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Performance of SIMO Networked Time-Delay Systems With Encoding–Decoding and Quantization Constraints

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ABSTRACT In this paper, the performance of the single-input multiple-output (SIMO) networked time-delay systems is investigated. The performance is related to the internal factors, communication parameters and reference signals, and the boundary is related to the adjustment factors. Some new results are derived according to the inner-outer factorization and Cauchy's Theorem of two degrees-of-freedom controller. The results show that the performance is in connection with the inner factor (unstable poles, non-minimum phase zeros of the system). It is also demonstrated that the modified performance will be badly degraded by feedback channel quantization and adjustment factor constraints, and encoding-decoding is beneficial to the modified performance.

INDEX TERMS Networked time-delay systems, modified performance, inner-outer factorization, SIMO.

I. INTRODUCTION

In the last decade, the research of neural networks [1]–[3] have become a hot topic, at the same time, researchers have been investigated the control problems of various systems, for example, physical systems [4], Takagi-Sugeno fuzzy systems [5], [6], nonlinear systems [7], multi-agent systems [8], [9], and networked systems [10]–[12]. Networked control systems (NCSs) have been well developed in many fields, such as telemedicine [13], [14], automatic current regulation [15], [16], industrial control [17]–[19]. As is known to all, the stability of NCSs have been studied extensively in feedback control systems [20]–[27]. In [20], the stability of continuous and discrete linear time-invariant (LTI) systems over the signal-to-noise ratio (SNR) constrained channels has been studied. In [21], the state feedback stability problem for multiple-input multiple-output (MIMO) systems over the memoryless fading noisy channels under the SNR constraints has been investigated. The feedback stabilization over an additive white gaussian noise (AWGN) channel has been established in [22]. In [23], [24], by using the Lyapunov stability theory and random analysis technique, the stability of the NCSs with the hybrid drive mechanism and probability of the network attack has been obtained. In [25], the minimum SNR of a linear system with the output feedback affected by AWGN has been studied. The application of the mean square stabilization controller to the discrete-time linear system with MIMO has been studied in [26]. The robust stability on the channel SNR constrained feedback control plant model has been considered in [27]. Thus, the stability of NCSs has been very mature. However, from the perspective of application, the stability of NCSs is as important as the tracking performance of NCss.

In recent years, there are many achievements in the tracking performance of NCSs [28]–[32]. The minimum tracking error of the MIMO NCSs has been investigated by considering the quantization, encoding-decoding, channel noise restraint in [28]. The minimum tracking error of NCSs was studied by spectral factorization and partial decomposition techniques with time-delay and encoding-decoding constraints in [29]. The modified minimum tracking error of the networked time-delay systems has been investigated with two-channel constraints in [30]. The limitation of the tracking performance of two kinds of network parameter

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systems, bandwidth and AWGN was studied in [31]. The error signal measured by the energy between the plant output and the reference signal of NCSs was studied in [32]. The optimal performance of NCSs under packet loss and channel noise was obtained by spectral decomposition in [33]. In [34], the modified tracking performance of unstable linear SIMO feedback control system was studied. In addition, considering the balance between the tracking error power and the control energy, the tradeoff performance was analyzed in [35]. The performance of MIMO NCSs under the multiple communication constraints was studied in [36]. The optimal modified performance of MIMO NCSs under the multiple constraints was obtained by means of the coprime factorization and partial fraction in [37]. The research on transforming multivariable systems into SIMO subsystems is still scarce, but SIMO systems are widespread in the process industry. Therefore, it is necessary to study the performance of SIMO systems. In [38], the vehicle suspension system was analyzed and designed as a typical SIMO nonlinear system. By designing a feed-forward compensator, a multi-objective tracking control algorithm for a nonlinear systems with SIMO was obtained in [39], [40]. Different from single-input single-output(SISO) and MIMO systems, SIMO problems are relatively complicated, and problems in SISO and MIMO systems also appear in SIMO systems, but the solutions are different. Based on the above research results, traditional references have described that the optimal tracking performance is related to non-minimum phase zeros, unstable poles and its directions. These results will be helpful for the design of control systems and communication networked. However, these results cannot be directly extended to the case of the SIMO plant since the plant must be right invertible in the existing results. Then, the study of SIMO plant is necessary. It is known that, in practical NCSs applications, the signal distortion is inevitably existed during the data transmission, in order to avoid the signal distortion or correct the error signal, the signal should be encoded before transmission and then decoded at the destination, so the signal is transmitted in the network by encoding and decoding, the design of coder-decoder will inevitably have effect on the control performance of the system. In order to improve the SNR, it is necessary to amplify the attenuation signal in the process of signal transmission, and the noise which is inevitably superimposed on the signal during transmission is also amplified, with the increase of transmission distance, more and more noise is accumulated, resulting in serious deterioration of transmission quality, as it is well known, the quantization is convenient for encryption, storage, processing and exchange, and equipment integration and miniaturization, at the same time, quantization ensures strong anti-interference ability and robustness to noise accumulation, so the quantitative design will inevitably affect the performance of NCSs. Thus, the quantization, encoding-decoding of networked time-delay systems should also be considered.

Inspired by the above works, the modified tracking performance of the SIMO networked time-delay systems with

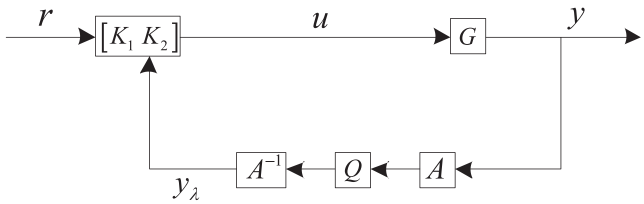

FIGURE 1. Networked systems with encoding-decoding and quantization constraints.

the quantization, encoding-decoding constraints is investigated in this paper. The contributions of this paper include: (1) Developing a general formula for the minimal tracking error, which is expressed in terms of the inner factor of the plant, the derive explicit expressions for the modified tracking performance concerning SIMO systems. (2) In order to calculate minimal modified tracking performance of NCSs, some parameters of designed controllers are determined from internal-external factorization and two-degree-of-freedom (2DF) controller, the minimal modified tracking performance is obtained by using the H_2 norm technique. (3) For unstable SIMO plants, unlike traditional NCSs, the results show that the modified tracking performance is constrained not only dependent on non-minimum phase (NMP) zeros and unstable poles of a given plant but also dependent on channel noise, quantization noise, encoding-decoding and other correction factors.

The structure of this paper is as follows: In Section II, the description of the problem has been given. The minimal modification tracking error is derived in Section III. In Section V, numerical examples are given to illustrate the validity of the results. Section IV draws some conclusions.

II. PROBLEM DESCRIPTION

In this paper, A^H is the complex conjugate of a vector A. The open right-half and left-half planes are respectively denoted by $\mathbb{C}_+ = \{s : \mathrm{Re}(s) > 0\}$, and $\mathbb{C}_- = \{s : \mathrm{Re}(s) < 0\}$, respectively. And imaginary axis by $\mathbb{C}_0 = \{s : Re(s) = 0\}$. The space L_2 is defined by $\left\{f : f(s) \text{ analytic in} \mathbb{C}_0, \|f\|_2^2 = \frac{1}{2\pi}\left\|f\left(e^{j\theta}\right)\right\|^2 d\theta < \infty\right\}$. The inner product of Hilbert space is defined by: $\langle f, g\rangle := \frac{1}{2\pi}\int_{\pi}^{\pi} f^H\left(e^{j\theta}\right) g\left(e^{j\theta}\right) d\theta$. As we know, the orthogonal decomposition L_2 is H_2 and $H_2^{\perp}$.

In Fig. 1, we consider the SIMO networked time-delay systems with encoding-decoding and quantization constraints, G denotes the controllable plant, $G(s)$ denotes transfer function matrix; $[K_1(s) \quad K_2(s)]$ denotes the transfer function matrix of the 2DOF controller $[K_1 \quad K_2]$. In addition, the transfer function matrices $A(z)$ and $A^{-1}(z)$ respectively denote encoder A and decoder A^{-1}. Laplace transform $\hat{r}$ denotes a reference input r, Laplace transform $\hat{y}$ denotes the system output y, and Laplace transform $\hat{u}$ denotes a control input u. Q denotes the uniform quantizer of communication channel, with quantization noise of $n(t) = (n_1(t), n_2(t), \cdots n_m(t))^T$. It is supposed that the quantization noise obeys the uniform distribution in the interval $\left[-\frac{\Delta_i}{2}, \frac{\Delta_i}{2}\right]$, $i \in 1, 2, \cdots m$, in order to simplify the analysis. Δ_i represents the quantization interval in each channel. The expectation of noise n is 0, and the variance is $\frac{\Delta_i^2}{12}$, and $V^2 = \mathrm{diag}\left(\frac{\Delta_1^2}{12}, \frac{\Delta_2^2}{12}, \cdots, \frac{\Delta_m^2}{12}\right)$. Signals r and n are unrelated, and the matrix is given by $U^2 = \mathrm{diag}\left(\alpha_1^2, \alpha_2^2, \cdots \alpha_m^2\right)$.

To obtain the minimal modification tracking error of SIMO networked time-delay systems, we define its performance index as follows:

$$J_\lambda = E\left\{\left[e^{-\lambda t} e^T(t)\right]\left[e^{-\lambda t} e(t)\right]\right\} \tag{1}$$

When the controllers are chosen among all the possible stabilizing controllers, we can get the minimal modification tracking error by

$$J^* = \inf_{\mathcal{K} \in K} J \tag{2}$$

In this work, we consider the coprime factorization of G given by

$$G(s) = G_1(s)e^{-\tau s}G = e^{-\tau s}NM^{-1} \tag{3}$$

where $N, M \in RH_\infty$, and satisfies the double Bezout identity, which is given by

$$MX - e^{-\tau s}NY = I \tag{4}$$

For $X, Y \in RH_\infty$, using the Youla parameterization, any stabilizing controllers $\mathcal{K}$ can be characterized as [40]:

$$\begin{aligned} \mathcal{K} := \{K : K = [K_1\ K_2] \\ = \left(X - e^{-\tau s}RN\right)^{-1}[Q \quad Y - RM] \quad Q, R \in RH_\infty\right\} \end{aligned} \tag{5}$$

A non-minimum phase transfer function may factorize a minimum phase part and an all pass factor [41]:

$$N = \Theta_i \Lambda, M = B_p M_m \tag{6}$$

where $\Theta_i^H \Theta_i = I$, $\Theta_i(s) = \Theta_i^T(s)$, $\Phi_i = \begin{pmatrix} \Theta_i^H \\ I - \Theta_i \Theta_i^H \end{pmatrix}$, $\Phi_i^H \Phi_i = I$, it is easy to verify that Θ_i and B_p are all pass factors, Λ and M_m are minimum phase, $\begin{pmatrix} \Theta_i^H \\ I - \Theta_i \Theta_i^H \end{pmatrix}$ is an inner matrix. Specifically, B_p can be constructed as $B_p(z) = \prod_{j=1}^{m} B_j(z)$, $B_j(z) = \frac{z - p_j}{1 - p_j z}\omega_j \omega_j^H + W_j W_j^H$, ω_j is the unitary vectors in the direction of unstable poles, moreover, $\omega_j \omega_j^H + W_j W_j^H = I$.

The minimal modification tracking error of SIMO networked time-delay systems is defined as

$$e = r - y \tag{7}$$

From Fig. 1, we get

$$u = K_1 r + K_2 y_\lambda \tag{8}$$

$$y_\lambda = A^{-1}(n + Ay) = A^{-1}n + y \tag{9}$$

$$y = Gu \tag{10}$$

According to (8), (9) and (10), we have

$$u = (I - K_2G)^{-1}K_1 r + (I - K_2G)^{-1}K_2A^{-1}n \tag{11}$$

$$y = G(I - K_2G)^{-1}K_1 r + G(I - K_2G)^{-1}K_2A^{-1}n \tag{12}$$

From (7) and (12), we have

$$\begin{aligned} e = r - y = \left[I - G(I - K_2G)^{-1}K_1\right] r \\ - G(I - K_2G)^{-1}K_2A^{-1}n = T_1 r + T_2 n \end{aligned} \tag{13}$$

where $T_1 = I - G(I - K_2G)^{-1}K_1$,
$T_2 = G(I - K_2G)^{-1}K_2A^{-1}$.

By using (3), (4) and (5), we obtain

$$\begin{aligned} T_1 &= I - G(I - K_2G)^{-1}K_1 \\ &= I - e^{-\tau s}NM^{-1} \\ &\quad \times \left[I - \left(X - e^{-\tau s}RN\right)^{-1}(Y - RM)e^{-\tau s}NM^{-1}\right]^{-1} \\ &\quad \times \left(X - e^{-\tau s}RN\right)^{-1}Q \\ &= I - e^{-\tau s}N \\ &\quad \times \left[\left(X - e^{-\tau s}RN\right)M - (Y - RM)e^{-\tau s}N\right]Q \\ &= I - e^{-\tau s}N \\ &\quad \times \left[XM - e^{-\tau s}RNM - e^{-\tau s}NY + e^{-\tau s}RNM\right]Q \\ &= I - e^{-\tau s}NQ \end{aligned} \tag{14}$$

$$\begin{aligned} T_2 &= G(I - K_2G)^{-1}K_2A^{-1} \\ &= e^{-\tau s}NM^{-1} \\ &\quad \times \left[I - \left(X - e^{-\tau s}RN\right)^{-1}(Y - RM)e^{-\tau s}NM^{-1}\right]^{-1} \\ &\quad \times \left(X - e^{-\tau s}RN\right)^{-1}(Y - RM)A^{-1} \\ &= e^{-\tau s}N\left[\left(X - e^{-\tau s}RN\right)M - (Y - RM)e^{-\tau s}N\right] \\ &\quad \times (Y - RM)A^{-1} \\ &= e^{-\tau s}N(Y - RM)A^{-1} \end{aligned} \tag{15}$$

Then, from (1), we get

$$\begin{aligned} J_\lambda &= E\left\{\left[e^{-\lambda t}e^T(t)\right]\left[e^{-\lambda t}e(t)\right]\right\} \\ &= \|e\|_2^2 = \left\|\left(I - e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2 \\ &\quad + \left\|e^{-\tau s}N(Y - RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2 \end{aligned} \tag{16}$$

III. MINIMAL MODIFICATION TRACKING ERROR OF SIMO NETWORKED TIME-DELAY SYSTEMS

According to (2), we have:

$$J_\lambda^* = \inf_{\mathcal{K} \in K} J_\lambda \tag{17}$$

Combineing with(16) and (17), the minimal modification tracking error can be rewritten as

$$\begin{aligned} J_\lambda^* = \inf_{Q \in RH_\infty} \left\|\left(I - e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2 \\ + \inf_{R \in RH_\infty} \left\|e^{-\tau s}N(Y - RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2 \end{aligned} \tag{18}$$

Theorem 1: Assume that $p_j \in \mathbb{C}_+, j = 1, \ldots, m$ is an unstable pole, and $z_i \in \mathbb{C}_+, i = 1, \ldots, n$ is an NMP zero of G, respectively. According to the structure presented in Fig. 1, we have

$$J_\lambda^* = \frac{1}{2\lambda}\sum_{i,j=1}^{n} e^{\tau(\bar{z}_i+z_j-2\lambda)}\alpha_j^2\left[1-[\Theta_i(\lambda)]_j^2\right] + \sum_{i,j=1}^{N_p}\frac{\Delta_i^2}{12}e^{\tau(\bar{p}_j+p_i-2\lambda)}\frac{4Re(p_j)Re(p_i)}{\bar{p}_j p_i(\bar{p}_j+p_i-2\lambda)}tr\left(\gamma_j\gamma_i^H\right)$$

where $\lambda = -s$,

$$\gamma_j = \Theta_i^{-1}(p_j-\lambda)A^{-1}(p_j-\lambda)G_j\varepsilon_j\varepsilon_j^H H_j,$$

$$H(jw) = tr\left\{U^H\left[I-\Theta_i(jw)\Theta_i^H(-\lambda) - e^{\tau jw}\Theta_i(-\lambda)\Theta_i^H(jw)+e^{\tau jw}\Theta_i(-\lambda)\Theta_i^H(-\lambda)\right]U\right\}.$$

Proof: First, we can define

$$J_1^* = \inf_{Q\in RH_\infty}\left\|\left(I-e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2 \quad (19)$$

$$J_2^* = \inf_{R\in RH_\infty}\left\|e^{-\tau s}N(Y-RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2 \quad (20)$$

Because (5) and $e^{-\tau s}$ are the all pass factors, we get

$$\begin{aligned}J_1^* &= \inf_{Q\in RH_\infty}\left\|\Phi_i\left(I-e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2\\
&= \inf_{Q\in RH_\infty}\left\|\Theta_i^H\left(I-e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2 + \inf_{Q\in RH_\infty}\left\|\left(I-\Theta_i\Theta_i^H\right)\left(I-e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2\\
&= \inf_{Q\in RH_\infty}\left\|\Theta_i^H\left(e^{\tau s}-\Theta_i\Lambda Q\right)\frac{U}{s+\lambda}\right\|_2^2 + \left\|e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\frac{U}{s+\lambda}\right\|_2^2\\
&= \left\|e^{\tau s}\left[\Theta_i^H-\Theta_i^H(-\lambda)\right]\frac{U}{s+\lambda}\right\|_2^2 + \inf_{Q\in RH_\infty}\left\|\left[e^{\tau s}\Theta_i^H(-\lambda)+\Lambda Q\right]\frac{U}{s+\lambda}\right\|_2^2 + \left\|e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\frac{U}{s+\lambda}\right\|_2^2\end{aligned}$$

Define

$$J_{11}^* = \left\|e^{\tau s}\left[\Theta_i^H-\Theta_i^H(-\lambda)\right]\frac{U}{s+\lambda}\right\|_2^2 + \left\|e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\frac{U}{s+\lambda}\right\|_2^2$$

$$J_{12}^* = \inf_{Q\in RH_\infty}\left\|\left[e^{\tau s}\Theta_i^H(-\lambda)+\Lambda Q\right]\frac{U}{s+\lambda}\right\|_2^2$$

Because of $\Theta_i^H(s) = \Theta_i^T(-s), \Theta_i^H(jw) = \Theta_i^T(-jw)$, we have

$$J_{11}^* = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{H(jw)}{(jw+\lambda)(-jw+\lambda)}dw$$

where

$$H(jw) = tr\left\{U^H\left[I-\Theta_i(jw)\Theta_i^H(-\lambda) - e^{\tau jw}\Theta_i(-\lambda)\Theta_i^H(jw)+e^{\tau jw}\Theta_i(-\lambda)\Theta_i^H(-\lambda)\right]U\right\}$$

Define $J_{11}^* = J_a^* + J_b^* + J_c^*$

$$J_a^* = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{tr\left\{U^H e^{\tau jw}\left[I+\Theta_i(-\lambda)\Theta_i^H(-\lambda)\right]U\right\}}{(jw+\lambda)(-jw+\lambda)}dw$$

$$J_b^* = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{tr\left\{U^H e^{\tau jw}\left[-\Theta_i(jw)\Theta_i^H(-\lambda)\right]U\right\}}{(jw+\lambda)(-jw+\lambda)}dw$$

$$J_c^* = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{tr\left\{U^H e^{\tau jw}\left[-\Theta_i(-\lambda)\Theta_i^H(jw)\right]U\right\}}{(jw+\lambda)(-jw+\lambda)}dw$$

Since $s = -\lambda$, we have

$$J_c^* = -\frac{1}{2\lambda}tr\left\{U^H e^{\tau jw}\left[-\Theta_i(-\lambda)\Theta_i^H(jw)\right]U\right\}$$

$$J_a^* = \frac{1}{2\lambda}\times\left[tr\left\{U^H U\right\}+tr\left\{U^H e^{\tau jw}\left[-\Theta_i(-\lambda)\Theta_i^H(jw)\right]U\right\}\right]$$

In J_2^*, there is only one unstable pole $s = \lambda$, by Cauchy's Theorem, we have

$$J_b^* = -\frac{1}{2\lambda}tr\left\{U^H e^{\tau jw}\left[\Theta_i(\lambda)\Theta_i^T(\lambda)\right]U\right\} = -\frac{e^{\tau(\bar{z}_i+z_j-2\lambda)}}{2\lambda}\alpha_j^2\left[\Theta_i(\lambda)\right]_j^2$$

Then, we can get

$$J_1^* = \frac{1}{2\lambda}\sum_{i,j=1}^{n}e^{\tau(\bar{z}_i+z_j-2\lambda)}\alpha_j^2\left[1-[\Theta_i(\lambda)]_j^2\right]$$

From (6), we derive

$$\begin{aligned}J_2^* &= \inf_{R\in RH_\infty}\left\|e^{-\tau s}N(Y-RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2\\
&= \inf_{R\in RH_\infty}\left\|\Phi_i\left(\Theta_i\Lambda YA^{-1}-\Theta_i\Lambda RMA^{-1}\right)\frac{V}{s+\lambda}\right\|_2^2\\
&= \inf_{R\in RH_\infty}\left\|(\Lambda Y-\Lambda RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2 + \inf_{R\in RH_\infty}\left\|\left(I-\Theta_i\Theta_i^H\right)(\Theta_i\Lambda Y-\Theta_i\Lambda RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2\\
&= \inf_{R\in RH_\infty}\left\|(\Lambda Y-\Lambda RM)A^{-1}\frac{V}{s+\lambda}\right\|_2^2\\
&= \inf_{R\in RH_\infty}\left\|\left(\Lambda YB_p^{-1}-\Lambda RM_m\right)A^{-1}\frac{V}{s+\lambda}\right\|_2^2\end{aligned}$$

By employing the partial fraction method, we have

$$\begin{aligned}\Lambda Y B_p^{-1} &= \Lambda\left(p_j-\lambda\right) Y\left(p_j-\lambda\right) G_j \\ &\quad \times\left(I+\frac{2Re\left(p_j\right)}{s+\lambda-p_j}\varepsilon_j\varepsilon_j^H-I+\frac{2Re\left(p_j\right)}{p_j}\varepsilon_j\varepsilon_j^H\right) H_j \\ &\quad +R_1-\Lambda\left(p_j-\lambda\right) Y\left(p_j-\lambda\right) G_j\frac{2Re\left(p_j\right)}{p_j}\varepsilon_j\varepsilon_j^H H_j\end{aligned}$$

Therefore, it can be written

$$\begin{aligned}J_2^* &= \left\|\Lambda\left(p_j-\lambda\right) Y\left(p_j-\lambda\right) V G_j\right. \\ &\quad\left.\times\left(I+\frac{2Re\left(p_j\right)}{s+\lambda-p_j}\varepsilon_j\varepsilon_j^H-I+\frac{2Re\left(p_j\right)}{p_j}\varepsilon_j\varepsilon_j^H\right) H_j\frac{1}{s+\lambda}\right\|_2^2 \\ &\quad+\inf_{R\in RH_\infty}\left\|\left[R_1-\Lambda\left(p_j-\lambda\right) Y\left(p_j-\lambda\right) V G_j\frac{2Re\left(p_j\right)}{p_j}\varepsilon_j\varepsilon_j^H H_j\right.\right. \\ &\quad\left.\left.-\Lambda R M_m A^{-1} V\right]\frac{1}{s+\lambda}\right\|_2^2\end{aligned}$$

By choosing the proper R_1

$$\begin{aligned}\inf_{R\in RH_\infty}\left\|\left[R_1-\Lambda\left(p_j-\lambda\right) Y\left(p_j-\lambda\right) V G_j\frac{2Re\left(p_j\right)}{p_j}\varepsilon_j\varepsilon_j^H H_j\right.\right. \\ \left.\left.-\Lambda R M_m A^{-1} V\right]\frac{1}{s+\lambda}\right\|_2^2=0\end{aligned}$$

From(4) and $M\left(p_j\right)=0$, we get $Y=-e^{\tau s}N^{-1}$, and $Y\left(p_j-\lambda\right)=-e^{\tau\left(p_j-\lambda\right)}\Lambda^{-1}\Theta_i^{-1}$. Thus,

$$\begin{aligned}J_2^* &= \left\|\sum_{j=1}^{N_p} e^{\tau\left(p_j-\lambda\right)}\Theta_i^{-1}\left(p_j-\lambda\right) A^{-1}\left(p_j-\lambda\right) V G_j\right. \\ &\quad\left.\times\left(I+\frac{2Re\left(p_j\right)}{s+\lambda-p_j}\varepsilon_j\varepsilon_j^H-I+\frac{2Re\left(p_j\right)}{p_j}\varepsilon_j\varepsilon_j^H\right) H_j\frac{1}{s+\lambda}\right\|_2^2 \\ &= \sum_{i,j=1}^{N_p}\frac{\Delta_i^2}{12} e^{\tau\left(\bar{p}_j+p_i-2\lambda\right)}\frac{4Re\left(p_j\right) Re\left(p_i\right)}{\bar{p}_j p_i\left(\bar{p}_j+p_i-2\lambda\right)} tr\left(\gamma_j\gamma_i^H\right)\end{aligned}$$

where $\gamma_j=\Theta_i^{-1}\left(p_j-\lambda\right) A^{-1}\left(p_j-\lambda\right) G_j\varepsilon_j\varepsilon_j^H H_j$. So, we can get

$$\begin{aligned}J_\lambda^* &= \frac{1}{2\lambda}\sum_{i,j=1}^{n} e^{\tau\left(\bar{z}_i+z_j-2\lambda\right)}\alpha_j^2\left[1-\left[\Theta_i(\lambda)\right]_j^2\right] \\ &\quad+\sum_{i,j=1}^{N_p}\frac{\Delta_i^2}{12} e^{\tau\left(\bar{p}_j+p_i-2\lambda\right)}\frac{4Re\left(p_j\right) Re\left(p_i\right)}{\bar{p}_j p_i\left(\bar{p}_j+p_i-2\lambda\right)} tr\left(\gamma_j\gamma_i^H\right)\end{aligned}$$

Generally speaking, it is need to pay attention to the channel input to meet the power constraint

$$E\left\{\left[e^{-\lambda t}e^T(t)\right]\left[e^{-\lambda t}e(t)\right]\right\}<\Gamma$$

for some input power level $\Gamma>0$. The power limitations may come from electronic hardware limitations or regulatory restrictions introduced to minimize interference with users of other communication systems. The minimal modification tracking error of channel input power constraint is defined as

$$\begin{aligned}J_\lambda(K,\varepsilon) &= (1-\varepsilon) E\left\{\left[e^{-\lambda t}e^T(t)\right]\left[e^{-\lambda t}e(t)\right]\right\} \\ &\quad+\varepsilon\left(E\left\{\left[e^{-\lambda t}y^T(t)\right]\left[e^{-\lambda t}y(t)\right]\right\}-\Gamma\right)\end{aligned}\tag{21}$$

where ε is the adjustment factor $0\le\varepsilon<1$, which represents the trade-off between the minimal modification tracking error and the channel input power, and $\varepsilon=0$ means that the power constraint does not exist in the systems.

Here, $\mathcal{K}$ represents all stabilizing controllers, we can get the minimal modification tracking error by

$$J_\lambda^*=\inf_{\mathcal{K}\in K} J_\lambda(K,\varepsilon)$$

Theorem 2: Assume that $p_j\in\mathbb{C}_+, j=1,\ldots,m$ is an unstable pole, and $z_i\in\mathbb{C}_+, i=1,\ldots,n$ is an NMP zero of G, respectively. According to the structure presented in Fig. 1, we have

$$\begin{aligned}J_\lambda^* &= \frac{1}{2\lambda}\sum_{i,j=1}^{n} e^{\tau\left(\bar{z}_i+z_j-2\lambda\right)}\alpha_j^2\left[1-\left[\Theta_i(\lambda)\right]_j^2\right] \\ &\quad+\sum_{i,j=1}^{N_p}\frac{\Delta_i^2}{12} e^{\tau\left(\bar{p}_j+p_i-2\lambda\right)}\frac{4Re\left(p_j\right) Re\left(p_i\right)}{\bar{p}_j p_i\left(\bar{p}_j+p_i-2\lambda\right)} tr\left(\gamma_j\gamma_i^H\right)\end{aligned}$$

where $\lambda=-s$,

$$\gamma_j=\Theta_i^{-1}\left(p_j-\lambda\right) A^{-1}\left(p_j-\lambda\right) G_j\varepsilon_j\varepsilon_j^H H_j,$$

and

$$\begin{aligned}H(jw) &= tr\left\{U^H\left[I-\Theta_i(jw)\Theta_i^H(-\lambda)\right.\right. \\ &\quad\left.\left.-e^{\tau jw}\Theta_i(-\lambda)\Theta_i^H(jw)+e^{\tau jw}\Theta_i(-\lambda)\Theta_i^H(-\lambda)\right] U\right\}.\end{aligned}$$

Proof: From (1), (7) and (16), we can get

$$\begin{aligned}&E\left\{\left[e^{-\lambda t}e^T(t)\right]\left[e^{-\lambda t}e(t)\right]\right\} \\ &= \left\|\left(I-e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2+\left\|e^{-\tau s}N(Y-RM) A^{-1}\frac{V}{s+\lambda}\right\|_2^2 \\ &E\left\{\left[e^{-\lambda t}y^T(t)\right]\left[e^{-\lambda t}y(t)\right]\right\} \\ &= \left\|e^{-\tau s}NQ\frac{U}{s+\lambda}\right\|_2^2+\left\|e^{-\tau s}N(Y-RM) A^{-1}\frac{V}{s+\lambda}\right\|_2^2\end{aligned}$$

From (21), we can get

$$\begin{aligned}J_\lambda^* &= \inf_{K\in K}\left\{(1-\varepsilon)\left[\left\|\left(I-e^{-\tau s}NQ\right)\frac{U}{s+\lambda}\right\|_2^2\right.\right. \\ &\quad\left.\left.+\left\|e^{-\tau s}N(Y-RM) A^{-1}\frac{V}{s+\lambda}\right\|_2^2\right]\right.\end{aligned}$$

$$+\varepsilon\left[\left\|e^{-\tau s}NQ\frac{U}{s+\lambda}\right\|_2^2+\right.$$
$$\left.\left.+\left\|e^{-\tau s}N\left(Y-RM\right)A^{-1}\frac{V}{s+\lambda}\right\|_2^2-\Gamma\right]\right\}$$
$$=\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(I-e^{-\tau s}NQ\right)\\ \sqrt{\varepsilon}e^{-\tau s}NQ\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$+\inf_{R\in RH_\infty}\left\|e^{-\tau s}N\left(Y-RM\right)A^{-1}\frac{V}{s+\lambda}\right\|_2^2$$

Define

$$J_1^*=\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(I-e^{-\tau s}NQ\right)\\ \sqrt{\varepsilon}e^{-\tau s}NQ\end{matrix}\frac{U}{s+\lambda}\right\|_2^2 \quad (22)$$
$$J_2^*=\inf_{R\in RH_\infty}\left\|e^{-\tau s}N\left(Y-RM\right)A^{-1}\frac{V}{s+\lambda}\right\|_2^2 \quad (23)$$

So, we can get

$$J_\lambda^*=J_1^*+J_2^*-\varepsilon\Gamma$$

Through simple calculation, J_1^* can be expressed as follows:

$$J_1^*=\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}-NQ\right)\\ \sqrt{\varepsilon}NQ\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$=\inf_{Q\in RH_\infty}\left\|\Phi_i\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}-NQ\right)\\ \sqrt{\varepsilon}NQ\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$=\inf_{Q\in RH_\infty}\left\|\begin{matrix}\Theta_i^H\sqrt{1-\varepsilon}\left(e^{\tau s}-\Theta_i\Lambda Q\right)\\ \Theta_i^H\sqrt{\varepsilon}\Theta_i\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$+\inf_{Q\in RH_\infty}\left\|\begin{matrix}\left(I-\Theta_i\Theta_i^H\right)\sqrt{1-\varepsilon}\left(e^{\tau s}-\Theta_i\Lambda Q\right)\\ \left(I-\Theta_i\Theta_i^H\right)\sqrt{\varepsilon}\Theta_i\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$=\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}\Theta_i^H-\Lambda Q\right)\\ \sqrt{\varepsilon}\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$+\left\|\begin{matrix}\sqrt{1-\varepsilon}e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\\ 0\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$=\left\|\begin{matrix}\sqrt{1-\varepsilon}e^{\tau s}\left(\Theta_i^H-\Theta_i^H\left(-\lambda\right)\right)\\ 0\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$+\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}\Theta_i^H\left(-\lambda\right)-\Lambda Q\right)\\ \sqrt{\varepsilon}\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$+\left(1-\varepsilon\right)\left\|e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\frac{U}{s+\lambda}\right\|_2^2$$
$$=\left(1-\varepsilon\right)\left\|e^{\tau s}\left(\Theta_i^H-\Theta_i^H\left(-\lambda\right)\right)\frac{U}{s+\lambda}\right\|$$
$$+\left(1-\varepsilon\right)\left\|e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\frac{U}{s+\lambda}\right\|_2^2$$
$$+\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}\Theta_i^H\left(-\lambda\right)-\Lambda Q\right)\\ \sqrt{\varepsilon}\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$

Here, we define

$$J_{11}^*=\inf_{Q\in RH_\infty}\left\|\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}\Theta_i^H\left(-\lambda\right)-\Lambda Q\right)\\ \sqrt{\varepsilon}\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$

By J_1^*, we have

$$J_{11}^*=\inf_{Q\in RH_\infty}\left\|\Phi_i\begin{matrix}\sqrt{1-\varepsilon}\left(e^{\tau s}\Theta_i^H\left(-\lambda\right)-\Lambda Q\right)\\ \sqrt{\varepsilon}\Lambda Q\end{matrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$=\inf_{Q\in RH_\infty}\left\|\begin{pmatrix}\Theta_i^H\\ I-\Theta_i\Theta_i^H\end{pmatrix}\right.$$
$$\left.\times\left[\begin{pmatrix}\sqrt{1-\varepsilon}Ie^{\tau s}\\ 0\end{pmatrix}+\begin{pmatrix}\sqrt{1-\varepsilon}I\\ \sqrt{\varepsilon}I\end{pmatrix}\Lambda Q\right]\frac{U}{s+\lambda}\right\|_2^2$$
$$=\left\|\left(I-\Theta_i\Theta_i^H\right)\begin{pmatrix}\sqrt{1-\varepsilon}Ie^{\tau s}\\ 0\end{pmatrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$+\inf_{Q\in RH_\infty}\left\|\left[\Theta_i^H\begin{pmatrix}\sqrt{1-\varepsilon}Ie^{\tau s}\\ 0\end{pmatrix}+\Lambda Q\right]\frac{U}{s+\lambda}\right\|_2^2$$

Because $Q\in RH_\infty$, Q can choose appropriately, so

$$\inf_{Q\in RH_\infty}\left\|\left[\Theta_i^H\begin{pmatrix}\sqrt{1-\varepsilon}Ie^{\tau s}\\ 0\end{pmatrix}+\Lambda Q\right]\frac{U}{s+\lambda}\right\|_2^2=0$$

we can obtain

$$J_{11}^*=\left\|\left(I-\Theta_i\Theta_i^H\right)\begin{pmatrix}\sqrt{1-\varepsilon}Ie^{\tau s}\\ 0\end{pmatrix}\frac{U}{s+\lambda}\right\|_2^2$$
$$=\frac{\left(1-\varepsilon\right)\varepsilon}{2\lambda}\sum_{i,j=1}^{n}e^{\tau\left(\bar{z}_i+z_j-2\lambda\right)}\alpha_j^2\left[1-\left[\Theta_i\left(\lambda\right)\right]_j^2\right]$$
$$J_1^*=\left(1-\varepsilon\right)\left\|e^{\tau s}\left(\Theta_i^H-\Theta_i^H\left(-\lambda\right)\right)\frac{U}{s+\lambda}\right\|$$
$$+\left(1-\varepsilon\right)\left\|e^{\tau s}\left(I-\Theta_i\Theta_i^H\right)\frac{U}{s+\lambda}\right\|_2^2$$
$$+\frac{\left(1-\varepsilon\right)\varepsilon}{2\lambda}\sum_{i,j=1}^{n}e^{\tau\left(\bar{z}_i+z_j-2\lambda\right)}\alpha_j^2\left[1-\left[\Theta_i\left(\lambda\right)\right]_j^2\right]$$

From Theorem 1, we can get

$$J_1^*=\frac{1-\varepsilon^2}{2\lambda}\sum_{i,j=1}^{n}e^{\tau\left(\bar{z}_i+z_j-2\lambda\right)}\alpha_j^2\left[1-\left[\Theta_i\left(\lambda\right)\right]_j^2\right]$$
$$J_2^*=\sum_{i,j=1}^{N_p}\frac{\Delta_i^2}{12}e^{\tau\left(\bar{p}_j+p_i-2\lambda\right)}\frac{4Re\left(p_j\right)Re\left(p_i\right)}{\bar{p}_jp_i\left(\bar{p}_j+p_i-2\lambda\right)}tr\left(\gamma_j\gamma_i^H\right)$$
$$J_\lambda^*=\frac{1-\varepsilon^2}{2\lambda}\sum_{i,j=1}^{n}e^{\tau\left(\bar{z}_i+z_j-2\lambda\right)}\alpha_j^2\left[1-\left[\Theta_i\left(\lambda\right)\right]_j^2\right]$$
$$+\sum_{i,j=1}^{N_p}\frac{\Delta_i^2}{12}e^{\tau\left(\bar{p}_j+p_i-2\lambda\right)}\frac{4Re\left(p_j\right)Re\left(p_i\right)}{\bar{p}_jp_i\left(\bar{p}_j+p_i-2\lambda\right)}tr\left(\gamma_j\gamma_i^H\right)-\varepsilon\Gamma$$
$$\gamma_j=\Theta_i^{-1}\left(p_j-\lambda\right)A^{-1}\left(p_j-\lambda\right)G_j\varepsilon_j\varepsilon_j^HH_j$$

This completes the proof.

Remark: The modified factor should satisfy: $Re\left(p_j\right)-\lambda>0$, $Re\left(z_i\right)-\lambda>0$. The proposed method for networked time-delay systems with encoding-decoding and quantization constraints assume that the parameters of the systems are known. For the systems with unknown parameters, one can obtain the system paraemters first by using some identification algorithms [42]–[46] such as the iterative algorithms [47]–[50] and the recursive algorithms [51]–[54].

IV. ILLUSTRATIVE EXAMPLE

Example 1: In this section, the plant is given as

$$G(s) = \begin{pmatrix} \dfrac{s-z}{(s+4)(s-p_1)} \\ \dfrac{s+3}{(s+1)(s-p_2)} \end{pmatrix} e^{-\tau s}$$

where $z > 0, p_1 > 0, p_2 > 0, N = \Theta_i \Lambda$, then

$$\Theta_i(s) = \begin{pmatrix} \dfrac{(s-z+\lambda)(s-p_2+\lambda)}{\sqrt{2}(s-z_1)(s-z_2)} \\ \dfrac{(s+3+\lambda)(s-p_1+\lambda)}{\sqrt{2}(s-z_1)(s-z_2)} \end{pmatrix},$$

$$\Lambda = \frac{\sqrt{2}(s-z_1)(s-z_2)}{(s+2+\lambda)^2}$$

First, we suppose $z = 2, z_1 = 3, z_2 = 4, p_1 = 3.5, p_2 = 5.5, \tau = 0.5, \alpha_j^2 = 5, \lambda = 1$, when the NMP zeros $p = k$, from Theorem 1, we can obtain

$$J_\lambda^* = \frac{\Delta_i^2 e^{k-1}}{3(k-1)} \left[\frac{(k+3)^2(k-4)^2(k-5)^2}{(k-2)^2(k-5.5)^2} + \frac{(k+3)^2(k-4)^2(k-5)^2}{(k+2)^2(k-3.5)^2} \right] + \frac{315}{576}e$$

Then, we assume the quantization interval Δ_i^2 for three different values of $\Delta_1^2 = 1, \Delta_2^2 = 3, \Delta_3^2 = 6$, then we have

$$J_1^* = \frac{e^{k-1}}{3(k-1)} \left[\frac{(k+3)^2(k-4)^2(k-5)^2}{(k-2)^2(k-5.5)^2} + \frac{(k+3)^2(k-4)^2(k-5)^2}{(k+2)^2(k-3.5)^2} \right] + \frac{315}{576}e$$

$$J_2^* = \frac{e^{k-1}}{(k-1)} \left[\frac{(k+3)^2(k-4)^2(k-5)^2}{(k-2)^2(k-5.5)^2} + \frac{(k+3)^2(k-4)^2(k-5)^2}{(k+2)^2(k-3.5)^2} \right] + \frac{315}{576}e$$

$$J_3^* = \frac{2e^{k-1}}{(k-1)} \left[\frac{(k+3)^2(k-4)^2(k-5)^2}{(k-2)^2(k-5.5)^2} + \frac{(k+3)^2(k-4)^2(k-5)^2}{(k+2)^2(k-3.5)^2} \right] + \frac{315}{576}e$$

The minimal modification tracking error of SIMO networked time-delay systems with different quantization error and unstable poles is shown in Fig. 2. Fig. 2 shows that the larger the unstable pole is, the larger the minimal modification tracking error will be. Also, Fig. 2 shows that the minimal modification tracking error is affected by the quantization error, and the smaller the quantization error is, the smaller the minimal modified tracking error will be.

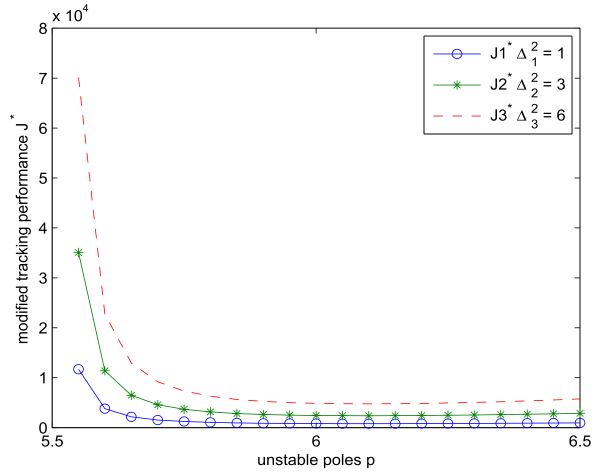

FIGURE 2. Performance with different quantization and unstable poles.

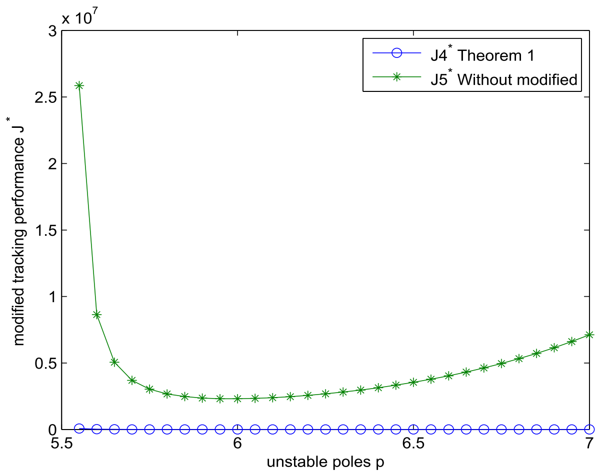

FIGURE 3. Modified performance.

Next, if $\Delta_i^2 = 6$, and other conditions do not change, from Theorem 1, we can obtain

$$J_4^* = \frac{2e^{k-1}}{(k-1)} \left[\frac{(k+3)^2(k-4)^2(k-5)^2}{(k-2)^2(k-5.5)^2} + \frac{(k+3)^2(k-4)^2(k-5)^2}{(k+2)^2(k-3.5)^2} \right] + \frac{315}{576}e$$

If the modification performance does not exist, we can obtain

$$J_5^* = e^k \left(k^2 - 1\right) \left[\frac{(k+3)^2(k-4)^2(k-5)^2}{(k-2)^2(k-5.5)^2} + \frac{(k+3)^2(k-4)^2(k-5)^2}{(k+2)^2(k-3.5)^2} \right] + \frac{315}{576}e$$

The minimum modified tracking error of the networked time-delay systems with different unstable poles is shown as Fig. 3. Fig. 3 shows the effects of the modified performance whether there is modification. It can be seen from the Fig. 3 that

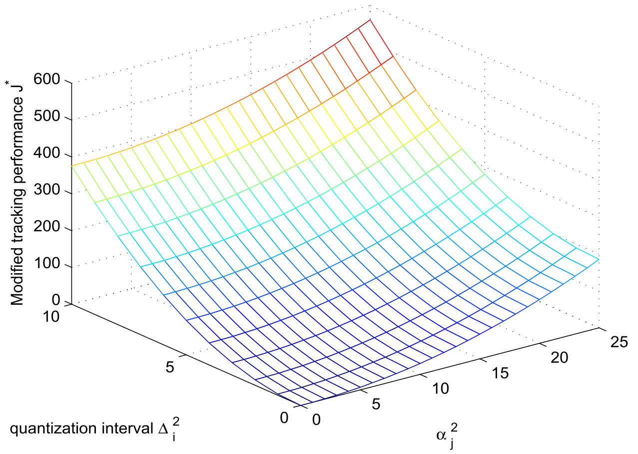

FIGURE 4. Performance with the quantization interval and reference signal.

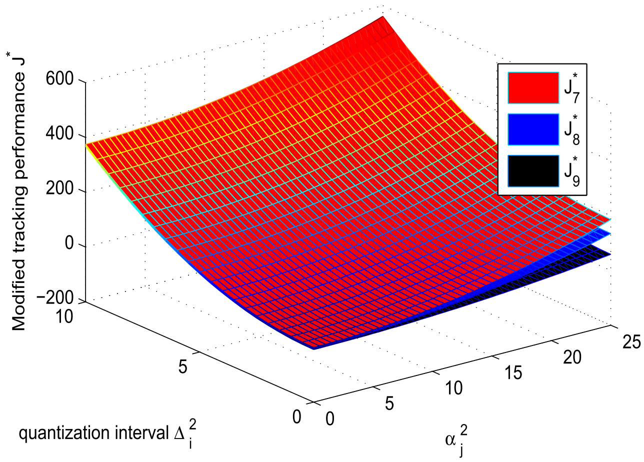

FIGURE 5. Performance with different adjustment factors.

the minimum modified tracking error of the networked time-delay systems using the modified performance index is limited, while the minimum tracking error using the previous performance index is continuously increased to infinity by the number of non-minimum phase zeros.

Example 2: In this section, the other plant is given by

$$G(s) = \begin{pmatrix} \dfrac{s-z}{(s+4)(s-p_1)} \\ \dfrac{s+3}{(s+1)(s-p_2)} \end{pmatrix} e^{-\tau s}$$

where $z > 0, p_1 > 0, p_2 > 0, N = \Theta_i \Lambda$, then

$$\Theta_i(s) = \begin{pmatrix} \dfrac{(s-z+\lambda)(s-p_2+\lambda)}{\sqrt{2}(s-z_1)(s-z_2)} \\ \dfrac{(s+3+\lambda)(s-p_1+\lambda)}{\sqrt{2}(s-z_1)(s-z_2)} \end{pmatrix},$$

$$\Lambda = \frac{\sqrt{2}(s-z_1)(s-z_2)}{(s+2+\lambda)^2}$$

we suppose $z = 2, z_1 = 3, z_2 = 4, p_1 = 3.5, p_2 = 5.5, \tau = 0.5, \alpha_j^2 = 5, \lambda = 1$, when the NMP zeros $p = k$, from Theorem 1, we have

$$J_6^* = \frac{63}{576} e\alpha_j^2 + \frac{37}{120} e^{2.5} \Delta_i^2$$

Fig. 4 shows the influence of the the reference signal and quantization error. In Fig. 4, it can be seen that the reference signal and the quantization error could degrade the minimum modified tracking error.

If the channel constraint $\Gamma = 10$, the for three different values of $\varepsilon_1 = 0, \varepsilon_2 = \frac{1}{2}, \varepsilon_3 = \frac{4}{5}$, from Theorem 2, we can obtain

$$J_7^* = \frac{63}{576} e\alpha_j^2 + \frac{37}{120} e^{2.5} \Delta_i^2$$
$$J_8^* = \frac{63}{768} e\alpha_j^2 + \frac{37}{120} e^{2.5} \Delta_i^2 - 5$$
$$J_9^* = \frac{63}{1600} e\alpha_j^2 + \frac{37}{120} e^{2.5} \Delta_i^2 - 8$$

Fig. 5 show the effects of the the different adjustment factors. In Fig. 5 shows that ε represents the tradeoff factor between the minimum modified tracking error and channel input power, the increase of ε indicates that more channel input power is used for the adjustment of reference input signal, and the better the minimum modified tracking error of the plant is.

V. CONCLUSION

This paper studies the minimum modified tracking error of SIMO networked time-delay systems with quantization and encoding-decoding constraints. Some new results are obtained according to the inner-outer factorization and Cauchy's Theorem with two degrees of freedom controller. The results show that the minimum modified tracking error are related to the inherent properties of the given system, such as non-minimum phase zeros, unstable poles and time delay. In addition, they are also affected by different parameters, such as the scale factor, quantization noise, reference input signal and encoding-decoding. Finally, some illustrative examples are given to illustrate the obtained results. The methods proposed in this paper can combine some parameter estimation approaches [55]–[58] to explore and study the performance of networked time-delay systems with unknown parameters and can be applied to other literatures [59]–[63].

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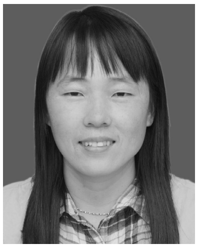

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