IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 2, 2020, accepted March 10, 2020, date of publication March 16, 2020, date of current version March 25, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2980879

Robust Finite-Time Output Feedback Control for Systems With Unpredictable Time-Varying Disturbances

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This work was supported in part by the National Natural Science Foundation of China under Grant 61803059 and Grant 61773083, in part by the Foundation of Chongqing University of Posts and Telecommunications under Grant A2017-74 and Grant A2017-15, and in part by the Innovation Team Project of Chongqing Education Committee under Grant CXTDX201601019.

ABSTRACT This work presents a robust finite-time output feedback control method for a number of nonlinear systems that suffer unpredictable time-varying disturbances. The developed method takes advantage of control techniques based on a finite-time state observer used to estimate the lumped disturbance, where the estimator plays an important role in online compensation after being transmitted to a corresponding controller. Besides, with the state estimation provided by the finite-time state observer, the finite-time output feedback controller can guarantee the finite-time stability of the closed-loop system. The experimental results validate the effectiveness of the developed method on the trajectory tracking control of a robot manipulator.

INDEX TERMS Disturbance compensation, finite-time state observer, robust finite-time output feedback control, time-varying disturbance.

I. INTRODUCTION

In many industrial applications, such as robotics systems [1]–[3], servo-control systems [4], [5], and power commutators [6], the performance of control systems mainly suffers from uncertainties like parameter uncertainties, external disturbances, and unmodeled dynamics. To cope with this issue, a number of approaches have been explored, where the concept of uncertainty estimation, disturbance compensation, and observer-based control strategies, have been increasingly studied and employed successfully in a wide range of this fields [7]–[9], [10], [11]. These control algorithms aim to estimate the uncertainties and disturbances that exerts on uncertain nonlinear systems (UNS). Meanwhile, the estimation plays an important role in the on-line feedforward compensation of a controller, which directly and effectively eliminates the unexpected effects caused by disturbances. Thus, disturbance estimation techniques are vital to disturbance compensation.

In recent years, multiple methods for estimating disturbances have been proposed, such as disturbance observer [12], [13], perturbation observer [14], unknown input observer [15], and equivalent input disturbance approach [16]. Notice that all these methods are based on the design of plant models. Hence, the accuracy of dynamic models is highly-related to the used observers and control systems. Furthermore, most of the modern feedback controllers rely on the availability of the system states, though multiple aforementioned disturbance observers may perform disturbance compensation. Furthermore, measuring all states is sometimes impossible or costly, inspiring us to design a new state observer for alleviating this issue.

The concept of extended state observer (ESO) has recently gained increasing attention in research and industrial applications due to their capabilities on the estimation of uncertainties, disturbances, and system states, under the knowledge of the number of system order [17]–[19]. ESO considers

The associate editor coordinating the review of this manuscript and approving it for publication was Sing Kiong Nguang.

all uncertainties as a lumped disturbance [20] and estimates this disturbance for disturbance compensation in the controller design. Although ESO-based control methods are well employed in multiple actual plants, the performance of ESO is unsatisfied while facing time-varying perturbations because the basic design of ESO can only estimate constant or slow-changing disturbances [21], [22]. In many applications, industrial control systems are subject to time-varying disturbances that ESO-based control method cannot alleviate, including changing environmental disturbances, internal parameter uncertainties, and unmodeled dynamics, which can be the forms of step disturbance, ramp disturbance, and parabolic disturbance. To observe such time-varying disturbances, the generalized proportional integral observer (GPIO) was proposed [23], [24], where the observer treats the estimated lump disturbance as a form of time polynomial function. The GPIO -based control method can successfully suppress time-varying disturbances, which has been widely used in various systems [25]–[28]. However, GPIO can only estimate the disturbances in an asymptotic manner, and the corresponding feedback control method is an asymptotically stable controller for a system.

With the increasing needs of higher precision control, the design of novel controllers based on high-performance disturbance observers becomes a challenging task for practitioners. Since the finite-time stable systems have many useful properties [29]–[31], [32], [33], such as faster convergence, higher precision, better anti-disturbance capability. Reference [34]–[36] proposed a finite-time disturbance observer (FTDO) that can estimate the time-varying disturbance in a finite-time. This kind of FTDO, however, brings computing costs due to the discontinuity of the observer state. These facts inspire us to design a robust finite-time controller, which will consist of a finite-time observer and a finite-time feedback controller, and the anti-disturbance performance of such a controller will superior to that of the aforementioned GPIO-based control scheme.

This brief studies the robust finite-time output feedback (FTOF) control problem for UNS with unpredictable time-varying disturbances. The developed method is intuitive, summarized as follows. First of all, based on the system output and studied perturbation, a simplified model is proposed to reduce the design complexity of the observer and controller. The lump disturbances mainly include parameter uncertainty and external disturbance and unmodeled dynamics, which are approximated to a time-varying polynomial model. Next on, a finite-time state observer (FTSO) is proposed to estimate the system time-varying disturbances and system states. A design of the FTSO-based feedforward compensation and related control techniques are used to construct a robust FTOF controller, which eliminates the adverse effects caused by unknown disturbances and ensures the finite-time stability of the closed-loop system. To verify the effectiveness of the developed robust control method, a robot manipulator is used for the experiments on the trajectory tracking control. Compared with the basic GPIO-based control method, the results show the superiority of the controller.

The remainder of this paper is organized as follows. Section II briefly introduces the dynamic model and the solution to the problem. Section III gives the design of the robust controller, including an FTSO, an FTOF control law, and a related stability analysis. Section IV gives the experimental application of the control method on the manipulator. Section V summarizes the paper.

II. BACKGROUND AND PRELIMINARY KNOWLEDGE

A. PROBLEM FORMULATION

Consider an n-dimensional single-input single-output (SISO) nonlinear system [24], [36], [37]:

$$x^{(n)} = \psi(x, t)u + \varphi(x, \dot{x}, \ldots, x^{(n-1)}, t) + w(t), \tag{1}$$

where $\psi(x, t) \neq 0$ is a nonlinear term, and u refers to the control input, $\varphi(\cdot)$ refers to the dynamics of the plant, which is completely unknown, x refers to the system output, $w(t)$ refers to an unknown disturbance (contains external disturbance and unmodeled dynamics). Let x_r be the desired output and it is assumed that $\dot{x}_r, \ddot{x}_r, \ldots, x_r^{(n)}$ exist.

This work aims to use an FTSO and finite-time control techniques to develop a robust feedback controller which can solve the nonlinear uncertainty/time-varying disturbance suppression problem of the system (1).

B. DEFINITIONS AND LEMMAS

Definition 1: $\lceil z \rfloor^{\rho} = \mathrm{sign}(z) \mid z \mid^{\rho}$, $\rho > 0, \forall z \in \mathbb{R}$.

Definition 2 [38]: Consider a system

$$\dot{z} = f(t, z, u), \tag{2}$$

where $f : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is piece-wise continuous in t and locally Lipschitz in z and u. The input $u(t)$ is a piece-wise continuous, bounded function of t for all $t \geq 0$. If there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $z(t_0)$ and any bounded input $u(t)$, the solution $z(t)$ exists for all $t \geq t_0$ and satisfies

$$\|z(t)\| \leq \beta(\|z(t_0)\|, t - t_0) + \gamma\Big(\sup_{\epsilon \in [t_0, t]} \|u(\epsilon)\|\Big), \tag{3}$$

the system (2) is said to be input-to-state stable (ISS).

Lemma 1 [39]: Consider a system:

$$\begin{cases} \dot{z}_i = z_{i+1}, & i = 1, \ldots, n-1, \\ \dot{z}_n = u, & y = z_1, \end{cases} \tag{4}$$

where $z = (z_1, z_2, \ldots, z_n)^T \in \mathbb{R}^n$ is the desired system states; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are control input and system output, respectively. Besides, in the following observer

$$\begin{cases} \dot{\hat{z}}_i = \hat{z}_{i+1} + l_{n-i+1} \lceil z_1 - \hat{z}_1 \rfloor^{i\iota+1}, i = 1, \ldots, n-1, \\ \dot{\hat{z}}_n = u + l_1 \lceil z_1 - \hat{z}_1 \rfloor^{n\iota+1}, \end{cases} \tag{5}$$

for any $\iota = -p/q \in (-1/n, 0)$ with a positive even number p and a positive odd number q, there exist constant gains $(l_1, \ldots, l_n)$ such that the states $(\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_n)$ of the observer (5) will converge to the desired system states $(z_1, z_2, \ldots, z_n)$ in finite-time.

Lemma 2 [40]: Consider the system (4) with the following control:

$$u = -c_n\lceil z_n\rfloor^{\rho_n} - \ldots - c_2\lceil z_2\rfloor^{\rho_2} - c_1\lceil z_1\rfloor^{\rho_1}, \tag{6}$$

where $\rho_1, \rho_2, \ldots, \rho_n$ satisfy the following conditions:

$$\begin{cases} \rho_1 = \rho, n = 1 \\ \rho_{n-1} = \dfrac{\rho_i\rho_{i+1}}{2\rho_{i+1} - \rho_i}, \quad i = 2, \ldots, n, \ \forall n \geq 2 \end{cases} \tag{7}$$

with $\rho_{n+1} = 1$, $\rho_n = \rho$, $\rho \in (1-\delta, 1)$, $\delta \in (0, 1)$. If $c_i > 0$, then the system (4) is finite-time stable.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

A. PLANT PRE-PROCESSING

For better understanding, we rewrite the UNS (1) into (8). Considering various sources of the system disturbances, we use a so-called lumped disturbance $\xi(t)$ to represent modeling errors, external disturbance, and unmodeled dynamics. The studied system (1) can be rewritten in a state-space form:

$$\begin{aligned} x^{(n)} &= \underbrace{\varphi(x, \dot{x}, \ldots, x^{(n-1)}, t) + w(t) + \Delta\psi(x, t)u}_{\textit{lumped}\ \text{time-varying}\ \textit{disturbance}\ \xi(t)} \\ &\quad + \psi_0(x, t)u \\ &= \psi_0(x, t)u + \xi(t), \end{aligned} \tag{8}$$

where $\psi_0(x, t) = \psi(x, t) - \Delta\psi(x, t)$ refers to the estimation; $\Delta\psi(x, t)$ refers to the uncertainties; $\xi(t)$ is the lumped time-varying disturbance that is completely unknown, representing internal dynamics, model uncertainties, and external disturbances of the system. The same treatment of (8) has been applied and reported in literature such as [27], [36], [37].

Assumption 1: Suppose that the first m time derivative of the lumped disturbance $\xi(t)$ of system (8) exits, and $\xi(t)$ can be represented in the following time-varying form [41]:

$$\xi(t) = \sum_{i=0}^{m-1} a_i t^i, \tag{9}$$

where all of the coefficients a_i, $i = 0, 1, \ldots, m-1$ are completely unknown, and m is a positive integer.

Remark 1: As shown in (9), the lumped disturbance $\xi(t)$ can be modeled locally using Taylor polynomials of degree $m-1$. This model can be regarded as an unknown internal model for application in the observer design. The order m can be determined according to the nature of disturbances $\xi(t)$. For example, it is easy to verify that the unknown step disturbance, ramp disturbance, and parabolic disturbance can be expressed by (9) with $m = 1$, 2, and 3, respectively. In general, a higher order m can guarantee a better estimation accuracy, but the computing cost is increased accordingly. Therefore, in practical applications, there is a trade-off between the accuracy of observers and the computing resource [27], [28], [41].

According to (9), an extended state-space model (8) can be constructed by

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad i = 0, 1, \ldots, n-2, \\ \dot{x}_{n-1} = \xi_0 + \psi_0(x, t)u, \\ \dot{\xi}_j = \xi_{j+1}, \quad j = 0, 1, \ldots, m-2, \\ \dot{\xi}_{m-1} = 0, \end{cases} \tag{10}$$

where $x_i = x^{(i)}$, $i = 0, 1, \ldots, n-1$ and $\xi_j = \xi^{(j)}(t)$, $j = 0, 1, \ldots, m-1$.

With (10), the following section illustrates the detailed process of our control scheme design: an FTSO and a finite-time controller.

B. FTSO

Enlightened by the developed method in *Lemma 1*, we design a FTSO for system (10):

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + l_{m+n-i}\lceil x - \hat{x}_0\rfloor^{\varrho_{m+n-i}}, \\ i = 0, 1, \ldots, n-2, \\ \dot{\hat{x}}_{n-1} = \hat{\xi}_0 + \psi_0(x, t)u + l_{m+1}\lceil x - \hat{x}_0\rfloor^{\varrho_{m+1}}, \\ \dot{\hat{\xi}}_j = \hat{\xi}_{j+1} + l_{m-j}\lceil x - \hat{x}_0\rfloor^{\varrho_{m-j}}, \\ j = 0, 1, \ldots, m-2, \\ \dot{\hat{\xi}}_{m-1} = l_1\lceil x - \hat{x}_0\rfloor^{\varrho_1}, \end{cases} \tag{11}$$

where $\hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{n-1}$ refer to the estimations of x_0, x_1, $\ldots$, x_{n-1}, respectively; $\hat{\xi}_0$, $\hat{\xi}_1$, $\ldots$, $\hat{\xi}_{m-1}$ refer to the estimations of ξ_0, ξ_1, $\ldots$, ξ_{m-1}, respectively; $l_1, l_2, \ldots, l_{n+m}$ refer to the parameters of the observer, respectively, which are well-chosen so that the roots of the characteristic polynomial

$$p_o(s) = s^{n+m} + l_{m+n}s^{n+m-1} + \cdots + l_2 s + l_1,$$

are situated in the left-hand side of the complex plane s; $[\varrho_1, \varrho_2, \ldots, \varrho_{m+n-1}, \varrho_{m+n}] = [(m+n)\iota + 1, (m+n-1)\iota + 1, \ldots, 2\iota+1, \iota+1]$ with $\iota = -p/q \in (-1/(n+m), 0)$, where p and q are positive even and odd numbers, respectively.

The developed FTSO is constructed to estimate the disturbances and its m time derivatives, as well as the states of the system, which is quite similar to the traditional GPIO. However, compared with the asymptotic performance of the GPIO, all of the estimations under the FTSO converge to a real value in finite-time. This advantage improves the performance of closed-loop systems because the lumped disturbance can be estimated and then compensated in finite-time by the following developed robust control method.

C. FTOF CONTROLLER

With the estimations of (11) and *Lemma 2*, we design a robust FTOF controller:

$$u = \psi_0^{-1}(x, t)\Big[x_r^{(n)} - \sum_{i=0}^{n-1}\Big(c_{i+1}\lceil \hat{x}_i - x_r^{(i)}\rfloor^{\rho_{i+1}}\Big) - \hat{\xi}_0\Big], \tag{12}$$

where $c_1, c_2, \ldots, c_n$ are the coefficients of the controller, which can be chosen to guarantee that the roots of the following characteristic equation $p_c(s)$ in the complex variable s,

$$p_c(s) = s^n + c_n s^{n-1} + \cdots + c_2 s + c_1, \tag{13}$$

are situated in the left-hand side of the complex plane, and $\rho_1, \rho_2, \ldots, \rho_n$ satisfy (7).

The developed FTOF controller (12) is composed of an FTSO and finite-time feedback control techniques, where the FTSO is used to estimate and suppress the time-varying disturbance $\xi(t)$, and the finite-time feedback control techniques are employed to achieve the finite-time stability of the studied system. Next, we will show the finite-time stability of the closed-loop system under this control method while facing an unpredictable time-varying disturbance.

D. STABILITY ANALYSIS

Herein, the closed-loop stability of the studied system (10) under the developed finite-time controller is established, where the observer is described by (11) and the controller is described by (12).

To this end, we define $e_{x_i} = x_i - \hat{x}_i$ and $e_{\xi_j} = \xi_j - \hat{\xi}_j$. With (10) and (11) in mind, the error dynamics of the observer can be written by

$$\begin{cases} \dot{e}_{x_i} = e_{x_{i+1}} - l_{m+n-i}\lceil e_{x_0} \rfloor^{\varrho_{m+n-i}}, \\ i = 0, 1, \ldots, n-2, \\ \dot{e}_{x_{n-1}} = e_{\xi_0} - l_{m+1}\lceil e_{x_0} \rfloor^{\varrho_{m+1}}, \\ \dot{e}_{\xi_j} = e_{\xi_{j+1}} - l_{m-j}\lceil e_{x_0} \rfloor^{\varrho_{m-j}}, \\ j = 0, 1, \ldots, m-2, \\ \dot{e}_{\xi_{m-1}} = -l_1\lceil e_{x_0} \rfloor^{\varrho_1}. \end{cases} \tag{14}$$

Letting $e_i = x_i - x_r^{(i)}, i = 1, 2, \ldots, n$ and $e = x - x_r = e_0$, the error dynamics is given after substituting (12) to (10):

$$\begin{cases} \dot{e}_i = e_{i+1}, \\ \dot{e}_{n-1} = -\sum_{i=0}^{n-1} \left(c_i \lceil e_i - e_{x_i} \rfloor^{\rho_{i+1}} \right) + e_{\xi_0}. \end{cases} \tag{15}$$

Then, combining (15) and (14), the closed-loop system is governed by

$$\begin{cases} \textit{System } (15), \\ \textit{System } (14). \end{cases} \tag{16}$$

Theorem 1: Considering the system (10) under the unpredictable time-varying disturbances that satisfy *Assumption 1*, a robust finite-time control scheme in the form of (11) and (12) is given. If the gains of l_i in (11) and c_i in (12) satisfy $l_i > 0$ and $c_i > 0$, respectively, the error system (16) will converge to the desired equilibrium in finite-time.

Proof: Given the following system,

$$\begin{cases} \dot{e}_i = e_{i+1}, \\ \dot{e}_{n-1} = -\sum_{i=0}^{n-1} \left(c_{i+1} \lceil e_i \rfloor^{\rho_{i+1}} \right). \end{cases} \tag{17}$$

According to *Lemma 2*, the system (17) is stable in finite-time. Meanwhile, due to *Lemma 1*, the system (14) is also stable in finite-time. Based on the aforementioned knowledge, we know that e_{x_i} and e_{ξ_0} are bounded. Then, by the *Lemma 2* in [42], the system (16) is ISS with respect to the input e_{x_i} and e_{ξ_0}. By the stability of (14) and the ISS of (15), there exists $\gamma \in \mathcal{K}$ and $\alpha, \beta \in \mathcal{KL}$ such that for any $t_0 \geq 0$ and all $t \geq t_0$,

$$\begin{aligned} \|e_o(t)\| &\leq \alpha(\|e_o(t_0)\|, t - t_0), \\ \|e_c(t)\| &\leq \beta(\|e_c(t_0)\|, t - t_0) + \gamma\Big(\| \sup_{\epsilon \in [t_0, t]} e_o(\epsilon)\|\Big), \end{aligned}$$

where e_o is the state of (14) and e_c is the state of (15). We have the estimates $\sup_{\epsilon \geq 0} \|e_o(\epsilon)\| \leq \alpha(\|e_o(0)\|, 0)$, and then

$$\|e_c(t)\| \leq \beta(\|e_c(0)\|, 0) + \gamma(\alpha(\|e_o(0)\|, 0)).$$

Finally, denoting $\|(e_c, e_o)\| = \|e_c\| + \|e_o\|$, we find that

$$\begin{aligned} \|(e_c(t), e_o(t))\| \leq\ & \beta(\|e_c(0)\|, 0) + \alpha(\|e_o(0)\|, 0) \\ & + \gamma(\alpha(\|e_o(0)\|, 0)). \end{aligned}$$

From the above formula, the system (16) is ISS. Then, the finite-time convergence of the system (16) is a direct result for the finite-time convergence of the observer error e_o and the finite-time convergence of the system (17).

Remark 2: In this work, the finite-time stabilization is achieved by using finite-time control scheme based on a FTSO. Since the FTSO can estimate the information of all states and the time-varying disturbance in finite-time, meanwhile, by delivering the estimations to the FTOF control law, the finite-time stability of the closed-loop system can be guaranteed. However, the control law developed in [24], [26], [27], which is so-called GPI control (GPIC) method, only obtain the asymptotic stabilization for a system in the presence of the time-varying disturbance. This promising property will be shown in the following experiments through the comparison of these two methods.

Remark 3: It is known that the existing finite-time control methods mainly include terminal sliding mode control (TSMC) [43], finite-time control using adding a power integrator technique [44], and finite-time control mentioned in Lemma 2. However, TSMC can cause a chattering phenomenon for a system due to its design structure, adding a power integrator technique-based control will increase the computational burden, and the method in Lemma 2 is a feedback controller for systems without uncertainties. The proposed controller in this paper is continuous and can reduce the computational burden rather than adding a power integrator technique-based controller. Moreover, it consists of a finite-time observer-based feedforward item and a finite-time controller-based feedback item, and the anti-disturbance performance of such a controller is superior to that in Lemma 2.

IV. EXPERIMENTAL RESULTS

A. DESCRIPTION OF DYNAMIC MODEL

The general dynamics for an $n-$link manipulator is formed by

$$\mathcal{M}(q)\ddot{q} + \mathcal{C}(q, \dot{q})\dot{q} + \mathcal{G}(q) = \tau + w(t), \tag{18}$$

where $q = [q_1, q_2, \ldots, q_n]^T \in \mathbb{R}^n$ refers to the measurable robot's joint positions; $\mathcal{M}(q) \in \mathbb{R}^{n\times n}$ refers to its inertia matrix; $\mathcal{C}(q, \dot{q}) \in \mathbb{R}^{n\times n}$ refers to its centripetal-Coriolis matrix; $\mathcal{G}(q) \in \mathbb{R}^n$ refers to the exerted gravitational torques; $\tau \in \mathbb{R}^n$ refers to its control torques; $w(t)$ refers to modeling error and external disturbances. The objective of this experiment is to develop a robust finite-time control strategy for unpredictable time-varying disturbance rejection problem, where the developed controller can drive a manipulator (18) to track the desired trajectory q_r efficiently and accurately. It is known that $\dot{q}_r$ and $\ddot{q}_r$ exist according to the aforementioned information.

B. DESIGN OF CONTROLLER

To design the observer and controller, the used system (18) is rearranged to the form of (8):

$$\ddot{q} = \mathcal{M}_0^{-1}(q)\tau + \xi(t), \tag{19}$$

where $\xi(t) = \Delta\mathcal{M}^{-1}(q)\tau + \mathcal{M}^{-1}(q)(w(t) - \mathcal{C}(q,\dot{q})\dot{q} - \mathcal{G}(q))$ refers to the lumped time-varying disturbance, including unknown system dynamics, parameter uncertainties, and external disturbances.

Next, the developed robust FTOF controller is given as follows.

$$\begin{aligned}
\tau &= \mathcal{M}_0(q)\Big(\ddot{q}_r - c_0\lceil \hat{q}_0 - q_r\rfloor^{\rho_1} \\
&\quad -c_1\lceil \hat{q}_1 - \dot{q}_r\rfloor^{\rho_2} - \hat{\xi}_0\Big), \\
\dot{\hat{q}}_0 &= \hat{q}_1 + l_0\lceil q - \hat{q}_0\rfloor^{\varrho_1}, \\
\dot{\hat{q}}_1 &= \hat{\xi}_0 + \mathcal{M}_0^{-1}(q)\tau + l_1\lceil q - \hat{q}_0\rfloor^{\varrho_2}, \\
\dot{\hat{\xi}}_j &= \hat{\xi}_{j+1} + l_{2+j}\lceil q - \hat{q}_0\rfloor^{\varrho_{2+j+1}}, \\
\dot{\hat{\xi}}_{m-1} &= l_{2+m-1}\lceil q - \hat{q}_0\rfloor^{\varrho_{2+m}}.
\end{aligned} \tag{20}$$

In order to verify the performance of the developed controller, the basic GPIC method developed is given for later comparison:

$$\begin{aligned}
\tau &= \mathcal{M}_0(q)\Big(\ddot{q}_r - c_0[\hat{q}_0 - q_r] \\
&\quad -c_1[\hat{q}_1 - \dot{q}_r] - \hat{\xi}_0\Big), \\
\dot{\hat{q}}_0 &= \hat{q}_1 + l_0(q - \hat{q}_0), \\
\dot{\hat{q}}_1 &= \hat{\xi}_0 + \mathcal{M}_0^{-1}(q)\,\tau + l_1(q - \hat{q}_0), \\
\dot{\hat{\xi}}_j &= \hat{\xi}_{j+1} + l_{2+j}(q - \hat{q}_0), \\
\dot{\hat{\xi}}_{m-1} &= l_{2+m-1}(q - \hat{q}_0).
\end{aligned} \tag{21}$$

In this experiment, the used controller parameters, the developed controller (20) and the basic GPIC (21), are in a second-order polynomial form:

$$p_c(s) = s^2 + 2\zeta_c\omega_c s + \omega_c^2,$$

with $[c_0, c_c] = [\omega_c^2, 2\zeta_c\omega_c]$, where $\omega_c = 60$ and $\zeta_c = 1$. Then, considering a trade-off between the accuracy of observers and the computing resource mentioned on *Remark 1*, the time-varying disturbance is chosen as a third-order form for the second-order system (19), so the used observer gains in (20) and (21) are correspondingly set as a fifth-order form:

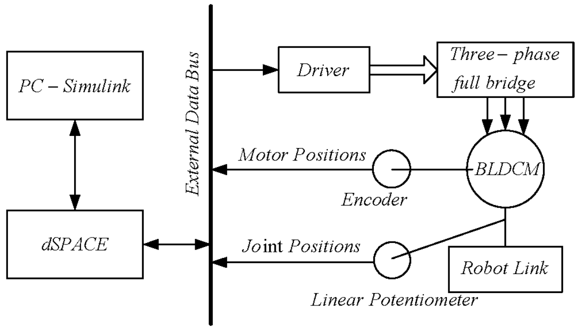

FIGURE 1. Configuration of experimental setup.

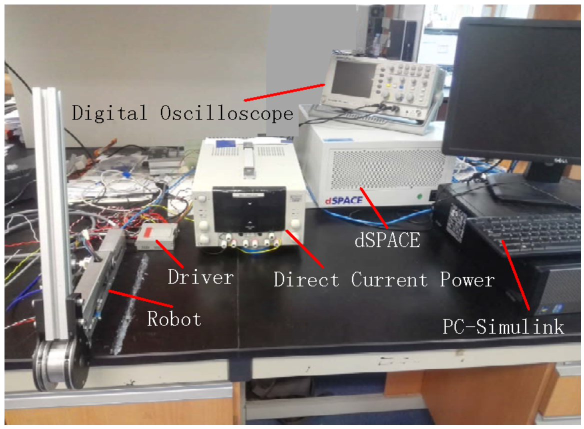

FIGURE 2. Scenario of experimental prototype.

$$p_o(s) = (s + \kappa_o)(s^2 + 2\zeta_o\omega_o s + \omega_o^2)^2,$$

with

$$\begin{aligned}
&[l_0, l_1, l_2, l_3, l_4] \\
&\quad = [\kappa_o + 5\zeta_o\omega_o, 2\omega_o^2 + 4\zeta_o^2\omega_o^2 + 4\zeta_o\omega_o\kappa_o, \\
&\qquad 4\zeta_o\omega_o^3 + 2\omega_o^2\kappa_o + 4\zeta_o^2\omega_o^2\kappa_o,\ 4\zeta_o\omega_o^3\kappa_o + \omega_o^4,\ \omega_o^4\kappa_o],
\end{aligned}$$

where $\kappa_o = \omega_o = 120$ and $\zeta_o = 1$. The others are $\rho_1 = 7/13$, $\rho_2 = 7/10$, $m = 3$, $[\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5] = [\iota + 1, 2\iota + 1, 3\iota + 1, 4\iota + 1, 5\iota + 1]$, and $\iota = -2/15$. The following three experiments are implemented to validate the effectiveness of the proposed control approach.

C. EXPERIMENTAL RESULTS

The experimental results validate the effectiveness of the developed control method. The RFTC and GPIC are used in a single-link rigid manipulator, which has 0.45 m and 0.20 kg on dimension and mass. The nominal parameters $\mathcal{M}_0(q)$ used in the experiment is selected as 4.15×10^{-6} kg·m^2.

The configuration of the experimental setup and scenario of the experimental prototype are shown in Fig. 1 and Fig. 2, respectively. The experimental device comprises a robot

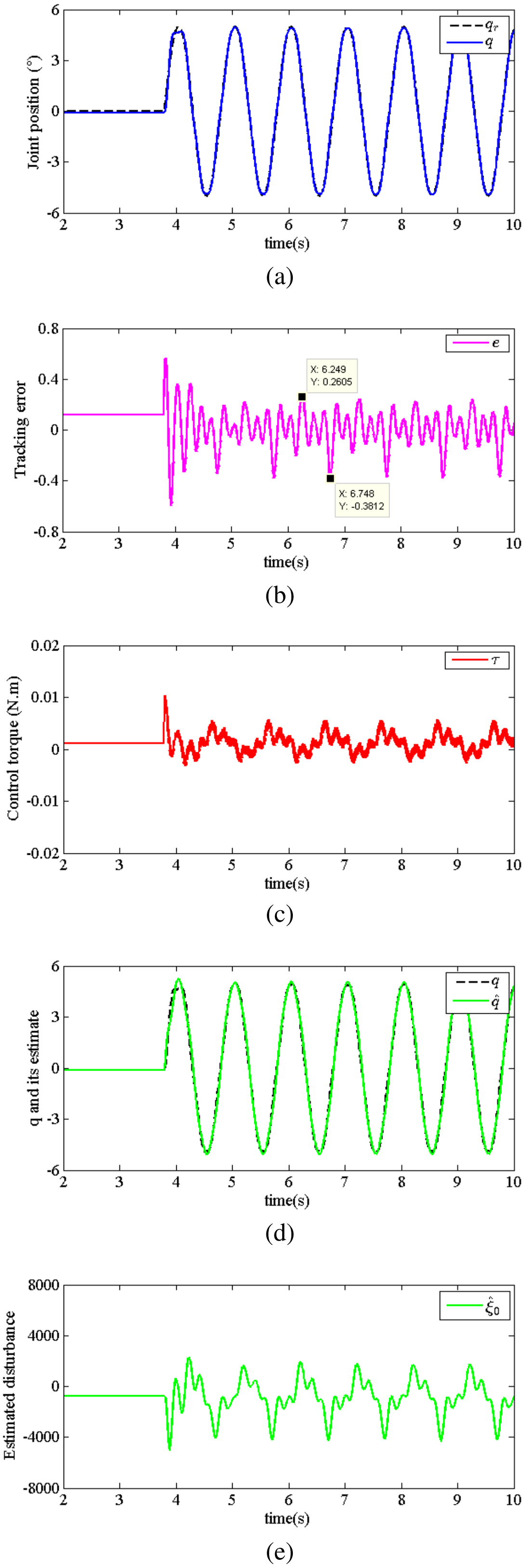

FIGURE 3. Tracking performance of GPIC (Offload).

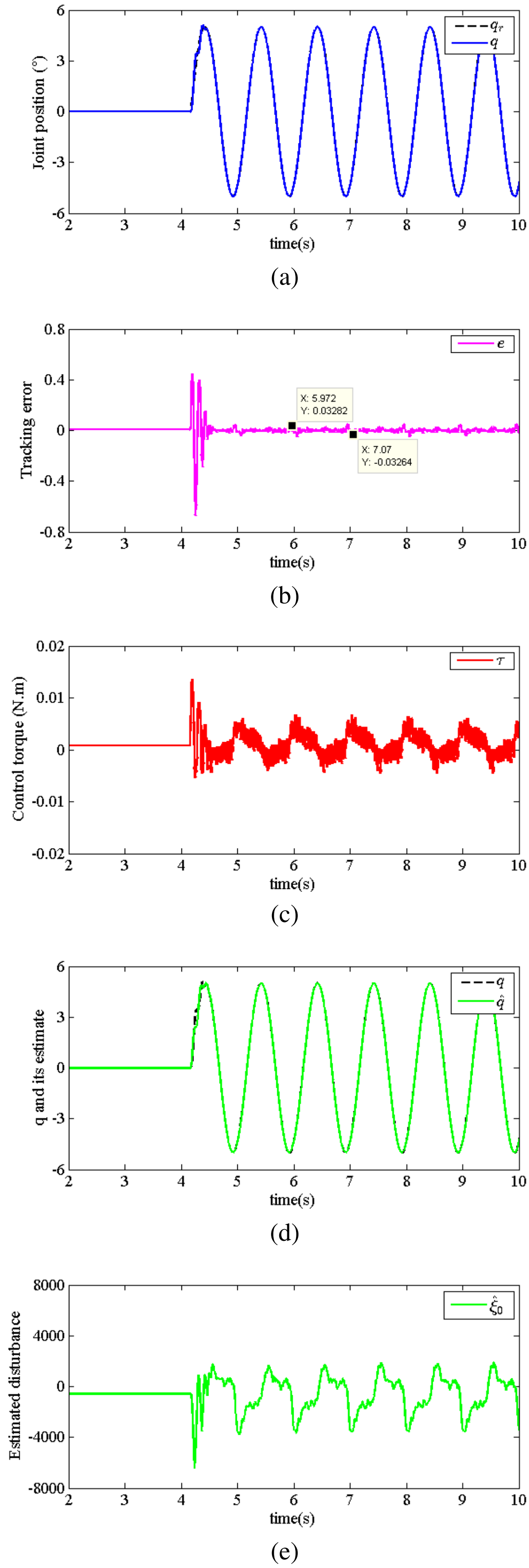

FIGURE 4. Tracking performance of RFTC (Offload).

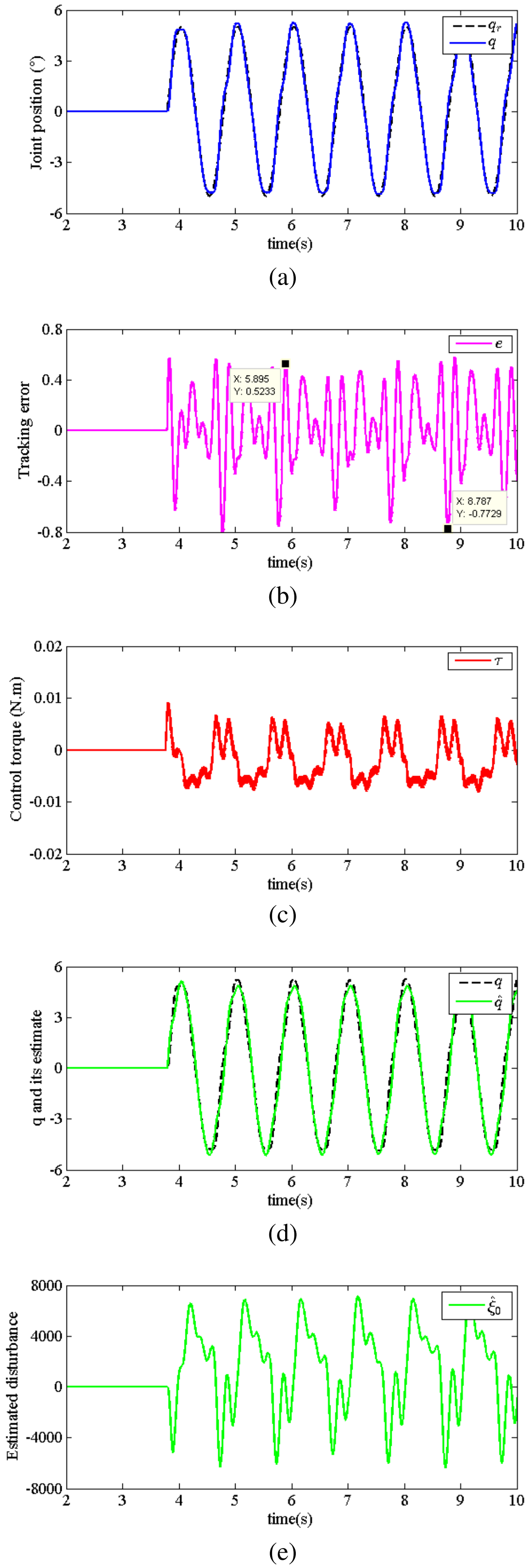

FIGURE 5. Tracking performance of GPIC (Onload).

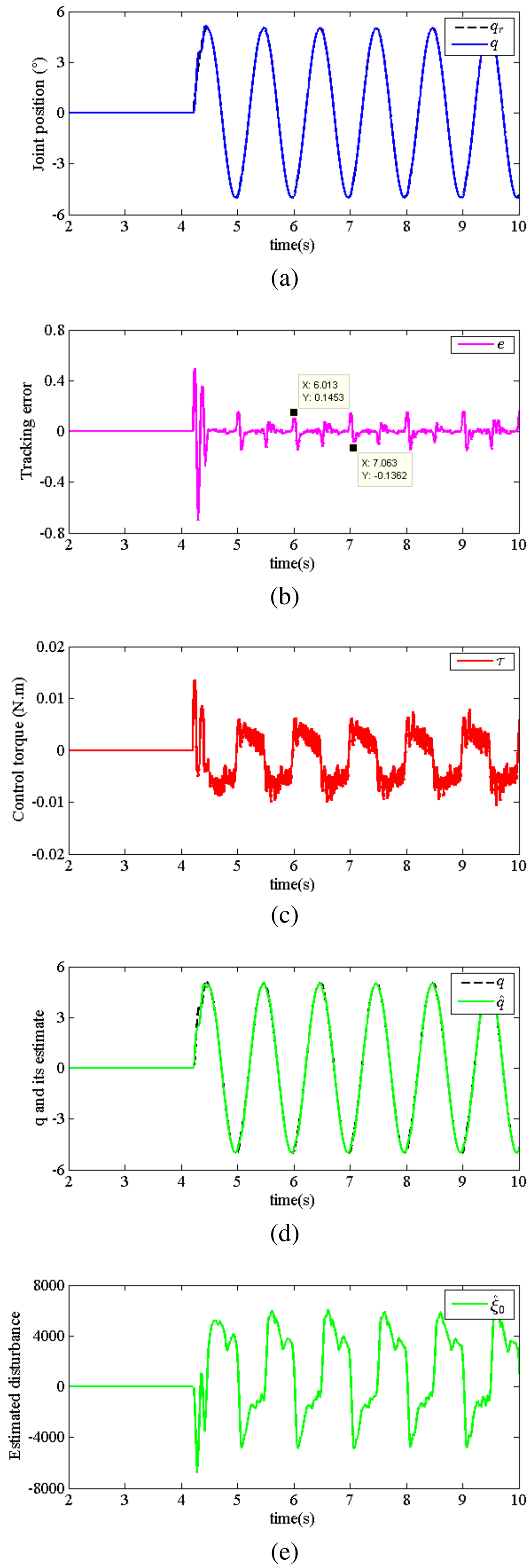

FIGURE 6. Tracking performance of RFTC (Onload).

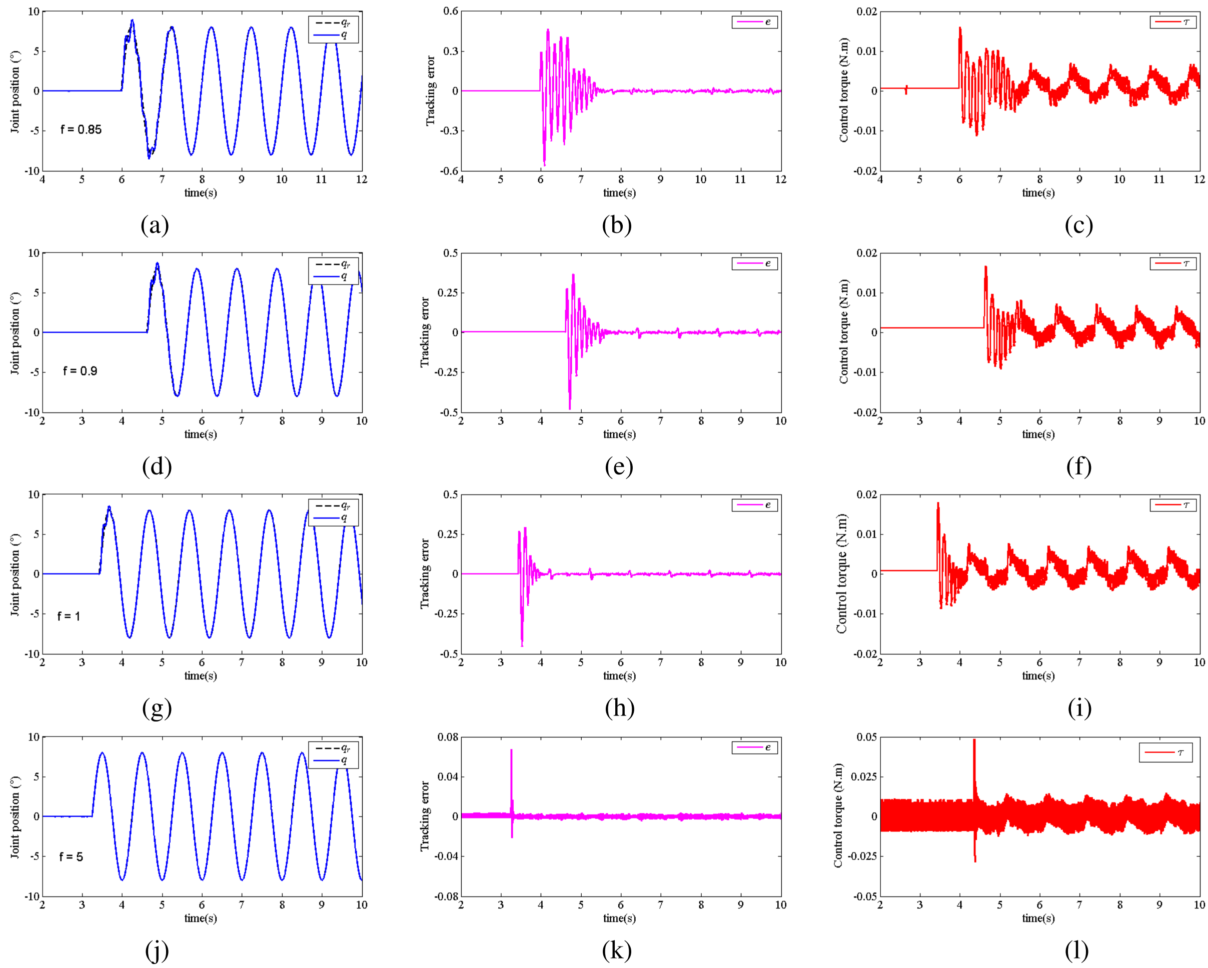

FIGURE 7. Robustness with respect to the control input gain under the RFTC scheme: Top row ($f = 0.85$), second row ($f = 0.9$), third row ($f = 1$), bottom row ($f = 5$).

arm, a dSPACE hardware kit, an Elmo Harmonica 12/60 motor driver, and a personal computer. We use a dSPACE DS1007 real-time hardware kit for implementation, where the sampling period is set to be 0.1 ms for the two control methods. We design three experiments to evaluate and validate the performance of the developed RFTC.

1) RFTC V.S. GPIC DESIGN STRATEGY (Offload)

In the first experiment, we use experimental results to compare the control performance of the developed RFTC method (20) against the GPIC method (21). Joint position trajectory tracking curve, position tracking error, control torque, q and its estimation, and disturbance estimation $\hat{\xi}_0$ are shown in Fig. 3 and Fig. 4, where the desired output is chosen as ($5°$, 1 Hz). As shown in Fig. 3(a) and Fig. 4(a), when the two control methods are used separately, the joint position of the robot can converge to the required trajectory. Nevertheless, it is obvious in Fig. 3(b) and Fig. 4(b) that the tracking error at steady state is in the interval $[-0.0326, 0.0328]$ under the RFTC method, which is significantly better than the GPIC method which is in the interval $[-0.3812, 0.2605]$. The reason is that the estimation of the time-varying disturbance $\hat{\xi}_0$ is observable by the FTSO, facilitating the online disturbance compensation through the feedforward mechanism in the designed controller. This validates that the developed RFTC is better than GPIC.

2) RFTC V.S. GPIC DESIGN STRATEGY (Onload)

In the second experiment, the robustness of the manipulator for external disturbances is studied. The trajectory tracking curves of joint position, position tracking error, control torque, q and its estimation, and disturbance estimation $\hat{\xi}_0$ are given in Fig. 5 and Fig. 6. Herein, external disturbances on the end of the manipulator are generated by the end-effector. It is obvious that both the anti-disturbance ability and trajectory tracking performance can be validated when the developed RFTC method is used.

3) CONTROL GAIN ROBUSTNESS OF RFTC

In the third experiment, we demonstrate the robustness of the RFTC method in controlling gain (20). For the previous two experimental results, the control gain $\mu = \mathcal{M}_0(q)$

was assumed to be known. If the control gain μ cannot be obtained accurately, it can be substituted by an estimation $\hat{\mu}$ that is available in advance. For experimental purposes, the following estimated gain $\hat{\mu} = f\mu$ is employed in the FTSO and RFTC of (20). It can be easily discovered that $f = 1$ represents the exact knowledge of the control gain. In this experiment, the accurate trajectory tracking of the desired profile q_r (chosen as 8°, 1 Hz) is verified in Fig. 7 when f changes in the scope $[0.85, 5]$. However, from the response curves of tracking error (see Fig. 7(b), Fig. 7(e), Fig. 7(h), Fig. 7(k)) and control torque (see Fig. 7(c), Fig. 7(f), Fig. 7(i), Fig. 7(l)), the experiment shows that the smaller f is chosen, the larger the initial tracking error is induced; the bigger f is chosen, the higher noise is induced in control torque. Hence, there is a trade-off for the choice of f between the tracking error and the noise of control torque in practice.

V. CONCLUSION

In this work, a robust FTOF control method is developed for a number of UNS with unpredictable time-varying disturbances. The developed method uses an FTSO to estimate the states and disturbances of the system. To design a robust controller that can eliminate the lumped time-varying disturbances in finite-time and guarantee the stability of closed-loop systems, a combination of FTOF control techniques and the feedforward compensation based on the FTSO are employed. Experiments on the robotic trajectory tracking control is conducted. The experimental results under three different testing conditions show that the developed method is better than the basic GPIC method on the effectiveness.

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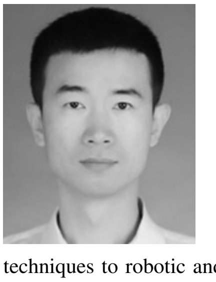

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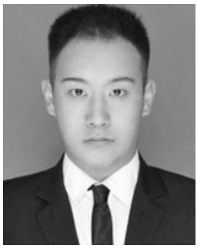

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