skewness, which represents quality or condition of being skew, can be calculated by,

$$\begin{aligned} u &= \frac{E\left(x - E\left(x\right)\right)^3}{\sigma^3} \\ E\left(x - E\left(x\right)\right)^3 &= \sum_{i=1}^{N_{pc}} \sum_{j=1}^{N_{pc}} \sum_{k=1}^{N_{pc}} E(\psi_i \cdot \psi_j \cdot \psi_k) \cdot \varphi_i \cdot \varphi_j \cdot \varphi_k \end{aligned} \tag{6}$$

The kurtosis, which is a measure of the concentration of a distribution around its mean, can be calculated by,

$$\begin{aligned} k &= \frac{E\left(x - E\left(x\right)\right)^4}{\sigma^4} \\ E\left(x - E\left(x\right)\right)^3 &= \sum_{i=1}^{N_{pc}} \sum_{j=1}^{N_{pc}} \sum_{k=1}^{N_{pc}} \sum_{l=1}^{N_{pc}} E(\psi_i \cdot \psi_j \cdot \psi_k \cdot \psi_l) \\ &\quad \cdot \varphi_i \cdot \varphi_j \cdot \varphi_k \cdot \varphi_l \end{aligned} \tag{7}$$

where,

$$\begin{aligned} E(\psi_i \cdot \psi_j \cdot \psi_k) &= D_{i1,j1,k1} \cdot D_{i2,j2,k2} \\ E(\psi_i \cdot \psi_j \cdot \psi_k \cdot \psi_l) &= D_{i1,j1,k1,l1} \cdot D_{i2,j2,k2,l2} \\ D_{i,j,k} &= C_{i,j,k} \cdot k! \\ D_{i,j,k,l} &= \sum_{q \geq 0} D_{i,j,q} \cdot C_{k,l,q} \end{aligned} \tag{8}$$

$$C_{i,j,k} = \frac{i!j!}{[(i+j-k)/2]![(j+k-i)/2]![(i-j+k)/2]!} \quad if \begin{cases} (i+j+k) even \\ k \in [|i-j|, i+j] \end{cases} \tag{9}$$

The calculus results show that the statistics are related with coefficients of polynomial chaos and weight function of polynomial basis. The weight function of polynomial basis contains probabilistic properties of random variables, which is reason that NIPC can calculate statistics based on small sample of random variables.

## III. RANDOMIZATION METHOD OF S-N MODEL

Basquin *S-N* model [8] is most widely used to predict middle fatigue life for metallic materials, which is written as,

$$S^m \cdot N = C \tag{10}$$

where $S$ is stress amplitude, $N$ is fatigue life, $m$ and $C$ are model parameters. Eq. (10) can be written in terms of logarithm,

$$\lg N = -m \lg S + \lg C \tag{11}$$

Fatigue life has great dispersion under given stress amplitude loading, and is always thought to obey lognormal distribution. It is reasonable to regard parameter of *S-N* model as random variable. In this paper, we regard $C$ as random variable to present randomness of fatigue life while $m$ is still material constant, which can be determined by fitting mean value of fatigue life through Eq. (11) in advance. One sample of lg$C$ can be obtained for one fatigue test life through Eq. (11). In the view of statistics, numerous samples of fatigue test life are needed to obtain probabilistic properties of lg$C$ with enough accuracy. Non-intrusive polynomial chaos method introduced in Section 2 is used to lower test cost in this study. Now we will use non-intrusive polynomial chaos to establish stochastic analysis framework on the basis of small sample of fatigue test life.

For our problem, the random functions $\varphi$ in Eq. (1) is the *S-N* model parameter lg$C$, and the random variables $\xi = \{\xi_1, \ldots, \xi_n\}$ are one-dimensional, because there is only one random variable, i.e. fatigue life $N$, which is assumed to obey lognormal distribution in this study. As listed in Table 1, there is no direct polynomial basis for lognormal distribution. So we should calculate the logarithm of fatigue life $N$ in advance, which obeys normal distribution. Then

**TABLE 2. Fatigue test results under constant amplitude loading.**

| Group | *S*/MPa | Test life |
|---|---|---|
| A | 88.59 | 22226, 23435, 22135, 23410, 23752, 22651, 23067, 24808 |
| B | 52.78 | 164457, 189686, 180280, 187271, 227964, 199233, 155843, 178365 |
| C | 33.47 | 931175, 1017854, 1065166, 1069414, 1188311, 1257910, 772159, 792045 |
| D | 70.97 | 47271, 52777, 50778, 47325, 52663, 53352, 55004, 52272 |

polynomial chaos expansion can be established with Hermit orthogonal polynomial basis. The samples of fatigue life $\boldsymbol{N}\{N_1, N_2, \ldots, N_n\}$ with an appropriate sample size(no less than $N_{pc}$) are obtained by fatigue test under some constant amplitude loading. The corresponding samples of lg$C$\{ lg$C_1$, lg$C_2$,…,lg$C_n$\} are calculated through Eq. (11). Then, the coefficients $\{C_0, C_1, \ldots, C_{Npc-1}\}$ of the polynomial chaos expansion for lg$C$ are achieved by solving Eq. (3). At this point, the polynomial chaos expansion for the random function lg$C$ with respect to random variable $N$ is constituted. The statistics of lg$C$, such as the mean, variance, skewness and kurtosis, can be calculated directly from the polynomial coefficients.

Only the statistics are not enough. The distribution type is also needed to determine. It can be seen from Eq. (11) that lg$N$ is linearly related to lg$C$. In the view of probabilistic properties of normal distribution, it is reasonable to assume lg$C$ also obeys normal distribution. Besides, a fast large-sample of the parameter lg$C$ can be achieved based on the NIPC response surface at a significantly lower computational cost. Then probabilistic distribution type of lg$C$ can also be determined by EDF test [24]. EDF test is more effective than $\chi^2$ test, especially for small sample. We will primarily do the EDF test of normal distribution for lg$C$. The analysis process of non-intrusive polynomial chaos approach used for stochastic analysis is given in Fig.1.

Once probabilistic properties of lg$C$ are achieved, big sample of which can be easily obtained. One sample of lg$C$ represents one 'specimen'. Substituting samples of lg$C$ into *S-N* model Eq. (11), we can get samples of fatigue life, and fatigue life under different reliabilities can be calculated for fitting *p-S-N* curve.

## IV. VALIDATION OF PROPOSED METHOD

In general, *p-S-N* curve for metallic material is obtained by fatigue test of simply standard specimen, such as standard coupon or plate with a hole. In this section, fatigue test of aluminum alloy 2024-T3 plate with hole, which is representative component in engineering structure, under 4 groups of constant amplitude stress loading are performed. Stress ratio $R = 0.06$. Test machine is Instron 8802. The specimens are shown in Fig.2. The fractured specimens are illustrated in Fig.3. Test results are listed in Table 2, where the stress amplitude is nominal.

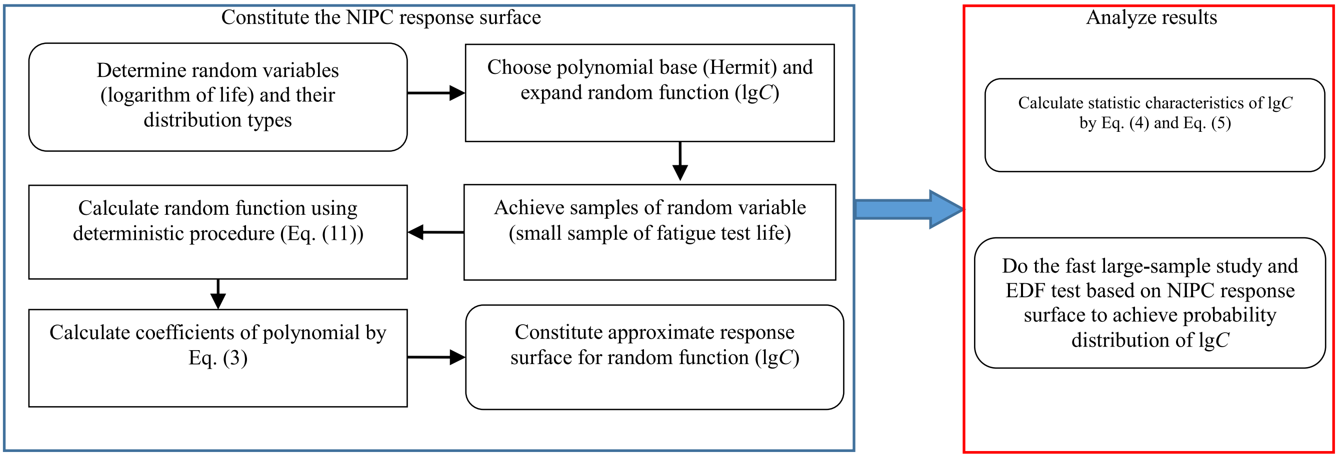


**FIGURE 1. Stochastic analysis process of NIPC method.**

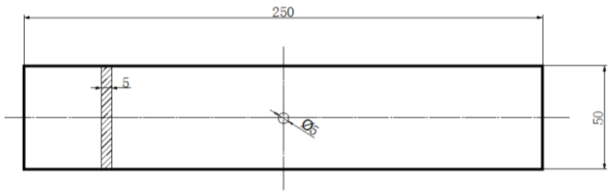


**FIGURE 2. Configuration of specimen.**

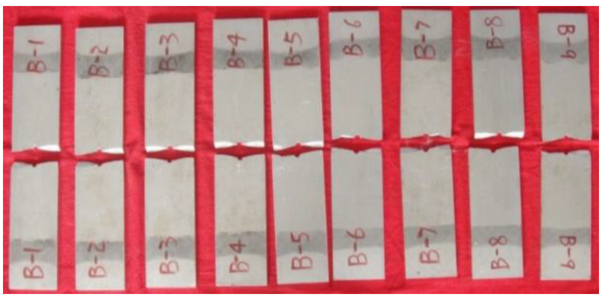

**FIGURE 3. The fractured specimens.**

The minimum number of specimens to constitute NIPC is $N_{pc} = 4$ under each stress amplitude, which is calculated by Eq. (2) with number of random variable $n = 1$ and polynomial expansion order $p = 3$. We make 2 plans to verify proposed method. All of the 8 test lives in each group are used to constitute NIPC in plan 1. Only the first 4 test lives in each group are used to constitute NIPC in plan 2.

For plan 1, the NIPC of lg$C$ with respect to logarithm of fatigue life are constituted under 4 groups. The statistics mean and standard deviation of lg$C$ are calculated and listed in Table 3. A fast large-sampling of lg$C$ is performed based on NIPC. The EDF tests of normal distribution for lg$C$ based on large-sampling are achieved and results are also listed in Table 3, where √ means passing the EDF test with 95% confidence. It can't be rejected that lg$C$ obeys normal distribution. So lg$C$ is regarded as normal distribution random variable.

Now big sample of lg$C$ under each stress level can be obtained by random sampling. The sample size is 5000 in this paper. Substitute samples of lg$C$ into Eq. (11) to calculate

**TABLE 3. Statistics of lgC and EDF test results (plan 1).**

| Group | $\mu$ | $\sigma$ | $D_n$ | $W^2$ | $A^2$ |
|---|---|---|---|---|---|
| A | 11.9539 | 0.0163 | 0.3718√ | 0.0296√ | 0.2243√ |
| B | 11.9779 | 0.0506 | 0.4296√ | 0.0301√ | 0.2015√ |
| C | 11.9408 | 0.0765 | 0.4770√ | 0.0411√ | 0.2757√ |
| D | 11.9304 | 0.0251 | 0.7061√ | 0.0751√ | 0.4817√ |

**TABLE 4. Predicted life *N* for different reliabilities by NIPC (plan 1).**

| Group | Reliability $p$ | | | |
|---|---|---|---|---|
| | 95% | 75% | 50% | 5% |
| A | 21858 | 22679 | 23267 | 24769 |
| B | 152407 | 170487 | 184304 | 222878 |
| C | 751774 | 888313 | 997574 | 1323741 |
| D | 47445 | 50167 | 52151 | 57324 |

**TABLE 5. Fitted *m* and lgC for different reliabilities (plan 1).**

| $p$ | Traditional method | | Proposed method | | Relative error/% | |
|---|---|---|---|---|---|---|
| | $m$ | lg$C$ | $m$ | lg$C$ | $\delta_m$ | $\delta_{\lg C}$ |
| 0.95 | 3.654 | 11.453 | 3.657 | 11.460 | 0.082 | 0.061 |
| 0.75 | 3.797 | 11.746 | 3.795 | 11.744 | -0.053 | -0.017 |
| 0.5 | 3.897 | 11.950 | 3.892 | 11.942 | -0.128 | -0.067 |
| 0.05 | 4.140 | 12.447 | 4.127 | 12.425 | -0.314 | -0.177 |

samples of fatigue life. Then fatigue life under different reliabilities can be achieved by statistical analysis. Results are listed in Table 4.

$m_p$ and lg$C_p$, coefficients of $p$-$S$-$N$ curve under different reliabilities $p$, can be fitted by least square method with corresponding fatigue life in Table 4. Results are listed in Table 5. Besides, traditional method uses all test fatigue lives to fit $p$-$S$-$N$ curve. $m_p$ and lg$C_p$ obtained by traditional method are also listed in Table 5.

Based on results in Table 5, the logarithmic $p$-$S$-$N$ curve is illustrated in Fig.4, where scatters are test fatigue lives.

For plan 2, NIPC can also be constituted with sample size $N_{pc} = 4$ in the same way. Fatigue life $N$ under different reliabilities are predicted and listed in Table 6.

The fitted $m_p$ and lg$C_p$ are listed in Table 7. It can be seen that almost all relative errors are less than 5%, which

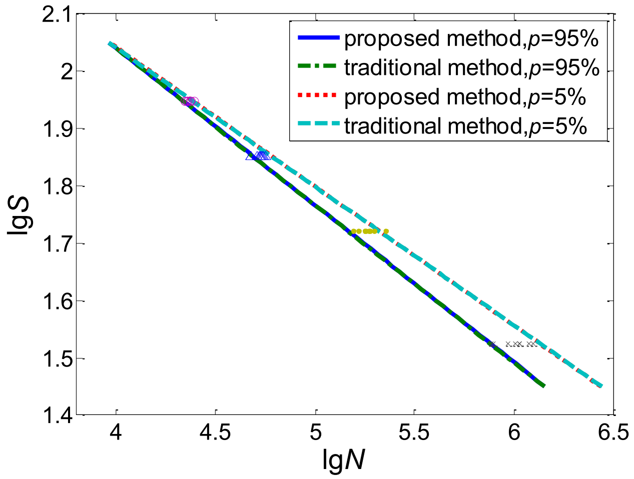


**FIGURE 4.** Fitted $p-S-N$ curves by proposed method and traditional method (plan 1).

**TABLE 6.** Predicted life $N$ for different reliabilities by NIPC (plan 2).

| Group | Reliability $p$ | | | |
|---|---|---|---|---|
| | 95% | 75% | 50% | 5% |
| A | 21766 | 22427 | 22897 | 24088 |
| B | 161754 | 172245 | 179936 | 200162 |
| C | 914865 | 974719 | 1018616 | 1134133 |
| D | 45209 | 47641 | 49408 | 53998 |

**TABLE 7.** Fitted $m$ and lg$C$ for different reliabilities (plan 2).

| $p$ | Traditional method | | Proposed method | | Relative error/% | |
|---|---|---|---|---|---|---|
| | $m$ | lg$C$ | $m$ | lg$C$ | $\delta_m$ | $\delta_{\lg C}$ |
| 0.95 | 3.654 | 11.453 | 3.890 | 11.893 | 6.459 | 3.842 |
| 0.75 | 3.797 | 11.746 | 3.921 | 11.970 | 3.266 | 1.907 |
| 0.5 | 3.897 | 11.950 | 3.943 | 12.025 | 1.180 | 0.627 |
| 0.05 | 4.140 | 12.447 | 3.996 | 12.156 | -3.478 | -2.338 |

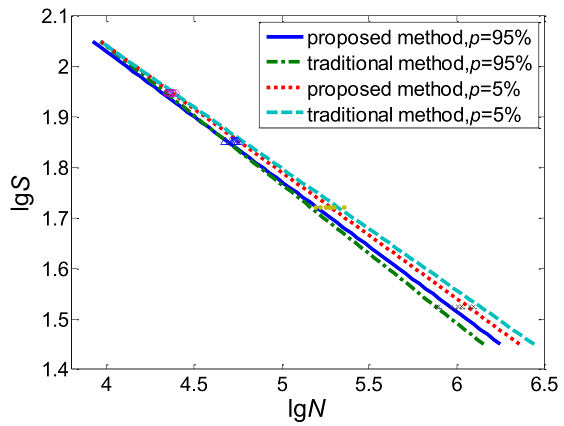


**FIGURE 5.** Fitted $p-S-N$ curves by proposed method and traditional method (plan 2).

is acceptable in engineering. The logarithmic $p$-$S$-$N$ curve is illustrated in Fig.5.

Compare results obtained by 2 plans. The $p$-$S$-$N$ curves are more accurate when the samples used to constitute NIPC are closer to population. Besides, it should be noticed that the two-parameter Basquin model studied in this paper is always applied for middle fatigue life. For long fatigue life, it is suggested to use bilinear Basquin model. The segmented $p$-$S$-$N$ curve can be fitted independently by the proposed method.

## V. CONCLUSIONS

In this paper, Basquin $S$-$N$ model is randomized by regarding model parameter lg$C$ as random vaiable. For the purpose of lowering test cost, non-intrusive polynomial chaos method is implemented to obtain statistical characteristics of lg$C$. The fast large-sample of lg$C$ is achieved based on constituted NIPC response surface at significantly lower computational cost. Then distribution type of lg$C$ are determined by EDF test.

4 groups of fatigue test of Al 2024-T3 plate with hole are performed. Two plans of sample size are designed to constitute NIPC and obtain $p$-$S$-$N$ curves under different reliabilities. The results comparision with traditional method show that the proposed randomization method of $S$-$N$ model based on small sample of fatigue test life is effective. $p$-$S$-$N$ curve can be obtained based on small smaple by proposed method.

## REFERENCES

[1] M. R. Sriraman and R. M. Pidaparti, "Life prediction of aircraft aluminum subjected to pitting corrosion under fatigue conditions," *J. Aircr.*, vol. 46, no. 4, pp. 1253–1259, Jul. 2009.
[2] T. Okada, S. Machida, T. Nakamura, H. Tanaka, K. Kuwayama, and M. Asakawa, "Fatigue crack growth of Friction-Stir-Welded aluminum alloy," *J. Aircr.*, vol. 54, no. 2, pp. 737–746, Mar. 2017.
[3] X. Wen, Z. Wang, Q. Wu, and C. Liu, "Reliability analysis for strain-based fatigue incorporating dispersity variation," *IEEE Access*, vol. 7, pp. 180804–180817, 2019.
[4] K. B. Rao, M. Anoop, G. Raghava, M. Prakash, and A. Rajadurai, "Probabilistic fatigue life analysis of welded steel plate railway bridge girders using S–N curve approach," *Proc. Inst. Mech. Eng., O, J. Risk Rel.*, vol. 227, no. 4, pp. 385–404, Aug. 2013.
[5] X. Chai, Z. Sun, J. Wang, Y. Zhang, and Z. Yu, "A new kriging-based learning function for reliability analysis and its application to fatigue crack reliability," *IEEE Access*, vol. 7, pp. 122811–122819, 2019.
[6] X. Liu, W. Zhang, X. Liu, W. Dai, and G. Fu, "A novel RLS-KS method for parameter estimation in particle filtering-based fatigue crack growth prognostics," *IEEE Access*, vol. 7, pp. 156764–156778, 2019.
[7] C. Luo, J. Wei, M. Parra-Garcia, A. Chattopadhyay, and P. Peralta, "Fatigue damage prediction in metallic materials based on multiscale modeling," *AIAA J.*, vol. 47, no. 11, pp. 2567–2576, Nov. 2009.
[8] O. H. Basquin, "The exponential law of endurance tests," *Proc. Amer. Soc. Test. Mater.*, vol. 10, no. 2, pp. 625–630, 1910.
[9] M. A. Miner, "Cumulative damage in fatigue," *J. Appl. Mech.*, vol. 12, no. 3, pp. 159–164, 1945.
[10] J. Ling, "A maximum likelihood method for estimating P-S-N curves," *Int. J. Fatigue*, vol. 19, no. 5, pp. 415–419, May 1997.
[11] S. Shimizu, K. Tosha, and K. Tsuchiya, "New data analysis of probabilistic stress-life (P–S–N) curve and its application for structural materials," *Int. J. Fatigue*, vol. 32, no. 3, pp. 565–575, Mar. 2010.
[12] R. Fouchereau, G. Celeux, and P. Pamphile, "Probabilistic modeling of S-N curves," *Int. J. Fatigue*, vol. 68, no. 11, pp. 217–223, 2014.
[13] D. Shi and H. Ma, "A Bayesian inference method and its application in fatigue crack life prediction," *IEEE Access*, vol. 7, pp. 118381–118387, 2019.
[14] L. Xie, "Principle of sample polymerization and method of P-S-N curve fitting," *J. Mech. Eng.*, vol. 49, no. 15, p. 96, 2013.
[15] J. Gao, Z. An, and B. Liu, "A new method for obtaining P-S-N curves under the condition of small sample," *Proc. Inst. Mech. Eng, O, J. Risk Rel.*, vol. 231, no. 2, pp. 130–137, Apr. 2017.
[16] L. Xie, J. Liu, N. Wu, and W. Qian, "Backwards statistical inference method for P-S-N curve fitting with small-sample experiment data," *Int. J. Fatigue*, vol. 63, pp. 62–67, Jun. 2014.

[17] J. J. Xiong and R. A. Shenoi, "A practical randomization approach of deterministic equation to determine probabilistic fatigue and fracture behaviours based on small experimental data sets," *Int. J. Fatigue*, vol. 145, no. 4, pp. 273–283, Oct. 2007.

[18] S. Hosder, R. Walters, and R. Perez, "A non-intrusive polynomial chaos method for uncertainty propagation in CFD simulations," in *Proc. 44th AIAA Aerosp. Sci. Meeting Exhibit*, Jan. 2006, p. 891.

[19] S. Hosder, R. W. Walters, and M. Balch, "Point-collocation nonintrusive polynomial chaos method for stochastic computational fluid dynamics," *AIAA J.*, vol. 48, no. 12, pp. 2721–2730, Dec. 2010.

[20] Y. Wang, S. Zeng, and J. Guo, "Improved non-intrusive polynomial chaos for reliability analysis under hybrid uncertainty," in *Proc. Int. Conf. Qual., Rel., Risk, Maintenance, Saf. Eng. (QR2MSE)*, Jul. 2013, pp. 460–464.

[21] S. S. Isukapalli, A. Roy, and P. G. Georgopoulos, "Stochastic response surface methods (SRSMs) for uncertainty propagation: Application to environmental and biological systems," *Risk Anal.*, vol. 18, no. 3, pp. 351–363, Jun. 1998.

[22] R. Ghanem and P. D. Spanos, "Polynomial chaos in stochastic finite elements," *J. Appl. Mech.*, vol. 57, no. 1, pp. 197–202, Mar. 1990.

[23] R. Askey and J. A. Wilson, *Some Basic Hypergeomietric Orthogonal Polynomials that Generalize Jacobi Polynomials*. Rhode Island: American Mathematical Society, 1985.

[24] M. A. Stephens, "EDF statistics for goodness of fit and some comparisons," *J. Amer. Stat. Assoc.*, vol. 69, no. 347, pp. 730–737, Sep. 1974.

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