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Resource Allocation in Multi-User Cognitive Radio Network With Stackelberg Game

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ABSTRACT Resource allocation with sensing-based interference price is considered for multi-users cognitive radio (CR) network, in which the primary base station (PBS) controls the secondary users' (SUs) transmission by pricing the SUs' interference power. SUs firstly initiates data transmission based on the sensing decision and then PBS sets the interference price according to each SU's interference power. Stackelberg game is formulated to jointly obtain the maximum revenue for PBS and optimize the resource allocation to maximize the transmission gain for SUs. Two practical CR network models are investigated: the sensing based spectrum sharing(SBSS) and the opportunistic spectrum access(OSA). For each scenario, the resource allocation strategy is investigated under the two pricing schemes, namely uniform interference pricing and non-uniform interference pricing. Especially, the stackelberg equilibriums for the proposed games is characterized, and the distributed sensing based interference price bargaining algorithm is proposed according to different channel state information (CSI) for the non-uniform interference pricing case. Numerical examples are carried out to demonstrate the effectiveness of the proposed game algorithm under different pricing scheme.

INDEX TERMS Resource allocation, Stackelberg game, cognitive radio networks, interference management.

I. INTRODUCTION

Fast and reliable wireless communication is becoming a major part of our daily lives such as online shopping, e-health, social networking, etc. The increasing requirements for the wireless communication services will lead to an exponential growth in the spectrum demand. However, the traditional spectrum allocation policies have created the shortage of the availiable spectrum. The reason is that the conventional static spectrum allocation does not match the dynamic spectrum utiliztion and a large increace of mobile terminals are deployed in super density. In order to resolve the problem, cognitive radio [1] as one of the advanced technologies, has been widely used in the Internet of Things (IoT) [2] to utilize scarce spectrum efficiently and intelligently. Developing the CR-based IoT communication systems will be the future development directions for the fifth-generation (5G) wireless networks. There are three main spectrum access model in CR network: opportunistic spectrum access (OSA) [3], [4], spectrum sharing (SS) [5], [6], and sensing-based spectrum sharing (SBSS) [7], [8]. In OSA model, SUs should opportunistically access the spectrum holes without interferencing the PUs' transmission. The SS model allows SUs to simultaneously utilize the licensed band together with PUs if the interference caused by the SUs' transmission is below the interference constraint threshold. In SBSS model, SUs could adapt the transmission power according to the spectrum detectrion results. The SBSS model is especially attractive because the SUs could utilize the licensed spectrum flexibly and achieve high spectral efficiency. However, it has also

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encountered some challenges in its deployments. On one hand, due to the imperfect spectrum sensing, the interference caused by the SUs' transmission should greatly reduce the transmission performance of the primary network. On the other hand, due to the difference of the channel state information of each SU, different muti-user spectrum access strategies will affect the overall performance of the secondary network.

To overcome the performance degradation of the primary network due to the SUs' transmission interference, the interference constraint strategy has recently been introduced into wireless communication networks in [9]–[18]. For example, an adaptive interference alignment (IA) method is proposed to solve the low signal-to-noise-ratio problem caused by the interference power constraint in [9]. In [10], the interference constraint with the imperfect successive interference cancellation is studied to minimize the outage probability. In [11], a cooperative relay selection rule by exploiting spatial diversity is proposed to improve the performance of interference-constrained SUs and achieve the maximum fading-averaged transmission rate in the cooperative CR network. In [13], the orthogonal pilot sharing scheme is proposed and power allocation strategy under the PUs' signal-interference-plus-noise-ratio constraint is investigated to maximize the sum rate of the secondary network. Obviously, the interference power constraint has become one of the most effective methods to protect the quality of service (QoS) of PU in CR network. However, these studies didn't consider the PU's revenue due to the licensed spectrum occupied by SUs. Therefore, it is necessary to consider the optimization of the PUs' transmission benefit under the imperfect spectrum sensing.

Game theory can develop the low-complexity distributed algorithm to describe the relationships among different entities. Hence, game theory is an atractive tool to analyze the flexible and efficient spectrum access strategy in multi-user CR network. In [19], a spectrum pricing and allocation stratedgyies with the stackelberg game is studied for the cognitive multi-homing networks. In [20], the cooperative game based on power control strategies under the interference power constraint is proposed in the CR networks. In [21], a non-cooperative stochastic game is studied to model the path discovery process to resolve the path congestion problem due to mixed attacks in a multi-hop, multi-channel CR network. In [22], the Stackelberg game is used against the spectrum sensing data falsification attack for securing communication in the CR network. In [23], the non-transferable utility coalition formation game theory is considered to solve the SUs' grouping and power allocation problems. Game theory-based resource allocation strategy to maximize the utility of the CR network is also investigated in [24]–[30].

Different from the previous works, we propose a game-theoretic approach using the stackelberg game to jointly optimize the utility of PBS and SUs in the multi-user cognitive radio network. Considering the imperfect spectrum sensing and the interference caused by SUs, the sensing-based interference power price machanism is introduced to constrain the SU transmission and protect the PBS. Then the joint the sensing-based interference power price and resource allocation problem in the sensing-based spectrum sharing model is formulated as a two-stage stackelberg game between the PBS and SUs. The contributions of this paper are summarized as follows:

1). This paper proposes an interference power pricing-based resource allocation stratege for both the PBS and SUs under the imperfect spectrum sensing. Furthermore, to control the transmission power of SUs indirectly, the primary base station sets the interference power price under a maximum tolerable interference power constraint.

2). A stackelberg game is formulated to jointly maximize the revenue of the primary network and the utility of the secondary network for the proposed sensing-based interference pricing strategy. In the formulated stackelberg game, PBS as the leader provides the spectrum resoure when the interference margin could be tolerated and SUs as the followers compete for the accessing opportunity based on the sensing results of the PBS's status. To study the performance of CR network under the different transmission strategies, two different pricing schemes are proposed in this paper: non-uniform sensing-based interference pricing scheme, in which different accessing prices are supplied to the different SUs, and uniform sensing-based interference pricing scheme, in which all the SUs could access the licensed spectrum by using the same price.

3). The stackelberg equilibrium algorithms for the two different pricing strategies are studied under two types of CR network models under imperfect spectrum sensing: the SBSS model where the interference occurs when PBS is correctly detected to be active or incorrectly detected to be idle and the OSA model where the interference occurs only when PBS is incorrectly detected to be idle. The optimal interference price and resource allocation for the two models are obtained to maximize the revenue of PBS and the transmission utility of SUs. Simulation results shows that the revenue of the primary network for the SBSS model is higher than that for the OSA model.

The rest of this paper is organized as follows. The system model and spectrum sensing algorithm in the multi-user CR network are introduced in Section II. The stackelberg game formulated for SBSS and OSA system is investigated in Section III and Section IV, respectively. Section V provides numerical results to validate the proposed studies. Finally, Section VI concludes this paper.

II. SYSTEM MODEL

In this paper, we consider a multi-user CR network consisting of a PBS, a PU and N pairs of secondary transmitter and receiver. It is assumed that the secondary transmitters access the same frequency band licensed by PBS. The channel gain of link from PBS to the secondary transmitter i, from the secondary transmitter i to PU, and from the secondary transmitter i to the secondary receiver j is denoted by $h_{ps,i}$, $h_{sp,i}$,

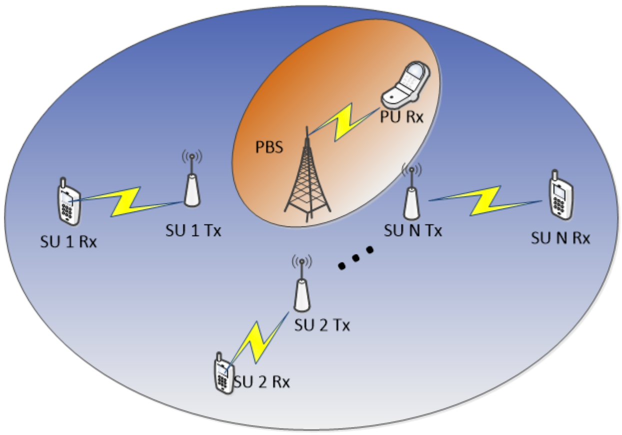

FIGURE 1. System model.

and $g_{i,j}$, $\forall i, j \in \{1, \ldots, N\}$. All the channels are assumed to be block-fading and remain constant. All the channel gains are assumed to be independent and identically distributed random variables. The additive noises are assumed to be independent circularly symmetric complex Gaussian (CSCG) noises with zero mean and variance σ^2. We assume that PU and SUs are synchronized in both time and frequency domains. The frame structure consists of a sensing slot τ and a data transmission slot $T-\tau$. N secondary transmitters detect the PBS's status cooperatively in the sensing slot, then access the same frequency band as PU in the transmission slot. In the sensing slot, we perform cooperative spectrum sensing using multiple distributed secondary transmitters. When PBS is active or inactive, the discrete received signal at the ith secondary transmitter can be represented as:

$$\begin{aligned} H_{i,0} &: Y_i(m) = Z_i(m), \\ H_{i,1} &: Y_i(m) = h_{ps,i}X(m) + Z_i(m) \end{aligned} \tag{1}$$

where $Y_i(m)$ is the mth received sample at the secondary transmitter, $Z_i(m)$ is the noise, and $X(m)$ is the transmitted signal at PU.

Accordingly, the final decision on whether PU is active will be dependent on the detection outcomes on all the N SUs. Suppose the channel from PBS to each SU are known. The maximal ratio combining by data fusion is given by:

$$Y(m) = \sum_{i=1}^{N} \frac{h^*_{ps,i}}{\sum_{i=1}^{N} |h_{ps,i}|^2} Y_i(m) \tag{2}$$

where $h^*_{ps,i}$ is the conjugate complex numbers of $h_{ps,i}$.

For a target probability of detection $\bar{p}_d$ and false alarm $\bar{p}_f$, the probability of false alarm and detection according to [26] is:

$$\begin{aligned} p_f &= Q\left(\xi Q^{-1}(\bar{p}_d) + \sqrt{f_s\tau}\gamma \sum_{i=1}^{N} |h_{ps,i}|^2\right) \\ p_d &= Q\left(\frac{1}{\xi Q^{-1}}(\bar{p}_f) - \sqrt{f_s\tau}\gamma \sum_{i=1}^{N} |h_{ps,i}|^2\right) \end{aligned} \tag{3}$$

where $\xi = \sqrt{2\gamma \sum_{i=1}^{N} |h_{ps,i}|^2 + 1}$, f_s is the sample rate, and γ is the received signa-to-noise ratio (SNR) of PU measured at SUs' interest.

According to the sensing resluts, there are four possible detection probablities: (1) the licensed band is detected to be idle correctly, as $\alpha_0 = p_0\left(1 - p_f\right)$; (2) the licensed band is detected to be busy incorrectly that PBS doesn't ocuppy the band actually, as $\alpha_1 = p_0 - \alpha_0$; (3) the licensed band is detected to be idle incorrectly that PBS ocuppys the band actually, as $\beta_0 = p_1\left(1 - p_d\right)$; (4) the licensed band is detected to be busy correctly, as $\beta_1 = p_1 - \beta_0$, where p_0 and p_1 denote the probability that the band is idle and busy.

III. THE STACKELBERG GAME FOR SBSS SYSTEM

In the sensing-based spectrum sharing model, the SUs adapt their transmission power based on the outcome of the cooperative spectrum sensing. If the licensed band is detected to be idle (H_0), the SU_i transmits with high power $P^{(0)}_{s,i}$, otherwise, the SU_i transmits with low power $P^{(1)}_{s,i}$ to reduce the interference to the primary network. The interference to the PU would occur in the following two cases: the PU is correctly detected to be active and falsely detected to be inactive (missed detection). As a result, the interference power constraints from all the N SUs can be formulated as follows:

$$\frac{T-\tau}{T} \sum_{i=1}^{N} \left(\beta_0 P^{(0)}_{s,i} h_{sp,i} + \beta_1 P^{(1)}_{s,i} h_{sp,i}\right) \leq \Gamma \qquad i \in \{1, \ldots, N\} \tag{4}$$

where Γ represents the maximum interference power that the PU can tolerate.

We assume that the secondary transmission pairs are sparsely in the multi-user CR network. Then the mutual interference between any secondary transmission pair is negligible. Thus, the closed-form price and resource allocation strategies for the stackelberg game model can be obtained easily. Considering that spectrum sensing is not perfect, when the licensed band is idle and PBS is correctly detected to be inactive, the transmission rate of SU_i is $r_{00,i} = \log_2(1 + \frac{P^{(0)}_{s,i} g_{i,i}}{\sigma^2})$; when the licensed band is idle but false alarm occurs, the transmission rate of SU_i is $r_{01,i} = \log_2(1 + \frac{P^{(1)}_{s,i} g_{i,i}}{\sigma^2})$; when the licensed band is busy but missed detection occurs, the transmission rate of SU_i is $r_{10,i} = \log_2(1 + \frac{P^{(0)}_{s,i} g_{i,i}}{P_p h_{ps,i} + \sigma^2})$, where P_p is the PBS's transmission power, $P_p h_{ps,i}$ is the interference to SU_i due to the PBS's transmission; when the licensed band is busy and PBS is correctly detected to be active, the transmission rate of SU_i is $r_{11,i} = \log_2(1 + \frac{P^{(1)}_{s,i} g_{i,i}}{P_p h_{ps,i} + \sigma^2})$.

The resource allocation in the sensing-based spectrum sharing model would be studied by a two-stage Stackelberg game. The Stackelberg game is a strategic game that consists of a leader and several followers competing with each other on certain resources. In this model, PBS is the leader, who protects itself by pricing the interference from SUs. PBS sets up a set of prices about the unit interference power from each SU to maximize its revenue. The SUs are the followers, who pay interference prices and optimize the sensing time and power allocation to maximize the transmission rate gain.

At SU side, the *SUs* compete on the interference price for accessing the licesed band and obtaining the transmission rate gain. The SU_i pays the interference cost to PBS when the licensed band is active, so the ulility of SU_i should only focus on the case that SU_i could cause interference to PBS. Then the utility function of the SU_i is defined as:

$$U_{s,i}\left(l_i,\tau,P_{s,i}^{(0)},P_{s,i}^{(1)}\right)=\eta_i\frac{T-\tau}{T}\left(\beta_0 r_{10,i}+\beta_1 r_{11,i}\right)\\ -l_i\frac{T-\tau}{T}(\beta_0 P_{s,i}^{(0)}h_{sp,i}+\beta_1 P_{s,i}^{(1)}h_{sp,i}) \tag{5}$$

where η_i is the utility gain per unit transmission rate for the SU_i, l_i is the ith SU's cost per unite interference to PBS.

The sensing time and power allocation strategy would be designed to maximize the utility function at SU side. An optimization problem $P1$ can be constructed to maximize the profit of SU_i:

$$\begin{aligned}&\max_{\tau,P_{s,i}^{(0)},P_{s,i}^{(1)}} U_{s,i}\left(\tau,P_{s,i}^{(0)},P_{s,i}^{(1)}\right)\\ &\text{subject to}:\ 0\le P_{s,i}^{(0)}\le P_T^{(0)},\\ &\qquad 0\le P_{s,i}^{(1)}\le P_T^{(1)},\\ &\qquad 0\le\tau\le T\end{aligned} \tag{6}$$

where $P_T^{(0)}$ and $P_T^{(1)}$ represent the transmission power threshold.

At PBS side, to maximize the interference power revenue from all the N SUs is the main objective. The PBS's revenue function is given by:

$$U_{PBS}(l_i)=\frac{T-\tau}{T}\sum_{i=1}^{N}\left(l_i\beta_0 P_{s,i}^{(0)}h_{sp,i}+l_i\beta_1 P_{s,i}^{(1)}h_{sp,i}\right)$$

The optimal interference prices of all the N SUs is needed to maximize the PBS's revenue under the condition that PBS can tolerate the interference from all the N SUs. The problem $P2$ to maximize the PBS's revenue under the SUs' interference constrants can be formulated as:

$$\begin{aligned}&\max_{l_i} U_{PBS}\left(l_i,\tau,P_{s,i}^{(0)},P_{s,i}^{(1)}\right)\\ &\text{subject to}:\ \frac{T-\tau}{T}\sum_{i=1}^{N}\left(\beta_0 P_{s,i}^{(0)}h_{sp,i}+\beta_1 P_{s,i}^{(1)}h_{sp,i}\right)\le\Gamma\\ &\qquad i\in\{1,\ldots,N\}\end{aligned} \tag{7}$$

According to the problem $P1$, the objective function is not convex with respect to the sensing time τ, therefore, convex optimization techniques can not be directly applied. However, for a fixed sensing time, the objective function is convex with respect to $P_{s,i}^{(0)}, P_{s,i}^{(1)}$. Let $P_{s,i}^{(0)*}$ and $P_{s,i}^{(1)*}$ be the optimal power allocation strategies under fixed sensing time. Since $0\le\tau\le T$, the optimal sensing time can be obtained using one-dimensional exhaustive search, that is, $\tau_{opt}=\arg\max_\tau\sum C_i(\tau,P_{s,i}^{(0)*},P_{s,i}^{(1)*})$.

To solve the problem for the stackelberg game, the objective is to find the Stackelberg Equilibrium (SE) point(s) from which neither PBS nor the N SUs have incentive to deviate. Then the SE definition is given as follows:

Definition 1: Let $\boldsymbol{l}^*$ be a solution for the problem $P1$ and $(\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*})$ be a solution for the problem $P2$ of the SU_i for a given sensing time $\hat{\tau}$. Then the point $(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*})$ is a Stackelberg Equilibrium (SE) for the Stackelberg game if for any $(\boldsymbol{l},\hat{\tau},P_{s,i}^{(0)},P_{s,i}^{(1)})$:

$$\begin{aligned}U_{PBS}\left(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*}\right)&\ge U_{PBS}\left(\boldsymbol{l},\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*}\right),\\ U_{s,i}\left(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*}\right)&\ge U_{s,i}\left(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0)},P_{s,i}^{(1)}\right)\\ &\quad i\in\{1,\ldots,N\}\end{aligned}$$

Generally, the SE for the proposed Stackelberg game strategy can be obtained by solving the the problem $P1$ and $P2$. It is easy to observe that the SUs compete for accessing the licensed band in a non-cooperative fashion. For the non-cooperative game between the SUs, the SE point at SUs side can be obtained when each SU cann't improve the utility by changing its strategy and use the current transmission power as the best strategy, which can be defined as $U_{s,i}\left(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*}\right)\ge U_{s,i}\left(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0)},P_{s,i}^{(1)}\right)$. For the stackerberg game, PBS as the only one leader could obtain the optimal revenue by solving the the problem $P2$ for the optimal tramsmission power in the problem $P1$, which is defined as $U_{PBS}\left(\boldsymbol{l}^*,\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*}\right)\ge U_{PBS}\left(\boldsymbol{l},\hat{\tau},P_{s,i}^{(0),*},P_{s,i}^{(1),*}\right)$. Therefore, the SE for the proposed Stackelberg game can be obtained as follows: for a given interference price of SU_i, the problem to maximize the the transmission rate gain is solved firstly, then with the optimal resource allocation of SUs, the optimal interference prices of SUs is obtained to maximize the total PBS's revenue.

The lagrangian with respect to the transmission power $P_{s,i}^{(0)*}$ and $P_{s,i}^{(1)*}$ can be written as:

$$L\left(l_i,P_{s,i}^{(0)},P_{s,i}^{(1)}\right)=\eta_i\frac{T-\hat{\tau}}{T}\left(\beta_0 r_{10,i}+\beta_1 r_{11,i}\right)\\ -l_i\frac{T-\hat{\tau}}{T}(\beta_0 P_{s,i}^{(0)}h_{sp,i}+\beta_1 P_{s,i}^{(1)}h_{sp,i})\\ -\lambda_i^0 P_{s,i}^{(0)}-\lambda_i^1 P_{s,i}^{(1)}$$

where $\hat{\tau}$ is the given sensing time, λ_i^0 and λ_i^1 are Lagrangian dual variables.

The dual objective function can be expressed as:

$$g\left(P_{s,i}^{(0)},P_{s,i}^{(1)},\lambda_i^0,\lambda_i^1\right)=\max_{P_{s,i}^{(0)},P_{s,i}^{(1)}} L\left(P_{s,i}^{(0)},P_{s,i}^{(1)},\lambda_i^0,\lambda_i^1\right)$$

Considering that the jointly optimization problem is convex with respect to the transmission power $P_{s,i}^{(0)}$ and $P_{s,i}^{(1)}$, respectively. Therefore, the problem can be solved by using the dual decomposition method. Then the joint optimization problem can be decomposed into two optimization subproblems p1 and p2:

$$
\begin{aligned}
p0: \max_{P_{s,i}^{(0)}} \eta_i \frac{T-\hat{\tau}}{T}\beta_0 r_{10,i} - l_i \frac{T-\hat{\tau}}{T}\beta_0 P_{s,i}^{(0)} h_{sp,i} - \lambda_i^0 P_{s,i}^{(0)} \\
p1: \max_{P_{s,i}^{(1)}} \eta_i \frac{T-\hat{\tau}}{T}\beta_1 r_{11,i} - l_i \frac{T-\hat{\tau}}{T}\beta_1 P_{s,i}^{(1)} h_{sp,i} - \lambda_i^1 P_{s,i}^{(1)}
\end{aligned} \tag{8}
$$

By writing the Lagrangian function $L_{p0}\left(P_{s,i}^{(0)}, \lambda_i^0\right)$ of the subproblem p0, the KKT conditions of the problem p1 can be given as:

$$
\begin{aligned}
&\frac{\partial L_{p0}\left(P_{s,i}^{(0)}, \lambda_i^0\right)}{\partial P_{s,i}^{(0)}} = 0 \\
&\lambda_i^0 (P_{s,i}^{(0)} - P_T^{(0)}) = 0 \\
&P_{s,i}^{(0)} - P_T^{(0)} \leq 0 \\
&P_{s,i}^{(0)} \geq 0, \lambda_i^0 \geq 0, \quad \forall i
\end{aligned} \tag{9}
$$

After applying the KKT condition, the optimal high power allocation, $P_{s,i}^{(0)*}$, for a given interference price l_i, is given by

$$
P_{s,i}^{(0)*} = \left(\frac{\eta_i \beta_0}{l_i \beta_0 h_{sp,i} + \lambda_i^0} - \frac{\sigma_1^2}{g_{i,i}}\right)^+ \tag{10}
$$

where $\sigma_1^2 = P_p h_{ps,i} + \sigma^2$ is the noise and the PBS interference to the SU_i.

The optimization algorithm for the low transmission power ($P_{s,i}^{(0)*}$) is achieved similarly as the high transmission power, and is thus omitted. Then the optimal low power allocation, $P_{s,i}^{(1)*}$, for a given interference price l_i, is given by

$$
P_{s,i}^{(1)*} = \left(\frac{\eta_i \beta_1}{l_i \beta_1 h_{sp,i} + \lambda_i^1} - \frac{\sigma_1^2}{g_{i,i}}\right)^+ \tag{11}
$$

From the equation (9) and (10), the SU_i is allowed to transmit only When the interference price satisfies, $l_i \geq \max\{\frac{\eta_i g_{i,i}}{\sigma_1^2 h_{sp,i}} - \frac{\sigma_1 \lambda_i^0}{\beta_0 h_{sp,i}}, \frac{\eta_i g_{i,i}}{\sigma_1^2 h_{sp,i}} - \frac{\sigma_1 \lambda_i^1}{\beta_1 h_{sp,i}}\}$. The problem P2 to maximize the PBS's revenue under the SUs' interference constrants can be formulated as:

$$
\begin{aligned}
\max_{l_i} \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \left(\frac{l_i h_{sp,i} \eta_i \beta_0^2}{l_i h_{sp,i} \beta_0 + \lambda_i^0} - \frac{l_i \beta_0 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \\
& + \left(\frac{l_i h_{sp,i} \eta_i \beta_1^2}{l_i h_{sp,i} \beta_1 + \lambda_i^1} - \frac{l_i \beta_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \\
\text{subject to}: \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \left(\frac{h_{sp,i} \eta_i \beta_0^2}{l_i h_{sp,i} \beta_0 + \lambda_i^0} - \frac{\beta_0 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \\
& + \left(\frac{h_{sp,i} \eta_i \beta_1^2}{l_i h_{sp,i} \beta_1 + \lambda_i^1} - \frac{\beta_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \leq \Gamma, \\
& l_i \geq 0, i \in \{1, \ldots, N\}
\end{aligned} \tag{12}
$$

Note that the above problem is non-convex, since the object function is a non-convex function of $\boldsymbol{l}$. In order to ensure the QoS of PBS, the transmission power of SU_i need to be lower than the transmission power threshold. According to the equations $\lambda_i^0(P_{s,i}^{(0)} - P_T^{(0)}) = 0$ and $\lambda_i^1(P_{s,i}^{(1)} - P_T^{(1)}) = 0$, we have that λ_i^0 and λ_i^1 are equal to zero. Then the problem P2 is reformulated as:

$$
\begin{aligned}
\max_{l_i} \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \left(\eta_i \beta_0 - \frac{l_i \beta_0 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \\
& + \left(\eta_i \beta_1 - \frac{l_i \beta_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \\
\text{subject to}: \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \left(\frac{\eta_i \beta_0}{l_i} - \frac{\beta_0 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \\
& + \left(\frac{\eta_i \beta_1}{l_i} - \frac{\beta_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right)^+ \leq \Gamma, \\
& l_i \geq 0, i \in \{1, \ldots, N\}
\end{aligned} \tag{13}
$$

It is obviously that the obove problem is also non-convex. However, the problem can be converted to a series of convex subproblems by introducing the following indicator function.

$$
\chi_i = \begin{cases} 1, & l_i < \dfrac{\eta_i g_{i,i}}{h_{sp,i} \sigma_1^2}, \\ 0, & \text{otherwise} \end{cases}
$$

With the above indicator functions, the optimal problem of the PBS's revenue can be given as:

$$
\begin{aligned}
\max_{l_i} \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \chi_i \left(\eta_i \beta_0 - \frac{l_i \beta_0 h_{sp,i} \sigma_1^2}{g_{i,i}}\right) \\
& + \chi_i \left(\eta_i \beta_1 - \frac{l_i \beta_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right) \\
\text{subject to}: \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \chi_i \left(\frac{\eta_i \beta_0}{l_i} - \frac{\beta_0 h_{sp,i} \sigma_1^2}{g_{i,i}}\right) \\
& + \chi_i \left(\frac{\eta_i \beta_1}{l_i} - \frac{\beta_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right) \leq \Gamma, \\
& \chi_i \in \{0, 1\}, i \in \{1, \ldots, N\}
\end{aligned} \tag{14}
$$

It is easy to find that the above problem is convex for a given indicator vector $[\chi_1, \chi_2, \ldots, \chi_N]$. If the interference power threshold Γ is set to be large enough by PBS, all the SUs is allowed to access the licensed band. Then the indicators for all SUs are equal to 1. Under the special case, the problem P2 is transformed to the following form.

$$
\begin{aligned}
\min_{l_i} \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \frac{l_i p_1 h_{sp,i} \sigma_1^2}{g_{i,i}} \\
\text{subject to}: \; & \frac{T-\hat{\tau}}{T} \sum_{i=1}^{N} \left(\frac{\eta_i p_1}{l_i} - \frac{p_1 h_{sp,i} \sigma_1^2}{g_{i,i}}\right) \leq \Gamma
\end{aligned} \tag{15}
$$

We can solve the problem using its lagrange dual. The Lagrangian can be written as:

$$L(\boldsymbol{l}, \lambda, \boldsymbol{\mu}) = \frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{l_i h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} - \lambda\left(\frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{\eta_i p_1}{l_i} - \frac{p_1 h_{sp,i}\sigma_1^2}{g_{i,i}} - \Gamma\right) + \sum_{i=1}^{N}\mu_i l_i$$

Then the KKT conditions can be formulated as:

$$\frac{\partial L(\boldsymbol{l}, \lambda, \boldsymbol{\mu})}{\partial l_i} = 0$$

$$\lambda\left(\frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{\eta_i p_1}{l_i} - \frac{p_1 h_{sp,i}\sigma_1^2}{g_{i,i}} - \Gamma\right) = 0$$

$$\frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{\eta_i p_1}{l_i} - \frac{p_1 h_{sp,i}\sigma_1^2}{g_{i,i}} - \Gamma \le 0$$

$$\lambda \ge 0, \quad \mu_i \ge 0, \; l_i \ge 0, \; \mu_i l_i = 0$$

Proposition 1: By applying the KKT conditions, the optimal solution to the problem P2 is given by

$$l_i^* = \sqrt{\frac{p_1\eta_i g_{i,i}}{h_{sp,i}\sigma_1^2}}\frac{\sum_{i=1}^{N}\sqrt{p_1\eta_i\frac{h_{sp,i}\sigma_1^2}{g_{i,i}}}}{\sum_{i=1}^{N}\frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} + \frac{T}{T-\hat{\tau}}\Gamma} \tag{16}$$

Proof: In order to obtain the interference price, the lagrangian dual variables λ, μ_i must be determined firstly. From the equation $\frac{\partial L(\boldsymbol{l},\lambda,\boldsymbol{\mu})}{\partial l_i} = 0$, we have:

$$l_i^2 = \frac{(T-\hat{\tau})\lambda p_1\eta_i g_{i,i}}{(T-\hat{\tau})l_i h_{sp,i}\sigma_1^2 p_1 - T\mu_i g_{i,i}}, \quad i \in 1,\ldots,N \tag{17}$$

where λ, μ_i are the non-negative dual variables.

If $\mu_i \neq 0$, it follows that $l_i = 0$ to the equation $\mu_i l_i = 0$. Due to the sensing time $\hat{\tau}$ is less than the transmission time slot T and $p_1\eta_i g_{i,i} \neq 0$, it indicates that $\lambda = 0$ from (14). However, this contradicts the inequation $\frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{p_1\eta_i}{l_i} - \frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} - \Gamma \le 0$. Then, we conclude that $\mu_i = 0, \lambda \neq 0$. According to the equation $\lambda(\frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{p_1\eta_i}{l_i} - \frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} - \Gamma) = 0$, we have $(\frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\frac{p_1\eta_i}{l_i} - \frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} - \Gamma) = 0$.

Thus, the lagrangian dual variable λ is equal to $\left(\frac{\sum_{i=1}^{N}\sqrt{p_1\eta_i\frac{h_{sp,i}\sigma_1^2}{g_{i,i}}}}{\sum_{i=1}^{N}\frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} + \frac{T}{T-\hat{\tau}}\Gamma}\right)^2$. Then the optimal sensing-based interference price is proved.

Proposition 2: Assuming that all the SUs are sorted by the order, $\frac{\eta_1 g_{1,1}}{h_{sp,1}} > \ldots > \frac{\eta_i g_{i,i}}{h_{sp,i}} > \ldots > \frac{\eta_N g_{N,N}}{h_{sp,N}}$, the interference price l_i^* for the ith SU is the optimal solution of (6) if and only if $\Gamma > \Psi_N$, where $\Psi_N = \frac{T-\hat{\tau}}{T}\left(\frac{\sum_{i=1}^{N}\sqrt{\frac{\eta_i h_{sp,i}\sigma_1^2 p_1}{g_{i,i}}}}{\sqrt{\frac{\eta_N g_{N,N} p_1}{h_{sp,N}\sigma_1^2}}} - \sum_{i=1}^{N}\frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}}\right)$

Proof: According to the designed interference price that $l_i \ge \max\{\frac{\eta_i g_{i,i}}{\sigma_1^2 h_{sp,i}} - \frac{\sigma_1\lambda_i^0}{\beta_0 h_{sp,i}}, \frac{\eta_i g_{i,i}}{\sigma_1^2 h_{sp,i}} - \frac{\sigma_1\lambda_i^1}{\beta_1 h_{sp,i}}\}$, the interference threshold can be obtained that $\Gamma > \Psi_N$ when the transmission power of SU_i need to be lower than the transmission power threshold, $\forall i \in \{1,\ldots,N\}$, where $\Psi_N = \frac{T-\hat{\tau}}{T}\left(\frac{\sum_{i=1}^{N}\sqrt{\frac{\eta_i h_{sp,i}\sigma_1^2 p_1}{g_{i,i}}}}{\sqrt{\frac{\eta_i g_{i,i} p_1}{h_{sp,i}\sigma_1^2}}} - \sum_{i=1}^{N}\frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}}\right)$

Furthermore, if all the SUs are sorted by the following order, $\frac{\eta_1 g_{1,1}}{h_{sp,1}} > \ldots > \frac{\eta_i g_{i,i}}{h_{sp,i}} > \ldots > \frac{\eta_N g_{N,N}}{h_{sp,N}}$, the inequalities can be rewritten as: $\Gamma > \Psi_N$. Thus, the "if" part is proved.

Assuming that the range of Γ is obtained as follows: $\Psi_{N-1} < \Gamma \le \Psi_{N-1}$, then it follows that $l_N < \frac{\eta_N g_{N,N}}{h_{sp,N}\sigma_1^2}$ and $P_{s,N}^{(0)*} = 0, P_{s,N}^{(1)*} = 0$. Thus, the "only if" part is proved.

With the above proposition, the optimal non-uniform interference price for Problem P2 is given in the following theorem.

Theorem 1: Assuming that all the SUs are sorted by the order, $\frac{\eta_1 g_{1,1}}{h_{sp,1}} > \ldots > \frac{\eta_i g_{i,i}}{h_{sp,i}} > \ldots > \frac{\eta_N g_{N,N}}{h_{sp,N}}$, the optimal solutions of (12) under different interference threshold values is given as follows.

$$\boldsymbol{l}^* = \begin{cases} q_N\left[\sqrt{\frac{\eta_1 g_{1,1}}{h_{sp,1}\sigma_1^2}},\ldots,\sqrt{\frac{\eta_N g_{N,N}}{h_{sp,N}\sigma_1^2}}\right]^T, & \Gamma > \Psi_N \\ q_{N-1}\left[\sqrt{\frac{\eta_1 g_{1,1}}{h_{sp,1}\sigma_1^2}},\ldots,\sqrt{\frac{\eta_{N-1} g_{N-1,N-1}}{h_{sp,N-1}\sigma_1^2}},\infty\right]^T, & \\ \quad \Psi_N \ge \Gamma > \Psi_{N-1} & \\ \vdots \quad \vdots & \\ q_1\left[\sqrt{\frac{\eta_1 g_{1,1}}{h_{sp,1}\sigma_1^2}},\ldots,\infty,\infty\right]^T, & \Psi_2 \ge \Gamma > \Psi_1 \end{cases}$$

where $q_m = \frac{\sum_{i=1}^{m}\sqrt{\eta_i\frac{h_{sp,i}\sigma_1^2}{g_{i,i}}}}{\sum_{i=1}^{m}\frac{h_{sp,i}\sigma_1^2 p_1}{g_{i,i}} + \frac{T}{T-\hat{\tau}}\Gamma}, \forall m \in \{1,\ldots,N\}$.

From the theorem 1, the interference pricing scheme is completely solved. If the interference price for SU_i is set to be ∞, SU_i is not allowed to transmit; If all the SUs are allowed to transmit, the interference threshold is set to be above Ψ_N. Then the SE for the Stackelberg game is given as follows.

Proposition 3: The point $(\tau^*, P_{s,i}^{(0)*}, P_{s,i}^{(1)*}, l_i^*)$ is the SE for the Stackelberg game in the problem P1 and P2, where $(\tau^*, P_{s,i}^{(0)*}, P_{s,i}^{(1)*})$ is the optimal solution of P1, and l_i^* is the optimal solution P2.

To this end, the proposed non-uniform interference pricing scheme for the SBSS model is summarized in Algorithm 1. In this algorithm, the complexity to obtain the optimal interference price and transmission is linear to the number of the SUs, i.e., $O(2N^2)$. The optimal sensing time can be obtained

Algorithm 1 The Non-Uniform Sensing-Based Interference Pricing Scheme in SBSS

Input:
Channel gain: $h_{sp,i}$, $h_{ps,i}$, $g_{i,i}$;
Unit transmission gain: η_i;
Noise variance: σ^2;
Probability of channel activity: p_1
Maximum iteration times: T_{max}
output:
Interference price: l_i^*;
Resource allocation: $P_{s,i}^{(0)*}$, $P_{s,i}^{(1)*}$, τ^*
While t<T_{max}, do

1 Obtain the optimal interference price as follows:
  (1) Set m = N, $1 \leq m \leq N$.
  (2) Sort the N SUs in the order $\frac{\eta_1 g_{1,1}}{h_{sp,1}} > \ldots > \frac{\eta_i g_{i,i}}{h_{sp,i}} > \ldots > \frac{\eta_N g_{N,N}}{h_{sp,N}}$.
  (3) Compute $Q_{0,m}$, $Q_{1,m}$, $Q_m = \max\{Q_{0,m}, Q_{1,m}\}$.
  (4) If $\Gamma \leq Q_m$ set $m = m-1$, repeat (3), (4); otherwise, the interference price for SU i is given by:

$$l_i = \begin{cases} q_m \sqrt{\frac{\eta_i g_{i,i}}{h_{sp,i}\sigma_1^2}}, & i \leq m \\ \infty, & \text{otherwise} \end{cases}$$

2 Obtain the optimal power allocation $P_{s,i}^{(0)*}$, $P_{s,i}^{(1)*}$ according to (10), (11);
3 Obtain the optimal sensing time τ^* by the bisection method;

by the bisection method with the complexity $O(2N^3 log_2(\varsigma))$, where ς is the required accuracy. Then, the complexity of the proposed picing scheme algorithm is $O(2T_{max}N^3 log_2(\varsigma))$, where T_{max} is the maximum iteration times.

IV. THE STACKELBERG GAME FOR OSA SYSTEM

In this section, the stackelberg game is introduced in the multi-user OSA model where the SUs simultaneously detect PBS's activity firstly, then access the licensed band only when the PBS is detected to be inactive. The interference is caused to PU when the PU is falsely detected to be inactive (missed detection). The interference power constraint can be written as follows:

$$\frac{T-\tau}{T}\sum_{i=1}^{N}\beta_0 P_{s,i}h_{sp,i} \leq \Gamma, \quad i \in \{1,\ldots,N\} \tag{18}$$

where Γ represents the maximum interference power that the PBS can tolerate.

For the Stackelberg game in the multi-user OSA model, the PBS as the leader obtains the revenue only when SUs falsely detect the PBS's status. The SUs as the followers compete on the interference price for the maximum throughput gain. We also assume the SU pairs are sparse and the mutual interference between any pair of SUs is negligible. Thus, the transmission rate of SU_i is $r_i = \log_2(1 + \frac{P_{s,i}g_{i,i}}{\sigma_1^2})$.

At SU side, SUs also compete with each other to access the licensed band. However, different from the SBSS model, The SU_i only need pay the interference cost to PBS when the miss detection occurs, Then the optimization problem of the utility function in the OAS model can be formulated as $P3$:

$$\max_{l_i,\tau,P_{s,i}} \frac{T-\tau}{T}(\eta_i\beta_0 r_i - l_i\beta_0 P_{s,i}h_{sp,i})$$
$$\text{subject to}: P_{s,i} \geq 0, \quad 0 \leq \tau \leq T \tag{19}$$

At the PBS side, the main objective to maximize the interference power revenue from all the N SUs under the SUs' interference constraints can be formulated as $P4$:

$$\max_{l_i} \frac{T-\tau}{T}\sum_{i=1}^{N} l_i\beta_0 P_{s,i}h_{sp,i}$$
$$\text{subject to}: \frac{T-\tau}{T}\sum_{i=1}^{N}\beta_0 P_{s,i}h_{sp,i} \leq \Gamma \tag{20}$$

To solve the problem for the stackelberg game in the OSA model, the stackelberg Equilibrium (SE) point(s) is also need to be found. Then the SE definition is given as follows:

Definition 2: Let $\boldsymbol{l}^*$ be a solution for the problem $P4$ and $(\hat{\tau}, P_{s,i}^*)$ be a solution for the problem $P3$ of the SU_i for a given sensing time $\hat{\tau}$. Then the point $(\boldsymbol{l}^*, \hat{\tau}, P_{s,i}^*)$ is a SE for the Stackelberg game if for any $(\boldsymbol{l}, \hat{\tau}, P_{s,i})$:

$$U_{PBS}\left(\boldsymbol{l}^*, \hat{\tau}, P_{s,i}^*\right) \geq U_{PBS}\left(\boldsymbol{l}, \hat{\tau}, P_{s,i}^*\right),$$
$$U_{s,i}\left(\boldsymbol{l}^*, \hat{\tau}, P_{s,i}^*\right) \geq U_{s,i}\left(\boldsymbol{l}^*, \hat{\tau}, P_{s,i}\right)$$
$$i \in \{1,\ldots,N\}$$

For the Stackelberg game, the SE can be obtained similar to the case in the SBSS model. The optimal sensing time is obtained by the one-dimensional exhaustive search method due to the non-convex problem, $\tau_{opt} = \arg\max_\tau \sum C_i\left(\tau, \epsilon, P_{s,i}^*\right)$.

By using the Lagrangian and applying the KKT conditions, the optimal transmit power for a given sensing time can be obtained as follows. The optimal power allocation in the opportunistic access model, $P_{s,i}^*$, for a given interference price l_i, is given by

$$P_{s,i}^* = \left(\frac{\eta_i}{l_i h_{sp,i}} - \frac{\sigma_1^2}{g_{i,i}}\right)^+ \tag{21}$$

It is easy to find that only if $l_i < \frac{\eta_i g_{i,i}}{\sigma_1^2 h_{sp,i}}$, the SU_i will transmit.

When the interference price is designed as, $\frac{\eta_i g_{i,i}}{\sigma_1^2 h_{sp,i}} > l_i$, the SU_i will transmit data when PBS is detected to be idle. The problem to maximize the PBS's revenue under the SUs' interference constraints can be formulated as:

$$\max_{l_i} \frac{T-\hat{\tau}}{T}\sum_{i=1}^{N} l_i\beta_0 h_{sp,i}\left(\frac{\eta_i}{l_i h_{sp,i}} - \frac{\sigma_1^2}{g_{i,i}}\right)$$

$$\text{subject to}: \frac{T-\hat{\tau}}{T}\sum_{i=1}^{N}\beta_0 h_{sp,i}\Big(\frac{\eta_i}{l_i\beta_0 h_{sp,i}}-\frac{\sigma_1^2}{g_{i,i}}\Big)\leq\Gamma$$
$$l_i\geq 0,\quad i\in\{1,\ldots,N\} \tag{22}$$

The optimization problem P4 can be solved by using the same method for Problem P2. Thus the solution of P4 can be achieved by the following proposition.

Proposition 4: The optimization problem can be solved by using Lagrange dual. According to the KKT conditions, the optimal interference price for the ith SU is given by:

$$l_i^*=\sqrt{\frac{\eta_i g_{i,i}}{h_{sp,i}\sigma_1^2}}\frac{\sum_{i=1}^{N}\sqrt{\eta_i\frac{h_{sp,i}\sigma_1^2}{g_{i,i}}}}{\sum_{i=1}^{N}\frac{h_{sp,i}\sigma_1^2\beta_0}{g_{i,i}}+\frac{T}{T-\hat{\tau}}\Gamma} \tag{23}$$

Due to difference in the channel state Information of SUs, the minimum interference threshold could be given by the following proposition.

Proposition 5: Assuming that all the N SUs are sorted by the order, $\frac{\eta_1 g_{1,1}}{h_{sp,1}}>\ldots>\frac{\eta_i g_{i,i}}{h_{sp,i}}>\ldots>\frac{\eta_N g_{N,N}}{h_{sp,N}}$, the interference price l_i^* for the ith SU is the optimal solution of (22) if and only if $\widetilde{Q}_{0,N}<\Gamma$, where $\widetilde{Q}_{0,N}=\frac{T-\hat{\tau}}{T}\left(\beta_0\frac{\sum_{i=1}^{N}\sqrt{\frac{\eta_i h_{sp,i}\sigma_1^2}{g_{i,i}}}}{\sqrt{\frac{\eta_N g_{N,N}}{h_{sp,N}\sigma_1^2}}}-\sum_{i=1}^{N}\frac{\beta_0 h_{sp,i}\sigma_1^2}{g_{i,i}}\right)$.

In order to maximize the PBS's revenue, the non-uniform interference pricing scheme should be further studied. Then the non-uniform interference price according to different interference threshold is given as follows.

Theorem 2: Assuming that all the N SUs are sorted by the order, $\frac{\eta_1 g_{1,1}}{h_{sp,1}}>\ldots>\frac{\eta_i g_{i,i}}{h_{sp,i}}>\ldots>\frac{\eta_N g_{N,N}}{h_{sp,N}}$, the optimal solutions of (10) under different interference threshold values is given as follows.

$$\boldsymbol{l}^*=\begin{cases}\tilde{q}_N[\sqrt{\frac{\eta_1 g_{1,1}}{h_{sp,1}\sigma_1^2}},\ldots,\sqrt{\frac{\eta_N g_{N,N}}{h_{sp,N}\sigma_1^2}}]^T,\\ \quad\Gamma>\widetilde{Q}_{0,N}\\ \tilde{q}_{N-1}[\sqrt{\frac{\eta_1 g_{1,1}}{h_{sp,1}\sigma_1^2}},\ldots,\sqrt{\frac{\eta_{N-1} g_{N-1,N-1}}{h_{sp,N-1}\sigma_1^2}},\infty]^T,\\ \quad\widetilde{Q}_{0,N}\geq\Gamma>\widetilde{Q}_{0,N-1}\\ \vdots\quad\vdots\\ \tilde{q}_1[\sqrt{\frac{\eta_1 g_{1,1}}{h_{sp,1}\sigma_1^2}},\ldots,\infty,\infty]^T,\\ \quad\widetilde{Q}_{0,2}\geq\Gamma>\widetilde{Q}_{0,1}\end{cases}$$

where $\tilde{q}_m=\frac{\sum_{n=1}^{m}\sqrt{\frac{\eta_i h_{sp,i}\sigma_1^2}{g_{i,i}}}}{\sum_{n=1}^{m}\frac{\beta_0\eta_i h_{sp,i}\sigma_1^2}{g_{i,i}}+\frac{T}{T-\hat{\tau}}\Gamma}\forall m\in\{1,\ldots,N\}$.

From the theorem, it is obviously that the optimal interference price is unique for a fixed interference threshold. Therefore, the SE for the proposed stackelberg game in the OSA model is also unique.

Proposition 6: The point $(\tau^*, P_{s,i}^*, \boldsymbol{l}^*)$ is SE point for the Stackelberg game in the problem P3 and P4, where $(\tau^*, P_{s,i}^*)$ is the optimal solution of P3, $(\boldsymbol{l}^*)$ is the optimal solution of P4.

Algorithm 2 The Non-Uniform Sensing-Based Interference Pricing Scheme in OSA

Input:
Channel gain: $h_{sp,i}$, $h_{ps,i}$, $g_{i,i}$;
Unit transmission gain: η_i;
Noise variance: σ^2;
Maximum iteration times: T_{max}
output:
Interference price: l_i^*;
Resource allocation: $P_{s,i}^*$, τ^*
While t<T_{max}, do

1 Obtain the optimal interference price as follows:
 (1) Set m = N, $1\leq m\leq N$.
 (2) Sort the N SUs in the order $\frac{\eta_1 g_{1,1}}{h_{sp,1}}>\ldots>\frac{\eta_i g_{i,i}}{h_{sp,i}}>\ldots>\frac{\eta_N g_{N,N}}{h_{sp,N}}$.
 (3) Compute $\widetilde{Q}_{0,m}$. If $\Gamma\leq Q_m$ set $m=m-1$, repeat (3); otherwise, the interference price for SU i is given by:

$$l_i=\begin{cases}\tilde{q}_m\sqrt{\frac{\eta_i g_{i,i}}{h_{sp,i}\sigma_1^2}}, & i\leq m\\ \infty, & \text{otherwise}\end{cases}$$

2 Obtain the optimal power allocation $P_{s,i}^*$ according to (18);
3 Obtain the optimal sensing time τ^* by the bisection method;

The optimal interference price for SU i can be obtained by the following algorithm 2. In the algorithm 2, the complexity to obtain the optimal interference price is linear to the number of the SUs, i.e., $O(N)$. Due to SUs competing with each other only when the spectrum is detected to be idle, the complexity to obtain the optimal transmission power is $O(2N^2)$. The optimal sensing time can be obtained by the bisection method with the complexity $O(N^3 log_2(\zeta))$, where ζ is the required accuracy. Then, the complexity of the proposed picing scheme algorithm is $O(\tilde{T}_{max}N^3 log_2(\zeta))$, where $\tilde{T}_{max}$ is the maximum iteration times.

V. SIMULATION RESULTS

In this section, simulation results are presented to evaluate the performance of the proposed strategies in multi-user CR networks. The channel gains are assumed to be ergodic, stationary and exponentially distributed with unit mean. The optimal sensing time is assumed to be 5ms.

In Fig.2 and 3, the PBS revenue and the sum rate of SUs versus the maximum interference power margin at PBS with non-uniform pricing strategy is presented in the SBSS or OSA model, respectively, when the probabilities that the licensed frequency bands are idle equal to $P(H_0)=0.6$ and $P(H_0)=0.8$. In physics, when the probability $P(H_0)$ is

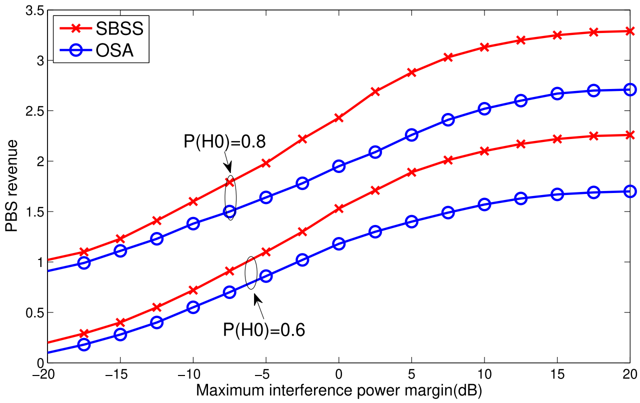

FIGURE 2. The PBS revenue vs Γ in different system model.

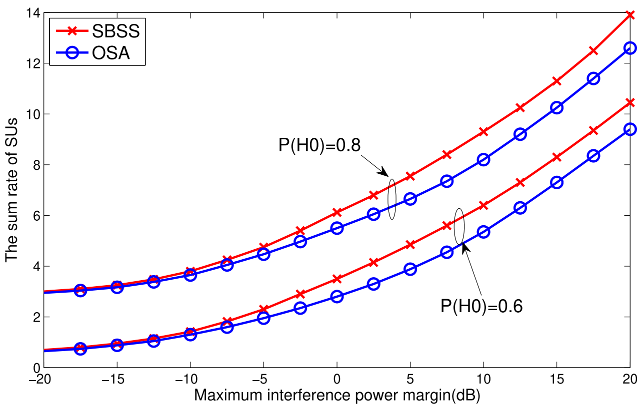

FIGURE 3. The sum rate of SUs vs Γ in different system model.

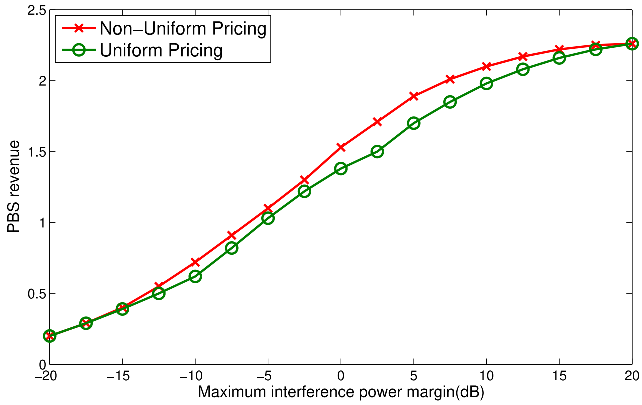

FIGURE 4. The PBS revenue vs Γ with different pricing strategy.

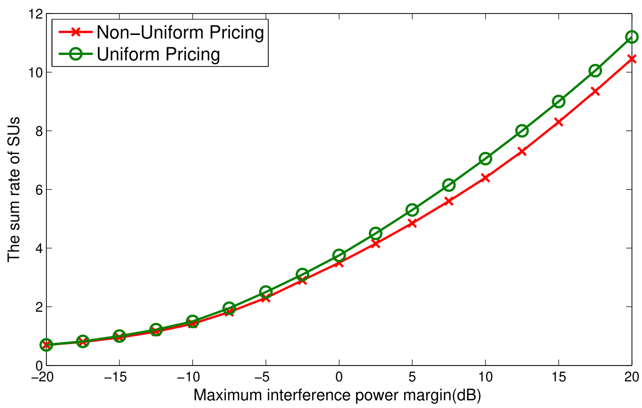

FIGURE 5. The sum rate of SUs vs Γ with different pricing strategy.

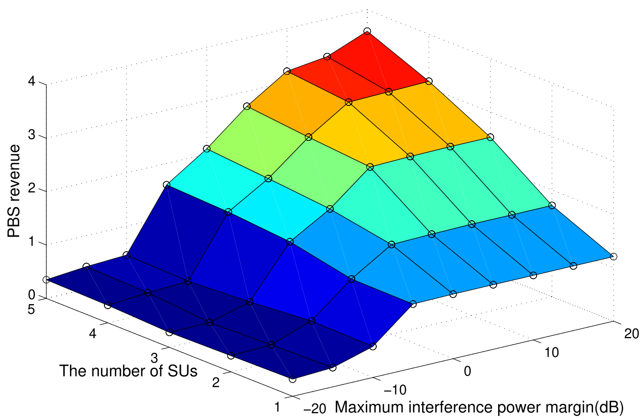

FIGURE 6. The PBS revenue vs Γ and the number of SUs with non-uniform pricing strategy.

0.8, SUs could have more oppotunity to access the licensed spectrum compared to the other cases when the probability $P(H_0)$ is 0.6. It indicates that the PBS revenue and the sum rate of SUs increases with the probability P(H0) increases before reaching the maximum point. When the maximum interference power margin receives low values, the performance of the two models is almost the same. It indicates that the licensed spectrum is only allowed to access at the period when the frequency band is idle. When the maximum interference power margin receives higher values, the more PU revenue and sum rate of SUs is obtained in SBSS model compared to that in OSA model. It indicates that PU control the use of the licensed spectrum by the given interference power margin. This also can be explained that PBS would like to given lower access threshold if it wants more benefits in SBSS model.

In Fig.4 and 5, the PBS revenue and the sum rate of SUs versus the maximum interference power margin at PBS with non-uniform pricing or uniform pricing strategy is presented in the SBSS model, respectively. The probability that the licensed frequency bands are idle equal to $P = 0.6$. It is observed that the PBS revenue and the sum rate of SUs is almost same for the two pricing schemes, when the interference power margin is sufficiently small. It indicates that the more sensitive PBS is to the interference of SUs, less SUs could be allowed to access the licensed spectrum, and thus the non-uniform pricing scheme is almost same as the uniform pricing scheme. It is also observed that for the same interference threshold Γ, the PBS's revenue under the uniform pricing scheme is general larger than that under uniform pricing scheme, whereas the sum rate of SUs under the same scheme is reverse. It is indicated that according to the non-uniform pricing scheme, the SU with weaker channel state should pay a higher price to consume the resource of CR network.

In Fig.6, the PBS's revenue versus the maximum interference power margin and the number of SUs with non-uniform pricing is presented in the SBSS model. The probability that the licensed frequency bands are idle equal to P = 0.6. It is observed that the PBS's revenue is almost same for the different number of SUs when the interfenrence threshold Γ is very small. That is because that only one SU is allowed to access the licensed band when PBS is strict about the performance of the primary network. It is also observed that when the interfenrence threshold Γ is sufficiantly large, the PBS's revenue is growing and converges to the same value for the different number of SUs. The reason is that the number of SUs that allowed to access the licensed band increases with the increase of interfence threshold. It is indicated that the interference price for SUs decreases with the increase of interference threshold and the PBS's revenue coverages to $\sum_{i=1}^{N} p_1 \eta_i$ when Γ goes to infinity according to the equation (12).

VI. CONCLUSION

In this paper, the stackelberg game is formulated in the sensing based spectrum sharing and opportunistic spectrum access model for multi-user CR networks. In the stackelberg game CR system, the sensing based interference price is studied to protect PU and enable the revenue of PU. The sensing based interference algorithms in the two models is proposed to solve the problem of the maximum revenue of PU and optimizing the resource allocation to maximize the transmission gain for the SUs. Simulation results have revealed that the SUs could obtain the higher throughput and the PU could obtain the higher interference power revenue in the SBSS model compared to that happen in OSA model with imperfect spectrum sensing.

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