IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received January 16, 2020, accepted February 29, 2020, date of publication March 11, 2020, date of current version March 19, 2020.
Digital Object Identifier 10.1109/ACCESS.2020.2979996

UAVs' Formation Keeping Control Based on Multi–Agent System Consensus

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This work was supported by the National Natural Science Foundation of China under Grant 61374011.

ABSTRACT In this paper, formation keeping control of unmanned aerial vehicles (UAVs) based on multi-agent system consensus is studied. Firstly, a leader–following model based multiple unmanned aerial vehicles' (multi–UAVs') formation system is proposed. In which, every UAV has a heading keep autopilot as standard flight controller, and the heading control signal is transmitted by a nonlinear feedback controller with time delays. Secondly, some criteria of stability and Hopf bifurcation conditions for the equilibrium point of the leader UAV are established by using the *Routh–Hurwitz* criterion. Then, the consensus protocol is designed. A model prediction controller is introduced to make followers predict the leader's status and maintain a relative position in the formation, and eventually reach a consensus with the leader. Finally, some simulation examples are given to verify the correctness of the conclusions.

INDEX TERMS UAV formation, time delay, Hopf bifurcation, multi–agent system, leader-following consensus.

I. INTRODUCTION

In recent years, UAV technology and its industry have developed rapidly and have been widely used in various fields. For example, in military, it can replace human–machines to accomplish certain tasks in harsh environments and high risks [1]. In civil applications, it has been widely used in agriculture, services and other areas such as spraying pesticides, geological exploration, archaeological exploration, logistics and transportation [2], [3]. Regardless of military career or civilian market, the breakthrough in technical of UAV means a significant increase in work efficiency [4]–[6].

Normally, a single UAV is autonomous and its motion control is relatively simple. In order to perform complex tasks better, there are usually required for UAVs' formation cooperation. In recent decades, multi–UAVs system has become an attractive and active topic. The information sharing method is studied to make full use of the advantages of each single UAV [7]–[9]. And in past researches, investigators constructed architecture of multi–UAVs' formation by simulating the cluster characteristics of different biomes [10], [11].

The associate editor coordinating the review of this manuscript and approving it for publication was Chao-Yang Chen.

As we all know, one of the typical UAVs' formation methods is 'leader–following' model [12]–[14]. Generally, in multi–UAVs system, a single UAV is designed as leader, while others are considered as followers [15], [16]. The role of the leader during formation navigation is to control the route of the entire formation. The formation can be controlled by maintaining a certain angle and distance between the followers and the leader. One of the main tasks of researching multi–UAVs' formation system is to keep the shape of the required formation. This requires a consensus flight status [10], [13]. Generally speaking, the purpose of consensus is to synchronize all UAVs into a common state through a consensus protocol based on the neighbor UAV's information exchange. Therefore, with the extensive research of multi–agent systems consensus problem. A growing number of interest drivers are multi–UAVs' consensus [17]–[20], especially its effects on UAVs' formation keeping. There are many findings about the formation of UAVs. For example, in [21], *Karimoddini et al.* established a hybrid formation control of the UAVs. They proposed a new approach of hybrid supervisory control of UAVs for a two-dimensional leader–following formation scenario. *Hafez et al.* solved the problem of multi–UAVs dynamic encirclement via model predictive control in [22]. And the model predictive controller is introduced to model and implement controllers for the problem of dynamic encirclement. Recently, *Dentler et al.* in [23] studied the model predictive control of multi–UAVs by using potential functional sensor constraints. They proposed a model predictive control method for this cooperative positioning scheme to solve the problem of limited sensor perception space. In [24], *Tang et al.* proposed an algorithm of leader–following consensus control of multiple fixed–wing UAVs' attitudes with time delays and unknown external disturbances. And a distributed controller based on undirected graph for leader–following consensus control of multi–UAVs' attitudes is proposed. Furthermore, more research results produce that model predictive control can be effectively applied in the leader–following multi–UAVs' formation problem. It can make the following UAVs advance estimate the leader's flight status, and each UAV updates its flight status that final all UAVs reach to an expectation formation [25]–[27]. However, few research has been done for this, so we will focus on this challenging issue in this paper.

In order to facilitate the study of the position status of UAVs, its body coordinate and inertial coordinate can be established. And the body coordinate after translation and rotation can be converted to inertial coordinate [7], [10]. Thus, the position of UAV can be directly expressed in inertial coordinate. As shown in FIGURE I. The blue coordinate system $X_1^* - Y_1^*$ constitutes the body coordinate system of follower W_1, and the red coordinate system $X_2^* - Y_2^*$ constitutes the body coordinate system of follower W_2.

Then the flight motion equations of leader (L) and followers ($W_1, \cdots, W_n$) are expressed as follows:

$$\begin{cases} \dot{x}_i(t) = V_i(t)\cos(\phi_i(t)), \\ \dot{y}_i(t) = V_i(t)\sin(\phi_i(t)), \\ \dot{z}_i(t) = V_{iz}(t). \end{cases} \tag{1}$$

where x_i, y_i, z_i ($i = L, W_1, W_2, \cdots, W_N$) are the position status of the i–th UAV in three–dimensional inertial space, respectively. And ϕ_i is the heading angle of the i–th UAV in inertial coordinate, V_i is the instantaneous linear velocity of the i–th UAV in inertial coordinate, V_{iz} is the flight speed of i–th UAV in vertically of inertial space. The heading angle rate has no component in the z–axis, and the height is just the simple height difference between the leader and followers.

Typically, each UAV in the formation is equipped with flight controllers: heading autopilot and velocity autopilot. Since there have time delays between the process of controller receiving signal and making adjustment response [28], [29]. And time delays always lead to complex dynamic behaviors such as bifurcations, chaos and many more [30]–[32]. So, the flight status of UAVs become more complicated. Therefore, we can discuss the UAVs' complex motions and formations based on time–delayed heading automatic control algorithms or velocity automatic control algorithms. After constructing the controller, in this paper, we are committed to design a consensus algorithm that makes UAVs to keep a

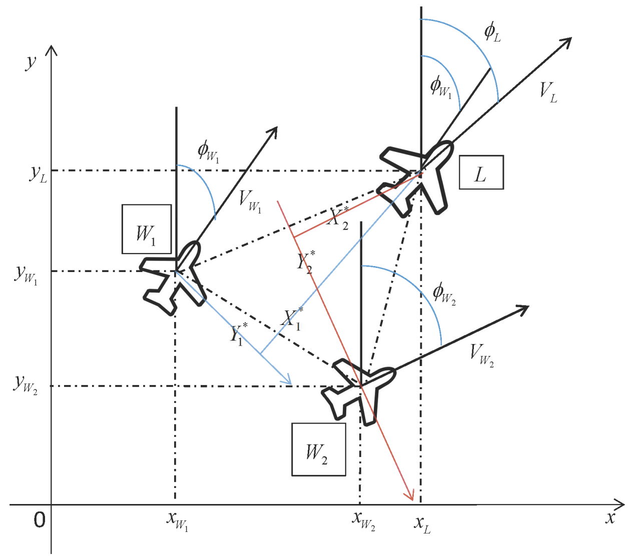

FIGURE 1. UAV formation relative motion relationship in the inertial coordinate.

relative position in formation, whether the heading changing is stable or unstable. Then, a model prediction controller is introduced to make followers to estimate the state information of the leader in case of incomplete communication. Finally, followers ultimately reach a consensus with the leader, and all followers stay in relative position. Specifically, the main contributions of this paper can be summarized as follows:

1) The multi–UAVs system is established based on leader–following structure, and feedback controller based heading control signal is designed.

2) Based on geometric formulation, the *Routh–Hurwitz* stability criterion, we obtain some criteria of stability and Hopf bifurcation conditions for the equilibrium point τ_0 of leader UAV.

3) Adopted the information transmission method of neighboring nodes and introduced a state predictive control strategy to formulate a consensus protocol. The above–obtained results make followers predicts the motion status of leader, and the UAVs' formation shape kept in a stable status. This completes the research of UAVs' bifurcation consensus problems.

This paper is organized as follows: Section *II* gives some preparatory works, and a leader–following system based multi–UAVs control model established. In Section *III*, the Hopf bifurcation problem of leader UAV is discussed. A status predictor is introduced to analyze the consensus of multi–UAVs, and a consensus protocol then designed. Simulation examples verify the validity of the status predictor in Section *IV*. Finally, Section *V* gives the conclusions of this paper.

II. BACKGROUND AND PRELIMINARIES

A. PRELIMINARIES

Let $G = (\nu, \epsilon, A)$ a graph consisted by node sets $\nu = \{\nu_0, \nu_1, \nu_2, \ldots, \nu_N\}$ and edge sets $\epsilon \subseteq \nu \times \nu$ [33], [34].

The adjacency matrix $A = (a_{ij})_{N\times N}$ of graph G is introduced as follows: If there is information transfer between the node i and the node j, then $a_{ij} = 1$, otherwise $a_{ij} = 0$, which is

$$a_{ij} = \begin{cases} 1, & (\nu_i, \nu_j) \in \epsilon, \\ 0, & otherwise, \end{cases} \tag{2}$$

and the diagonal elements of matrix A are defined by

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}, \quad i = 1, 2, \ldots, N. \tag{3}$$

The adjacency matrix in undirected graphs is symmetrical, and digraphs has no such property [34].

Then the corresponding *Laplacian* matrix $L = (l_{ij})_{N\times N}$ can be defined as

$$l_{ij} = \begin{cases} \sum_{j=1, j\neq i}^{N} a_{ij}, & i = 1, 2, \ldots, N, \\ -a_{ij} \leq 0, & i \neq j. \end{cases} \tag{4}$$

The *Laplacian* matrix in undirected graphs is symmetric semi–definite, and does not have such properties with digraphs [34].

Next, define a diagonal matrix of the connection relationship between leader and followers as $B = diag(b_1, b_2, \cdots, b_N)$, and

$$b_i = \begin{cases} 1, & (\nu_i, \nu_0) \in \epsilon, \\ 0, & otherwise, \end{cases} \tag{5}$$

where ν_0 represents the leader, and ν_i $(i = 1, 2, \cdots, N)$ represent the followers.

Lemma 1 [34]: The *Laplacian* matrix L has a simple zero eigenvalue and all the other eigenvalues have positive real parts if and only if the diagraph associated with L has a directed spanning tree.

We need some properties of *Laplacian* matrix from [35] as follows:

(i) 0 is the eigenvalue of matrix L and $\mathbf{1}$ is its corresponding eigenvector, where $\mathbf{1} = (1, 1, \cdots, 1)^T$.

(ii) If the directed graph G is strongly connected, then 0 is a single eigenvector of the matrix L.

(iii) If the digraph G is connected and symmetrical, then matrix L is symmetric and semi–positive. All feature values are non-negative real numbers and have following form: $0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_N(L)$.

Definition 1 [35]: A nonsingular matrix $\tilde{A}$ is called M–matrix if $\tilde{A} \in Z_{N\times N}$ and all the eigenvalues of $\tilde{A}$ have positive real parts.

Let

$$J(\tilde{A}) = \min_{1\leq i\leq n} Re(\lambda_i(\tilde{A})),$$

denote the minimum real part among all its eigenvalues [35], [36]. Then the following statements are equivalent:

(i) The matrix $\tilde{A}$ is an M–matrix.

(ii) The matrix $\tilde{A}$ can be expressed as $\tilde{A} = \zeta T_{N\times N} - \tilde{M}$, where $\tilde{M} > 0$, $\zeta > \rho(\tilde{M})$ and $I_{N\times N}$ is the $N \times N$ identity matrix.

(iii) All of $\tilde{A}'$ eigenvalues have positive real parts, implying that $J(\tilde{A}) > 0$.

B. MODEL

In this paper, every UAV equipped with a heading autopilot and velocity autopilot. The first–order heading autopilot and velocity autopilot respectively as follows:

$$\dot{\phi}_i = -\frac{1}{\tau_{\phi_i}}(\phi_i - \phi_{ic}), \tag{6}$$

$$\dot{V}_i = -\frac{1}{\tau_{V_i}}(V_i - V_{ic}), \tag{7}$$

where $i = 0, 1, 2, \cdots, N$, 0 represents leader UAV and $1, 2, \ldots, N$ represent follower UAVs, ϕ_i and ϕ_{ic} indicate the heading angle and heading control signal of the i–th UAV, respectively. V_i and V_{ic} indicate the velocity and velocity control signal of the i–th UAV, respectively.

Assuming that the linear velocity is not affected by the time delay, we only consider the design of heading control. According to nonlinear feedback control, define ϕ_{ic} as follows:

$$\phi_{ic} = \phi_i - \tau_{\phi_i}[k_1 \int_0^t f(\phi_i(s-\tau) - \phi^*)ds + k_2 \int_0^t f(\omega_i(s))ds], \tag{8}$$

where k_1 and k_2 are the feedback gain constant, $f(\cdot) \in R^4$ is a delay kernel function satisfies $f(0) = 0$ and $f'(0) \neq 0$, ϕ^* is an expected heading angle, $\dot{\phi}_i(t) = \omega_i(t)$ is the heading angular velocity change rate of the i–th UAV, u_i and v_i are the corresponding followers' control input. Then, from equations (6) and (8), we have

$$\begin{aligned} \ddot{\phi}_i &= \frac{1}{\tau_{\phi_i}}(\dot{\phi}_i - \dot{\phi}_{ic}) \\ &= \frac{1}{\tau_{\phi_i}}\{\dot{\phi}_i - \tau_{\phi_i}[k_1 f(\phi_i(t-\tau) - \phi^*) + k_2 f(\omega_i(t))] - \dot{\phi}_i\} \\ &= -k_1 f(\phi_i(t-\tau) - \phi^*) - k_2 f(\omega_i(t)). \end{aligned} \tag{9}$$

Next, we introduce the following model prediction controllers:

$$\dot{\phi}(t)^p = -(L+B)\phi(t) + B\phi_0(t)1, \tag{10}$$

$$\dot{\omega}(t)^p = -(L+B)\omega(t) + B\omega_0(t)1, \tag{11}$$

where

$$\phi(t) = (\phi_1(t), \phi_2(t), \cdots, \phi_N(t))^T,$$
$$\omega(t) = (\omega_1(t), \omega_2(t), \cdots, \omega_N(t))^T,$$

and

$$\dot{\phi}^p(t) = (\dot{\phi}_1^p(t), \dot{\phi}_2^p(t), \cdots, \dot{\phi}_N^p(t))^T,$$
$$\dot{\omega}^p(t) = (\dot{\omega}_1^p(t), \dot{\omega}_2^p(t), \cdots, \dot{\omega}_N^p(t))^T$$

Then, i–th UAV only computes $\dot{\phi}_i^p$ and $\dot{\omega}_i^p$, and passes them to its neighbors by communication, thereby achieving the purpose of predicting the state of the system.

Consider a multi–agent system consisting of N agents based leader–following model, defined as follows:

$$\begin{cases} \dot{X}_0(t) = f(t, X_0(t)), \\ \dot{X}_i(t) = f(t, X_i(t)) + U_i, \end{cases} \tag{12}$$

where $X_0(t) = [x_{01}(t), x_{02}(t), \ldots, x_{0n}(t)]^T$ is the state variable of leader agent, $X_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T$ is the state variable of i–th following agent. And $f(\cdot) \in R^n$ is a continuous vector–valued function, U_i is the following node control law with $i = 1, 2, \ldots, N$.

From equation (12), let $U_i = (u_i, v_i)^T$. Then, based on equations (10) and (11), the control rate of followers under the predictive controllers can be designed as follows:

$$\begin{aligned} u_i &= -\Sigma a_{ij}(\phi_i(t) - \phi_j(t)) - b_i(\phi_i(t) - \phi_0(t)) \\ &\quad + \eta_1 \Sigma a_{ij}(\dot{\phi}_i^p(t) - \dot{\phi}_j^p(t)) + \eta_1 b_i \dot{\phi}_i^p(t), \end{aligned} \tag{13}$$

$$\begin{aligned} v_i &= -\Sigma a_{ij}(\omega_i(t) - \omega_j(t)) - b_i(\omega_i(t) - \omega_0(t)) \\ &\quad + \eta_2 \Sigma a_{ij}(\dot{\omega}_i^p(t) - \dot{\omega}_j^p(t)) + \eta_2 b_i \dot{\omega}_i^p(t), \end{aligned} \tag{14}$$

where η_1 and η_2 are the influence factors of predictors.

From above discussions and based on equation (1), the leader of multi–UAVs' formation system can then be described by the following form:

$$\begin{cases} \dot{x}_0(t) = V_0(t)\cos(\phi_0(t)), \\ \dot{y}_0(t) = V_0(t)\sin(\phi_0(t)), \\ \dot{z}_0(t) = V_{0z}(t), \\ \dot{\phi}_0(t) = \omega_0(t), \\ \dot{\omega}_0(t) = -k_1 f(\phi_0(t-\tau) - \phi^*) - k_2 f(\omega_0(t)). \end{cases} \tag{15}$$

And the followers of multi–UAVs' formation system can be described by the following form:

$$\begin{cases} \dot{x}_i(t) = & V_i(t)\cos(\phi_i(t)), \\ \dot{y}_i(t) = & V_i(t)\sin(\phi_i(t)), \\ \dot{z}_i(t) = & V_{iz}(t), \\ \dot{\phi}_i(t) = & \omega_i(t) - \Sigma a_{ij}(\phi_i(t) - \phi_j(t)) - b_i(\phi_i(t) - \phi_0(t)) \\ & + \eta_1 \Sigma a_{ij}(\dot{\phi}_i^p(t) - \dot{\phi}_j^p(t)) + \eta_1 b_i \dot{\phi}_i^p(t), \\ \dot{\omega}_i(t) = & -k_1 f(\phi_i(t-\tau) - \phi^*) - k_2 f(\omega_i(t)) \\ & -\Sigma a_{ij}(\omega_i(t) - \omega_j(t)) - b_i(\omega_i(t) - \omega_0(t)) \\ & + \eta_2 \Sigma a_{ij}(\dot{\omega}_i^p(t) - \dot{\omega}_j^p(t)) + \eta_2 b_i \dot{\omega}_i^p(t), \\ & i = 1, 2, \ldots, N. \end{cases} \tag{16}$$

III. MAIN RESULTS

In this section, we first discussed the heading control bifurcation problem of leader. Then, studied the consensus of multi–UAVs' formation flight state and ensure the UAVs remain in a relative position, whether the heading state is stable or bifurcated.

A. HOPF BIFURCATION ANALYSIS OF LEADER

First, give a result proposed by Ruan and Wei [37] can be employed to analyze equation (15), which is stated as follows.

Lemma 2: Consider the exponential polynomial

$$\begin{aligned} &\rho(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m}) \\ &= \lambda^n + \rho_1^{(0)}\lambda^{n-1} + \ldots + \rho_{n-1}^{(0)}\lambda + \rho_n^{(0)} \\ &\quad + (\rho_1^{(1)}\lambda^{n-1} + \ldots + \rho_{n-1}^{(1)}\lambda + \rho_n^{(1)})e^{-\lambda\tau_1} + \ldots \\ &\quad + (\rho_1^{(m}\lambda^{n-1} + \ldots + \rho_{n-1}^{(m)}\lambda + \rho_n^{(m)})e^{-\lambda\tau_m}, \end{aligned}$$

where $\tau_j \geq 0$ $(j = 1, 2, \ldots, m)$ and $\rho_i^{(j)}$ $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ are constants. As $(\tau_1, \tau_2, \ldots, \tau_m)$ vary, the sum of the order of the zeros of $\rho(\lambda, e^{-\lambda\tau_1}, \ldots, e^{-\lambda\tau_m})$ on the right half plane can change only if zero appears on or cross the imaginary axis.

Remark 1: In this paper, we designed the heading automatic controller as equation (8). The position of each UAV is represented by its coordinates $(x_i(t), y_i(t), z_i(t))$, and which are determined by the linear velocity $V_i(t)$ and the heading angular velocity $\phi_i(t)$.

Therefore, under the control of heading automatic controller, we discuss the Hopf bifurcation and stability of the following controlled equation:

$$\begin{cases} \dot{\phi}_0(t) = \omega_0(t), \\ \dot{\omega}_0(t) = -k_1 f(\phi_0(t-\tau) - \phi^*) - k_2 f(\omega_0(t)). \end{cases} \tag{17}$$

Let $(\phi^*, 0)$ be the equilibrium point of system (17). We get the linearized system of (16) as

$$\begin{cases} \dot{\phi}_0(t) = \omega_0(t), \\ \dot{\omega}_0(t) = -k_1 f'(0)\phi_0(t-\tau) - k_2 f'(0)\omega_0(t). \end{cases} \tag{18}$$

Then, we have the following main conclusion of this paper.

Theorem 1: Let $\tau_0 = \frac{1}{\alpha_0}\arccos(\frac{\alpha_0^2}{k_1 f'(0)})$, we have
(i) The equilibrium $(\phi^*, 0)$ of system (18) is stable when $\tau < \tau_0$ and unstable for $\tau > \tau_0$.
(ii) A Hopf bifurcation occurs at the equilibrium point as τ passes through τ_0.

Proof: The characteristic equation associated with system (18) is

$$det \begin{bmatrix} \lambda & -1 \\ k_1 f'(0)e^{-\lambda\tau} & \lambda + k_2 f'(0) \end{bmatrix} = 0. \tag{19}$$

It can be transformed into the following form:

$$\lambda^2 + k_2 f'(0)\lambda + k_1 f'(0)e^{-\lambda\tau} = 0. \tag{20}$$

Obviously, when $\tau \neq 0$, $\lambda = i\alpha$ $(\alpha > 0)$ is a root of characteristic equation (20) if and only if α satisfies

$$-\alpha^2 + ik_2 f'(0)\alpha + k_1 f'(0)(\cos(\alpha\tau) - i\sin(\alpha\tau)) = 0. \tag{21}$$

Separating the real and imaginary parts and getting

$$\begin{cases} \cos(\alpha\pi) = \dfrac{\alpha^2}{k_1 f'(0)}, \\ \sin(\alpha\pi) = \dfrac{k_2 f'(0)\alpha}{k_1 f'(0)}. \end{cases} \tag{22}$$

Adding up the squares of the corresponding sides of the above equations will result in the following result as:

$$\alpha^4 + k_2^2[f'(0)]^2\alpha^2 - k_1^2[f'(0)]^2 = 0. \tag{23}$$

Let $\beta = \alpha^2$, then

$$\beta^2 + k_2^2[f'(0)]^2\beta - k_1^2[f'(0)]^2 = 0, \tag{24}$$

and denote

$$h(\beta) = \beta^2 + k_2^2[f'(0)]^2\beta - k_1^2[f'(0)]^2). \tag{25}$$

Then, equation (24) has two roots as follows:

$$\beta_1 = \frac{-k_2^2[f'(0)]^2 - \sqrt{k_2^4[f'(0)]^4 + 4k_1^2[f'(0)]^2}}{2} < 0,$$

and

$$\beta_2 = \frac{-k_2^2[f'(0)]^2 + \sqrt{k_2^4[f'(0)]^4 + 4k_1^2[f'(0)]^2}}{2} > 0.$$

So, equation (23) has a positive root as $\alpha_0 = \sqrt{\beta_2}$. From (21), we can get

$$\tau_0{}^{(j)} = \frac{1}{\alpha\, 0}\{arcos\frac{\alpha_0^2}{k_2 f'(0)} + 2\pi j\}, \quad j = 0, 1, 2, \cdots. \tag{26}$$

Therefore, $\pm i\alpha_k$ are a pair of purely imaginary roots of equation (20). Define $\tau_0 = \tau_0^{(0)}$, let $\lambda(\tau) = \xi(\tau) \pm i\eta(\tau)$ be the root of (20) during $\tau = \tau_0$ and meet the conditions: $\xi(\tau_0) = 0, \eta(\tau_0) = \alpha_0$. From (20), in terms of direct calculations, we have

$$\begin{aligned}
Re(\frac{d\lambda}{d\tau})|_{\lambda=i\alpha_0} &= Re(\frac{d\lambda}{d\tau})^{-1}|_{\lambda=i\alpha_0} \\
&= Re[\frac{2\lambda + k_2 f'(0) - \tau k_1 f'(0)e^{-\lambda\tau}}{\lambda k_1 f'(0)e^{-\lambda\tau}}]|_{\lambda=i\alpha_0} \\
&= Re[-\frac{\tau}{\lambda} + \frac{2}{k_1 f'(0)e^{-\lambda\tau}} + \frac{k_2}{\lambda k_1 e^{-\lambda\tau}}]|_{\lambda=i\alpha_0} \\
&= \frac{k_2 f'(0)\sin(\alpha_0\tau)}{\alpha_0 k_1 f'(0)} + \frac{2\alpha_0\cos(\alpha_0\tau)}{k_1\alpha_0 f'(0)} \\
&= \frac{2\alpha_0^2 + k_2^4[f'(0)]^2}{[k_1\alpha_0 f'(0)]^2} \\
&= \frac{h'(\beta_0)}{[k_1\alpha_0 f'(0)]^2}|_{\beta_0=\alpha_0^2} > 0.
\end{aligned}$$

Then, as τ increase that the root passing the imaginary axis from left to the right, the theorem is proved.

B. CONSENSUS ANALYSIS OF MULTI–AGENT SYSTEM WITH BIFURCATION

From equations (15) and (16), we get the multi–agent system based multiple heading control system as follows:

$$\begin{cases} \dot{\phi}_0(t) = \omega_0(t), \\ \dot{\omega}_0(t) = -k_1 f(\phi_0(t-\tau) - \phi^*) - k_2 f(\omega_0(t)). \end{cases} \tag{27}$$

$$\begin{cases} \dot{\phi}_i(t) = & \omega_i(t) \\ & -\Sigma a_{ij}(\phi_i(t) - \phi_j(t)) - b_i(\phi_i(t) - \phi_0(t)) \\ & +\eta_1\Sigma a_{ij}(\dot{\phi}_i^p(t) - \dot{\phi}_j^p(t)) + \eta_1 b_i\dot{\phi}_i^p(t), \\ \dot{\omega}_i(t) = & -k_1 f(\phi_i(t-\tau) - \phi^*) - k_2 f(\omega_i(t)) \\ & -\Sigma a_{ij}(\omega_i(t) - \omega_j(t)) - b_i(\omega_i(t) - \omega_0(t)) \\ & +\eta_2\Sigma a_{ij}(\dot{\omega}_i^p(t) - \dot{\omega}_j^p(t)) + \eta_2 b_i\dot{\omega}_i^p(t), \\ & \quad i = 1, 2, \ldots, N. \end{cases} \tag{28}$$

Definition 2 [38]: The multi–UAVs' heading control system (28) is said to be consensus if $\lim_{t\to\infty}|\phi_i(t) - \phi_0(t)| = 0$ and $\lim_{t\to\infty}|\omega_i(t) - \omega_0(t)| = 0$ for any $i = 1, 2, \ldots, N$.

Let $\delta_i(t) = \phi_i(t) - \phi_0(t)$ and $\theta_i(t) = \omega_i(t) - \omega_0(t)$ are the multi–UAVs' heading control errors with $i = 1, 2, \cdots, N$, where $\delta(t) = [\delta_1(t), \delta_2(t), \ldots, \delta_N(t)]^T$ and $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_N(t)]^T$. After some calculations we get

$$\begin{aligned}
\dot{\delta}_i(t) &= \dot{\phi}_i(t) - \dot{\phi}_0(t) \\
&= \omega_i(t) + u_i - \omega_0(t) \\
&= \theta_i(t) - \Sigma a_{ij}(\phi_i - \phi_j) - b_i(\phi_i - \phi_0) \\
&\quad + \eta_1\Sigma a_{ij}(\dot{\phi}_i^p - \dot{\phi}_j^p) + \eta_1 b_i\dot{\phi}_i^p \\
&= -\Sigma a_{ij}(\phi_i - \phi_j) - b_i(\phi_i - \phi_0) \\
&\quad - \eta_1[\Sigma a_{ij}b_i(\phi_i - \phi_j) + b_i^2(\phi_i - \phi_0)] \\
&\quad - \eta_1[\Sigma_{j=1}^N\Sigma_{k=1}^N a_{ij}a_{ik}(\phi_i - \phi_k) + \Sigma a_{ij}b_i(\phi_i - \phi_0)] \\
&\quad - \eta_1[\Sigma_{j=1}^N\Sigma_{p=1}^N a_{ij}a_{ip}(\phi_j - \phi_p) + \Sigma a_{ij}b_j(\phi_j - \phi_0)], \\
&\quad\quad i = 1, 2, \ldots, N.
\end{aligned} \tag{29}$$

And

$$\begin{aligned}
\dot{\theta}_i(t) &= \dot{\omega}_i(t) - \dot{\omega}_0(t) \\
&= -k_1 f'(0)[\phi_i(t) - \phi_0(t)] \\
&\quad - k_2 f'(0)[(\omega_i(t-\tau) - \omega_0(t-\tau)] + v_i(t) \\
&= -k_1 f'(0)\delta_i - k_2 f'(0)\theta_i(t-\tau) \\
&\quad - \Sigma a_{ij}(\omega_i(t) - \omega_j(t)) - b_i(\omega_i(t) - \omega_0(t)) \\
&\quad + \eta_2\Sigma a_{ij}(\dot{\omega}_i^p(t) - \dot{\omega}_j^p(t)) + \eta_2 b_i\dot{\omega}_i^p(t) \\
&= -k_1 f'(0)\delta_i - k_2 f'(0)\theta_i(t-\tau) \\
&\quad - \Sigma a_{ij}(\omega_i - \omega_j) - b_i(\omega_i - \omega_0) \\
&\quad - \eta_2[\Sigma a_{ij}b_i(\omega_i - \omega_j) + b_i^2(\omega_i - \omega_0)] \\
&\quad - \eta_2[\Sigma_{j=1}^N\Sigma_{k=1}^N a_{ij}a_{ik}(\omega_i - \omega_k) + \Sigma a_{ij}b_i(\omega_i - \omega_0)] \\
&\quad - \eta_2[\Sigma_{j=1}^N\Sigma_{p=1}^N a_{ij}a_{ip}(\omega_j - \omega_p) + \Sigma a_{ij}b_j(\omega_j - \omega_0)], \\
&\quad\quad i = 1, 2, \ldots, N.
\end{aligned} \tag{30}$$

Then, rewrite equations (29) and (30) to the following matrix form:

$$\begin{bmatrix} \dot{\delta}(t) \\ \dot{\theta}(t) \end{bmatrix} = \Gamma_1 \begin{bmatrix} \delta(t) \\ \theta(t) \end{bmatrix} + \Gamma_2 \begin{bmatrix} \delta(t-\tau) \\ \theta(t-\tau) \end{bmatrix}, \tag{31}$$

where $H = L + B$ and

$$\Gamma_1 = \begin{bmatrix} -(H + \eta_1 H^2) & I_{N\times N} \\ -k_1 f'(0)I_{N\times N} & -(H + \eta_2 H^2) \end{bmatrix}, \tag{32}$$

and

$$\Gamma_2 = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & -k_2 f'(0)I_{N\times N} \end{bmatrix}. \tag{33}$$

Remark 2: When solving the block matrix determinant, we have the following conclusions: when matrix A is an invertible matrix, then

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A||D - CA^{-1}B|.$$

Let $\xi_1, \xi_2, \cdots, \xi_N$ be the eigenvalues of H. Since H is a Hermite matrix, there must be an unitary matrix U such that $U^{-1}HU = diag(\xi_1, \xi_2, \cdots, \xi_N) = \Lambda_H$. So, let $\gamma_{ij}, (i = 1, 2, \cdots, N; j = 1, 2)$ are the eigenvalues of equation (31), we can find the characteristic equation of (31) as shown in (34), which at the bottom of this page.

Lemma 3 [38]: Equation (31) is globally and asymptotically stable if and only if

$$Re(\gamma_{ij}) < 0, (i = 1, 2, \cdots, N; j = 1, 2).$$

Theorem 2: The leader–following heading controlled system (28) is globally and asymptotically consensus with the leader (27) if

$$k_1 f'(0) > (\xi_i^2 + \eta_1\xi_i^2)^2,$$
$$k_2 f'(0) > -[2\xi_i + (\eta_1 + \eta_2)\xi_i^2], \quad i = 1, 2, \cdots, N. \quad (35)$$

Proof: From equation (34), eigenvalue γ_{ij} determined by the eigenvalue ξ_i of H. Then, the following equation is established

$$\gamma^2 + [2\xi_i + (\eta_1 + \eta_2)\xi_i^2]\gamma + k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) + k_2 f'(0)(\gamma + \xi_i + \eta_1\xi_i^2)e^{-\gamma\tau} = 0. \quad (36)$$

When $\tau = 0$, we get

$$\gamma^2 + [2\xi_i + (\eta_1 + \eta_2)\xi_i^2 + k_2 f'(0)]\gamma + k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) + k_2 f'(0)(\xi_i + \eta_1\xi_i^2) = 0. \quad (37)$$

Let $a = 1$, $b = 2\xi_i + (\eta_1 + \eta_2)\xi_i^2 + k_2 f'(0)$ and $c = k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) + k_2 f'(0)(\xi_i + \eta_1\xi_i^2)$. Then, equation (37) has two roots as follows:

$$\gamma_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}. \quad (38)$$

From equation (38), we have $Re(\gamma_{1,2}) < 0$, because

$$2\xi_i + (\eta_1 + \eta_2)\xi_i^2 + k_2 f'(0) > 0,$$

and

$$4[k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) + k_2 f'(0)(\xi_i + \eta_1\xi_i^2)] > 4(\xi_i^2 + \eta_1\xi_i^2)^2 + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) - 4[2\xi_i + (\eta_1 + \eta_2)\xi_i^2](\xi_i + \eta_1\xi_i^2) = 0.$$

When $\tau \neq 0$, $\gamma = i\varpi$, $(\varpi > 0)$ is a root of characteristic equation (34) if and only if ϖ satisfies

$$-\varpi^2 + i[2\xi_i + (\eta_1 + \eta_2)\xi_i^2]\varpi + k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) + k_2 f'(0)(i\varpi + \xi_i + \eta_1\xi_i^2)(\cos(\varpi\tau) - i\sin(\varpi\tau)) = 0. \quad (39)$$

After simple calculations we can get

$$\begin{cases} \cos(\varpi\tau) = \dfrac{K_1\varpi^2 + K_2}{F_1\varpi^2 + F_2}, \\ \sin(\varpi\tau) = \dfrac{H_1\varpi^3 + H_2\varpi}{F_1\varpi^2 + F_2}. \end{cases} \quad (40)$$

where

$$F_1 = k_2 f'(0),$$
$$F_2 = k_2 f'(0)(\xi_i + \eta_1\xi_i^2)^2,$$
$$K_1 = \xi_i + \eta_2\xi_i^2,$$
$$K_2 = -(\xi_i + \eta_1\xi_i^2)[k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2)],$$
$$H_1 = 1,$$
$$H_2 = (\xi_i + \eta_1\xi_i^2)[2\xi_i + (\eta_1 + \eta_2)\xi_1^2] - k_1 f'(0) - (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2).$$

According to $\cos^2(\varpi\tau) + \sin^2(\varpi\tau) = 1$, we obtain

$$h(\varpi) = H_1^2\varpi^6 + (2H_1H_2 - K_1^2 - F_1^2)\varpi^4 + (H_2^2 - 2K_1K_2 - 2F_1F_2)\varpi^2 + K_2^2 - F_2^2. \quad (41)$$

Then,

$$\begin{aligned} &K_2^2 - F_2^2 \\ &= (\xi_i + \eta_1\xi_i^2)^2[k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2)]^2 - [k_2 f'(0)]^2(\xi_i + \eta_1\xi_i^2)^4 \\ &= (\xi_i + \eta_1\xi_i^2)^2 \\ &\quad [k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) - k_2 f'(0)(\xi_i + \eta_1\xi_i^2)] \\ &\quad [k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) + k_2 f'(0)(\xi_i + \eta_1\xi_i^2)] \end{aligned}$$

$$\begin{aligned} &|\gamma I_{N\times N} - \Gamma_1 - \Gamma_2\, e^{-\gamma\tau}| \\ &= \begin{vmatrix} \gamma I_{N\times N} + H + \eta_1 H^2 & -I_{N\times N} \\ k_1 f'(0) I_{N\times N} & \gamma I_{N\times N} + H + \eta_2 H^2 + k_2 f'(0)e^{-\gamma\tau} I_{N\times N} \end{vmatrix} \\ &= |(\gamma I_{N\times N} + H + \eta_1 H^2)(\gamma I_{N\times N} + H + \eta_2 H^2 + k_2 f'(0)e^{-\gamma\tau} I_{N\times N}) + k_1 f'(0) I_{N\times N}| \\ &= |U[(\gamma I_{N\times N} + H + \eta_1 H^2)(\gamma I_{N\times N} + H + \eta_2 H^2 + k_2 f'(0)e^{-\gamma\tau} I_{N\times N}) + k_1 f'(0) I_{N\times N}]U^{-1}| \\ &= |(\gamma I_{N\times N} + UHU^{-1} + \eta_1 UHHU^{-1})(\gamma I_{N\times N} + UHU^{-1} + \eta_2 UHU^{-1} + k_2 f'(0)e^{-\gamma\tau} I_{N\times N}) + k_1 f'(0) I_{N\times N}| \\ &= |(\gamma I_{N\times N} + \Lambda_H + \eta_1 \Lambda_H^2)(\gamma I_{N\times N} + \Lambda_H + \eta_2 \Lambda_H^2 + k_2 f'(0)e^{-\gamma\tau} I_{N\times N}) + k_1 f'(0) I_{N\times N}| \\ &= \prod_{i=1}^{N}[(\gamma + \xi_i + \eta_1\xi_i^2)(\gamma + \xi_i + \eta_2\xi_i^2 + k_2 f'(0)e^{-\gamma\tau}) + k_1 f'(0)]. \end{aligned} \quad (34)$$

$$> (\xi_i + \eta_1\xi_i^2)^2 \\ [k_1 f'(0) + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) - k_2 f'(0)(\xi_i + \eta_1\xi_i^2)] \\ [(\xi_i + \eta_1\xi_i^2)^2 + (\xi_i + \eta_1\xi_i^2)(\xi_i + \eta_2\xi_i^2) \\ -((2\xi_i + (\eta_1 + \eta_2)\xi_i^2)(\xi_i + \eta_1\xi_i^2)] = 0.$$

Then, $h(\varpi) > 0$ for all $\varpi > 0$. In summary, all roots of characteristic equation (34) are negative or have negative real parts. From *Lemma 3*, the leader–following controlled system (28) globally and asymptotically consensus with the leader (27), the theorem is proved.

Remark 3: It is worth noting that the two inequalities in equation (35) do not need to hold for all of ξ_i at the same time, and the proof process is more complicated, which we will show in future work. In this paper, we specifically point out that when $\xi = \min\{\xi_i, i = 1, 2, \ldots, N\}$, the conclusion in *Theorem 2* holds, and we will further verify this conclusion in simulations.

Lemma 4 [39]: For system (12), if there exists a constant $\theta > 0$ such that

$$(Y - X)^T(f(t, X)) - f(t, Y)) \leq \theta(Y - X)^T(Y - X), \\ \forall X, Y \in R^n, \quad (42)$$

and the digraph has a directed spanning tree, the leader–following controlled system (12) globally and asymptotically synchronizes with the isolated node if

$$J(L + B) > \frac{\theta}{c},$$

where $J(L + B) = \min_{1 \leq i \leq n} Re(\lambda_i(L + B))$ and the constant c is known as the coupling strength eigenvalues.

Remark 4: If the undirected graph has an spanning tree and also exists a constant $\alpha > 0$ such that equation (42) holds. Then, the conclusion of *Lemma 3* still holds. We will focus on the discussion of undirected graph in this paper and get an more directed consensus protocol based on *Lemma 3*.

Lemma 5 [39]: The matrix $H = L + B$ in (32) is an $M-$matrix if and only if graph G has a spanning tree, where G is the graph of leader (27) and followers (28).

Theorem 3: Let $X = (x_r(t), y_r(t), z_r(t)) \in R^3$ and $Y = (x_s(t), y_s(t), z_s(t)) \in R^3$, $(r, s = 1, 2, \ldots, N)$ for system (16) if there exists a constant ϑ such that

$$(Y - X)^T(\begin{bmatrix} V_r(t)\cos(\phi_r(t)) \\ V_r(t)\sin(\phi_r(t)) \\ V_{rz}(t) \end{bmatrix} - \begin{bmatrix} V_s(t)\cos(\phi_s(t)) \\ V_s(t)\sin(\phi_s(t)) \\ V_{sz}(t) \end{bmatrix}) \\ \leq \vartheta(Y - X)^T(Y - X). \quad (43)$$

and the graph has a spanning tree, the leader–following controlled system (16) globally and asymptotically consensus with the leader (15) if

$$J(H) > \frac{\vartheta}{c},$$

where $J(H) = \min_{1 \leq i \leq N} Re(\xi_i(H))$, the constant $c = \max\{\eta_1, \eta_2\}$ is known as the coupling strength eigenvalues.

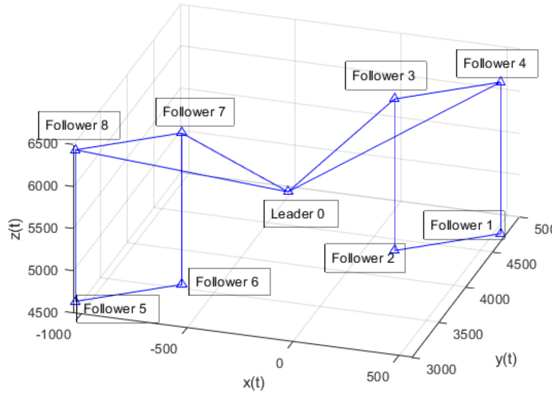

FIGURE 2. Multi–UAV system topology diagram with leader–following.

Remark 5: The proof of *Theorem 3* can be similar to the proof of *Lemma 4* in [39], which is omitted here.

Remark 6: Specially, if $\|V_i(t)\| = \|V_{iz}(t)\| = \|V_0(t)\| = V^*$, $(i = 1, 2, \ldots, N)$. Let $e_{1i} = x_i(t) - x_0(t)$, $e_{2i} = y_i(t) - y_0(t)$, $e_{3i}(t) = z_i(t) - z_0(t)$ and $e_i(t) = [e_{1i}(t), e_{2i}(t), e_{3i}(t)]^T$, we get an error system as follows:

$$\dot{e}_i(t) = \begin{bmatrix} V_i(t)\cos(\phi_i(t)) - V_0(t)\cos(\phi_0(t)) \\ V_i(t)\sin(\phi_i(t)) - V_0(t)\sin(\phi_0(t)) \\ V_{iz}(t) - V_{0z}(t) \end{bmatrix} \\ \leq V^* \begin{bmatrix} |\phi_i(t) - \phi_0(t)| \\ |\phi_i(t) - \phi_0(t)| \\ 0 \end{bmatrix}. \quad (44)$$

From *Theorem 2*, we get $\lim_{t\to\infty} |\phi_i(t) - \phi_0(t)| = 0$ and equation (44) is stability. Then it shows that the controlled followers (16) globally asymptotically consensus to the leader (15).

IV. NUMERICAL SIMULATION AND ANALYSIS

In order to verify bifurcation consensus conclusions of the multi–UAVs system, a simple multi–UAVs system topology diagram is given as FIGURE 2, and simulation examples are listed.

Then, we can get the adjacency matrix $A = (a_{ij})_{N\times N}$ as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}. \quad (45)$$

The corresponding *Laplacian* matrix $L = (l_{ij})_{(N-1)\times(N-1)}$ can be defined as

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix}. \tag{46}$$

The diagonal matrix $B = diag(b_i)_{(N-1)\times(N-1)}$ describing the interaction between the leader and followers as

$$B = diag(0, 0, 1, 1, 0, 0, 1, 1), \tag{47}$$

and let *Hermite* matrix H is

$$H = L + B = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 3 \end{bmatrix}. \tag{48}$$

Let $f(\cdot) = \tanh(\cdot)$, $k_1 = 1$, $k_2 = 0.5$ and $\phi^* = 2$. We consider the model prediction controller with $\eta_1 = \eta_2 = 5$. And for convenience, let $V_i = V_{iz} = 50$, $i = 0, 1, 2, \ldots, 8$. Then equations (27) and (28) can be converted into

$$\begin{cases} \dot{\phi}_0(t) = \omega_0(t), \\ \dot{\omega}_0(t) = -f(\phi_0(t-\tau) - 2) - 0.5f(\omega_0(t)). \end{cases} \tag{49}$$

$$\begin{cases} \dot{\phi}_i(t) &= \omega_i(t) - \Sigma a_{ij}(\phi_i(t) - \phi_j(t)) + 5\Sigma a_{ij}(\dot{\phi}_i^p(t) \\ &\quad -\dot{\phi}_j^p(t)) - b_i(\phi_i(t) - \phi_0(t)) + 5b_i\dot{\phi}_i^p(t), \\ \dot{\omega}_i(t) &= -f(\phi_i(t-\tau) - 2) - 0.5f(\omega_i(t)) \\ &\quad -\Sigma a_{ij}(\omega_i(t) - \omega_j(t)) + 5\Sigma a_{ij}(\dot{\omega}_i^p(t) \\ &\quad -\dot{\omega}_j^p(t)) - b_i(\omega_i(t) - \omega_0(t)) + 5b_i\dot{\omega}_i^p(t), \end{cases} \tag{50}$$

From equations (24) and (26), direct calculation can get $\alpha_0 = 0.4551$, $\tau_0 = 2.5132$. Based on *Theorem 1*, the equilibrium (2, 0) of system (49) is locally stable when $\tau \in (0, \tau_0)$ and undergoes a Hopf bifurcation when $\tau > \tau_0$. That is, they are stable when $\tau_1 = 2.2 < \tau_0 = 2.5132$, and Hopf bifurcation occurs when $\tau_2 = 2.6 > \tau_0 = 2.5132$. Which waveform plots and phase diagram are illustrated by the simulation is shown in FIGURE 3 and FIGURE 4.

After some conclusions, the eigenvalues of H are $\xi_1 = 0.382$, $\xi_2 = 2.382$, $\xi_3 = 2.618$ and $\xi_4 = 4.618$.

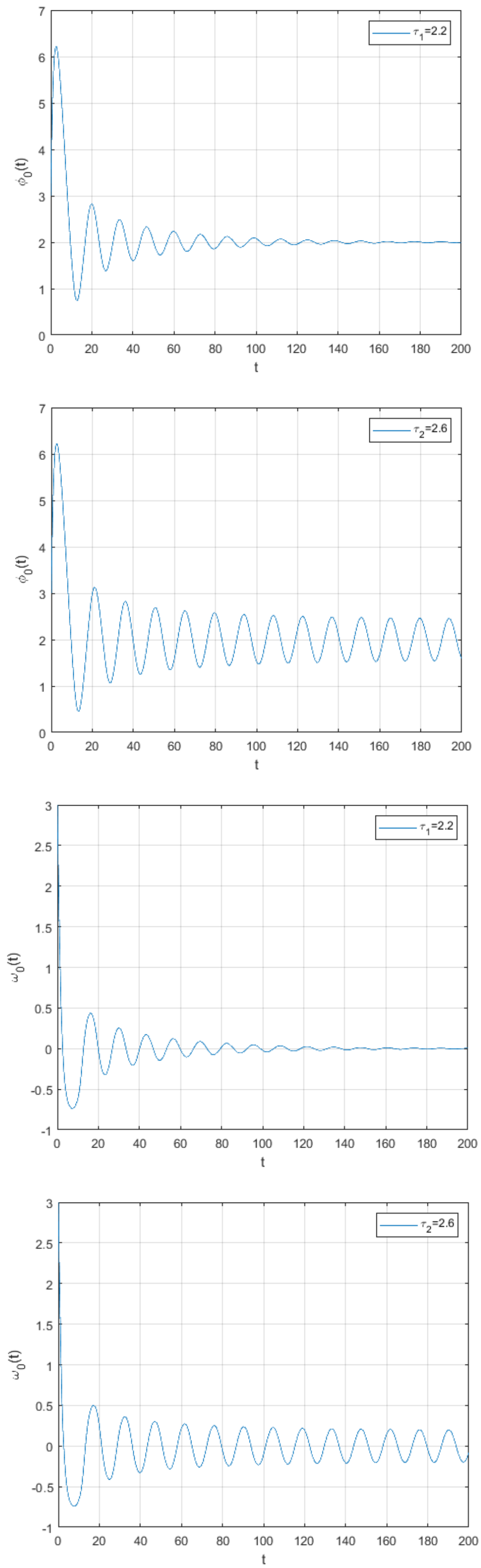

FIGURE 3. Waveform plots of system (49) with ω_0 and ϕ_0.

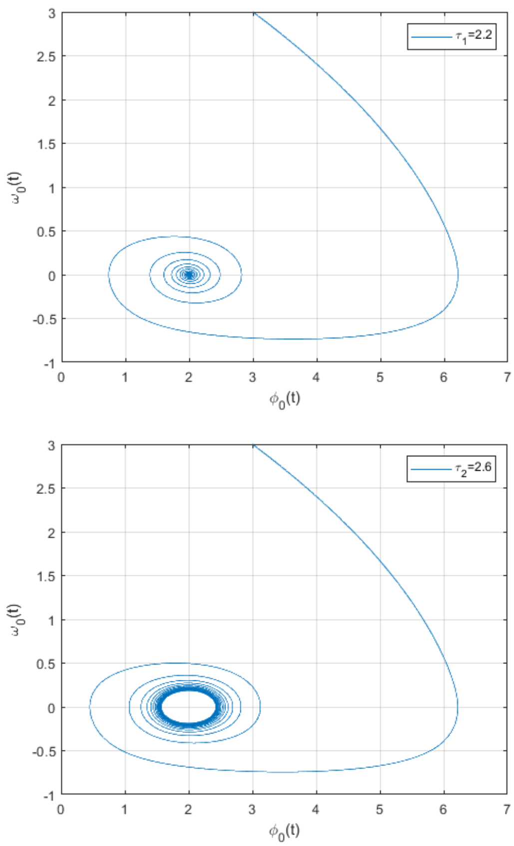

FIGURE 4. Phase diagrams of system (49) at ω_0–ϕ_0 phase plane.

From *Theorem 2* and *Remark 3*, we know when $\xi = 0.382 = \min\{\xi_i, i = 1, 2, 3, 4\}$, there have

$$k_1 f'(0) = 1 > (\xi^2 + \eta_1 \xi^2)^2 = 0.875544,$$
$$k_2 f'(0) = 0.5 > -[2\xi + (\eta_1 + \eta_2)\xi^2] = -2.22324.$$

Then, system (50) is globally and asymptotically synchronizes with the leader (49). And some simulation results of heading control system bifurcation consensus shown in FIGURE 5 and FIGURE 6. ϕ_i and ω_i are stable when $\tau_1 = 2.2 < \tau_0 = 2.5132$, and Hopf bifurcation occurs at $(2, 0)$ when $\tau_2 = 2.6 > \tau_0 = 2.5132$.

And FIGURE 7 shows the heading control system errors graph for multi–UAVs' consensus corresponding to leader and followers. Which further demonstrates the effectiveness of the model prediction controller for consensus control of multi–UAVs' formation system.

From equations (15) and (16), the leader of multi–UAVs' formation system can be rewritten as follows:

$$\begin{cases} \dot{x}_0(t) = & 50\cos(\phi_0(t)), \\ \dot{y}_0(t) = & 50\sin(\phi_0(t)), \\ \dot{z}_0(t) = & 50, \\ \dot{\phi}_0(t) = \omega_0(t), \\ \dot{\omega}_0(t) = -f(\phi_0(t-\tau) - 2) - 0.5 f(\omega_0(t)), \end{cases} \tag{51}$$

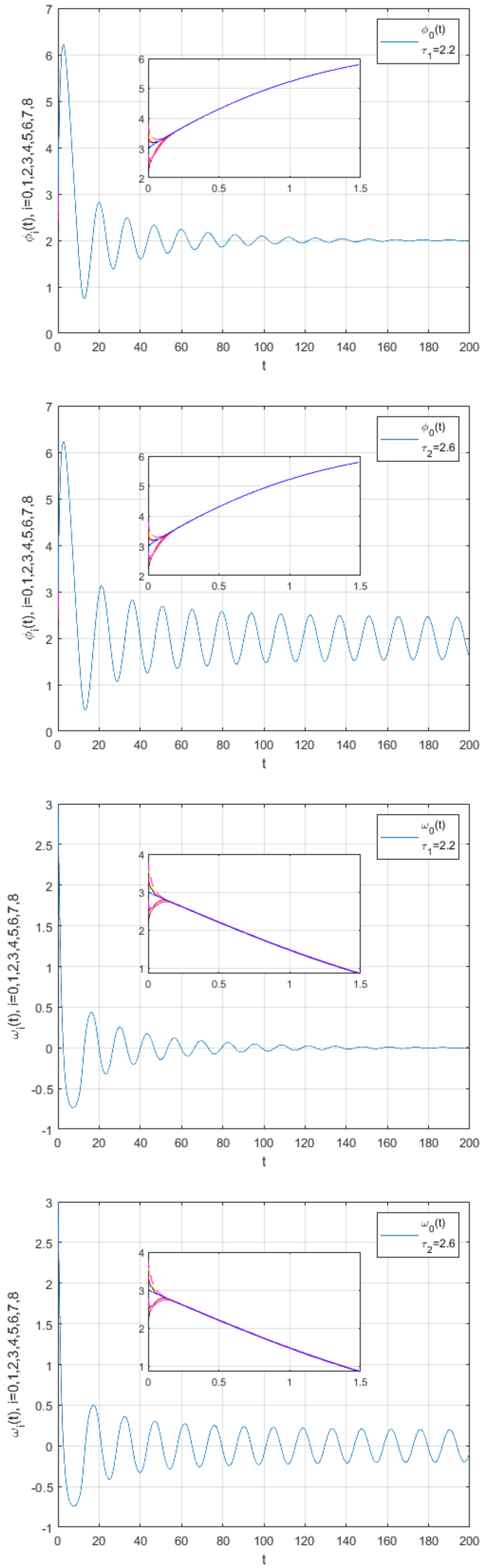

FIGURE 5. Waveform plots of heading velocity bifurcation consensus with system (50).

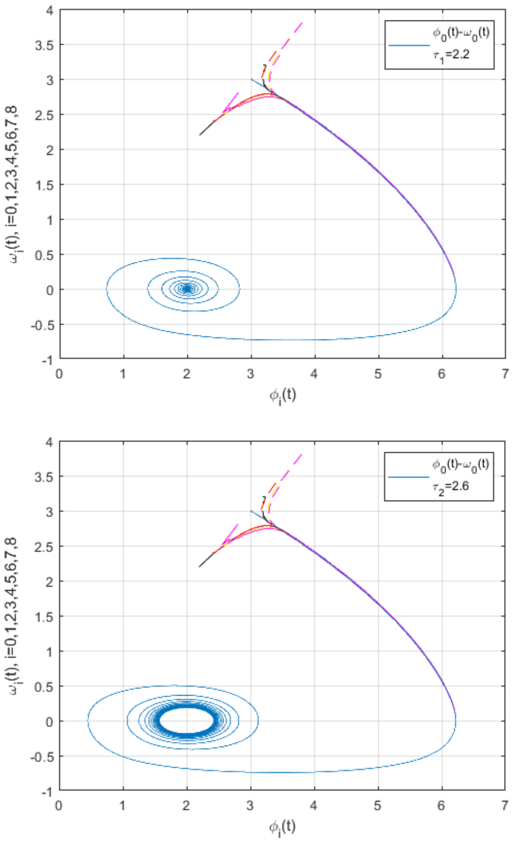

FIGURE 6. **Phase diagrams of heading bifurcation consensus with system (50).**

And the followers can then be written as follows:

$$\begin{cases} \dot{x}_i(t) = & 50\cos(\phi_i(t)), \\ \dot{y}_i(t) = & 50\sin(\phi_i(t)), \\ \dot{z}_i(t) = & 50, \\ \dot{\phi}_i(t) = & \omega_i(t) - \Sigma a_{ij}(\phi_i(t) - \phi_j(t)) + 5\Sigma a_{ij}(\dot{\phi}_i^p(t) \\ & -\dot{\phi}_j^p(t)) - b_i(\phi_i(t) - \phi_0(t)) + 5b_i\dot{\phi}_i^p(t), \\ \dot{\omega}_i(t) = & -f(\phi_i(t-\tau) - 2) - 0.5f(\omega_i(t)) \\ & -\Sigma a_{ij}(\omega_i(t) - \omega_j(t)) + 5\Sigma a_{ij}(\dot{\omega}_i^p(t) \\ & -\dot{\omega}_j^p(t)) - b_i(\omega_i(t) - \omega_0(t)) + 5b_i\dot{\omega}_i^p(t), \end{cases} \tag{52}$$

From *Theorem 3* and *Remark 6*, we can choose $\vartheta = 1.75$. Then,

$$J(H) = \min_{1 \leq i \leq n} Re(\gamma_i(H)) = 0.382 > \frac{1.75}{5} = 0.35.$$

And the consensus simulation of the multi–UAVs' position state can be seen in the FIGURE 8 and FIGURE 9. It shows that when the leader's heading control system (49) undergoes Hopf bifurcation, leader and followers all exhibit a periodic motion state. But whether (49) is stable or bifurcated, the shape of formation is maintained.

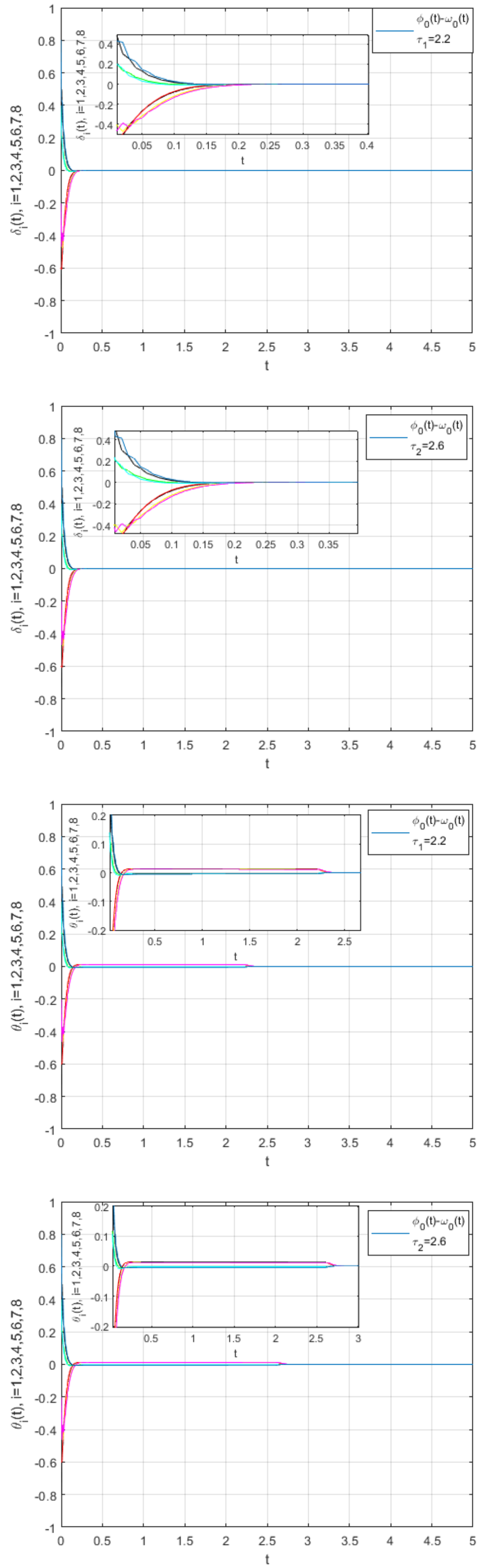

FIGURE 7. **Plots of heading control system consensus error for multi–UAVs.**

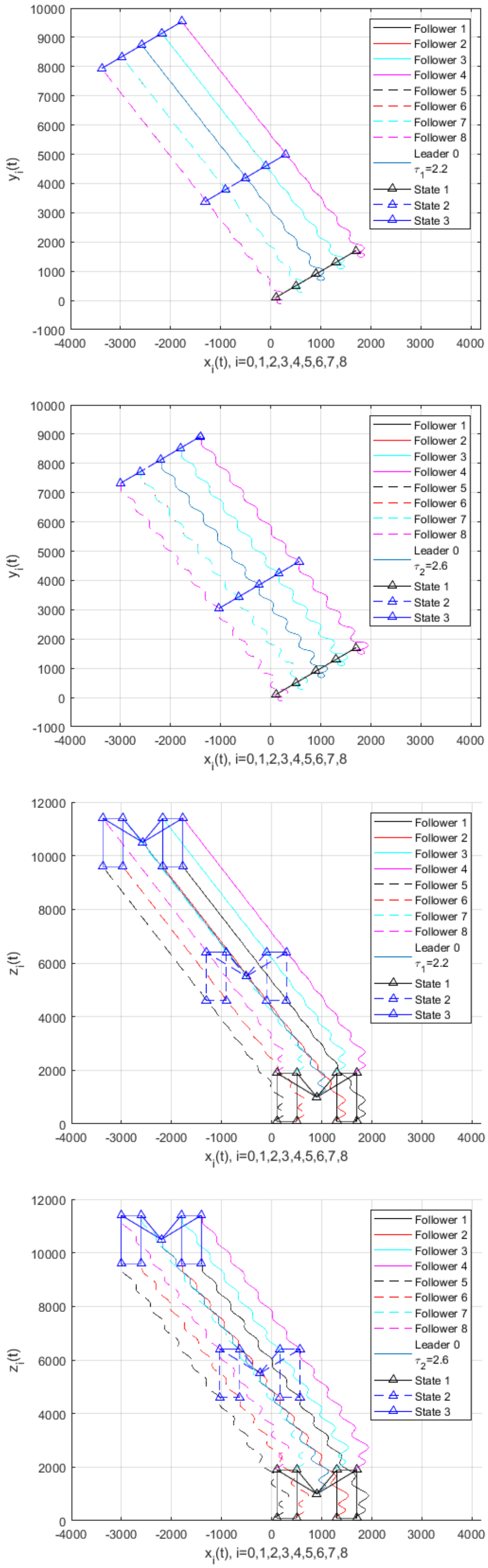

FIGURE 8. Position state consensus diagrams of multi–UAVs system (52).

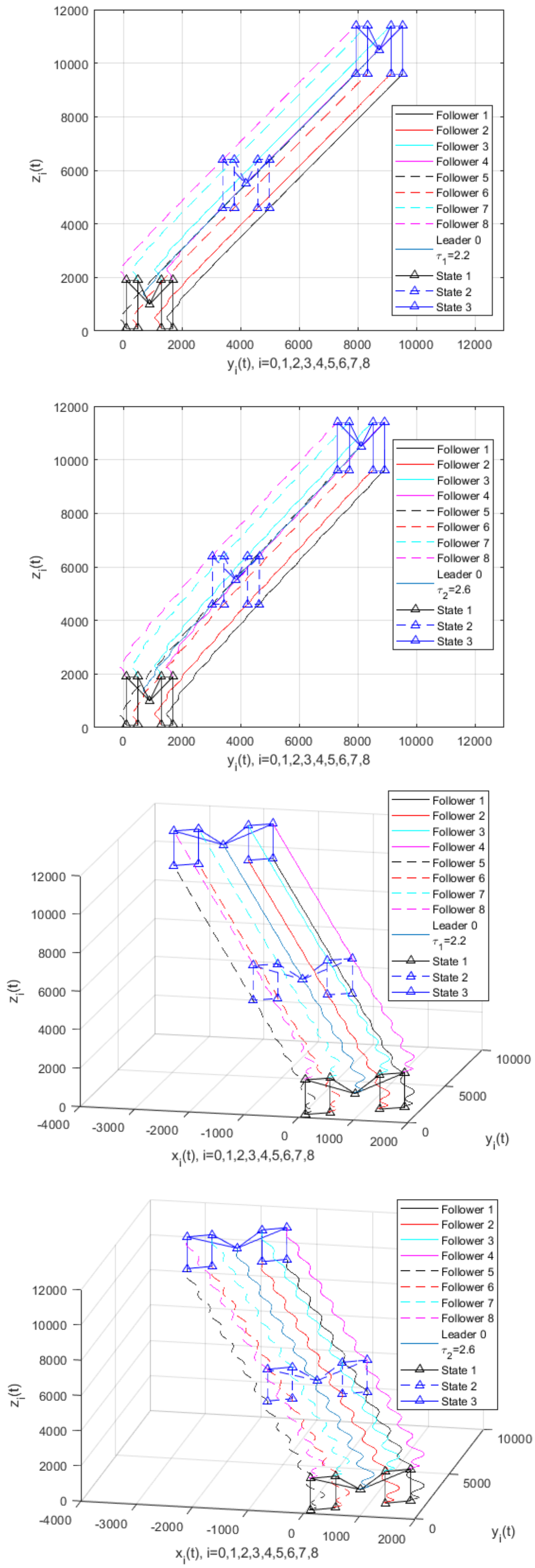

FIGURE 9. Position state consensus diagrams of multi–UAVs system (52).

V. CONCLUSION

This paper discusses the bifurcation consensus problem of a multi–UAVs' formation control system. The heading rate controller is introduced with discrete delay, and a model prediction controller for followers can predict the state change of the leader. This research reduces the time of achieving consensus and has great important for keeping the shape of UAVs' formation. Non-linear systems also exist widely in practice, it is necessary to extend the method proposed in this paper to nonlinear systems. For the sake of discussion, the discussion in this article has omitted the vertical direction controller. The research related to the challenges of increasing the computational complexity and spatial complexity of extending the two–dimensional configuration to three–dimensional space, in subsequently. Further research on the control and consensus convergence speed of such a multi–UAVs system with added disturbance is also a new problem worth exploring.

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