Effect of Plasma Sheath Velocity on Propagation of Electromagnetic Waves

LINJING GUO [1,2] **AND LIXIN GUO** [1,3], (Senior Member, IEEE)

[1] School of Physics and Optoelectronic Engineering, Xidian University, Xi'an 710071, China
[2] School of Electronic Engineering, Xidian University, Xi'an 710071, China
[3] State Key Laboratory of Integrated Service Networks, School of Telecommunications Engineering, Xidian University, Xi'an 710071, China

Corresponding author: Linjing Guo (guolinjing@163.com)

ABSTRACT Hypersonic velocities can theoretically affect the propagation of electromagnetic waves in a plasma sheath covering a vehicle. This article is the continuation of our previous study and, for the first time, investigates the reflection characteristics of electromagnetic waves by a moving bounded plasma slab and examines five typical plasma parameters in detail when the plasma moves parallel to the interface. The results show that the motion of the plasma slab significantly affects electromagnetic propagation, and the reflection coefficient oscillates with the velocity of the plasma sheath. In addition, different physical parameters cause different oscillation characteristics. Various interesting features of the reflection coefficient are obtained, which are very important for researching the propagation of electromagnetic waves in this extreme physical environment or providing reference for researching "blackouts" in the future.

INDEX TERMS Electromagnetic waves, moving nonuniform bounded plasma, reflection coefficient.

I. INTRODUCTION

When a reentry vehicle or a supersonic vehicle enters the dense atmosphere at hypersonic velocity (Mach 10–25), the very high kinetic energy of the vehicle is constantly converted into heat, the heat shield material detaches and is ionized along with the surrounding air in the stagnation zone, and a layer of high-temperature plasma shock fluid forms around the spacecraft. The plasma shock fluid, or plasma sheath, can temporarily disrupt radio signals and cause a communications blackout [1], [2].

The basic reason of this phenomenon is that several factors affect the propagation of electromagnetic waves in the plasma sheath, such as the physical characteristics of the plasma sheath itself, the radar antenna, and the flight attitude of the vehicle. The first two factors have been investigated at length [3]–[6]. Besides, the National Aeronautics and Space Administration conducted several experiments and performed extensive research to resolve this "communications blackout", including the Radio Attenuation Measurements Program, the Hyper-X Program and the Hypersonic Technology Vehicle (HTV) Program [7]–[10].

Although there were important results and discussions on relative velocity derived from those studies [11]–[13], there has been a new surge in hypersonic vehicle research in several countries since the beginning of this century. Furthermore, the results obtained during that time are relatively imprecise and cannot meet current technical requirements.

On the one hand, although the relativistic effect does not need to be considered and is completely negligible, the velocity of supersonic vehicles will increase in the future, and this effect must not be ignored. On the other hand, even in some extreme physical environments in space, the phenomenon and the interaction of moving medium and electromagnetic waves need to be described theoretically.

This paper not only provides a quantitative reference for information transmission technology for hypersonic vehicles but also analyzes the effects of several key physical parameters. The paper is organized as follows. The physical model is introduced in Section II. Section III presents five numerical examples involving the key physical parameters studied in this work. The conclusions of this research are discussed in Section IV.

II. PHYSICAL MODEL AND FORMULATION

A plasma sheath is a lossy medium. The key to studying the propagation of electromagnetic waves in a moving plasma

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sheath is establishing the relative permittivity. Because of relativistic effects, the relative permittivity of a moving plasma sheath is different from that of a stationary one.

A. PHYSICAL MODEL

Two sets of reference coordinates are established. One is the stationary reference system $\sum$, which is static with respect to free space. The other is the moving system $\sum'$, which moves with velocity v with respect to $\sum$. Figure 1 shows the geometry of this problem. The thickness of the plasma sheath on the supersonic vehicle surface is d. The velocity of the supersonic vehicle has a component v_x parallel to the interface and a component v_y perpendicular to it. This work only discusses the case of a plasma sheath moving at a uniform velocity v_x in the x direction.
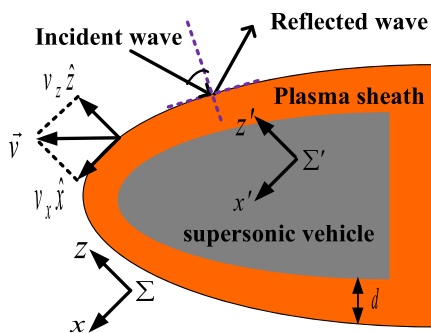

FIGURE 2. Incident frequency in Σ' as a function of β for different incident angles.

FIGURE 1. Geometry of the problem.
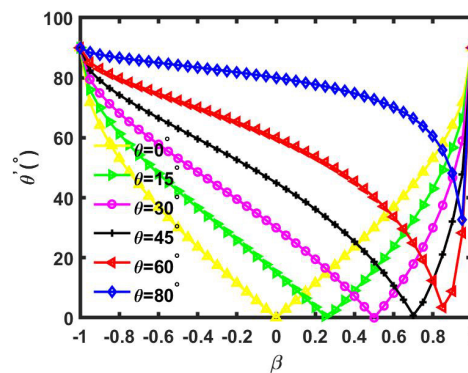
FIGURE 3. Incident angle in Σ' as a function of β for different incident frequencies.

Because of relativistic effects, the frequency of an incident wave is ω' in $\sum'$ and ω in $\sum$, and the angle of incidence is θ' in $\sum'$ and θ in $\sum$. The covariance of Maxwell's equations and phase invariance of a uniform plane wave yield the following transformations:

$$p_x = \gamma_x(1 - \beta_x \sin\theta) \tag{1}$$
$$\omega' = p_x\omega \tag{2}$$

and

$$\cos\theta' = \cos\theta/p_x \tag{3}$$

where $\beta_x = v_x/c$ is the normalized velocity, $\gamma_x = 1/\sqrt{1-\beta_x^2}$, and c is the speed of light in free space. The parameter p_x transforms the incident wave frequency to the moving system of the plasma. Figs. 2 and 3 show the incident angle and frequency of the electromagnetic wave in the moving system as obtained from (1)–(3). Electromagnetic waves move toward the moving plasma if $\beta_x > 0$ and away from it if $\beta_x < 0$, while $\beta_x = 0$ means that the plasma sheath is static.

Fig. 2 shows that the transformed incident frequency in the moving system first slowly decreases and then increases steadily when β_x varies from -1 to 1 for different incident frequencies. In Fig. 3, the angle of incidence in the moving system first decreases to 0° from 90° and returns to 90° afterwards for different fixed frequencies. There is an interesting
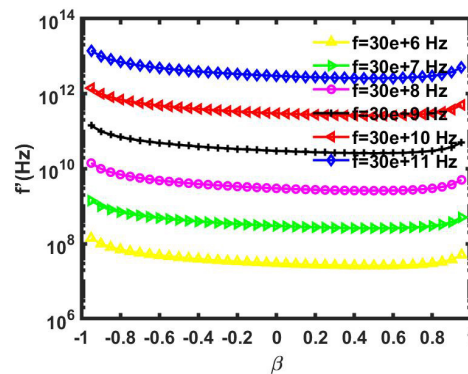
phenomenon here: the changed angle of incidence reaches its minimum at different values of β_x for different incident angles, and vice-versa. We call each of these values of β_x a critical velocity. Clearly, the critical velocities in Figs. 3 are 0, 0.25, 0.5, 0.7, 0.85, and 0.95. This result was mentioned in our last paper [14]; here, we will further research the effect of plasma sheath velocity on the reflection of electromagnetic waves and obtain new interesting results.

B. TRANSFORMED COMPLEX PERMITTIVITY

The complex permittivity for a plane wave propagating through the plasma sheath at an arbitrary incident angle in a stationary system is a function of incident frequency, which was given by Jamison et al. [15]. According to part A, the complex relative permittivity of the plasma sheath can also be transformed to $\varepsilon_r'(\omega')$ in the moving system owing to the transformed frequency and angle, so it can be expressed in terms of ω' and θ':

$$\varepsilon_r'(\omega') = 1 - \frac{\omega_p^2}{\omega'^2 + v_{en}^2} - \frac{jv_{en}}{\omega'}\frac{\omega_p^2}{(\omega'^2 + v_{en}^2)} \tag{4}$$

where $j = \sqrt{-1}$; $\omega' = 2\pi f'$, $v_{en} = 2\pi f_{en}'$, and ω_p is the radian plasma frequency given by $\omega_p = \sqrt{e^2n_e^2/m_e\varepsilon_0}$, where m_e and e are the mass and charge of an electron, ε_0 is the

free-space permittivity, n_e is the plasma density, and v_{en} is the collision frequency.

C. TRANSMISSION LINE METHOD

Basing on a transmission line analogy, the plasma sheath can be modeled as a series of two-dimensional sub-slabs along the direction perpendicular to the aircraft surface, where each sub-slab has a fixed electron density [16]; electromagnetic waves propagating in the n-layered plasma slab can be equivalent to the cascade of the transmission lines with different impedances.

It is assumed that the incident wave is a parallel polarized wave. The incident wave launches to the plasma slab at a normal incident angle.

For the n-th plasma slab, the propagation constant of the electromagnetic waves is k'_n, and the characteristic impedances of the parallel polarization are $Z_n = Z_n \cos \theta_n$, in which $Z_n = \sqrt{\mu_n / \varepsilon'(r)_n}$ is the characteristic impedance, θ_n is the complex refraction angle of electromagnetic waves propagating in the n-th plasma slab and in compliance with the Snell's law.

For the parallel polarized wave E^i, the transmission matrix of the n-th layer is:

$$\begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} = \begin{bmatrix} \cosh(jk'_n \cos \theta'_n d_n) & Z_n \sinh(jk'_n \cos \theta'_n d_n) \\ \sinh(jk'_n \cos \theta'_n d_n)/Z_n & \cosh(jk'_n \cos \theta'_n d_n) \end{bmatrix} \tag{5}$$

By multiplying each transport matrix separately, the total transmission matrix of the interaction between the n-th plasma of the parallel polarized wave and the perpendicular polarized wave can be obtained. They have the same form:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \tag{6}$$

From the total transmission matrix, the transmission coefficient of the EM waves incident on the plasma slab at any angle can be obtained. Due to the phase delay of the refracted waves propagating along the x direction in the plasma, phase correction is required for the transmission system [17]. The propagation distance of the refracted wave in the plasma slab along the x direction is $L = \sum_1^N d_n \tan \theta_n^t$. The phase delay along the x direction after the EM waves transmitted in the plasma is $e^{-jk_0 \sin \theta_0^t L}$. Finally, the reflection coefficient of the parallel polarized waves is as follows:

$$\tilde{R} = \frac{(A + B/Z_{N+1}) - Z_0(C + D/Z_{N+1})}{(A + B/Z_{N+1}) + Z_0(C + D/Z_{N+1})} \tag{7}$$

When using this method, we refer to the measured electron density distribution of the RAM-C reentry vehicle [18]. We assume that the electron density is a double Gaussian function of the horizontal coordinate z. The electron density distribution is given by

$$n_e(z) = \begin{cases} N_0 \exp\left[-a_1(z - z_p)^2\right] & (0 \leq z < z_p) \\ N_0 \exp\left[-a_2(z - z_p)^2\right] & (z_p \leq z < d) \end{cases} \tag{8}$$

where z_p determines the location of maximum electron density, d is the total thickness of the plasma sheath. N_0 is the peak electron density. a_1 and a_2 are the raise index and the fall-off index respectively.

III. RESULTS AND ANALYSIS

In this part, we discuss the behavior of the reflection coefficient as a function of velocity of the moving plasma sheath with different parameters. To simplify the multivariable problem, the raise index a_1 and the fall-off index a_2 are both set to 0.05/cm, and the position of the electron density peak N_0 is fixed at the center of the reentry plasma sheath. Table 1 shows the specific parameters of each numerical example.

TABLE 1. Parameters in each numerical example.

Number of examples	1	2	3	4	5
$N_0 \left(\times 10^{18} \, \text{m}^{-3}\right)$	0.5; 1; 1.5; 2	1	1	1	1
$f_{en} \left(\text{GHz}\right)$	1	0.5; 1; 2; 5	1	1	1
d(cm)	10	10	5; 8; 10; 30	10	10
$f \left(\text{GHz}\right)$	20	20	20	20; 25; 30; 35	20
$\theta \left(\degree\right)$	30	30	30	30	0; 15; 30; 45

Fig. 4 present the reflection coefficient, which is a complicated function of the normalized velocity of the moving slab when the plasma moves parallel to the interface. Several interesting phenomena can be observed in these figures. On the whole, the reflection coefficient oscillates as β_x varies from -1 to 1 from Fig. 4(a)–4(e). These oscillations are caused by the rapid changes in equivalent electrical thickness of the slab with velocity as viewed from the Σ system [19]. It is worth noting that different physical parameters cause different oscillation characteristics; a detailed analysis is conducted below.

As shown in Fig. 4(a), both the oscillation frequency and amplitude are proportional to the maximum electron density. Fig. 4(b) shows the oscillation amplitude is inversely proportional to the collision frequency, although the oscillation frequency is unaffected by the collision frequency.

As seen in Fig. 4(c), the oscillation frequency increases as the plasma thickens. By contrast, the second maximum increases as the plasma thickens until it becomes higher than the two major peaks and becomes the highest unique peak when the plasma thickness is 30 cm.
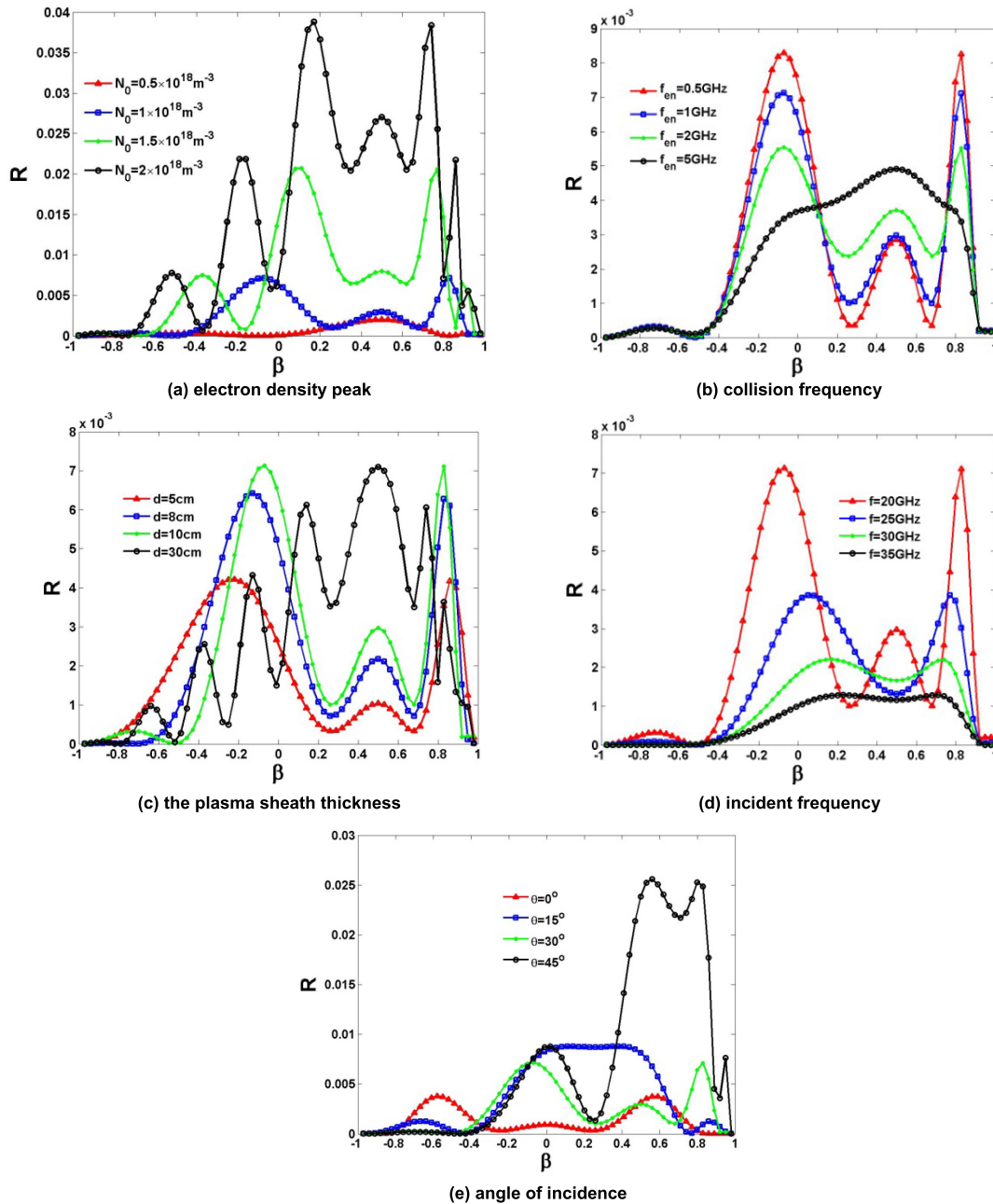
(a) electron density peak

(b) collision frequency

(c) the plasma sheath thickness

(d) incident frequency

(e) angle of incidence

FIGURE 4. Reflection spectra of waves for various parameters of plasma profiles.

Fig. 4(d) shows that the oscillation amplitude decreases as the incident frequency increases. The reflection coefficient has a second maximum between the two major maxima when the incident frequency is 20 GHz. However, this second maximum disappears at an incident frequency of 25 GHz. Finally, these two major maxima are no longer significant when the incident frequency is 35 GHz.

Another intriguing feature is observed. Let us take $\beta = 0.5$ as the cutoff point in Fig. 4(a)–4(d). The right reflection coefficient oscillates faster than the left one. Interestingly, this cutoff point is the critical speed we found for an incident angle

of 30° in Fig. 3. Thus, we speculate that the critical speed determined by the angle of incidence is the demarcation point of the oscillation frequency. Fig. 4(e) verifies our hypothesis. The critical speed is 0, 0.25, 0.5, and 0.7 when the angle of incidence is 0°, 15°, 30°, and 45°, respectively. The oscillation frequency is similar in both directions when the critical speed 0 is the cutoff point. However, the right oscillation frequency is higher than the left when the cutoff point is 0.25 or 0.7. Beyond that, all of the reflection are zero when the plasma sheath is at the speed of light regardless of the physical parameters, under this extreme physical condition, we may

infer that there is some extreme and complex interaction between the plasma sheath and the incident electromagnetic waves.

IV. CONCLUSION

In conclusion, the reflection coefficient is a complex function of normalized velocity. Specifically, the curves of the reflection coefficient are affected by the plasma sheath parameters; we can see that the angle of incidence is a key parameter when the plasma moves parallel to the interface because it determines the critical speed, and the critical speed determines the properties of reflection and transmission.

This paper is a theoretical study about the interaction between a moving medium and incident electromagnetic waves; it could be used as a quantitative reference for space exploration or communication "blackouts" in the future. Finally, although this research has revealed several interesting features, further studies on the microscopic mechanisms and energy involved will be conducted in our future work.

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LINJING GUO was born in Shaanxi, China, in 1986. She received the Ph.D. degree in radio science from the School of Physics and Optoelectronic Engineering, Xidian University, Xi'an, China.

She is currently a Postdoctoral Researcher with the School of Electronic Engineering, Xidian University. Her research interests include the characteristics of the plasma sheath covering the hypersonic vehicle, the relevant electromagnetic computation, and the propagation and scattering of the electromagnetic waves in complex and random media.

LIXIN GUO (Senior Member, IEEE) received the M.S. degree in radio science from Xidian University, Xi'an, China, and the Ph.D. degree in astrometry and celestial mechanics from the Chinese Academy of Sciences, China, in 1993 and 1999, respectively.

From 2001 to 2002, he was a Visiting Scholar with the School of Electrical Engineering and Computer Science, Kyungpook National University, Daegu, South Korea. He has been a Visiting Professor with d'Energétique des Systémes et Procédés (LESP), University of Rouen, Mont-Saint-Aignan, France, and the Faculty of Engineering and Physical Sciences, The University of Manchester, Manchester, U.K. He is currently a Professor and the Head of the School of Physics and Optoelectronic Engineering, Xidian University, China. He has authored or coauthored 4 books and over 300 journal articles. He has been in charge of and undertaken more than 30 projects. His research interests include electromagnetic wave propagation and scattering in complex and random media, computational electromagnetics, inverse scattering, and antenna analysis and design. He is a Fellow of the Chinese Institute of Electronics (CIE) and a Fellow of the Physics Institute of Shaanxi Province. He was a recipient of the National Science Fund for Distinguished Young Scholars, in 2012. He was also a Distinguished Professor of Changjiang Scholars Program, in 2014. He has been the Chief Professor of Innovative Research Team, Shaanxi, China, since 2014.

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