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Design of an Optimal Fractional Order PID for Constant Tension Control System

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ABSTRACT Winding system is a production system commonly used in industry. Its control goal is to achieve constant tension control of the system by controlling motor torque, and to cope with various external disturbances in industrial environment. The Fractional Order PID (FOPID) controller is well known for its simplicity with better tuning flexibility as well as robustness with respect to disturbances. Thus, this paper proposes a control strategy based on FOPID to achieve the control objectives. First, a linear approximation model is established for the constant tension control system. Then, a FOPID controller is designed based on the model. Four non-linear constraints and an objective function of the controller are proposed based on the frequency domain. Five tunable parameters of FOPID are determined by combining genetic algorithm (GA) with FMINCON non-linear optimization. Finally, through the simulation of MATLAB, the performance of the controller is compared with the classic integer order PID. The results show that the FOPID controller has better control characteristics and robustness.

INDEX TERMS Tension control, fractional order PID controller (FOPID), robustness analysis, nonlinear optimization, genetic algorithm.

I. INTRODUCTION

Tension control refers to the control of various strips to maintain constant tension in the process of crimping, jointing and drawing [3]. At present, it is widely used in printing, packaging, textile, dyeing and cutting industries [4]. The control precision of tension control system directly affects the quality of products. When the tension is less than the value required by the process, it will lead to the deformation between the layers of the winding material, such as wrinkles, irregular winding and other low-quality conditions, while excessive tension will cause fracture, machine damage and other situations. If the tension increases or decreases, the product yield will reduce. Therefore, high-precision tension control is extremely important for industrial production, and the control system is required to have a certain degree of robustness to overcome system uncertainty, such as sensor measurement of noise and external disturbances.

Because of its simple structure and easy implementation, the PID controller has become the most widely used controller in industrial occasions. The methods for debugging parameters of PID controller have been widely studied and relatively mature at present [5], [6]. However, the tension control system is greatly affected by external interference, so it is difficult to achieve the desired control effect only by using PID control. Therefore, in this paper, the idea of robustness is introduced to the classic PID controller, and a constrained FOPID control method is proposed, which achieves the control requirements without additional computation.

In recent years, fractional calculus has attracted more attention in many fields of engineering and science [7]. In addition to its advantages in modeling, fractional calculus is also applied to the design of controllers. FOPID control introduces fractional order differential and integral into classic PID control, which makes parameter debugging more flexible and makes it possible to obtain better controller performance and robustness [1], [2]. The properties of FOPID controller have been studied in much literature, and the control performance of FOPID controller and classic PID controller has been compared in different situations [8]–[14]. Much literature puts forward different debugging principles for five parameters of FOPID controller. Literature [8] gives a FOPID controller by solving a set of nonlinear equations. The debugging principle of fractional order controller for motion system is proposed

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in literature [9] and literature [10]. The FOPID controller is obtained by genetic algorithm in [11]. Some effective design principles are summarized in [8] and [14].

In this paper, a nominal model of tension control system is given, and the tension control is realized by designing FOPID controller. The parameters of the controller are obtained by solving a set of non-linear equations, and the numerical optimization algorithm is implemented in MATLAB. The controller needs to satisfy some constraints in frequency domain. At the same time, the performance of FOPID controller and classic PID controller is compared. Besides, this paper will conduct the robustness analysis of static gain variation, load interference, high frequency noise suppression and output interference suppression.

This paper is organized as follows: Section II introduces the basic knowledge of fractional calculus. The mathematical model of tension control system is described in section III. Then, we propose the design principle of FOPID controller in section IV. The performance of FOPID controller and classic PID controller is compared and analyzed in section V, and the conclusion is given in the final section.

II. FRACTIONAL CALCULUS

Fractional calculus extends the order of differential and integral from integer to any real number. The basic formula is ${}_0D_t^\alpha$,where α is the order and t denotes the limit of the formula. When $\alpha > 0$, this formula represents fractional differential. While $\alpha < 0$, it represents fractional integral [15]. The most commonly used definitions of fractional calculus are Riemann-Liouville(RL) definition and Grumwald-Letnikov(GL) definition.

In this paper, the fractional calculus definition we use is Grumwald-Letnikov(GL) definition [7]:

$${}_0D_t^\alpha = \lim_{t\to 0} \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{\frac{t-\alpha}{n}} \frac{\Gamma(k+\alpha)}{\Gamma(k+1)} f(t-kh) \tag{1}$$

where, $n \in N$, $\alpha \in R^+$. Γ is the famous Euler Gamma function.

The most commonly used algebraic tool for describing fractional order systems is the Laplace transform. When $t = 0$, the Laplace transform of differential of order $n(n \in R^+)$ with signal $x(t)$ is [16]

$$L\{D^n x(t)\} = s^n X(s) \tag{2}$$

For fractional calculus equation, when $t = 0$, the transfer function of input signal $u(t)$ and output signal $y(t)$can be expressed as follows:

$$\frac{Y(s)}{U(s)} = \frac{a_1 s^{\alpha_1} + a_2 s^{\alpha_2} + \cdots + a_{m_A} s^{\alpha_{m_A}}}{b_1 s^{\beta_1} + b_2 s^{\beta_2} + \cdots + b_{m_B} s^{\beta_{m_B}}} \tag{3}$$

where, $(a_m, b_m) \in R^2$, $(\alpha_m, \beta_m) \in R_+^2$, $\forall m \in N$.

III. CONTROL PROBLEM DESCRIPTION

Winding system is a kind of control system which is often used in industrial field, and the key issue is tension control. The system is generally composed of unwinding roller, coil roller and traction roller, as shown in FIGURE 1. The system keeps the tension constant by controlling the torque of the motor to control the rewinding speed.

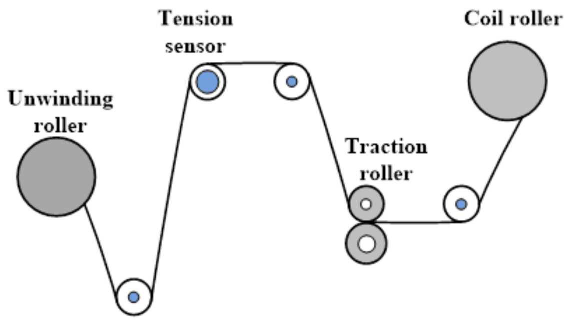

FIGURE 1. The structure chart of winding system.

FIGURE 2 is the frame diagram of the system. The closed-loop control of the system is realized by tension sensor.

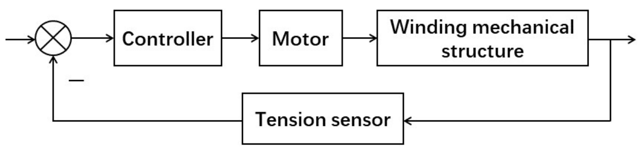

FIGURE 2. System frame diagram.

Through the system identification toolbox of MATLAB, the system transfer function is obtained as follows:

$$P(s) = \frac{90.5}{1 + 3.432s} \tag{4}$$

IV. PARAMETER DESIGN OF FOPID CONTROLLER

The general form of FOPID controller is $PI^\lambda D^\mu$. The transfer function is:

$$C(s) = \frac{Y(s)}{E(s)} = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \tag{5}$$

where, λ and μ separately represent integral order and differential order, which can be any real number.

In time domain, the control signal $y(t)$ can be expressed as follows:

$$y(t) = K_p e(t) + \frac{K_i}{D^\lambda} e(t) + K_d D^\mu e(t) \tag{6}$$

It can be noticed that the classic PID controller is a special case when $\lambda = \mu = 1$ in FOPID controller. Two adjustable parameters λ and μ are introduced to FOPID controller, which has greatly improved the flexibility of parameter debugging. By reasonable selection of parameters, FOPID controller will have better control effect [18].

References [8], [14], [17] have put forward some parameter debugging methods for FOPID controllers. It can be seen that numerical optimization has been widely used in parameter debugging of FOPID controllers.

The purpose of this paper is to design a FOPID controller for constant tension control systems, so that the output of the system can track the input quickly, overcome the external interference, and have a certain degree of robustness. The method of parameter debugging is based on a set of constraint equations in the frequency domain, and the controller satisfying the constraints is obtained by numerical optimization, so the parameter debugging is transformed into a numerical optimization problem with constraints. This design scheme based on frequency domain was first proposed in the [8]:

The dynamic controlled object P (s) and controller C (s) should satisfy the following relationships:

- Traversal frequency constraint

$$|G(j\omega_{gc})|_{dB} = |C(j\omega_{gc})P(j\omega_{gc})|_{dB} = 0dB \tag{7}$$

- Phase margin constraint

$$arg[G(j\omega_{gc})] = arg[C(j\omega_{gc})P(j\omega_{gc})] = \pi + \phi_m \tag{8}$$

- Robust constraint against gain variation

$$(\frac{d}{d\omega}(arg[G(j\omega)]))_{\omega=\omega_{gc}} = 0 \tag{9}$$

- Noise suppression constraint

$$\begin{aligned} |T(j\omega)|_{dB} &= |\frac{C(j\omega)P(j\omega)}{1+C(j\omega)P(j\omega)}|_{dB} \leq AdB \\ &\forall\omega \geq \omega_t \\ &\Rightarrow |T\ (j\omega_t)|_{dB} = AdB \end{aligned} \tag{10}$$

- Interference suppression constraint

$$\begin{aligned} |S(j\omega)|_{dB} &= |\frac{1}{1+C(j\omega)P(j\omega)}|_{dB} \leq BdB \\ &\forall\omega \leq \omega_s \\ &\Rightarrow |S(j\omega_s)|_{dB} = BdB \end{aligned} \tag{11}$$

- Steady-state error cancellation: the steady-state error of the closed-loop system is eliminated automatically because the FOPID controller still introduces the function of integration.

In this paper, the design principle of FOPID controller is introduced. Five non-linear constraints (7) $\sim$ (11) of the controller are proposed based on frequency domain to solve five unknown parameters (K_p, K_i, K_d, λ, μ) of FOPID controller $C(s)$. It is difficult to solve the above nonlinear equations directly. In this paper, we use the optimization toolbox of MATLAB to determine the objective function and constraints, and find the optimal solution by iteration optimization to minimize the error. Five parameters are debugged by MATLAB optimization toolbox, GA function and FMINCON function. Using FMINCON function to optimize needs to solve three problems: objective function, constraints and the determination of initial value. The selection of objective function will affect the convergence speed and optimization ability. Most studies regard the above Eq.(7) as objective function and the other four as constraints. This design only considers stability, while real difficulty of control system design should be how to obtain better robustness. In addition, the selection of the initial value also determines whether the FMINCON function can obtain a desired optimal solution, because the FMINCON function obtains a local optimal solution. However, the determination of initial value in existing designs mostly depends on experience or attempts. In view of the above two problems, the following improvements are made in determining the five parameters of FOPID controller. Firstly, the Eq.(9), which measures robustness, is regarded as the objective function and other four as constraints. Secondly, in order to obtain the global optimal solution for FMINCON function, the genetic algorithm is used to select the initial value. The genetic algorithm utilizes biological mechanisms and has wide applicability. Besides, the genetic algorithm searches the solution by probability, rather than using traditional paths, which possesses with the main advantage to avoid a local optimum. We execute the genetic algorithm and take the result as the initial search vector of FMINCON function, and then get the controller.

GEATPY toolbox and GA toolbox in MATLAB can be used to compile genetic algorithm. The former genetic algorithm toolbox runs fast, while the latter is more convenient to write the fitness function and constraint functions. This paper uses GA toolbox in MATLAB to realize genetic algorithm.

Firstly, the fitness function is compiled. In genetic algorithm, the selection of the fitness function is extremely crucial, which will greatly affect its convergence speed and optimization ability. As can be seen from the above, the Eq.(9) measuring robustness is taken as the objective function, from which the fitness function can be obtained as follows:

$$Fit[f(\omega)] = (\frac{d}{d\omega}(arg[G(j\omega)]))_{\omega=\omega_{gc}} \tag{12}$$

For the convenience of programming on MATLAB, we need to analyze it in frequency domain and simplify the function into the forms as follows:

1) The transfer function of the first-order system is:

$$P(j\omega) = \frac{K}{j\omega T+1} = \frac{K}{1+\omega^2T^2} + j\cdot\frac{-K\omega T}{1+\omega^2T^2} \tag{13}$$

2) The transfer function of the controller is:

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^{\lambda}} + K_d\cdot(j\omega)^{\mu} = r + j\cdot s \tag{14}$$

where, r is the real part and s is the imaginary part. According to Dimowei's theorem:

$$[r\cdot(cos\,\theta + j\cdot sin\,\theta)]^n = r^n\cdot(cos\,n\theta + j\cdot sin\,n\theta) \tag{15}$$

So there are:

$$\begin{aligned} (j\omega)^{\lambda} &= [w\cdot(cos\frac{\pi}{2} + j\cdot sin\frac{\pi}{2})]^{\lambda} \\ &= \omega^{\lambda}\cdot(cos\frac{\pi\lambda}{2} + j\cdot sin\frac{\pi\lambda}{2}) \\ (j\omega)^{\mu} &= [w\cdot(cos\frac{\pi}{2} + j\cdot sin\frac{\pi}{2})]^{\mu} \\ &= \omega^{\mu}\cdot(cos\frac{\pi\mu}{2} + j\cdot sin\frac{\pi\mu}{2}) \end{aligned} \tag{16}$$

Bring it into Eq.(14):

$$r = K_p + K_i \cdot \omega^{-\lambda} \cdot cos(\frac{\pi\lambda}{2}) + K_d \cdot \omega^{\mu} \cdot cos(\frac{\pi\mu}{2})$$
$$s = -K_i \cdot \omega^{-\lambda} \cdot sin(\frac{\pi\lambda}{2}) + K_d \cdot \omega^{\mu} \cdot sin(\frac{\pi\mu}{2}) \quad (17)$$

3) For open-loop transfer function, the amplitude condition $|G(j\omega_{gc})|$ is:

$$|G(j\omega_{gc})| = |C(j\omega_{gc})P(j\omega_{gc})| = \sqrt{r^2 + s^2} \cdot [\frac{K}{\sqrt{\omega_{gc}^2 \cdot T^2 + 1}}] \quad (18)$$

4) According to the phase angle formula $\varphi_\omega = arctan\frac{Q_\omega}{P_\omega}$, the phase angle can be calculated from the following equation:

$$arg[G(j\omega_{gc})] = arg[C(j\omega_{gc})P(j\omega_{gc})] = arctan(s_{gc}/r_{gc}) - arctan(\omega_{gc}T) \quad (19)$$

5) The robustness constraint of gain variation is calculated as follows:

$$(\frac{d}{d\omega}(arg[G(j\omega)]))_{\omega=\omega_{gc}} = \frac{s'_{gc}r_{gc} - r'_{gc}s_{gc}}{[1+(s_{gc}/r_{gc})^2]r_{gc}^2} - \frac{T}{1+(\omega_{gc}T)^2} \quad (20)$$

6) The constraint of high frequency noise suppression is calculated as follows:

$$|T(j\omega)| = \frac{K\sqrt{r^2+s^2}}{\sqrt{(1+K\cdot r)^2 + (\omega T + K \cdot s)^2}} \quad (21)$$

7) The calculation of output interference suppression is as follows:

$$|S(j\omega)| = \frac{\sqrt{(\omega T)^2+1}}{\sqrt{(1+K\cdot r)^2 + (\omega T + K \cdot s)^2}} \quad (22)$$

After analyzing the above constraints in frequency domain, we can debug and solve five parameters by GA and FMINCON function. A roulette sampling method is adopted to perform single-point crossover and random bit mutation on genes. Besides, an elite retention strategy is added and the parental cross is controlled based on the fitness value. The individual with the less fitness value will be more likely to be selected. The iterative procedure stops when the relative changes in all elements of the solution are less than 10^{-10}.

V. SYSTEM SIMULATION RESULTS

In this section, the FOPID controller and the classic PID controller are applied to the system model. The input reference value of the system is 50N, and the closed-loop response of the system is obtained.

Then this paper proposes the following constraints to design the FOPID controller:

- Traversal frequency constraint:

$$\omega_{gc} = 1.6\ rad/s \quad (23)$$

- Phase margin constraint:

$$\phi_m = 70° \quad (24)$$

- Robust constraints against gain variation:

$$(\frac{d}{d\omega}(arg[G(j\omega)]))_{\omega=\omega_{gc}} = 0 \quad (25)$$

- Noise suppression constraint:

$$|T(j\omega)| \leq -10dB, \quad \forall\omega \geq \omega_t = 10rad/s \quad (26)$$

- Interference suppression constraint:

$$|S(j\omega)| \leq -20dB, \quad \forall\omega \leq \omega_s = 0.1rad/s \quad (27)$$

According to the principle of debugging, the FOPID controller and the classic PID controller are respectively used to perform the simulation test. The parameters of both controllers are obtained by the non-linear optimization method. The constraints of values of parameters λ and μ are as follows:

$$0.01 \leq \lambda \leq 1, 0.01 \leq \mu \leq 1 \quad (28)$$

The parameters of FOPID controller and classic PID controller obtained by optimization are as follows:

FOPID controller

$$K_p = 0.0486,\ K_i = 0.0319,\ K_d = 0.01$$
$$\lambda = 0.949,\ \mu = 0.1 \quad (29)$$

Classic PID controller

$$K_p = 0.0206, K_i = 0.0486, K_d = 0.01 \quad (30)$$

A. ANALYSIS OF STEP RESPONSE PERFORMANCE OF CLOSED-LOOP SYSTEM

Two optimal controllers are used to realize the closed loop control of the system. The dynamic response of the system is shown in FIGURE 3. Specific performance indexes can refer to the data in TABLE 1. By comparing the performance indexes of FOPID controller and classic PID controller, it can be seen that the response performance of FOPID controller is better than that of classic PID controller. The rise time, steady

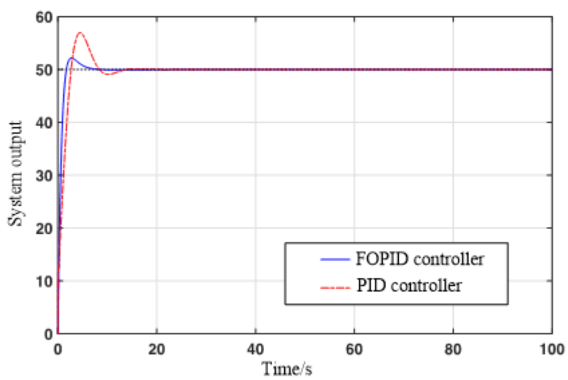

FIGURE 3. Comparison of step response curves.

TABLE 1. System response performance indexes.

Controller type	Rise time	Steady state time	Peak time	Overshoot
FOPID controller	1.1033s	4.5898s	2.75s	4.30%
PID controller	2.1528s	7.6652s	4.48s	13.85%

state time, peak time and overshoot of the former are less than those of the latter.

The Bode diagram of the open-loop system is shown in FIGURE 4, and it can be seen from the diagram that the FOPID controller fully satisfies the constraints of the controller design. The traversal frequency $\omega_{gc} = 1.6\ rad/s$, and phase margin $\phi_m = 84°$

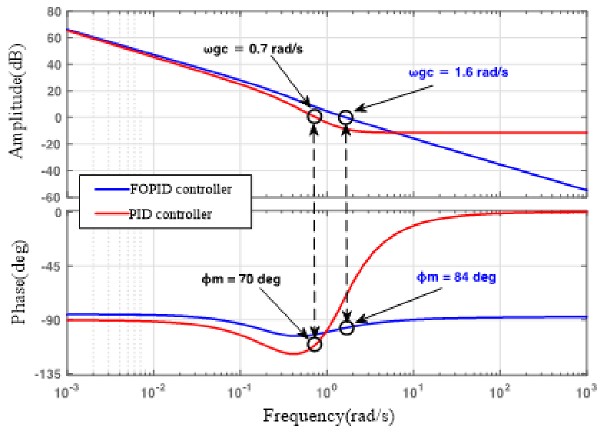

FIGURE 4. The bode diagram of open-loop system.

B. ROBUSTNESS ANALYSIS OF SYSTEM GAIN AND LOAD VARIATION

In this section, we mainly study the robustness of the controller to the static gain and load variation of the system. FIGURE 5 and FIGURE 6 are the response curves of the FOPID controller and the classic PID controller when the static gain and the input reference value of the system are changed, respectively. It can be seen from the diagrams that the FOPID controller has better response performance than the classic PID controller when the gain change is ±33% and the disturbance occurs in the 40s, and its rising time and steady state time are better than those of the classic PID controller. The peak time and overshoot are less than those of the classic PID controller.

More system response metrics are shown in Table 2. It can be seen that when the static gain is ±33%, the changes of overshoot of FOPID controller is less than those of classic PID controller. This means that the FOPID controller is more robust to gain variation.

C. ROBUSTNESS ANALYSIS OF HIGH FREQUENCY NOISE AND OUTPUT INTERFERENCE

In order to make the system more robust to noise and output interference, noise suppression constraint and output

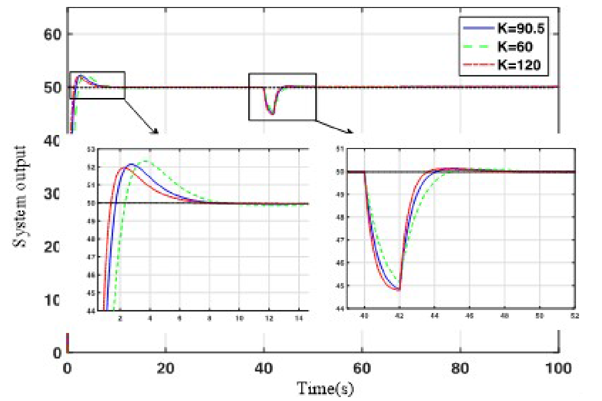

FIGURE 5. Step response curves of FOPID controller for different gain and load variation.

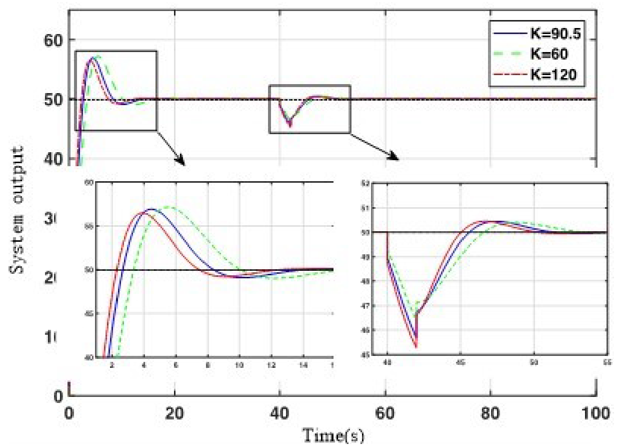

FIGURE 6. Step response curves of classic PID controller for different gain and load variation.

TABLE 2. Performance metrics of system response under different gain.

Gain variation	Controller type	Rise time	Steady state time	Peak time	Overshoot
K=90.5	FOPID	1.1033s	4.5898s	2.75s	4.30%
	PID	2.1528s	7.6652s	4.48s	13.85%
K=60	FOPID	1.5524s	5.8596s	3.65s	4.63%
	PID	2.7666s	12.285s	5.51s	14.33%
K=120	FOPID	0.8696s	3.8248s	2.26s	3.91%
	PID	1.8238s	6.7815s	3.92s	12.97%

interference suppression constraint are proposed. The Bode graphs of the sensitivity function $S(j\omega)$ and the complementary sensitivity function $T(j\omega)$ are shown in FIGURE 7 and FIGURE 8. As can be seen from the figures, both controllers satisfy the constraints we have proposed, namely (26) and (27).

It can be concluded that the two controllers are robust to both noise and output interference. In particular, the FOPID controller has better performance than the classic PID controller in noise suppression.

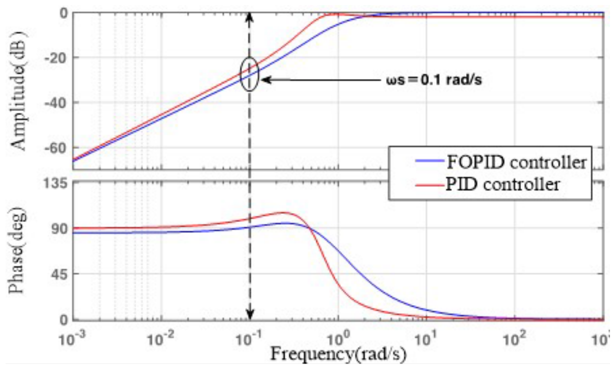

FIGURE 7. The bode diagram of sensitivity function (output interference suppression).

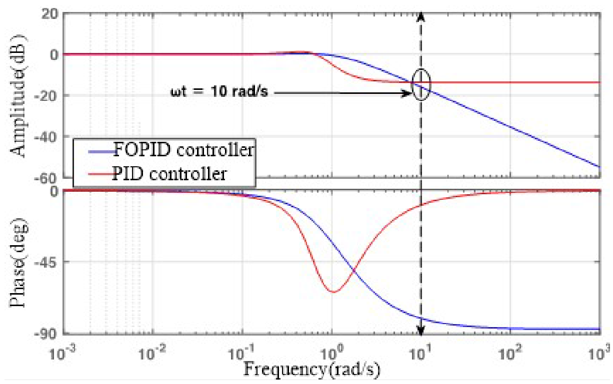

FIGURE 8. The bode diagram of complementary sensitivity function (noise suppression).

From the above results, we can see that the FOPID controller has better response performance and is more robust than the classic PID controller.

D. EFFECT OF FRACTIONAL ORDER ON CLOSED-LOOP RESPONSE

In order to study the effect of fractional order on the closed-loop response of the system, we increase μ and λ from 0.1 to 1 respectively, and obtain the Bode diagrams of the open-loop system as shown in FIGURE 9 and FIGURE 10.

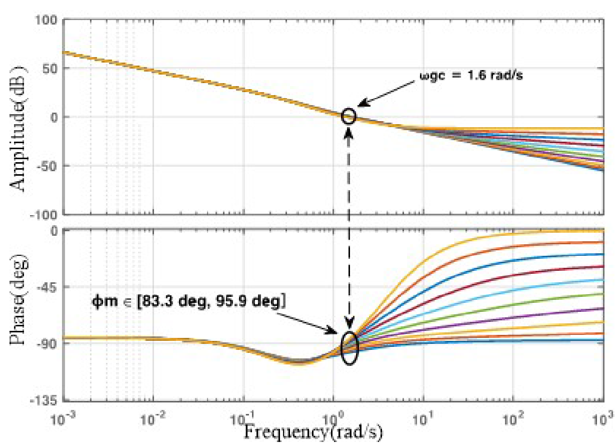

FIGURE 9. The bode diagram of open-loop system with μ.

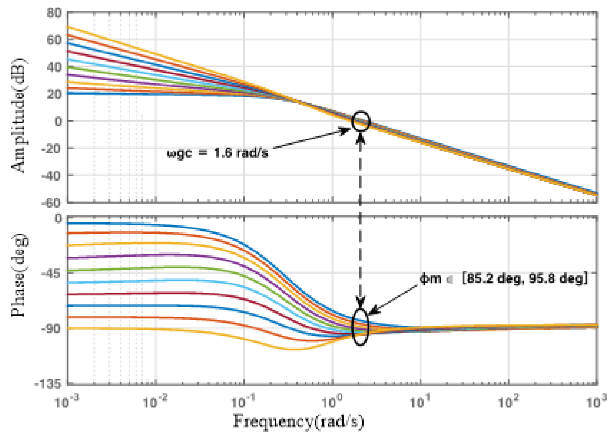

FIGURE 10. The bode diagram of open-loop system with λ.

From the diagrams, we can see that although μ and λ changes, the traversal frequency of the system is basically unchanged, and the phase angle has always met the requirements of the system. In addition, we can see that μ is mainly in the high-frequency part of the system, and λ is mainly in the low-frequency part of the system. Therefore, the parameter debugging of FOPID controller is more flexible than that of classic PID controller, and the response performance is better.

VI. CONCLUSION

In this paper, a design method of FOPID controller for constant tension control system by constrained nonlinear optimization is proposed. Five parameters of FOPID controller are debugged and solved by GA function of MATLAB and FMINCON function of non-linear optimization. The FOPID controller proposed is compared with classic integer order PID controller. The simulation results show that the FOPID controller has better control characteristics, robustness, tracking accuracy and time response ability, and the process is simple and easy to implement.

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