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Pinning Synchronization via Intermittent Control for Memristive Cohen-Grossberg Neural Networks With Mixed Delays

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ABSTRACT This paper presents the exponential synchronization for a class of memristive Cohen-Grossberg neural networks (MCGNNs) with mixed delays via a new hybrid control strategy. This new hybrid control strategy combines pinning control and periodic intermittent control. According to the feature of memristor, the memristive terms of the MCGNNs with mixed delays are normalized by a simple linear transformation. Then the designed periodic intermittent control is added to selected partial network nodes. Based on the stability theory of memristive neural networks and the exponential synchronization rule, the new synchronization conditions are given. Finally, numerical simulations are provided to show the effectiveness of the theoretical method.

INDEX TERMS Memristive Cohen-Grossberg neural networks, exponential synchronization, pinning control, periodic intermittent control.

I. INTRODUCTION

In the past few decades, neural networks have been extensively studied in such diverse fields as associative memory, classification, parallel computation, pattern recognition, signal processing, decision aid, and artificial intelligence [1]–[4]. In 1983, Cohen-Grossberg neural networks (CGNNs) were proposed by Cohen and Grossberg [5], which can be described by:

$$\dot{z}_l(t) = c_l\left(z_l(t)\right)\{-d_l\left(z_l(t)\right)z_l(t) + \sum_{k=1}^{N} a_{lk}\left(z_l(t)\right)f_k\left(z_k(t)\right) + J_l\}. \quad (1)$$

where $l, k = 1, 2, \cdots N$, $N \geq 2$ is the number of neurons in the CGNNs. $z_l(t)$ is the state variable associated with the l-th neuron at the time t. $c_l(\cdot)$ and $d_l(\cdot)$ represent the amplification function and the behaved function, respectively. $a_{lk}(\cdot)$ denotes the neuron interconnection weights. The activation functions $f_k(\cdot)$ map the input of the neuron to the output. The constants J_k represent the external inputs of the CGNNs. Since it includes a large class of models from the view of neurobiology, population biology, and evolutionary theory, as well as the well-known Hopfield neural networks [6]–[10] and recurrent neural network [11]–[18], the model of CGNNs [19]–[32] is quite general.

Due to the speed limitations of amplifier processing, signal transmission and conversion, time delays are ubiquitous in the hardware implementations of neural networks. Marcus and Westervelt pointed [33] out time delays were easily ignored in theoretical models and firstly proposed the stability of Hopfield neural networks with a constant time delay $\tau > 0$. After that, the stability of neural networks with multiple delays [19] $\tau_k > 0$ was studied. However, the unknown and bounded delay $0 \leq \tau_{lk}(t) \leq \tau$ is more suitable in the neural networks. Thus, the dynamics of various neural networks with the time-varying delays [15]–[17], [20]–[27], [34]–[37] are taken into consideration. On the other hand, due to a special characteristic of neural networks formed by a large number of neurons through parallel channels, the distributed delay exists

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in signal transmission between the different axon size and size of neurons. Thus, the unbounded distributed delay [18], [28], [38], [39] has also attracted much attention and has been extensively studied. Particularly, it can be used to solve some practical problems such as the dynamics analysis of neural networks with constant delays or time-varying delays [27], [29]–[32], [40]–[42].

In 1971, Chua [43] proposed the fourth basic circuit elements, called memristor. However, the physical model of the memristor has not been proposed in the next few decades. Until 2008, HP researchers observed memristance in nanoscale electronic devices [44]. Due to its characteristic, memristor has wide application prospect such as: non-volatile random access memory (NVRAM), synapses of artificial neural network, chaos control and secret communication.

Especially, the studies on memristive CGNNs (MCGNNs) in the stability [27], [29], [30] and synchronization [24], [25], [31], [32] attract more attention. Among them, it is worth studying on synchronization for a class of MCGNNs in the field of secure communications, image and data encryption. Based on a simplified MCGNNs with mixed delays obtained by a non-linear transformation, Yang *et al.* [32] deals with the problem of exponential synchronization for the networks through a state feedback controller. Abdurahman *et al.* [25] proposed a controller consisting of three switching open-loop controls and a linear feedback control to achieve exponential function projective synchronization for MCGNNs with time-varying delays. Chen *et al.* [31] designed a feedback control to complete the finite-time synchronization for MCGNNs with mixed delays by using the same non-linear transformation as Yang.

Noted that there are different control methods have been applied to achieve synchronization for neural neworks such as feedback control [13], [15], [23]–[25], [31], [32], adaptive control [45], pinning control [46]–[48], impulsive control [49], sliding mode control [50], fuzzy control [51] and intermittent control [23], [51]–[54]. Abdurahman *et al.* [23] studied that the exponential lag synchronization of CGNNs with mixed time-delays via periodically intermittent control. In [53], the exponential synchronization of memristive neural networks with time-varying delays was proposed via designing a pinning aperiodic intermittent control. Feng *et al.* [54] considered the asymptotic synchronization of memristive neural networks with mixed delays via quantized intermittent control. To achieve synchronization of MCGNNs, the continuous control is added in every nodes of networks [24], [25], [31], [32]. Obviously, in practical neural networks, it is impossible to control each node. And the continuous control may cost more. Thus, the use of pinning intermittent control can solve this problem. Pinning control enables the entire MCGNNs to have the desired behavior by selectively applying control to partial nodes in the MCGNNs. In order to further reduce resource consumption, intermittent control is applied to selected nodes to achieve synchronization.

Based on the description above, the exponential synchronization was presented via hybrid control for a class of MCGNNs with mixed delays. Currently, the new study will arise new challenges. (1)How to reduce the complexity of the MCGNNs solution caused by the time-varying delays and distribution delays. (2)How to determine the control gain and the control duration to achieve exponential synchronization for MCGNNs with mixed delays. In this paper, to balance the control parameters, some new synchronization conditions are given. The main contributions are described as follows:

(1) The memristive terms of the MCGNNs with mixed delays are normalized by the feature of memristor and a simple linear transformation. Compared with the previous methods, the control gain is more accurate.

(2) The hybrid control is designed by pinning control and periodic intermittent control. By selecting part of the MCGNNs with mixed delays for periodic intermittent control, the exponential synchronization of the networks are achieved. Moreover, the hybrid control is general. According to changing the control parameter and amplification function, the exponential synchronization of the MCGNNs with time-varying delays or the memristive neural networks with time-varying delays or mixed delays can be achieved.

(3) At present, there are few researches on the synchronization control system by combining pinning control and intermittent control. In addition, as far as we known, there is no literature study the synchronization of MCGNNs via a pinning periodic intermittent control. To expand synchronization researches on memristive neural networks, it is of great significance on the synchronization on MCGNNs with mixed delays by a hybrid control.

The rest of this paper is organized as follows. The model of the MCGNNs with mixed delays and some definitions, lemmas and assumption are given in Section 2. Then, in Section 3, the exponential synchronization conditions are given. And the control gain and the control duration are proposed. In Section 4, to illustrate the feasibility of the main results, some simulations are given. Finally, the conclusion is given in Section 5.

Notation: $\mathbb{R}$ and $\mathbb{R}^n$ donate the space of real numbers and the n-dimensional Euclidean space. $exp\{\cdot\}$ means exponential function with base natural exponential e. $\|x_i\|= \sum_{i=0}^{n}(x_i^2)^{1/2}$ means 2-norm.

II. PRELIMINARY

In this paper, the MCGNNs with mixed delays is chosen, which can be described by:

$$\begin{aligned}\dot{z}_i(t) = c(z_i(t))\Bigg\{&-d_i(z_i(t))z_i(t) + \sum_{j=1}^{N}\Bigg[a_{ij}(z_i(t))f_j(z_j(t))\\ &+ b_{ij}(z_i(t))g_j(z_j(t-\tau_{ij}(t))) + w_{ij}(z_i(t))\\ &\times \int_{t-\tau_{ij}(t)}^{t} h_j(z_j(s))\,ds\Bigg] + J_i\Bigg\}, \quad i,j = 1,2,\cdots,N.\end{aligned} \tag{2}$$

with the initial conditions: $z(s) = \Psi^z(s) \in (\Psi_1^z(s), \Psi_2^z(s)), \cdots, \Psi_n^z(s) \in \mathbb{C}(s, \mathbb{R}^n)$, $s \in (-\tau, 0]$, where term $g_j(z_j(t-\tau_{ij}(t)))$

and $h_j(z_j(s))$ denote the activation functions. Term $\tau_{ij}(t)$ represents the time-varying delay function. Term $b_{ij}(z_i(t))$ and $w_{ij}(z_i(t))$ describe the connection weights of MCGNNs. And $d_i(z_i(t))$, $a_{ij}(z_i(t))$, $b_{ij}(z_i(t))$ and $w_{ij}(z_i(t))$ can be described as follows:

$$d_i(z_i(t)) = \frac{1}{C_i}[\sum_{j=1}^{n}(\mathfrak{M}_{ij}^f + \mathfrak{M}_{ij}^g + \mathfrak{M}_{ij}^h) \times sgn_{ij} + \frac{1}{R_i}],$$

$$a_{ij}(z_i(t)) = \frac{\mathfrak{M}_{ij}^f}{C_i} \times sgn_{ij},$$

$$b_{ij}(z_i(t)) = \frac{\mathfrak{M}_{ij}^g}{C_i} \times sgn_{ij}, \quad w_{ij}(z_i(t)) = \frac{\mathfrak{M}_{ij}^h}{C_i} \times sgn_{ij},$$

$$sgn_{ij} = \begin{cases} 1, & \text{if } i = j \\ -1, & \text{if } i \neq j. \end{cases}$$

where $\mathfrak{M}_{ij}^f$, $\mathfrak{M}_{ij}^g$ and $\mathfrak{M}_{ij}^h$ donate the memristance of memristors W_{ij}^f, W_{ij}^g and W_{ij}^hrespectively. W_{ij}^f represents the memristor between $f_j(z_j(t))$ and $z_j(t)$. W_{ij}^g represents the memristor between $g_j(z_j(t-\tau_{ij}(t)))$ and $z_j(t-\tau_{ij}(t))$. W_{ij}^h represents the memristor between $h_j(z_j(t))$ and $z_j(t)$, R_i represents the resistance corresponding to the capacitor C_i.

According to the feature of the memristor and current-voltage characteristics, we can brief these MCGNNs and get:

$$d_i(z_i(t)) = \begin{cases} d_i^{\circ}, & \text{if } |z_i(t)| \leq T_i \\ d_i^{\circ\circ}, & \text{if } |z_i(t)| > T_i, \end{cases} \tag{3}$$

$$a_{ij}(z_i(t)) = \begin{cases} a_{ij}^{\circ}, & \text{if } |z_i(t)| \leq T_i \\ a_{ij}^{\circ\circ}, & \text{if } |z_i(t)| > T_i \end{cases} \tag{4}$$

$$b_{ij}(z_i(t)) = \begin{cases} b_{ij}^{\circ}, & \text{if } |z_i(t)| \leq T_i \\ b_{ij}^{\circ\circ}, & \text{if } |z_i(t)| > T_i. \end{cases} \tag{5}$$

$$w_{ij}(z_i(t)) = \begin{cases} w_{ij}^{\circ}, & \text{if } |z_i(t)| \leq T_i \\ w_{ij}^{\circ\circ}, & \text{if } |z_i(t)| > T_i. \end{cases} \tag{6}$$

where d_i°, $d_i^{\circ\circ}$, a_{ij}°, $a_{ij}^{\circ\circ}$, $b_{ij}^{\circ\circ}$, b_{ij}° w_{ij}°, $w_{ij}^{\circ\circ}$ and T_i are known constants, $T_i > 0$ are the voltage thresholds.

To finish the main results in this paper, the MCGNNs satisfies the following assumptions:

Assumption 1: The amplification functions $c_i(\cdot)$ are continuous and bounded, i.e., for each $i \in I$ there exist two positive constants $\underline{c}_i$ and $\overline{c}_i$ such that: $0 < \underline{c}_i \leq c_i(u) \leq \overline{c}_i$, $\forall u \in \mathbb{R}$.

Assumption 2: The activation functions $f_j(\cdot)$, $g_j(\cdot)$ and $h_j(\cdot)$ are Lipschitz continuous and bounded, i.e., for each $j \in I$, there exist positive constants $L_j^{f_1}$, $L_j^{f_2}$, $L_j^{g_1}$, $L_j^{g_2}$, $L_j^{h_1}$ and $L_j^{h_2}$ such that:

$$L_j^{f_1} \leq \frac{f_j(u) - f_j(v)}{u - v} \leq L_j^{f_2},$$

$$L_j^{g_1} \leq \frac{g_j(u) - g_j(v)}{u - v} \leq L_j^{g_2},$$

$$L_j^{h_1} \leq \frac{h_j(u) - h_j(v)}{u - v} \leq L_j^{h_2}.$$

Assumption 3: The delay functions $\tau_{ij}(t)$ are bounded and satisfy the inequations $0 \leq \tau_{ij}(t) \leq \tau$ and $\dot{\tau}_{ij}(t) \leq \tau_{ij}$.

Based on previous works [29], [59], the memristive behaved function is defined as follow:

$$d_i(z_i(t)) = d_i^{\circ}\frac{1 - sgn(z_i(t))}{2} + d_i^{\circ\circ}\frac{1 + sgn(z_i(t))}{2},$$

$$\text{where } d_i(z_i(t)) = \begin{cases} d_{1i}, & |z_i(t)| \leq 0 \\ d_{2i}, & |z_i(t)| > 0. \end{cases}$$

$$\text{Let } x_i(t) = z_i(t) - T_i \quad \text{and } \dot{z}_i(t) = \dot{x}_i(t),$$

$$sgn(x_i(t)) = \begin{cases} -1, & |x_i(t)| \leq 0 \\ 1, & |x_i(t)| > 0. \end{cases}$$

Similarly, the memristive functions $d_i(z_i(t))$, $a_{ij}(z_i(t))$, $b_{ij}(z_i(t))$ and $w_{ij}(z_i(t))$ can be rewritten:

$$\begin{aligned} d_i(z_i(t)) &= d_i^{\circ}\frac{1 - sgn(x_i(t))}{2} + d_i^{\circ\circ}\frac{1 + sgn(x_i(t))}{2} \\ &= \frac{1}{2}(d_i^{\circ} + d_i^{\circ\circ}) + \frac{1}{2}(d_i^{\circ\circ} - d_i^{\circ}) \times sgn(x_i(t)), \end{aligned} \tag{7}$$

$$\begin{aligned} a_{ij}(z_i(t)) &= a_{ij}^{\circ}\frac{1 - sgn(x_i(t))}{2} + a_{ij}^{\circ\circ}\frac{1 + sgn(x_i(t))}{2} \\ &= \frac{1}{2}(a_{ij}^{\circ} + a_{ij}^{\circ\circ}) + \frac{1}{2}(a_{ij}^{\circ\circ} - a_{ij}^{\circ}) \times sgn(x_i(t)), \end{aligned} \tag{8}$$

$$\begin{aligned} b_{ij}(z_i(t)) &= b_{ij}^{\circ}\frac{1 - sgn(x_i(t))}{2} + b_{ij}^{\circ\circ}\frac{1 + sgn(x_i(t))}{2} \\ &= \frac{1}{2}(b_{ij}^{\circ} + b_{ij}^{\circ\circ}) + \frac{1}{2}(b_{ij}^{\circ\circ} - b_{ij}^{\circ}) \times sgn(x_i(t)). \end{aligned} \tag{9}$$

$$\begin{aligned} w_{ij}(z_i(t)) &= w_{ij}^{\circ}\frac{1 - sgn(x_i(t))}{2} + w_{ij}^{\circ\circ}\frac{1 + sgn(x_i(t))}{2} \\ &= \frac{1}{2}(w_{ij}^{\circ} + w_{ij}^{\circ\circ}) + \frac{1}{2}(w_{ij}^{\circ\circ} - w_{ij}^{\circ}) \times sgn(x_i(t)). \end{aligned} \tag{10}$$

For simplicity, the notations shown below are given to brief the parameters:

$$\begin{aligned} o_i^{d_1} &= \frac{1}{2}(d_i^{\circ} + d_i^{\circ\circ}), \quad & o_i^{d_2} &= \frac{1}{2}(d_i^{\circ\circ} - d_i^{\circ}), \\ o_{ij}^{a_1} &= \frac{1}{2}(a_{ij}^{\circ} + a_{ij}^{\circ\circ}), \quad & o_{ij}^{a_2} &= \frac{1}{2}(a_{ij}^{\circ\circ} - a_{ij}^{\circ}), \\ o_{ij}^{b_1} &= \frac{1}{2}(b_{ij}^{\circ} + b_{ij}^{\circ\circ}), \quad & o_{ij}^{b_2} &= \frac{1}{2}(b_{ij}^{\circ\circ} - b_{ij}^{\circ}), \\ o_{ij}^{w_1} &= \frac{1}{2}(w_{ij}^{\circ} + w_{ij}^{\circ\circ}), \quad & o_{ij}^{w_2} &= \frac{1}{2}(w_{ij}^{\circ\circ} - w_{ij}^{\circ}). \end{aligned}$$

Based on the description above, the MCGNNs(2) can be rewritten as follows:

$$\begin{aligned} \dot{x}_i(t) = c_i(x_i(t) + T_i)\Bigg\{ & -(o_i^{d_1} + o_i^{d_2} \times sgn(x_i(t))) \\ & \times (x_i(t) + T_i) + \sum_{j=1}^{N}\Bigg[(o_{ij}^{a_1} + o_{ij}^{a_2} \times sgn(x_i(t))) \\ & \times f_j(x_j(t) + T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2} \times sgn(x_i(t))) \\ & \times g_j(x_j(t - \tau_{ij}(t)) + T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2} \\ & \times sgn(x_i(t)))\int_{t-\tau_{ij}(t)}^{t} h_j(x_j(s) + T_j)\,ds\Bigg] + J_i\Bigg\}, \\ & i, j = 1, 2, \cdots, N. \end{aligned} \tag{11}$$

Obviously, the differential equation of the MCGNNs(11) is discontinuous at the right hand side and solutions of the classical sense are not applicable to all MCGNNs in this paper. So in the following we define solutions of the discontinuous MCGNNs in FilippovÌŕs sense [55], [56]. By using the theories of set-valued maps and differential inclusions [57], [58], from the MCGNNs(11) we can obtain the following differential inclusion:

$$\dot{x}_i(t) \in c_i(x_i(t)+T_i)\Bigg\{-(o_i^{d_1}+o_i^{d_2}\times[-1,1]) \times (x_i(t)+T_i)+\sum_{j=1}^{N}\Bigg[(o_{ij}^{a_1}+o_{ij}^{a_2}\times[-1,1]) \times f_j(x_j(t)+T_j)+(o_{ij}^{b_1}+o_{ij}^{b_2}\times[-1,1]) \times g_j(x_j(t-\tau_{ij}(t))+T_j)+(o_{ij}^{w_1}+o_{ij}^{w_2}\times[-1,1] \times\int_{t-\tau_{ij}(t)}^{t}h_j(x_j(s)+T_j)\,ds\Bigg]+J_i\Bigg\}, \quad i,j=1,2,\cdots,N. \tag{12}$$

Or equivalently, there exists $\varsigma_x\in[-1,1]$, such that

$$\dot{x}_i(t) = c_i(x_i(t)+T_i)\Bigg\{-(o_i^{d_1}+o_i^{d_2}\varsigma_x)(x_i(t)+T_i) +\sum_{j=1}^{N}\Bigg[(o_{ij}^{a_1}+o_{ij}^{a_2}\varsigma_x)f_j(x_j(t)+T_j)+(o_{ij}^{b_1}+o_{ij}^{b_2}\varsigma_x) \times g_j(x_j(t-\tau_{ij}(t))+T_j)+(o_{ij}^{w_1}+o_{ij}^{w_2}\varsigma_x \times\int_{t-\tau_{ij}(t)}^{t}h_j(x_j(s)+T_j)\,ds\Bigg]+J_i\Bigg\}, \quad i,j=1,2,\cdots,N. \tag{13}$$

Consider networks (2) as the drive networks and the response networks are given as follows:

$$\dot{v}_i(t) = c_i(v_i(t))\Bigg\{-d_i(v_i(t))z_i(t)+\sum_{j=1}^{N}\Bigg[a_{ij}(v_i(t))f_j(v_j(t)) +b_{ij}(v_i(t))g_j(v_j(t-\tau_j(t)))+w_{ij}(v_i(t)) \times\int_{t-\tau_{ij}(t)}^{t}h_j(v_j(s))\,ds\Bigg]+J_i\Bigg\}+U_i(t), \quad i,j=1,2,\cdots,N. \tag{14}$$

with the initial conditions: $v(s)=\Psi^v(s)\in(\Psi_1^v(s),\Psi_2^v(s)),\cdots,\Psi_n^v(s)\in\mathbb{C}(s,\mathbb{R}^n)$, $s\in(-\tau,0]$, where $U_i(t)$ denotes the control law that will be designed to achieve the synchronization between MCGNNs (2) and (14).

Let $y_i(t)=v_i(t)-T_i$ and $\dot{v}_i(t)=\dot{y}_i(t)$, $sgn(y_i(t))=\begin{cases}-1, & |y_i(t)|\le 0\\ 1, & |y_i(t)|>0\end{cases}$, we have:

$$\dot{y}_i(t) = c_i(y_i(t)+T_i)\Bigg\{-(o_i^{d_1}+o_i^{d_2}\times sgn(y_i(t))) \times(y_i(t)+T_i)+\sum_{j=1}^{N}\Bigg[(o_{ij}^{a_1}+o_{ij}^{a_2}\times sgn(y_i(t))) \times f_j(y_j(t)+T_j)+(o_{ij}^{b_1}+o_{ij}^{b_2}\times sgn(y_i(t))) \times g_j(y_j(t-\tau_{ij}(t))+T_j)+(o_{ij}^{w_1}+o_{ij}^{w_2} \times sgn(y_i(t)))\int_{t-\tau_{ij}(t)}^{t}h_j(y_j(s)+T_j)\,ds\Bigg]+J_i\Bigg\} +U_i(t), \quad i,j=1,2,\cdots,N. \tag{15}$$

Similarly, we can obtain the following differential inclusion from MCGNNs(15):

$$\dot{y}_i(t) \in c_i(y_i(t)+T_i)\Bigg\{-(o_i^{d_1}+o_i^{d_2}\times[-1,1])(y_i(t)+T_i) +\sum_{j=1}^{N}\Bigg[(o_{ij}^{a_1}+o_{ij}^{a_2}\times[-1,1])f_j(y_j(t)+T_j) +(o_{ij}^{b_1}+o_{ij}^{b_2}\times[-1,1])g_j(y_j(t-\tau_{ij}(t))+T_j)+(o_{ij}^{w_1} +o_{ij}^{w_2}\times[-1,1]\int_{t-\tau_{ij}(t)}^{t}h_j(y_j(s)+T_j)\,ds\Bigg]+J_i\Bigg\} +U_i(t), \quad i,j=1,2,\cdots,N. \tag{16}$$

Or equivalently, there exists $\varsigma_y\in[-1,1]$, such that

$$\dot{y}_i(t) = c_i(y_i(t)+T_i)\Bigg\{-(o_i^{d_1}+o_i^{d_2}\varsigma_y)(y_i(t)+T_i) +\sum_{j=1}^{N}\Bigg[(o_{ij}^{a_1}+o_{ij}^{a_2}\varsigma_y)f_j(y_j(t)+T_j) +(o_{ij}^{b_1}+o_{ij}^{b_2}\varsigma_y)g_j(y_j(t-\tau_{ij}(t))+T_j)+(o_{ij}^{w_1} +o_{ij}^{w_2}\varsigma_y\times\int_{t-\tau_{ij}(t)}^{t}h_j(y_j(s)+T_j)\,ds\Bigg]+J_i\Bigg\} +U_i(t), \quad i,j=1,2,\cdots,N. \tag{17}$$

Remark 1: Usually, the neuron interconnection weights of memristive neural networks [13], [15], [16], [24], [25], [29]–[32], [34], [35] are expressed as $\overline{co}[a_{ij}(\cdot)]$ or $co[a_{ij}(\cdot)]$ in the sense of Filippov. To obtain more accurate control gain, the memristive terms of MCGNNs with mixed delays are normalized by a simple linear transformation. And the memristive terms can be expressed as $\varsigma_x\in[-1,1]$. Moreover, the control gain depends on ς_x and each weight of MCGNNs with mixed delays, instead of picking the maximum or minimum value of the memristive terms.

Then, the synchronization error of networks (2) and (14) can be represented by: $e_i(t)=v_i(t)-z_i(t)=y_i(t)-x_i(t)$ with the initial conditions $e(s)=\Psi^v(s)-\Psi^z(s)$.

Definition 1: Networks (2) and Networks (14) are said to be globally exponentially synchronized, if there exist constants $M\ge 1$ and ϱ such that

$|v_i(t)-z_i(t)|\le M\exp\{-\varrho t\}, i=1,2,\cdots,n,$

where $M=\gamma\|\Psi^v(s)-\Psi^z(s)\|$, γ is a constant and the constant $\varrho>0$ is said to be the degree of exponential synchronization.

Lemma 1 ([32]): If the Assumption2 holds,then for any $i, j \in I$, $f(0) = g(0) = h(0) = 0$, there exist the following inequations:

$\varsigma_y f_j(y_i(t)) - \varsigma_x f_j(x_i(t)) \leq \varsigma L_j^f |y_i(t) - x_i(t)|$,
$\varsigma_y g_j(y_i(t)) - \varsigma_x g_j(x_i(t)) \leq \varsigma L_j^g |y_i(t) - x_i(t)|$,
$\varsigma_y \int_{t-\tau_{ij}}^t h_j(y_i(s))ds - \varsigma_x \int_{t-\varrho_j}^t h_j(x_i(s))ds$
$\leq \varsigma L_j^h \int_{t-\tau_{ij}}^t |y_i(s) - x_i(s)|$,

where $\varsigma = max\{\varsigma_y, \varsigma_x\}$, $L_j^f = max\{L_j^{f_1}, L_j^{f_2}\}$, $L_j^g = max\{L_j^{g_1}, L_j^{g_2}\}$ and $L_j^h = max\{L_j^{h_1}, L_j^{h_2}\}$.

III. MAIN RESULTS

In this paper, the hybrid control with periodic intermittent control and pinning control is designed. The first $n(1 \leq n < N)$ nodes of the response networks (14) are selected and pinned. Moreover, the periodic intermittent control is applied to n nodes networks to achieve the synchronization between drive networks (2) and response networks (14). The hybrid control is designed as follows:

$$U_i(t) = \begin{cases} u_i(t), & |e_i(t)| > 0 \\ 0, & |e_i(t)| = 0 \end{cases} \tag{18}$$

where $u_i(t) = -\beta_i(t)(v_i(t) - z_i(t))sgn(v_i(t) - z_i(t))$, $1 \leq i \leq n$, the control gain $\beta_i(t) = \begin{cases} \beta_i, & \mu T \leq t < \mu T + \theta \\ 0, & \mu T + \theta \leq t < (\mu + 1)T \end{cases}$, $T > 0$ is the control period, θ is the control duration, $\mu = 0, 1, 2, \cdots$.

Remark 2: At present, the synchronization of neural networks often uses global feedback control [13], [15], [23]–[25], [31], [32]. And the control is continuously applied to the response networks. Firstly, in practical applications, the use of continue control will increase costs. Intermittent control can reduce control costs. Secondly, in a large and complex network, it is impractical to control each network node. The use of pinning control can reduce network control nodes and control complexity. Obviously, combining intermittent control with pinning strategy can further reduce the control cost and control complexity.

If $|e_i(t)| > 0$ and $\mu T \leq t < \mu T + \theta$, the error dynamical networks can be written:

$$\begin{aligned}
\dot{e}_i(t) &= c_i(y_i(t) + T_i)\Bigg\{ -(o_i^{d_1} + o_i^{d_2}\varsigma_y)(y_i(t) + T_i) \\
&\quad + \sum_{j=1}^n \Bigg[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma_y) f_j(y_j(t) + T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma_y) \\
&\quad \times g_j(y_j(t - \tau_{ij}(t)) + T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma_y) \\
&\quad \times \int_{t-\tau_{ij}(t)}^t h_j(y_j(s) + T_j)\, ds \Bigg] + J_i \Bigg\} - \beta_i |e_i(t)| \\
&\quad - c_i(x_i(t) + T_i)\Bigg\{ -(o_i^{d_1} + o_i^{d_2}\varsigma_x)(x_i(t) + T_i) \\
&\quad + \sum_{j=1}^n \Bigg[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma_x) f_j(x_j(t) + T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma_x) \\
&\quad \times g_j(x_j(t - \tau_{ij}(t)) + T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma_x) \\
&\quad \times \int_{t-\tau_{ij}(t)}^t h_j(x_j(s) + T_j)\, ds \Bigg] + J_i \Bigg\}.
\end{aligned} \tag{19}$$

If $|e_i(t)| = 0$ or $\mu T + \theta \leq t < (\mu + 1)T$, the error dynamical networks can be written:

$$\begin{aligned}
\dot{e}_i(t) &= c_i(y_i(t) + T_i)\Bigg\{ -(o_i^{d_1} + o_i^{d_2}\varsigma_y)(y_i(t) + T_i) \\
&\quad + \sum_{j=1}^n \Bigg[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma_y) f_j(y_j(t) + T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma_y) \\
&\quad \times g_j(y_j(t - \tau_{ij}(t)) + T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma_y) \\
&\quad \times \int_{t-\tau_{ij}(t)}^t h_j(y_j(s) + T_j)\, ds \Bigg] + J_i \Bigg\} \\
&\quad - c_i(T_i x_i(t) + T_i)\Bigg\{ -(o_i^{d_1} + o_i^{d_2}\varsigma_x)(x_i(t) + T_i) \\
&\quad + \sum_{j=1}^n \Bigg[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma_x) f_j(x_j(t) + T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma_x) \\
&\quad \times g_j(x_j(t - \tau_{ij}(t)) + T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma_x) \\
&\quad \times \int_{t-\tau_{ij}(t)}^t h_j(x_j(s) + T_j)\, ds \Bigg] + J_i \Bigg\}.
\end{aligned} \tag{20}$$

Theorem 1: Let the Assumptions1–Assumptions3 hold, under the hybrid control $U_i(t)$, the drive networks(2) can be globally exponentially synchronized with response networks(14), if there exists constants $\beta > \alpha_2 + \alpha_3 + \delta$, $\rho_1 > \delta$ and ρ_2 satisfying the following conditions:

$$\begin{aligned}
&(C1) - \alpha_1 + \alpha_2 + \alpha_3 + \rho_1 - \beta < 0, \\
&(C2) - \alpha_1 + \alpha_2 + \alpha_3 - \rho_2 < 0, \\
&(C3) \varrho = \epsilon - (\rho_1 + \rho_2)(1 - \frac{\theta}{T}) > 0.
\end{aligned} \tag{21}$$

where $\epsilon > 0$ is the unique positive solution of the equation $\epsilon - \rho_1 + \delta \exp\{\epsilon\tau\} = 0$. *Proof:* Consider a proper Lyapunov function as follows:

$$V(t) = V_1(t) + V_2(t) \tag{22}$$

where

$$\begin{aligned}
V_1(t) &= \sum_{i=1}^n \int_{x_i(t)+T_i}^{y_i(t)+T_i} \frac{sgn(y_i(t) - x_i(t))}{c_i(s)}\, ds, \\
V_2(t) &= \sum_{i=1}^n \sum_{j=1}^n \left(o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma\right) L_j^h \\
&\quad \times \int_{-\tau_{ij}(t)}^0 \int_{t+s}^t e^{\rho_1(u-t)} \frac{y_j(u) - x_j(u)}{sgn(y_j(u) - x_j(u))} duds.
\end{aligned}$$

According to Assumption1, the boundary values of $\int_{x_i(t)+T_i}^{y_i(t)+T_i} \frac{sgn(y_i(t)-x_i(t))}{c_i(s)}\, ds$ can be obtained:

$$\frac{|e_i(t)|}{\overline{c}_i} \leq \int_{x_i(t)+T_i}^{y_i(t)+T_i} \frac{sgn(y_i(t) - x_i(t))}{c_i(s)}\, ds \leq \frac{|e_i(t)|}{\underline{c}_i}.$$

Firstly, calculating the upper right derivative of $V_m(t)$, $m = 1, 2$ along the trajectory of error networks $e_i(t)$, we have

$$\begin{aligned}
&\mathfrak{D}^+V_1(t)\\
&= \sum_{i=1}^{n} sgn(e_i(t))\left(\frac{\dot{y}_i(t)}{c_i(y_i(t)+T_i)} - \frac{\dot{x}_i(t)}{c_i(x_i(t)+T_i)}\right)\\
&= \sum_{i=1}^{n} sgn(e_i(t))\Bigg\{ -(o_i^{d_1} + o_i^{d_2}\varsigma_y)(y_i(t)+T_i)\\
&\quad + \sum_{j=1}^{n}\Big[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma_y)f_j(y_j(t)+T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma_y)\\
&\quad \times g_j(y_j(t-\tau_{ij}(t))+T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma_y\\
&\quad \times \int_{t-\tau_{ij}(t)}^{t} h_j(y_j(s)+T_j)\,ds\Big] + \frac{U_i(t)}{c_i(y_i(t)+T_i)}\\
&\quad + (o_i^{d_1} + o_i^{d_2}\varsigma_x)(x_i(t)+T_i) - \sum_{j=1}^{n}\Big[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma_x)\\
&\quad \times f_j(x_j(t)+T_j) + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma_x)\\
&\quad g_j(x_j(t-\tau_{ij}(t))+T_j) + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma_x\\
&\quad \times \int_{t-\tau_{ij}(t)}^{t} h_j(x_j(s)+T_j)\,ds\Big]\Bigg\}
\end{aligned}\tag{23}$$

According to Assumption1, Assumption2, Assumption3 and Lemma1, we get

$$\begin{aligned}
&\mathfrak{D}^+V_1(t)\\
&\le \sum_{i=1}^{n}\Bigg\{-(o_i^{d_1} + o_i^{d_2}\varsigma)|e_i(t)| + \sum_{j=1}^{n}\Big[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma)\\
&\quad \times L_j^f|e_j(t)| + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma)L_j^g|e_j(t-\tau_{ij}(t))|\\
&\quad + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma)L_j^h \times \int_{t-\tau_{ij}(t)}^{t}|e_j(s)|\,ds\Big]\\
&\quad + \frac{T_iU_i(t)}{\underline{c}_i}\Bigg\}\\
&= -\sum_{i=1}^{n}(o_i^{d_1} + o_i^{d_2}\varsigma)|e_i(t)| + \sum_{i=1}^{n}\sum_{j=1}^{n}\Big[(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma)\\
&\quad \times L_j^f|e_j(t)| + (o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma)L_j^g|e_j(t-\tau_{ij}(t))|\\
&\quad + (o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma)L_j^h \times \int_{t-\tau_{ij}(t)}^{t}|e_j(s)|\,ds\Big]\\
&\quad + \sum_{i=1}^{n}\frac{U_i(t)}{\underline{c}_i}.
\end{aligned}\tag{24}$$

Let $\alpha_1 = \min\{(o_i^{d_1} + o_i^{d_2}\varsigma)\underline{c}_i\}$, $\alpha_2 = \max\{\sum_{j=1}^{n}\left(o_{ji}^{a_1} + o_{ji}^{2}\varsigma\right)L_i^f\underline{c}_i\}$, and $\delta = \max\{\sum_{j=1}^{n}\left(o_{ji}^{b_1} + o_{ji}^{b_2}\varsigma\right)L_i^g\underline{c}_i\}$, we have

$$\begin{aligned}
\mathfrak{D}^+V_1(t) &\le (-\alpha_1 + \rho_1 + \alpha_2)V_1(t) + \delta\sum_{i=1}^{n}\frac{|e_i(t-\tau_{ji}(t))|}{\underline{c}_i}\\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}(o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma)L_j^h\int_{t-\tau_{ij}(t)}^{t}|e_j(s)|\,ds\\
&\quad + \sum_{i=1}^{n}\frac{U_i(t)}{\underline{c}_i} - \rho_1V_1(t).
\end{aligned}\tag{25}$$

Based on Assumption3, we have

$$\begin{aligned}
&\mathfrak{D}^+V_2(t)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\left(o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma\right)L_j^h\int_{-\tau_{ij}(t)}^{0}|e_j(s)|ds\\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{n}\left(o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma\right)L_j^h\int_{-\tau_{ij}(t)}^{0}e^{\rho_1 s}|e_j(t+s)|ds\\
&\quad - \rho_1V_2(t)\\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n}\left(o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma\right)L_j^h\tau|e_j(t)| - \sum_{i=1}^{n}\sum_{j=1}^{n}\Big(o_{ij}^{w_1}\\
&\quad + o_{ij}^{w_2}\varsigma\Big)L_j^h\int_{t-\tau_{ij}(t)}^{t}e^{\rho_1 s}|e_j(s)|ds - \rho_1V_2(t).
\end{aligned}\tag{26}$$

Let $\alpha_3 = \max\{\sum_{j=1}^{n}\left(o_{ji}^{w_1} + o_{ji}^{w_2}\varsigma\right)L_i^h\tau\underline{c}_i\}$, we have

$$\begin{aligned}
&\mathfrak{D}^+V(t)\\
&\le (-\alpha_1 + \alpha_2 + \alpha_3 + \rho_1)V_1(t) + \delta\sup_{t-\tau_{ij}(t)\le s\le t}(V(s))\\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(o_{ij}^{w_1} + o_{ij}^{w_2}\varsigma\right)L_j^h\int_{t-\tau_{ij}(t)}^{t}(1-e^{\rho_1 s})|e_j(s)|ds\\
&\quad + \sum_{i=1}^{n}\frac{U_i(t)}{\underline{c}_i} - \rho_1(V_1(t) + V_2(t)).
\end{aligned}\tag{27}$$

Obviously, $1 - e^{\rho_1 s} < 0$, we have

$$\begin{aligned}
\mathfrak{D}^+V(t) &\le (-\alpha_1 + \alpha_2 + \alpha_3 + \rho_1)V_1(t) - \rho_1V(t)\\
&\quad + \delta\sup_{t-\tau_{ij}(t)\le s\le t}(V(s)) + \sum_{i=1}^{n}\frac{U_i(t)}{\underline{c}_i}.
\end{aligned}\tag{28}$$

Then for $|e_i(t)| > 0$ and $\mu T \le t < \mu T + \theta, \mu = 0, 1, 2, \cdots$, applying the control system $U_i(t) = u_i(t)$, we have

$$\begin{aligned}
\mathfrak{D}^+V(t) &\le (-\alpha_1 + \alpha_2 + \alpha_3 + \rho_1 - \beta)V_1(t)\\
&\quad - \rho_1V(t) + \delta\sup_{t-\tau_{ij}(t)\le s\le t}(V(s)).
\end{aligned}\tag{29}$$

where $\beta = \max\{\beta_i\}$.

Applying the condition C1, we have

$$\mathfrak{D}^+V(t) \le -\rho_1V(t) + \delta\sup_{t-\tau_{ij}(t)\le s\le t}(V(s))\tag{30}$$

Similarly, for $|e_i(t)| > 0$ and $\mu T + \theta \leq (\mu+1)T$, $\mu = 0, 1, 2, \cdots$, applying the control system $U_i(t) = 0$ and the condition C2, we have

$$\begin{aligned}\mathfrak{D}^+V(t) &\leq (-\alpha_1+\alpha_2+\alpha_3-\rho_2)V_1(t)+\rho_2V_1(t)\\ &\quad -\rho_1V_2(t)+\delta \sup_{t-\tau_{ij}(t)\leq s\leq t}(V(s))\\ &\leq \rho_2V(t)+\delta \sup_{t-\tau_{ij}(t)\leq s\leq t}(V(s)). \qquad (31)\end{aligned}$$

Secondly, we will prove the MCGNNs (2) and (14) can achieve the exponential synchronization, if there exist three constants ρ_1, ρ_2 and δ such that:

$$\mathfrak{D}^+V(t) \leq \begin{cases}-\rho_1V(t)+\delta \sup\limits_{t-\tau_{ij}(t)\leq s\leq t}(V(s)), \mu T\leq t<\mu T+\theta\\ \rho_2V(t)+\delta \sup\limits_{t-\tau_{ij}(t)\leq s\leq t}(V(s)), \mu T+\theta\leq(\mu+1)T.\end{cases} \qquad (32)$$

Define two functions $q(\epsilon) = \epsilon - \rho_1 + \delta\exp\{\epsilon\tau\}$ and $W(t) = \exp\{\epsilon t\}V(t)$. Using $\rho_1 > \delta$, we have $q(0) < 0$. When $\epsilon \to +\infty$, we have $q(\epsilon) = +\infty$ and $q'(\epsilon) = 1 + \delta\epsilon\exp\{\epsilon\tau\} > 0$. There exists a unique positive solution $\epsilon > 0$ satisfying the equation $\epsilon - \rho_1 + \delta\exp\{\epsilon\tau\} = 0$. Let $M_0 = \sup\limits_{-\tau\leq s\leq 0}(V(s))$ and $P(t) = W(t) - hM_0$, where $h > 1$ is a constant. It can be easily obtained:

$$P(t) < 0, \textit{for all } t \in [-\tau, 0] \qquad (33)$$

Step1. According to inequations (32) and Lemma1, the exponential synchronization for MCGNNs (2) and (14) can be proved in the first period $0 \leq t < T$.

For $|e_i(t)| > 0$ and $0 \leq t < T$, we prove that $P(t) < 0$ holds for all $t \in [0, \theta)$, otherwise there exist a $0 \leq t_0 < \theta$ such that

$$\begin{cases}P(t_0)=0,\\ P(t)<0, \quad -\tau\leq t<t_0\\ \dot{P}(t_0)\geq 0,\end{cases} \qquad (34)$$

It can be easily obtain from (30) and (34) that

$$\begin{aligned}\dot{P}(t_0) &= \epsilon\exp\{\epsilon t_0\}W(t_0)+\exp\{\epsilon t_0\}\mathfrak{D}^+V(t_0)\\ &\leq \epsilon\exp\{\epsilon t_0\}W(t_0)-\rho_1\exp\{\epsilon t_0\}W(t_0)\\ &\quad +\delta\exp\{\epsilon t_0\}\sup_{t_0-\tau\leq s\leq t_0}(V(s))\\ &= (\epsilon-\rho_1)W(t_0)+\delta\exp\{\epsilon t_0\}\sup_{t_0-\tau\leq s\leq t_0}(V(s)) \qquad (35)\end{aligned}$$

Using(34), we have $W(t) < hM_0$, for all $-\tau \leq t < t_0$, and $W(t_0) = hM_0$. Then we obtain $V(t) < hM_0\exp\{-\epsilon t\}$, for all $-\tau \leq t < t_0$, and so $\sup\limits_{t_0-\tau\leq s\leq t_0}(V(s)) < \exp\{\epsilon\tau\}hM_0\exp\{-\epsilon t_0\}$. Therefore, we have $\exp\{\epsilon t_0\}\sup\limits_{t_0-\tau\leq s\leq t_0}(V(s)) < \exp\{\epsilon\tau\}hM_0 = \exp\{\epsilon\tau\}W(t_0)$.

It follows from (34) that

$$\dot{P}(t_0) < (\epsilon-\rho_1+\delta\exp\{\epsilon\tau\})W(t_0) = 0. \qquad (36)$$

which contradicts with (34) and $P(t) < 0$ holds for all $t \in [-\tau, t_0)$. Therefore,

$$V(t) < hM_0\exp\{-\epsilon t\}, \textit{for all } t\in[-\tau,\theta). \qquad (37)$$

In the following, we prove that for all $\theta \leq t < T$,

$$Q(t) = W(t) - hM_0\exp\{\varpi(t-\theta)\} < 0. \qquad (38)$$

where $\varpi = \rho_1 + \rho_2$.

Otherwise, there exist a $t_1 \in [\theta, T)$ such that

$$\begin{cases}Q(t_1)=0,\\ Q(t)<0, \quad \text{if } \theta\leq t<t_1\\ \dot{Q}(t_1)\geq 0,\end{cases} \qquad (39)$$

For $\tau > 0$, if $\theta \leq t_1 - \tau < t_1$, it follows from (39) that

$$\sup_{t_1-\tau\leq s\leq t_1}(V(s)) < \exp\{\epsilon\tau\}\exp\{-\epsilon t_1\}hM_0\exp\{\varpi(t-\theta_1)\}. \quad (40)$$

and if $-\tau \leq t_1 - \tau < \theta$, by (34) and (39), we have

$$\begin{aligned}\sup_{t_1-\tau\leq s\leq t_1}(V(s)) &= \max\left\{\sup_{t_1-\tau\leq s\leq\theta}(V(s)), \sup_{\theta\leq s\leq t_1}(V(s))\right\}\\ &< \max\{\exp\{\epsilon\tau\}hM_0\exp\{-\epsilon t_1\},\\ &\quad \exp\{\epsilon(t_1-\theta)\}V(t)\}\\ &< \exp\{\epsilon\tau\}V(t_1) = W(t_1). \qquad (41)\end{aligned}$$

Therefore, by (31) and (41), we have

$$\begin{aligned}&\dot{Q}(t_1)\\ &= \epsilon W(t_1)+\exp\{\epsilon t_1\}\mathfrak{D}^+V(t_1)-hM_0\varpi\exp\{\varpi(t-\theta)\}\\ &\leq (\epsilon+\rho_2-\varpi)W(t_1)+\delta\exp\{\epsilon t_1\}\sup_{t_1-\tau\leq s\leq t_1}(V(s))\\ &< (\epsilon+\rho_2-\varpi+\delta\exp\{\epsilon\tau\})W(t_1)\\ &\leq (\epsilon-\rho_1+\delta\exp\{\epsilon\tau\})W(t_1) = 0. \qquad (42)\end{aligned}$$

which contradicts with (39) and $Q(t) < 0$ holds for all $t \in [-\theta, T)$. Therefore,

$$W(t) < hM_0\exp\{\varpi(t-\theta)\} < hM_0\exp\left\{\varpi(1-\frac{\theta}{T})T\right\}, \quad \textit{for all } t\in[-\tau,T). \qquad (43)$$

Step2. Similar to above proof, in the second period $T \leq t < 2T$, the exponential synchronization for MCGNNs (2) and (14) can be proved.

For all $t \in [T, (1+\frac{\theta}{T})T)$

$$W(t) < hM_0\exp\left\{\varpi(1-\frac{\theta}{T})T\right\}. \qquad (44)$$

and for all $t \in [(1+\frac{\theta}{T})T, 2T)$

$$\begin{aligned}W(t) &< hM_0\exp\{\varpi(T-\theta)\}\exp\left\{\varpi(t-(1+\frac{\theta}{T})T)\right\}\\ &= hM_0\exp\{\varpi(t-2\theta)\}. \qquad (45)\end{aligned}$$

Step3. By mathematical induction, in the $\mu + 1$ period $\mu T \leq t < (\mu + 1)T$, the exponential synchronization for MCGNNs (2) and (14) can be proved.

For *for all* $t \in [\mu T, \mu T + \theta)$

$$W(t) < hM_0 \exp\left\{\mu\varpi(1-\frac{\theta}{T})T\right\} \leq hM_0 \exp\left\{\varpi(1-\frac{\theta}{T})t\right\}. \quad (46)$$

and for all $t \in [\mu T + \theta, (\mu + 1)T)$,

$$W(t) < hM_0 \exp\{\varpi(t-(\mu+1)\theta)\} \leq hM_0 \exp\left\{\varpi(1-\frac{\theta}{T})t\right\}. \quad (47)$$

Setp4. Let $h \to 1$, according to $W(t) = \exp\{\epsilon t\}V(t)$ and $\varpi = \rho_1 + \rho_2$, we have

$$V(t) \leq M_0 \exp\left\{-(\epsilon - \varpi(1-\frac{\theta}{T}))t\right\}, t > 0. \quad (48)$$

Because of $\frac{|e_i(t)|}{\bar{c}_i} \leq V(t)$, we have

$$\begin{aligned}|y_i(t) - z_i(t)| &\leq M_0\bar{c}_i \exp\left\{-(\epsilon - \varpi(1-\frac{\theta}{T}))t\right\} \\ &= M\exp\{-\varrho t\}, t > 0. \end{aligned} \quad (49)$$

where $M = M_0\bar{c}_i$ and $\varrho = \epsilon - \varpi(1 - \frac{\theta}{T})$.

According to C3 and Lemma1, the drive networks(2) can be be globally exponentially synchronized with response networks(14) under the hybrid control $U_i(t)$. Finally, the proof is completed. □

Remark 3: Based on the existing works [13], [15], [16], [24]–[32], [34], [35], the control parameters can be expressed as follows:

$$\mathfrak{a}_1 = \min\{\underline{d}_i\underline{c}_i\}, \quad \mathfrak{a}_2 = \max\{\sum_{j=1}^{n} \bar{a}_{ji}L_i^f\underline{c}_i\},$$
$$\mathfrak{d} = \max\{\sum_{j=1}^{n} \bar{b}_{ji}L_i^g\underline{c}_i\}, \quad \mathfrak{a}_3 = \max\{\sum_{j=1}^{n} \bar{w}_{ji}L_i^h\tau\underline{c}_i\}.$$

where

$$\begin{aligned}\underline{d}_i &= \min\{d_i^{\circ}, d_i^{\circ\circ}\}, & \bar{d}_i &= \max\{d_i^{\circ}, d_i^{\circ\circ}\},\\ \underline{a}_{ji} &= \min\{a_{ji}^{\circ}, a_{ji}^{\circ\circ}\}, & \bar{a}_{ji} &= \max\{a_{ji}^{\circ}, a_{ji}^{\circ\circ}\},\\ \underline{b}_{ji} &= \min\{b_{ji}^{\circ}, b_{ji}^{\circ\circ}\}, & \bar{b}_{ji} &= \max\{b_{ji}^{\circ}, b_{ji}^{\circ\circ}\},\\ \underline{w}_{ji} &= \min\{w_{ji}^{\circ}, w_{ji}^{\circ\circ}\}, & \bar{w}_{ji} &= \max\{w_{ji}^{\circ}, w_{ji}^{\circ\circ}\}.\end{aligned}$$

In this paper, the control parameters can be expressed as follows:

$$\begin{aligned}\alpha_1 &= \min\{(o_i^{d_1} + o_i^{d_2}\varsigma)\underline{c}_i\},\\ \delta &= \max\{\sum_{j=1}^{n}(o_{ji}^{b_1} + o_{ji}^{b_2}\varsigma)L_i^g\underline{c}_i\},\\ \alpha_2 &= \max\{\sum_{j=1}^{n}\left(o_{ij}^{b_1} + o_{ij}^{b_2}\varsigma\right)L_i^g\underline{c}_i\},\\ \alpha_3 &= \max\{\sum_{j=1}^{n}\left(o_{ji}^{w_1} + o_{ji}^{w_2}\varsigma\right)L_i^h\tau\underline{c}_i\}.\end{aligned}$$

Obviously, $\underline{d}_i \leq o_i^{d_1} + o_i^{d_2}\varsigma \leq \bar{d}_i$, $\underline{a}_{ji} \leq o_{ji}^{a_1} + o_{ji}^{a_2}\varsigma \leq \bar{a}_{ji}$, $\underline{b}_{ji} \leq o_{ji}^{b_1} + o_{ji}^{b_2}\varsigma \leq \bar{b}_{ji}$, $\underline{w}_{ji} \leq o_{ji}^{w_1} + o_{ji}^{w_2}\varsigma \leq \bar{w}_{ji}$.

Therefore, the more small control gain β in this paper can achieve the exponential synchronization on MCGNNs (2) and (14). This shows the control is more accurate.

From Theorem1, the control gain $\beta_i(t)$ satisfies $\beta > \alpha_2 + \alpha_3 + \delta$ and $\beta > -\alpha_1 + \alpha_2 + \alpha_3 + \rho_1$. If $\beta > \max\{\alpha_2 + \alpha_3 + \delta, -\alpha_1 + \alpha_2 + \alpha_3 + \rho_1\}$, the C1 and $\beta > \alpha_2 + \alpha_3 + \delta$ can be satisfied. Moreover, ρ_1 is determined by $\rho_1 > \delta$ and C3. $\epsilon = \Theta(\rho_1)$ is yielded from the equation $\epsilon - \rho_1 + \delta\exp\{\epsilon\tau\} = 0$, then the Corollary1 can be obtained as follows:

Corollary 1: Let the Assumptions1–Assumptions3 hold, under the hybrid control $U_i(t)$, $\rho_1 > \delta$, the drive networks(2) can be be globally exponentially synchronized with response networks(14), if there exists constants, β and θ satisfying the following conditions:

$$\begin{aligned}&(C4)\beta > \max\{\alpha_2 + \alpha_3 + \delta, -\alpha_1 + \alpha_2 + \alpha_3 + \rho_1\},\\ &(C5)1 - \frac{\Theta(\rho_1)}{\rho_1 + \rho_2} < \frac{\theta}{T} \leq 1.\end{aligned} \quad (50)$$

where δ, ρ_1 and ρ_2 are same as in Theorem1.

Remark 4: In Corollary1, the control period and gain can be quickly determined. After determining the control nodes,to achieve the exponential synchronization for a class of MCGNNs, it is feasible to just confirm the control period and gain. This facilitates practical operation.

Let $c_i(x_i(t)) \equiv 1$, we have

$$\begin{aligned}\dot{z}_i(t) = &-d_i(z_i(t))z_i(t) + \sum_{j=1}^{n}\Big[a_{ij}(z_i(t))f_j(z_j(t)) + b_{ij}(z_i(t))\\ &\times g_j(z_j(t-\tau_{ij}(t))) + w_{ij}(z_i(t))\\ &\times \int_{t-\tau_{ij}(t)}^{t} h_j(z_j(s))\,ds\Big] + J_i, \quad i, j = 1, 2, \cdots, n. \end{aligned} \quad (51)$$

Networks (50), a special case of MCGNNs (2), are called memristive neural networks. Let $\alpha_1' = \max\{o_i^{d_1} + o_i^{d_2}\varsigma\}$, $\alpha_2' = \max\{\sum_{j=1}^{n}\left(o_{ji}^{a_1} + o_{ji}^{a_2}\varsigma\right)L_i^f\}$, $\delta' = \max\{\sum_{j=1}^{n}(o_{ji}^{b_1} + o_{ji}^{b_2}\varsigma)L_i^g\}$, $\alpha_3' = \max\{\sum_{j=1}^{n}\left(o_{ji}^{w_1} + o_{ji}^{w_2}\varsigma\right) \times L_j^h\tau\}$ Thus, the Corollary2 can be obtained as follows:

Corollary 2: Let the Assumptions1–Assumptions3 hold, under the hybrid control $U_i(t)$, $\rho_1 > \delta$, the drive networks(2) can be be globally exponentially synchronized with response networks(14), if there exists constants, β and θ satisfying the following conditions:

$$\begin{aligned}&(C6)\beta > \max\{\alpha_2' + \alpha_3' + \delta', -\alpha_1' + \alpha_2' + \alpha_3' + \rho_1\},\\ &(C7)1 - \frac{\Theta(\rho_1)}{\rho_1 + \rho_2} < \frac{\theta}{T} \leq 1.\end{aligned} \quad (52)$$

where ρ_1 and ρ_2 are same as in Theorem1, $\epsilon = \Theta(\rho_1)$ is yielded from the equation $\epsilon - \rho_1 + \delta'\exp\{\epsilon\tau\} = 0$.

Remark 5: In the synchronization conditions, the parameter δ denotes the impact of time-varying delay and the parameter α_3 denotes the impact of unbounded distributed delay. If $\alpha_3 = 0$, the result can be applied in the synchronization MCGNNs with time-varying delay [24], [25]. If $\delta = 0$ and $\alpha_3 = 0$, the result can be applied in the synchronization MCGNNs without delays. In corollary2, if $c_i(x_i(t)) \equiv 1$, memristive neural networks can be obtained. Similarly, the result *can be applied in the synchronization MNNs with mixed delays [16] or with time-varying delay [15] or without delays. Thus, the method in this paper is universal.*

Remark 6: The following steps can fastly determine the parameters of the hybrid control (18) to achieve the exponential synchronization on MCGNNs with mixed delays (2) and (14):

Step1.Determine the parameters of MCGNNs with mixed delays and choose some proper nodes to add the intermittent control.

Step2.Calculate the value of α_1, α_2, α_3 *and* δ. *Recall* $\alpha_1 = \max\{(o_i^{d_1} + o_i^{d_2}\varsigma)\underline{c}_i\}$, $\alpha_2 = \max\{\sum_{j=1}^{n}\left(o_{ji}^{a_1} + o_{ji}^{2}\varsigma\right)L_i^f\underline{c}_i\}$, $\delta = \max\{\sum_{j=1}^{n}\left(o_{ji}^{b_1} + o_{ji}^{b_2}\varsigma\right)L_i^g\underline{c}_i\}$ *and* $\alpha_3 = \max\{\left(o_{ji}^{w_1} + o_{ji}^{w_2}\varsigma\right)L_i^h\tau\underline{c}_i\}$.

Step3.Determine the value of ρ_1, ρ_2, $\frac{\theta}{T}$ *and* β. *Firstly, according to C2,* ρ_2 *satisfies* $\rho_2 > -\alpha_1 + \alpha_2 + \alpha_3$. *Then, according to C3 and the equation* $\epsilon - \rho_1 + \delta\exp\{\epsilon\tau\} = 0$, *the relationship between* ρ_1 *and the control ratio* $\frac{\theta}{T}$ *can be obtained. And choose* ρ_1 *by the minimum value of* $\frac{\theta}{T}$. *Finally, according to C1 and C4, the control gain* β *satisfies* $\beta > \max\{\alpha_2 + \alpha_3 + \delta, -\alpha_1 + \alpha_2 + \alpha_3 + \rho_1\}$. *Therefore, the parameters of hybrid control (18) are determined.*

IV. NUMERICAL SIMULATION

The following numerical examples are given to show the effectiveness of the above theoretical results.Cosider the three-dimensional memristive Cohen-Grossberg neural network model with mixed delays:

$$\begin{cases} \frac{dz_1(t)}{dt} = c_1(z_1(t))\Big\{-d_1(z_1(t)) + \sum_{j=1}^{3}[a_{1j}(z_1(t)) \\ \quad \times f_j(z_j(t)) + \sum_{j=1}^{3} b_{1j}(z_1(t))g_j(z_j(t-\tau_{ij}(t))) \\ \quad + w_{1j}(z_1(t))\int_{t-\tau_{1j}(t)}^{t} h_j(z_j(s))] + J_1\Big\} \\ \frac{dz_2(t)}{dt} = c_2(z_2(t))\Big\{-d_2(z_2(t)) + \sum_{j=1}^{3}[a_{2j}(z_2(t)) \\ \quad \times f_j(z_j(t)) + \sum_{j=1}^{3} b_{2j}(z_2(t))g_j(z_j(t-\tau_{ij}(t))) \\ \quad + w_{2j}(z_2(t))\int_{t-\tau_{2j}(t)}^{t} h_j(z_j(s))] + J_2\Big\} \\ \frac{dz_3(t)}{dt} = c_3(z_3(t))\Big\{-d_3(z_3(t)) + \sum_{j=1}^{3}[a_{3j}(z_3(t)) \\ \quad \times f_j(z_j(t)) + \sum_{j=1}^{3} b_{3j}(z_3(t))g_j(z_j(t-\tau_{ij}(t))) \\ \quad + w_{3j}(z_3(t))\int_{t-\tau_{3j}(t)}^{t} h_j(z_j(s))] + J_3\Big\}. \end{cases} \tag{53}$$

where

$$J_1 = 0.1, \quad J_2 = 0.2, \quad J_3 = 0.3,$$

$$d_1 = \begin{cases} 1.8 & |z_1(t)| \le 1 \\ 1.34 & |z_1(t)| > 1, \end{cases} \quad d_2 = \begin{cases} 1.65 & |z_2(t)| \le 1 \\ 2.95 & |z_2(t)| > 1, \end{cases}$$

$$d_3 = \begin{cases} 2.67 & |z_3(t)| \le 1 \\ 2.38 & |z_3(t)| > 1, \end{cases} \quad a_{11} = \begin{cases} -1.4 & |z_1(t)| \le 1 \\ -2.1 & |z_1(t)| < 1 \end{cases},$$

$$a_{21} = \begin{cases} 1.2 & |z_2(t)| \le 1 \\ 1.5 & |z_2(t)| < 1, \end{cases} \quad a_{31} = \begin{cases} -3.6 & |z_3(t)| \le 1 \\ -2.1 & |z_3(t)| < 1 \end{cases},$$

$$a_{12} = \begin{cases} 1.8 & |z_1(t)| \le 1 \\ -0.92 & |z_1(t)| < 1, \end{cases} \quad a_{22} = \begin{cases} 1.6 & |z_2(t)| \le 1 \\ -1.2 & |z_2(t)| < 1, \end{cases}$$

$$a_{32} = \begin{cases} -2.1 & |z_3(t)| \le 1 \\ 1.3 & |z_3(t)| < 1, \end{cases} \quad a_{13} = \begin{cases} -4.3 & |z_1(t)| \le 1 \\ 3.2 & |z_1(t)| < 1, \end{cases}$$

$$a_{23} = \begin{cases} -2.6 & |z_2(t)| \le 1 \\ 1.1 & |z_2(t)| < 1, \end{cases} \quad a_{33} = \begin{cases} 2.2 & |z_3(t)| \le 1 \\ 2.6 & |z_3(t)| < 1, \end{cases}$$

$$b_{11} = \begin{cases} -2.3 & |z_1(t)| \le 1 \\ -2.6 & |z_1(t)| < 1, \end{cases} \quad b_{21} = \begin{cases} -1.1 & |z_2(t)| \le 1 \\ -1.7 & |z_2(t)| < 1, \end{cases}$$

$$b_{31} = \begin{cases} -4.4 & |z_3(t)| \le 1 \\ -4.3 & |z_3(t)| < 1, \end{cases} \quad b_{12} = \begin{cases} 1.4 & |z_1(t)| \le 1 \\ 1.1 & |z_1(t)| < 1, \end{cases}$$

$$b_{22} = \begin{cases} 1.8 & |z_2(t)| \le 1 \\ -1.3 & |z_2(t)| < 1, \end{cases} \quad b_{32} = \begin{cases} -3.9 & |z_3(t)| \le 1 \\ 5.3 & |z_3(t)| < 1, \end{cases}$$

$$b_{13} = \begin{cases} -3.9 & |z_1(t)| \le 1 \\ -1.5 & |z_1(t)| < 1, \end{cases} \quad b_{23} = \begin{cases} -2.7 & |z_2(t)| \le 1 \\ -3.4 & |z_2(t)| < 1, \end{cases}$$

$$b_{33} = \begin{cases} -3.6 & |z_3(t)| \le 1 \\ -1.8 & |z_3(t)| < 1, \end{cases} \quad w_{11} = \begin{cases} 0.9 & |z_1(t)| \le 1 \\ -0.7 & |z_1(t)| < 1, \end{cases}$$

$$w_{21} = \begin{cases} -3.5 & |z_2(t)| \le 1 \\ -2.1 & |z_2(t)| < 1, \end{cases} \quad w_{31} = \begin{cases} -0.6 & |z_3(t)| \le 1 \\ 1.3 & |z_3(t)| < 1, \end{cases}$$

$$w_{12} = \begin{cases} 0.3 & |z_1(t)| \le 1 \\ -0.9 & |z_1(t)| < 1, \end{cases} \quad w_{22} = \begin{cases} 1.3 & |z_2(t)| \le 1 \\ -0.9 & |z_2(t)| < 1, \end{cases}$$

$$w_{32} = \begin{cases} -0.7 & |z_3(t)| \le 1 \\ 0.5 & |z_3(t)| < 1, \end{cases} \quad w_{13} = \begin{cases} -1.7 & |z_1(t)| \le 1 \\ 1.6 & |z_1(t)| < 1, \end{cases}$$

$$w_{23} = \begin{cases} 1.7 & |z_2(t)| \le 1 \\ 0.3 & |z_2(t)| < 1, \end{cases} \quad w_{33} = \begin{cases} -1.2 & |z_3(t)| \le 1 \\ -1.1 & |z_3(t)| < 1, \end{cases}$$

The amplifications function $c_1(z_1(t)) = 2+\sin(|z_1(t)|-1)$, $c_2(z_2(t)) = 1.8+\frac{0.5}{1+z_2^2(t)}$, $c_3(z_3(t)) = 1+\frac{0.4}{2+\tanh(z_3(t))}$, the time-varying delays $\tau_{ij}(t) = 0.5 + 0.2\sin t$, $i, j = 1, 2, 3$ and the activation functions $f_j(x) = 0.5\tanh x$, $g_j(x) = 1.18\cos(|x| - 0.2) + 0.2sign(x)$, $h_j(x) = \frac{|x+1|-|x-1|}{2}$, $j = 1, 2, 3$.

Consider networks (52) as the drive networks and corresponding response networks. Also from A1, A2 and A3, we have $\bar{c}_1 = 3$, $\underline{c}_1 = 1$, $\bar{c}_2 = 2.3$, $\underline{c}_2 = 1.8$, $\bar{c}_3 = 1.4$, $\underline{c}_3 = 1.33$ and $L_j^{f_1} = 0$, $L_j^{f_2} = 0.5$, $L_j^{g_1} = -1.179$, $L_j^{g_2} = 1.179$, $L_j^{h_1} = 0$, $L_j^{h_2} = 1$ and $0.3 \le \tau_{ij}(t) \le \tau = 0.7$, $-0.2 \le \dot{\tau}_{ij}(t) \le \tau_{ij} = 0.2$. With the random initial conditions z_1,z_2, z_3 and v_1,v_2, v_3. Let $\varsigma = 0.8$. The chaotic attractor and trajectory of networks(52) is shown in Fig.1.

Case(1): According to networks(52), Theorem1 and Corollary1, we can obtain the following numerical results to show the effectiveness of the hybrid control (18).

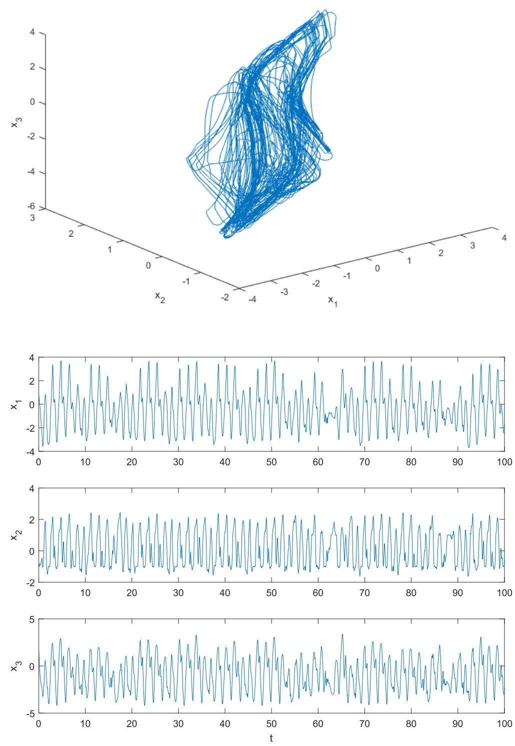

FIGURE 1. The chaotic attractors and trajectory of MCGNNs.

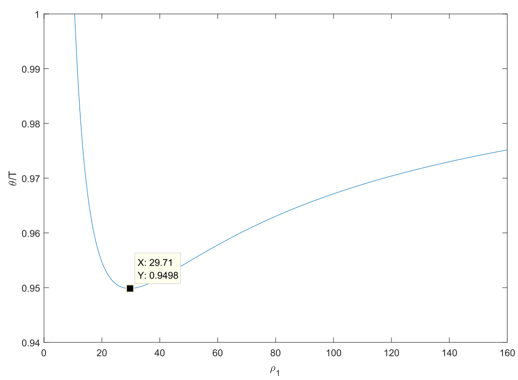

FIGURE 2. The relationship between $\frac{\theta}{T}$ and ρ_1.

By simple calculation, $\alpha_1 = \min\{(o_i^{d_1} + o_i^{d_2}\varsigma)\underline{c}_i\} = 1.386$, $\alpha_2 = \max\{\sum_{j=1}^{n}\left(o_{ij}^{a_1} + o_{ij}^{a_2}\varsigma\right)L_i^g\underline{c}_i\} = 3.8171$, $\delta = \max\{\sum_{j=1}^{n}(o_{ji}^{b_1} + o_{ji}^{b_2}\varsigma)L_i^g\underline{c}_i\} = 9.592$, $\alpha_3 = \max\{\sum_{j=1}^{n}\left(o_{ji}^{w_1} + o_{ji}^{w_2}\varsigma\right)L_i^h\tau\underline{c}_i\} = 0.5586$. The relationship between $\frac{\theta}{T}$ and ρ_1 are shown in Fig.2. From Fig.2, we have $\rho_1 = 29.71$, $0.9498 < \frac{\theta}{T} \leq 1$. To satisfy C1 and C2, we have $\beta = 32.7$ and $\rho_2 = 1$. Choosing $\frac{\theta}{T} = 0.95$ and the control period $T = 3s$, the condition C3 hold.

The first two network nodes are selected and pinned. And the corresponding response networks are controlled. Moreover, the C1, C2 and C3 can be satisfied. Therefore, the drive

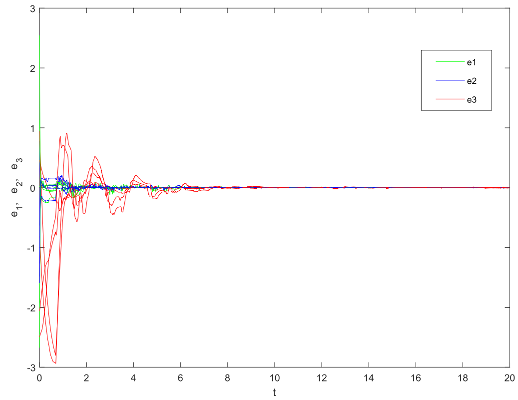

FIGURE 3. The synchronization error curves of MCGNNs with mixed delays.

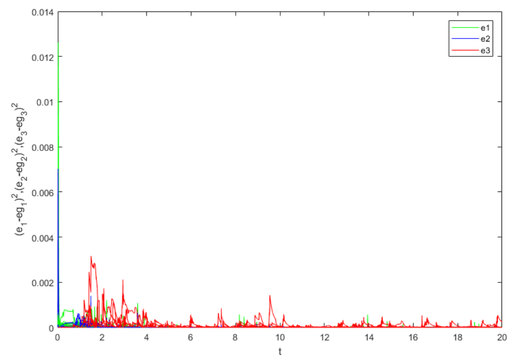

FIGURE 4. The residual analysis curves of error between control gain $\beta_g = 35.7$ and control gain $\beta = 32.7$.

networks (52) can be be globally exponentially synchronized with the corresponding response networks. Under random initial conditions, the error curves are shown in Fig.3. The initial conditions can be any value from -2 to 2.

Case(2): Considered the same networks(52), the accuracy of control gain is shown by comparing the existing methods of processing memristive terms [16], [27], [29]–[32] for memristive weights and the work in this paper.

Based on the existing method, the control parameters are given as follows:

$$\mathfrak{a}_1 = \min\{\underline{d}_i\underline{c}_i\} = 1.34 < \alpha_1 = 1.386,$$
$$\mathfrak{a}_2 = \max\{\sum\nolimits_{j=1}^{n} \overline{a}_{ji}L_i^f\underline{c}_i\} = 4.5885 > \alpha_2 = 3.8171,$$
$$\mathfrak{d} = \max\{\sum\nolimits_{j=1}^{n} \overline{b}_{ji}L_i^g\underline{c}_i\} = 18.0387 > \delta = 9.592,$$
$$\mathfrak{a}_3 = \max\{\sum\nolimits_{j=1}^{n} \overline{w}_{ji}L_i^h\tau\underline{c}_i\} = 2.772 > \alpha_3 = 0.5586.$$

Considered the same $\rho_1 = 29.71$ and $\frac{\theta}{T} = 0.95$, the control gain is $\beta_g = 35.7 > \beta = 32.7$. Under the same initial conditions with any value ranging from -2 and 2, the two error networks are compared by residual analysis. The analysis results are shown in Fig.4. Obviously, the effect of control gain $\beta = 32.7$ is almost the same as control gain $\beta_g = 35.7$. Therefore, under the same control effect, the control gain by a simple linear transformation on the memristive terms is more accurate.

V. CONCLUSION

This paper addressed the issue of exponential synchronization for a class of memristive Cohen-Grossberg neural networks with mixed delays by using a hybrid controller. Firstly, according to the memristive characteristics, the memristive terms were normalized by a simple linear transformation. Then a hybrid controller is designed by the strategies of pinning control and intermittent control. Based on the stability theory of memristive neural networks and the exponential synchronization rule, the new synchronization conditions are established. Finally, to show the effectiveness of the theoretical synchronization conditions, numerical simulations are presented. In the future, we will focus on practical applications of neural networks, such as traffic network control, signal encryption, and image encryption.

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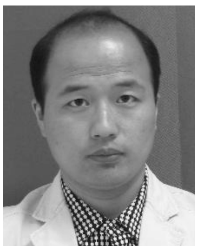

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