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A Fast Nonsingular Terminal Sliding Mode Control Method for Nonlinear Systems With Fixed-Time Stability Guarantees

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ABSTRACT This paper proposes a nonsingular terminal sliding mode control scheme with fast fixed-time convergence for a class of second-order nonlinear systems in the presence of matched uncertainties and perturbations. First, based on fixed-time stability theory, a novel stable system is proposed. Then, using the fixed-time stable system, a fast fixed-time nonsingular terminal sliding surface is derived. The settling time is independent of the initial system state and can be set in advance with the design parameters; the upper-bound of convergence time is derived from the Lyapunov theory. Moreover, the proposed control scheme has an advantage in convergence rate over existing results and achieves better control performance with low control energy cost. The simulation results for a tracking system with a single inverted pendulum are presented to validate the effectiveness and superiority of the proposed control method.

INDEX TERMS Fixed-time stability, nonsingular terminal sliding mode control, nonlinear system.

I. INTRODUCTION

Sliding mode control (SMC) is a nonlinear control method that alters the dynamics of a system by using a discontinuous control signal and forces the system to slide along a prescribed switching manifold [1]. Compared to other control methods, SMC has attracted significant interest due to its simplicity, low sensitivity to system parameter variations and high robustness to external disturbances [2]. Therefore, SMC has been developed and applied widely in many applications, including robot manipulators [3], cable-driven manipulators [4], [5], power systems [6], multiagent systems [7] and guidance law design [8].

It is notable that in standard SMC schemes for nonlinear systems, a linear hyperplane-based SM manifold is used to drive the system states to the equilibrium point. However, the standard SMC approach can only guarantee that the system asymptotically converges. To achieve finite-time stability [9], some other kinds of SMC methods are developed, such as terminal sliding mode control (TSMC), integral sliding mode control (ISMC) [10], [11], discrete-time sliding mode control (DSMC) [12] and so forth. The TSMC [13]–[16] was developed by adopting a nonlinear sliding hyperplane to achieve fast finite-time convergence in the sliding phase, it has a good control performance for a class of nonlinear systems with uncertainties. Nevertheless, standard TSMC suffers a singularity problem in some areas of the state space due to the use of negative fractional power terms. Hence, various control strategies were developed to solve this problem.

In [17], an indirect approach is adopted to transform the system trajectory to a prespecified region where no singularity occurs. In [18], a modified sliding surface is proposed for second-order systems to avoid the singularity domain. To achieve a faster convergence rate, Yang and Yang [19] constructed the new concept of nonsingular fast TSMC, whose convergence time is smaller than that of the conventional TSM in [16]. Feng *et al.* [20] introduced a saturation function into TSM controller design for nonlinear systems to avoid the singularity phenomenon. A new adaptive nonsingular integral terminal sliding mode control method is developed to avoid the singularity phenomenon and guarantee fast transient convergence for the trajectory tracking control of autonomous underwater vehicles in [21], [22]. In [23] and [24], a fractional-order nonsingular terminal sliding mode

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manifold was proposed to ensure fast dynamical response and overcome the singularity problem. However, explicit estimates of the settling time for the NTSMC design were not given in the above mentioned schemes.

One of the key issues with finite-time stability is the estimation of the settling time, which is a function of the initial conditions of the system. Generally, different initial values result in different estimations of convergence time. In addition, the initial conditions of practical systems may be difficult to accurately obtain in advance, which makes settling time inaccessible and deteriorates the system performance.

In addition to the finite-time stability, a fixed-time stabilization concept has been proposed [25]. In contrast to finite-time stability, fixed-time stability provides an explicit estimation of the settling time independently of initial conditions. Reference [26] revealed the essence of finite-time stability and fixed-time stability. Polyakov and Fridman [27] surveyed the mathematical tools required for fixed-time stability and convergent analysis of modern SMC systems, and the generalized Lyapunov theorems for stability analysis and the convergence time estimation were presented. In [28], theorems on implicit Lyapunov functions (ILF) for finite-time and fixed-time stability analysis of nonlinear systems were presented, which define Lyapunov functions implicitly as solutions to an algebraic equation. The control design problem for finite-time and fixed-time stabilizations of linear multi-input systems with nonlinear uncertainties is discussed in [29]. Zuo and Tie [30] proposed a fixed-time TSMC surface, which suffers from a singularity problem, and the control input cannot be guaranteed to be bounded during the reaching phase. In [31], a fixed-time nonsingular TSMC approach for a class of second-order nonlinear systems was proposed and then applied in the consensus protocol of a multiagent system [32]. Based on [31], Li *et al.* [33] proposed a singularity-free terminal sliding mode control scheme with fast fixed-time convergence. However, the convergence time of the nonsingular fixed-time terminal sliding mode controller in [31] and [33] is not optimal, and the methods of circumventing the singularity are complicated. In [34], a fixed-time NTSM approach is proposed that uses a modified fixed-time stable system to improve the convergence rate. However, the designed control law does not consider external disturbances. Hu *et al.* [35] presented a new theorem of fixed-time stability by reductio ad absurdum, and a high-precision estimation of the settling time is given. Yang *et al.* [36] proposed a nonrecursive fixed-time convergence observer to deal with state estimation and tracking differentiation. Fixed-time stabilization control has also been investigated for high-order regulators [37], synchronization problems in neural networks [38], double integrator systems [39], group tracking problems for multiagent systems [40] and so forth. Inspired by this attractive feature, some applications of fixed-time stabilization in engineering have also been developed. In [34], a fixed-time NTSM controller for chaos suppression in power systems was proposed. Fixed-time SMC surfaces were developed for the attitude control of a rigid spacecraft in [41] and [42]. Zhang *et al.* [43] proposed an adaptive NTSM guidance law with a terminal angle constraint based on the fixed-time convergence theory.

Motivated by the above discussion, a novel fast fixed-time nonsingular terminal sliding mode (FFNTSM) control scheme is proposed in this paper, which is for second-order nonlinear systems with matched uncertainties and external disturbances, and the preset settling time is independent of initial conditions. The proposed control scheme has an advantage in convergence rate and control energy cost over the existing results of fixed-time stable control methods.

The main contributions of this paper are as follows: (1) Based on fixed-time stability theory, a novel fixed-time stable system is presented; (2) The fixed-time stability is guaranteed with the proposed FFNTSM; (3) By using the saturation function method, the FFNTSM structure is nonsingular; and (4) The convergence time is independent of the initial state and can be preset by the design parameters.

The organization of the paper is as follows. In Section II, the problem statement is given. In Section III, the main algorithm is derived. In Section IV, numerical simulations are presented to evaluate the performance and superiority of the proposed control scheme. Finally, a brief summary of this work is given in Section V.

II. PROBLEM STATEMENT

A. PRELIMINARIES

Consider the system defined by

$$\dot{x}(t) = F(t, x), \quad x(0) = x_0 \tag{1}$$

where $x \in R^n$ is the vector of system states and $F(t, x) : R^+ \times D \to R^n$ is a continuous nonlinear function that is on an open neighborhood $D \in R^n$ of the origin. The solutions of (1) are understood in the sense of Filippov [44], assume the origin is an equilibrium point of (1).

Definition 1 [9]: The origin is a 'finite-time stable' equilibrium of eq. (1) if the origin is Lyapunov stable and there exists an open neighborhood $N \subseteq D$ of the origin and a positive definite function $T(x_0) : N \to R$ called the settling time function such that, for all $x(0) \in N \setminus \{0\}$,

$$\begin{cases} \lim\limits_{t \to T(x_0)} x(t, x_0) \to 0 \\ x(t, x_0) = 0 \quad \forall t > T(x_0) \end{cases} \tag{2}$$

Furthermore, the origin is a 'globally finite-time stable' equilibrium if it is finite-time stable with $N = R^n$. Additionally, finite-time stability of the origin implies asymptotic stability of the origin.

However, the finite settling time T in Definition 1 depends on the initial state x_0 of the system. The initial conditions of many practical systems may be unavailable or difficult to obtain accurately, which restricts its practical application. Moreover, a fixed-time stability concept is developed by Polyakov [25]. In contrast to finite-time stable systems, fixed-time stable systems can guarantee stabilization within bounded time independent of the initial condition.

Definition 2 [25]: The origin is a 'fixed-time stable' equilibrium of system (1) if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded by a constant $T_{\max} > 0$, s.t. $T(x_0) \le T_{\max}, \forall x_0 \in R^n$.

B. CONTROL OBJECTIVE

In this paper, a second-order nonlinear system with matched lumped perturbations is considered:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f(t,x) \\ \quad + b(t,x)u(t) + d(t,x) \end{cases} \quad x(0) = x_0 \tag{3}$$

where $\boldsymbol{x} = [x_1, x_2]^T \in R^2$ is the system state, f and $b \ne 0$ are sufficiently smooth nonlinear functions, $u \in R$ is the control input and d denotes the model uncertainty, which satisfies the following assumption.

Assumption 1: The model uncertainty $d(t,x)$ is assumed to be bounded. For all $x \in R^2$ and $t \ge 0$, there exists a constant $D > 0$ such that $|d(t,x)| \le D$.

Remark 1: It should be pointed out that plenty of practical dynamical systems, such as the mechanical systems, power systems, and missile-target engagement dynamics model, can be expressed in (3) satisfying the above conditions. For example, the robotic manipulator system mentioned in [16], engagement dynamics in [45] are not exactly in the form of (3), but they were transformed into such a form by the coordinates' change. In the transformed robotic manipulator system in [16], x and $\dot{x}$ denote the vectors and angular position of joint, respectively. Therefore, the proposed control algorithm in the work can be applied to such plant, which can be transformed to (3).

The objective is to design a control law $u(t)$ such that the origin of (3) is a fixed-time stable equilibrium.

III. MAIN RESULT

A novel fixed-time nonsingular terminal sliding mode control methodology is proposed in this section.

A. A NOVEL FAST FIXED-TIME CONVERGENCE SYSTEM

To further improve the convergence rate, a novel fast fixed-time stable system is designed in this subsection, to be used in the controller design, whose settling time is smaller than that of existing fixed-time stable systems.

Lemma 1 [31]: Consider a scalar dynamic system

$$\dot{y} = -l_1 \mathrm{sign}^{m_1} y - l_2 \mathrm{sign}^{m_2} y, \quad y(0) = y_0 \tag{4}$$

where $\mathrm{sign}^{m_1} y = |y|^{m_1} \cdot \mathrm{sign}(y)$, $l_1 > 0$, $l_2 > 0$, $m_1 > 1$, and $0 < m_2 < 1$. Then, the equilibrium of (4) is fixed-time stable and the settling time T is bounded by

$$T_f < T_{\max} = \frac{1}{l_1(m_1-1)} + \frac{1}{l_2(1-m_2)} \tag{5}$$

Lemma 1 provides a fixed-time stable system whose predefined global settling-time estimate $T_{\max}$ does not rely on the system initial state y_0 but only on the design parameters l_1, l_2, m_1, and m_2. This implies that the convergence time can be guaranteed through selecting appropriate parameters. The fixed-time stable system (4) is used to construct the fixed-time nonsingular terminal sliding mode surface and control law in [31] and [33].

In addition to Lemma 1, a different fast fixed-time stable system is proposed in this paper, and the convergence time of the system is smaller than T_f in eq. (5). Moreover, the upper bound of the system convergence time is given.

Theorem 1: Consider the following scalar system

$$\dot{z} = -l_1 \mathrm{sign}^{k_1} z - l_2 \mathrm{sign}^{k_2} z, \quad z(0) = z_0 \tag{6}$$

where $l_1 > 0$, $l_2 > 0$, $k_1 = \frac{m_1+1}{2} + \frac{m_1-1}{2}\mathrm{sign}(|z|-1)$, $k_1 = \frac{m_2+1}{2} + \frac{1-m_2}{2}\mathrm{sign}(|z|-1)$, $\mathrm{sign}^{k_1} z = |z|^{k_1} \cdot \mathrm{sign}(z)$, and $m_1 > 1$, $1/2 < m_2 < 1$. Then, system (6) is fixed-time stable, and the settling time T is bounded by

$$T \le T_{\max} = \frac{1}{l_2(m_1-1)} \ln\left(\frac{l_1+l_2}{l_1}\right) + \frac{1}{l_1(1-m_2)} \ln\left(\frac{l_1+l_2}{l_2}\right) \tag{7}$$

Proof: Eq. (6) can be rewritten as

$$\begin{cases} \dot{z} = -l_1 \mathrm{sign}^{m_1} z - l_2 z, & |z| > 1 \\ \dot{z} = -l_1 z - l_2 \mathrm{sign}^{m_2} z, & |z| \le 1 \end{cases} \tag{8}$$

Let $y = 1 + \ln|z|$ for $|z| > 1$ and $y = |z|^{1-m_2}$ for $|z| \le 1$. Then, eq. (8) can be written as

$$\begin{cases} \dot{y} = -l_1 e^{(m_1-1)(y-1)} - l_2, & y > 1 \\ \dot{y} = -(1-m_2) l_1 y - (1-m_2) l_2, & 0 < y \le 1 \end{cases} \tag{9}$$

Therefore, the upper bound of the convergence time can be estimated by solving eq. (9).

$$\begin{aligned} T_{\max} &= \lim_{z(0)\to\infty} T(z(0)) \\ &= \lim_{y_0\to\infty} \left(\int_1^{y_0} \frac{1}{l_1 e^{(m_1-1)(y-1)} + l_2} dy \right. \\ &\quad \left. + \int_0^1 \frac{1}{(1-m_2)(l_1 y + l_2)} dy \right) \\ &= \lim_{y_0\to\infty} \int_1^{y_0} \frac{1}{l_1 e^{(m_1-1)(y-1)} + l_2} dy \\ &\quad + \frac{1}{l_1(1-m_2)} \ln\left(\frac{l_1+l_2}{l_2}\right) \end{aligned} \tag{10}$$

Let $T_1 = \lim_{y_0\to\infty} \int_1^{y_0} \frac{1}{l_1 e^{(m_1-1)(y-1)} + l_2} dy$ and $\rho = e^{(m_1-1)(y-1)}$; then

$$\begin{aligned} T_1 &= \frac{1}{(m_1-1)} \lim_{\rho_0\to\infty} \int_1^{\rho_0} \frac{1}{\rho(l_1\rho + l_2)} d\rho \\ &= \frac{1}{(m_1-1)} \lim_{\rho_0\to\infty} \int_1^{\rho_0} \left(\frac{1}{l_2 \rho} - \frac{l_1}{l_2(l_1\rho + l_2)} \right) d\rho \\ &= \frac{1}{l_2(m_1-1)} \ln\left(\frac{l_1+l_2}{l_1}\right) \end{aligned} \tag{11}$$

That is,

$$T_{\max} = \frac{1}{l_2(m_1-1)} \ln\left(1+\frac{l_2}{l_1}\right) + \frac{1}{l_1(1-m_2)} \ln\left(1+\frac{l_1}{l_2}\right)$$

The proof is completed.

Remark 2: Compared with eq. (4), since $\ln(1+(l_2/l_1)) \le (l_2/l_1)$ and $\ln(1+(l_1/l_2)) \le (l_1/l_2)$, the fixed-time stable system (6) achieves a faster convergence rate than the system presented in Lemma 1. In detail, the proposed system uses the variable power terms y^{k_1} and y^{k_2}, which can be adjusted according to the system states, instead of y^{m_1} and y^{m_2} in eq. (4); thereby, it achieves a fast convergence rate both far from and at a close permissible range to the origin.

B. FAST FIXED-TIME NONSINGULAR TERMINAL SLIDING MODE CONTROL

First, the singularity problem for the conventional FTSM is discussed to clarify the motivations for this work.

Lemma 2: Consider the FTSM surface [19]

$$s = x_2 + \bar{k}_1 \mathrm{sign}^{\bar{a}_1} x_1 + \bar{k}_2 \mathrm{sign}^{\bar{a}_2} x_1 \tag{12}$$

where $\bar{k}_1 > 0$, $\bar{k}_2 > 0$, $\bar{a}_1 > 1$, and $0 < \bar{a}_2 < 1$. Let the control input be

$$u = -b^{-1}(f + \bar{k}_1\bar{a}_1 |x_1|^{\bar{a}_1 - 1} x_2 + \bar{k}_2\bar{a}_2 |x_1|^{\bar{a}_2-1} x_2 + \alpha \mathrm{sign}^{\gamma_1} s + \beta \mathrm{sign}^{\gamma_2} s + k \mathrm{sign} s) \tag{13}$$

The FTSM dynamics can be obtained as

$$\dot{s} = -\alpha \mathrm{sign}^{\gamma_1} s - \beta \mathrm{sign}^{\gamma_2} s - k \mathrm{sign} s + d \tag{14}$$

where $\alpha > 0$, $\beta > 0$, $\gamma_1 > 1$, $0 < \gamma_2 < 1$ and $k = D$ is the switching gain.

It can be concluded from [19] that the FTSM (12) is finite-time convergent, and the convergence time is smaller than that of the conventional TSM [16]. Moreover, by applying Lemma 1 twice, it can be shown that the origin of (3) is fixed-time stable and the settling time is bounded by

$$T < T_{\max} = T_1 + T_2 \tag{15}$$

where

$$T_1 = \frac{1}{\alpha(\gamma_1 - 1)} + \frac{1}{\beta(1-\gamma_2)}$$
$$\text{and } T_2 = \frac{1}{\bar{k}_1(\bar{a}_1 - 1)} + \frac{1}{\bar{k}_2(1-\bar{a}_2)}$$

Unfortunately, the negative fractional power term $|x_1|^{\bar{a}_2 - 1}$ in the control input (13) may cause a singularity when $x_1 = 0$ and $x_2 \neq 0$. Therefore, the control input cannot be guaranteed bounded during the reaching phase.

Inspired by [20], the saturation function method is adopted to construct the control input to avoid the singularity problem. Based on Theorem 1 and Lemma 2, a novel fast fixed-time nonsingular terminal sliding mode (FFNTSM) is constructed as

$$s = x_2 + \alpha_1 \mathrm{sign}^{k_1} x_1 + \beta_1 \mathrm{sign}^{k_2} x_1 \tag{16}$$

where $\alpha_1 > 0$, $\beta_1 > 0$, $k_1 = \frac{m_1+1}{2} + \frac{m_1-1}{2}\mathrm{sign}(|x_1|-1)$, $k_2 = \frac{m_2+1}{2} + \frac{1-m_2}{2}\mathrm{sign}(|x_1|-1)$, $m_1 > 1$, and $1/2 < m_2 < 1$.

The dynamics can be obtained as

$$\dot{s} = -\alpha_2 \mathrm{sign}^{\gamma_1} s - \beta_2 \mathrm{sign}^{\gamma_2} s - k \mathrm{sign} s + d \tag{17}$$

where $\alpha_2 > 0$, $\beta_2 > 0$, $\gamma_1 = \frac{n_1+1}{2} + \frac{n_1-1}{2}\mathrm{sign}(|s|-1)$, $\gamma_2 = \frac{n_2+1}{2} + \frac{1-n_2}{2}\mathrm{sign}(|s|-1)$, $n_1 > 1$, and $1/2 < n_2 < 1$.

According to (17), the control law can be designed as

$$u = -b^{-1}[f + \alpha_1 k_1 |x_1|^{k_1-1} x_2 + \mathrm{sat}\left(\beta_1 k_2 |x_1|^{k_2-1} x_2, h\right) + \alpha_2 \mathrm{sign}^{\gamma_1} s + \beta_2 \mathrm{sign}^{\gamma_2} s + k \mathrm{sign} s] \tag{18}$$

In the controller design, the saturation function is applied to limit the amplitude of the singularity term $|x_1|^{k_2-1} x_2$ in the control input. The saturation function is defined as

$$\mathrm{sat}(x, y) = \begin{cases} x & \text{if } |x| < y \\ y\,\mathrm{sign}(x) & \text{if } |x| \ge y \end{cases} \tag{19}$$

It can be concluded from (16) that when the system state is far from the equilibrium, $\alpha_1 \mathrm{sign}^{k_1} x_1$ dominates over $\beta_1 \mathrm{sign}^{k_2} x_1$, which guarantees a high convergence rate; when the system state is close to the origin, the dominant term $\beta_1 \mathrm{sign}^{k_2} x_1$ determines fixed-time convergence. Thereby, the dynamics converge very quickly in the whole FFNTSM (16), and the form of the control law is concise.

Theorem 2: Consider the second-order system (3). The states will converge to the origin within fixed time if the sliding-mode surface is chosen as the proposed FFNTSM (16) and the control law is taken to be (18); in this case, the settling time T is bounded by

$$T < T_{\max} = T_1 + T_2 \tag{20}$$

where

$$T_1 = \frac{1}{\beta_2(n_1-1)} \ln\left(1 + \frac{\beta_2}{2^{(n_1-1)/2}\alpha_2}\right) + \frac{1}{\alpha_2(1-n_2)} \ln\left(1 + \frac{\alpha_2}{2^{(n_2-1)/2}\beta_2}\right)$$
$$T_2 = \frac{1}{\beta_1(m_1-1)} \ln\left(1 + \frac{\beta_1}{2^{(m_1-1)/2}\alpha_1}\right) + \frac{1}{\alpha_1(1-m_2)} \ln\left(1 + \frac{\alpha_1}{2^{(m_2-1)/2}\beta_1}\right)$$

Proof: Consider the Lyapunov candidate function

$$V = \frac{1}{2}s^2 \tag{21}$$

whose time derivative along (3) yields

$$\begin{aligned}\dot{V} &= s\dot{s} = s(f + bu + d + \alpha_1 k_1 |x_1|^{k_1-1} x_2 + \beta_1 k_2 |x_1|^{k_2-1} x_2) \\ &= s(d - k\mathrm{sign} s + \beta_1 k_2 |x_1|^{k_2-1} x_2 - \mathrm{sat}(\beta_1 k_2 |x_1|^{k_2-1} x_2, h) \\ &\quad - \alpha_2 \mathrm{sign}^{\gamma_1} s - \beta_2 \mathrm{sign}^{\gamma_2} s)\end{aligned} \tag{22}$$

According to Theorem 2.2 in [16], it follows that

$$\dot{V} \le s((D-k)|s| - \alpha_2 |s|^{\gamma_1} - \beta_2 |s|^{\gamma_2})$$

$$\begin{aligned} &\leq -\alpha_2 |s|^{\gamma_1+1} - \beta_2 |s|^{\gamma_2+1} \\ &= -\alpha_2 (2V)^{(\gamma_1+1)/2} - \beta_2 (2V)^{(\gamma_2+1)/2} \end{aligned} \tag{23}$$

when $|s| \geq 1$, one has

$$\dot{V} \leq -\alpha_2 (2V)^{(n_1+1)/2} - \beta_2 (2V) \tag{24}$$

when $|s| < 1$, (23) becomes

$$\dot{V} \leq -\alpha_2 (2V) - \beta_2 (2V)^{(n_2+1)/2} \tag{25}$$

Note that $V = 0$ implies $s = 0$. It follows from Theorem 1 that the system states can reach the sliding surface $s = 0$ within a fixed time:

$$\begin{aligned} t_1 < T_1 = &\frac{1}{\beta_2 (n_1 - 1)} \ln \left(1 + \frac{\beta_2}{2^{(n_1-1)/2}\alpha_2}\right) \\ &+ \frac{1}{\alpha_2 (1 - n_2)} \ln \left(1 + \frac{\alpha_2}{2^{(n_2-1)/2}\beta_2}\right) \end{aligned} \tag{26}$$

It can be concluded that the sliding surface $s = 0$ can be reached from anywhere in the phase plane within a fixed time $t_1 < T_1$. Then, when the system reaches the sliding surface $s = 0$, the ideal sliding motion of the system satisfies the following nonlinear differential equation:

$$\dot{x}_1 = x_2 = -\alpha_1 \mathrm{sign}^{k_1} x_1 - \beta_1 \mathrm{sign}^{k_2} x_1 \tag{27}$$

Similarly, according to Theorem 1, the settling time is bounded by

$$\begin{aligned} t_2 < T_2 = &\frac{1}{\beta_1 (m_1 - 1)} \ln \left(1 + \frac{\beta_1}{2^{(m_1-1)/2}\alpha_1}\right) \\ &+ \frac{1}{\alpha_1 (1 - m_2)} \ln \left(1 + \frac{\alpha_1}{2^{(m_2-1)/2}\beta_1}\right) \end{aligned} \tag{28}$$

When state variable x_1 settles to the origin, the state variable x_2 also converges to the origin.

Hence, the settling time T for system (3) can be estimated by $T < T_{\max} = T_1 + T_2$. The proof is completed.

Remark 3: The situation of $\left|\beta_1 k_2 |x_1|^{k_2-1} x_2\right| > h$ is ignored in the proof process. Define the singularity area as the region where $\left|\beta_1 k_2 |x_1|^{k_2-1} x_2\right| > h$. According to system (3), the solution of x_1 can be expressed as $x_1(t) = x_1(0) + \int_0^t x_2(\tau)\, d\tau$. Therefore, if $x_2(t) > 0$, $x_1(t)$ will increase monotonically until it leaves the singularity area. Similarly, if $x_2(t) < 0$, $x_1(t)$ will decrease monotonically until it leaves the singularity area. From the above, it can be concluded that the system states will not stay in the singularity region forever, but will leave the area within finite time. Therefore, as pointed out in [20], the existence of a singularity region does not influence the results of the stability analysis, and the time to travel through the singularity area is a very small proportion of the total settling time. It can be concluded in [20] that the introduction of the saturation function does not significantly degenerate the control performance. Therefore, an upper bound of settling time exists and can be estimated by (20).

Remark 4: To guarantee that $s = 0$ lies outside the singularity area, h should be selected according to $\beta_1 k_2 |x_{1\max}|^{k_2-1} \left(\alpha_1 \mathrm{sign}^{k_1} x_{1\max} + \beta_1 \mathrm{sign}^{k_2} x_{1\max}\right) < h$, where $|x_1| < x_{1\max}$.

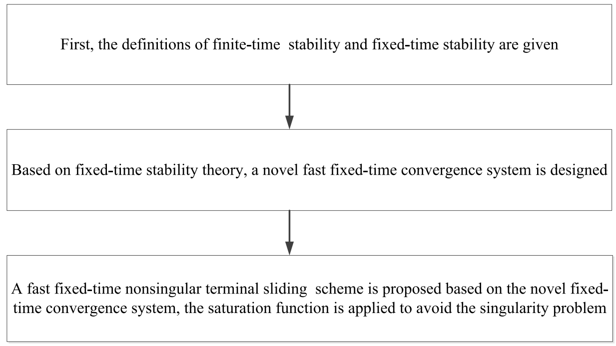

FIGURE 1. Design flow of the FFNTSM scheme.

Remark 5: To determine the sliding manifold and achieve good control performance, the parameters $\alpha_1, \beta_1, m_1, m_2, \alpha_2, \beta_2, n_1, n_2$ and h can be selected based on the trade-off between the convergence time and the steady-state tracking precision. According to Theorem 2, parameter values should be selected in the following ranges $m_1 > 1$, $n_1 > 1$, $1/2 < m_2 < 1$, and $1/2 < n_2 < 1$.

Remark 6: There exists a chattering phenomenon because of the discontinuity of the sign function in the control law (18). To alleviate chattering, the sign function is replaced by

$$\rho(\lambda, s) = \frac{e^{\lambda s} - 1}{e^{\lambda s} + 1} \tag{29}$$

where λ is a small positive constant. The function is used to approximate the sign function, if λ is given properly, and then the better approximating effect will be obtained.

In order to better understand the proposed control scheme, a block diagram of the FFNTSM scheme is exhibited as follows:

IV. SIMULATION RESULTS AND ANALYSIS

In this section, numerical simulation results are presented to evaluate the performance of the proposed FFNTSM.

Simulation 1:

To investigate the effect of the design parameters on the performance of FFNTSM, we consider the following four settings of the sliding surface parameters with the same initial state $x_1(0) = 1$: Case 1. $\alpha_1 = \beta_1 = 1$, $m_1 = 9/5$, $m_2 = 5/9$; Case 2. $\alpha_1 = \beta_1 = 2.5$, $m_1 = 9/5$, $m_2 = 5/9$; Case 3. $\alpha_1 = \beta_1 = 1$, $m_1 = 11/5$, $m_2 = 5/9$; and Case 4. $\alpha_1 = \beta_1 = 1$, $m_1 = 9/5$, $m_2 = 7/9$.

Figure 2 shows how the convergence rates of FFNTSM are influenced by the different sliding surface parameters. By comparing case 1 with case 2, it is noted that when α_1, β_1 increase, the convergence time decreases. By comparing case 1 with case 4, it is noted that the smaller m_2 is, the faster the convergence rate becomes. The curves for case 1 and case 3 are almost the same, which means the controller is more sensitive to the parameter m_2 than to m_1.Through the above analysis, we can conclude that the parameters can be

TABLE 1. SIP parameters.

Symbol	Parameter	Value
g	acceleration of gravity	9.8 m/s^2
m_c	mass of the cart	1 kg
m	mass of the pole	0.1 kg
l	half-length of the pole	0.5 m

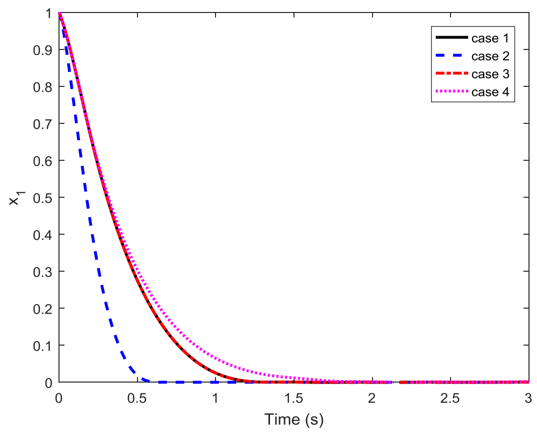

FIGURE 2. Convergence rate with different sliding surface parameters.

selected according to the requirements of speed or control effort.

Simulation 2:

A single inverted pendulum (SIP) system is considered to verify the effectiveness of the proposed FFNTSM controller. The dynamic system is formulated as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f + bu + d \end{cases} \tag{30}$$

where

$$f = \frac{g\sin x_1 - mlx_2^2\cos x_1 \sin x_1/(m_c+m)}{l\left[4/3 - m\cos^2 x_1/(m_c+m)\right]}$$
$$\text{and } b = \frac{\cos x_1/(m_c+m)}{l\left[4/3 - m\cos^2 x_1/(m_c+m)\right]}.$$

x_1 and x_2 denote the angular position and velocity, respectively. u denotes the applied force, and d denotes the external disturbance. The SIP parameters are listed in Table 1. The objective is to design a control law such that the SIP motion tracks the given desired trajectory x_{1d}. The system is controllable only if $b \neq 0$. Hence, the conditions $|x_1(0)| \leq \pi/2 - \xi$ and $|x_{1d}(t)| \leq \pi/2 - \xi$ are necessary, where $\xi > 0$ is a small constant.

The trajectory tracking can be converted to a regulation problem, and then we can apply the fixed-time control methodology proposed in this paper. Define $e_1 = x_1 - x_{1d}$ and $e_2 = x_2 - \dot{x}_{1d}$; then

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f - \ddot{x}_{1d} + bu + d \end{cases} \tag{31}$$

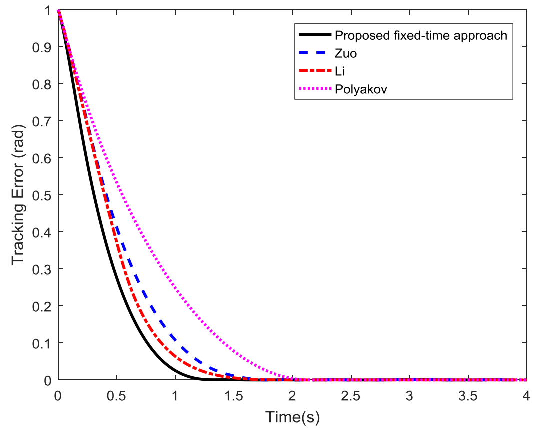

FIGURE 3. Tracking error of each fixed-time controller.

The FFNTSM surface is defined by

$$s = e_2 + \alpha_1 \text{sign}^{k_1} e_1 + \beta_1 \text{sign}^{k_2} e_1 \tag{32}$$

The fixed-time controller for SIP can be constructed as

$$u = -b^{-1}[f - \ddot{x}_{1d} + \alpha_1 k_1 |e_1|^{k_1-1} x_2 + \text{sat}\left(\beta_1 k_2 |e_1|^{k_2-1} x_2, h\right) + \alpha_2 \text{sign}^{\gamma_1} s + \beta_2 \text{sign}^{\gamma_2} s + k\rho(\lambda, s)] \tag{33}$$

In the simulation, the external lumped disturbance is chosen as $d(x_1, x_2) = \sin(10x_1) + \cos(x_2)$ to demonstrate the robustness of the proposed controller. The desired trajectory is $x_{1d}(t) = \sin(0.5\pi t)$ and the initial state is $x_1(0) = 1$, $x_2(0) = 0.5$.

To verify the effectiveness of the proposed FFNTSM surface and controller for enhancing control performance, three typical fixed-time controllers are selected for comparison: (1) Polyakov's fixed-time controller [25], (2) Zuo's fixed-time controller [31], and (3) Li's fixed-time controller [33].

Polyakov's fixed-time controller for SIP can be expressed as

$$u = -b^{-1}[f - \ddot{x}_{1d} + \frac{\alpha_1 + 3\beta_1 e_1^2 + 2\gamma}{2}\text{sign}s + \text{sign}s^{0.5}(\alpha_2 s + \beta_2 \text{sign}^3 s)] \tag{34}$$

with the sliding surface

$$s = e_2 + \text{sign}^{0.5}(\text{sign}^2 e_2 + \alpha_1 e_1 + \beta_1 \text{sign}^3 e_2) \tag{35}$$

Without any retuning of the well-designed control parameters in [25], the settling time of the system is bounded by the constant 8 s.

In addition, Zuo's fixed-time controller can be expressed as

$$u = -\frac{f - \ddot{x}_{1d} + \gamma\,\text{sign}(s)}{b} + \frac{1}{bk} \times \left[\alpha_1\left(\frac{m_1}{n_1} - \frac{p_1}{q_1}\right) e^{\frac{m_1}{n_1} - \frac{p_1}{q_1} - 1} k^2 e_2^2 - \frac{p_1}{q_1} k^{1-\frac{q_1}{p_1}} e_2^{2-\frac{q_1}{p_1}}\right] - \frac{1}{b}\frac{p_1}{q_1} k^{-\frac{q_1}{p_1}} \mu_\tau e_2^{1-\frac{q_1}{p_1}} \left(\alpha_2 s^{\frac{m_2}{n_2}} + \beta_2 s^{\frac{p_2}{q_2}}\right) \tag{36}$$

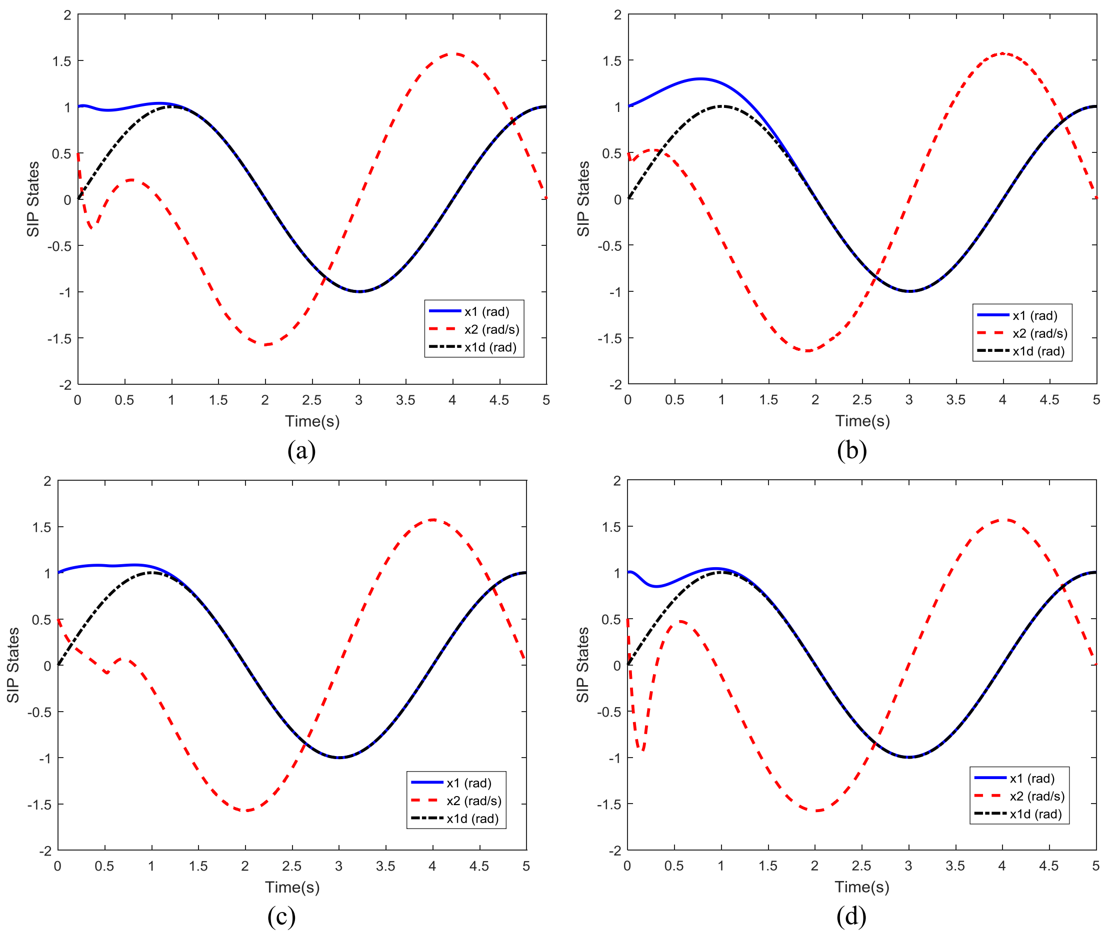

FIGURE 4. Angular position and velocity of the inverted pendulum for (a) The proposed fixed-time controller. (b) Polyakov's fixed-time controller. (c) Zuo's fixed-time controller. (d) Li's fixed-time controller.

with the sliding surface

$$s = e_1 + (ke_2)^{q_1/p_1} \tag{37}$$

where $k(e_1) = 1/\left(\alpha_1 |e_1|^{m_1/n_1 - p_1/q_1} + \beta_1\right) > 0$,

$$\mu_\tau\left(e_2^{q_1/p_1-1}\right) = \begin{cases} \sin\left(\dfrac{\pi}{2}\cdot\dfrac{e_2^{q_1/p_1-1}}{\tau}\right) & \text{if } e_2^{q_1/p_1-1} \le \tau \\ 1 & \text{otherwise} \end{cases} \tag{38}$$

Without any retuning of the well-designed control parameters in [31], the settling time of the system is bounded by the constant 6.142 s.

In addition, Li's controller can be formulated as

$$u = -b^{-1}\left[f - \ddot{x}_{1d} + k_1 a_1 |e_1|^{a_1-1}\left(\frac{\phi}{k_1} + e_2\right) + \alpha \text{sign}^{\gamma_1} s + \beta \text{sign}^{\gamma_2} s + k\psi(\rho, s)\right] \tag{39}$$

with the sliding surface

$$s = \text{sign}^{a_1} e_1 + \frac{k_2 a_2}{2a_2 - 1}\text{sign}^{2-1/a_2}\left(e_2 + k_1 \text{sign}^{a_1} e_1\right) \tag{40}$$

where

$$\phi = \frac{1}{k_2}\text{sign}^{1/a_2}\left(e_2 + k_1 \text{sign}^{a_1} e_1\right) + \frac{k_1 a_2}{2a_2 - 1}\left(e_2 + k_1 \text{sign}^{a_1} e_1\right)$$

Without any retuning of the well-designed control parameters in [33], the settling time of the closed-loop system is bounded by the constant 6.076 s.

The design parameters of the fixed-time controllers (34), (36) and (39) have been selected properly in these references to achieve good performance of the control scheme. Therefore, these fixed-time controllers are chosen without any retuning of the control parameters in the references, and then their settling time are given.

In order to achieve good control performance and make the comparison fair, the design parameters of the proposed controller should be selected appropriately. According to the requirements of the sliding surface parameters in Theorem 2, parameter values should be selected within the following ranges: $m_1 > 1$, $n_1 > 1$, $1/2 < m_2 < 1$, and $1/2 < n_2 < 1$. From the conclusion of simulation 1, we can get that the proposed controller is more sensitive to the parameter m_2 than to m_1. The appropriate m_2 is selected firstly, and then m_1 is properly selected within the specified range. It is noted

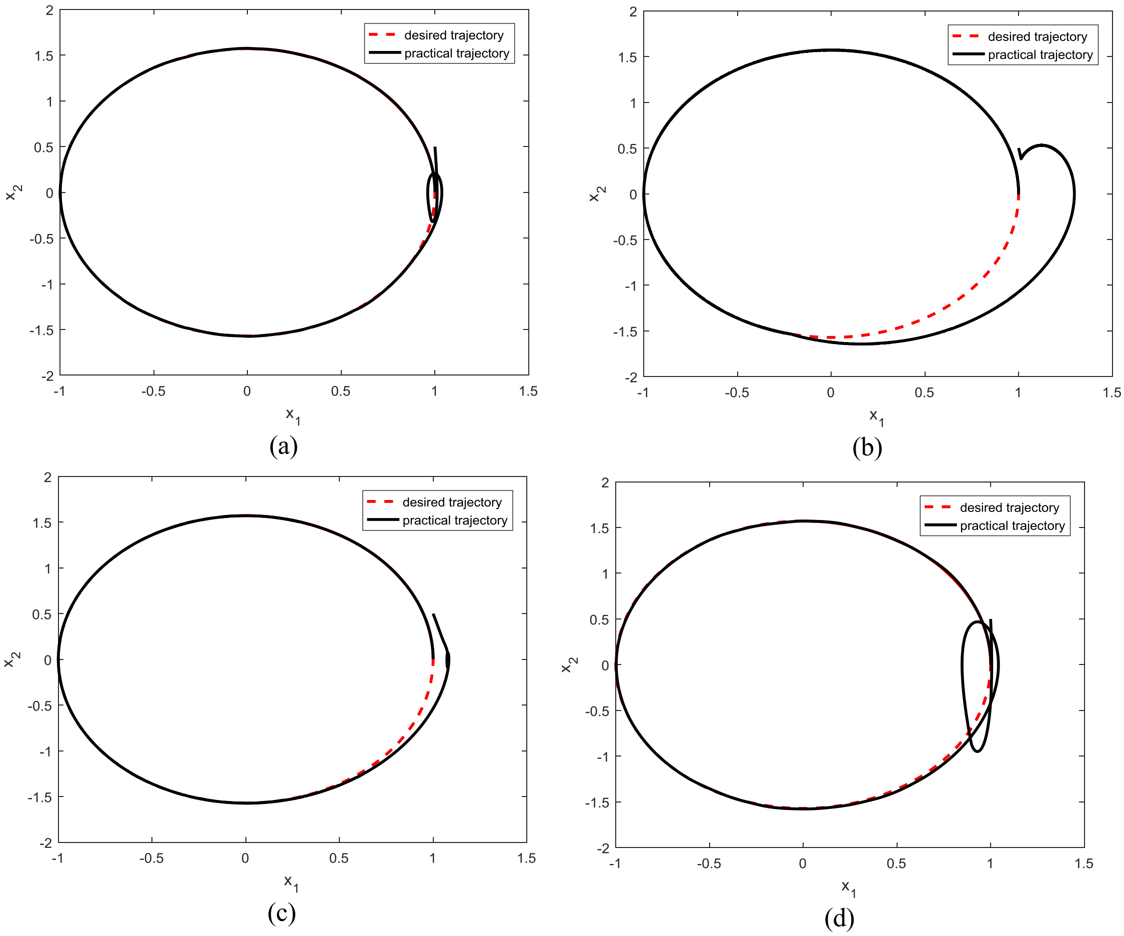

FIGURE 5. Phase portraits for (a) The proposed fixed-time controller. (b)Polyakov's fixed-time controller. (c) Zuo's fixed-time controller. (d) Li's fixed-time controller.

that when α_1, β_1 increase, the convergence time decreases. However, faster convergence speed may cause the curves of the angular position and the trajectories of phase portraits to be non-smooth, and it will lead to the degeneration of the control performance. Therefore, these parameters should be determined based on the requirements for the fastness or the steady-state tracking precision of the system. Analogously, the value of h is determined according to remark 4, λ is selected according to remark 6.

In summary, the design parameters of controller (33) are selected as $\alpha_1 = \beta_1 = 1, \alpha_2 = \beta_2 = 1, m_1 = 9/5, m_2 = 5/9, n_1 = 9/5, n_2 = 5/9, k = 2, h = 0.1$ and $\lambda = 100$. The estimate of settling time T in (20) is 4.881 s.

The simulation results of tracking error under controllers (33), (34), (36) and (39) are shown in Figure 3. All the controllers can ensure good control performance under bounded time-varying disturbance. The convergence time of the closed-loop system under controller (33) is approximately 1.3 s, which is the shortest time; the convergence times of different approaches are listed in Table 2. The effectiveness of the upper-bound estimates in Theorem 2 is verified. Moreover, the transient response under our controller (33) is also the fastest. The settling time of the proposed controller

TABLE 2. Comparison of tracking error convergence time.

Controller	Convergence Time of Tracking Error
Proposed controller	1.32 s
Polyakov's controller	2.11 s
Zuo's controller	1.84 s
Li's controller	1.79 s

achieves the best control performance. The trajectories of the angular position and velocity of the SIP are presented in Figure 4. It can be observed from Figure 4(a)-4(d) that these controllers achieve good control performances in the steady state, and the angular position tracks the time-varying reference input quickly under all the controllers. Meanwhile, the angular position in Figure 4(a) tracks the reference input faster than the others. Figure 5 displays the phase plots of the four closed-loop systems, and it is obvious that the proposed controller has the fastest rate of convergence to equilibrium and retains a relatively high tracking precision. The curves of the control input are shown in Figure 6. Due to the introduction of the discontinuous sign function, a chattering

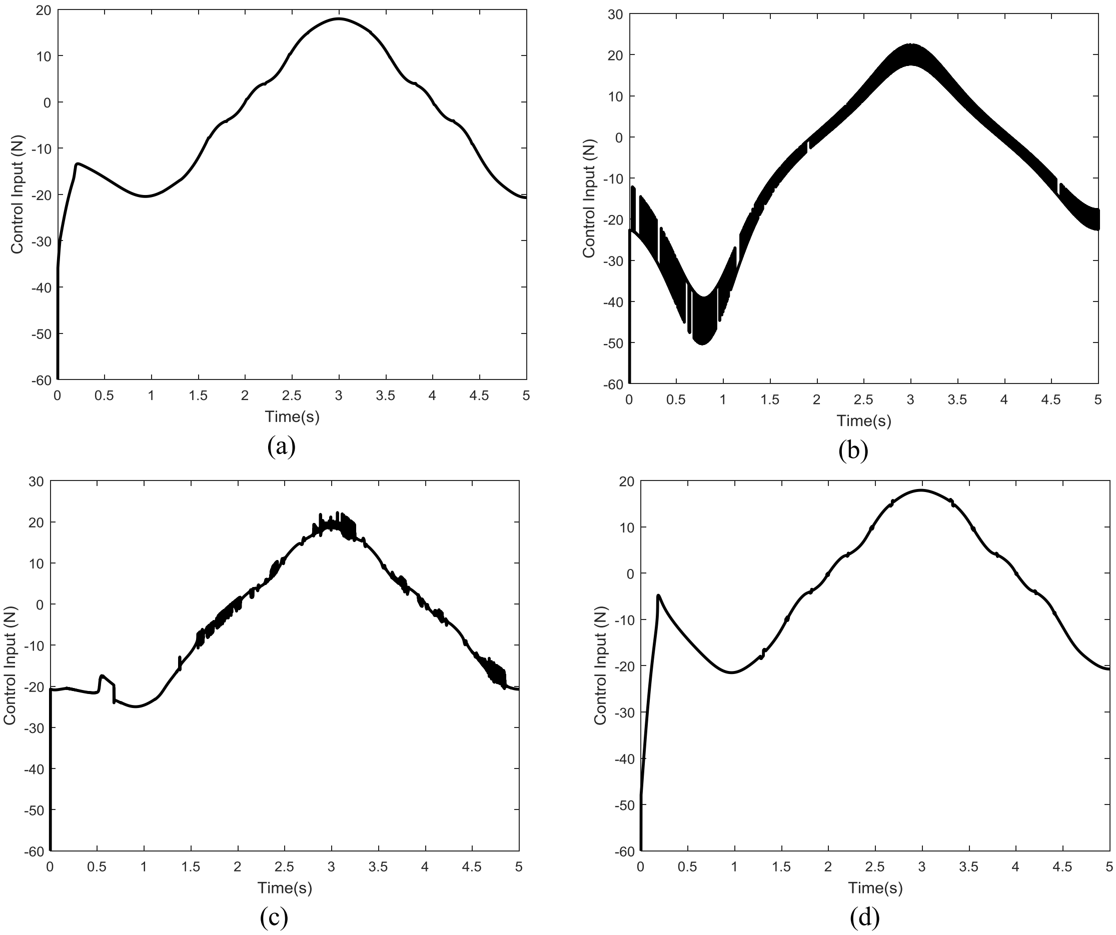

FIGURE 6. **Control inputs for (a) The proposed fixed-time controller. (b)Polyakov's fixed-time controller. (c) Zuo's fixed-time controller. (d) Li's fixed-time controller.**

phenomenon occurs in Figure 6(b) and Figure 6(c). On the other hand, the control input curve of the proposed controller is smoother than those of the other three. It is notable that the oscillation amplitude and the control force in Figure 6(a) are smaller than those in Figure 6(b) - Figure 6(d).

The total variation (TV) [46] of the input $u(t)$ is introduced to evaluate the control input performance, which can indicate the smoothness of the control input signal.

$$\mathrm{TV} = \sum_{i=1}^{N-1} |u_{i+1} - u_i| \tag{41}$$

The effort of the control input is calculated by the 2-norm method. For the desired control performance, the control energy should be as small as possible.

The comparison of the TVs and efforts are listed in Table 3. It can be observed that the controller proposed in this paper has the minimum input variation and achieves the least control energy.

From the above simulation results and analyses, it can be observed that the proposed controller is effective against matched lumped time-varying disturbance. Moreover, it has a bounded settling time and achieves better control performance than the other three controllers with low control energy cost. The control signals of the proposed controller are continuous, without a chattering phenomenon, and the singularity problem is avoided. Therefore, the proposed controller can guarantee a fast convergence rate and relatively high precision.

TABLE 3. **Input performance comparison for the four controllers.**

Controller	TV	Control Effort
Proposed controller	131.62	983.80
Polyakov's controller	8095.13	1402.28
Zuo's controller	2935.33	1082.83
Li's controller	168.22	1004.25

V. CONCLUSION

In this paper, an FFNTSM control problem for second-order nonlinear systems in the presence of matched model uncertainty and external disturbance is investigated. First, a novel fixed-time convergence system is developed, and then, the FFNTSM surface is derived based on this fixed-time system.

With the proposed control law, the closed-loop settling time is independent of the initial state and can be estimated in advance. The singularity problem can be avoided, and the control scheme achieves good control performance with low control energy cost, which greatly facilitates practical applications. Simulations validate the effectiveness of this method. In future work, a less conservative upper-bound estimation for the settling time and the extensions of the obtained results for higher-order nonlinear systems will be considered; based on the proposed fixed-time nonsingular terminal sliding mode control scheme, we will further carry on research of the fixed-time guidance in aerospace engineering and the fixed-time consensus for second-order multi-agent systems.

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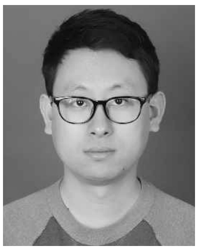

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