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Separation Principle for Linear Sampled-Data Control Systems Under Observer-Based Output Feedback With Aperiodic Sampling

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ABSTRACT This paper studies a separation principle for a class of linear sampled-data control systems with aperiodic sampling. The concerned problem is to show that in aperiodic sampling, the design of observer-based output feedback controller can be broken into two separate designs: aperiodic sampled-data static state feedback controller and observer designs. It is proved that asymptotic (respectively, exponential) stabilization by the sampled-data static state feedback and asymptotic (respectively, exponential) observation by the sampled-data Luenberger observer imply asymptotic (respectively, exponential) stabilization by the sampled-data observer-based output feedback. An example is given to check the validity of the proposed theoretical claims.

INDEX TERMS Separation principle, sampled-data, aperiodic, Lyapunov, observer, output feedback, asymptotic stability, exponential stability.

I. INTRODUCTION

A prominent research field in control theories, a sampled-data control has been studied widely. Its research focus has changed from periodic (or constant) sampling (see, [12]–[20], and references therein) to aperiodic (or arbitrary varying) one (see, [1]–[7], [21], [24]–[27], and references therein). Analytic approaches to aperiodic sampling can be categorized into two groups: the first one is to treat the aperiodic sampled-data system in the discrete-time domain (see, [8]–[11], and references therein), while the other is to analyze it as an input delay system via Lyapunov-Krasovskii theorems (see, [1]–[4], [6], [7], and references therein). Especially, [1] analyzed the rigorous stability relation between the discrete-time Lyapunov and the Lyapunov-Krasovskii stability theorems.

A sampled-data observer-based output feedback control is practically appealing because of not utilizing all internal states of a given real system. In aperiodic sampling, observer should provide an estimate of the state used in the sampled-data control, from aperiodic sampled measurements, which makes its design perplexing. Several observers have been presented in the literatures, such as a discrete-time observer [21], a sampled-data observer [3], [5]–[7], and a continuous-discrete observer [4]. Among them, [7], [21] introduced simple design strategies under the separation principle.

The separation principle is an important issue on the observer-based output feedback control as discussed in [28]. If the separation principle holds, the overall design of output feedback controller can be separated into the state feedback controller and observer designs. There are valuable theoretical results on the separation principle for various classes of linear or nonlinear sampled-data systems with periodic sampling in [16]–[20], and references therein. However, in the case of aperiodic sampling, the separation principle has not been studied fully, except for [21]. Reference [21] proved the separation principle for a class of aperiodic sampled-data systems under the discrete-time observer of which gains depends on sampling instants. In particular, very few researches have been done to prove the separation principle achieved with not discrete-time but sampled-data observer.

This paper addresses a separation principle for a class of linear aperiodic sampled-data control systems. Within the framework of the stability theorems [1], the separation principle is proved by showing that the asymptotic (respectively, exponential) state feedback stabilization and the asymptotic (respectively, exponential) observation yields

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the asymptotic (respectively, exponential) stabilization by the sampled-data observer-based output feedback. Contrary to [21], the observer adopted here is not in the discrete-time form but in the aperiodic sampled-data form. Also, the controller and observer gains adopted here are not dependent on the aperiodic sampling interval. Finally, an example, a separate design for the sampled-data observer-based output feedback is given to verify the theoretical claims.

Notations: The relation $P \succ Q$ ($P \prec Q$) means that the matrix $P - Q$ is positive (negative) definite. $\mathbb{N}$ and $\mathbb{R}$ denote the sets of natural and real numbers, respectively. $\mathbb{R}^n$ means the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ stands for the field of real matrices of dimension $m \times n$. For simplicity, x is used in place of $x(t)$ for the continuous-time signal vectors unless otherwise indicated. Δ indicates the forward difference operator, e.g., $\Delta V(x(t_k)) := V(x(t_{k+1})) - V(x(t_k))$ for $k \in \mathbb{N}$. Also, when $a \in \mathbb{R}$ and $b \in \mathbb{R}$, $\mathbb{R}_{>a} := \{x \in \mathbb{R} \mid x > a\}$, $\mathbb{R}_{\geqslant a} := \{x \in \mathbb{R} \mid x \geqslant a\}$, $\mathbb{R}_{<a} := \{x \in \mathbb{R} \mid x < a\}$, $\mathbb{R}_{\leqslant a} := \{x \in \mathbb{R} \mid x \leqslant a\}$, $\mathbb{R}_{[a,b]} := \{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}$, $\mathbb{R}_{(a,b]} := \{x \in \mathbb{R} \mid a < x \leqslant b\}$, $\mathbb{R}_{[a,b)} := \{x \in \mathbb{R} \mid a \leqslant x < b\}$, and $\mathbb{R}_{(a,b)} := \{x \in \mathbb{R} \mid a < x < b\}$.

II. PROBLEM FORMULATION

Consider the linear time-invariant system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

together with the sampled-data observer-based output feedback controller

$$u = u(t_k) = K\hat{x}(t_k) \tag{2}$$

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu(t_k) + L(y(t_k) - \hat{y}(t_k)) \\ \hat{y}(t_k) = C\hat{x}(t_k) \end{cases} \tag{3}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, where $x \in \mathbb{R}^n$ the state; $u \in \mathbb{R}^m$ the sampled-data control input; $y \in \mathbb{R}^p$ the measurement output; $\hat{x} \in \mathbb{R}^n$ the estimation of x; $\hat{y} \in \mathbb{R}^p$ the observer output; $t_k, k \in \mathbb{N}$ the kth sampling instant; $A \in \mathbb{R}^{n\times n}$; $B \in \mathbb{R}^{n\times m}$; $C \in \mathbb{R}^{p\times n}$; $K \in \mathbb{R}^{m\times n}$ the static control gain; $L \in \mathbb{R}^{n\times p}$ the static observer gain.

Assumption 1: Let the kth aperiodic sampling interval be $\tilde{h}_k := t_{k+1} - t_k$. Assume that for the given $h_1 \in \mathbb{R}_{>0}$ and $h_2 \in \mathbb{R}_{\geqslant h_1}$, $\tilde{h}_k \in \mathbb{R}_{[h_1,h_2]}$.

By defining $e := x - \hat{x}$ and augmenting (1), (2), and (3), the closed-loop system becomes

$$\Sigma_1 : \dot{x} = Ax + BKx(t_k) - BKe(t_k) \tag{4}$$

$$\Sigma_2 : \dot{e} = Ae - LCe(t_k) \tag{5}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$.

The main interest is addressed as follows:

Problem 1 (Separation Principle): Consider the closed-loop system (4) and (5) and its nominal system

$$\Sigma_1' : \dot{x} = Ax + BKx(t_k) \tag{6}$$

Suppose that (6) and (5) are asymptotically (or exponentially) stable, respectively. Then, what can we say about the stability of the overall closed-loop system (4) and (5)?

Remark 1: Within the discrete-time domain, the separation principle, for sampled-data control systems under the discrete-time observer-based output feedback, has been investigated by [16]–[20] (for the periodic case) and [21] (for the aperiodic case). Because the discretized system is not usually time-invariant in the aperiodic sampling case, unlike the periodic case, [21] consider the non-static control and observer gains. This is why within the continuous-time domain, most of studies [1]–[7], [21], [24]–[27] design the controller and the observer of the aperiodic sampled-data control system via Lyapunov-Krasovskii functional under the assumption that it can be represented as an equivalent continuous-time system with the input delay. Therefore, within the continuous-time domain, we need to examine the separation principle for the aperiodic sampled-data control system via Lyapunov-Krasovskii functional in design.

III. MAIN RESULTS

Before proceeding to our main results, the following lemmas will be needed throughout the proof:

Lemma 1: Let $\mathscr{D}_x \subset \mathbb{R}^n$ be domain containing $x = 0$. Let $V : \mathscr{D}_x \to \mathbb{R}$ be a continuously differentiable function such that $W_1(x) \leqslant V(x) \leqslant W_2(x)$ for all $x \in \mathscr{D}_x$, where $W_1(x)$ and $W_2(x)$ are continuous-positive definite functions on $\mathscr{D}_x$. Then two following statements are equivalent:

i) There exists a positive definite function $W_3(x(t_k))$ such that

$$\Delta V(x(t_k)) \leqslant -W_3(x(t_k)) \tag{7}$$

for all $k \in \mathbb{N}$ and $x(t_k) \in \mathscr{D}_x$.

ii) There exist a differentiable function $V_a : [0,\infty) \times \mathscr{D}_x \to \mathbb{R}$ and a positive definite function $W_4(x(t_k))$ such that

$$\Delta V_a(t_k, x(t_k)) \geqslant 0 \tag{8}$$

for all $k \in \mathbb{N}$, and

$$\dot{\tilde{V}}(t,x) \leqslant -W_4(x(t_k)) \tag{9}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, where $\tilde{V}(t,x) = V(x) + V_a(t,x)$.

Proof: See Appendix A. ■

Lemma 2: Let $\mathscr{D}_x \subset \mathbb{R}^n$ be domain containing $x = 0$. Let $V : \mathscr{D}_x \to \mathbb{R}$ be a continuously differentiable function such that $\upsilon_1 \|x\|^b \leqslant V(x) \leqslant \upsilon_2 \|x\|^b$ for all $x \in \mathscr{D}_x$, where $\upsilon_1 \in \mathbb{R}_{>0}$, $\upsilon_2 \in \mathbb{R}_{>0}$, and $b \in \mathbb{R}_{>0}$. Then two following statements are equivalent:

i) There exists $\upsilon_3 \in \mathbb{R}_{>0}$ such that

$$\Delta V(x(t_k)) \leqslant -\upsilon_3 \|x(t_k)\|^b \tag{10}$$

for all $k \in \mathbb{N}$ and $x(t_k) \in \mathscr{D}_x$.

ii) There exist differentiable functions $V_a : [0,\infty) \times \mathscr{D}_x \to \mathbb{R}$ and $\upsilon_4 \in \mathbb{R}_{>0}$ such that (8) holds for all $k \in \mathbb{N}$, and

$$\dot{\tilde{V}}(t,x) \leqslant -\upsilon_4 \|x(t_k)\|^b \tag{11}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, where $\tilde{V}(t,x) := V(x) + V_a(t,x)$.

Proof: See Appendix B. ■

Remark 2: Note that Lemmas 1 and 2 directly follow from Theorems 1 and 4 [1], respectively. The difference is that Theorems 1 and 4 [1] require $\Delta V_a(t_k, x(t_k)) = 0$ which is a special case of (8) in Lemmas 1 and 2.

Lemma 3: Consider (4) and (5). Define $\xi := \mathrm{col}\{x, e\}$. There exist $\mu \in \mathbb{R}_{>0}$ such that

$$\|\xi\| \leqslant \mu \|\xi(t_k)\| \tag{12}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$.

Proof: See Appendix C. ■

The main results on Problem 1 are summarized in the following theorems:

Theorem 1: Consider the closed-loop system system (4) and (5) and its nominal system (6). Let $\mathscr{D}_x \subset \mathbb{R}^n$, $\mathscr{D}_e \subset \mathbb{R}^n$, and $\mathscr{D}_\xi \subset \mathbb{R}^{2n}$ be domains containing $x = 0$, $e = 0$, and $\xi = 0$, respectively. Let $V_x : \mathscr{D}_x \to \mathbb{R}$ and $V_e : \mathscr{D}_e \to \mathbb{R}$ be a continuously differentiable function such that $W_{1x}(x) \leqslant V_x(x) \leqslant W_{2x}(x)$ and $W_{1e}(e) \leqslant V_e(e) \leqslant W_{2e}(e)$ for all $x \in \mathscr{D}_x$ and $e \in \mathscr{D}_e$, respectively, where $W_{1x}(x)$, $W_{2x}(x)$, $W_{1e}(e)$, and $W_{2e}(e)$ are continuous-positive definite functions on $\mathscr{D}_x$ and $\mathscr{D}_e$.

Suppose that

A1. There exist a differentiable function $\tilde{V}_x(t,x) : [0,\infty) \times \mathscr{D}_x \to \mathbb{R}$, positive definite functions $W_{4x}(x(t_k))$ and $W_{5x}(x)$, $\varpi_1 \in \mathbb{R}_{>0}$, and $\varpi_2 \in \mathbb{R}_{>0}$ such that

$$\Delta V_{ax}(t_k, x(t_k)) \geqslant 0$$
$$\dot{\tilde{V}}_x(t,x)|_{(6)} \leqslant -W_{4x}(x(t_k)) - W_{5x}(x)$$
$$\left\| \frac{\partial \tilde{V}_x(t,x)}{\partial x} \right\| \leqslant \varpi_1 W_{4x}(x(t_k))^{\frac{1}{2}} + \varpi_2 W_{5x}(x)^{\frac{1}{2}}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, where $\tilde{V}_x(t,x) = V_x(x) + V_{ax}(t,x)$.

A2. There exist a differentiable function $\tilde{V}_e(t,e) : [0,\infty) \times \mathscr{D}_e \to \mathbb{R}$, positive definite function $W_{4e}(e(t_k))$, and $\varpi_3 \in \mathbb{R}_{>0}$ such that

$$\Delta V_{ae}(t_k, e(t_k)) \geqslant 0$$
$$\dot{\tilde{V}}_e(t,e)|_{(5)} \leqslant -W_{4e}(e(t_k))$$
$$\|BKe(t_k)\| \leqslant \varpi_3 W_{4e}(e(t_k))^{\frac{1}{2}}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, where $\tilde{V}_e(t,e) = V_e(e) + V_{ae}(t,e)$.

Then, the following hold.

i) There exists a positive definite function $W_{3\xi}(\xi(t_k))$ such that

$$\Delta V(\xi(t_k))|_{(4),(5)} \leqslant -W_{3\xi}(\xi(t_k)) \tag{13}$$

for all $k \in \mathbb{N}$ and $\xi(t_k) \in \mathscr{D}_\xi$, where $V(\xi) = V_x(x) + \alpha V_e(e)$ with $\alpha \in \mathbb{R}_{>\underline{\alpha}}$ and

$$\underline{\alpha} = \frac{\varpi_3^2}{4} \lambda_{\max}\left(\begin{bmatrix} \varpi_1 \\ \varpi_2 \end{bmatrix} \begin{bmatrix} \varpi_1 \\ \varpi_2 \end{bmatrix}^T \right).$$

ii) the closed-loop system (4) and (5) is asymptotically stable at the origin. Moreover, if the assumptions hold globally, $\xi = 0$ is globally asymptotically stable.

Proof: A Lyapunov function for (4) and (5) can be taken as

$$\tilde{V}(t,\xi) = V(\xi) + V_a(t,\xi) \tag{14}$$

where $V_a(t,\xi) = V_{ax}(t,x) + \alpha V_{ae}(t,e)$ with $\alpha \in \mathbb{R}_{>0}$. The derivative of $\tilde{V}$ along the trajectories of (4) and (5) becomes

$$\begin{aligned}\dot{\tilde{V}}|_{(4),(5)} &= \frac{\partial \tilde{V}_x(t,x)}{\partial t} + \frac{\partial \tilde{V}_x(t,x)}{\partial x}(Ax + BKx(t_k)) \\ &\quad - \frac{\partial \tilde{V}_x(t,x)}{\partial x} BKe(t_k) + \alpha \dot{\tilde{V}}_e(t,e)|_{(5)} \\ &\leqslant \dot{\tilde{V}}_x|_{(6)} + \alpha \dot{\tilde{V}}_e|_{(5)} + \left\| \frac{\partial \tilde{V}_x}{\partial x} \right\| \|BKe(t_k)\|\end{aligned}$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$. Using the assumptions A1 and A2 lead to

$$\begin{aligned}&\dot{\tilde{V}}|_{(4),(5)} \\ &\leqslant -W_{4x}(x(t_k)) - W_{5x}(x) - \alpha W_{4e}(e(t_k)) \\ &\quad + \varpi_1\varpi_3 W_{4x}(x(t_k))^{\frac{1}{2}} W_{4e}(e(t_k))^{\frac{1}{2}} \\ &\quad + \varpi_2\varpi_3 W_{5x}(x)^{\frac{1}{2}} W_{4e}(e(t_k))^{\frac{1}{2}} \\ &= \begin{bmatrix} W_{4x}^{\frac{1}{2}} \\ W_{5x}^{\frac{1}{2}} \\ W_{4e}^{\frac{1}{2}} \end{bmatrix}^T \begin{bmatrix} -1 & 0 & \frac{1}{2}\varpi_1\varpi_3 \\ 0 & -1 & \frac{1}{2}\varpi_2\varpi_3 \\ \frac{1}{2}\varpi_1\varpi_3 & \frac{1}{2}\varpi_2\varpi_3 & -\alpha \end{bmatrix} \begin{bmatrix} W_{4x}^{\frac{1}{2}} \\ W_{5x}^{\frac{1}{2}} \\ W_{4e}^{\frac{1}{2}} \end{bmatrix}\end{aligned}$$

from which we see that, by Schur complement,

$$\begin{aligned}\dot{\tilde{V}}|_{(4),(5)} < 0 &\Leftarrow \begin{bmatrix} -1 & 0 & \frac{1}{2}\varpi_1\varpi_3 \\ 0 & -1 & \frac{1}{2}\varpi_2\varpi_3 \\ \frac{1}{2}\varpi_1\varpi_3 & \frac{1}{2}\varpi_2\varpi_3 & -\alpha \end{bmatrix} \prec 0 \\ &\Leftrightarrow \begin{cases} \frac{\varpi_3^2}{4\alpha} \begin{bmatrix} \varpi_1 \\ \varpi_2 \end{bmatrix} \begin{bmatrix} \varpi_1 \\ \varpi_2 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \prec 0 \\ \alpha > 0 \end{cases}\end{aligned}$$

on $[0,\infty) \times \mathscr{D}_1/\{0\} \times \mathscr{D}_2/\{0\}$. By choosing $\alpha \in \mathbb{R}_{>\underline{\alpha}}$, there exists a positive definite function $W_{4\xi}(\xi(t_k))$ such that $\dot{\tilde{V}}(t,\xi)|_{(4),(5)} \leqslant -W_{4\xi}(\xi(t_k))$. Also, $\Delta V_{ax}(t_k, x(t_k)) \geqslant 0$ and $\Delta V_{ae}(t_k, e(t_k)) \geqslant 0$ yield $\Delta V_a(t_k, \xi(t_k)) \geqslant 0$. Therefore, from Lemma 1, we see that there exists a positive definite function $W_{3\xi}(\xi(t_k))$ such that (13) holds on $\mathscr{D}_\xi$, which means that $\xi(t_k) = 0, k \in \mathbb{N}$ is asymptotically stable. Moreover, from (12) in Lemma 3, $\xi = 0$ is also asymptotically stable. ■

Theorem 2: Consider the closed-loop system system (4) and (5) and its nominal system (6). Let $\mathscr{D}_x \subset \mathbb{R}^n$, $\mathscr{D}_e \subset \mathbb{R}^n$, and $\mathscr{D}_\xi \subset \mathbb{R}^{2n}$ be domains containing $x = 0$, $e = 0$, and $\xi = 0$, respectively. Let $V_x : \mathscr{D}_x \to \mathbb{R}$ and $V_e : \mathscr{D}_e \to \mathbb{R}$ be a continuously differentiable function such that $\upsilon_{1x}\|x\|^2 \leqslant V_x(x) \leqslant \upsilon_{2x}\|x\|^2$ and $\upsilon_{1e}\|e\|^2 \leqslant V_e(e) \leqslant \upsilon_{2e}\|e\|^2$ for all $x \in \mathscr{D}_x$ and $e \in \mathscr{D}_e$, respectively, where $\upsilon_{1x} \in \mathbb{R}_{>0}$, $\upsilon_{2x} \in \mathbb{R}_{>0}$, $\upsilon_{1e} \in \mathbb{R}_{>0}$, and $\upsilon_{2e} \in \mathbb{R}_{>0}$.

Suppose that

A1. There exist a differentiable function $\widetilde{V}_x(t,x) : [0,\infty) \times \mathscr{D}_x \to \mathbb{R}$, $\upsilon_{4x} \in \mathbb{R}_{>0}$, $\upsilon_{5x} \in \mathbb{R}_{>0}$, $\upsilon_{6x} \in \mathbb{R}_{>0}$, and $\upsilon_{7x} \in \mathbb{R}_{>0}$ such that

$$\Delta V_{ax}(t_k, x(t_k)) \geqslant 0$$
$$\dot{\widetilde{V}}_x(t,x)|_{(6)} \leqslant -\upsilon_{4x}\|x(t_k)\|^2 - \upsilon_{5x}\|x\|^2$$
$$\left\|\frac{\partial \widetilde{V}_x(t,x)}{\partial x}\right\| \leqslant \upsilon_{6x}\|x(t_k)\| + \upsilon_{7x}\|x\|$$

for all $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, where $\widetilde{V}_x(t,x) = V_x(x) + V_{ax}(t,x)$.

A2. There exist a differentiable function $\widetilde{V}_e(t,e) : [0,\infty) \times \mathscr{D}_e \to \mathbb{R}$, $\upsilon_{4e} \in \mathbb{R}_{>0}$, and $\upsilon_{5e} \in \mathbb{R}_{>0}$ such that

$$\Delta V_{ae}(t_k, e(t_k)) \geqslant 0$$
$$\dot{\widetilde{V}}_e(t,e)|_{(5)} \leqslant -\upsilon_{4e}\|e(t_k)\|^2$$
$$\|BKe(t_k)\| \leqslant \upsilon_{5e}\|e(t_k)\|$$

for all $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, where $\widetilde{V}_e(t,e) = V_e(e) + V_{ae}(t,e)$.

Then, the following hold.

1) there exists $\upsilon_{3\xi} \in \mathbb{R}_{>0}$ such that

$$\Delta V(\xi(t_k))|_{(4),(5)} \leqslant -\upsilon_{3\xi}\|\xi(t_k)\|^2 \tag{15}$$

for all $k \in \mathbb{N}$ and $\xi(t_k) \in \mathscr{D}_\xi$, where $V(\xi) = V_x(x) + \alpha V_e(e)$ with

$$\underline{\alpha} = \frac{\upsilon_{5e}^2}{4\upsilon_{4e}}\lambda_{\max}\left(\begin{bmatrix} \upsilon_{4x}^{-\frac{1}{2}} & 0 \\ 0 & \upsilon_{5x}^{-\frac{1}{2}} \end{bmatrix}\begin{bmatrix} \upsilon_{6x} \\ \upsilon_{7x} \end{bmatrix} \times \begin{bmatrix} \upsilon_{6x} \\ \upsilon_{7x} \end{bmatrix}^T \begin{bmatrix} \upsilon_{4x}^{-\frac{1}{2}} & 0 \\ 0 & \upsilon_{5x}^{-\frac{1}{2}} \end{bmatrix}\right).$$

2) the closed-loop system (4) and (5) is exponentially stable at the origin. Moreover, if the assumptions hold globally, $\xi = 0$ is globally exponentially stable.

Proof: Consider the Lyapunov function (14) with $\alpha \in \mathbb{R}_{>0}$. Following the similar line of proof of Theorem 1, it can be shown that

$$\begin{aligned}
&\dot{\tilde{V}}|_{(4),(5)} \\
&\leqslant -\upsilon_{4x}\|x(t_k)\|^2 - \upsilon_{5x}\|x\|^2 - \alpha\upsilon_{4e}\|e(t_k)\|^2 \\
&\quad + \upsilon_{6x}\upsilon_{5e}\|x(t_k)\|\,\|e(t_k)\| + \upsilon_{7x}\upsilon_{5e}\|x\|\,\|e(t_k)\| \\
&= \begin{bmatrix} \|x(t_k)\| \\ \|x\| \\ \|e(t_k)\| \end{bmatrix}^T \begin{bmatrix} -\upsilon_{4x} & 0 & \frac{1}{2}\upsilon_{6x}\upsilon_{5e} \\ 0 & -\upsilon_{5x} & \frac{1}{2}\upsilon_{7x}\upsilon_{5e} \\ \frac{1}{2}\upsilon_{6x}\upsilon_{5e} & \frac{1}{2}\upsilon_{7x}\upsilon_{5e} & -\alpha\upsilon_{4e} \end{bmatrix}\begin{bmatrix} \|x(t_k)\| \\ \|x\| \\ \|e(t_k)\| \end{bmatrix}
\end{aligned}$$

under the assumptions A1 and A2. By choosing $\alpha \in \mathbb{R}_{>\underline{\alpha}}$, we see that $\dot{\tilde{V}}|_{(4),(5)} \leqslant -\upsilon_{4\xi}\|\xi(t_k)\|^2$ for some $\upsilon_{4\xi} \in \mathbb{R}_{>0}$. Also, it follows from $\Delta V_{ax}(t_k, x(t_k)) \geqslant 0$ and $\Delta V_{ae}(t_k, e(t_k)) \geqslant 0$ that $\Delta V_a(t_k, \xi(t_k)) \geqslant 0$. From Lemma 2, there exists $\upsilon_{3\xi} \in \mathbb{R}_{>0}$ such that (15) holds for all $k \in \mathbb{N}$ and $\xi \in \mathscr{D}_\xi$. Under the assumption that $\upsilon_{1\xi}\|\xi(t_k)\|^2 \leqslant V(\xi(t_k)) \leqslant \upsilon_{2\xi}\|\xi(t_k)\|^2$ with $\upsilon_{1\xi} \in \mathbb{R}_{>0}$ and $\upsilon_{2\xi} \in \mathbb{R}_{>0}$, it follows from the foregoing inequality that $\Delta V(\xi(t_k)) \leqslant -\frac{\upsilon_{3\xi}}{\upsilon_{2\xi}}V(\xi(t_k)) \Rightarrow V(\xi(t_k)) \leqslant \left(1 - \frac{\upsilon_{3\xi}}{\upsilon_{2\xi}}\right)^k V(\xi(t_0))$ for all $k \in \mathbb{N}$. Hence,

$$\begin{aligned}
\|\xi(t_k)\| &\leqslant \left(\frac{1}{\upsilon_{1\xi}}\left(1 - \frac{\upsilon_{3\xi}}{\upsilon_{2\xi}}\right)^k V(\xi(t_0))\right)^{\frac{1}{2}} \\
&\leqslant \left(\frac{\upsilon_{2\xi}}{\upsilon_{1\xi}}\left(1 - \frac{\upsilon_{3\xi}}{\upsilon_{2\xi}}\right)^k \|\xi(t_0)\|^2\right)^{\frac{1}{2}} \\
&= \left(\frac{\upsilon_{2\xi}}{\upsilon_{1\xi}}\right)^{\frac{1}{2}} \mathrm{e}^{-\frac{\gamma}{2}k}\|\xi(t_0)\|
\end{aligned}$$

where $\gamma = -\ln\left(1 - \frac{\upsilon_{3\xi}}{\upsilon_{2\xi}}\right)$. By using (12) in Lemma 3, the properties $t_k \in \mathbb{R}_{[kh_1,kh_2]}$, $\forall\, k \in \mathbb{N}/\{0\}$ and $(t - t_k) \in \mathbb{R}_{[h_1,h_2]}$, $\forall\, t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, and referring to the exponential stability analysis ([19], [23]), it can be shown that

$$\begin{aligned}
\|\xi\| &\leqslant \mu\left(\frac{\upsilon_{2\xi}}{\upsilon_{1\xi}}\right)^{\frac{1}{2}} \mathrm{e}^{-\frac{\gamma}{2}\frac{k}{t_k}t_k}\|\xi(t_0)\| \\
&\leqslant \mu\left(\frac{\upsilon_{2\xi}}{\upsilon_{1\xi}}\right)^{\frac{1}{2}} \mathrm{e}^{\frac{\gamma}{2h_2}(t-t_k)}\,\mathrm{e}^{-\frac{\gamma}{2h_2}t}\|\xi(t_0)\| \\
&\leqslant \mu\left(\frac{\upsilon_{2\xi}}{\upsilon_{1\xi}}\right)^{\frac{1}{2}} \mathrm{e}^{\frac{\gamma}{2}}\,\mathrm{e}^{-\frac{\gamma}{2h_2}t}\|\xi(t_0)\|
\end{aligned}$$

for all $t \in \mathbb{R}_{\geqslant 0}$. Therefore, $\xi = 0$ is exponentially stable. ■

Remark 3: I would like to emphasize that

i) In Theorems 1 and 2, it is shown that stabilizability by sampled-data state feedback and detectability by sampled-data Luenberger observer imply stabilizability by sampled-data observer-based output feedback with aperiodic sampling.

ii) Within the continuous-time domain differently from the previous results [16]–[21], Theorems 1 and 2 provide the separation principles for the asymptotic and the exponential stability of the sampled-data control system with aperiodic sampling, respectively. Therefore, we can examine the separation principles directly via the continuous-time Lyapunov function like the Lyapunov-Krasovskii functional in design.

iii) Also, we know that if there exist the Lyapunov-Krasovskii functionals to guarantee A1 and A2 of Theorem 1 (or 2) for (6) and (5), respectively, then there exists the discrete-time Lyapunov function to ensure the asymptotic (or exponential) stability for the closed-loop system (4) and (5).

iv) From Theorems 1 and 2, the separation principle in the periodic sampling can be proved directly by setting $h := t_{k+1} - t_k$ and replacing $\tilde{h}$, h_1, and h_2 by h.

v) If there exist Lyapunov functions satisfying A1 and A2 in Theorems 1 or 2, the controller and the observer will be designed separately for the concerned sampled-data systems without introducing any additional conservatism and without considering the perturbation $BKe(t_k)$ in (4).

vi) It is conceivable that Theorems 1 and 2 are extendable to the separation principle for the polytopic uncertain systems or the fuzzy systems.

Example 1: Consider the closed-loop system (4) and (5) with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
$$K = \begin{bmatrix} 3.75 & 11.5 \end{bmatrix}, \quad L = \begin{bmatrix} 3.9 \\ 3.61 \end{bmatrix}$$

for all $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, where A, B, and K are borrowed from [22], and $\tilde{h}_k \in \mathbb{R}_{[0.1,0.5]}$. To examine the separation principle for the exponential stability, we should show that there exist differential functions $\widetilde{V}_x(t,x)$ and $\widetilde{V}_e(t,e)$, $\upsilon_{4x} \in \mathbb{R}_{>0}$, $\upsilon_{5x} \in \mathbb{R}_{>0}$, $\upsilon_{6x} \in \mathbb{R}_{>0}$, $\upsilon_{7x} \in \mathbb{R}_{>0}$, $\upsilon_{4e} \in \mathbb{R}_{>0}$, and $\upsilon_{5e} \in \mathbb{R}_{>0}$ to guarantee A1 and A2 in Theorem 2.

By solving the LMIs presented in [1, Th. 2]

$$\Pi_1 + h_i(\Pi_2 + \Pi_3) \prec 0$$
$$\begin{bmatrix} \Pi_1 - h_i\Pi_3 & * \\ h_i M & -h_i R_3 \end{bmatrix} \prec 0$$

for all $i \in \{1, 2\}$ and the given A, B, and K, we see that the LMIs are feasible in

$$P = \begin{bmatrix} 12.9989 & 5.2806 \\ 5.2806 & 17.4412 \end{bmatrix}$$
$$R_1 = \begin{bmatrix} 0.1088 & -1.4982 \\ -1.4982 & 4.1918 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -1.1762 & -1.5999 \\ 5.8791 & 0.3134 \end{bmatrix}$$
$$R_3 = \begin{bmatrix} 16.5883 & -0.1875 \\ -0.1875 & 16.6261 \end{bmatrix}, \quad R_4 = \begin{bmatrix} 1.5975 & -0.0703 \\ -0.0703 & -15.4542 \end{bmatrix}$$
$$M = \begin{bmatrix} 4.0991 & 8.9841 & -2.7278 & -0.8245 \\ 4.7999 & 9.3412 & -5.0086 & -7.5235 \end{bmatrix}$$

where

$$\Pi_1 = \mathrm{He}\left\{E_1^T P A_c - \frac{1}{2}E_3^T R_1 E_3 - E_3^T R_2 E_2 - E_3^T M\right\}$$
$$\Pi_2 = A_c^T R_3 A_c + \mathrm{He}\left\{A_c^T R_1 E_3 + A_c^T R_2 E_2\right\}$$
$$\Pi_3 = E_2^T R_4 E_2, \quad A_c = \begin{bmatrix} A & BK \end{bmatrix}$$
$$E_1 = \begin{bmatrix} I & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & I \end{bmatrix}, \; E_3 = \begin{bmatrix} I & -I \end{bmatrix}.$$

Thus, from [1, Th. 2], (6) is globally exponentially stable. That is, taking

$$\widetilde{V}_x = x^T P x + (t_{k+1} - t)(x - x(t_k))^T R_1 (x - x(t_k)) + 2(t_{k+1} - t)(x - x(t_k))^T R_2 x(t_k) + (t_{k+1} - t)\int_0^{t-t_k} \dot{x}(s + t_k)^T R_3 \dot{x}(s + t_k)\mathrm{d}s + (t_{k+1} - t)(t - t_k)x(t_k)^T R_4 x(t_k)$$

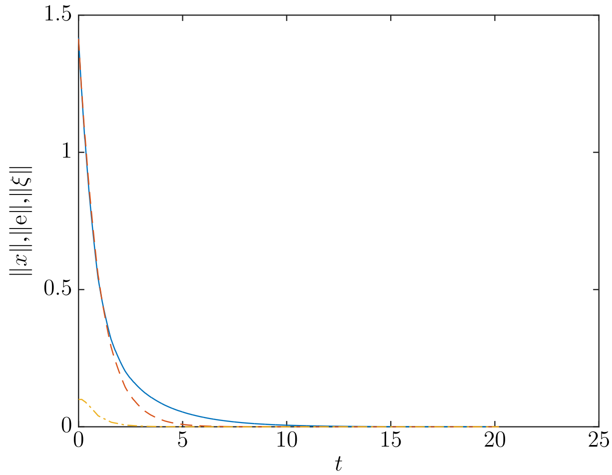

FIGURE 1. $\|x\|$ **of (6) (dash-dot),** $\|e\|$ **of (5) (dashed), and** $\|\xi\|$ **of (4) and (5) (solid).**

there exist $\upsilon_{4x} \in \mathbb{R}_{>0}$ and $\upsilon_{5x} \in \mathbb{R}_{>0}$ such that $\dot{\widetilde{V}}_x|_{(6)} \leqslant -\upsilon_{4x}\|x(t_k)\|^2 - \upsilon_{5x}\|x\|^2$. Moreover, we know that $\left\|\frac{\partial \widetilde{V}_x}{\partial x}\right\| \leqslant \upsilon_{6x}\|x(t_k)\| + \upsilon_{7x}\|x\|$ for $\upsilon_{6x} \in \mathbb{R}_{\geqslant 2h_2\|R_1 - R_2\|}$ and $\upsilon_{7x} \in \mathbb{R}_{\geqslant 2(\|P\| + h_2\|R_1\|)}$ since

$$\frac{\partial \widetilde{V}_x}{\partial x} = 2x^T P + 2(t_{k+1} - t)((x - x(t_k))^T R_1 + x(t_k)^T R_2).$$

In the similar way, we know that (5) is exponential stable because there exist

$$Q = \begin{bmatrix} 18.6128 & -11.9065 \\ -11.9065 & 10.8459 \end{bmatrix}$$
$$S_1 = \begin{bmatrix} 1.7680 & -0.4232 \\ -0.4232 & -1.8542 \end{bmatrix}, \quad S_2 = \begin{bmatrix} -2.1770 & -2.3488 \\ 14.6919 & 5.9363 \end{bmatrix}$$
$$S_3 = \begin{bmatrix} 15.7683 & -7.1053 \\ -7.1053 & 12.1057 \end{bmatrix}, \quad S_4 = \begin{bmatrix} -18.8270 & 25.8112 \\ 25.8112 & 8.5025 \end{bmatrix}$$
$$N = \begin{bmatrix} 3.4894 & 14.4677 & -25.3414 & -0.1459 \\ -1.3443 & -3.6786 & -7.9210 & -13.2503 \end{bmatrix}$$

such that

$$\Sigma_1 + h_i(\Sigma_2 + \Sigma_3) \prec 0$$
$$\begin{bmatrix} \Sigma_1 - h_i\Sigma_3 & * \\ h_i N & -h_i S_3 \end{bmatrix} \prec 0$$

for all $i \in 1, 2$ and the given A, C, and L, where

$$\Sigma_1 = \mathrm{He}\left\{E_1^T Q A_o - \frac{1}{2}E_3^T S_1 E_3 - E_3^T S_2 E_2 - E_3^T N\right\}$$
$$\Sigma_2 = A_o^T S_3 A_o + \mathrm{He}\left\{A_o^T S_1 E_3 + A_o^T S_2 E_2\right\}$$
$$\Sigma_3 = E_2^T S_4 E_2, \; A_o = \begin{bmatrix} A & -LC \end{bmatrix}.$$

From [1, Th. 2], there exists $\upsilon_{4e} \in \mathbb{R}_{>0}$ such that $\dot{\widetilde{V}}_e(t,e)|_{(5)} \leqslant -\upsilon_{4e}\|e(t_k)\|^2$ when $\widetilde{V}_e(t,e) : [0,\infty) \times$

$\mathbb{R}^n \to \mathbb{R}$ is defined as

$$\begin{aligned}\widetilde{V}_e|_{(5)} &= e^T Q e + (t_{k+1} - t)(e - e(t_k))^T S_1 (e - e(t_k)) \\ &\quad + 2(t_{k+1} - t)(e - e(t_k))^T S_2 e(t_k) \\ &\quad + (t_{k+1} - t)\int_0^{t-t_k} \dot{e}(s + t_k)^T S_3 \dot{e}(s + t_k) \mathrm{d}s \\ &\quad + (t_{k+1} - t)(t - t_k) e(t_k)^T S_4 e(t_k).\end{aligned}$$

Therefore, by Theorem 2, we can conclude that the exponential stability of the closed-loop system (4) and (5) is assured. Fig. 1 shows the simulation results when $(x_1(0), x_2(0), e_1(0), e_2(0)) = (1, -1, 0, 0.1)$. As shown in Fig. 1, $\|\xi\|$ of the closed-loop system (4) and (5) globally exponentially converges to the origin when both (6) and (5) are globally exponentially stable. Moreover, we know that the controller and the observer can be designed separately for the sampled-data observer-based output feedback control of the concerned system with aperiodic sampling.

IV. CONCLUSION

This paper proved a separation principle for linear sampled-data control system with aperiodic, rather than periodic, sampling. The theoretical results show that the stabilizability by observer-based output feedback can be deduced from the stabilizability by sampled-data state feedback and the detectability by sampled-data observer, namely, sampled-data controller and observer can be designed separately. Simulation results have validated all of theoretical statements successfully.

APPENDIXES

APPENDIX A
THE PROOF OF LEMMA 1

i) $\Rightarrow$ *ii)* Similar to V_a defined in [1], consider $V_a(t, x) := -V(x) + \delta \frac{t-t_k}{\tilde{h}_k} \Delta V(x(t_k))$ for $\delta \in \mathbb{R}_{(0,1]}$, which it satisfies (8) if (7) holds for $k \in \mathbb{N}$. It follows from $\widetilde{V}(t, x) = \delta \frac{t-t_k}{\tilde{h}_k} \Delta V(x(t_k))$, $\tilde{h}_k \in \mathbb{R}_{[h_1,h_2]}$, and (7) that $\dot{\widetilde{V}}(t, x) = \frac{\delta}{\tilde{h}_k} \Delta V(x(t_k)) \leqslant -\frac{\delta}{h_2} W_3(x(t_k))$ for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$. Thus, defining $W_4(x(t_k)) := \frac{\delta}{h_2} W_3(x(t_k))$ implies (9).

ii) $\Rightarrow$ *i)* Integrating on both sides of (9) over $[t_k, t_{k+1})$ and using (8) and $\tilde{h}_k \in \mathbb{R}_{[h_1,h_2]}$ yield $\Delta V(x(t_k)) + \Delta V_a(t_k, x(t_k)) \leqslant -\tilde{h}_k W_4(x(t_k)) \Rightarrow \Delta V(x(t_k)) \leqslant -\tilde{h}_k W_4(x(t_k)) \Rightarrow \Delta V(x(t_k)) \leqslant -h_1 W_4(x(t_k))$ for $k \in \mathbb{N}$. By defining $W_3(x(t_k)) := h_1 W_4(x(t_k))$, we see that (7) holds for $k \in \mathbb{N}$.

APPENDIX B
THE PROOF OF LEMMA 2

i) $\Rightarrow$ *ii)* In the similar manner to the proof of Lemma 1, we see that $\dot{\widetilde{V}}(t, x) = \frac{\delta}{\tilde{h}_k} \Delta V(x(t_k)) \leqslant -\frac{\delta \upsilon_3}{h_2} \|x(t_k)\|^b$ for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$, and hence, choosing $\upsilon_4 \in \mathbb{R}_{\left(0, \frac{\delta \upsilon_3}{h_2}\right]}$ implies (11).

ii) $\Rightarrow$ *i)* By integrating on both sides of (11) over $[t_k, t_{k+1})$ and applying (8) and $\tilde{h}_k \in \mathbb{R}_{[h_1,h_2]}$, it can be shown that $\Delta V(x(t_k)) \leqslant -h_1 \upsilon_4 \|x(t_k)\|^b$ for $k \in \mathbb{N}$. By choosing $\upsilon_3 \in \mathbb{R}_{(0,h_1\upsilon_4]}$, we see that (10) holds for $k \in \mathbb{N}$.

APPENDIX C
THE PROOF OF LEMMA 3

By using the state vector ξ, rewrite the closed-loop system (4) and (5) as an augmented one $\dot{\xi} = A_{\xi 1}\xi + A_{\xi 2}\xi(t_k)$, where $A_{\xi 1} = \mathrm{diag}\{A, A\}$ and $A_{\xi 2} = \begin{bmatrix} BK & -BK \\ 0 & -LC \end{bmatrix}$. Premultiplying $e^{-A_{\xi 1}t}$ on both sides of the augmented system, integrating over $[t_k, t)$, and taking the norms yield

$$\|\xi\| \leqslant \left(\mathrm{e}^{\|A_{\xi 1}\| h_2} + \int_{t_k}^{t} \mathrm{e}^{\|A_{\xi 1}\|(t-s)} \mathrm{d}s \, \|A_{\xi 2}\| \right) \|\xi(t_k)\|$$

for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$. Therefore, by using $\tilde{h}_k \in \mathbb{R}_{[h_1,h_2]}$, we can choose $\mu \in \mathbb{R}_{\geqslant \underline{\mu}}$ with $\underline{\mu} = \mathrm{e}^{\|A_{\xi 1}\| h_2} + \frac{1}{\|A_{\xi 1}\|} \left(\mathrm{e}^{\|A_{\xi 1}\| h_2} - 1 \right) \|A_{\xi 2}\|$ such that (12) holds for all $t \in [t_k, t_{k+1}), k \in \mathbb{N}$.

REFERENCES

[1] A. Seuret, "A novel stability analysis of linear systems under asynchronous samplings," *Automatica*, vol. 48, no. 1, pp. 177–182, Jan. 2012.

[2] E. Fridman, "A refined input delay approach to sampled-data control," *Automatica*, vol. 46, no. 2, pp. 421–427, Feb. 2010.

[3] A. Selivanov and E. Fridman, "Observer-based input-to-state stabilization of networked control systems with large uncertain delays," *Automatica*, vol. 74, pp. 63–70, Dec. 2016.

[4] F. Mazenc and E. Fridman, "Predictor-based sampled-data exponential stabilization through continuous–discrete observers," *Automatica*, vol. 63, pp. 74–81, Jan. 2016.

[5] M. Wakaiki, K. Okano, and J. P. Hespanha, "Stabilization of systems with asynchronous sensors and controllers," *Automatica*, vol. 81, pp. 314–321, Jul. 2017.

[6] T. Ahmed-Ali, E. Fridman, F. Giri, L. Burlion, and F. Lamnabhi-Lagarrigue, "Using exponential time-varying gains for sampled-data stabilization and estimation," *Automatica*, vol. 67, pp. 244–251, May 2016.

[7] A. Seuret, M. Termens-Ballester, A. Toguyeni, S. El Khattabi, and J.-P. Richard, "Implementation of an Internet-controlled system under variable delays," in *Proc. IEEE Conf. Emerg. Technol. Factory Autom.*, Sep. 2006, pp. 675–680.

[8] L. Hetel, A. Kruszewski, W. Perruquetti, and J.-P. Richard, "Discrete and intersample analysis of systems with aperiodic sampling," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1696–1701, Jul. 2011.

[9] N. van de Wouw, D. Nešić, and W. P. M. H. Heemels, "A discrete-time framework for stability analysis of nonlinear networked control systems," *Automatica*, vol. 48, no. 6, pp. 1144–1153, Jun. 2012.

[10] Y. Oishi and H. Fujioka, "Stability and stabilization of aperiodic sampled-data control systems: An approach using robust linear matrix inequalities," in *Proc. 48h IEEE Conf. Decis. Control (CDC).*, Dec. 2009, pp. 8142–8147.

[11] Y. S. Suh, "Stability and stabilization of nonuniform sampling systems," *Automatica*, vol. 44, no. 12, pp. 3222–3226, Dec. 2008.

[12] D. W. Kim, J. B. Park, and Y. H. Joo, "Effective digital implementation of fuzzy control systems based on approximate discrete-time models," *Automatica*, vol. 43, no. 10, pp. 1671–1683, Oct. 2007.

[13] D. W. Kim, J. B. Park, and Y. H. Joo, "Theoretical justification of approximate norm minimization method for intelligent digital redesign," *Automatica*, vol. 44, no. 3, pp. 851–856, Mar. 2008.

[14] D. W. Kim and H. J. Lee, "Stability connection between sampled-data fuzzy control systems with quantization and their approximate discrete-time model," *Automatica*, vol. 45, no. 6, pp. 1518–1523, Jun. 2009.

[15] D. W. Kim, H. J. Lee, and M. Tomizuka, "Fuzzy stabilization of nonlinear systems under sampled-data feedback: An exact discrete-time model approach," *IEEE Trans. Fuzzy Syst.* vol. 18, no. 2, pp. 251–260, Apr. 2010.

[16] H. Jae Lee, J. Bae Park, and Y. Hoon Joo, "An efficient observer-based sampled-data control: Digital redesign approach," *IEEE Trans. Circuits Syst. I. Fundam. Theory Appl.*, vol. 50, no. 12, pp. 1595–1601, Dec. 2003.

[17] H. J. Lee, J. B. Park, and Y. H. Joo, "Digitalizing a fuzzy observer-based output-feedback control: Intelligent digital redesign approach," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 5, pp. 701–716, Oct. 2005.

[18] A. M. Dabroom and H. K. Khalil, "Output feedback sampled-data control of nonlinear systems using high-gain observers," *IEEE Trans. Autom. Control*, vol. 46, no. 11, pp. 1712–1725, Nov. 2001.

[19] H. K. Khalil, "Performance recovery under output feedback sampled-data stabilization of a class of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 49, no. 12, pp. 2173–2184, Dec. 2004.

[20] D. S. Laila, M. Lovera, and A. Astolfi, "A discrete-time observer design for spacecraft attitude determination using an orthogonality-preserving algorithm," *Automatica*, vol. 47, no. 5, pp. 975–980, May 2011.

[21] A. Sala, "Computer control under time-varying sampling period: An LMI gridding approach," *Automatica*, vol. 41, no. 12, pp. 2077–2082, Dec. 2005.

[22] W. Zhang, M. S. Branicky, and S. M. Phillips, "Stability of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 84–99, Feb. 2001.

[23] H. Jae Lee, J. Bae Park, and Y. Hoon Joo, "Further refinement on LMI-based digital redesign: Delta-operator approach," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 53, no. 6, pp. 473–477, Jun. 2006.

[24] S. Li, C. K. Ahn, and Z. Xiang, "Sampled-data adaptive output feedback fuzzy stabilization for switched nonlinear systems with asynchronous switching," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 200–205, Jan. 2019.

[25] S. Li, J. Guo, and Z. Xiang, "Global stabilization of a class of switched nonlinear systems under sampled-data control," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 9, pp. 1912–1919, Sep. 2019.

[26] S. Li, C. K. Ahn, and Z. Xiang, "Decentralized stabilization for switched large-scale nonlinear systems via sampled-data output fe edback," *IEEE Syst. J.*, vol. 13, no. 4, pp. 4335–4343, Dec. 2019.

[27] M. S. Ali, N. Gunasekaran, C. K. Ahn, and P. Shi, "Sampled-data stabilization for fuzzy genetic regulatory networks with leakage delays," *IEEE/ACM Trans. Comput. Biol. Bioinf.*, vol. 15, no. 1, pp. 271–285, Jan. 2018.

[28] L. Cheng, Z.-G. Hou, Y. Lin, M. Tan, and W. Zhang, "Solving a modified consensus problem of linear multi-agent systems," *Automatica*, vol. 47, no. 10, pp. 2218–2223, Oct. 2011.

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