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Improved MDD Algorithm for Mission Reliability Estimation of an Escort Formation

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ABSTRACT An escort formation is a phased mission system of systems (PMSoS), which is composed of multiple ships with different functions. The configuration of the formation and success criteria for a mission may vary in different phases. Reliability estimation for PMSoS is complicated due to the strong phase dependence of multiple systems. This paper proposes an improved multiple-valued decision diagram (MDD) algorithm to perform reliability estimation of a nonrepairable escort formation. First, a phased fault tree is established to describe the failure mode of an escort formation throughout a mission, which is simplified according to the common failure basic mission (module) (CFBM). Bottom events are sorted based on the CFBM, and the case method is adopted to generate an MDD from the simplified fault tree model. On this basis, the MDD method is adopted to estimate mission reliability. The performance of the improved MDD method is compared with that of a binary decision diagram (BDD) method and a general MDD method. The results show that the improved MDD method can offer lower computational complexity as well as a simpler model construction over the BDD method and general MDD method. A case study of an escort formation PMSoS is analyzed to illustrate the proposed MDD method, and the sensitivity and composite importance measure (CIM) of each system are evaluated.

INDEX TERMS Escort formation, common failure basic mission (module), multiple-valued decision diagram, phased mission system of systems, reliability.

ACRONYM

BDD	binary decision diagram
CIM	composite importance measure
CFBM	common failure basic mission (module)
DFLM	depth-first-left-most
EOOPN	extended object-oriented Petri net
FV	Fussell-Vesely
MAD	mean absolute deviation
MDD	multiple-valued decision diagram
MMAW	mean multistate risk achievement worth
MMFV	mean multistate FV
MSS	multistate system
PDO	phase-dependent operation
PMS	phased mission system
PMSoS	phased mission system of systems

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I. INTRODUCTION

The escort formation is a typical phased mission system of systems (PMSoS), the corresponding platforms include destroyers, frigates, comprehensive supply ships and shipborne helicopters. The system of each platform is quite complex, and the requirements for completing the mission in various phases are also different; thus, the reliability of the whole system of systems is prioritized. To improve the reliability of escort missions and protect the safety of ships and personnel in navigational seas, this paper analyses the mission reliability of an escort formation to provide an analytical basis for the mission plan of the formation.

A phased mission system (PMS) involves multiple, consecutive and nonoverlapping mission durations [1], [2]. Compared to that for a single-phase mission system, the reliability analysis of a PMS is more complicated due to the correlation among different phases of the system (component). Specifically, assuming that the system and its components are unrepairable, the system (component) should have the same state at the beginning of a phase as at the end of the previous phase. The existing PMS reliability analysis methods mainly include simulation methods and analytical models [3]. Simulation methods are very versatile for system representation, but the calculations can be extensive, and only an approximate system reliability result can be obtained [1], [4], [5]. Analytical models increase the flexibility of system representation and provide applicable solutions; such models can be further classified into three types: combinatorial methods [1], [6]–[10], state-space-oriented methods based on Markov chains or Petri nets [6], [7], [9]–[16], and hybrid methods that integrate combinatorial and state-space-oriented methods as appropriate [17], [18]. In [11], [13], [15], the Markov model was proposed; [12] used the hierarchical method; [14] used the Markov and Petri net models; and [16] proposed an extended object-oriented Petri net (EOOPN) model to analyze a PMS. These models are dynamic models based on state space and can represent the complex dependence among the platforms or systems of a PMS, but they have significant solution limitations. The hybrid method [17] used a binary decision diagram (BDD) and Markov models to calculate the static and dynamic modules of the system, respectively, and combines the advantages of both modeling methods; however, hybrid methods are only applicable in cases in which the lifetime follows an exponential distribution.

The BDD, an important type of combinatorial method, employs a special Boolean algebra called phase algebra to deal with the statistical dependence among Boolean variables of the same component but different phases [1], [14]. The BDD method is more computationally efficient than other existing methods for PMS analysis. However, this method requires a large number of Boolean variables, making BDD-based methods inefficient in analyses of large-scale PMSs. Mo *et al.* [18]–[20] proposed the multiple-valued decision diagram (MDD) method, in which a phased fault tree model is built and is then converted to a phased MDD. The overall MDD model is obtained based on logic gate combination with the phased MDD. Compared with the BDD method, the MDD method can offer lower computational complexity for analyzing large-scale PMSs by producing smaller models using fewer multivalued variables and by avoiding special operations for handling the dependence among variables in the MDD generation and evaluation processes.

In [18] and [19], the MDD algorithm was proposed to estimate the reliability of a general PMS, and the research focused on the components that make up a PMS. However, an escort formation, including multiple ships, is a system of systems consisting of multiple platforms. Compared with a general PMS, this system has many features, such as multiple components, complex functions, and strong phase dependence. Also, the system provides new types of reliability logic, such as a) the network relationship due to the multipurpose characteristics of each ship in the communication and command network, in our work, the network structure is transformed into a shared reliability block diagram; b) a plus system and its components may appear at the same or different phases, we have not studied it yet in this paper. We call such a situation a phased mission system of systems (PMSoS) in this paper. In addition, there are many redundant structures in phased fault tree, and the same system appears repeatedly in multiple phases. Thus, the scale and complexity are larger than that in [18] and [19]. All of these increase the difficulty of reliability assessment if the phased fault tree is directly transformed into a phased MDD for combination.

To solve the above problem, this paper makes improvements to the existing MDD method [18]–[20]. The phased fault tree is simplified according to the common failure basic mission (module) (CFBM), which simplifies the fault tree model of the PMSoS. The flow of transforming fault tree into an MDD with the case method is given, followed by the simplification of the edges in the generated MDD model, which reduces the number of calculations. The flow chart and pseudocode of the MDD algorithm are given to improve the efficiency of reliability estimation.

The remainder of this paper is organized as follows. Section II describes the reliability problem of the escort formation PMSoS. Section III introduces the MDD algorithm to analyze the reliability of this PMSoS. Section IV describes the advantages of the improved MDD method compared with the BDD method and the existing MDD method. Section V presents a case study of the escort formation to further illustrate the proposed MDD method. Finally, in Section VI, we present our conclusions and directions for future work.

II. RELIABILITY BLOCK DIAGRAM MODEL OF THE ESCORT FORMATION PMSoS

The typical escort formation consists of a destroyer, a frigate, a comprehensive supply ship and two helicopters. The destroyer serves as the command ship of the formation; in contrast with the frigate and the comprehensive supply ship, it is equipped with a to-shore satellite communicate system and a formation command system. The destroyer and frigate are also equipped with a sea radar search system, which the comprehensive supply ship lacks. In addition, all three ships are equipped with a maneuvering, location and communication system. The mission includes a navigation phase, a ''sea warning when entering a pirate area'' phase, a ''helicopter moving away from the attack'' phase and a return phase.

A. MODEL DESCRIPTION

The numbers 1, 2 and 3 represent the destroyer, frigate and comprehensive supply ship, respectively. The letters *A*-*O* represent each equipment system, and t_1, t_2, t_3 and t_4 represent the end times of the four phases.

B. MISSION RELIABILITY BLOCK DIAGRAM OF EACH PHASE

Before analyzing the reliability of the PMSoS, the duration of each phase and the reliability logic relationships among the systems in each phase should be clarified. Fig. 1 shows the reliability block diagram of each phase mission for the escort formation.

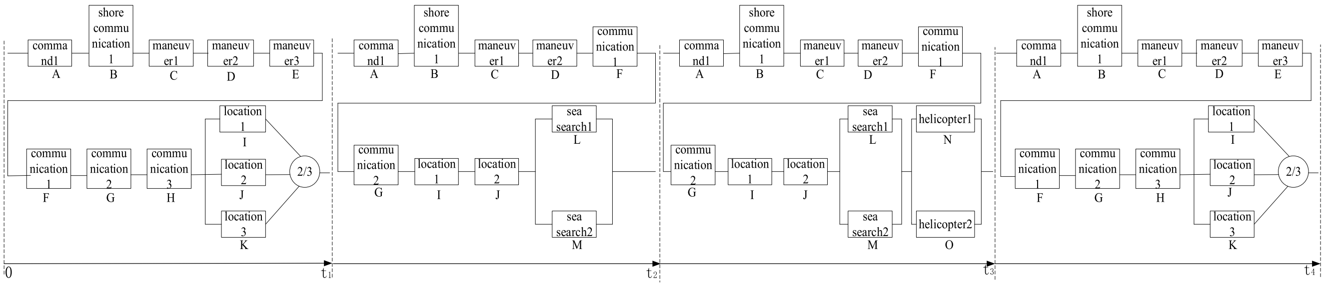

FIGURE 1. Reliability block diagram of each phase mission for the escort formation.

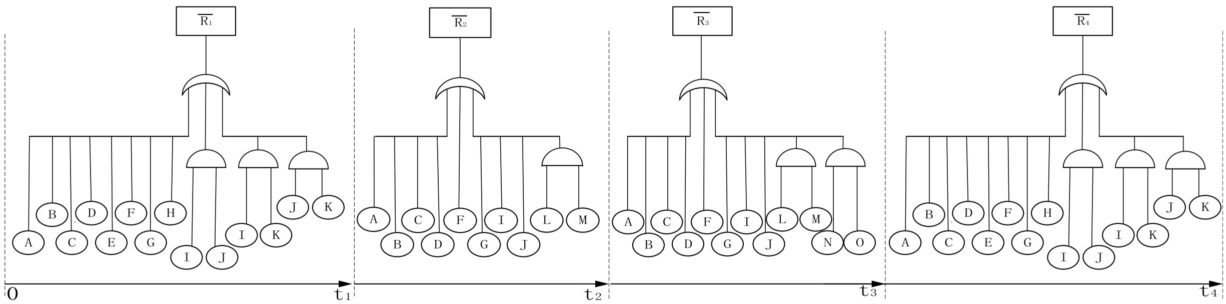

FIGURE 2. Fault tree of each phase for the escort formation PMSoS.

During the navigation phase, the completion of the mission requires the command and to-shore communication systems of the destroyer and the maneuvering and formation internal communication systems of the three ships to be in a normal state; additionally, at least two of the three location systems on the ships must be in the normal state to complete the positioning of the entire formation, and the mission duration is $(0, t_1)$. In the sea warning phase, completing the mission requires the command and to-shore communication systems of the destroyer and the maneuvering, formation internal communication and location systems of the destroyer and frigate to be in a normal state; as long as one of the two ships' surface radar search systems is in a normal state, information sharing can be completed through the formation internal communication system, the mission duration is (t_1, t_2). In the "helicopter moving away from attack" phase, completing the mission requires that one helicopter be removed according to the size of the pirate force after the target is detected by surface radar, the mission duration is (t_2, t_3). In the return phase, the completion requirements are the same as those in the navigation phase, and the duration is (t_3, t_4).

III. PMSoS ANALYSIS USING AN MDD

The MDD method for PMSoS reliability analysis mainly includes the following four steps: (1) construct the phased mission reliability calculation model; (2) draw the corresponding phased fault tree according to the mission; (3) transform the fault tree into an MDD; and (4) simplify the model, and obtain quantitative calculations.

A. PHASED MISSION RELIABILITY CALCULATION MODEL

R is a phased mission that can be divided into Q phases; R_q indicates that the mission of phase q is reliable, and $\overline{R_q}$ indicates that the mission of phase q fails. S_q indicates that the mission to phase q is reliable, and F_q indicates that the mission to phase q fails. When S_q occurs, we have:

$$S_q = R_1 \cap R_2 \cap \cdots \cap R_q \tag{1}$$

Formula (1) indicates that when the mission is reliable for phase q , the phases before phase q (including phase q) must all be reliable. Furthermore, the reliability of the mission for each phase is:

$$P(S_q) = P(R_1 \cap R_2 \cap \cdots \cap R_q) \tag{2}$$

Thus, the reliability of the last phase is also the reliability of the overall mission.

B. FAULT TREE OF EACH PHASE

According to the reliability block diagram of each phase for the escort formation in Fig. 1, the corresponding phased fault tree can be obtained by transformation, as shown in Fig. 2.

C. PMSoS MDD MODEL GENERATION

In the reliability model of the escort formation PMSoS, there are many redundant structures, such as k out of n, as well as the CFBM, which cause the same system to appear repeatedly in different phases. Formula (1) shows that the MDD of S_q is generated by combining the MDD of R_q with the reliability MDD of the previous phases through a logical "AND" relationship. Therefore, to simplify the calculation, the fault tree is firstly simplified according to the CFBM, and the simplified phased MDDs are then combined to obtain the MDD of S_q.

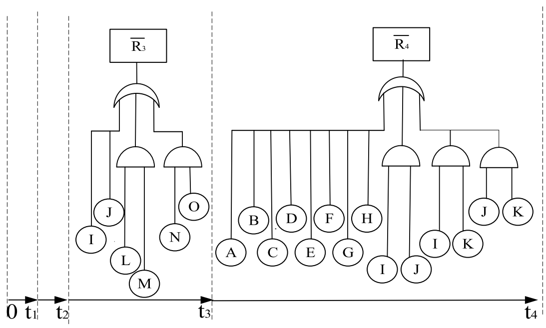

FIGURE 3. **Simplified phased fault tree of F_4.**

1) SIMPLIFY THE PHASED MISSION FAULT TREE

The CFBM is defined as follows. In a PMSoS, if a basic mission (module) directly causes the PMSoS to fail in phase q, then a basic mission (module) failure would directly cause the PMSoS to fail in all phases of the basic mission (module) occurring before phase q.

Taking A for example, assuming that it directly causes the PMSoS to fail in phase q, then in the fault tree from the first phase to the phase $q-1$, if A is under the OR gate, delete the bottom event of A; if A is under the AND gate, delete the entire AND gate where A is located.

For F_4, Fig. 2 shows that there are CFBMs A, B, C, D, E, F, G, H in $\overline{R_4}$. Delete A, B, C, D, F, G in the first three phases and E, H in the first phase. Then, there are CFBMs $I, J, \{L, M\}$ in $\overline{R_3}$, delete $I, J, \{L, M\}$ in the second phase and $\{I, J\}, \{I, K\}, \{J, K\}$ in the first phase to obtain the simplified phased fault tree of F_4, as shown in Fig. 3.

2) SORT THE BOTTOM EVENTS

The key to the application of the MDD method is to sort the bottom events when transitioning the fault tree to the MDD model, and the node number and computational complexity of the final MDD depend largely on the input order of the bottom events. Referring to the current research, sorting methods based on the CFBM and the minimum adjacent bottom event are more suitable for the PMSoS than other methods. By sorting the minimum adjacent bottom events adjacently and giving low-ranking priority to the nodes with a large number of repetitions increase the number of shared nodes in the MDD, further improving the reliability calculation efficiency of the PMSoS [21].

Suppose that there are n systems in the PMSoS composed of k phases; T represents the CFBM set, C_p represents the $p-th$ ordered minimum adjacent bottom events set in a certain phase, O_i represents the order result $(i \leq k)$ in phase i of the PMSoS fault tree, and $\max(C_p)$ represents the maximum repetition number of the bottom events in C_p. The sorting steps are shown in Fig. 4.

The process of determining the order of all bottom events in the PMSoS is as follows.

(1) Prioritize the CFBMs in T. If there were multiple CFBMs in the set, sort them in order from left to right.

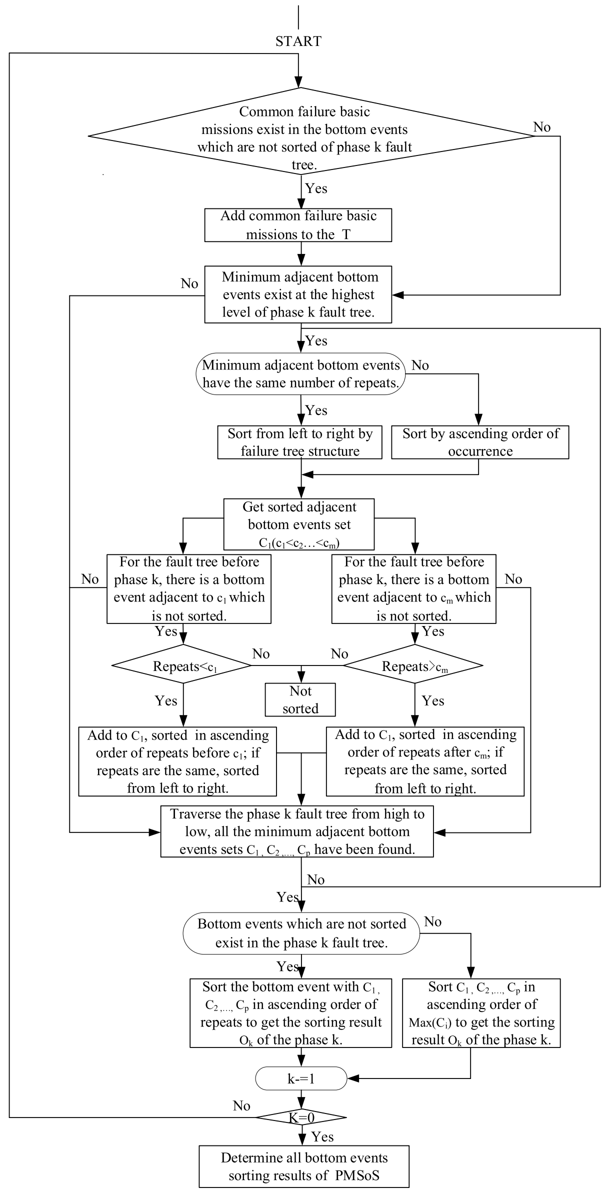

FIGURE 4. **Bottom events sorting steps for the PMSoS.**

(2) Determine the order of the bottom events in the various phases. For the sorting results of the $l(l \leq k)$ groups $O_1, O_2, \cdots, O_l$, find the maximum number of repetitions of all bottom events in each group, denoted as $\max(O_l)$. Sort $O_1, O_2, \cdots, O_l$ in ascending order according to $\max(O_l)$. If the two values are equal, sort in ascending order of l, that is, the generation order of the sorting group. Finally, the final bottom events order of the PMSoS is obtained.

The following steps are applied for the simplified fault tree F_4 in Fig. 3.

Step 1. Sort from phase 4. There are CFBMs A, B, C, D, E, F, G, H joined in set T since there are no minimum adjacent bottom events at the highest level where these CFBMs are located; thus, we trace downward to find the minimum adjacent bottom events. I, J, and K are the minimum adjacent bottom events, and the number of repetitions is $K < I = J$;

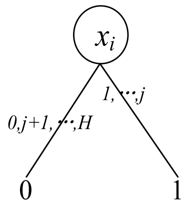

FIGURE 5. C_i **fails at phase** j**.**

thus, I and J are sorted in order from left to right, so the first minimum adjacent bottom event sorting set $C_1(K \prec I \prec J)$ is obtained. Track all previous phases for K; if no minimum adjacent bottom events of K are found, track all previous phases of I and J to find those that are the minimum adjacent bottom events in the third phase. The sorting order of this phase is $O_1(K \prec I \prec J)$.

Step 2. Sort phase 3. In phase 3, I and J participate in the sorting process, and the CFBM is obtained. Start from the highest level of the fault tree, look for the minimum adjacent bottom events from top to bottom. If no minimum adjacent bottom events are found at the highest level, then trace the events downward. L and M, and N and O are adjacent bottom events with the same number of repetitions. The minimum adjacent ordering sets $C_1(L \prec M)$ and $C_2(N \prec O)$ are obtained according to the sorting principle from left to right. In addition, when $\max(C_1) = \max(C_2) = 1$, the number of repetitions is the same, so the sorting order of phase 3 is $O_2(L \prec M \prec N \prec O)$ according to the sorting principle from left to right.

Step 3. All the bottom events of F_4 are involved in the sorting process to determine the final order of the bottom events. The CFBMs A, B, C, D, E, F, G, H are first sorted, and O_1 and O_2 are then sorted; $\max(O_1) = 2$, and $\max(O_2) = 1$, so O_2 is sorted earlier. Therefore, the sorting order is $A \prec B \prec C \prec D \prec E \prec F \prec G \prec H \prec L \prec M \prec N \prec O \prec K \prec I \prec J$.

3) GENERATE THE PHASED MISSION MDD USING THE CASE METHOD

An MDD consists of non-sink decision nodes and two sink nodes labeled '0' and '1', representing system success and failure, respectively. Each non-sink decision node is labeled with a multivalued variable and has multiple outgoing directed edges. For the PMSoS, each system C_i with H phases is represented using an $(H+1)$-valued variable x_i, and each non-sink node associated with x_i in the MDD model has $H+1$ outgoing edges; a 0-edge indicates that the system has survived all the H phases ($x_i = 0$), and a jth edge indicates that the system fails within phase j given that it is working at the beginning of the phase j ($x_i = j$).

Fig. 5 shows the MDD encoding for a basic failure event in the jth phase fault tree of the H-phase system C_i.

The case structure is defined as

$$\begin{aligned} f &= case(A, F_1, \cdots, F_r) = case(A, F_{xA} = 1, \cdots, F_{xA} = r) \\ &\equiv A_1 \cdot F_{xA} = 1 + \cdots Ar \cdot F_{xA} = r \end{aligned} \tag{3}$$

Suppose two logic expressions representing sub-MDDs G and H are expressed as follows:

$$\begin{aligned} G &= case(x, G_1, G_2, \cdots, G_r) \\ H &= case(y, H_1, H_2, \cdots, H_r) \end{aligned} \tag{4}$$

Then, the logical operation between G and H is represented by the MDD operation as follows:

$$\begin{aligned} I &= G \diamond H = case(x, G_1, \cdots, G_r) \diamond case(y, H_1, \cdots, H_r) \\ &= \begin{cases} case(x, G_1 \diamond H_1, \cdots, G_r \diamond H_r) & index(x) = index(y) \\ case(x, G_1 \diamond H, \cdots, G_r \diamond H) & index(x) < index(y) \\ case(y, G \diamond H_1, \cdots, G \diamond H_r) & index(x) > index(y) \end{cases} \end{aligned} \tag{5}$$

The logic operation $\diamond$ can be either a logic AND or a logic OR; $index(v)$ is the sequence number of the variable v. The OR gate in the fault tree is represented as an addition, and the AND gate is represented as a multiplication. The above operation is repeated to process the sub-MDD expressions until the system finally becomes a constant expression of 0 or 1.

Note that no special algebra is needed, unlike in the BDD method, the state dependence across phases is automatically considered by the MDD manipulation operations in (5). For example, consider the AND(+) and OR(·) operations in the two sub-MDDs $G = case(x, 0, 1, 0, 0)$ and $H = case(x, 0, 1, 1, 0)$:

$$\begin{aligned} G \cdot H &= case(x, 0, 1, 0, 0) \cdot case(x, 0, 1, 1, 0) \\ &= case(x, 0, 1, 0, 0) = G \\ G + H &= case(x, 0, 1, 0, 0) + case(x, 0, 1, 1, 0) \\ &= case(x, 0, 1, 1, 0) = H \end{aligned} \tag{6}$$

The physical meaning of formula (6) is that the system has failed in phases 1 and 2, equivalent to the system failing in phase 1; the system having failed in phase 1 or 2 (it might be operational in phase 1) is equivalent to the system having failed in phase 2.

The main idea of using the case method to transform the fault tree into an MDD is to recursively use the operations in formula (5). The algorithm flow is as shown in Fig. 6. Starting from the bottom gate event of the simplified fault tree, replace the gate event with the bottom events from bottom to top according to the order of the basic events in the fault tree, and encode each replacement step at the same time according to the case structure until all the gate events are encoded with the bottom events. In this way, the MDD of the top event is obtained.

The phased MDD model after simplification is shown in Fig. 7.

4) PHASED MDD COMBINATION TO OBTAIN THE MDD OF THE PMSoS

According to the operation rules of formula (5), the MDD of the PMSoS can be obtained by combining the phased MDD with the phased mission reliability calculation model in 3.1, as shown in Fig. 8.

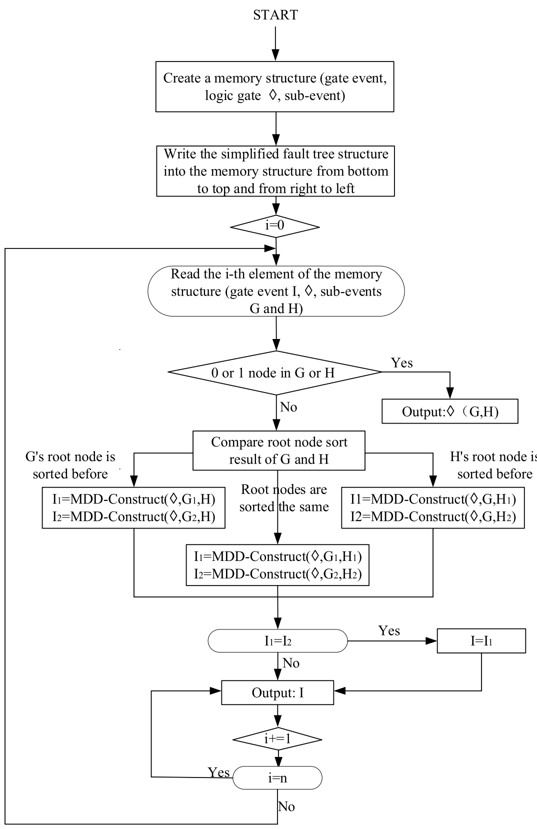

FIGURE 6. Case method flow.

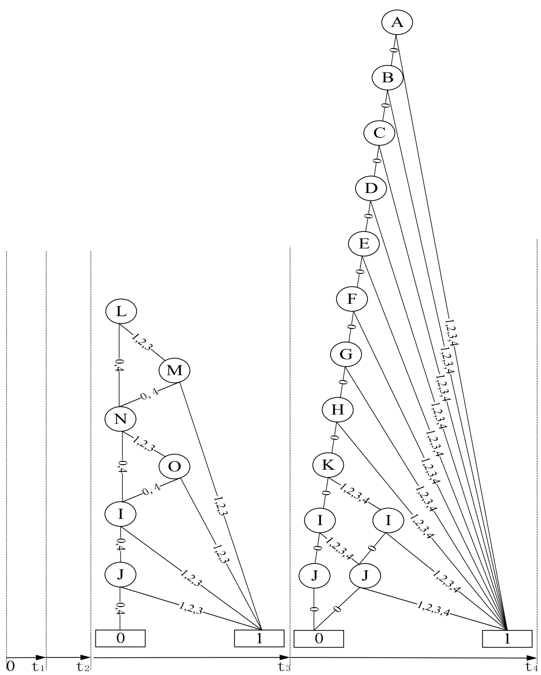

FIGURE 7. Simplified phased MDD.

5) SIMPLIFY THE MDD MODEL

From the MDD of the PMSoS, the nonintersecting path set L_q can be obtained to indicate that the PMSoS satisfies the reliable conditions. However, the expression of L_q obtained by traversing the path is often too complicated and needs to be simplified when performing quantitative calculations. The subscripts of each variable are simplified: A_j indicates that system A fails during (t_{j-1}, t_j), $A_{1,2,\cdots,j}$ indicates that A fails during (t_0, t_j), $\overline{A_j}$ represents A operations during (t_{j-1}, t_j), $\overline{A_{1,2,\cdots,j-1}}$ represents A operations during (t_0, t_{j-1}), and A_0 represents A operations in all phases.

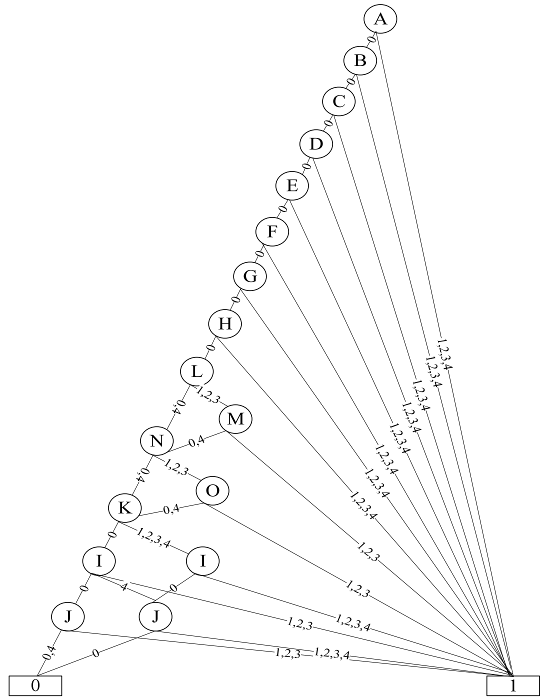

FIGURE 8. MDD of the PMSoS.

Combined with Fig. 5, $P(A_j)(1 \leq j \leq H)$ reflects the probability that the system fails in phase j given that it is working at the beginning of phase j. Then:

$$P(A_0) = 1 - F_A(t_H) \tag{7}$$

$$P(A_j) = F_A(t_j) - F_A(t_{j-1}) \tag{8}$$

$$P(A_{1,2,\cdots,j}) = 1 - F_A(t_j) \tag{9}$$

$$P(A_{0,H}) = 1 - F_A(t_{H-1}) \tag{10}$$

where $F(t)$ is the failure probability function.

Since the paths obtained by the MDD are disjoint, the reliability of the overall mission can be obtained:

$$P(S_4) = \sum P(L_q) \tag{11}$$

6) ALGORITHM FLOW OF THE MDD

According to the principle of the simplified model, the process of the MDD algorithm is shown in Fig. 9.

IV. COMPARISION TO EXISTING METHODS

In this section, the proposed MDD model is compared to the BDD method and the existing MDD method.

A. BDD METHOD

The order of input variables has a considerable impact on the size of the models generated by the MDD and BDD methods;

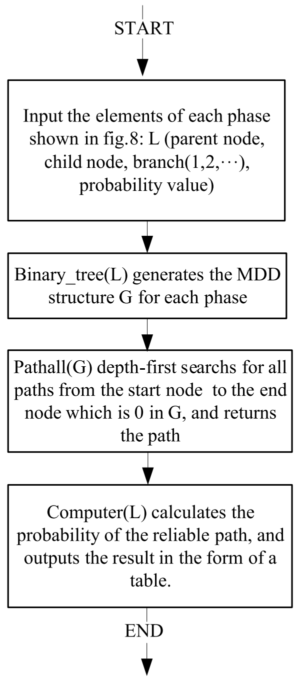

FIGURE 9. MDD algorithm flow.

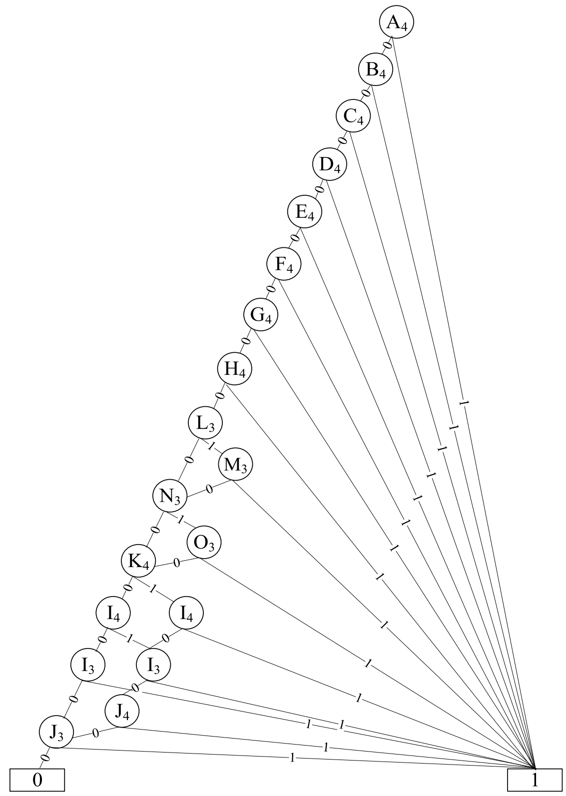

FIGURE 10. BDD model of the PMSoS.

moreover, the size of the BDD model is related not only to the order among various systems but also among different phases of the same system. The ordering method in this paper is used to determine the order among systems on the basis of simplifying the fault tree based on the CFBM; previous research has shown that using the backward PDO ordering method on the same component can decrease the size of the BDD model. Therefore, the order of generated BDDs is $A_4 \prec B_4 \prec C_4 \prec D_4 \prec E_4 \prec F_4 \prec G_4 \prec H_4 \prec L_3 \prec M_3 \prec N_3 \prec O_3 \prec K_4 \prec I_4 \prec I_3 \prec J_4 \prec J_3$, and the BDD model of the PMSoS is shown in Fig. 10.

TABLE 1. MDD Versus BDD.

Method	Model size
Existing MDD	17
Improved MDD	17
BDD	19

B. EXISTING MDD METHOD

The existing MDD method first uses the depth-first-left-most (DFLM) heuristic algorithm to sort the bottom events of each phase of the PMSoS and then transforms the phased fault tree into a phased MDD model according to the order of bottom events. Finally, the method combines the phased MDD to obtain the MDD of the PMSoS according to the operation rules in formula (5). Fig. 11 shows the phased MDD model using the existing method.

Then, the phased MDDs are combined to obtain the overall mission MDD model shown in Fig. 8.

C. RESULTS AND DISCUSSION

In the PMSoS, the size of the model (the number of non-sink nodes) is generally used to represent the complexity of the model as a benchmark for comparing performance. Table 1 shows the model sizes of the three methods.

Based on a comparison of results, we summarize the performance of the proposed MDD method as follows.

1) Table 1 shows that the MDD method can provide remarkably smaller system models than the BDD approach. Because the complexity of the model generation and evaluation algorithms highly depends on the number of nodes in the system model, the MDD method is computationally more efficient than the BDD method, especially for large PMSoSs.
2) By comparing Figs. 7 and 11, it is found that the improved MDD method proposed in this paper by simplifying the process of generating the MDD of the overall mission from the phased MDDs and simplifying the edges in the MDD model significantly improves the PMSoS evaluation efficiency compared with using the existing MDD method, even if the MDDs of the overall mission obtained by these two methods are the same.
3) For a PMSoS, the MDD process is simplified based on CFBMs. Since the system in the working state in the previous phases is removed in this simplified approach, the reliability of the overall mission can only be estimated by obtaining the reliability instead of the failure probability of the overall mission using the disjoint path set algorithm. The principle of this approach is as follows. Because the system is non-repairable, a CFBM

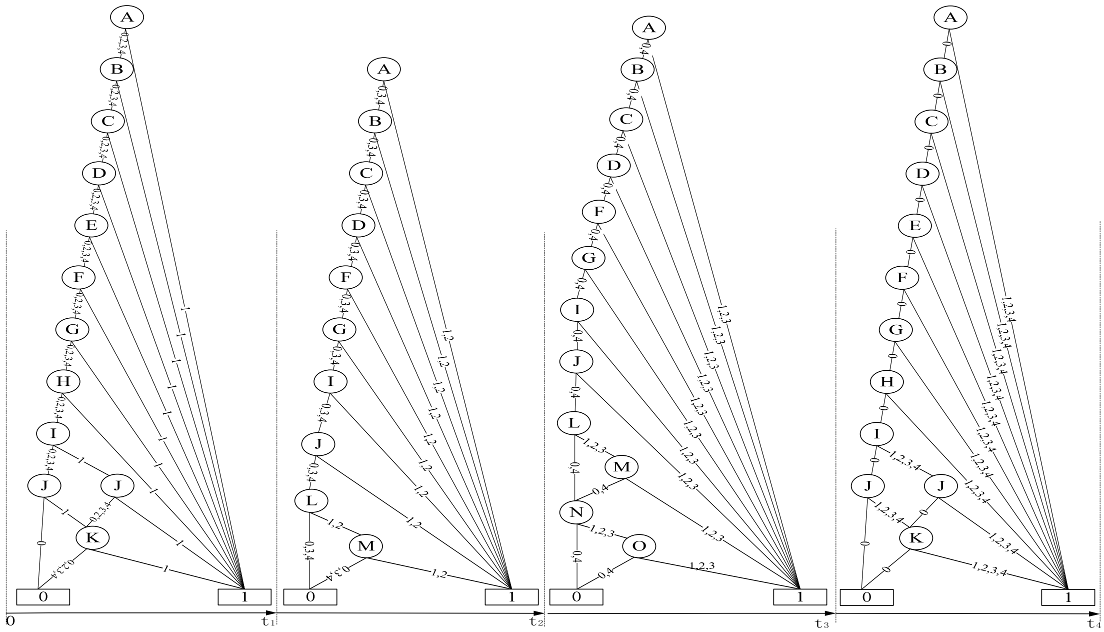

FIGURE 11. Phased MDD model.

TABLE 2. System code and failure rate.

Code	Failure rate ($10^{-4}/h$)	Code	Failure rate ($10^{-4}/h$)
A	4	H	2
B	3	I	2
C	2	J	2
D	2	K	2
E	2	L	5
F	2	M	5
G	2		

works in a later phase indicates that the system is in a working state in the previous phases. Therefore, for a CFBM, when calculating the reliability of the overall mission, the system appeared in the previous phases can be removed to directly calculate the reliability of the CFBM in later phase. The simplification process in this paper does not affect the reliability of the overall mission, but if the method is used to calculate the failure probability of the overall mission and then determine the reliability, there would be an error, because some paths of failure were lost during simplification.

V. CASE STUDY

It is assumed that the starting times of the four phases are 0, 15, 20 and 24 hours and the ending times are 15, 20, 24 and 40 hours, respectively.

A. RELIABILITY CALCULATION OF THE PMSoS

In the escort formation PMSoS, the lives of command, shore communication, location, maneuvering, and formation internal communication systems follow exponential distributions. The failure rates are shown in Table 2.

TABLE 3. System code and Weibull distribution parameters.

Code	m	$\eta(h)$
N	2	2000
O	2	2000

The life of the helicopter obeys a Weibull distribution, and the values of m and η are shown in Table 3.

For $A - M$, the lifetimes of which also follow an exponential distribution,

$$F(t) = 1 - e^{-\lambda t} \tag{12}$$

For $N - O$, with lifetimes that follow a Weibull distribution,

$$F(t) = 1 - \exp\left\{-(\frac{t}{\eta})^m\right\} \tag{13}$$

Then, the probabilities of each state for a system are calculated from formulas (7)-(8), as shown in Table 4. In a PMSoS, the states of a system are not only binary: function and failure, for the multiple-valued decision diagram (MDD) algorithm in this paper, each system has five states: 0, 1, 2, 3, and 4, which represents "works in all phases", "fails in the first phase", "fails in the second phase", "fails in the third phase" and "fails in the fourth phase", respectively.

The reliability of the escort formation PMSoS is 0.9178 using the MDD algorithm according to formulas (9)-(11). Table 5 compares the MDD and BDD algorithms for mission estimation.

Both the BDD and MDD methods are used to evaluate reliability and theoretically yielded accurate results. However, when the disjoint path set algorithm is used to calculate the reliability of the overall mission, the obtained paths would differ, and the calculation result of the PMSoS varies from

TABLE 4. Reliability of each system in various phases.

System	Probability of each state				
	0	1	2	3	4
A	0.98412	0.00598	0.00199	0.00159	0.00632
B	0.98807	0.00449	0.00149	0.00119	0.00475
C-K	0.99203	0.00299	0.00099	0.00080	0.00318
L-M	0.98020	0.00747	0.00248	0.00198	0.00787
N-O	0.99960	0.00006	0.00004	0.00004	0.00003

TABLE 5. MDD versus BDD.

	MDD	BDD
Reliability	0.91777476	0.92214175
Evaluation time (s)	0.03870440	0.04666680

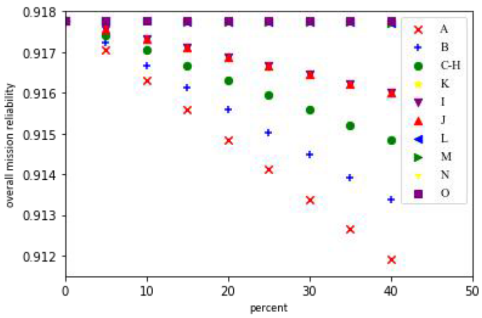

FIGURE 12. Reliability sensitivity of each system.

the actual value due to algorithm operations such as preserving decimals and the arithmetic operations among the input systems along a path. Therefore, the accuracy of the calculation results increases as the number of decimals of each system increases. The results indicate that the relative error of the MDD method relative to that of the BDD algorithm for mission reliability is calculated to be 0.47%, and it is too small to be ignored; for the evaluation time, the efficiency of the MDD algorithm relative to that of the BDD method is 17.06% higher. Thus, the MDD method is better for mission reliability estimation of an escort formation.

B. RELIABILITY SENSETIVITY ANALYSIS OF EACH SYSTEM

To determine the impact of each system on the escort formation PMSoS, a reliability sensitivity analysis of each system is required. In the case where the failure rate of systems *A*-*M* increases by 5%, 10%, 15%, 20%, 25%, 30%, 35%, or 40% and the η of *N*-*O* is reduced by 5%, 10%, 15%, 20%, 25%, 30%, 35%, or 40%, Fig. 12 shows the coordinate graph of the impact of system reliability variations on overall mission reliability.

From a functional perspective, the reliability of the overall mission is obtained by combining Fig. 8 and formula (11):

$$\begin{aligned} P(S_4) = &P(A_0)\cdot P(B_0)\cdot P(C_0)\cdot P(D_0)\cdot P(E_0)\\ &\cdot P(F_0)\cdot P(G_0)\cdot P(H_0)\cdot (P(L_{0,4})\\ &+P(L_{1,2,3})\cdot P(M_{0,4})\cdot (P(K_0)\cdot P(I_0)\cdot P(J_{0,4})\\ &+((P(K_0)\cdot P(I_4)+P(K_{1,2,3,4})\cdot\\ &P(I_0))\cdot (P(N_{0,4})+P(N_{1,2,3})\cdot P(N_{0,4})) \end{aligned} \tag{14}$$

Taking system *A* with an exponential life distribution as an example, to calculate the impact of the failure rate on the overall mission reliability, the failure rates of other systems must remain unchanged; thus, let

$$\begin{aligned} k = &P(B_0)\cdot P(C_0)\cdot P(D_0)\cdot P(E_0)\cdot P(F_0)\cdot P(G_0)\cdot P(H_0)\\ &\cdot (P(L_{0,4})+P(L_{1,2,3})\cdot P(M_{0,4})\cdot (P(K_0)\cdot P(I_0)\\ &\cdot P(J_{0,4})+((P(K_0)\cdot P(I_4)+P(K_{1,2,3,4})\cdot P(I_0))\\ &\cdot (P(N_{0,4})+P(N_{1,2,3})\cdot P(N_{0,4})), \end{aligned}$$

expressed as

$$P(S_4) = k\cdot P(A_0) \tag{15}$$

Here, k is a constant, and $P(A_0) = e^{-40\lambda_A}$, so:

$$\frac{\partial P(S_4)}{\partial \lambda_A} = -40ke^{-40\lambda_A} \tag{16}$$

The Taylor expansion of $e^{-40\lambda_A}$ is obtained, which leads to:

$$\frac{\partial P(S_4)}{\partial \lambda_A} = -40k(1-40\lambda_A+o(\lambda_{A^2})) \tag{17}$$

and the magnitude of λ_A is 10^{-4}, thus:

$$\frac{\partial P(S_4)}{\partial \lambda_A} = -40k \tag{18}$$

Because $k > 0$, it is concluded that the overall mission reliability decreases approximately linearly with an increasing system *A* failure rate under the condition that other system failure rate parameters remain unchanged, and $40k$ is the slope of the line, reflecting the reliability sensitivity of the system. This conclusion is consistent with the trend presented in Fig. 12, and the impact order of system reliability variations on overall mission reliability is $A > B > C-H > J > L(M) > K(I) > N(O)$, therefore, the command system of the destroyer has the most significant impact on the overall mission reliability.

C. IMPORTANCE ANALYSIS OF EACH SYSTEM

Importance measures facilitate the prioritization of various reliability improvement tasks by quantifying criticalities of the system. Specifically, these measures identify the systems that contribute the most to the overall PMSoS reliability (or susceptibility to failure) and thus help identify good candidates to upgrade so the reliability of the entire system can be improved. There are two methods to measure the importance of an multistate system (MSS) [22], [23]: type-1 measures, or composite importance measures (CIMs), assess the contribution of specific polymorphic elements to the reliability of the MSS, and type-2 measures identify the most important component performance level (state) related to MSS reliability.

Type-1 measures can be divided into two categories: general CIMs and alternative CIMs [22], [24]. The general CIMs focus on the possible state levels of a component but do

TABLE 6. Results for $P(U'|x_i = b_{ij})$ using the MDD approach.

System	$P(U' \mid x_i = b_{ij})$				
	0	1	2	3	4
A	0.067423	1	1	1	1
B	0.071146	1	1	1	1
C-H	0.074854	1	1	1	1
I-J	0.077777	1	1	1	0.088025
K	0.082179	0.088024	0.088024	0.088024	0.088024
L-M	0.082095	0.093044	0.093044	0.093044	0.082095
N-O	0.082225	0.082357	0.082357	0.082357	0.082225

TABLE 7. Relative ranking for systems using CIMS.

System	Rank	MAD	Rank	MMAW	Rank	MMFV
A	1	0.029142	1	1.177248	1	0.177165
B	2	0.021897	2	1.133159	2	0.13314
C-H	3	0.014628	3	1.088959	3	0.088937
I-J	4	0.008819	4	1.053577	4	0.053672
K	6	0.000092	5	1.000562	6	0.000562
L-M	5	0.000258	6	1.000157	5	0.001569
N-O	7	3.7519E-08	7	1.00000023	7	2.3133E-07

not account for the probability of the component being in a particular state $p_{ij} = P(x_i = b_{ij})$. The alternate CIMs incorporate both possible state levels and state probabilities p_{ij} into the computation of MSS component importance. That is, the alternative CIMs account for both the effects of changes in component states on overall system unreliability and the probability of such changes.

This paper uses the MDD algorithm to obtain alternative CIMs to evaluate the importance of each system in the PMSoS.

Step 1. Generate the MDD model. The MDD model of the escort formation PMSoS is shown in Fig. 8.

Step 2. Calculate the unreliability $P(U)$ of the overall mission in the current state. The reliability of the mission is 0.9178, so $P(U) = 1 - 0.9178 = 0.0822$.

Step 3. Calculate the conditional probability $P(U'|x_i = b_{ij})$. By setting the probability of the system x_i in state b_{ij} to 1 and the probability associated with the other states to 0, the required conditional probability is calculated using the MDD generated in step 1, as shown in Table 6.

Step 4. Calculate each alternative CIM according to formulas (19)-(21). Table 7 lists the CIM results.

The ranking is $A \prec B \prec C = D = E = F = G = H \prec I = J \prec L = M \prec K \prec N = O$ for the MAD and MMFV measures and $A \prec B \prec C = D = E = F = G = H \prec I = J \prec K \prec L = M \prec K \prec N = O$ for the MMAW measures. This result implies that system A is the best candidate for an upgrade to improve the PMSoS reliability. Note that $C - H$ have the same ranking, and so do L and M, N and O, because they are in similar structural positions and have the same probability distribution.

VI. CONCLUSION AND FUTURE WORK

This paper investigates an escort formation and performs mission reliability estimations on this complex system of systems. The definition of the PMSoS is given based on the PMS. For such large-scale and complex systems, the phased fault tree is simplified according to the CFBM to reduce the size of the fault tree structure, and the flow of transforming the fault tree into an MDD with the case method is given; then, a phased mission reliability calculation model is adopted, and the phased MDD is combined to generate the MDD model of the PMSoS. Then, the edges in the model are simplified, which further reduces the complexity of the MDD calculations. Finally, the MDD algorithm is used to calculate the reliability of the PMSoS, which improves the efficiency of mission reliability analysis. The performance of the improved MDD method is compared with that of the BDD method and general MDD method, showing that the improved MDD method can offer lower computational complexity as well as a simpler model construction over the BDD method and general MDD method. A case study of an escort formation PMSoS demonstrates the effectiveness of the proposed MDD method.

In fact, large-scale systems of systems involve many platforms and systems, and the corresponding logic relationships are complicated. This complexity is reflected mainly in the dynamic repairability of the system, the common causes of failure and intercorrelations. To improve the accuracy of reliability analyses of complex equipment systems, we will conduct in-depth research on the above factors.

APPENDIXES

APPENDIX A

PROOF OF FORMULAS (8), (9) AND (10)

$P(A_j)(1 \leq j \leq H)$ represents the probability that the system fails in phase j given that it is working at the beginning of phase j. Then,

$$P(A_j) = P(\overline{A_1}) \cdot \cdots \cdot P(\overline{A_{1,2,\cdots,j-1}}|\overline{A_{1,2,\cdots,j-2}}) \cdot P(A_j|\overline{A_{1,2,\cdots,j-1}}) \quad (19)$$

and,

$$P(A_1) = F_A(t_1), \quad P(\overline{A_1}) = 1 - P(A_1) \tag{20}$$

$$P(\overline{A_{1,2}}|\overline{A_1}) = \frac{P(\overline{A_{1,2}}, \overline{A_1})}{P(\overline{A_1})} \tag{21}$$

$$P(\overline{A_{1,2}}, \overline{A_1}) = 1 - \int_0^{t_2} f_A(t)dt = 1 - F_A(t_2) \tag{22}$$

$$P(\overline{A_{1,2,\cdots,j-1}}|\overline{A_{1,2,\cdots,j-2}}) = \frac{P(\overline{A_{1,2,\cdots,j-1}}, \overline{A_{1,2,\cdots,j-2}})}{P(\overline{A_{1,2,\cdots,j-2}})} \tag{23}$$

$$P(\overline{A_{1,2,\cdots,j-1}}, \overline{A_{1,2,\cdots,j-2}}) = 1 - \int_0^{t_{j-1}} fA(t)dt = 1 - F_A(t_{j-1}) \tag{24}$$

$$P(\overline{A_{1,2,\cdots,j-2}}) = 1 - F_A(t_{j-2}) \tag{25}$$

$$P(A_j|\overline{A_{1,2,\cdots,j-1}}) = \frac{P(A_j, \overline{A_{1,2,\cdots,j-1}})}{P(\overline{A_{1,2,\cdots,j-1}})} \tag{26}$$

$$P(A_j, \overline{A_{1,2,\cdots,j-1}}) = \int_{t_{j-1}}^{t_j} f_A(t)dt = F_A(t_j) - F_A(t_{j-1}) \tag{27}$$

where $f(t)$ is the failure probability density function and $F(t)$ is the failure probability function. Therefore,

$$P(A_j) = F_A(t_j) - F_A(t_{j-1}) \tag{28}$$

where $P(A_{1,2,\cdots,j})$ represents the probability that the system fails within (t_0, t_j), then,

$$\begin{aligned} P(A_{1,2,\cdots,j}) &= P(A_1) + P(\overline{A_1}) \cdot P(A_2|\overline{A_1}) + \cdots \\ &+ P(\overline{A_1}) \cdot \cdots \cdot P(\overline{A_{1,2,\cdots,j-1}}|\overline{A_{1,2,\cdots,j-2}}) \\ &\cdot P(A_j|\overline{A_{1,2,\cdots,j-1}}) \end{aligned} \tag{29}$$

In addition,

$$P(A_2|\overline{A_1}) = \frac{P(A_2, \overline{A_1})}{P(\overline{A_1})} \tag{30}$$

$$P(A_2, \overline{A_1}) = \int_{t_1}^{t_2} fA(t)dt = F_A(t_2) - F_A(t_1) \tag{31}$$

Thus,

$$P(A_{1,2,\cdots,j}) = F_A(t_j) \tag{32}$$

For $P(A_{0,H})$,

$$\begin{aligned} P(A_{0,H}) = P(A_0) + P(\overline{A_1}) \cdot \cdots &\cdot P(\overline{A_{1,2,\cdots,H-1}}|\overline{A_{1,2,\cdots,H-2}}) \\ &\cdot P(A_H|\overline{A_{1,2,\cdots,H-1}}) \end{aligned} \tag{33}$$

and

$$P(A_0) = 1 - F_A(t_H) \tag{34}$$

$$\begin{aligned} &P(\overline{A_1}) \cdot \cdots \cdot P(\overline{A_{1,2,\cdots,H-1}}|\overline{A_{1,2,\cdots,H-2}}) \\ &\cdot P(A_H|\overline{A_{1,2,\cdots,H-1}}) = F_A(t_H) - F_A(t_{H-1}) \end{aligned} \tag{35}$$

Therefore,

$$P(A_{0,H}) = 1 - F_A(t_{H-1}) \tag{36}$$

APPENDIX B
PSEUDOCODE OF THE MDD ALGORITHM

```
FUNCTION Binary_tree (L):
BEGIN
    G = nx. DiGraph () /* G is the network graph structure
    */
    Show the elements in list L in G
    return G
END
FUNCTION Pathall (G, A, B, visited, path, paths):
BEGIN
    FOR each neighbor node i of A:
      IF (A, i) in G:
          Set the neighbor node to the already visited
          nodes
          Add (A, i) to the path
          Pathall (G, i, B, visited, path, paths)
          /* Recursive */
          Reset the neighbor nodes to unaccessed nodes
          Clear path
    return paths
END
FUNCTION Computer(L)
BEGIN
    L (parent node, child node, branch, probability, stage)
    G = Binary_tree (L)
    Read branch probability and phase representation
    from G
    S = Pathall (G, root node, '0', visited nodes, path,
    paths) /* DFS obtains all the reliable paths */
    The probability of each successful path = the product
    of the probability of each branch on the path
    probability.clear () /* Clear the above assignment
    when calculating the probability of the next path */
    The reliability of the mission = the sum of the
    probability of each successful path
END
```

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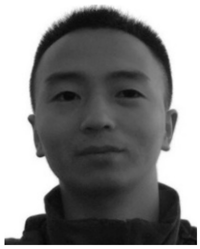
REFERENCES
[1] X. Zang, N. Sun, and K. S. Trivedi, "A BDD-based algorithm for reliability analysis of phased-mission systems," *IEEE Trans. Rel.*, vol. 48, no. 1, pp. 50–60, Mar. 1999.
[2] S. Zhang, S. Sun, S. Si, and P. Wang, "A decision diagram based reliability evaluation method for multiple phased-mission systems," *Eksploatacja i Niezawodnosc-Maintenance Rel.*, vol. 19, no. 3, pp. 485–492, Jun. 2017.
[3] L. Xing and S. V. Amari, "Reliability of phased-mission systems," in *Handbook of Performability Engineering*, K. B. Misra, Eds. London, U.K.: Springer, 2008.

[4] X. Qi, B. Guo, and Y. Liu, "Simulation based reliability analysis of phased-mission system considering maintenance in system-level," in *Proc. Int. Conf. Qual.*, 2012, pp. 1435–1438.
[5] A. K. Somani, J. A. Ritcey, and S. H. L. Au, "Phased mission reliability analysis," *ACM SIGMETRICS Perform. Eval. Rev.*, vol. 18, no. 1, pp. 247–248, 1990.
[6] J. D. Andrews, "Birnbaum and criticality measures of component contribution to the failure of phased missions," *Rel. Eng. Syst. Saf.*, vol. 93, no. 12, pp. 1861–1866, Dec. 2008.
[7] J. B. Dugan, "Automated analysis of phased-mission reliability," *IEEE Trans. Rel.*, vol. 40, no. 1, pp. 45–52, Apr. 1991.
[8] Y. Mo, "New insights into the BDD-based reliability analysis of phased-mission systems," *IEEE Trans. Rel.*, vol. 58, no. 4, pp. 667–678, Dec. 2009.
[9] L. Xing, S. V. Amari, and C. Wang, "Reliability of k-out-of-n systems with phased-mission requirements and imperfect fault coverage," *Rel. Eng. Syst. Saf.*, vol. 103, pp. 45–50, Jul. 2012.
[10] L. Xing and G. Levitin, "BDD-based reliability evaluation of phased-mission systems with internal/external common-cause failures," *Rel. Eng. Syst. Saf.*, vol. 112, pp. 145–153, Apr. 2013.
[11] I. Mura and A. Bondavalli, "Hierarchical modeling and evaluation of phased-mission systems," *IEEE Trans. Rel.*, vol. 48, no. 4, pp. 360–368, Dec. 1999.
[12] I. Mura and A. Bondavalli, "Markov regenerative stochastic Petri nets to model and evaluate phased mission systems dependability," *IEEE Trans. Comput.*, vol. 50, no. 12, pp. 1337–1351, Dec. 2002.
[13] M. Smotherman and K. Zemoudeh, "A non-homogeneous Markov model for phased-mission reliability analysis," *IEEE Trans. Rel.*, vol. 38, no. 5, pp. 585–590, Dec. 1989.
[14] Z. Tang and J. B. Dugan, "BDD-based reliability analysis of phased-mission systems with multimode failures," *IEEE Trans. Rel.*, vol. 55, no. 2, pp. 350–360, Jun. 2006.
[15] X. Wu, H. Yan, and L. Li, "Numerical method for reliability analysis of phased-mission system using Markov chains," *Commun. Statist.-Theory Methods*, vol. 41, no. 21, pp. 3960–3973, Nov. 2012.
[16] X. Wu and X. Wu, "Mission reliability modeling and evaluation of multi-mission phased mission system based on an extended object-oriented Petri net," *Eksploatacja i Niezawodnosc-Maintenance Rel.*, vol. 19, no. 2, pp. 244–253, Mar. 2017.
[17] J.-M. Lu and X.-Y. Wu, "Reliability evaluation of generalized phased-mission systems with repairable components," *Rel. Eng. Syst. Saf.*, vol. 121, pp. 136–145, Jan. 2014.
[18] Y. Mo, L. Xing, and S. V. Amari, "A multiple-valued decision diagram based method for efficient reliability analysis of non-repairable phased-mission systems," *IEEE Trans. Rel.*, vol. 63, no. 1, pp. 320–330, Mar. 2014.
[19] Y. Mo, L. Xing, and J. B. Dugan, "MDD-based method for efficient analysis on phased-mission systems with multimode failures," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 44, no. 6, pp. 757–769, Jun. 2014.
[20] A. Shrestha, L. Xing, and Y. Dai, "Decision diagram based methods and complexity analysis for multi-state systems," *IEEE Trans. Rel.*, vol. 59, no. 1, pp. 145–161, Mar. 2010.
[21] W. Nan and D. Suguo, "A new method of BDD sorting for phased-mission system," (in Chinese), *Sci. Technol. Eng.*, vol. 17, pp. 111–118, Jun. 2010.
[22] J. E. Ramirez-Marquez and D. W. Coit, "Composite importance measures for multi-state systems with multi-state components," *IEEE Trans. Rel.*, vol. 54, no. 3, pp. 517–529, Sep. 2005.
[23] J. E. Ramirez-Marquez and D. W. Coit, "Multi-state component criticality analysis for reliability improvement in multi-state systems," *Rel. Eng. Syst. Saf.*, vol. 92, no. 12, pp. 1608–1619, Dec. 2007.
[24] A. Shrestha, L. Xing, and D. W. Coit, "An efficient multistate multivalued decision diagram-based approach for multistate system sensitivity analysis," *IEEE Trans. Rel.*, vol. 59, no. 3, pp. 581–592, Sep. 2010.



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