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Invariants of Space Line Element Structure Based on Projective Geometric Algebra

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ABSTRACT Based on the theory of Conformal Geometric Algebra, this paper presents a geometric constraint structure consisting of seven straight lines on three adjacent planes and its projective invariants, which can be obtained from a single frame image. Comparing with the use of multi-frame images to calculate invariants, our method is more convenient. First, this paper uses the projective property of points as an index to introduce the projective transformation properties of geometric structure of three lines on two adjacent planes. Then using it as an index, the invariant of the geometric structure of seven lines on three adjacent planes is proposed, and the invariant of the geometric structure of five lines on three adjacent planes is obtained by using the algorithm in this paper. Finally, the accuracy and stability of the invariant geometry are verified by experiments.

INDEX TERMS Geometric invariant, space line element structure, projective geometric algebra.

I. INTRODUCTION

The essence of computer vision to recognize three-dimensional (3D) objects is to retrieve invariant information from images, to reduce the 3D spatial information to two-dimensional(2D) plane, and to find a fast and effective retrieval method. W. Cao *et al*. summarized several current quick retrieval methods [1]. Since object imaging could be affected by various factors such as shooting angle, light, camera parameters, etc., the images obtained by shooting the same object under different shooting conditions will be different, which will cause difficulty in recognition. One of the retrieval methods discovered by scholars is to use geometric invariants.

The earliest invariant theory was originated in the 19th century. Proposed by Cayley, Salmon, Clebsh and other mathematicians, it was introduced into computer vision because of its similarity to human vision, and preliminarily formed a new framework of visual invariant theory, also known as geometric structure invariant theory. D. Hestenes studied the language expression of projective geometry by using the unified mathematical language of Clifford Algebra, and promoted the connection with other mathematics [2]. H. Li studied the mathematical theory and main properties of conformal Geometric Algebra [3]. Based on the theory of Geometric Algebra, R. Wang *et al*. proposed a feature extraction algorithm GA-SURF for multispectral images [4]. With the development of computer vision, the theory of invariant has interpenetrated and integrated with modern science. The historical significance of invariant research has been highlighted and it also attracts more and more researchers' attention. The understanding and application of invariants have also been deepened.

At present, there are two ways to obtain projective invariants of 3D structures from 2D perspective images:

(1) Using the theory of projective geometry, the geometric constraints among multiple projection views are established, or the 3D projective invariants are obtained from multiple images under the condition of known point correspondences. Their goal is to reconstruct 3D images without calibrating the camera [5]. For example, if the corresponding relations of the points in the images are known, three invariants can be calculated from six pairs of corresponding points in three uncalibrated images, or three 3D perspective invariants can be calculated from seven points in two images [6], [7], [8]. The core of these methods is to establish imaging equations by using the corresponding relations between 2D image points and 3D image points, and then to calculate the

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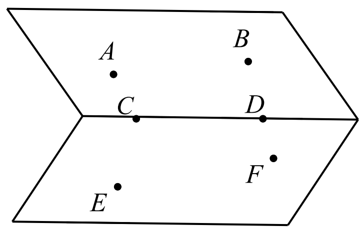

FIGURE 1. Geometric structure of six points in space.

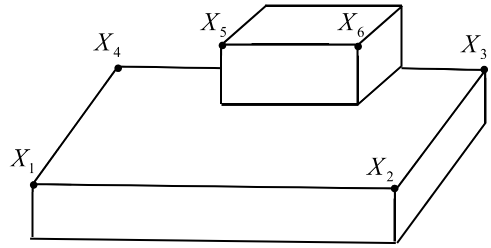

FIGURE 2. Roh's six-point geometry in space.

invariants by eliminating camera parameters. In general, it is a very complex problem to solve this relationship, because the correspondence of points is unknown [9], [10].

(2) Calculating perspective invariants from a single perspective image;

Although it has been proved theoretically that the 3D projective invariants of general geometric structures cannot be obtained from a single frame image without any constraints [11], [12], [13], Roh pointed out that there are three ways [14] to solve this problem: First, use two images and assume that the priori relationship of the epipolar geometry of the two images has been determined [15], [16]. Second, if three images are used, the invariants can be obtained without calculating the epipolar geometry [17]. Third, for some special geometric structures, the three-dimensional projective invariants can still be calculated from a single fame image. For example, Zhu and Earles *et al.* proposed a geometric structure in which they intersected two planes in 3D space, taking two points at the intersection line and two points outside the intersection line on the two planes respectively, to form a geometric structure consisting of six points on two adjacent planes [18] (as shown in Figure 1 below), and calculated its invariants from it. Because there are two kinds of four-point coplanar cases, the structure has only one three-dimensional projective invariant, so the invariant can be calculated from a single frame. On this basis, they generalized the method to a geometric structure composed of seven points in 3D space [19].

Rothwell *et al.* presented two special structural invariants from a single view gray image [20], which can be obtained from a single frame image: one is using the geometric constraints between points and surfaces to calculate the invariant of the point structure on a polyhedron, that is, the projective invariants of seven vertices on three adjacent planes; the other is the invariant of the bilateral symmetric structure, or the eight-point or four-point bilateral linear symmetric structure. Weiss *et al.* obtained invariant relations from a single frame image by using algebraic relations [21]. They pointed out that although the 3D projective invariants of a general geometric structure cannot be calculated from a single frame image, some ratios between independent 3D projective invariants of a geometric structure can be obtained from the single frame image, and these ratios are composed entirely of independent invariants, so they also belong to the category of 3D projective constants of geometric structures. Experiments show that these invariants have high stability and reliability, and can meet the needs of model-based object recognition, target tracking and the like in some applications. Quan studied a geometric structure consisting of six points in a general position in 3D space [22], this geometric structure has three projective invariants, according to the information provided by the three frames of images, these 3D projective invariants can be obtained directly by calculation, avoiding the explicit 3D reconstruction process. This method is simple and fast, reduces the complexity of the algorithm, and has high application value. But in fact, in Quan's method, the final solution is the high-order equation about the invariant, so the solution obtained by this method is not unique, and there is a false solution. At the same time, Quan's method introduces a coordinate transformation process, and additional numerical calculation will lead to increased calculation error [23].On the basis of his method, many scholars had improved the method of calculating 3D projective invariants based on six sets of image point features, and put forward various opinions [24], [25], [26]. Roh *et al.* Proposed a six-point geometric structure [27], which consists of four points in the same plane and two points out of the plane (as shown in Figure 2.) Roh *et al.* got a kind of invariant relation between the 3D object points and the 2D plane points by using this structure, and constructed the model base index by using this invariant relation. However, the invariant relation need to deal with more combination relation than the invariant when retrieving models, so the efficiency of the structure is not very high, but this structure is more universal than the structure proposed by Rothwell *et al.* and Zhu *et al.*

In addition to the point structures mentioned above, Sugimoto and Song *et al.* also proposed the perspective invariants possessed by some line structures in space respectively [28], [29]. Sugimoto derived a perspective invariant from six lines in three planes by using the method of calculating the ratio of determinants (its structure is shown in Figure 3).

On this basis, Song *et al.* further obtained the perspective invariants of five lines in two adjacent planes and six lines in four planes by using a similar method (as shown in Figure 4).

In addition, the study of invariants has also got some attention in China. For example, Z. Chen *et al.* proposed the invariants of five-line geometric structure on three adjacent planes [30], W. Cao *et al.* also obtained the GA-sift invariants by using concurrency geometric algebra theory [31], [32], [33].

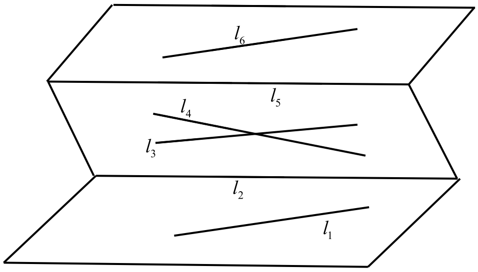

FIGURE 3. Space line structure.

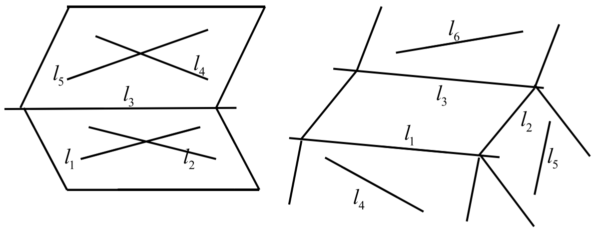

FIGURE 4. Two spatial line structures proposed by Song *et al.*

H. Wei *et al.* studied the recognition method of man-made object with geometric structure line segment feature [34]. L. Ding *et al.* studied the geometric correction of 3D human face with geometric invariance [35]. R. Wang *et al.* proposed the joint projective series invariants (GA-surf, GA-ORB etc.), and summarized the number and type of invariants, [36], [37], [38]. J. Gu *et al.* used a fast registration method with geometric feature space constraint for 3D medical image registration [39]. These invariants are continuously proposed and their applications are being developed, gradually improving the development of invariants.

The advantages of the above methods are that they do not require the corresponding relationship between the 3D features and 2D features of the object, are ingenious in conception, have intuitive geometric invariant structure, are simple and fast in calculation, and can truly describe the 3D structure characteristics of the object when recognizing the object. Their disadvantages are limited in scope of application and lack of universal applicability, so they have no great referential significance for the calculation of geometric invariants of general geometric structures. For this reason, for the general 3D scene, because the shooting angle changes or the illumination changes, with some other possible changes such as the change of camera parameter, the image noise or the background and so on, it will cause great differences in its formation of the image. Although the human eye can still distinguish the object accurately, it is quite difficult for the computer to do so. At the same time, there is no universal invariant of 3D structure, therefore it is very important to find a simple and easy-to-extract invariant. 3D invariants can be calculated directly from the 3D information of the geometric structure of the object, and the relationship between the 3D geometric structure and its 2D projection in the image can be analyzed and described by the projection imaging model. In this way, by analyzing the acquired images, locating the extracted invariant structure and obtaining the expression of object invariant features, and then matching the invariant features with the models in the model library, we can easily complete the target object recognition and image matching. In this paper, based on the theory of geometric algebra, the geometric structure of invariants, feature extraction and calculation of invariants are studied. The main contents are as follows: firstly, in the conformal space, based on the expression and mutual transformation of point, line, surface and other elements in conformal geometric algebra, the projective transformation relations of these geometric elements between 3D objects and 2D images were studied, and the projective invariant relations of 3-line structure of two adjacent planes in the space were found. In this paper, the invariant of geometric structure consisting of two planes and eight points in 3D space was proposed by using the projection transformation between 3D points and 2D points. The algorithm was compared with the invariant of cross ratio of five points in the same plane and the invariant algorithm of six points in space proposed by Zhu *et al.* The simplicity of the algorithm was proved. Then a 3D model was constructed by AutoCAD to verify the correctness of the space eight points projective invariant, at the same time, the reliability and practicality of the invariant was verified by real experiments. On the basis of the projective invariance of the geometric structure of three lines on two adjacent planes in space, the invariant of geometric structure of seven lines on three adjacent planes was proposed, and its correctness was verified by simulation model, and the model was put into various environments in Google Earth to verify its stability and robustness..

II. THREE-LINE PROJECTION RELATION BETWEEN TWO ADJACENT PLANES IN SPACE

In projective geometric algebra, space point and line can express each other, so this paper introduces the projection of space line through the projection of space point. Firstly, this paper introduces the expression of invariant and projective transformation of space point.

In projective geometry, let a point $\tilde{\mathbf{M}}_i^{\Gamma} = (X_i, Y_i, Z_i)^T$ to be the i-th point on a plane Γ in a 3D space, and its projection onto the plane XY is $\tilde{\mathbf{m}}_{i'} = (x, y)^T$. The secondary coordinates of $\tilde{\mathbf{M}}_i^{\Gamma}$ and $\tilde{\mathbf{m}}_{i'}$ are respectively $\tilde{\mathbf{M}}_i^{\Gamma} = (X, Y, Z, 1)^T$ and $\tilde{\mathbf{m}}_{i'} = (x, y, 1)^T$. The relationship between a 3D point and a point on its 2D projection can expressed as

$$\lambda_i \mathbf{m}_{i'} = \mathbf{T}_{\Gamma}\, \mathbf{k}_i^{\Gamma} \tag{1}$$

where, λ is a scale factor, $\mathbf{T}_{\Gamma}$ is a 3×3 matrix. $\mathbf{k}_i^{\Gamma} = (X_i^{\Gamma}, Y_i^{\Gamma}, 1)^T$

Now we use the above relation to deduce the 3-line projection relation of two adjacent planes.

In order to apply the projection of a space point onto a straight line, we first introduce the relationship between a point and a straight line on the plane. We can see that a line **P** on the plane can be represented by any two points on the straight line. If the straight line **P** passes through the points A

and B, which are represented by vectors $\mathbf{m}_A$ and $\mathbf{m}_B$, then $\mathbf{P}$ can be represented by the vector product of these two vectors, that is

$$\omega \mathbf{P} = \mathbf{m}_A \times \mathbf{m}_B \tag{2}$$

where, ω is a constant.

The intersection $\mathbf{m}_{12}$ of two lines $\mathbf{P}_1$ and $\mathbf{P}_2$, it is also possible to represent by their vector product:

$$\beta \mathbf{m}_{12} = \mathbf{P}_1 \times \mathbf{P}_2 \tag{3}$$

where, $\beta$is constant.

The above definition is still valid for points and lines in space. Let $\mathbf{k}_A^\Gamma = (x_A, y_A, z_A)^T_,$ $\mathbf{k}_B^\Gamma = (x_B, y_B, z_B)^T_,$ $\mathbf{T}_\Gamma = \left[\mathbf{t}_1^\Gamma, \mathbf{t}_2^\Gamma, \mathbf{t}_3^\Gamma\right]_,$ substitute (1) into (2).

$$\begin{aligned}
\mathbf{P}' &= \frac{1}{\omega}(\mathbf{m}_A \times \mathbf{m}_B) \\
&= \frac{1}{\omega}\left(\frac{1}{\lambda_A}\mathbf{T}_\Gamma \mathbf{k}_A^\Gamma \times \frac{1}{\lambda_B}\mathbf{T}_\Gamma \mathbf{k}_B^\Gamma\right) \\
&= \frac{1}{\omega\lambda_A\lambda_B}\left(\mathbf{T}_\Gamma \mathbf{k}_A^\Gamma \times \mathbf{T}_\Gamma \mathbf{k}_B^\Gamma\right) \\
&= \frac{1}{\omega\lambda_A\lambda_B}\{\left(x_A\mathbf{t}_1^\Gamma + y_A\mathbf{t}_2^\Gamma + z_A\mathbf{t}_3^\Gamma\right) \\
&\quad \times \left(x_B\mathbf{t}_1^\Gamma + y_B\mathbf{t}_2^\Gamma + z_B\mathbf{t}_3^\Gamma\right)\} \\
&= \frac{1}{\omega\lambda_A\lambda_B}\left(x_Ay_B - x_By_A\right)\left(\mathbf{t}_1^\Gamma \times \mathbf{t}_2^\Gamma\right) \\
&\quad + \left(y_Az_B - y_Bz_A\right)\left(\mathbf{t}_2^\Gamma \times \mathbf{t}_3^\Gamma\right) + \left(z_Ax_B - x_Az_B\right)\left(\mathbf{t}_3^\Gamma \times \mathbf{t}_1^\Gamma\right) \\
&= \frac{1}{\omega\lambda_A\lambda_B}\left[\mathbf{t}_2^\Gamma \times \mathbf{t}_3^\Gamma \;\; \mathbf{t}_3^\Gamma \times \mathbf{t}_1^\Gamma \;\; \mathbf{t}_1^\Gamma \times \mathbf{t}_2^\Gamma\right]\left(\mathbf{k}_A^\Gamma \times \mathbf{k}_B^\Gamma\right) \\
&= \frac{\omega'}{\omega\lambda_A\lambda_B}\left[\mathbf{t}_2^\Gamma \times \mathbf{t}_3^\Gamma \;\; \mathbf{t}_3^\Gamma \times \mathbf{t}_1^\Gamma \;\; \mathbf{t}_1^\Gamma \times \mathbf{t}_2^\Gamma\right]\mathbf{P}' \\
&= \varepsilon^\Gamma \mathbf{V}^\Gamma \mathbf{P}^\Gamma
\end{aligned}$$

Among, $\varepsilon^\Gamma = \frac{\omega'}{\omega\lambda_A\lambda_B}$,

$$\mathbf{V}_\Gamma = \left[\mathbf{t}_2^\Gamma \times \mathbf{t}_3^\Gamma \;\; \mathbf{t}_3^\Gamma \times \mathbf{t}_1^\Gamma \;\; \mathbf{t}_1^\Gamma \times \mathbf{t}_2^\Gamma\right]$$

That is, for a line $\mathbf{P}^\Gamma$ in space Γ whose projective plane straight line $\mathbf{P}'$, then $\mathbf{P}^\Gamma$ and $\mathbf{P}'$ can be represented by a projective matrix $\mathbf{V}_\Gamma$, i.e.

$$\mathbf{P}' = \varepsilon^\Gamma \mathbf{V}_\Gamma \mathbf{P}^\Gamma \tag{4}$$

where $\mathbf{V}_\Gamma = \left[\mathbf{t}_2^\Gamma \times \mathbf{t}_3^\Gamma \mathbf{t}_3^\Gamma \times \mathbf{t}_1^\Gamma \mathbf{t}_1^\Gamma \times \mathbf{t}_2^\Gamma\right]$, $\mathbf{t}_i^\Gamma$is the$i$-th column of the matrix $\mathbf{T}_\Gamma$,and ε^Γ is a constant. It is obvious that the point and plane projective matrix $\mathbf{T}_\Gamma$and $\mathbf{V}_\Gamma$ satisfy the following relation:

$$\mathbf{V}_\Gamma^T \mathbf{T}_\Gamma = |\mathbf{T}_\Gamma|\, \mathbf{I} \tag{5}$$

$\mathbf{I}$ is the 3×3 identity matrix.

As for three straight line on two adjacent planes, any two of which are not parallel and that three straight line do not intersect at a point, as shown in Figure 5, it consists of two sets of coplanar straight lines $\mathbf{P}_1^\Gamma$, $\mathbf{P}_2^\Gamma$ and $\mathbf{P}_2^\Omega$, $\mathbf{P}_3^\Omega$ ($\mathrm{P}_2^\Gamma = \mathrm{P}_2^\Omega$), let $\mathbf{P}'_\mathrm{i}$ denote the i-th straight line on the j plane, and the corresponding plane projective straight line is $\mathbf{P}'_\mathrm{i}$, $\mathbf{H}_{123}$ represents the determinant consisting of $\mathbf{P}_1^\Gamma$, $\mathbf{P}_2^\Gamma$, $\mathbf{P}_3^\Omega$ and $\mathbf{H}'_{123}$ represents the determinant consisting of $\mathbf{P}'_1$, $\mathbf{P}'_2$, $\mathbf{P}'_3$, ie

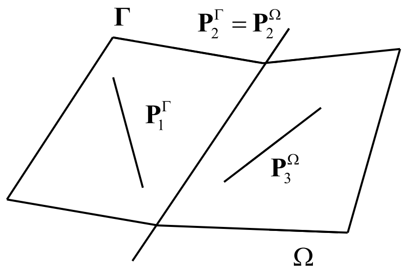

FIGURE 5. 3-line geometric structure of two adjacent planes.

$$\begin{aligned}
\mathbf{H}_{123} &= \left|\mathbf{P}_1^\Gamma \quad \mathbf{P}_2^\Gamma \quad \mathbf{P}_3^\Omega\right| \\
\mathbf{H}'_{123} &= \left|\mathbf{P}'_1 \quad \mathbf{P}'_2 \quad \mathbf{P}'_3\right|
\end{aligned}$$

substitute (1),(3),(4) and (5)into $\mathbf{H}'_{123}$:

$$\begin{aligned}
\mathbf{H}'_{123} &= \left|\mathbf{P}'_1\mathbf{P}'_2\mathbf{P}'_3\right| \\
&= \mathbf{P}'^T_1\left(\mathbf{P}'_2 \times \mathbf{P}'_3\right) \\
&= \varepsilon_1^\Gamma(\mathbf{V}_\Gamma \mathbf{P}_1^\Gamma)^T\left(\mathbf{P}'_2 \times \mathbf{P}'_3\right) \\
&= \varepsilon_1^\Gamma \beta'_{23}(\mathbf{V}_\Gamma \mathbf{P}_1^\Gamma)^T \mathbf{m}_{23} \\
&= \frac{\varepsilon_1^\Gamma \beta'_{23}}{\lambda_{23}}\left(\mathbf{P}_1^\Gamma\right)^T \mathbf{V}_\Gamma^T \mathbf{T}_\Gamma \mathbf{k}_{23}^\Omega \\
&= \frac{\varepsilon_1^\Gamma \beta'_{23}}{\lambda_{23}\beta_{23}}|\mathbf{T}_\Gamma|\left(\mathbf{P}_1^\Gamma\right)^T\left(\mathbf{P}_2^\Omega \times \mathbf{P}_3^\Omega\right) \\
&= \frac{\varepsilon_1^\Gamma \beta'_{23}}{\lambda_{23}\beta_{23}}|\mathbf{T}_\Gamma|\left(\mathbf{P}_1^\Gamma\right)^T\left(\mathbf{P}_2^\Gamma \times \mathbf{P}_3^\Omega\right) \\
&= \frac{\varepsilon_1^\Gamma \beta'_{23}}{\lambda_{23}\beta_{23}}|\mathbf{T}_\Gamma|\, \mathbf{H}_{123}
\end{aligned}$$

where $\mathbf{m}_{23}$ is the intersection of $\mathbf{P}'_2$and $\mathbf{P}'_3$, $\mathbf{m}_{23}$ it is derived from the formula (3).

That is the projection relationship of the three lines:

$$\mathbf{H}'_{123} = \frac{\varepsilon_1^\Gamma \beta'_{23}}{\lambda_{23}\beta_{23}}|\mathbf{T}_\Gamma|\, \mathbf{H}_{123} \tag{6}$$

III. INVARIANTS OF GEOMETRIC STRUCTURES OF THREE ADJACENT PLANES AND SEVEN LINES

A. CALCULATION OF GEOMETRIC STRUCTURE INVARIANTS OF SEVEN LINES IN SPACE

Through the above discussion on the projective property of three straight lines on two adjacent planes, this section proposes a new invariant of geometric structure, that is, the invariant of 3-adjacent plane 7-line structure. The geometric structure is shown in Figure 6. In 3D space, three adjacent planes Ω_1, Ω_2 and Ω_3 intersect each other, Ω_1 and Ω_2 intersect L_1, Ω_1 and Ω_3 intersect L_2, Ω_2 and Ω_3 intersect L_3. In addition, the straight lines L_4 and L_5 lie on the plane

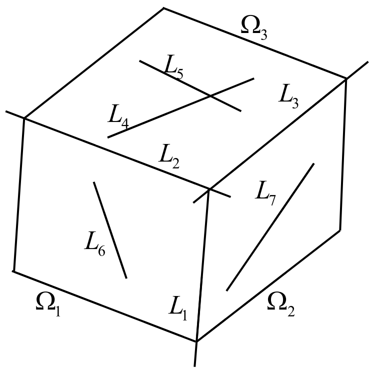

FIGURE 6. Geometric structure of three adjacent planes and seven lines.

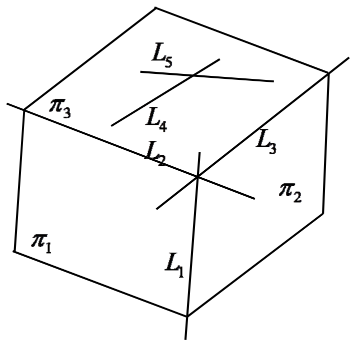

FIGURE 7. Geometric structure diagram of 3 adjacent planes and 5 lines.

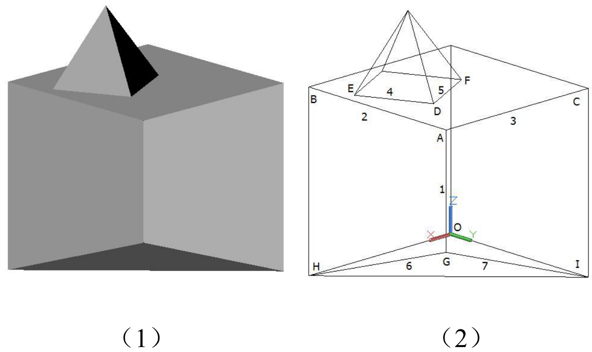

FIGURE 8. 3D model and its wireframe structure.

TABLE 1. True values of 3D invariants calculated from the 3D model.

No.	Coordinates (X, Y, Z)	No.	Coordinates (X, Y, Z)
A	(100,100,100)	F	(35.97,44.96,100)
B	(100,0,100)	G	(100,100,35.15)
C	(0,100,100)	H	(100,0,0)
D	(74.04,64.02,100)	I	(0,100,0)
E	(93.11,25.96,100)		
3D invariance Quantity (I'_{3D})		-0.5911	

Ω_3, the straight line L_6 lies on the plane Ω_1, the straight lineL_7 lies on the plane Ω_2.

At this time, using the conclusion of the previous section, the 7-line structure in Figure 6 can be viewed in such a way that Ω_1 and Ω_2 are regarded as two adjacent planes, and then (L_4, L_2, L_6), (L_4, L_2, L_1), (L_5, L_2, L_6), (L_5, L_2, L_1) constitute four sets of two-adjacent planar 3-line structures, and similarly for the planes Ω_2 and Ω_3, (L_4, L_3, L_7) , (L_4, L_3, L_1), (L_5, L_3, L_7) constitute four groups of two adjacent plane 3-line structure. In eight straight line groups (L_4, L_2, L_6) , (L_4, L_2, L_1) , (L_4, L_3, L_7) , (L_4, L_3, L_1) , (L_5, L_2, L_6) , (L_5, L_2, L_1) (L_5, L_3, L_7), (L_5, L_3, L_1) any two straight lines are not parallel. And, any of the three straight lines in any of the groups do not intersect at a point. Let $L_i (i = 1, 2, 3 \ldots, 7)$ corresponds to the projective straight line L'_i.

Using the conclusion of (6) in the previous section, for two adjacent planar 3-line structures, it can be deduced that the geometric relationship satisfied by the eight straight line groups (L_4, L_2, L_6) , (L_4, L_2, L_1) , (L_4, L_3, L_7) , (L_4, L_3, L_1) , (L_5, L_2, L_6) , (L_5, L_2, L_1) , (L_5, L_3, L_7) , (L_5, L_3, L_1) is as follows:

$$\mathbf{H}'_{426} = \frac{\varepsilon_4^{\Omega_3}\beta'_{26}}{\lambda_{26}\beta_{26}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{426}$$

$$\mathbf{H}'_{421} = \frac{\varepsilon_4^{\Omega_3}\beta'_{21}}{\lambda_{21}\beta_{21}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{421}$$

$$\mathbf{H}'_{437} = \frac{\varepsilon_4^{\Omega_3}\beta'_{37}}{\lambda_{37}\beta_{37}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{437}$$

$$\mathbf{H}'_{431} = \frac{\varepsilon_4^{\Omega_3}\beta'_{31}}{\lambda_{31}\beta_{31}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{431}$$

$$\mathbf{H}'_{526} = \frac{\varepsilon_5^{\Omega_3}\beta'_{26}}{\lambda_{26}\beta_{26}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{526}$$

$$\mathbf{H}'_{521} = \frac{\varepsilon_5^{\Omega_3}\beta'_{21}}{\lambda_{21}\beta_{21}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{521}$$

$$\mathbf{H}'_{537} = \frac{\varepsilon_5^{\Omega_3}\beta'_{37}}{\lambda_{37}\beta_{37}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{537}$$

$$\mathbf{H}'_{531} = \frac{\varepsilon_5^{\Omega_3}\beta'_{31}}{\lambda_{31}\beta_{31}} \left|\mathbf{T}_{\Omega_3}\right| \mathbf{H}_{531}$$

Among, $\mathbf{H}_{ijk} = \left|\mathbf{L}_i \quad \mathbf{L}_j \quad \mathbf{L}_k\right|$, $\mathbf{H}'_{ijk} = \left|\mathbf{L}'_i \quad \mathbf{L}'_j \quad \mathbf{L}'_k\right|$.

Thus, it is possible to obtain the invariants of the 3-adjacent plane 7-line structure as

$$I_{line} = \frac{\mathbf{H}'_{426}\mathbf{H}'_{437}\mathbf{H}'_{521}\mathbf{H}'_{531}}{\mathbf{H}'_{421}\mathbf{H}'_{431}\mathbf{H}'_{526}\mathbf{H}'_{537}} = \frac{\mathbf{H}_{426}\mathbf{H}_{437}\mathbf{H}_{521}\mathbf{H}_{531}}{\mathbf{H}_{421}\mathbf{H}_{431}\mathbf{H}_{526}\mathbf{H}_{537}} \tag{7}$$

TABLE 2. Seven straight lines represented by the vector product of 2D homogeneous coordinate points in Figures (1) ~ (12).

Image NO.	L_1	L_2	L_3	L_4
1	(-161,0,39123)	(125,-189,19521)	(104,229,-85728)	(28,-116,15720)
2	(-151,-1,55101)	(28,-322,82572)	(224,40,-92832)	(-32,-131,36945)
3	(-33,0,6864)	(336,-176,23040)	(174,340,-215712)	(95,-131,25382)
4	(-246,0,43788)	(35,-120,16090)	(11,362,-69290)	(11,-115,17157)
5	(-71,-1,10288)	(128,-25,-11123)	(18,168,-49074)	(44,-42,1662)
6	(-192,0,59136)	(64,-267,44902)	(97,174,-71984)	(6,-134,24320)
7	(-11,101,-19306)	(-178,-96,78656)	(121,-115,-12730)	(-92,-14,27164)
8	(-109,147,-7445)	(-147,-180,71661)	(196,62,-57100)	(-94,-80,33716)
9	(138,21,-45147)	(-205,131,37060)	(-74,-273,72631)	(-64,102,-10858)
10	(138,112,-78958)	(-135,24,27021)	(145,-233,48893)	(-79,54,-3913)
11	(174,-73,-33040)	(-8,264,-69376)	(-196,-166,104736)	(34,131,-54810)
12	(-93,0,16740)	(82,-127,19149)	(79,132,-49464)	(17,-74,13185)

TABLE 2. ***Continued*** Seven straight lines represented by the vector product of 2D homogeneous coordinate points in Figures (1) ~ (12).

Image NO.	L_5	L_6	L_7
1	(64,52,-24960)	(-39,189,-70848)	(17,229,-101456)
2	(90,-46,-13146)	(54,321,-160467)	(142,42,-69842)
3	(130,97,-65274)	(-320,176,-32176)	(157,340,-223396)
4	(11,116,-22344)	(99,121,-69894)	(-123,359,-133194)
5	(30,58,-18360)	(-90,23,4596)	(-21,169,-55872)
6	(49,16,-15681)	(41,266,-128072)	(-8,174,-73052)
7	(12,-62,12924)	(184,38,-86470)	(117,-60,-35037)
8	(45,-10,-5660)	(206,97,-107745)	(136,144,-97120)
9	(-67,-78,40187)	(132,-145,-35425)	(1,-261,11947)
10	(30,-85,26445)	(59,-86,-1893)	(219,-171,-42303)
11	(-76,-13,31294)	(-86,-224,41818)	(-101,-206,43519)
12	(46,27,-13633)	(-32,127,-39960)	(30,132,-52920)

B. COMPARISON WITH OTHER METHODS

For the three-adjacent plane five-line structure proposed by Z. Chen as shown in Figure 7, the invariants can also be calculated using the algorithm in this paper. In Figure 7, $\pi_1$and π_3 are regarded as two adjacent planes, then (L_4, L_2, L_1), (L_5, L_2, L_1) form two sets of two-adjacent plane 3-line structures. Similarly, for the planes π_2 and π_3, (L_4, L_3, L_1), (L_5, L_3, L_1) constitute two sets of two adjacent planes and 3-line structures. Using the conclusion of (6), for the two adjacent planes and 3-line structures, the geometric relationship that 4 straight line groups satisfy can be deduced as follows:

$$\mathbf{H}'_{421} = \frac{\alpha_4^{\pi_3}\gamma'_{21}}{\rho_{21}\gamma_{21}}\left|\mathbf{T}_{\pi_3}\right|\mathbf{H}_{421}$$

$$\mathbf{H}'_{521} = \frac{\alpha_5^{\pi_3}\gamma'_{21}}{\rho_{21}\gamma_{21}}\left|\mathbf{T}_{\pi_3}\right|\mathbf{H}_{521}$$

$$\mathbf{H}'_{431} = \frac{\alpha_4^{\pi_3}\gamma'_{31}}{\rho_{31}\gamma_{31}}\left|\mathbf{T}_{\pi_3}\right|\mathbf{H}_{431}$$

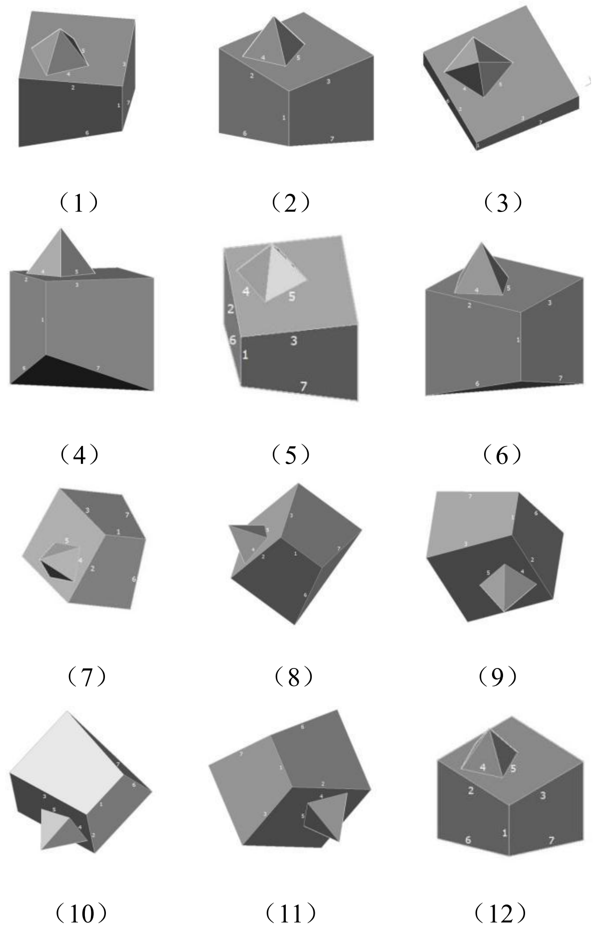

FIGURE 9. 3D views taken from each transformed perspective view.

$$\mathbf{H}'_{531} = \frac{\alpha_5^{\pi_3}\gamma'_{31}}{\rho_{31}\gamma_{31}}\left|\mathbf{T}_{\pi_3}\right|\mathbf{H}_{531}$$

Among, $\mathbf{H}_{ijk} = \left|\mathbf{L}_i \quad \mathbf{L}_j \quad \mathbf{L}_k\right|$, $\mathbf{H}'_{ijk} = \left|\mathbf{L}'_i \quad \mathbf{L}'_j \quad \mathbf{L}'_k\right|$.

Therefore, the invariant of the 3 adjacent planar 5 line structure proposed by Chen Zhe is obtained as follows:

$$I_c = \frac{\mathbf{H}'_{421}\mathbf{H}'_{531}}{\mathbf{H}'_{431}\mathbf{H}'_{521}} = \frac{\mathbf{H}_{421}\mathbf{H}_{531}}{\mathbf{H}_{431}\mathbf{H}_{521}} \tag{8}$$

Using this method, the invariants of other structures can be calculated conveniently, which shows that the invariant calculation method proposed in this paper has certain generalizability.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. EXPERIMENTAL PLATFORM

Hardware environment: a personal computer (Intel(R) T2400, 1.83GHz processor, 1 GB of memory), a camera (8 million pixels).

Software platform: Windows XP operating system, 3D graphics software AutoCAD, Google Earth, image processing platform software matlab7.5.0 (R2007b).

B. SIMULATION EXPERIMENT

In this section, based on the above derivation, in order to verify the accuracy and robustness of the above obtained invari-

TABLE 3. 2D invariant values calculated from 12 images.

Image NO.	Invariant value (I'_{2D})	Error Rate (%)
1	-0.6088	2.9944
2	-0.5985	1.2519
3	-0.6019	1.8271
4	-0.5658	4.2802
5	-0.5935	0.4060
6	-0.6067	2.6391
7	-0.5855	0.9474
8	-0.5981	1.1842
9	-0.5830	1.3703
10	-0.6129	3.6880
11	-0.5975	1.0827
12	-0.6001	1.5226
Mean value	-0.5960	0.8290
Standard Deviation	0.0129	-

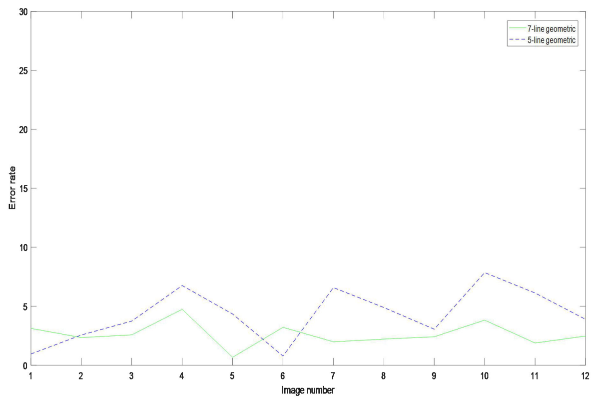

FIGURE 10. 7-line and 5-line geometric structure.

ants, a 3D model was constructed using AutoCAD software, as shown in Figure 8, (1) the 3D model, (2) the 3D wireframe structure of the model extracted from (1). Seven of the straight lines were selected, as shown in the Figure 8(2).Respectively, use the vector product of A and G to represent the straight line 1, the vector product of A and B to represent the straight line 2, the vector product of A and C to express the straight line 3, the vector product of D and E to express the line 4, the vector product of D and F to express the line 5, the vector product of G and H to express the straight line 6, and the vector product of G and I to express the straight line 7.

First, the 3D invariant value of the model was calculated, that is, the true value. If O is selected as the origin, the coordinates of each point obtained from the 3D model are shown in Table 1 below. The 3D projective invariant $I'_{3D}$of the model was calculated as -0.5911 by taking each coordinate into the algorithm proposed in this paper.

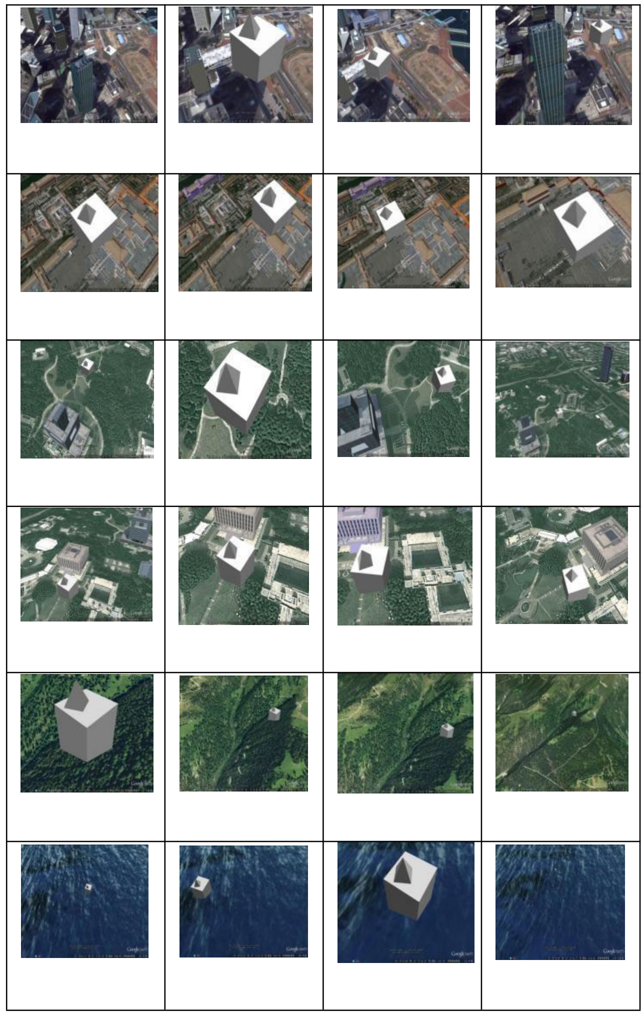

FIGURE 11. 24 images from putting the 3D model into Google Earth.

Secondly, in order to calculate its 2D projective invariants, this paper used the same camera to select 12 frames (1)~(12) obtained by the model under the conditions of rotation, translation, scaling, changing the viewpoint and so on, as shown in Figure 9, and calculated their 2D invariants respectively.

Because the main advantage of Hough transform is that it is less affected by noise and curve discontinuity, we used Hough transform to extract straight line, and then tested the extracted straight line. If it meets the constraint conditions, we extract the coordinates of each edge image and get the straight line (expressed by the outer product of two points), as shown in Table 2. The first column represents the image number, and $L_1 \sim L_7$represent line 1 to line 7. Then use (6) to calculate the projective invariant, and the result is shown in the following table 3. I'_{2D} represents the 2D projective invariant values calculated from (1) to (12). The last column represents the absolute value of the relative error between each 2D invariant and the 3D invariant true value -0.5911 calculated above. The penultimate row represents the mean value of all 2D invariants and the obtained mean value relative to the true value error, and the last row is the standard deviation of the obtained invariant values.

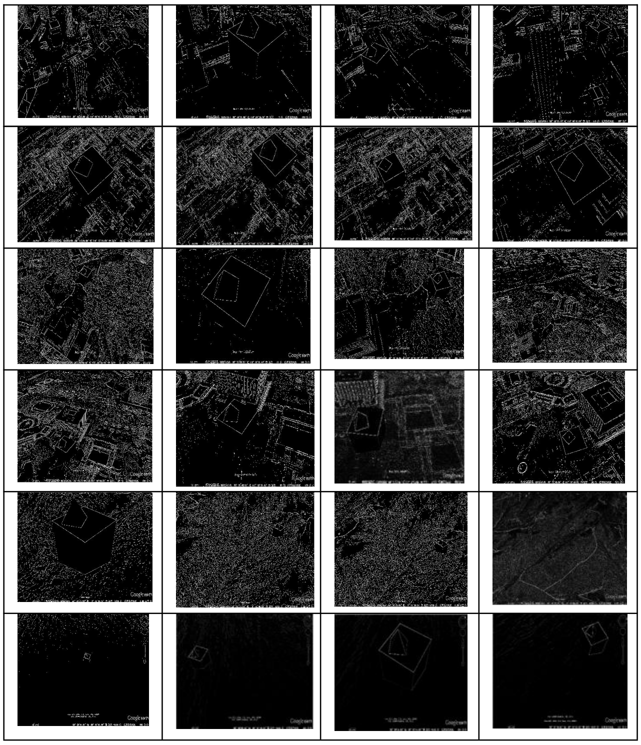

FIGURE 12. Edge extraction image.

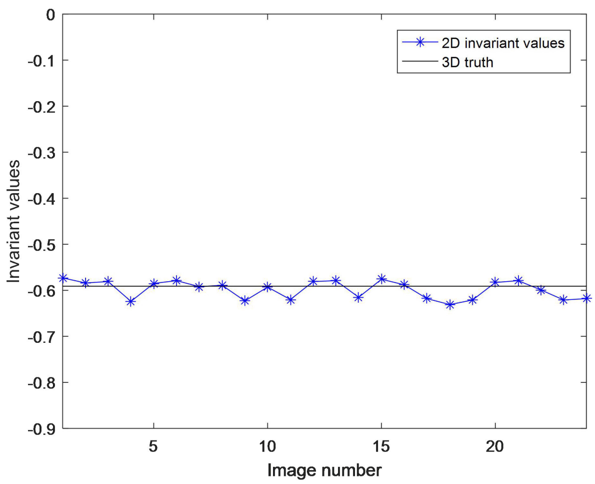

FIGURE 13. Distribution of Invariant Values of Subimages.

Comparing the results of Table 1 and Table 3, we can see that the 3D invariant calculated by the algorithm in this paper is -0.5911, while the average value of 2D invariant is -0.5960, that is, although the viewpoint position of the camera is different and the 3D model is changed, the result of invariant calculation is relatively ideal, and the projective invariant basically remains unchanged in 3D and 2D, with small error.

In section III(B), this paper introduced the invariants of five-line geometric structure of three adjacent planes in space proposed by Z. Chen and other mathematicians, now we will compare their accuracy. The error of the invariants of five-line geometric structure relative to the true value and the error of the invariant of seven-line geometric structure relative to the true value were calculated respectively, as shown in Figure 10. From the figure, we can see that the accuracy of the invariant of seven-line geometric structure proposed by this paper is improved and its stability is relatively good.

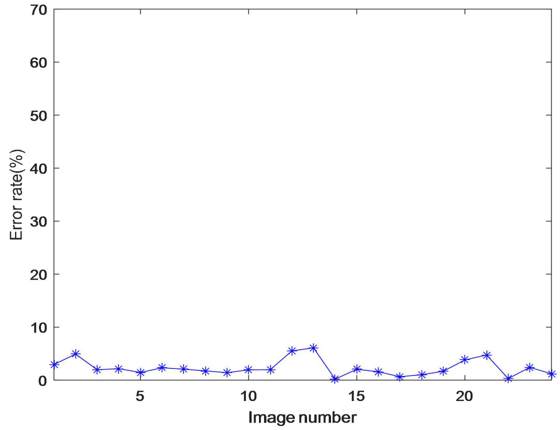

FIGURE 14. Error rate of invariant relative to 3D true value with increasing interference.

C. NOISE-ADDING EXPERIMENT

In order to test the robustness of the invariants of the seven-line geometric structure proposed in this paper, the following 24 images were obtained from different viewpoints by importing the 3D model into different environments in Google Maps through Google Earth, as shown in Figure 11. Figure 12 is the straight line extraction image, and Figure 13 shows the distribution of the invariant values of this geometric structure in the case of interference, with 24 images on the horizontal axis and invariant values on the vertical axis. Figure 14 shows the error rate of the invariant relative to the true value in the case of increased interference.

From Figure 13, we can see that the 2D invariants obtained from the image group fluctuate around -0.5911, which is close to the 3D true value. Figure 14 shows that the error rate of the invariants obtained in this experiment is within 10%, and the accuracy rate is high, which proves that the invariants of the seven-line structure proposed in this paper have good robustness and anti-interference performance.

V. CONCLUSION

In this paper, we extracted the projective invariant of geometric structure composed of seven straight lines on three adjacent planes from a single frame image. The invariant has the property of perspective projection. It has been proved that there is no universal invariant of projection from three to two dimensions, so we deduced a kind of projection invariant from three to two dimensions of constrained structure. Because the extracted line features and calculations are simple, fast, reliable and stable, the value of the invariant is relatively stable. In the application of specific target recognition and tracking, obtaining 3D projective invariants of special geometric structure from a single frame image can effectively improve the effect of advanced computer vision applications such as object recognition and target tracking.

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