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SDP Relaxation Methods for RSS/AOA-Based Localization in Sensor Networks

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ABSTRACT With the fast development of new array technology and intelligent antenna, it is easier to obtain angle of arrival (AOA) measurements. Hybrid received signal strength (RSS) and AOA measurement techniques are proposed for the position computing in sensor networks. By converting the measurement equations and relaxing the optimization function, range-based square semidefinite programming (RLS-SDP) and squared range-based square semidefinite programming (SRLS-SDP) algorithms are put forward to obtain the source position estimate by considering the transmit power to be known or unknown. The proposed RLS-SDP and SRLS-SDP algorithms provide accurate solution to the source position estimate and avoid the initialization process of numerical calculation. The simulations show that the proposed RLS-SDP and SRLS-SDP algorithms perform better than the linear estimator and provide the accuracy performance which is very close to the Cramér-Rao Lower Bound (CRLB) of position estimation. The proposed SRLS-SDP algorithm shows its advantages in the computational complexity compared with the RLS-SDP, since the complexity of SRLS-SDP is independent of the number of anchor nodes.

INDEX TERMS Localization, received signal strength, angle of arrival, semidefinite programming.

I. INTRODUCTION

Sensor network has been playing a key role in many applications, such as surveillance, emergency services, friend finding, and tracking of the elderly [1]–[5]. Position obtaining is an indispensable component of sensor network since the readings from a large number of sensor nodes are meaningful only when the positions of these readings are known. So position computing becomes a crucial problem when all kinds of information resources will be automatically collected from the sensor nodes or terminals [6]–[8]. To obtain the position information, sensor nodes are categorized into anchor node with known position and source node which is required to be localized. A localization scheme tries to localize the source node using the ranging information extracted from the signaling between anchor node and source node [9], [10]. Most of the accurate localization techniques are based on the ranging information by using the techniques such as, time of arrival (TOA) [11], [12], time difference of arrival (TDOA) [13], [14], received signal strength (RSS) [15]–[17], and angle of arrival (AOA) [18], [19].

Among these ranging methods, RSS-based localization scheme is very popular for its easier implementation and less complexity [16], [20], [21]. However, the noise of RSS measurements is large, so the positioning performance is not very well. Hybrid positioning techniques can greatly improve the reliability of the position estimation and reduce the dependence on the number of anchor nodes, so they are becoming a popular positioning method [22], [23]. Due to the limitations of RSS ranging method, some hybrid technologies are also proposed in recent years, such as hybrid RSS and TDOA [24], RSS and TOA positioning methods [25]. Electronic compass or vision sensor provides the possibility of AOA measurements [26], [27], but it requires additional hardware configuration and adds the hardware cost of the node. Recently, AOA measurement becomes easier with the development of new array technology and smart antenna, so it provides a broad space to realize the position obtaining [28], [29] in sensor networks. Some new hybrid AOA positioning methods are also put forward and include the hybrid AOA and

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TDOA [30], AOA and TOA [22], [31], and the hybrid AOA and RSS positioning methods [32], [33].

To locate the source node by using these different ranging methods, some algorithms including maximum likelihood (ML) estimator [34], [35], linear estimator [14], [36], and convex semidefinite programming (SDP) method [37], [38] are proposed. The ML estimator is always solved by the numerical method which requires initial solution to ensure the convergence. When the selected initial solution is far from the actual, it will be trapped in the local optimum [39]. To overcome the shortcoming of the ML estimator, the linear estimator and convex SDP algorithms are proposed to obtain the position estimate of the source node. The linear estimator provides a closed-form solution for the source position estimate, but the constraint conditions are difficult to be exploited. So the solution of linear estimator is often suboptimal. By relaxing the non-convex optimization model into convex problem, the convex SDP method also provides robust solution to the position estimation problem [40]–[42]. However, the computational complexity of SDP is higher than that of linear estimator.

The received RSS is determined by the transmit power which depends on the height and orientation of the node antenna, antenna gain, and its battery [43] and will be subject to a large fluctuation. In many cases, the RSS-based localization problem always assumes the transmit power to be unknown. When the transmit powers are unavailable and assumed to be unknown, the RSS-based scheme is designed to estimate the positions of the source nodes in [44]. The convex optimization algorithms are proposed to estimate the position parameters by considering the transmit powers to be known or unknown [21]. In [45], the linear least square approach is designed to determine the positions of the source nodes, when path loss model parameters are unknown.

Recently, some researches focus on the localization by using the hybrid RSS and AOA measurements [32], [46]. Compared with the single RSS or AOA method, the hybrid RSS and AOA measurements provide more ranging information which leads to more accurate position estimate. In non-cooperative or cooperative approach, the convex relaxation algorithms are proposed for hybrid RSS and AOA localization [33]. However, the correlation between RSS and AOA measurements is not exploited in [33]. The linear estimator [47] is also proposed for the hybrid RSS and AOA source position estimate problem. However, the proposed linear estimator performs not very well, since the constraint condition is not considered in the optimization model.

In this paper, range-based least square SDP (RLS-SDP) and squared range-based least square SDP (SRLS-SDP) algorithms are proposed for the hybrid RSS and AOA localization problem. Then the RLS-SDP and SRLS-SDP algorithms are also extended to the situation of unknown transmit power. By relaxing the non-convex optimization problem into the convex optimization, the proposed RLS-SDP and SRLS-SDP algorithms provide a solution for the source position estimate and avoid the initialization of the numerical calculation. The main contributions of this paper are listed as follows,

1) By exploiting the correlation between RSS and AOA measurements, weighted least square (WLS) solution is proposed to obtain the source position estimate. Then two robust SDP algorithms (i.e., RLS-SDP and SRLS-SDP) are put forward to estimate the source position by availing of the RLS or SRLS-based cost function.
2) When the transmit power is unavailable and assumed to be an unknown parameter, the RLS-SDP and SRLS-SDP algorithms are redesigned by relaxing the optimization problem into convex form. The RLS-SDP and SRLS-SDP algorithms are extended to the situation of unknown transmit power.
3) The computational complexity of these proposed algorithms are compared by using the variables, equality constraints, and SDP cones produced in the process of convex relaxation.

The rest of this paper is structured as follows. Section II presents the problem specification of hybrid RSS and AOA localization. Section III in detail describes the proposed RLS-SDP and SRLS-SDP algorithms by assuming transmit power to be known. In Section IV, the RLS-SDP and SRLS-SDP algorithms are extended to the situation of unknown transmit power. Section V derives the computational complexity of these proposed algorithms. Section VI analyzes the simulation results. The conclusion is presented in Section VII. This paper contains a number of symbols. Following the convention, we represent the matrices as bold case letters. If the matrix is denoted by $(*)$, $(*)^{-1}$ and $(*)^T$ represent the matrix inverse and transpose operator, respectively. $\| * \|$ denotes ℓ_2 norm. $\mathbf{A}_{i,j}$ denotes the element at the ith row and jth column of matrix $\mathbf{A}$. $\text{diag}\{\mathbf{a}_i\}$ constructs a diagonal matrix with principal diagonal element $\mathbf{a}_i$. For arbitrary symmetric matrix $\mathbf{A}$, $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is positive semidefinite.

II. PROBLEM SPECIFICATION

In a three-dimensional space, N anchor nodes are deployed with known positions which are denoted as $\mathbf{a}_i = [a_{i,x} \quad a_{i,y} \quad a_{i,z}]^T$, $i = 1, 2, \ldots, N$. In the same region, a source node is required to be located. The position of the source node is denoted as $\mathbf{x} = [x_x \quad x_y \quad x_z]^T$. To derive the position of the source node, the signals transmitted by the source node reach the anchor nodes, then the RSS of the signals is received and measured by the anchor nodes. Assuming that the RSS obeys the logarithmic decay model, the received RSS in anchor node i is denoted as p_i, which is given by [13], [15]

$$p_i = p_0 - 10\beta \log_{10} d_i + n_{i,p} \tag{1}$$

where $i = 1, 2, \ldots, N$, β is called as path loss exponent (PLE) and generally varied from 2 to 5. p_0 is called as transmit power and related with the antenna gain and energy supply of the source node. d_i is the range of the source node respect to

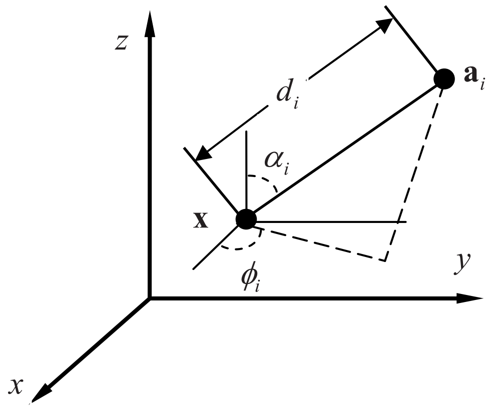


FIGURE 1. AOA measurements between anchor node and source node.

the ith anchor node. $n_{i,p}$ represents the noise which conforms to the Gaussian distribution with zero mean and variance $\delta_{i,p}^2$.

In the three-dimensional space, the unknown source position parameter includes three direction of x, y and z. It is possible to be unreliable for the position obtaining only by using the RSS measurements. To ensure the reliability of the position estimation and improve the positioning performance, the direction angle and elevation angle are also measured and shown in Fig. 1. The direction angle and elevation angle are denoted as ϕ_i and α_i, respectively. By using the geographical position relationship between the nodes, the direction angle ϕ_i and elevation angle α_i can be written as

$$\phi_i = \arctan(\frac{a_{i,y} - x_y}{a_{i,x} - x_x}) + n_{i,\phi} \tag{2}$$

$$\alpha_i = \arccos(\frac{a_{i,z} - x_z}{d_i}) + n_{i,\alpha} \tag{3}$$

where $n_{i,\phi}$ and $n_{i,\alpha}$ are noise of the direction and elevation measurements, respectively. Without loss of generality, it is assumed that the noise $n_{i,\phi}$ and $n_{i,\alpha}$ are gaussian with zero mean and variance $\delta_{i,\phi}^2$ and $\delta_{i,\alpha}^2$, respectively.

To derive the unknown position of the source node, the well known weighted least square (WLS) solution to hybrid RSS and AOA localization is given by

$$\min_{\mathbf{x}} \quad \mathbf{e}^T \Sigma^{-1} \mathbf{e} \tag{4}$$

where $\Sigma = E(\mathbf{n}\mathbf{n}^T)$, $\mathbf{n} = [\mathbf{n}_p^T \quad \mathbf{n}_\phi^T \quad \mathbf{n}_\alpha^T]^T$, $\mathbf{n}_p = [n_{1,p} \quad n_{2,p} \quad \ldots \quad n_{N,p}]^T$, $\mathbf{n}_\phi = [n_{1,\phi} \quad n_{2,\phi} \quad \ldots \quad n_{N,\phi}]^T$, $\mathbf{n}_\alpha = [n_{1,\alpha} \quad n_{2,\alpha} \quad \ldots \quad n_{N,\alpha}]^T$, $\mathbf{e} = [\mathbf{e}_p^T \quad \mathbf{e}_\phi^T \quad \mathbf{e}_\alpha^T]^T$, $\mathbf{e}_p = [e_{1,p} \quad e_{2,p} \quad \ldots \quad e_{N,p}]^T$, $\mathbf{e}_\phi = [e_{1,\phi} \quad e_{2,\phi} \quad \ldots \quad e_{N,\phi}]^T$, $\mathbf{e}_\alpha = [e_{1,\alpha} \quad e_{2,\alpha} \quad \ldots \quad e_{N,\alpha}]^T$, $i = 1, 2, \ldots, N$, $e_{i,p}$, $e_{i,\phi}$, and $e_{i,\alpha}$ denote the error of the RSS, direction and elevation measurements, respectively. $e_{i,p}$, $e_{i,\phi}$, and $e_{i,\alpha}$ are written as

$$\begin{cases} e_{i,p} = p_i - p_0 + 10\beta \log_{10} d_i \\ e_{i,\phi} = \phi_i - \arctan(\dfrac{a_{i,y} - x_y}{a_{i,x} - x_x}) \\ e_{i,\alpha} = \alpha_i - \arccos(\dfrac{a_{i,z} - x_z}{d_i}) \end{cases} \tag{5}$$

where $i = 1, 2, \ldots, N$, it is obviously noted that $d_i = \|\mathbf{x} - \mathbf{a}_i\|$. Problem (4) is a nonlinear and non-convex optimization model, so it always be solved by numerical calculation method which requires an initial point. When the initial point is not enough close to the actual solution, the estimate will be trapped in the local optimum. To overcome the shortcoming of the numerical calculation method and fasten the iterative calculation, the non-convex model of (4) is relaxed into the convex optimization problem when the transmit power p_0 is assumed to be known in section III and unknown in section IV.

III. KNOWN TRANSMIT POWER

In the section, the source position $\mathbf{x}$ is estimated by using the hybrid RSS and AOA measurements when the transmit power p_0 is assumed to be available. It is possible to convert the WLS formulation to a convex SDP optimization problem, to provide an approximate solution that can be obtained in a globally optimum fashion with reduced computational efforts. To obtain the convex SDP optimization form, the RSS, direction and elevation angle measurement equations are approximately linearized by considering the small noise level. In the following, we in detail describe the proposed convex RLS-SDP and SRLS-SDP algorithms for the hybrid RSS and AOA localization.

A. RLS-SDP ALGORITHM

Firstly, (1) is rewritten as

$$d_i = 10^{\frac{p_0 - p_i + n_{i,p}}{10\beta}} \tag{6}$$

where $i = 1, 2, \ldots, N$. Expanding right side of (6) with Taylor series and neglecting the high order terms at small noise level, we can obtain that

$$d_i \approx \lambda_i + \frac{\lambda_i \ln 10}{10\beta} n_{i,p} \tag{7}$$

where $\lambda_i = 10^{\frac{p_0 - p_i}{10\beta}}$, $i = 1, 2, \ldots, N$. (7) is obtained by linearizing (1) and considered as an equivalent RSS measurement equation. A new unknown vector is defined by $\mathbf{u} = [\mathbf{x}^T \quad \mathbf{d}^T \quad 1]^T \in \mathbb{R}^{N+4}$ and $\mathbf{d} = [d_1 \quad d_2 \quad \ldots \quad d_N]^T$. Then stacking the N equivalent RSS measurement expressions of (7), we can obtain the linear matrix form

$$\mathbf{C}_1 \mathbf{u} = \varepsilon_1 \tag{8}$$

where $\mathbf{C}_1 = [\mathbf{0}_{N\times 3} \quad \mathbf{1}_N^i \quad -\lambda] \in \mathbb{R}^{N\times(N+4)}$, $\mathbf{1}_N^i$ is an $N \times N$ square matrix with 1 at the ith row and the ith column element and 0's elsewhere, $\lambda = [\lambda_1 \quad \lambda_2 \quad \ldots \quad \lambda_N]^T$, $\varepsilon_1 = [\varepsilon_{1,p} \quad \varepsilon_{2,p} \quad \ldots \quad \varepsilon_{N,p}]^T$, $\varepsilon_{i,p} = \frac{\lambda_i \ln 10}{10\beta} n_{i,p}$, $i = 1, 2, \ldots, N$.

By moving $n_{i,\phi}$ to the left side and doing the tangent operation, (2) is also rewritten as

$$\tan(\phi_i - n_{i,\phi}) = \frac{a_{i,y} - x_y}{a_{i,x} - x_x} \tag{9}$$

Expanding both sides of (9) and neglecting the high order terms, we can obtain that

$$-\sin\phi_i x_x + \cos\phi_i x_y + b_{i,\phi} \approx \lambda_i \sin\alpha_i n_{i,\phi} \tag{10}$$

where $(a_{ix}-x_x)\cos\phi_i+(a_{iy}-x_y)\sin\phi_i \approx \lambda_i \sin\alpha_i$, $b_{i,\phi}=\sin\phi_i a_{i,x}-\cos\phi_i a_{i,y}$, $i=1,2,\ldots,N$. (10) is derived from the direction expression of (2) and considered as an equivalent direction measurement equation. Then by stacking the equivalent direction measurement equation of (10), the linear matrix form is written as

$$\mathbf{C}_2\mathbf{u}=\varepsilon_2 \tag{11}$$

where $\mathbf{C}_2 \in \mathbb{R}^{N\times(N+4)}$, and

$$\mathbf{C}_2=\begin{bmatrix} -\sin\phi_1 & \cos\phi_1 & 0 & \ldots & 0 & b_{1,\phi}\\ -\sin\phi_2 & \cos\phi_2 & 0 & \ldots & 0 & b_{2,\phi}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ -\sin\phi_N & \cos\phi_N & \underbrace{0}_{} & \underbrace{\ldots}_{N+1} & \underbrace{0}_{} & b_{N,\phi}\end{bmatrix} \tag{12}$$

$\varepsilon_2=[\varepsilon_{1,\phi}\quad \varepsilon_{2,\phi}\quad \ldots\quad \varepsilon_{N,\phi}]^T$, $\varepsilon_{i,\phi}=\lambda_i\sin\alpha_i n_{i,\phi}$, $i=1,2,\ldots,N$.

Similarly, by transforming the elevation measurement equation of (3), it yields

$$d_i\cos(\alpha_i-n_{i,\alpha})=a_{i,z}-x_z \tag{13}$$

Substituting (7) into (13) and expanding the left side of (13), we can obtain that

$$-x_z+b_{i,\alpha}\approx\lambda_i\sin\alpha_i n_{i,\alpha}+\frac{\lambda_i\cos\alpha_i \ln 10}{10\beta}n_{i,p} \tag{14}$$

where $b_{i,\alpha}=a_{iz}-\lambda_i\cos\alpha_i$, $i=1,2,\ldots,N$. (14) is obtained with the elevation expression of (3) and considered as an equivalent elevation measurement equation. Similarly, the linear matrix form of (14) is expressed as

$$\mathbf{C}_3\mathbf{u}=\varepsilon_3 \tag{15}$$

where $\mathbf{C}_3\in\mathbb{R}^{N\times(N+4)}$, and

$$\mathbf{C}_3=\begin{bmatrix} 0 & 0 & -1 & 0 & \ldots & 0 & b_{1,\alpha}\\ 0 & 0 & -1 & 0 & \ldots & 0 & b_{2,\alpha}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & -1 & \underbrace{0}_{} & \underbrace{\ldots}_{N} & \underbrace{0}_{} & b_{N,\alpha}\end{bmatrix} \tag{16}$$

$\varepsilon_3=[\varepsilon_{1,\alpha}\quad\varepsilon_{2,\alpha}\quad\ldots\quad\varepsilon_{N,\alpha}]^T$, $\varepsilon_{i,\alpha}=\lambda_i\sin\alpha_i n_{i,\alpha}+\frac{\lambda_i\cos\alpha_i\ln 10}{10\beta}n_{i,p}$, $i=1,2,\ldots,N$. The equations (8), (11), and (15) provide the equivalent RSS, direction, and elevation measurement expressions, so the WLS solution to the hybrid RSS and AOA localization can be formulated as

$$\begin{aligned}&\min_{\mathbf{u}}\ (\mathbf{Cu})^T\mathbf{P}^{-1}(\mathbf{Cu})\\ &\text{s.t. } d_i=\|\mathbf{x}-\mathbf{a}_i\|=\|\mathbf{A}_i\mathbf{u}\|\quad i=1,2,\ldots,N\end{aligned} \tag{17}$$

where $\mathbf{C}=[\mathbf{C}_1^T\quad\mathbf{C}_2^T\quad\mathbf{C}_3^T]^T$, $\mathbf{A}_i=[\mathbf{I}_3\quad \mathbf{0}_{3\times N}\quad -\mathbf{a}_i]\in\mathbb{R}^{3\times(N+4)}$, $\mathbf{P}=E(\varepsilon\varepsilon^T)$, $\varepsilon=[\varepsilon_1^T\quad\varepsilon_2^T\quad\varepsilon_3^T]^T$, $\mathbf{P}$ is further given by

$$\mathbf{P}=\begin{bmatrix}E(\varepsilon_1\varepsilon_1^T) & E(\varepsilon_1\varepsilon_2^T) & E(\varepsilon_1\varepsilon_3^T)\\ E(\varepsilon_2\varepsilon_1^T) & E(\varepsilon_2\varepsilon_2^T) & E(\varepsilon_2\varepsilon_3^T)\\ E(\varepsilon_3\varepsilon_1^T) & E(\varepsilon_3\varepsilon_2^T) & E(\varepsilon_3\varepsilon_3^T)\end{bmatrix} \tag{18}$$

ε_2 is independent of ε_1 and ε_3, so $E(\varepsilon_1\varepsilon_2^T)$, $E(\varepsilon_2\varepsilon_1^T)$, $E(\varepsilon_2\varepsilon_3^T)$, and $E(\varepsilon_3\varepsilon_2^T)$ are all equal to $\mathbf{0}_{N\times N}$. The others in $\mathbf{P}$ are

$$\begin{cases}E(\varepsilon_1\varepsilon_1^T)=\text{diag}\{\frac{\lambda_i^2\ln^2 10}{100\beta^2}\delta_{i,p}^2\}\\ E(\varepsilon_2\varepsilon_2^T)=\text{diag}\{\lambda_i^2\sin^2\alpha_i\delta_{i,\phi}^2\}\\ E(\varepsilon_3\varepsilon_3^T)=\text{diag}\{\lambda_i^2\sin^2\alpha_i\delta_{i,\alpha}^2+\frac{\lambda_i^2\cos^2\alpha_i\ln^2 10}{100\beta^2}\delta_{i,p}^2\}\\ E(\varepsilon_1\varepsilon_3^T)=E(\varepsilon_3\varepsilon_1^T)=\text{diag}\{\frac{\lambda_i^2\cos\alpha_i\ln^2 10}{100\beta^2}\delta_{i,p}^2\}\end{cases} \tag{19}$$

where $i=1,2\ldots,N$. To relax the optimization problem (17) into a convex model, we define $\mathbf{U}=\mathbf{uu}^T$. Then problem (17) can be rewritten as

$$\begin{aligned}&\min_{\mathbf{U}}\ \text{Tr}(\mathbf{DU})\\ &\text{s.t. } \text{Tr}(\mathbf{B}_i\mathbf{U})=\mathbf{U}_{3+i,3+i}\quad i=1,2,\ldots,N && (20a)\\ &\quad\ \mathbf{U}\succeq\mathbf{0}_{N+4},\ \mathbf{U}_{N+4,N+4}=1 && (20b)\\ &\quad\ \text{rank}(\mathbf{U})=1 && (20c)\end{aligned}$$

where $\mathbf{D}=\mathbf{C}^T\mathbf{P}^{-1}\mathbf{C}$, $\mathbf{B}_i=\mathbf{A}_i^T\mathbf{A}_i$. Dropping the rank 1 constraint of (20c), we can obtain a range-based least square SDP (RLS-SDP) form

$$\begin{aligned}&\min_{\mathbf{U}}\ \text{Tr}(\mathbf{DU})\\ &\text{s.t. (20a), (20b)}\end{aligned} \tag{21}$$

It is fortunately found that matrix $\mathbf{U}$ must be rank 1 when the SDP solution is an optimal solution of the original problem (21). In Appendix, we demonstrate that matrix $\mathbf{U}$ must be rank 1 even if dropping the constraint of (20c). The RLS-SDP optimization problem of (21) can be solved with well known algorithms such as interior point methods which are self initialized and require no initialization from the user. Extracting from defined vector $\mathbf{U}$, we can obtain the source position estimate $\mathbf{x}=\mathbf{U}_{1:3,N+4}$.

B. SRLS-SDP ALGORITHM

By squaring both sides of (7) and neglecting the second order terms, it yields

$$d_i^2\approx\lambda_i^2+\frac{\lambda_i^2\ln 10}{5\beta}n_{i,p} \tag{22}$$

Since $d_i^2=\|\mathbf{x}-\mathbf{a}_i\|^2$, (22) can be further rewritten as

$$-2\mathbf{a}_i^T\mathbf{x}+\mathbf{x}^T\mathbf{x}+\mathbf{a}_i^T\mathbf{a}_i-\lambda_i^2\approx\frac{\lambda_i^2\ln 10}{5\beta}n_{i,p} \tag{23}$$

To derive the source position, a new unknown vector $\mathbf{v}$ is defined by $\mathbf{v}=[\mathbf{x}^T\quad \mathbf{x}^T\mathbf{x}\quad 1]^T\in\mathbb{R}^5$. Then by stacking the expressions of (22), the linear matrix form is given by

$$\mathbf{G}_1\mathbf{v}=\gamma_1 \tag{24}$$

where $\mathbf{G}_1 \in \mathbb{R}^{N\times 5}$, and

$$\mathbf{G}_1 = \begin{bmatrix} -2\mathbf{a}_1^T & 1 & \mathbf{a}_1^T\mathbf{a}_1 - \lambda_1^2 \\ -2\mathbf{a}_2^T & 1 & \mathbf{a}_2^T\mathbf{a}_2 - \lambda_2^2 \\ \vdots & \vdots & \vdots \\ -2\mathbf{a}_N^T & 1 & \mathbf{a}_N^T\mathbf{a}_N - \lambda_N^2 \end{bmatrix} \quad (25)$$

$\gamma_1 = [\gamma_{1,p} \quad \gamma_{2,p} \quad \dots \quad \gamma_{N,p}]^T$, $\gamma_{i,p} = \frac{\lambda_i^2 \ln 10}{5\beta} n_{i,p}$, $i = 1, 2, \dots, N$.

When the unknown vector $\mathbf{v}$ is defined, the linear matrix forms of direction and elevation measurement equations (10) and (14) are given by

$$\begin{cases} \mathbf{G}_2\mathbf{v} = \gamma_2 \\ \mathbf{G}_3\mathbf{v} = \gamma_3 \end{cases} \quad (26)$$

where $\mathbf{G}_2 \in \mathbb{R}^{N\times 5}$, $\mathbf{G}_3 \in \mathbb{R}^{N\times 5}$, and

$$\mathbf{G}_2 = \begin{bmatrix} -\sin\phi_1 & \cos\phi_1 & 0 & 0 & b_{1,\phi} \\ -\sin\phi_2 & \cos\phi_2 & 0 & 0 & b_{2,\phi} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\sin\phi_N & \cos\phi_N & 0 & 0 & b_{N,\phi} \end{bmatrix} \quad (27)$$

and

$$\mathbf{G}_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & b_{1,\alpha} \\ 0 & 0 & -1 & 0 & b_{2,\alpha} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -1 & 0 & b_{N,\alpha} \end{bmatrix} \quad (28)$$

$\gamma_2 = \varepsilon_2$, $\gamma_3 = \varepsilon_3$.

By using the linear expression forms of (24) and (26), a constrained WLS optimization problem is formulated as

$$\min_{\mathbf{v}} \ (\mathbf{G}\mathbf{v})^T\mathbf{Q}^{-1}(\mathbf{G}\mathbf{v})$$
$$\text{s.t. } \mathbf{v}_4 = \|\mathbf{v}_{1:3}\|^2 = \|\mathbf{E}\mathbf{v}\| \quad (29)$$

where $\mathbf{G} = [\mathbf{G}_1^T \quad \mathbf{G}_2^T \quad \mathbf{G}_3^T]^T$, $\mathbf{E} = [\mathbf{I}_3 \quad \mathbf{0}_{3\times 2}]$, $\mathbf{Q} = E(\gamma\gamma^T)$, $\gamma = [\gamma_1^T \quad \gamma_2^T \quad \gamma_3^T]^T$, $\mathbf{Q}$ is further given by

$$\mathbf{Q} = \begin{bmatrix} E(\gamma_1\gamma_1^T) & E(\gamma_1\gamma_2^T) & E(\gamma_1\gamma_3^T) \\ E(\gamma_2\gamma_1^T) & E(\gamma_2\gamma_2^T) & E(\gamma_2\gamma_3^T) \\ E(\gamma_3\gamma_1^T) & E(\gamma_3\gamma_2^T) & E(\gamma_3\gamma_3^T) \end{bmatrix} \quad (30)$$

where $E(\gamma_1\gamma_2^T)$, $E(\gamma_2\gamma_1^T)$, $E(\gamma_2\gamma_3^T)$, and $E(\gamma_3\gamma_2^T)$ are all equal to $\mathbf{0}_{N\times N}$. The others in $\mathbf{Q}$ are

$$\begin{cases} E(\gamma_1\gamma_1^T) = \text{diag}\{\frac{\lambda_i^4 \ln^2 10}{25\beta^2}\delta_{i,p}^2\} \\ E(\gamma_2\gamma_2^T) = \text{diag}\{\lambda_i^2 \sin^2\alpha_i \delta_{i,\phi}^2\} \\ E(\gamma_3\gamma_3^T) = \text{diag}\{\lambda_i^2 \sin^2\alpha_i \delta_{i,\alpha}^2 + \frac{\lambda_i^2\cos^2\alpha_i \ln^2 10}{100\beta^2}\delta_{i,p}^2\} \\ E(\gamma_1\gamma_3^T) = E(\gamma_3\gamma_1^T) = \text{diag}\{\frac{\lambda_i^3 \cos\alpha_i \ln^2 10}{50\beta^2}\delta_{i,p}^2\} \end{cases} \quad (31)$$

where $i = 1, 2 \dots, N$. To relax the optimization problem (29) into the convex model, we define $\mathbf{V} = \mathbf{v}\mathbf{v}^T$. Then problem (29) can be rewritten as

$$\min_{\mathbf{V}} \ \text{Tr}(\mathbf{HV})$$
$$\text{s.t. } \text{Tr}(\mathbf{FV}) = \mathbf{V}_{4,5} \quad (32a)$$
$$\mathbf{V} \succeq \mathbf{0}_5, \mathbf{V}_{5,5} = 1 \quad (32b)$$
$$\text{rank}(\mathbf{V}) = 1 \quad (32c)$$

where $\mathbf{H} = \mathbf{G}^T\mathbf{Q}^{-1}\mathbf{G}$, $\mathbf{F} = \mathbf{E}^T\mathbf{E}$.

Dropping the rank 1 constraint in (32), we can also obtain a convex squared range-based least square SDP (SRLS-SDP) form

$$\min_{\mathbf{U}} \ \text{Tr}(\mathbf{GV})$$
$$\text{s.t. (32a), (32b)} \quad (33)$$

Similar to (21), we can found that matrix $\mathbf{V}$ must be rank 1 when the SDP solution to (33) is an optimal solution for $\mathbf{F}_{i,i} > 0$ $(i = 1, 2, \dots, 5)$. So the SDP solution to (33) is also an optimal solution to the original problem (32) even if dropping the rank 1 constraint. Extracting from defined vector $\mathbf{V}$, we can also obtain the source position estimate $\mathbf{x} = \mathbf{V}_{1:3,5}$.

IV. UNKNOWN TRANSMIT POWER

According to the description of RSS measurement in (1), the received RSS value p_i is determined by the transmit power p_0. However, the transmit power is unavailable in many cases, since it depends on the battery and antenna gain of transmitting node. In addition, the transmit power might change with time, e.g., when the battery would begin to exhaust. Consequently, if each transmitting node has to report its transmit power to anchor nodes constantly during RSS measurements, it also requires additional communication overhead in both anchor nodes and source nodes and makes the network more convoluted. In this section, the transmit powers are considered as nuisance parameters and assumed to be unknown, then the transmit power of the source node is estimated jointly with the source position.

A. RLS-SDP ALGORITHM

When the transmit power of source node is assumed to be unknown, the convex SDP optimization follows the same procedure as described previously for the known transmit power case but with a slightly different expression. When the transmit power is considered as to be unknown, we define a new RSS-related parameter μ_i and a new variable ρ_0, which are given by

$$\begin{cases} \mu_i = 10^{\frac{-p_i}{10\beta}} \\ \rho_0 = 10^{\frac{p_0}{10\beta}} \end{cases} \quad (34)$$

So (7) can be rewritten as

$$d_i = \mu_i\rho_0 + \frac{\lambda_i \ln 10}{10\beta} n_{i,p} \quad (35)$$

where $\lambda_i = \mu_i \rho_0$, $i = 1, 2, \ldots, N$. By defining a new unknown vector $\widetilde{\mathbf{u}} = [\mathbf{x}^T \quad \mathbf{d}^T \quad \rho_0 \quad 1]^T \in \mathbb{R}^{N+5}$ and $\mathbf{d} = [d_1 \quad d_2 \quad \ldots \quad d_N]^T$, the linear matrix form of (35) is rewritten as

$$\widetilde{\mathbf{C}}_1 \widetilde{\mathbf{u}} = \varepsilon_1 \tag{36}$$

where $\widetilde{\mathbf{C}}_1 = [\mathbf{0}_{N\times 3} \quad \mathbf{1}_N^i \quad -\mu \quad 0]$, $\mu = [\mu_1 \quad \mu_2 \quad \ldots \quad \mu_N]^T$, ε_1 is same with that in (8).

The direction expression is unrelated with the transmit power, so we do not need rewrite the equivalent direction equations. Using the equivalent direction measurement equation of (10), we can obtain with

$$\widetilde{\mathbf{C}}_2 \widetilde{\mathbf{u}} = \varepsilon_2 \tag{37}$$

where ε_2 has been defined in (11), $\widetilde{\mathbf{C}}_2 \in \mathbb{R}^{N\times(N+5)}$, and

$$\widetilde{\mathbf{C}}_2 = \begin{bmatrix} -\sin\phi_1 & \cos\phi_1 & 0 & \ldots & 0 & b_{1,\phi} \\ -\sin\phi_2 & \cos\phi_2 & 0 & \ldots & 0 & b_{2,\phi} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\sin\phi_N & \cos\phi_N & \underbrace{0}_{} & \underbrace{\ldots}_{N+2} & \underbrace{0}_{} & b_{N,\phi} \end{bmatrix} \tag{38}$$

By assuming the transmit power to be unknown, the equivalent elevation measurement equation of (14) is modified as

$$-x_z - \mu_i \cos\alpha_i \rho_0 + a_{i,z} \approx \lambda_i \sin\alpha_i n_{i,\alpha} + \frac{\lambda_i \cos\alpha_i \ln 10}{10\beta} n_{i,p} \tag{39}$$

So the linear matrix form of (39) is given by

$$\widetilde{\mathbf{C}}_3 \widetilde{\mathbf{u}} = \varepsilon_3 \tag{40}$$

where ε_3 is same with that in (15), $\widetilde{\mathbf{C}}_3 \in \mathbb{R}^{N\times(N+5)}$, and

$$\widetilde{\mathbf{C}}_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & \ldots & 0 & -\mu_1\cos\alpha_1 & a_{1,z} \\ 0 & 0 & -1 & 0 & \ldots & 0 & -\mu_2\cos\alpha_2 & a_{2,z} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -1 & \underbrace{0}_{} & \underbrace{\ldots}_{N} & \underbrace{0}_{} & -\mu_N\cos\alpha_N & a_{N,z} \end{bmatrix} \tag{41}$$

So by using the matrix expressions of (36), (37), and (40), the WLS solution can be written as

$$\begin{aligned} &\min_{\widetilde{\mathbf{u}}} \ (\widetilde{\mathbf{C}}\widetilde{\mathbf{u}})^T \mathbf{P}^{-1} (\widetilde{\mathbf{C}}\widetilde{\mathbf{u}}) \\ &\text{s.t. } d_i = \|\mathbf{x} - \mathbf{a}_i\| = \|\widetilde{\mathbf{A}}_i \widetilde{\mathbf{u}}\| \quad i = 1, 2, \ldots, N \end{aligned} \tag{42}$$

where $\widetilde{\mathbf{C}} = [\widetilde{\mathbf{C}}_1^T \quad \widetilde{\mathbf{C}}_2^T \quad \widetilde{\mathbf{C}}_3^T]^T$, $\widetilde{\mathbf{A}}_i = [\mathbf{I}_3 \quad \mathbf{0}_{3\times(N+1)} \quad -\mathbf{a}_i]$, $\mathbf{P} = E(\varepsilon\varepsilon^T)$ is same with the definition in (18).

To obtain the convex SDP form, a new matrix is defined by $\widetilde{\mathbf{U}} = \widetilde{\mathbf{u}}\widetilde{\mathbf{u}}^T$. So problem (42) can be rewritten as

$$\begin{aligned} &\min_{\mathbf{U}} \ \mathrm{Tr}(\widetilde{\mathbf{D}}\widetilde{\mathbf{U}}) \\ &\text{s.t. } \mathrm{Tr}(\widetilde{\mathbf{B}}_i \widetilde{\mathbf{U}}) = \widetilde{\mathbf{U}}_{3+i,3+i} \quad i = 1, 2, \ldots, N && \text{(43a)} \\ &\quad \widetilde{\mathbf{U}} \succeq \mathbf{0}_{N+5}, \widetilde{\mathbf{U}}_{N+5,N+5} = 1 && \text{(43b)} \\ &\quad \mathrm{rank}(\widetilde{\mathbf{U}}) = 1 && \text{(43c)} \end{aligned}$$

where $\widetilde{\mathbf{D}} = \widetilde{\mathbf{C}}^T \mathbf{P}^{-1} \widetilde{\mathbf{C}}$, $\widetilde{\mathbf{B}}_i = \widetilde{\mathbf{A}}_i^T \widetilde{\mathbf{A}}_i$. Dropping the rank 1 constraint of (43c), we can also obtain a squared range-based least square SDP (SRLS-SDP) form

$$\begin{aligned} &\min_{\widetilde{\mathbf{U}}} \ \mathrm{Tr}(\widetilde{\mathbf{D}}\widetilde{\mathbf{U}}) \\ &\text{s.t. (43a), (43b)} \end{aligned} \tag{44}$$

Similar to (21), $\widetilde{\mathbf{U}}$ must be rank 1 when the solution to (44) is optimal for $\widetilde{\mathbf{D}}_{i,i} > 0$ $(i = 1, 2, \ldots, N+5)$.

By extracting from defined vector $\widetilde{\mathbf{U}}$, the source position is estimated by $\mathbf{x} = \widetilde{\mathbf{U}}_{1:3,N+5}$ along with $\rho_0 = \widetilde{\mathbf{U}}_{N+4,N+5}$ which is further used to derive the transmit power.

B. SRLS-SDP ALGORITHM

When the transmit power p_0 is unknown, the expression of (22) is rewritten as

$$-2\mathbf{a}_i^T \mathbf{x} + \mathbf{x}^T \mathbf{x} - \mu_i^2 \rho_0^2 + \mathbf{a}_i^T \mathbf{a}_i \approx \frac{\lambda_i \ln 10}{5\beta} n_{i,p} \tag{45}$$

Then a new unknown vector is defined by $\widetilde{\mathbf{v}} = [\mathbf{x}^T \quad \mathbf{x}^T\mathbf{x} \quad \rho_0 \quad \rho_0^2 \quad 1]^T \in \mathbb{R}^7$, so the linear matrix form of (45) is expressed by

$$\widetilde{\mathbf{G}}_1 \widetilde{\mathbf{v}} = \gamma_1 \tag{46}$$

where γ_1 is same with that in (24), $\widetilde{\mathbf{G}}_1 \in \mathbb{R}^{N\times 7}$, and

$$\widetilde{\mathbf{G}}_1 = \begin{bmatrix} -2\mathbf{a}_1^T & 1 & 0 & -\mu_1^2 & \mathbf{a}_1^T\mathbf{a}_1 \\ -2\mathbf{a}_2^T & 1 & 0 & -\mu_2^2 & \mathbf{a}_2^T\mathbf{a}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -2\mathbf{a}_N^T & 1 & 0 & -\mu_N^2 & \mathbf{a}_N^T\mathbf{a}_N \end{bmatrix} \tag{47}$$

When the new unknown vector $\widetilde{\mathbf{v}}$ is defined, the linear matrix forms of direction and elevation expressions are also written as

$$\begin{cases} \widetilde{\mathbf{G}}_2 \widetilde{\mathbf{v}} = \gamma_2 \\ \widetilde{\mathbf{G}}_3 \widetilde{\mathbf{v}} = \gamma_3 \end{cases} \tag{48}$$

where γ_2 and γ_3 have been defined in (26), $\widetilde{\mathbf{G}}_2 \in \mathbb{R}^{N\times 7}$, $\widetilde{\mathbf{G}}_3 \in \mathbb{R}^{N\times 7}$, $\widetilde{\mathbf{G}}_2$ and $\widetilde{\mathbf{G}}_3$ are defined by

$$\widetilde{\mathbf{G}}_2 = \begin{bmatrix} -\sin\phi_1 & \cos\phi_1 & 0 & \ldots & 0 & b_{1,\phi} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\sin\phi_N & \cos\phi_N & \underbrace{0}_{} & \underbrace{\ldots}_{4} & \underbrace{0}_{} & b_{N,\phi} \end{bmatrix} \tag{49}$$

$$\widetilde{\mathbf{G}}_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & -\mu_1\cos\alpha_1 & 0 & a_{1,z} \\ 0 & 0 & -1 & 0 & -\mu_2\cos\alpha_2 & 0 & a_{2,z} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & -1 & 0 & -\mu_N\cos\alpha_N & 0 & a_{N,z} \end{bmatrix} \tag{50}$$

Using the linear forms of (46) and (48), we obtain the WLS optimization problem

$$\begin{aligned} &\min_{\widetilde{\mathbf{v}}} \ (\widetilde{\mathbf{G}}\widetilde{\mathbf{v}})^T \mathbf{Q}^{-1} (\widetilde{\mathbf{G}}\widetilde{\mathbf{v}}) \\ &\text{s.t. } \widetilde{\mathbf{v}}_4 = \|\widetilde{\mathbf{v}}_{1:3}\|^2 = \|\widetilde{\mathbf{E}}\widetilde{\mathbf{v}}\| \\ &\quad \widetilde{\mathbf{v}}_5^2 = \widetilde{\mathbf{v}}_6 \end{aligned} \tag{51}$$

where $\widetilde{\mathbf{G}} = [\widetilde{\mathbf{G}}_1^T \quad \widetilde{\mathbf{G}}_2^T \quad \widetilde{\mathbf{G}}_3^T]^T$, $\widetilde{\mathbf{E}} = [\mathbf{I}_3 \quad \mathbf{0}_{3\times 4}]$, $\mathbf{Q} = E(\gamma\gamma^T)$ has been defined in (30).

A new unknown matrix is defined by $\widetilde{\mathbf{V}} = \widetilde{\mathbf{v}}\widetilde{\mathbf{v}}^T$, then problem (51) is rewritten as

$$\min_{\widetilde{\mathbf{V}}} \quad \mathrm{Tr}(\widetilde{\mathbf{H}}\widetilde{\mathbf{V}})$$
$$\text{s.t. } \mathrm{Tr}(\widetilde{\mathbf{F}}\widetilde{\mathbf{V}}) = \widetilde{\mathbf{V}}_{4,7} \tag{52a}$$
$$\widetilde{\mathbf{V}}_{5,5} = \widetilde{\mathbf{V}}_{6,7} \tag{52b}$$
$$\widetilde{\mathbf{V}} \succeq \mathbf{0}_7, \widetilde{\mathbf{V}}_{7,7} = 1 \tag{52c}$$
$$\mathrm{rank}(\widetilde{\mathbf{V}}) = 1 \tag{52d}$$

where $\widetilde{\mathbf{H}} = \widetilde{\mathbf{G}}^T\mathbf{Q}^{-1}\widetilde{\mathbf{G}}$, $\widetilde{\mathbf{F}} = \widetilde{\mathbf{E}}^T\widetilde{\mathbf{E}}$. Similarly, by dropping the rank 1 constraint, an SDP optimization problem of (52) is expressed by

$$\min_{\mathbf{V}} \ \mathrm{Tr}(\widetilde{\mathbf{H}}\widetilde{\mathbf{V}})$$
$$\text{s.t. (52a)-(52c)} \tag{53}$$

It can also be demonstrated that $\widetilde{\mathbf{V}}$ must be rank 1 when the solution to (53) is optimal for $\widetilde{\mathbf{H}}_{i,i} > 0$ $(i = 1, 2, \ldots, 7)$. The detailed proof process of rank 1 is not listed here due to too many similarities. Extracting from the estimated $\widetilde{\mathbf{V}}$, we can obtain the source position estimate $\mathbf{x} = \widetilde{\mathbf{V}}_{1:3,7}$ and the transmit power.

The weight $\mathbf{P}$ and $\mathbf{Q}$ are determined by λ_i. However, λ_i is determined by the transmit power and not available in the beginning. Preliminarily considering λ_i to be identical we obtain the initial estimate λ_i. Then putting the initial estimate into the optimization problem would produce better solution for the source position estimate along with the transmit power.

V. COMPLEXITY ANALYSIS

In the section, the computation complexity of the proposed convex SDP algorithms is analyzed. Firstly, when the transmit power is considered to be known, three unknown parameters are required to be estimated in 3-dimensional scenario. Only if the number of measurements is larger than the unknown parameters, the unknown parameters can be uniquely determined. Since three measurements including RSS, direction, and elevation are provided for each anchor node, the source position can be estimated theoretically only by using one anchor node. When the transmit power is considered as an unknown parameter and estimated along with source position, there are totally four unknown parameters. So at least two anchor nodes are required to obtain the unique solutions to the source position and transmit power.

A convex optimization problem can be solved by iterative optimization techniques, e.g., interior-point methods. As is known that the worst-case complexity of solving the SDP algorithm is $O\big((m^2\sum_{i=1}^{N_{sdp}}(n_i^{sdp})^2 + m\sum_{i=1}^{N_{sdp}}(n_i^{sdp})^3 + m^3)\sqrt{\psi}\log(1/\epsilon)\big)$, where N_{sdp} is the number of SDP cone constraints, n_i^{sdp} is the corresponding dimension of the ith SDP cone, m is the number of equality constraints in the convex optimization model, ϵ is the accuracy of convex optimization solution. ψ is called as barrier parameter and measures the geometric complexities of the cones involved. If only existing the SDP constraints, ψ is given by

$$\psi = \sum_{i=1}^{N_{sdp}} n_i^{sdp} \tag{54}$$

TABLE 1. Parameters in computing the computational complexity.

algorithms	variables	m	N_{sdp}	n_i^{sdp}
RLS-SDP-KTP	$(N+4)^2$	$N+1$	1	$N+4$
SRLS-SDP-KTP	25	2	1	5
RLS-SDP-UTP	$(N+5)^2$	$N+1$	1	$N+5$
SRLS-SDP-UTP	49	3	1	7

TABLE 2. Positions of four anchors (m).

x	87.6	92.9	99.2	9.1
y	31.7	49.6	39.5	38.2
z	96.0	49.2	94.7	73.1

The parameters in the computational complexity are listed in Tab. 1 where KTP or UTP represents the situation of known or unknown transmit power, respectively. It can be seen from Tab. 1 that the variables in RLS-SDP-KTP and RLS-SDP-UTP are quadratic $N + 4$ and $N + 5$, respectively, since the range parameter d_i is integrated into the unknown vector. However, the variables in SRLS-based SDP algorithms are irrelevant with N, the number of anchor nodes, so the computational complexity of the proposed SRLS-SDP-KTP and SRLS-SDP-UTP algorithms is also independent of the number of anchor nodes. Although the complexity of cost function in the SDP optimization model of (33) or (53) is slightly higher with the increasing of the anchor nodes, it does not affect the computational complexity of the proposed SRLS-SDP-KTP and SRLS-SDP-UTP. So the proposed SRLS-based SDP algorithms show their advantages in the computational complexity when compared with the RLS-SDP.

VI. EVALUATION

To evaluate the performance of the proposed convex optimization algorithms, the simulations are conducted by using the Matlab toolbox CVX, where the solver is SeDuMi. In a 3-dimensional space, four anchor nodes are randomly placed at the positions listed in Tab. 2. The position of the source node is set at (31.2, 52.4, 80.2) m. The noise powers of RSS, direction and elevation are set to δ_p^2, δ_ϕ^2, and δ_α^2, respectively. The true transmit power p_0 is randomly drawn from the range $[-40, -50]$ dB. Unless specifically mentioned, the PLE β is always set to 4. The accuracy performance of estimated parameter is evaluated with root mean square error (RMSE) which is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{M_c}\sum_{i=1}^{M_c} \| \mathbf{x}_i - \mathbf{x}^o \|^2} \tag{55}$$

where M_c is called as the Monte Carlo times, $\mathbf{x}_i$ and $\mathbf{x}^o$ denotes the estimate and the true position of the source node in the ith Monte Carlo run, respectively. In our simulation, we use the average of 1000 Monte Carlo runs to evaluate the accuracy performance of the proposed algorithms.

A. KNOWN TRANSMIT POWER

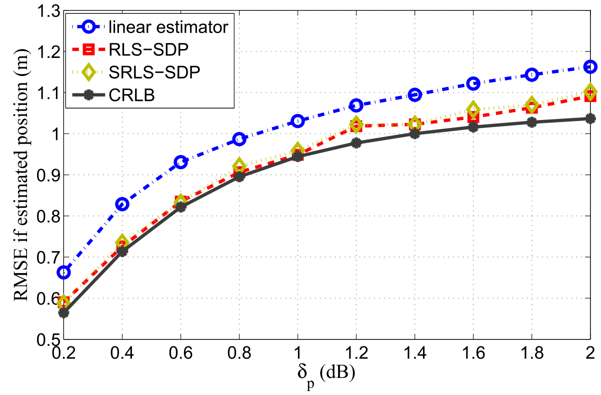
Firstly, when the transmit power is assumed to be known, the RMSE performance of different algorithms is compared with increasing of the RSS measurement noise. Both the direction noise δ_ϕ and elevation noise δ_α are set to 1.0 degree. When the RSS noise δ_p is varied from 0.2 dB to 2.0 dB, Fig. 2(a) plots the RMSE performance of the linear estimator proposed in [47], our proposed RLS-SDP, SRLS-SDP algorithms and its CRLB. It can be seen that the RMSE performance of all algorithms degrades as the RSS noise increases. When the RSS noise δ_p is set to 2.0 dB, the RMSEs of the RLS-SDP and SRLS-SDP algorithms are 1.10 m and 1.09 m, respectively. However, the proposed linear estimator proposed in [47] achieve 1.16 m. Compared with the linear estimator, our proposed RLS-SDP and SRLS-SDP algorithms perform better due to the exploitation of the constraint condition. The performance of our proposed RLS-SDP and SRLS-SDP is very close to the CRLB especially when the RSS noise is less than 1.0 dB.

Similarly, the RSS noise δ_p and the elevation noise δ_α are set to 1.0 dB and 1.0 degree, respectively. When the direction noise δ_ϕ is increased from 0.5 degree to 5.0 degree, Fig. 2(b) illustrates the RMSE performance of three different algorithms. As can be seen that the RMSE performance of three algorithms become worse as the direction noise increases. For instance, when the direction noise is varied from 0.5 degree to 5.0 degree, the RMSE of SRLS-SDP algorithm is increased from 0.64 m to 2.05 m. The RMSE of the RLS-SDP or SRLS-SDP algorithm is always less than that of the linear estimator when the elevation noise is increased from 0.5 degree to 5.0 degree. However, the bias between the linear estimator and RLS-SDP or SRLS-SDP is largen as the direction noise increases. So our proposed RLS-SDP or SRLS-SDP performs better especially at larger direction noise.

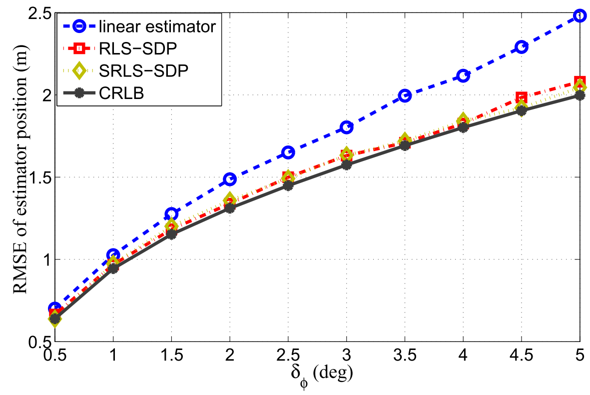
The RSS noise δ_p and direction noise δ_α are set to 1.0 dB and 1.0 degree, respectively. When the elevation noise δ_α is increased from 0.5 degree to 5.0 degree, Fig. 2(c) plots the RMSE performance of three different algorithms. When the elevation noise δ_α is set to 0.5 degree, the RMSEs of RLS-SDP, SRLS-SDP, and linear estimator are 0.77 m, 0.78 m, and 0.82 m, respectively. However, the RMSE of RLS-SDP, SRLS-SDP, and linear estimator reach to 2.13 m, 2.06 m, and 2.23 m, respectively, when the elevation noise δ_α is increased to 5.0 degree. The linear estimator provides worst performance among three algorithms, since it does not avail of the constraint conditions.

B. UNKNOWN TRANSMIT POWER

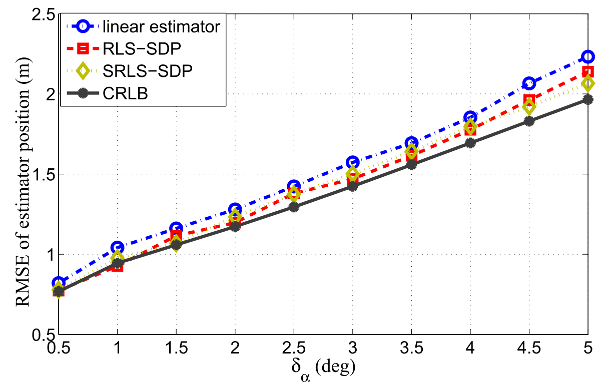
When the transmit power is assumed to be unknown, the transmit power is estimated along with the position of the source node. Similarly, both the direction noise δ_ϕ and elevation noise δ_α are kept at 1.0 degree. Fig. 3(a) plots the RMSE of the estimated source position with the linear estimator, RLS-SDP, and SRLS-SDP algorithms, when the RSS noise is also increased from 0.2 dB to 2.0 dB. As can be seen that the RMSE performance of three proposed algorithms also becomes worse as the RSS noise increases. For instance, the RMSE of the SRLS-SDP is 0.65 m when the RSS noise is set to 0.2 dB. However, when the RSS noise is increased to 2.0 dB, the RMSE of the SRLS-SDP is also increased to



(a) RMSE of estimated position with different RSS noise, $\delta_\phi^2 = 1^2$, $\delta_\alpha^2 = 1^2$.

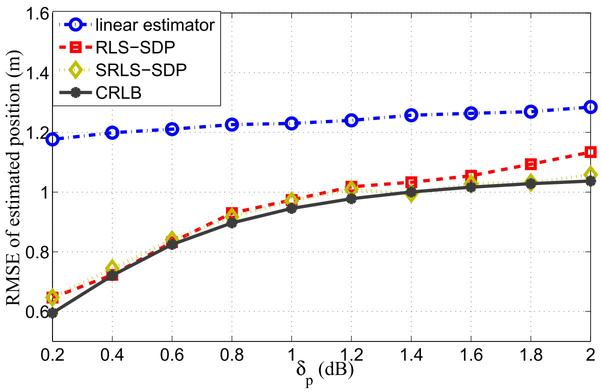


(b) RMSE of estimated position with different direction noise, $\delta_p^2 = 1^2$, $\delta_\alpha^2 = 1^2$.

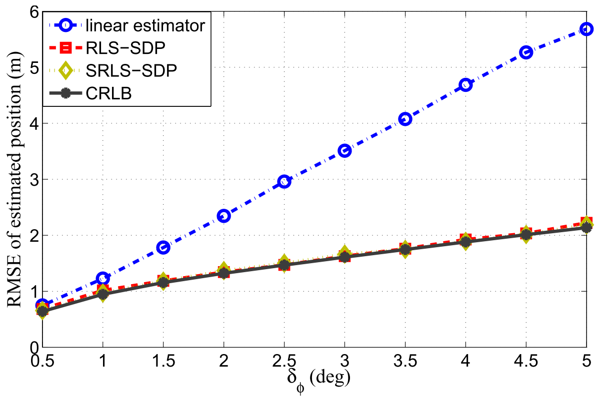


(c) RMSE of estimated position with different elevation noise, $\delta_p^2 = 1^2$, $\delta_\phi^2 = 1^2$.

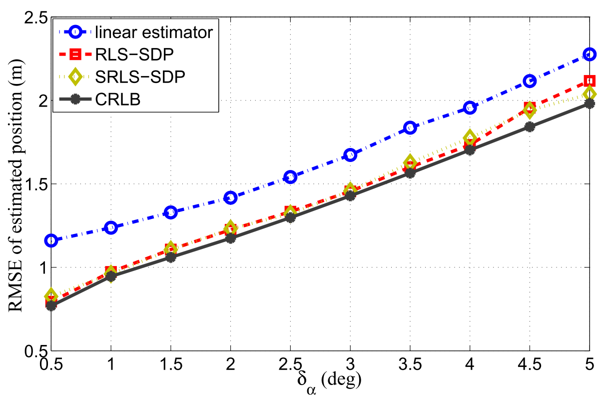
FIGURE 2. Performance comparison under known transmit power.



(a) RMSE of estimated position with different RSS noise, $\delta_\phi^2 = 1^2$, $\delta_\alpha^2 = 1^2$.

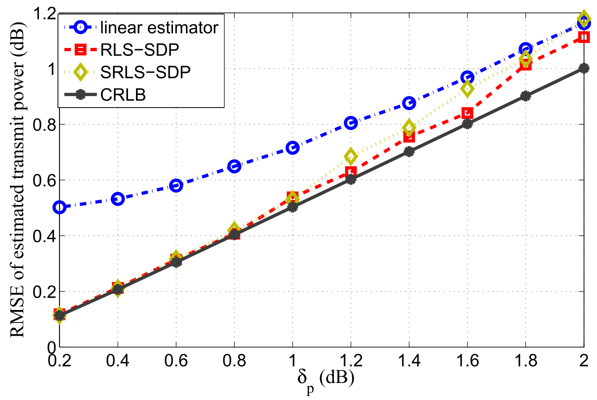


(b) RMSE of estimated position with different direction noise, $\delta_p^2 = 1^2$, $\delta_\alpha^2 = 1^2$.

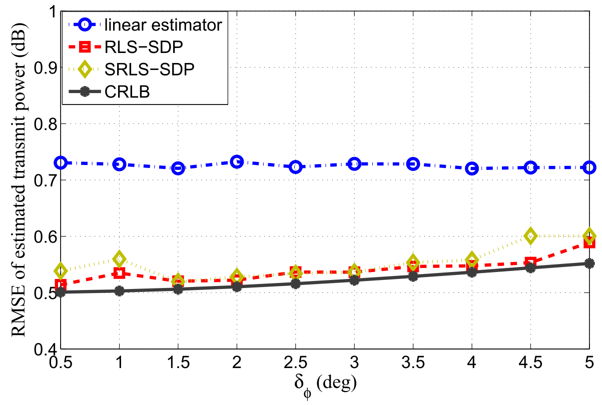


(c) RMSE of estimated position with different elevation noise, $\delta_p^2 = 1^2$, $\delta_\phi^2 = 1^2$.

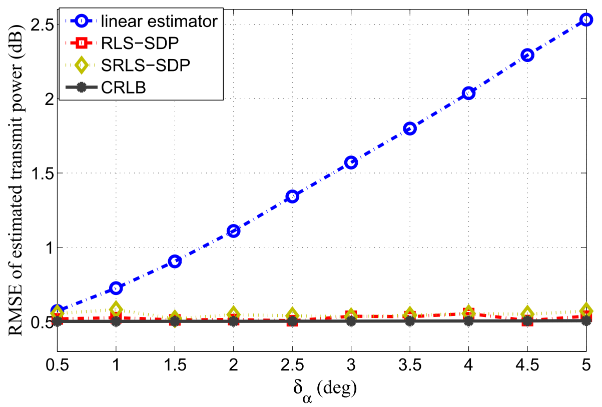
FIGURE 3. Position performance comparison under unknown transmit power.



(a) RMSE of estimated transmit power with different RSS noise, $\delta_\phi^2 = 1^2$, $\delta_\alpha^2 = 1^2$.



(b) RMSE of estimated transmit power with different direction noise, $\delta_p^2 = 1^2$, $\delta_\alpha^2 = 1^2$.



(c) RMSE of estimated transmit power with different elevation noise, $\delta_p^2 = 1^2$, $\delta_\phi^2 = 1^2$.

FIGURE 4. Performance comparison of estimated transmit power.

1.06 m. When δ_p is set at 0.2 dB, the RMSE of linear estimator is 1.17 m, which is the greatly larger than 0.59 m, its CRLB. However, when δ_p is increased to 2.0 dB, the RMSE of linear estimator is 1.28 m, which is the slightly larger than 1.03 m, its CRLB. The bias between the RMSE and its CRLB is much larger for the linear estimator at small noise level.

The RSS noise δ_p and elevation noise δ_α are also set to 1.0 dB and 1.0 degree, respectively. The RMSEs of three algorithms are plotted in Fig. 3(b), when the direction noise δ_ϕ is increased from 0.5 degree to 5.0 degree. As can be seen that the RMSE performance of these algorithms becomes worse as δ_ϕ increases. When δ_ϕ is increased from 0.5 degree to 5.0 degree, the RMSE of linear estimator is sharply increased from 0.75 m to 5.68 m. The RMSE of linear estimator is dramatically deviated from its CRLB at larger direction noise. However, the RMSE of our proposed RLS-SDP or SRLS-SDP is less and closer to the CRLB than that of the linear estimator. It is shown from Fig. 2(b) that the RMSE of SRLS-SDP is 0.64 m, when the transmit power is assumed to be known. It can also seen from Fig. 3(b) that the RMSE of SRLS-SDP is only 0.65 m under unknown transmit power. So the SRLS-SDP provides more robust estimation performance than the linear estimator, when the transmit power is adjusted to be unknown.

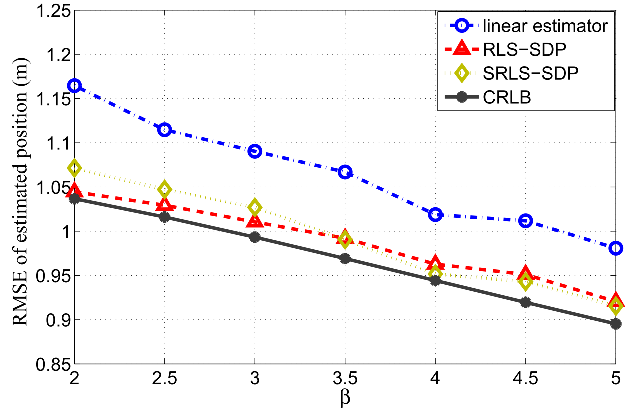
Similarly, the RSS noise δ_p and direction noise δ_α are set to 1.0 dB and 1.0 degree, respectively. Fig. 3(c) plots the RMSE of the estimated source position with these algorithms, when the elevation noise is also increased from 0.5 degree to 5.0 degree. The RMSE performance of there algorithms becomes worse as the elevation noise increases. For instance, the RMSE of SRLS-SDP is increased from 0.79 m to 2.11 m, when the elevation noise is varied from 0.5 degree to 5.0 degree. Our proposed SRLS-SDP has a bias level about 0.25 m better than the linear estimator for the estimate of source position.

C. RMSE OF ESTIMATED TRANSMIT POWER

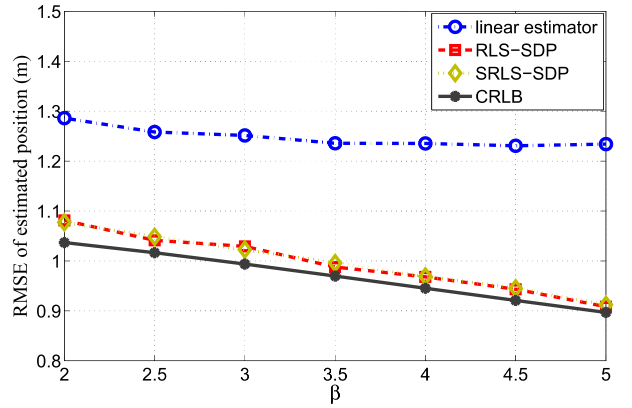
The transmit power is also estimated along with the position of source node when it is assumed to be unknown. In this subsection, the RMSE of estimated transmit power is evaluated with these different algorithms. To evaluate the impact of RSS noise, both the direction noise δ_ϕ and elevation noise δ_α are set to 1.0 degree. Fig. 4(a) plots the RMSE of the estimated transmit power with these algorithms, when the RSS noise is also increased from 0.2 dB to 2.0 dB. Similar to the results shown in Fig. 3(a), the linear estimator performs worse at small RSS noise level. When the RSS noise is increased to from 0.2 dB to 2.0 dB, the bias between the RMSE of linear estimator and its CRLB is reduced from 0.39 dB to 0.16 dB.

The parameter setting is same with that in Fig. 3(b), Fig. 4(b) illustrates the RMSE of estimated transmit power with different direction noise δ_ϕ. Differ from the position RMSE shown in Fig. 3(b), the RMSE of estimated transmit power almost has no change as the direction noise δ_ϕ increases. It is illustrated that different positions of anchor nodes have impact on the RMSE performance. The proposed RLS-SDP and SRLS-SDP have comparable performance in reaching their CRLB accuracy. For instance, the RMSE of SRLS-SDP is 0.53 dB, which is very close to 0.50 dB, its CRLB at 0.5 degree direction noise.

The RSS noise δ_p and direction noise δ_ϕ are same with those in Fig. 3(c). Fig. 4(c) illustrates the RMSE of estimated transmit power when the elevation noise δ_α is also increased from 0.5 degree to 5.0 degree. As can be seen that the RMSE of linear estimator is greatly increased from 0.57 m to 2.53 m, when δ_ϕ is varied from 0.5 degree to 5.0 degree. In contrast with the linear estimator, the performance of our proposed RLS-SDP and SRLS-SDP is almost stable with the increasing of elevation noise δ_α. For instance, the RMSE of SRLS-SDP is 0.55 dB when δ_α is set at 0.5 degree. However, the RMSE of SRLS-SDP is only 0.57 dB, when δ_ϕ is increased to 5.0 degree.



(a) RMSE of estimated position under known transmit power, $\delta_p^2 = 1^2$, $\delta_\phi^2 = 1^2$, $\delta_\alpha^2 = 1^2$.



(b) RMSE of estimated position under unknown transmit power, $\delta_p^2 = 1^2$, $\delta_\phi^2 = 1^2$, $\delta_\alpha^2 = 1^2$.

FIGURE 5. Impact of PLE.

D. PATH LOSS EXPONENT

In this subsection, we investigate the effect of path loss exponent (PLE) on the performance of the proposed algorithms. The RSS noise δ_p, direction noise δ_ϕ, and elevation noise δ_α are set to 1.0 dB, 1.0 degree, and 1.0 degree, respectively. When the PLE is varied from 2.0 to 5.0, Fig. 5(a) plots the position RMSE performance versus different PLE under known transmit power. As can be seen that the RMSE performance of these algorithms degrades, especially when the PLE is small. Compared with the linear estimator, our proposed RLS-SDP or SRLS-SDP algorithm performs better. For instance, when the PLE is set to 2.0, the RMSEs of estimated position are 1.16 m with the linear estimator, 1.04 m with the RLS-SDP, and 1.07 m with the SRLS-SDP, respectively.

When the transmit power is considered to be unknown, the impact of PLE on the position RMSE performance are also investigated. The parameter setting is same with that in Fig. 5(a). Fig. 5(b) illustrates the position RMSE performance versus different PLE under unknown transmit power. The results shown in Fig. 5(b) are almost consist with those in Fig. 5(a). The position RMSE performs better with the increasing of PLE. For instance, the RMSE of SRLS-SDP is 1.07 m when the PLE is set at 2.0. However, the PLE is increased to 5.0, the RMSE of SRLS-SDP is reduced to 0.91 m. When the PLE is increased from 2.0 to 5.0, the RLS-SDP or the SRLS-SDP always provide better performance than the linear estimator.

VII. CONCLUSION

Using the hybrid RSS and AOA measurements, we introduce the convex optimization RLS-SDP and SRLS-SDP algorithms for the source position estimates. The RMSE performance of the proposed RLS-SDP and SRLS-SDP degrades as the RSS, direction, and elevation noise increases. When the PLE becomes larger, the RMSE of the estimated position would be reduced at a given noise condition. The proposed RLS-SDP or the SRLS-SDP also provides accurate position estimate of the source node and performs better than the linear estimator, although the computational complexity of the linear estimator is lower than that of the convex RLS-SDP or the SRLS-SDP algorithm. The RLS-SDP and SRLS-SDP have almost the same accuracy performance, but the computational complexity of the SRLS-SDP is irrelevant with the number of anchor nodes. So the SRLS-SDP algorithm shows its advantage in the complexity compared with the RLS-SDP algorithm for the hybrid RSS and AOA localization.

APPENDIX

DERIVATION FOR RANK 1 OF U

When the square matrix $\mathbf{U}$ is positive semidefinite, its 2×2 principal submatrix must be a positive semidefinite matrix. So we have

$$\mathbf{U}_{i,i}\mathbf{U}_{N+4,N+4} \geq \mathbf{U}_{i,N+4}^2 \tag{56}$$

where $i = 1, 2, \ldots, N+4$. Since $\mathbf{U}_{N+4,N+4} = 1$, (56) is further given by

$$\mathbf{U}_{i,i} \geq \mathbf{U}_{i,5}^2 \tag{57}$$

It is noted that $\mathbf{D}_{i,i} > 0$ $(i = 1, 2, \ldots, N+4)$. So to minimize the cost function of (21), $\mathbf{U}_{i,i}$ must be equal to the least value $\mathbf{U}_{i,N+4}^2$. Then we further conclude that $\mathbf{U}_{i,N+4} = \sqrt{\mathbf{U}_{i,i}}$ $(i = 1, 2, \ldots, N+4)$.

Similarly, when $\mathbf{U}_{i,i}$, $\mathbf{U}_{j,j}$ and $\mathbf{U}_{N+4,N+4}$ are selected as principal diagonal elements and used to construct a 3×3 submatrix, it must also be positive semidefinite. So we can obtain that

$$\begin{bmatrix} \mathbf{U}_{i,i} & \mathbf{U}_{i,j} & \sqrt{\mathbf{U}_{i,i}} \\ \mathbf{U}_{j,i} & \mathbf{U}_{j,j} & \sqrt{\mathbf{U}_{j,j}} \\ \sqrt{\mathbf{U}_{i,i}} & \sqrt{\mathbf{U}_{j,j}} & 1 \end{bmatrix} \succeq \mathbf{0}_3 \tag{58}$$

where $i, j = 1, 2, \ldots, N+4$, $i < j$. Then the equivalent expression of (58) is given by

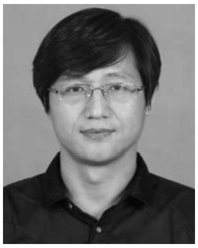
$$\begin{bmatrix} 0 & \mathbf{U}_{i,j} - \sqrt{\mathbf{U}_{i,i}\mathbf{U}_{j,j}} \\ \mathbf{U}_{i,j} - \sqrt{\mathbf{U}_{i,i}\mathbf{U}_{j,j}} & 0 \end{bmatrix} \succeq \mathbf{0}_2 \tag{59}$$

where $i, j = 1, 2, \ldots, N+4$, $i < j$. The expression of (59) holds if and only if $\mathbf{U}_{i,j} = \sqrt{\mathbf{U}_{i,i}\mathbf{U}_{j,j}}$. So we can further conclude that the square matrix $\mathbf{U}$ must be rank 1 when the solution to (21) is optimal.

REFERENCES

[1] E. L. Souza, E. F. Nakamura, and R. W. Pazzi, "Target tracking for sensor networks: A survey," *ACM Comput. Surv.*, vol. 49, no. 2, pp. 1–35, 2016.
[2] C. Zhang, H. Zhao, and S. Deng, "A density-based offloading strategy for IoT devices in edge computing systems," *IEEE Access*, vol. 6, pp. 73520–73530, 2018.
[3] X. Wu, X. Zhu, S. Wang, and L. Mo, "Cooperative motion parameter estimation using RSS measurements in robotic sensor networks," *J. Netw. Comput. Appl.*, vol. 136, pp. 57–70, Jun. 2019.
[4] H. Gao, W. Huang, and X. Yang, "Applying probabilistic model checking to path planning in an intelligent transportation system using mobility trajectories and their statistical data," *Intell. Autom. Soft Comput.*, vol. 25, no. 3, pp. 547–559, 2019.
[5] Y. Yin, J. Xia, Y. Li, Y. Xu, W. Xu, and L. Yu, "Group-wise itinerary planning in temporary mobile social network," *IEEE Access*, vol. 7, pp. 83682–83693, 2019.
[6] J. Yu, C. Hong, Y. Rui, and D. Tao, "Multitask autoencoder model for recovering human poses," *IEEE Trans. Ind. Electron.*, vol. 65, no. 6, pp. 5060–5068, Jun. 2018.
[7] L. Mo, X. Wu, and G. Wang, "Convex optimization algorithm for wireless localization by using hybrid RSS and AOA measurements," presented at the 10th Int. Conf. Broadband Commun., Netw., Syst., Xi'an, China, Oct. 2019.
[8] H. Gao, Y. Xu, Y. Yin, W. Zhang, R. Li, and X. Wang, "Context-aware QoS prediction with neural collaborative filtering for Internet-of-Things services," *IEEE Internet Things J.*, early access, Dec. 2, 2019, doi: 10.1109/JIOT.2019.2956827.
[9] X. Shi, G. Mao, B. D. O. Anderson, Z. Yang, and J. Chen, "Robust localization using range measurements with unknown and bounded errors," *IEEE Trans. Wireless Commun.*, vol. 16, no. 6, pp. 4065–4078, Jun. 2017.
[10] H. Chen, G. Wang, and X. Wu, "Cooperative multiple target nodes localization using TOA in mixed LOS/NLOS environments," *IEEE Sensors J.*, vol. 20, no. 3, pp. 1473–1484, Feb. 2020.
[11] J. Shen, A. F. Molisch, and J. Salmi, "Accurate passive location estimation using TOA measurements," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2182–2192, Jun. 2012.
[12] X. Wu, S. Wang, H. Feng, J. Hu, and G. Wang, "Motion parameter capturing of multiple mobile targets in robotic sensor networks," *IEEE Access*, vol. 6, pp. 24375–24390, 2018.
[13] T.-K. Le and N. Ono, "Closed-form and near closed-form solutions for TDOA-based joint source and sensor localization," *IEEE Trans. Signal Process.*, vol. 65, no. 5, pp. 1207–1221, Mar. 2017.
[14] X. Qu and L. Xie, "An efficient convex constrained weighted least squares source localization algorithm based on TDOA measurements," *Signal Process.*, vol. 119, pp. 142–152, Feb. 2016.
[15] L. Lin, H. C. So, and Y. T. Chan, "Received signal strength based positioning for multiple nodes in wireless sensor networks," *Digit. Signal Process.*, vol. 25, pp. 41–50, Feb. 2014.
[16] Y. Zhang, S. Xing, Y. Zhu, F. Yan, and L. Shen, "RSS-based localization in WSNs using Gaussian mixture model via semidefinite relaxation," *IEEE Commun. Lett.*, vol. 21, no. 6, pp. 1329–1332, Jun. 2017.
[17] Z. Wang, H. Zhang, T. Lu, and T. A. Gulliver, "Cooperative RSS-based localization in wireless sensor networks using relative error estimation and semidefinite programming," *IEEE Trans. Veh. Technol.*, vol. 68, no. 1, pp. 483–497, Jan. 2019.
[18] H.-J. Shao, X.-P. Zhang, and Z. Wang, "Efficient closed-form algorithms for AOA based self-localization of sensor nodes using auxiliary variables," *IEEE Trans. Signal Process.*, vol. 62, no. 10, pp. 2580–2594, May 2014.
[19] H. Huang and Y. R. Zheng, "Node localization with AoA assistance in multi-hop underwater sensor networks," *Ad Hoc Netw.*, vol. 78, pp. 32–41, Sep. 2018.
[20] S. Tomic, M. Beko, and R. Dinis, "Distributed RSS-based localization in wireless sensor networks based on second-order cone programming," *Sensors*, vol. 14, no. 10, pp. 18410–18432, 2014.
[21] J. Zheng and X. Wu, "Convex optimization algorithms for cooperative RSS-based sensor localization," *Pervas. Mobile Comput.*, vol. 37, pp. 78–93, Jun. 2017.
[22] Y.-Y. Li, G.-Q. Qi, and A.-D. Sheng, "Performance metric on the best achievable accuracy for hybrid TOA/AOA target localization," *IEEE Commun. Lett.*, vol. 22, no. 7, pp. 1474–1477, Jul. 2018.

[23] Y. Wang and K. C. Ho, "Unified near-field and far-field localization for AOA and hybrid AOA-TDOA positionings," *IEEE Trans. Wireless Commun.*, vol. 17, no. 2, pp. 1242–1254, Feb. 2018.
[24] E. Kazikli and S. Gezici, "Hybrid TDOA/RSS based localization for visible light systems," *Digit. Signal Process.*, vol. 86, pp. 19–28, Mar. 2019.
[25] S. Tomic and M. Beko, "A robust NLOS bias mitigation technique for RSS-TOA-based target localization," *IEEE Signal Process. Lett.*, vol. 26, no. 1, pp. 64–68, Jan. 2019.
[26] J. Yu, C. Zhu, J. Zhang, Q. Huang, and D. Tao, "Spatial pyramid-enhanced NetVLAD with and weighted triplet loss for place recognition," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 2, pp. 661–674, Feb. 2020, doi: 10.1109/TNNLS.2019.2908982.
[27] H. Gao, Y. Duan, L. Shao, and X. Sun, "Transformation-based processing of typed resources for multimedia sources in the IoT environment," *Wireless Netw.*, to be published, doi: 10.1007/s11276-019-02200-6.
[28] A. Fascista, G. Ciccarese, A. Coluccia, and G. Ricci, "A localization algorithm based on V2I communications and AOA estimation," *IEEE Signal Process. Lett.*, vol. 24, no. 1, pp. 126–130, Jan. 2017.
[29] T. Jia, H. Wang, X. Shen, Z. Jiang, and K. He, "Target localization based on structured total least squares with hybrid TDOA-AOA measurements," *Signal Process.*, vol. 143, pp. 211–221, Feb. 2018.
[30] J. Yin, Q. Wan, S. Yang, and K. C. Ho, "A simple and accurate TDOA-AOA localization method using two stations," *IEEE Signal Process. Lett.*, vol. 23, no. 1, pp. 144–148, Jan. 2016.
[31] R. Kalpana and M. Baskaran, "TAR: TOA-AOA based random transmission directed localization," *Wireless Pers. Commun.*, vol. 90, no. 2, pp. 889–902, Sep. 2016.
[32] S. Tomic, M. Beko, and R. Dinis, "Distributed RSS-AoA based localization with unknown transmit powers," *IEEE Wireless Commun. Lett.*, vol. 5, no. 4, pp. 392–395, Aug. 2016.
[33] S. Tomic, M. Beko, and R. Dinis, "3-D target localization in wireless sensor networks using RSS and AoA measurements," *IEEE Trans. Veh. Technol.*, vol. 66, no. 4, pp. 3197–3210, Apr. 2017.
[34] A. Simonetto and G. Leus, "Distributed maximum likelihood sensor network localization," *IEEE Trans. Signal Process.*, vol. 62, no. 6, pp. 1424–1437, Mar. 2014.
[35] E. Dranka and R. F. Coelho, "Robust maximum likelihood acoustic energy based source localization in correlated noisy sensing environments," *IEEE J. Sel. Topics Signal Process.*, vol. 9, no. 2, pp. 259–267, Mar. 2015.
[36] Y. Sun, K. C. Ho, and Q. Wan, "Solution and analysis of TDOA localization of a near or distant source in closed form," *IEEE Trans. Signal Process.*, vol. 67, no. 2, pp. 320–335, Jan. 2019.
[37] S. Tomic, M. Beko, and R. Dinis, "RSS-based localization in wireless sensor networks using convex relaxation: Noncooperative and cooperative schemes," *IEEE Trans. Veh. Technol.*, vol. 64, no. 5, pp. 2037–2050, May 2015.
[38] Z. Su, G. Shao, and H. Liu, "Semidefinite programming for NLOS error mitigation in TDOA localization," *IEEE Commun. Lett.*, vol. 22, no. 7, pp. 1430–1433, Jul. 2018.
[39] P. Oguz-Ekim, J. P. Gomes, J. Xavier, M. Stosic, and P. Oliveira, "An angular approach for range-based approximate maximum likelihood source localization through convex relaxation," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7, pp. 3951–3964, Jul. 2014.
[40] K. W. K. Lui, W.-K. Ma, H. C. So, and F. K. W. Chan, "Semi-definite programming algorithms for sensor network node localization with uncertainties in anchor positions and/or propagation speed," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 752–763, Feb. 2009.
[41] P. M. Ghari, R. Shahbazian, and S. A. Ghorashi, "Wireless sensor network localization in harsh environments using SDP relaxation," *IEEE Commun. Lett.*, vol. 20, no. 1, pp. 137–140, Jan. 2016.
[42] G. Wang and K. C. Ho, "Convex relaxation methods for unified near-field and far-field TDOA-based localization," *IEEE Trans. Wireless Commun.*, vol. 18, no. 4, pp. 2346–2360, Apr. 2019.
[43] H. Gao, W. Huang, Y. Duan, X. Yang, and Q. Zou, "Research on cost-driven services composition in an uncertain environment," *J. Internet Technol.*, vol. 20, no. 3, pp. 755–769, 2019.
[44] R. M. Vaghefi, M. R. Gholami, R. M. Buehrer, and E. G. Strom, "Cooperative received signal strength-based sensor localization with unknown transmit powers," *IEEE Trans. Signal Process.*, vol. 61, no. 6, pp. 1389–1403, Mar. 2013.
[45] Y. Xu, J. Zhou, and P. Zhang, "RSS-based source localization when path-loss model parameters are unknown," *IEEE Commun. Lett.*, vol. 18, no. 6, pp. 1055–1058, Jun. 2014.
[46] Y. T. Chan, F. Chan, W. Read, B. R. Jackson, and B. H. Lee, "Hybrid localization of an emitter by combining angle-of-arrival and received signal strength measurements," in *Proc. IEEE 27th Can. Conf. Electr. Comput. Eng. (CCECE)*, May 2014, pp. 1–5.
[47] S. Tomic, M. Beko, and M. Tuba, "A linear estimator for network localization using integrated RSS and AOA measurements," *IEEE Signal Process. Lett.*, vol. 26, no. 3, pp. 405–409, Mar. 2019.



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