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A Cooperative Lane Change Model for Connected and Automated Vehicles

TINGTING LI[1], JIANPING WU[1], CHING-YAO CHAN[2], MINGYU LIU[1], CHUNLI ZHU[1], WEIXIN LU[3], AND KEZHEN HU[4]

[1]Department of Civil Engineering, Tsinghua University, Beijing 100084, China
[2]California PATH, University of California at Berkeley, CA 94804, USA
[3]School of Electronic Engineering and Computer Science, Peking University, Beijing 100871, China
[4]China Academy of Information and Communication Technology, Beijing 100191, China

Corresponding author: Jianping Wu (jianpingwu@mail.tsinghua.edu.cn)

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ABSTRACT The emerging technology of vehicle-to-vehicle (V2V) communication, vehicle-to-infrastructure (V2I) communication makes it possible for vehicles to sense the environment information, which can be exploited to assist the vehicle in cooperative motion planning. In this paper, we focus on the cooperative trajectory planning of lane changes for connected and automated vehicles (CAVs). The proposed model considers the traffic scene with multiple mandatory lane change demands and completes the trajectory planning for vehicles by taking the safety and efficiency into consideration. The model solves two critical issues: the vehicle grouping and the motion planning. In the first issue, CAVs in the cooperative zone are divided into different groups. Then the problem is simplified and divided into several subproblems. In the second issue, the trajectory planning is conducted in each group. Trajectories are generated for vehicles with and without lane change demands. Besides, these two steps are iterated and updated in the fixed time interval, which makes full use of the dynamic cooperation ability of vehicles. Extensive simulation tests are conducted to validate the performance of the model. Results show that the cooperation of vehicles realizes safe and effective lane changes.

INDEX TERMS Lane change, connected and automated vehicles, motion planning, cooperative driving, collision avoidance.

I. INTRODUCTION

The lane change is one of the common and essential operations of vehicles, which has significant influence on the traffic safety and efficiency [1]. In the aspect of the safety influence, the study by Dijck and Heijden [2] demonstrated that about 4% to 10% accidents were caused by inappropriate lane changes. The Netherlands transportation statistics showed that 12.6% of traffic accidents resulted from lane changes [3]. Analysis of traffic accidents in Canada revealed that lane changes were responsible for about 10% of crash fatalities [4]. Data analysis by the Highway Traffic Safety Administration in China indicated that 60% of traffic crashes on freeway could be attributed to the lane change [5]. The crash data in US show that annually there were between 240,000 to 610,000 reported lane change crashes, leading to 60 thousand injured [6]. In the aspect of efficiency influence, Laval and Daganzo [7] treated the lane change as an obstacle on the road, which interfered the stability of traffic flow. You *et al.* [8] also put forward that lane change was a key factor for the traffic congestion. It is evident from the large body of literatures that the execution of safe and proper lane changes can play an important role in controlling and maintaining the smooth and efficient traffic flow.

This paper focuses on the problem of multi-vehicle cooperative lane change maneuvers for connected and automated vehicles (CAVs). The emerging technology of vehicle-to-vehicle (V2V) communication, vehicle-to-infrastructure (V2I) communication and environmental sensing enable CAVs drive cooperatively on the road [9]–[11]. Based on the communication among vehicles, Kim *et al.* [12] researched a multimodal cooperative perception system which could provide broader views for the early obstacle avoidance during the lane change. Considering different degrees of V2V

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collaboration, Bai *et al.* [13] proposed an accelerated lane change trajectory model, which turned out to be feasible and useful. Kato *et al.* [14] validated that the cooperative lane change could not only improve the road capacity, but also decreased the time for the lane change. Even a single vehicle is not able to change lane successfully in the congested traffic without cooperation with surrounding vehicles [15].

There is a wide spectrum of the previous research for the lane change maneuver of a single vehicle [10], [16]–[20]. As for cooperative lane changes, the existing work can be classified into five categories. In the first category, there is no specific lane change cooperation rule. Each lane change is achieved according to the specific envelop [21] or the vehicular kinematics equation [8], [22]–[25]. Only when limitation equations of the inter-vehicle traffic gap and time instance are met, the lane change is allowed. To improve the traffic efficiency, the variable desired gap is also put forward, which considers the information of immediate surrounding vehicles and ensures the flexible gap [26], [27]. However, if there is no appropriate lane change gap or cooperative zone, the lane change operation will be delayed or cancelled. As a result, the lane change demand of vehicles may not be met and the cooperation capacity is limited.

In the second category, the lane change sequence is determined by the longitudinal coordinate [28]. The cooperation among vehicles means the execution of the lane change sequence. Vehicles wait for their turns to change lanes in sequence. This method ensures the avoidance of conflict, but it is not suitable for a long road segment. Besides, the cooperation capability of CAV is not fully utilized, which is limited by simplified rules.

In the third category, the CAV platoon is reconfigured to a sufficient sparse formation firstly. Then all lane changes are executed at the same time without avoiding collisions [29]. The sparse formation is generated by widening the inter-vehicle gap via acceleration or deceleration. Although all lane change demands are met, the traffic flow is interfered seriously. Furthermore, the original lane change demand may be eliminated when the traffic flow is sparse enough.

In the fourth category, the vehicle platoon is divided into several groups [1]. Lane changes in different groups are executed simultaneously. In each group, the vehicle changes lane one by one according to the longitudinal coordinate. Thus, there exists unnecessary wait for vehicles in the group. In this case, the kinematic capability of CAVs is limited.

The fifth category is related to the data-driven method, the cooperative lane change is realized by the deep learning model. The Gaussian mixture model, Continuous Hidden Markov Model, Deep Belief Network and Long Short-term Memory neural network are utilized to model and predict the lane change process [30]–[32]. The reinforcement learning model is also deployed to train the motion of vehicles [33]. In the training process, the velocity and acceleration of following vehicles on the target lane and the current lane is adjusted to yield to the lane-change operation of the ego vehicle, which may lead to the large oscillation on the traffic flow.

According to previous studies, the cooperative motion planning is required for vehicles to realize safe and effective lane changes. The main contribution of this paper is to develop a dynamic cooperative planning model for lane changes of autonomous driving. It divides the traffic flow into groups which simplifies the problem. Another advantage of the model over other models is that it allows vehicles to cooperate with others, which means that vehicles accelerate and decelerate to create the proper gap for the mandatory lane change operation. It exploits the cooperative ability of CAVs and makes full use of time and space resources.

To give a better explanation of the proposed model, the remainder of this paper is organized as follows. Section II explains the problem formulation. Section III presents the model of the vehicle grouping. Section IV shows the trajectory planning model. Section V depicts the collision avoidance rule. Section VI elaborates the process of the cooperative trajectory planning. Section VII analyzes simulation experiments and results. Section VIII exhibits conclusions. Fig. 1 shows the flowchart of the proposed model.

II. PROBLEM FORMULATION

The lane change is classified as the discretionary lane change or the mandatory lane change depending on the decision-making process and the impact on the surrounding traffic. A discretionary lane change occurs when a vehicle tries to obtain a better driving environment for the speed advantage. The objective of a mandatory lane change is to reach the planned destination by following a certain path [17]. In this paper, we focus on the mandatory lane change.

The schematic representation of the lane change demand scene is depicted in Fig. 2. Before arriving at the intersection, vehicles should change to the specific lane. There are possible conflicts among trajectories of these vehicles. Thus, it is necessary to develop a cooperative lane change trajectory planning model.

To facilitate the model development, we make some assumptions:

(1) The high-quality V2V and V2I communications are assumed to be available. That is, there is no data loss and delay for V2V and V2I connections.

(2) CAVs are equipped with sensors which measure the dynamic information of the position, velocity, acceleration, jerk and direction angle.

(3) The real-time velocity, acceleration, jerk, position, direction angle and lane change demand of vehicles are shared among all vehicles on the road.

III. VEHICLE GROUPING

Imaging that there are several CAVs travelling on the road and they are far from each other. If they change lane simultaneously, they will have little influence on each other and do not need to consider the trajectory of another. However, when a vehicle is surrounded by other vehicles, its actions will exert influence on the subsequent motion of adjacent vehicles. It is difficult to realize the cooperative driving of all

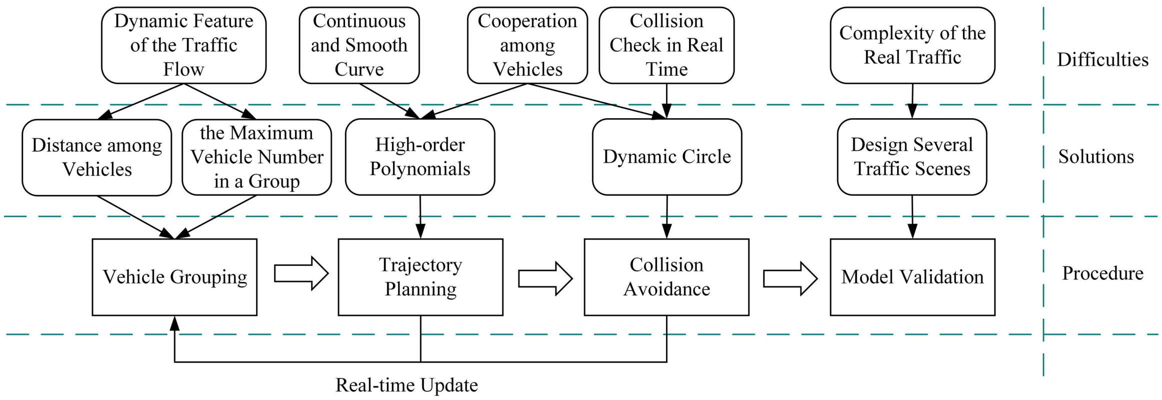

FIGURE 1. Flowchart of the proposed model.

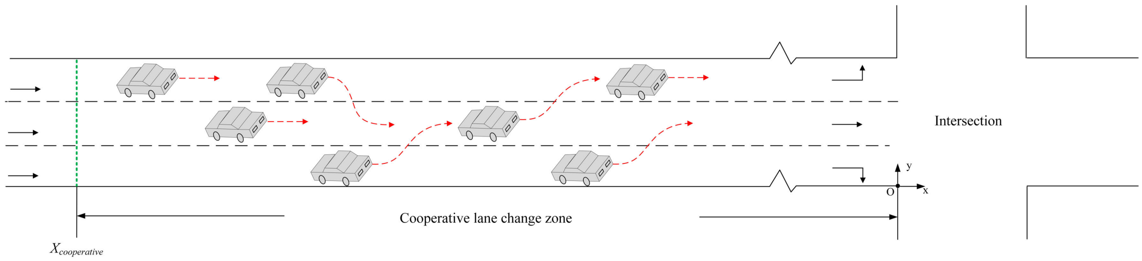

FIGURE 2. Schematic of the mandatory lane change problem.

vehicles on the road owing to the complexity of the traffic flow. Thus, we propose a grouping method which divides CAVs into groups. Then the problem of the traffic flow is broken up into subproblems of vehicle groups, which simplifies the problem.

For the vehicle on the road, parameters of its surrounding vehicles will vary all the time. It is difficult to ensure the effectiveness of the long-time planning trajectory. To solve the problem, we propose the on-line vehicle grouping and trajectory planning. The vehicle grouping is updated in the fixed time interval T_{update}. The trajectory is also planned in the fixed time gap T_{update}. As Fig. 2 shows, the dynamic cooperative lane change zone is defined, which is aimed to assist vehicles to realize the cooperative lane change. The variable $X_{cooperative}$ is defined as the bound of the cooperative zone. Once the vehicle enters into the zone, the action will be planned in the cooperative mandatory lane change model. Otherwise, it will not be considered in the cooperative lane change operation and can drive without the limit of the cooperative model. That is, for vehicles in the cooperative zone, the vehicle grouping and trajectory planning are conducted in each T_{update} seconds.

The detailed process of grouping is expressed as follows:

(1) All vehicles in the cooperative zone are sorted by longitudinal coordinates in the ascending order. The number of vehicles is assigned as V.
(2) Define the maximum vehicle number in one group as N.
(3) The vehicle with maximum longitudinal coordinate is divided into the Group 1.
(4) For the vehicle 2, if the longitudinal distance between the vehicle 1 and the vehicle 2 is less than $G_{2,desired}$ and the vehicle number in the group 1 is less than N. Then the vehicle 2 is a part of the group 1. Otherwise, the vehicle 2 belongs to the group 2.
(5) For the vehicle 3, if the longitudinal distance between the vehicle 2 and 3 is less than $G_{3,desired}$ and the vehicle number in the group of the vehicle 2 is less than N. Then the vehicle 3 is divided into the same group with the vehicle 2. Otherwise, the vehicle 3 belongs to a new group.
(6) Repeat the grouping rule for the vehicle $4, 5, 6, \ldots, V$, which is similar with (5). The grouping process ends when all vehicles belong to specific groups.
(7) Repeat the step (1)-(6) and update the vehicle grouping result in each T_{update} seconds.

The variable N is the maximum vehicle number of the group, which ensures the size of the group is in the reasonable range. The variable $G_{i,desired}$ is determined by the desired gap in the intelligent driver model (IDM) [27], [34], which ensures that vehicles with possible conflicts are divided into the same group. $G_{i,desired}$ is defined in Eq (1),

$$G_{i,desired}(t) = G_{\min} + \max\{T_{safe} v_{i,x}(t) + \frac{v_{i,x}(t)\Delta v_{i,x}(t)}{2\sqrt{a_{x,\max} b_{x,\max}}}, 0\} \tag{1}$$

where $G_{\min}$ is the static safe distance, T_{safe} is the safe time interval, $v_{i,x}$(t) is the velocity of the vehicle i, $\Delta v_{i,x}$(t) is the velocity difference between the vehicle i and the vehicle in front. Based on the previous study, values of parameters are set as follows: $G_{\min} = 2m$, $T_{safe} = 1.5s$, $b_{x,\max} = 1.67m/s^2$ [35], [36].

Once the grouping is completed, the trajectory planning is conducted. Trajectories of vehicles in the preceding group are planned firstly. Then trajectory planning of vehicles in the following group should take trajectories of the preceding groups into consideration. That is, the trajectory conflict among and in groups is considered.

IV. TRAJECTORY PLANNING

The trajectory planning has influence on the safety and comfortability of the driving and the efficiency of the traffic flow. Thus, the generated trajectory is required to be continuous and smooth. Besides, the computation cost is expected to be as low as possible. In our model, the high-order polynomials are applied to determine the trajectory, which has advantages over other trajectory planning models. On the one hand, it ensures the smoothness of the motion with the continuous second derivative. On the other hand, the motion is generated with a small number of points, requiring the low computational cost.

Relying on initial and target states, trajectories are generated with polynomials. The trajectory in x and y coordinates are described by the function $f(x,t)$ and $f(y,t)$. Combining the vehicle dynamic feature, the lane structure, the collision avoidance and the state of the vehicle we select the most appropriate trajectory from a class of functions. Moreover, considering the maximum available longitudinal velocity (30m/s) is much higher than the lateral velocity (2.5m/s), we define the one more freedom in the x direction. Specifically, the 6th and 5th order polynomials are employed in the x and y direction respectively.

$$f(x,t) = T_{1\times 7} \cdot A \tag{2}$$
$$f(y,t) = T_{1\times 6} \cdot B \tag{3}$$

where

$$\begin{aligned} A^T &= (a_6, a_5, a_4, a_3, a_2, a_1, a_0) \\ T_{1\times 7} &= (t^6, t^5, t^4, t^3, t^2, t^1, 1) \\ B^T &= (b_5, b_4, b_3, b_2, b_1, b_0) \\ T_{1\times 6} &= (t^5, t^4, t^3, t^2, t^1, 1) \end{aligned}$$

Initial and target states of the vehicle are defined as

$$\begin{aligned} S_{in} &= (f(x,t_{in}), f(\dot{x},t_{in}), f(\ddot{x},t_{in}), f(y,t_{in}), f(\dot{y},t_{in}), f(\ddot{y},t_{in})) \\ S_{fin} &= (f(x,t_{fin}), f(\dot{x},t_{fin}), f(\ddot{x},t_{fin}), f(y,t_{fin}), f(\dot{y},t_{fin}), \\ &\quad f(\ddot{y},t_{fin})) \end{aligned}$$

where variables $x, \dot{x}, \ddot{x}$ are the longitudinal location, velocity and acceleration of the vehicle. $y, \dot{y}, \ddot{y}$ are the lateral location, velocity and acceleration of the vehicle. In this paper, we focus on the mandatory lane change, which divides the decision making and trajectory planning into two parts. The result of the lane-change decision making is known before the trajectory planning. That is, the expected final state in the y direction is known in advance. Then the trajectory planning problem is transferred into the boundary condition problem: exploring a smooth path that guides the vehicle from the initial state to the final state. The key of the problem is to solve the coefficient matrix A^T, B^T and the final state depending on Eqs. (4) - (5). Variables t_{in} and t_{fin} are the initial and final time instant for the trajectory.

$$\begin{aligned} T_{6\times 7} \cdot A^T &= [f(x,t_{in}), f(\dot{x},t_{in}), f(\ddot{x},t_{in}), f(x,t_{fin}), f(\dot{x},t_{fin}), \\ &\quad f(\ddot{x},t_{fin})] \end{aligned} \tag{4}$$
$$\begin{aligned} T_{6\times 6} \cdot B^T &= [f(y,t_{in}), f(\dot{y},t_{in}), f(\ddot{y},t_{in}), f(y,t_{fin}), f(\dot{y},t_{fin}), \\ &\quad f(\ddot{y},t_{fin})] \end{aligned} \tag{5}$$

Matrixes $T_{6\times 7}$ and $T_{6\times 6}$ are derived as

$$T_{6\times 7} = \begin{bmatrix} t_{in}^6 & t_{in}^5 & t_{in}^4 & t_{in}^3 & t_{in}^2 & t_{in} & 1 \\ 6t_{in}^5 & 5t_{in}^4 & 4t_{in}^3 & 3t_{in}^2 & 2t_{in} & 1 & 0 \\ 30t_{in}^4 & 20t_{in}^3 & 12t_{in}^2 & 6t_{in} & 2 & 0 & 0 \\ t_{fin}^6 & t_{fin}^5 & t_{fin}^4 & t_{fin}^3 & t_{fin}^2 & t_{fin} & 1 \\ 6t_{fin}^5 & 5t_{fin}^4 & 4t_{fin}^3 & 3t_{fin}^2 & 2t_{fin} & 1 & 0 \\ 30t_{fin}^4 & 20t_{fin}^3 & 12t_{fin}^2 & 6t_{fin} & 2 & 0 & 0 \end{bmatrix}$$

$$T_{6\times 6} = \begin{bmatrix} t_{in}^5 & t_{in}^4 & t_{in}^3 & t_{in}^2 & t_{in} & 1 \\ 5t_{in}^4 & 4t_{in}^3 & 3t_{in}^2 & 2t_{in} & 1 & 0 \\ 20t_{in}^3 & 12t_{in}^2 & 6t_{in} & 2 & 0 & 0 \\ t_{fin}^5 & t_{fin}^4 & t_{fin}^3 & t_{fin}^2 & t_{fin} & 1 \\ 5t_{fin}^4 & 4t_{fin}^3 & 3t_{fin}^2 & 2t_{fin} & 1 & 0 \\ 20t_{fin}^3 & 12t_{fin}^2 & 6t_{fin} & 2 & 0 & 0 \end{bmatrix}$$

In the proposed cooperative lane change model, the trajectory of the vehicle with and without the lane change demand will be generated based on the polynomial model. According Eqs. (4)-(5), the value of matrixes A^T, B^T and the value of the final position, velocity and time are unknown, i.e., 16 parameters. But there are 12 equations available. The 4 unknown parameters are determined based on the collision avoidance algorithm and vehicle dynamic constraints.

V. COLLISION AVOIDANCE

There are numerous approaches to solve the collision avoidance problem. In this paper, we regard the vehicle as the combination of infinite dynamic circles. Compared with other methods, the dynamic circle model transforms the problem into the computation of the geometry relationship between circles, which approximates the vehicle into the 2-D space and simplifies the computation. Furthermore, the real-time collision checking is realized relying on the high-order polynomial functions. As Fig. 3 shows, the rectangular region, which represents the shape of the vehicle, is depicted by the region swept by infinite dynamic circles with the radius R. The diameter of the circle equals to the width of the vehicle. Then the coordinate of the center for each circle follows the Eq. (6)

$$\begin{cases} x = x_{c0} + \lambda(x_{cn} - x_{c0}) \\ y = y_{c0} + \lambda(y_{cn} - y_{c0}) \end{cases} \tag{6}$$

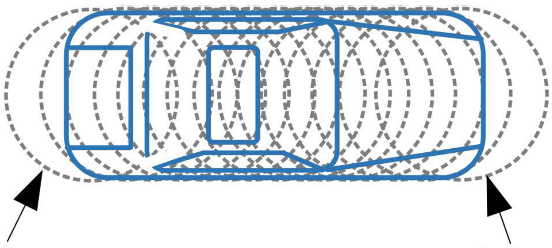

FIGURE 3. Vehicle representation by infinite dynamic circles.

where (x_{c0}, y_{c0}) and (x_{cn}, y_{cn}) are coordinates of the center of circles C_0 and C_n. The value of λ varies from 0 to 1.

The collision avoidance of the vehicle 1 and 2 is the collision avoidance of arbitrary two circles of the corresponding vehicle representation, that is

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 > (R_1 + R_2)^2 \tag{7}$$

where x_1 and x_2 are the center of the arbitrary circle for the vehicle 1 and 2. R_1 and R_2 are radii of dynamic circles of these two vehicles. Combining Eq. (6)-(7), we can obtain the Eq. (8),

$$\begin{aligned}&((x_{c01}(t) + \lambda_1 \cdot (x_{cn1}(t) - x_{c01}(t))) - (x_{c02}(t)\\&\quad +\lambda_2 \cdot (x_{cn2}(t) - x_{c02}(t))))^2\\&\quad +((y_{c01}(t) + \lambda_1 \cdot (y_{cn1}(t) - y_{c01}(t))) - (y_{c02}(t)\\&\quad +\lambda_2 \cdot (y_{cn2}(t) - y_{c02}(t))))^2\\&\quad > (R_1 + R_2)^2\end{aligned} \tag{8}$$

where $\lambda_1 \in [0, 1]$, $\lambda_2 \in [0, 1]$, $x_{c01}(t)$ and $x_{c02}(t)$ are coordinates of centers of the circle C_0 in the x direction, $y_{c01}(t)$ and $y_{c02}(t)$ are coordinates of centers of the circle C_0 in the y direction, $x_{cn1}(t)$ and $x_{cn2}(t)$ are coordinates of centers of the circle C_n in the x direction, $y_{cn1}(t)$ and $y_{cn2}(t)$ are coordinates of centers of the circle C_n in the y direction.

Then the collision avoidance is ensured if the Eq. (8) is satisfied during the lane-change period. Thus, the admissible value for variables in Eqs. (4)-(5) can be acquired. In order to find the optimum value of parameters, the problem is considered as the non-linear optimization problem.

VI. COOPERATIVE TRAJECTORY PLANNING IN THE GROUP

A. TRAJECTORY PLANNING FOR A SINGLE VEHICLE

Considering the vehicle 1 with the mandatory lane change demand, the motion planning for the lane change operation should take the safety, comfortability and efficiency into consideration. Thus, the objective function and constraints are determined by these factors.

1) DESIGN OF THE OBJECTIVE FUNCTION

Except for the safety, the comfortability is the first requirement for the driving. The comfortability can be evaluated by the acceleration and jerk of the motion in Eq. (9),

$$\min J_1 = w_1 \frac{\int_{t_{in}}^{t_{fin}} j_x^2(\tau)d\tau}{j_{x,\max} a_{x,\max}} + w_2 \frac{\int_{t_{in}}^{t_{fin}} j_y^2(\tau)d\tau}{j_{y,\max} a_{y,\max}} \tag{9}$$

where J_1 is the value of the comfortability function, w_1 and w_2 are weight coefficients, j_x(t) and j_y(t) are the value of the jerk at time t, $j_{x,\max}$ and $j_{y,\max}$ are the maximum available jerk in x and y directions, $a_{x,\max}$ and $a_{y,\max}$ are the maximum acceleration in x and y directions.

The ideal lane change operation has low influence on the surrounding traffic. Vehicles tend to accelerate to accomplish the lane change in a shorter time gap in the real traffic flow. The longitudinal velocity is expected to approach the desired velocity. Therefore, the second part of the objective function is determined as

$$\min J_2 = w_3(\dot{x}_{fin} - \dot{x}_{desired})^2 + w_4(t_{fin} - t_{in}) \tag{10}$$

where J_2 is the value of the function of the velocity and time, w_3 *and* w_4 are weight coefficients, $\dot{x}_{fin}$ is the final velocity for the planning trajectory, $\dot{x}_{desired}$ is the longitudinal desired velocity of the vehicle.

2) CONSTRAINTS

It is necessary to satisfy some constraints which consider characteristics of the vehicle and the traffic. Firstly, the traffic rule must to be obeyed, so the velocity of the vehicle should satisfy the requirement of the speed limit of the road. Secondly, the feature of the vehicular dynamics determines the maximum acceleration of the motion. Thirdly, the comfortability of driving limits the maximum jerk. Fourthly, the lateral motion in each planning should be smaller than the width of a lane, that is, the consecutive two lane-change operations are forbidden. The rule is consistent with the traffic rule. Fifthly, the collision avoidance criterion with other vehicles should be met. Sixthly, the final state should allow the vehicle to accelerate to 0 before the stop line of the intersection. Constraints are depicted as follows,

$$\begin{aligned}&|\dot{x}_i(t)| \le v_{x,\max}\\&|\dot{y}_i(t)| \le v_{y,\max}\\&|\ddot{x}_i(t)| \le a_{x,\max}\\&|\ddot{y}_i(t)| \le a_{y,\max}\\&|\dddot{x}_i(t)| \le j_{x,\max}\\&\left|\dddot{y}_i(t)\right| \le j_{y,\max}\\&|y_i(t) - y_i(t_{in})| < L_{width}\\&((x_{c01}(t) + \lambda_1 \cdot (x_{cn1}(t) - x_{c01}(t))) - (x_{c0h}(t)\\&+\lambda_h \cdot (x_{cnh}(t) - x_{c0h}(t))))^2\\&+((y_{c01}(t) + \lambda_1 \cdot (y_{cn1}(t) - y_{c01}(t))) - (y_{c0h}(t)\\&+\lambda_h \cdot (y_{cnh}(t) - y_{c0h}(t))))^2\\&> (R_1 + R_h)^2\\&f(\dot{x}_i, t_{fin})^2/(2 \cdot a_{x,\max}) < \left|f(x_i, t_{fin})\right|\end{aligned} \tag{11}$$

where $v_{x,\max}$ and $v_{y,\max}$ are the maximum velocities in the x and y directions, x_{c0h}(t) and x_{cnh}(t) are lateral coordinates of the center for the front and rear-end dynamic circle of the surrounding vehicle, L_{width} is the width of the vehicle, R_h is the radius of dynamic circles of the vehicle h which is one of the surrounding vehicle of the vehicle 1.

The trajectory planning problem is transformed into the non-linear optimization problem. Relying on the objective function and constraints, the lane-change trajectory of the single vehicle can be obtained.

B. TRAJECTORY PLANNING FOR MULTIPLE VEHICLES

1) TRAJECTORY IN THE SAME GROUP

In the section, trajectories of vehicles in the same group are planned simultaneously. Possible collisions in the one group are classified into two types: the first one is the trajectory conflict between vehicles with and without lane change demands. In this scene, the lateral motion of the vehicle without lane change demand keeps 0. The second one is the motion conflict between vehicles with lane change demands. In the scene, lateral trajectories of vehicles tend to be smooth curves. Considering these two scenes, we design the final state and lane change time of vehicles as unknown variables. Besides, trajectories of vehicles with and without lane change demands are generated together under the collision avoidance criterion. Then the trajectory planning problem is solved effectively. Based on the motion planning for a single vehicle, we define the trajectory planning model for multiple vehicles as follows,

$$
\begin{aligned}
\min & J(t_{fin1}, x_{fin1}, t_{fin2}, x_{fin2}, \ldots, t_{finG}, x_{finG}) \\
&= w_{11}\frac{\int_{t_{in1}}^{t_{fin1}} j_{x1}^2(\tau)d\tau}{j_{x1,\max}a_{x1,\max}} + w_{12}\frac{\int_{t_{in1}}^{t_{fin1}} j_{y1}^2(\tau)d\tau}{j_{y1,\max}a_{y1,\max}} \\
&\quad + w_{13}(\dot{x}_{fin1} - \dot{x}_{desired1})^2 + w_{14}(t_{fin1} - t_{in1}) \\
&\quad + w_{21}\frac{\int_{t_{in2}}^{t_{fin2}} j_{x2}^2(\tau)d\tau}{j_{x2,\max}a_{x2,\max}} + w_{22}\frac{\int_{t_{in2}}^{t_{fin2}} j_{y2}^2(\tau)d\tau}{j_{y2,\max}a_{y2,\max}} \\
&\quad + w_{23}(\dot{x}_{fin2} - \dot{x}_{desired2})^2 + w_{24}(t_{fin2} - t_{in2}) \\
&\quad + \ldots + w_{G1}\frac{\int_{t_{inG}}^{t_{finG}} j_{xG}^2(\tau)d\tau}{j_{xG,\max}a_{xG,\max}} + w_{G2}\frac{\int_{t_{inG}}^{t_{finG}} j_{yG}^2(\tau)d\tau}{j_{yG,\max}a_{yG,\max}} \\
&\quad + w_{G3}(\dot{x}_{finG} - \dot{x}_{desiredG})^2 + w_{G4}(t_{finG} - t_{inG}) \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t. } & |\dot{x}_i(t)| \le v_{x,\max} \\
& |\dot{y}_i(t)| \le v_{y,\max} \\
& |\ddot{x}_i(t)| \le a_{x,\max} \\
& |\ddot{y}_i(t)| \le a_{y,\max} \\
& |\dddot{x}_i(t)| \le j_{x,\max} \\
& |\dddot{y}_i(t)| \le j_{y,\max} \\
& |y_i(t) - y_i(t_{in})| < L_{width} \\
& ((x_{c0i}(t) + \lambda_i \cdot (x_{cni}(t) - x_{c0i}(t))) - (x_{c0h}(t) \\
& + \lambda_h \cdot (x_{cnh}(t) - x_{c0h}(t))))^2 \\
& + ((y_{c0i}(t) + \lambda_i \cdot (y_{cni}(t) - y_{c0i}(t))) - (y_{c0h}(t) \\
& + \lambda_h \cdot (y_{cnh}(t) - y_{c0h}(t))))^2 \\
& > (R_i + R_h)^2 \\
& f(\dot{x}_i, t_{fin})^2/(2 \cdot a_{x,\max}) < |f(x_i, t_{fin})| \qquad (13)
\end{aligned}
$$

where the number of vehicles in the group is assigned as G, $i \in [1, \mathrm{G}]$, h represents vehicles except the vehicle i in the group, $a_{xi,\max}$ and $j_{xi,\max}$ are the maximum acceleration and jerk of the vehicle i in the x direction, $a_{yi,\max}$ and $j_{yi,\max}$ are the maximum acceleration and jerk of the vehicle i in the y direction, t_{ini} and t_{fini} are initial and final time instant for the trajectory of the vehicle i, $\dot{x}_{fini}$ is the velocity of the vehicle i at the time instant t_{fini}, $\dot{x}_{desiredi}$ is the desired velocity of the vehicle i in the x direction. Variables w_{i1}, w_{i2}, w_{i3} and w_{i4} are weight coefficients for the vehicle i.

Then the essence of the problem equals to find the feasible solution of the optimization problem. In this paper, the objective function of the nonlinear optimization problem is the continuous function. Besides, the objective function has second-order derivatives. We solve the problem depending on the interior point algorithm [37]. The principle of the interior point algorithm is to construct a new unconstrained objective function. The penalty function is defined in the trust region [38]. That is, the searching point to solve the unconstrained problem is located in the trust region. Thus, the solution is guaranteed to be feasible all the time. The solution approached the optimal solution in the trust region [39].

2) TRAJECTORY IN DIFFERENT GROUPS

Vehicle groups are sorted in the descending order of the longitudinal coordinate. Trajectories of different groups are planned one by one according to the group number, which means that the trajectory planning of following groups should take trajectories of preceding groups into account.

For the vehicle group 1, trajectories are planned according to the Eq. (12) and constraints (13). For the group 2, the trajectory planning should take the collision avoidance between trajectories of the group 1 and the group 2 into account. That is, trajectories of the group 2 should stay the safe distance with generated trajectories of the group 1. Trajectories of the group K should stay the safe distance with trajectories of the group $K-1, K-2, K-3, \ldots, 1$. Therefore, for the trajectory planning of the group K, the objective function Eq. (12) is remained, but the constraint (13) are changed into the constraint (14):

$$
\begin{aligned}
& |\dot{x}_i(t)| \le v_{x,\max} \\
& |\dot{y}_i(t)| \le v_{y,\max} \\
& |\ddot{x}_i(t)| \le a_{x,\max} \\
& |\ddot{y}_i(t)| \le a_{y,\max} \\
& |\dddot{x}_i(t)| \le j_{x,\max} \\
& |\dddot{y}_i(t)| \le j_{y,\max} \\
& |y_i(t) - y_i(t_{in})| < L_{width} \\
& ((x_{c0i}(t) + \lambda_i \cdot (x_{cni}(t) - x_{c0i}(t))) - (x_{c0h}(t) \\
& + \lambda_h \cdot (x_{cnh}(t) - x_{c0h}(t))))^2 \\
& + ((y_{c0i}(t) + \lambda_i \cdot (y_{cni}(t) - y_{c0i}(t))) - (y_{c0h}(t) \\
& + \lambda_h \cdot (y_{cnh}(t) - y_{c0h}(t))))^2 \\
& > (R_i + R_h)^2
\end{aligned}
$$

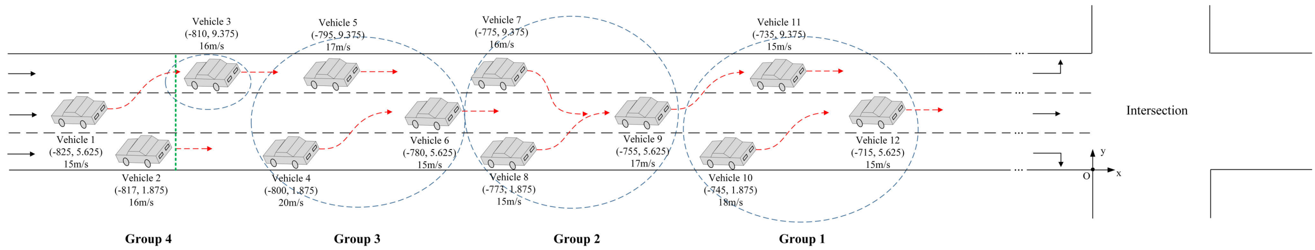

FIGURE 4. Simulation experiment of multiple vehicles on the road (t = 0, the maximum vehicle number in the group is 3).

$$\begin{aligned}
&((x_{c0i}(t) + \lambda_i \cdot (x_{cni}(t) - x_{c0i}(t))) - (x_{c0g}(t) \\
&+\lambda_g \cdot (x_{cng}(t) - x_{c0g}(t))))^2 + ((y_{c0i}(t) \\
&+\lambda_i \cdot (y_{cni}(t) - y_{c0i}(t))) - (y_{c0g}(t) + \lambda_g \cdot (y_{cng}(t) \\
&-y_{c0g}(t))))^2 \\
&> (R_i + R_g)^2 \\
&f(\dot{x}_i, t_{fin})^2/(2 \cdot a_{x,\max}) < \left|f(x_i, t_{fin})\right| \qquad (14)
\end{aligned}$$

where the number of vehicles in the group K is assigned as G, $i \in [1, \mathrm{G}]$, h represents vehicles except the vehicle i in the group K, g means vehicles in the group K-1, K-2, K-3, ..., 1. The collision avoidance of multiple groups is ensured based on the constraint (14). Then the intergroup cooperative trajectory planning is realized.

3) TRAJECTORY UPDATE

In order to realize the dynamic cooperative trajectory planning, the nonlinear optimization problem is solved in each T_{update} seconds. In each update, the grouping result maybe change. Then values of variables in objective functions and constraint functions should change correspondingly. The initial point of the trajectory is determined by the real-time trajectory coordinate. By solving the problem, the trajectory planning result is updated.

The trajectory update divides the whole task into multiple progressive sliding windows, which not only realizes the feasible trajectory planning, but also ensures the on-line planning.

VII. SIMULATION AND RESULTS

In order to evaluate the effectiveness of the proposed model, we conducted the simulation experiment in three different cases. To facilitate the discussion, parameters of the road and vehicle are defined in Table 1. The variable T_{update} is defined as 3s. The value of $X_{cooperative}$ is set as -815m according to the previous study [40]. When the longitudinal coordinate of the vehicle is between -815m to 0, the vehicle will be considered in the cooperative trajectory planning.

In this part, the performance of the proposed model in the traffic flow of multiple vehicles is evaluated. The vehicle number in one group has influence on the cooperation among vehicles. The cooperative ability of CAVs cannot be utilized for the small group size. Large group size may increase the complexity of the problem. Thus, we change the maximum vehicle number for the group from 3-5 and conduct the simulation experiment.

TABLE 1. Threshold for parameters of the vehicle trajectory.

Symbol	Definition (Unit)	Value
L_{width}	the width of the lane (m)	3.75
$width$	the width of the vehicle (m)	2.00
len	the length of the vehicle (m)	4.80
$v_{x,\max}$	the maximum velocity in the x direction (m/s)	30.00
$v_{y,\max}$	the maximum velocity in the y direction (m/s)	2.50
$a_{x,\max}$	the maximum acceleration in the x direction (m/s^2)	4.00
$a_{y,\max}$	the maximum acceleration in the y direction (m/s^2)	2.00
$j_{x,\max}$	the maximum jerk in the x direction (m/s^3)	2.00
$j_{y,\max}$	the maximum jerk in the y direction (m/s^3)	1.00

As Fig. 4 shows, there are 12 vehicles on the road and 6 vehicles have mandatory lane change demands. Initial and final states of vehicles are expressed as

$$\begin{aligned}
S_{in1} &= (-825, 15, 0, 5.625, 0, 0), \\
S_{fin1} &= (x_1, \dot{x}_1, 0, 9.375, 0, 0) \\
S_{in2} &= (-817, 16, 0, 1.875, 0, 0), \\
S_{fin2} &= (x_2, \dot{x}_2, 0, 1.875, 0, 0) \\
S_{in3} &= (-810, 16, 0, 9.375, 0, 0), \\
S_{fin3} &= (x_3, \dot{x}_3, 0, 9.375, 0, 0) \\
S_{in4} &= (-800, 20, 0, 1.875, 0, 0), \\
S_{fin4} &= (x_4, \dot{x}_4, 0, 5.625, 0, 0) \\
S_{in5} &= (-795, 17, 0, 9.375, 0, 0), \\
S_{fin5} &= (x_5, \dot{x}_5, 0, 9.375, 0, 0) \\
S_{in6} &= (-780, 15, 0, 5.625, 0, 0), \\
S_{fin6} &= (x_6, \dot{x}_6, 0, 5.625, 0, 0) \\
S_{in7} &= (-775, 16, 0, 9.375, 0, 0), \\
S_{fin7} &= (x_7, \dot{x}_7, 0, 5.625, 0, 0) \\
S_{in8} &= (-773, 15, 0, 1.875, 0, 0), \\
S_{fin8} &= (x_8, \dot{x}_8, 0, 5.625, 0, 0) \\
S_{in9} &= (-755, 17, 0, 5.625, 0, 0), \\
S_{fin9} &= (x_9, \dot{x}_9, 0, 9.375, 0, 0)
\end{aligned}$$

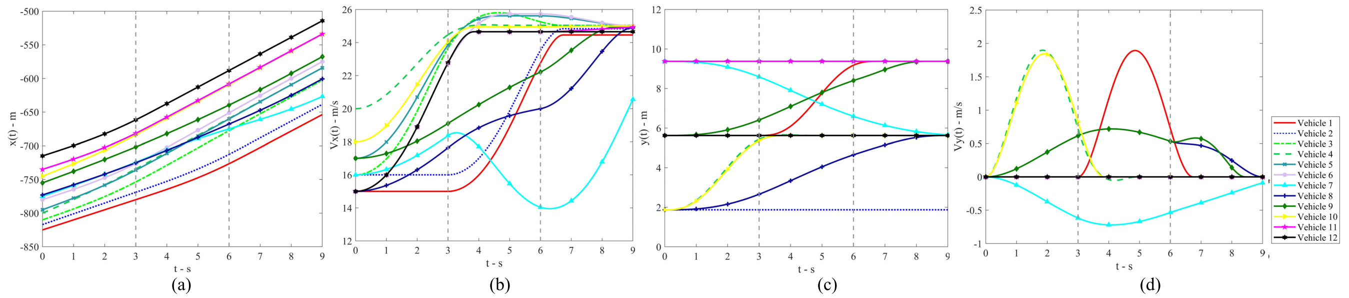

FIGURE 5. Trajectories of vehicles (the maximum group size is 3) (a) Longitudinal position (b) Longitudinal velocity (c) Lateral position (d) Lateral velocity.

$$
\begin{aligned}
S_{in10} &= (-745, 18, 0, 1.875, 0, 0),\\
S_{fin10} &= (x_{10}, \dot{x}_{10}, 0, 5.625, 0, 0)\\
S_{in11} &= (-735, 15, 0, 9.375, 0, 0),\\
S_{fin11} &= (x_{11}, \dot{x}_{11}, 0, 9.375, 0, 0)\\
S_{in12} &= (-715, 15, 0, 5.625, 0, 0),\\
S_{fin12} &= (x_{12}, \dot{x}_{12}, 0, 5.625, 0, 0)
\end{aligned}
$$

The total simulation time is 9 seconds. Corresponding to the different group size and the vehicle grouping rule, the grouping result and cooperative trajectory planning consequence are depicted as follows.

A. MAXIMUM GROUP SIZE IS 3

At time instant $t = 0$, the vehicle 1 and vehicle 2 are out of the cooperative zone. The vehicle 3 to 12 are considered in the vehicle grouping. When the maximum group size is 3 ($N = 3$), the grouping result at $t = 0$ is shown in Fig. 4. Then the trajectory planning is conducted. The grouping and trajectory planning are repeated in each 3 seconds.

The grouping results change at different time instants. When the value of t is 0, the vehicle 10, 11 and 12 are divided into the group 1. The vehicle 7, 8 and 9 belong to the group 2. The vehicle 4, 5 and 6 are considered in the group 3. The vehicle 3 belongs to the group 4. The vehicle 1 and 2 are not considered in the cooperative trajectory planning because they are out of the cooperative zone. When the value of t comes to 3s, the vehicle grouping is conducted again relying on the value of $G_{i,desired}$ for each vehicle. The vehicle number in the group 1, 2 and 3 remains unchanged. At the time instant, the vehicle 1 and 2 have entered into the cooperative zone, then they are divided into the group 4. The vehicle 3 still belongs to the group 4. When it comes to the 6s, the grouping result changes a lot. The vehicle 9, 10 and 12 belong to the group 1. The vehicle 4, 7 and 11 are divided into the group 2. The vehicle 5, 6 and 8 are considered in the group 3. The vehicle 1, 2 and 3 are divided into the group 4. It is clear that when longitudinal trajectories of vehicles vary, the vehicle grouping results also change, which ensures the effective vehicle grouping.

Trajectories of vehicles in one group are planned cooperatively and simultaneously. Besides, trajectories of vehicles are updated in each time interval T_{update}. For different groups, they are considered in the descending order of the longitudinal coordinate. The trajectory planning of following groups should take generated trajectories of preceding groups into account. For instance, trajectories of the vehicle 10, 11 and 12 are planned at first in Fig 4. Then trajectories of the vehicle 7, 8 and 9 are planned considering the collision avoidance with existing trajectories of the vehicle 10, 11 and 12. Subsequently, trajectories of the vehicle 4, 5 and 6 are generated, which ensures the collision avoidance with existing trajectories. The collision avoidance between groups is also realized relying on the dynamic circle model of the Section V.

When the maximum group size is 3, trajectories of vehicles are shown in Fig. 5. According to Fig. 5 (a) – (d), smooth and continuous trajectories of vehicles are generated, which satisfy the limitation function. Vehicles cooperate with each other to realize the lane change demand.

During 0 to 3 seconds, the vehicle 3 to 12 belong to 4 groups. In the group 1, only the vehicle 10 has the lane change demand, which will not have influence on the trajectory of the vehicle 11. Then the vehicle 11 and 12 accelerate to the desired velocity. For the vehicle 10, it stays the safe distance with the vehicle 12 when it approaches to the same lane with the vehicle 12. In the group 2, the vehicle 7, 8 and 9 have lane change demands. The vehicle 8 and 9 change to the left lane, so they both accelerate in the longitudinal direction to change lanes. The vehicle 9 accelerates and ensures the safe distance with the vehicle 11. Besides, the longitudinal distance between the vehicle 8 and 9 guarantees the safe driving. For the vehicle 7, its target lane is the same with the vehicle 8. Besides, its initial longitudinal coordinate is adjacent to the vehicle 8. Thus, the vehicle 7 accelerates slowly in the longitudinal coordinate during 0 to 3 seconds. In the group 3, the vehicle 5 and 6 accelerate and keep the safe gap with existing trajectories in the group 1 and 2. The vehicle 4 keeps the enough gap with the vehicle 6 and changes lane smoothly. In the group 4, the vehicle 3 accelerates and stays the lower velocity than the vehicle 5 to obtain the safe gap.

During 3 to 6 seconds, vehicles in the group 1, 2 and 3 remain unchanged. The vehicle 1 and 2 are divided into the group 4 because they are in the range of the cooperative zone. In the group 1, the vehicle 10 completes the lane change

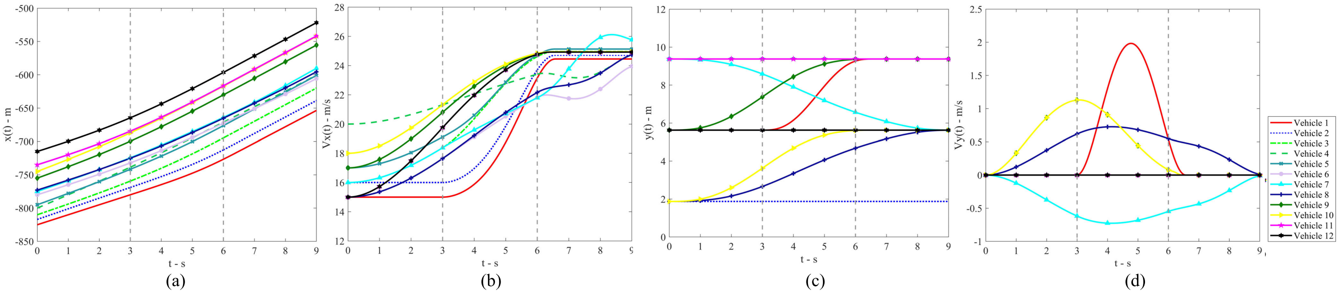

FIGURE 6. **Trajectories of vehicles (the maximum group size is 4) (a) Longitudinal position (b) Longitudinal velocity (c) Lateral position (d) Lateral velocity.**

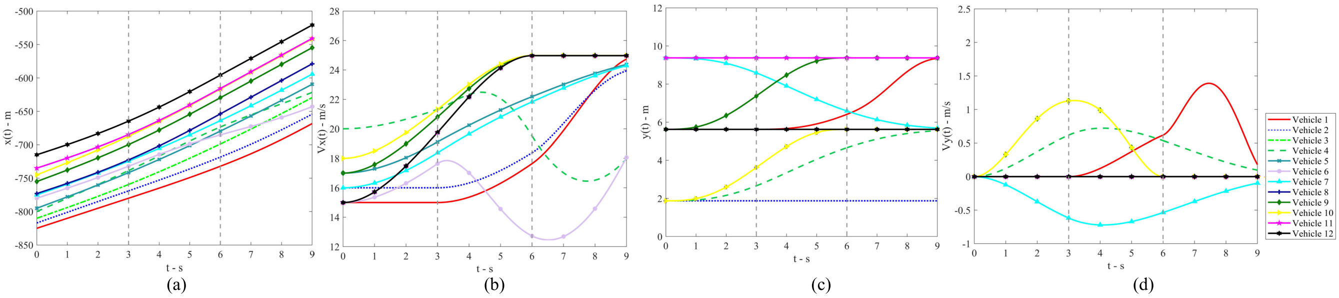

FIGURE 7. **Trajectories of vehicles (the maximum group size is 5) (a) Longitudinal position (b) Longitudinal velocity (c) Lateral position (d) Lateral velocity.**

operation at the 3.85s. Then the vehicle 10, 11 and 12 keep the stable velocity. In the group 2, the vehicle 7 slows down dramatically to leave the enough gap to avoid the possible collision with the vehicle 8. The motion in the longitudinal direction of the vehicle 7 ensures not only the safety of three vehicles but also the smooth lateral motion. In the group 3, the vehicle 4 completes the lane change at 4.90s and keeps the safe distance with the vehicle 6. In the group 4, the vehicle 1 and 2 begin to accelerate. The vehicle 1 also conducts the lane change operation.

During 6 to 9 seconds, the result of the vehicle grouping changes a lot. The group 1 consists of the vehicle 9, 10 and 12. The vehicle 9 accelerates and completes the lane change at 8.43s. The vehicle 4, 7 and 11 belong to the group 2. The vehicle 7 follows the vehicle 8 and continue changing lane. In the group 3, the vehicle 8 stays the safe distance with the vehicle 10 and completes the lane change operation at 9.00s. In the group 4, the vehicle 1 accelerates to the desired velocity and completes the lane change at 6.74s.

The cooperative trajectory planning makes sure safe and smooth trajectories of all vehicles and utilizes the acceleration and deceleration motion to create the proper gap for lane change operations. The cooperation among vehicles makes full use of time and space resources.

B. MAXIMUM GROUP SIZE IS 4

When the maximum group size is 4, the result of the cooperative trajectory planning is depicted in Fig. 6. During 0 to 3 seconds, there are 3 groups. The group 1 consists of the vehicle 9 to 12. The group 2 contains the vehicle 5 to 8. The group 3 is composed by the vehicle 3 and the vehicle 4. In the group 1, the vehicle 9 and 10 accelerate to change lane. In the group 2, the vehicle 7 and 8 both accelerate to change lane. In the group 3, the vehicle 4 plans to change to the same lane with the vehicle 6. In order to avoid the possible collision, the vehicle 4 accelerates to change lane and stays a safe distance with the vehicle 6. During 3 to 6s, the grouping result of the group 1 remains unchanged. The vehicle 9 and 10 continue changing lanes. The group 2 consists of the vehicle 4, 6, 7 and 8. The new member vehicle 4 accelerates to overtake the vehicle 6 and stays a safe distance with the vehicle 6. The group 3 contains the vehicle 1, 2, 3 and 5. The vehicle 1 begins to change lane. During 6 to 9s, the grouping result is the same with the previous one. In the group 1, the vehicle 9 and 10 complete the lane change operation at 6.44s. In the group 2, the vehicle 7 gets a higher longitudinal velocity than the vehicle 8 to obtain the safe distance with the vehicle 8. In the group 3, the vehicle 1 stays the enough longitudinal distance with the preceding vehicle on the target lane and completes the lane change operation at 6.57s.

C. MAXIMUM GROUP SIZE IS 5

When the maximum group size is 5, the generated trajectories are shown in Fig. 7. During 0 to 3s, there are two groups. The group 1 consists of the vehicle 8 to 12. The vehicle 8, 9 and 10 accelerate to change lane. The group 2 contains the vehicle 3 to 7. The vehicle 4 and 7 move to the target lane gradually. During 3 to 6s, the group 1 and 2 remain unchanged. The vehicle 1 and 2 are divided into the group 3. In the group 1, all vehicles accelerate to the desired velocity.

TABLE 2. Time instant when the lane change is completed during 0-9s.

Vehicle Number	Time (s)		
	Max group size = 3	Max group size = 4	Max group size = 5
1	6.74	6.57	—
4	4.90	—	—
7	—	—	—
8	9.00	—	6.00
9	8.43	6.44	6.00
10	3.85	6.44	6.00

TABLE 3. Average velocity of vehicles during 0-9s (m/s^2).

Max group size = 3	Max group size = 4	Max group size = 5
22.74	23.52	22.85

The vehicle 8 to 10 accomplish the lane change operation at 6.00s. In the group 2, the vehicle 4 decelerates to stay a safe distance with the preceding vehicle 7. At the same time, the vehicle 6 also decelerates to obtain a safe gap with the vehicle 4. In the group 3, the vehicle 1 begins to cooperate with the vehicle 2. During 6 to 9s, the lane change demand of the group 1 has been satisfied. Vehicles drive at the desired velocity. In the group 2, the vehicle 4 and 7 stay the safe distance with the preceding vehicle and continue changing lane. In the group 3, the vehicle 1 accelerates to change lane.

According to the result, the lane change time of each vehicle when the maximum group size equals to 3, 4, and 5 is shown in Table 2. When the maximum group size is 3, the vehicle 1, 4, 7, 8, 9 and 10 completes the lane change operation in 9s. The vehicle 7 approaches to the center line of the target lane when it comes to 9s, but the longitudinal acceleration is not zero. Thus, the lane change operation of the vehicle 7 is considered to be unaccomplished. For the experiment with the maximum group size 4, the vehicle 1, 9 and 10 accomplish the lane change in 9s. For the experiment with the maximum group size 5, the vehicle 8, 9 and 10 achieve the lane change in 6s. It can be known that the most number of vehicles achieve the lane change in the limited time when the maximum group size equals to 3.

Trajectories of vehicles in the cooperative planning are affected by the maximum vehicle number in the group. The average longitudinal velocity of vehicles in the simulation experiment is depicted in Table 3. The average longitudinal velocity of the experiment with the maximum group size 4 is higher than other two experiments. According to the Fig. 5, the vehicle 7 decelerates sharply to realize the cooperation with other vehicles. In the Fig. 7, the fluctuation of the vehicle 4 and 6 is huge in the cooperative trajectory planning. Thus, it is clear that vehicles drive in the highest average longitudinal velocity in the experiment with maximum group size 4. The fluctuation of the longitudinal velocity in the experiment with the group size 4 is smaller than the experiment with the group size 3 and 5.

VIII. CONCLUSION

The lane change is one of the most general operations of vehicles. Inappropriate lane changes are demonstrated to cause traffic congestion and accidents. This paper proposes a trajectory planning model of lane changes for multiple vehicles. The main contribution of the model contains two parts. The first part is the vehicle grouping, in which vehicles in the cooperative zone are divided into groups. Then groups are sorted by longitudinal coordinates. Trajectories of vehicles in the same group are planned at the same time. Trajectories in different groups are considered by the group number. Vehicles in following groups should leave the enough space for vehicles in preceding groups. By the vehicle grouping, the problem is simplified into several subproblems. Besides, considering the influence of the grouping on trajectory planning results, simulation experiments based on the different maximum vehicle numbers of the group are conducted. To realize the dynamic cooperative trajectory planning, results of the vehicle grouping and trajectory planning are updated in the fixed time interval. The result shows that the most number of vehicles achieves the lane change in the experiment with the maximum vehicle numbers 3. The smallest fluctuation in longitudinal velocity is realizes in experiment with the maximum vehicle number 4.

The second part is the model to generate trajectories for vehicles with and without lane change demands. Most authors have put forward the trajectory planning model for vehicles only with lane change demands. Vehicles without lane change demands are supposed to drive at a constant speed. As a result, the cooperative ability of vehicles is not fully utilized. As the improvement, the proposed model realizes the trajectory planning for all vehicles. That is, vehicles cooperate with each other by adjusting the velocity to leave the enough gap for the safe driving of all vehicles. In the model, the high order polynomials make sure smooth and continuous trajectories of vehicles. The dynamic circle algorithm ensures the collision avoidance. Thus, cooperative trajectories are generated.

The proposed method provides an innovative way to solve the trajectory planning problem for the lane change of CAVs. The cooperative trajectory planning model is also applicable for other traffic scenes where there are mandatory lane change demands. The model can be integrated into the driving control system of CAVs in the future. However, there are some limitations in the model. Firstly, the model only focuses on the mandatory lane changes. As for the traffic scene with discretionary lane changes, the model should be optimized. Secondly, simulation experiments of more complex traffic scenes should be considered. Therefore, one of the topics for further studies is to conduct the experiments in the traffic flow with different parameters, such as considering the traffic scene with discretionary lane-change demands, changing the flow rate and combining the signal information of the intersection with the communication among vehicles. What's more, we will explore the effect of the different vehicle grouping parameter such as the larger and smaller distance, the safe car-following distance and so on. We plan to compare the trajectory planning result when the vehicle belongs to one group or several groups, which lays the foundation of the vehicle grouping optimization. Besides, we will combine the V2V and V2I approaches to realize the trajectory planning for vehicles when they change lanes and go through the intersection.

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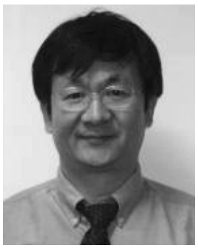
TINGTING LI was born in Sichuan, China, in 1992. She received the B.E. degree in transportation engineering from Beijing Jiaotong University, Beijing, China, in 2015. She is currently pursuing the Ph.D. degree with the Department of Civil Engineering, Tsinghua University, China.

Her research interests include traffic big data, intelligent transport systems, machine learning, and applications in decision-making and trajectory planning for autonomous vehicles.

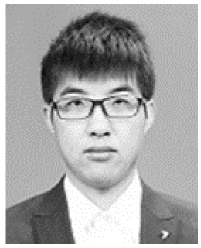


JIANPING WU was born in Zhejiang, China, in 1957. He received the B.S. and M.S. degrees in civil engineering from Zhejiang University, Hangzhou, China, in 1982 and 1984, respectively, and the Ph.D. degree in transportation engineering from the University of Southampton, Southampton, U.K., in 1994.

He is currently a Professor with the Department of Civil Engineering, Tsinghua University, Beijing, China, and the Director of Tsinghua University–Cambridge University and the MIT Low Carbon Alliance Centre for future transport research. He is also a Visiting Professor with the University of Southampton. He has been a main investigator for over 50 research projects and is the author or coauthor of over 250 articles in international journals and conferences. His research interests include road-user behavior and microscopic simulation, sustainable transport systems and low carbon transport, and smart city and intelligent transport systems (ITS). He received the prestigious "Chong Kong Scholar Professorship" awarded by the Ministry of Education of China. He is also in the Editorial Board of *Transportation Research Part C: Emerging Technologies* and *IET Intelligent Transport Systems* journal.

CHING-YAO CHAN received the Ph.D. degree in mechanical engineering from UC Berkeley, in 1988.

He worked in the private sector before returning to Berkeley, in 1994. Prior to joining PATH, he worked in the field of vehicular passive safety systems. While being involved in the research and development of crash sensing technologies, he also gained first-hand knowledge on general passive restraint systems, where he worked on automotive tier-one supplies and automotive OEMs. From 1990 to 1994, he worked in litigation support on accident reconstruction and participated in numerous cases of vehicle crashes, through which he gained insights on the interaction of drivers, vehicle characteristics, roadway environment, and their impacts on driving risks. He is the Co-Director of Berkeley Deep Drive (BDD), BDD is a research consortium dedicated to the research and development of technologies for intelligent autonomy, with applications for autonomous driving and robotics. He leads research projects on machine learning applications in autonomous driving and manages the BDD infrastructure projects in data and experimental vehicles. He is also a Researcher and a Program Manager at California PATH (Partners for Advanced Transportation Technology). PATH has been a pioneering organization spearheading the field of research on intelligent transportation systems, since 1986. At PATH, he leads research projects in vehicle automation, advanced vehicular technologies, human factors, and traffic systems.

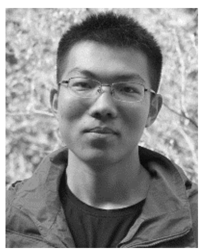


MINGYU LIU was born in Jilin, China, in 1993. He received the B.E. degree in civil engineering from Zhejiang University, Hangzhou, China, in 2016. He is currently pursuing the the Ph.D. degree with the Department of Civil Engineering, Tsinghua University, China.

His research interests include traffic flow theory, traffic big data, machine learning, and traffic management.

CHUNLI ZHU was born in Jiangsu, China, in 1990. She received the B.S. and M.S. degrees in traffic and transportation engineering from Central South University, Changsha, China, in 2013 and 2016, respectively. She is currently pursuing the Ph.D. degree with the Department of Civil Engineering, Tsinghua University, Beijing, China.

She was doing her research work at the University of Birmingham, Birmingham, U.K., as a Visiting Student, from 2015 to 2016. Her research interests include intelligent transport systems, urban resilience, and cooperative traffic control.



WEIXIN LU was born in Jiangsu, China, in 1998. He is currently pursuing the bachelor's degree in computer science from the School of Electronic Engineering and Computer Science, Peking University, Beijing, China. His research interests include graph database, graph embedding, and reinforcement learning.

KEZHEN HU was born in Zhejiang, China, in 1992. He received the Ph.D. degree in civil engineering from Tsinghua University, Beijing, China, in 2018. He was a Visiting Scholar with the University of Oxford. He is currently a Researcher with the China Academy of Information and Communications Technology (CAICT), Beijing. His research interests include connected vehicles, VR/AR, and smart city.