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Empirical Bayes Based on Squared Error Loss and Precautionary Loss Functions in Sequential Sampling Plan

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ABSTRACT An acceptance sampling plans are statistical tools in quality control which often used for lot inspection in several areas such as industry, engineering and business. It can be applied for preserving the quality of products in industry process and preserving the producer's risk and consumer's risk in the production process of manufactures. The objective of this study is to utilize the Empirical Bayes approach based on squared error loss and precautionary loss functions for parameter estimation in sequential sampling plans. The parameters are estimated using Lindley's approximation technique, and hyper-parameters can be obtained via Gibbs sampling technique. Data are normally distributed under an unknown mean and variance. The proposed plans are compared with traditional approaches including a single sampling plan and sequential sampling plan. The probability of acceptance (P_a) and average sample number (ASN) are used as criterion for comparison. Results show that the proposed plans yielded the smaller ASN and higher P_a than both classical methods.

INDEX TERMS Empirical Bayes, sequential sampling plan, single sampling plan, squared error loss function, precautionary loss function.

I. INTRODUCTION

Statistical quality control can be classified as control chart and acceptance sampling plans. An acceptance sampling plans have been widely used in lot inspection of productions in the industries, when can be divided into attributes and variables sampling plans. The quality characteristics of the variables sampling plans are measured on a continuous scale. This can be specified by the statistical hypothesis testing for variables process parameters which is utilized as part of the quality assurance concerning the average quality of products such as bulk materials in bags and drums whereas the one is the estimation of percentage of defective units out of the specification limits. Variables sampling plan provide more information regarding production in the lots than the attributes ones with a small sample size [1]. There are various types of sampling plan, such as a single sampling plan, double sampling plan (DSP), multiple sampling plans (MSP), sequential sampling plan (SSP) and skip lot sampling plan (SkSP). The sequential sampling plan often provides a smaller average sample number (ASN) than the single sampling plan, DSP and MSP [2], [3].

Bayesian approach is extensively applied in statistical inferences as an alternative to classical approaches. It was used to estimate parameter in SSP, which can be shown in [4]–[7]. Its principle is to incorporate information in the history about parameters through a prior distribution, assuming a known form of distribution. The parameters of prior distribution, called hyper-parameters, are usually assumed to be known or can be assessed regardless of the observed data. In contrast, when the hyper-parameters are unknown and estimated from the observed data, it is called the Empirical Bayes (EB) approach [8]. The researches of EB have been performed by many authors [9]–[14]. The EB approach can be implemented with various types of loss function such as absolute error loss (AEL) function, squared error loss (SEL) function and precautionary loss (PL) function [15]. It is applied in quality assurance of production and used to decide

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for producers and statisticians. The researches of EB method using loss functions, can be seen in [16]–[23]. In addition, the EB method was also applied to estimate the percentage of defective units when the lots were accepted [24]. Ohta and Ogawa [25], Shin and Shin [26], Karunamuni [27] utilized the EB approach, combined with a specified cost function, for testing destructive items of high quality products in Poisson process.

As seen that, the EB approach in a sequential sampling plan (EB in SSP) was usually performed to estimate proportion of defective items in the lot rather than test the hypothesis of processing mean in the variables sampling plan. In this paper, we propose the use of EB approach with squared error loss (SEL) and precautionary loss (PL) functions for the lot inspection in sequential sampling plans to test variables sampling plan process mean. Data are distributed as normal with unknown mean and variance. The proposed plans are then compared with the classical approaches, single sampling plan and sequential sampling plan. The probability of acceptance (P_a) and average sample number (ASN) are criterion for comparison. The Gibbs sampling technique is then applied to obtain hyper-parameters due to the complexity of posterior predictive distribution function. The outline of this paper is as follows. The variables sampling plan for process mean is presented in Section 2. In Section 3 and 4, single sampling plan and the sequential sampling plan by variables are shown, respectively. Section 5 the EB approach based on squared error loss and precautionary loss functions are explained. Section 6 covers the Gibbs sampling procedure. The result of simulation study is shown in Section 7. Section 8 provides an example and final section presents the conclusion.

II. VARIABLES SAMPLING FOR PROCESS MEAN TESTING

It considers under variable sampling plan for process mean testing which is used in the statistical hypothesis under a six-sigma quality level, then the process mean has 3.4 defective units per million opportunities (p) where $p = P(X > USL|\mu)$ and it is assumed to shift to $\pm 1.5\sigma$. Thus, the hypothesis testing under an upper specification limit (USL), it can be written as follows: H_0: $\mu_2 \leq \mu_1$ vs. H_1: $\mu_2 > \mu_1$ where μ_1 is acceptable process level (APL) and μ_2 is rejectable process level (RPL). This is considered under normal distribution and known variance. The μ_1 and μ_2 can be obtained from

$$\text{APL} = \mu_1 = P(X > USL|\mu_1) = Mean + 1.5\sigma_x$$
$$\text{and RPL} = \mu_2 = P(X > USL|\mu_2) = ACL + Z_\beta \sigma_{\overline{x}} \quad (1)$$

where ACL $= \mu_1 + Z_\alpha \sigma_{\overline{x}}$ is acceptance control limits, $Z \sim N(0, 1)$, $\overline{x}$ is the sample mean, the producer's risk (α) is the probability of rejection at μ_1 and the consumer's risk (β) is the probability of acceptance at [29].

III. SINGLE SAMPLING PLAN

This plan depends on two parameters according to sample size (n) and acceptance criterion (K). The lot is accepted if z $\geq$ K and rejected if z $<$ K where z $=$ (USL $- x)/\sigma$ for known variance and using standard deviation (s) for unknown variance. The criteria for comparison of the single sampling plan can be shown as follows.

$$P_a(\mu) = P(Z \leq z), \quad (2)$$
$$\text{and } ASN(\mu) = n. \quad (3)$$

IV. SEQUENTIAL SAMPLING PLAN BY VARIABLES

It is modified from the double sampling plan and multiple sampling plans. One random sample is taken sequentially from the lot [1]. For USL testing, the criteria for inspection uses the acceptance limit line (Y_1), and rejection limit line (Y_2) for rejecting the batch, accepting the batch and continuing sampling, respectively.

$$Y_1 = -h_1 + s \cdot n \quad \text{and } Y_2 = h_2 + s \cdot n. \quad (4)$$

Thus, the lot is accepted if $\sum_{i=1}^{n} x_i \leq Y_1$, the lot is to continue sampling if $Y_1 < \sum_{i=1}^{n} x_i < Y_2$ and the lot is rejected if $\sum_{i=1}^{n} x_i \geq Y_2$. The P_a and ASN are criterion for comparison [29] which are calculated as follows:

$$P_a(\mu) = \frac{\left[(1-\beta/\alpha)^w - 1\right]}{(1-\beta/\alpha)^w - (\beta/1-\alpha)^w}, \quad (5)$$
$$\text{and } ASN(\mu) = \frac{\left[\frac{La\sigma^2}{\mu_2-\mu_1} + P_a \cdot \left(\frac{L\sigma^2 A - L\sigma^2 a}{\mu_2-\mu_1}\right)\right]}{\left(\frac{2\mu-\mu_2-\mu_1}{2}\right)}. \quad (6)$$

where $w = (\mu_2+\mu_1-2\mu)/(\mu_2-\mu_1)$, $a = \log[(1-\beta)/\alpha]$, $b = \log[(1-\alpha)/\beta]$, $s = (\mu_2+\mu_1)/2$ is the slope of the lines, $h_1 = Lb\sigma^2/(\mu_2-\mu_1)$ is the intercept of acceptance line, $h_2 = La\sigma^2/(\mu_2-\mu_1)$ is the intercept of rejection line and $L = 2.3026$.

V. EMPIRICAL BAYES PREDICTION APPROACH

Bayesian approach is applied in statistical inference that is the difference from the classical approach. The traditional method assumes constant parameters and the parameters are the random sampling from the probability distribution function. However, the Bayesian approach is unknown parameters δ are considered as a random variable, depending on information in the history of parameters, called prior probability density function, assuming known prior distribution, $\pi(\delta|\omega)$ and known hyper-parameter ω. Thus, inference concerning δ is performed using the Bayes' theorem which can be expressed up to proportionality as the product of likelihood function, $L(\delta)$, and the prior distribution, $\pi(\delta|\omega)$. The posterior distribution, $h(\delta|\underline{x})$ is obtained as following

$$h(\delta|\underline{x}) = \frac{L(\delta) \cdot \pi(\delta|\omega)}{M(\underline{x}|\omega)} \propto L(\delta) \cdot \pi(\delta), \quad (7)$$

where $M(\underline{x}|\omega)$ denotes the marginal distribution function of $\underline{x}$.

If the hyper-parameter (ω) is unknown that is estimated from the observed data. This is called Empirical Bayes (EB) approach which the hyper-parameter can be determined from the marginal distribution of $\underline{x}$, provided by

$$M\left(\underline{x}|\omega\right)=\int_{\delta} f\left(\underline{x}|\delta\right)\cdot\pi\left(\delta|\omega\right)d\delta. \tag{8}$$

The observed data $\underline{x}$ are continuous random sample. The predictive distribution function is then developed to estimate newly observed data or a future observation (x_{n+1}), based on previous observed data $x_1, x_2, x_3, \ldots, x_n$ or $\underline{x}$, which can be derived from

$$h(x_{n+1}|\underline{x})=\int_{\delta} f\left(x_{n+1}|\delta\right)\cdot h\left(\delta|\underline{x}\right)d\delta. \tag{9}$$

Suppose that $f\left(x_{n+1}|\delta\right)$ is a function of the new observed data [8].

In this paper, we propose the use of Empirical Bayes prediction in sequential sampling plan (EB in SSP) in case of unknown mean μ and unknown variance σ^2. The parameter estimators are determined by squared error loss (SEL) and precautionary loss (PL) functions. Data are assumed: $X\sim N(\mu,\sigma^2)$, informative priors: $\mu\sim N(\theta,\tau^2)$, $\sigma^2\sim IG(a,b)$ where μ and σ^2 are parameters, θ,τ^2,a and b are hyper-parameters which can be estimated from marginal distribution function as follows.

A. THE MARGINAL LIKELIHOOD DISTRIBUTION FUNCTION

$$\begin{aligned}
&M\left(\underline{x}|\theta,\tau^2,a,b\right)\\
&=\int_{\sigma^2}\int_{\mu} f\left(\underline{x}|\mu,\sigma^2\right)\cdot\pi\left(\mu|\theta,\tau^2\right)\pi\left(\sigma^2|a,b\right)d\mu d\sigma^2\\
&=\int_0^{\infty}\int_{-\infty}^{\infty}\frac{1}{\left(2\pi\sigma^2\right)^{n/2}}e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}\times\frac{1}{\left(2\pi\tau^2\right)^{1/2}}e^{-\frac{1}{2\tau^2}(\mu-\theta)^2}\\
&\quad\times\frac{b^a}{\Gamma(a)}\left(\sigma^2\right)^{-(a+1)}e^{-\frac{b}{\sigma^2}}d\mu d\sigma^2\\
&\propto\int_0^{\infty}\frac{1}{(n\tau^2+\sigma^2)^{1/2}}\left(\sigma^2\right)^{-\left(a+\frac{n-1}{2}+1\right)}\\
&\quad\times e^{-\frac{1}{2(n\tau^2+\sigma^2)}\left[\sum_{i=1}^{n}(x_i-\theta)^2-\frac{n\tau^2}{\sigma_0^2}\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)\right]-\frac{b}{\sigma^2}}d\sigma^2.
\end{aligned}\tag{10}$$

It can see that the marginal distribution function does not have a closed form, the hyper-parameters thus cannot be estimated directly by classical method, such as maximum likelihood method (ML). Alternatively, the hyper-parameters θ,τ^2,a and b are determined using Gibbs sampler [30]. After that, the estimators θ,τ^2,a and b will be substituted into the posterior distribution function.

B. THE POSTERIOR DISTRIBUTION FUNCTION OF μ AND σ^2

The posterior distribution function of μ and σ^2 can be shown by

$$h\left(\mu,\sigma^2|\underline{x}\right)\propto L\left(\mu,\sigma^2|\underline{x}\right)\cdot\pi\left(\mu|\theta,\tau^2\right)\pi\left(\sigma^2|a,b\right)$$

Then,

$$\begin{aligned}
&h\left(\mu,\sigma^2|\underline{x}\right)\\
&\propto\frac{1}{\left(2\pi\sigma^2\right)^{n/2}}e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}\times\frac{1}{\left(2\pi\hat{\tau}^2\right)^{1/2}}e^{-\frac{1}{2\hat{\tau}^2}(\mu-\hat{\theta})^2}\\
&\quad\times\frac{\hat{b}^{\hat{a}}}{\Gamma(\hat{a})}\left(\sigma^2\right)^{-(\hat{a}+1)}e^{-\frac{\hat{b}}{\sigma^2}}\\
&\propto\left(\sigma^2\right)^{-\left(\hat{a}+\frac{n}{2}+1\right)}e^{-\frac{1}{2\sigma^2\hat{\tau}^2}\left[(n\hat{\tau}^2+\sigma^2)[\mu-\mu_n]^2\right]}\\
&\quad\times e^{-\frac{1}{2\sigma^2\hat{\tau}^2}\left[\left[(n-1)s^2+(2\hat{b}+n\bar{x}^2)\right]\hat{\tau}^2+\hat{\theta}^2\sigma^2-\frac{\left(n\bar{x}\hat{\tau}^2+\hat{\theta}\sigma^2\right)^2}{\left(n\hat{\tau}^2+\sigma^2\right)}\right]}.
\end{aligned}\tag{11}$$

Consider the following term

$$\begin{aligned}
&n\tau^2(\mu-\bar{x})^2+\sigma^2(\mu-\hat{\theta})^2\\
&\quad=\left(n\tau^2+\sigma^2\right)[\mu-\mu_n]^2-\frac{\left(n\bar{x}\tau^2+\hat{\theta}\sigma^2\right)^2}{\left(n\tau^2+\sigma^2\right)}\\
&\qquad+\hat{\theta}^2\sigma^2+n\tau^2\bar{x}^2,
\end{aligned}$$

$\mu_n=\frac{n\bar{x}\hat{\tau}^2+\hat{\theta}\sigma^2}{n\hat{\tau}^2+\sigma^2}$, $s^2=\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i-\bar{x})^2$ and $\sum_{i=1}^{n}(x_i-\mu)^2=s^2(n-1)+n(\mu-\bar{x})^2$.

Thus, the joint posterior distribution functions of μ and σ^2 do not have a closed form, which can be obtained using Gibbs sampler.

C. THE EB ESTIMATOR OF μ AND σ^2 WITH RESPECT TO THE SEL FUNCTION

The SEL function format is given by

$$L\left(t;\mu,\sigma^2\right)=\left[\left(\mu,\sigma^2\right)-t\right]^2, \tag{12}$$

where t is estimated value of parameters of μ and σ^2. Thus, the EB estimator of μ and σ^2 are the mean of the posterior distribution [20] which can determine by

$$\begin{aligned}
\hat{\mu}_{SEL}&=E\left(\mu|\underline{x}\right),\\
\hat{\sigma}^2_{SEL}&=E\left(\sigma^2|\underline{x}\right).
\end{aligned}$$

In generally, the posterior expectation is provided by

$$E\left[u\left(\mu,\sigma^2\right)|\underline{x}\right]=\int_0^{\infty}\int_{-\infty}^{\infty}u\left(\mu,\sigma^2\right)\cdot h\left(\mu,\sigma^2|\underline{x}\right)d\mu d\sigma^2, \tag{13}$$

where $u\left(\mu,\sigma^2\right)$ is any function for μ and σ^2 [18]. We can use Lindley's approximation procedure to estimate the parameters. The estimators μ and σ^2 can be obtained from the posterior expectation by Lindley's approximation [31] for two parameters as follows.

$$E\left(\mu,\sigma^2|\underline{x}\right) = u + \frac{1}{2}\left(u_{11}\sigma_{11} + u_{22}\sigma_{22}\right) + P_1u_1\sigma_{11} + P_2u_2\sigma_{22} + \frac{1}{2}\left[\sigma_{11}\sigma_{22}\left(u_1L_{12} + u_2L_{21}\right) + u_1\sigma_{11}^2L_{30} + u_2\sigma_{22}^2L_{03}\right]. \tag{14}$$

1) OBTAIN THE EB ESTIMATOR OF μ WITH RESPECT TO THE SEL FUNCTION

Let $u\left(\mu,\sigma^2\right)$ then

$$u_1 = = \frac{\partial u\left(\mu,\sigma^2\right)}{\partial\mu} = 1, \quad u_{11} = \frac{\partial^2 u\left(\mu,\sigma^2\right)}{\partial\mu^2} = 0,$$
$$u_2 = \frac{\partial u\left(\mu,\sigma^2\right)}{\partial\sigma^2} = 0, \quad u_{22} = \frac{\partial^2 u\left(\mu,\sigma^2\right)}{\partial\left(\sigma^2\right)^2} = 0.$$

The likelihood function and prior distribution of μ *and* σ^2 are written as follows:

$$\pi\left(\mu,\sigma^2\right) = \frac{1}{\left(2\pi\hat{\tau}^2\right)^{1/2}}e^{-\frac{1}{2\hat{\tau}^2}(\mu-\hat{\theta})^2}\cdot\frac{\hat{b}^{\hat{a}}}{\Gamma(\hat{a})}\left(\sigma^2\right)^{-(\hat{a}+1)}e^{-\frac{\hat{b}}{\sigma^2}},$$
$$L\left(\mu,\sigma^2|\underline{x}\right) = \frac{1}{\left(2\pi\sigma^2\right)^{n/2}}e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}.$$

where

$$P = \ln\pi\left(\mu,\sigma^2\right) = \hat{a}\ln\hat{b} - \ln\Gamma\left(\hat{a}\right) - \left(\hat{a}+1\right)\ln\sigma^2 - \frac{\hat{b}}{\sigma^2} - \frac{1}{2}\ln\left(2\pi\hat{\tau}^2\right) - \frac{1}{2\hat{\tau}^2}\left(\mu-\hat{\theta}\right)^2,$$
$$P_1 = \frac{\partial P}{\partial\mu} = -\frac{1}{\hat{\tau}^2}(\mu-\hat{\theta}), \quad P_2 = \frac{\partial P}{\partial\sigma^2} = -\frac{(\hat{a}+1)}{\sigma^2} + \frac{\hat{b}}{\sigma^4}$$
$$\ln L = -\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2,$$
$$L_{20} = \frac{\partial^2\ln L}{\partial\mu^2} = -\frac{n}{\sigma^2}$$
$$L_{02} = \frac{\partial^2\ln L}{\partial\left(\sigma^2\right)^2} = \frac{n}{2\sigma^4} - \frac{1}{\sigma^6}\sum_{i=1}^{n}(x_i-\mu)^2,$$
$$L_{30} = \frac{\partial^3\ln L}{\partial\mu^3} = 0$$
$$L_{03} = \frac{\partial^3\ln L}{\partial\left(\sigma^2\right)^3} = -\frac{n}{2\sigma^6} + \frac{3}{\sigma^8}\sum_{i=1}^{n}(x_i-\mu)^2,$$
$$L_{21} = \frac{\partial^3\ln L}{\partial\mu^2\partial\sigma^2} = \frac{n}{2\sigma^4}$$
$$L_{12} = \frac{\partial^3\ln L}{\partial\mu\partial\left(\sigma^2\right)^2} = \frac{2}{\sigma^6}\sum_{i=1}^{n}(x_i-\mu)^2,$$
$$\sigma_{11} = -\frac{1}{L_{20}} \text{ and } \sigma_{22} = -\frac{1}{L_{02}}$$

From equation (14), it can see that the estimator of μ with respect to the SEL reduce to

$$E\left(\mu|\underline{x}\right) = u + P_1u_1\sigma_{11} + \frac{1}{2}\left(\sigma_{11}\sigma_{22}u_1L_{12} + u_1\sigma_{11}^2L_{30}\right)$$

Then,

$$E\left(\mu|\underline{x}\right) = \hat{\mu} - \frac{\left(\hat{\mu}-\hat{\theta}\right)\hat{\sigma}^2}{n\hat{\tau}^2} - \frac{2\hat{\sigma}^2}{\left[n^2\hat{\sigma}^2 - 2n\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)\right]}. \tag{15}$$

Therefore, the EB estimator of μ for SEL can be provided by

$$\hat{\mu}_{SEL} = \hat{\mu} - \frac{(\hat{\mu}-\hat{\theta})\hat{\sigma}^2}{n\hat{\tau}^2} - \frac{2\hat{\sigma}^2}{\left[n^2\hat{\sigma}^2 - 2n\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)\right]} \tag{16}$$

where $\hat{\mu} = \bar{x}$ and $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{1}{n}\left(\sum_{i=1}^{n}x^2 - n\bar{x}^2\right)$ are ML estimators.

2) SPECIFY THE EB ESTIMATOR OF σ^2 WITH RESPECT TO THE SEL FUNCTION

Let $u\left(\mu,\sigma^2\right) = \sigma^2$ then the EB estimator of σ^2 with respect to the SEL reduce to

$$E\left(\sigma^2|\underline{x}\right) = u + P_2u_2\sigma_{22} + \frac{1}{2}\left(\sigma_{11}\sigma_{22}u_2L_{21} + u_2\sigma_{22}^2L_{03}\right) = \hat{\sigma}^2 - \left(\frac{\hat{\sigma}^4}{D}\right)\left\{1 - 2(\hat{a}+1) + \frac{2\hat{b}}{\hat{\sigma}^2} - \frac{n}{\hat{\sigma}^4} + \frac{3\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)}{\hat{\sigma}^6}\right\} \tag{17}$$

Therefore, the EB estimator of σ^2 for SEL function is provided by

$$\hat{\sigma}^2_{SEL} = \hat{\sigma}^2 - \left(\frac{\hat{\sigma}^4}{D}\right)\left\{1 - 2(\hat{a}+1) + \frac{2\hat{b}}{\hat{\sigma}^2} - \frac{n}{\hat{\sigma}^4} + \frac{3\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)}{\hat{\sigma}^6}\right\} \tag{18}$$

where

$$u_1 = \frac{\partial u\left(\mu,\sigma^2\right)}{\partial\mu} = 0, \quad u_{11} = \frac{\partial^2 u\left(\mu,\sigma^2\right)}{\partial\mu^2} = 0,$$
$$u_2 = \frac{\partial u\left(\mu,\sigma^2\right)}{\partial\sigma^2} = 1,$$
$$u_{22} = \frac{\partial^2 u\left(\mu,\sigma^2\right)}{\partial\left(\sigma^2\right)^2} = 0, \quad D = n\hat{\sigma}^2 - 2\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right).$$

After that, the estimators $\hat{\mu}_{SEL}$ and $\hat{\sigma}^2_{SEL}$ will be replaced into the posterior predictive distribution function.

D. THE EB ESTIMATOR OF μ AND σ^2 WITH RESPECT TO THE PL FUNCTION

The PL form is written as

$$L\left(t;\mu,\sigma^2\right)=\frac{\left[\left(\mu,\sigma^2\right)-t\right]^2}{t}. \quad (19)$$

The EB estimator of μ *and* σ^2 for PL [20] is determined by

$$\hat{\mu}_{PL}=\sqrt{E\left(\mu^2|\underline{x}\right)},$$
$$\hat{\sigma}^2_{PL}=\sqrt{E\left[\left(\sigma^2\right)^2|\underline{x}\right]}.$$

1) OBTAIN THE EB ESTIMATOR OF μ WITH RESPECT TO THE PL FUNCTION

Let $u\left(\mu,\sigma^2\right)=\mu^2$, the EB estimator of μ with respect to the PL function can be reduce as follows.

$$E\left(\mu^2|\underline{x}\right)$$
$$=u+\frac{1}{2}u_{11}\sigma_{11}+P_1u_1\sigma_{11}+\frac{1}{2}\left(\sigma_{11}\sigma_{22}u_1L_{12}+u_1\sigma_{11}^2L_{30}\right)$$
$$=\hat{\mu}^2+\frac{\hat{\mu}\hat{\sigma}^2}{n}\left[1-\frac{2\left(\hat{\mu}-\hat{\theta}\right)}{\hat{\tau}^2}-\frac{4\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)}{D}\right]. \quad (20)$$

Therefore, the EB estimator of μ for PL function is expressed as

$$\hat{\mu}_{PL}=\sqrt{\hat{\mu}^2+\frac{\hat{\mu}\hat{\sigma}^2}{n}\left[1-\frac{2\left(\hat{\mu}-\hat{\theta}\right)}{\hat{\tau}^2}-\frac{4\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)}{D}\right]} \quad (21)$$

where

$$u_1=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\mu}=2\mu,\quad u_{11}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\mu^2}=2,$$
$$u_2=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\sigma^2}=0,\quad u_{22}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\left(\sigma^2\right)^2}=0.$$

2) SPECIFY THE EB ESTIMATOR OF σ^2 WITH RESPECT TO THE PL FUNCTION

Let $u\left(\mu,\sigma^2\right)=\left(\sigma^2\right)^2$, the EB estimator of σ^2 with respect to the PL function can calculate from

$$E\left[\left(\sigma^2\right)^2|\underline{x}\right]$$
$$=u+\frac{1}{2}u_{22}\sigma_{22}+P_2u_2\sigma_{22}+\frac{1}{2}\left(\sigma_{11}\sigma_{22}u_2L_{21}+u_2\sigma_{22}^2L_{03}\right)$$
$$=\hat{\sigma}^4-\left(\frac{2\hat{\sigma}^6}{D}\right)\left\{1+\hat{\sigma}^2-2\left(\hat{a}+1\right)+\frac{2\hat{b}}{\hat{\sigma}^2}+\left(\frac{2\hat{\sigma}^6}{D}\right)\right.$$
$$\left.\times\left[\frac{3\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)}{\hat{\sigma}^6}-\frac{n}{\hat{\sigma}^4}\right]\right\}. \quad (22)$$

Therefore, the EB estimator of σ^2 for PL function is shown by (23), as shown at the bottom of the next page where

$$u_1=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\mu}=0,\quad u_{11}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\mu^2}=0,$$
$$u_2=\frac{\partial u\left(\mu,\sigma^2\right)}{\partial\sigma^2}=2\sigma^2,\quad u_{22}=\frac{\partial^2u\left(\mu,\sigma^2\right)}{\partial\left(\sigma^2\right)^2}=2.$$

After that, the estimators $\hat{\mu}_{PL}$ and $\hat{\sigma}^2_{PL}$ will be replaced into the posterior predictive distribution function.

E. THE POSTERIOR PREDICTIVE DISTRIBUTION FUNCTION

The posterior predictive distribution function of $x_{n+1}|\underline{x}$, which can be derived as

$$h\left(x_{n+1}|\underline{x}\right)$$
$$=\int_0^\infty\int_{-\infty}^\infty f\left(x_{n+1}|\mu,\sigma^2\right)\cdot h\left(\mu,\sigma^2|\underline{x}\right)d\mu d\sigma^2$$
$$=\int_0^\infty\int_{-\infty}^\infty\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}\left(x_{n+1}-\mu\right)^2}$$
$$\times\frac{\hat{b}^{\hat{a}}}{\Gamma\left(\hat{a}\right)\left(2\pi\right)^{n/2}\left(2\pi\hat{\tau}^2\right)^{1/2}}\left(\sigma^2\right)^{-\left(\hat{a}+\frac{n}{2}+1\right)}$$
$$\times e^{-\frac{1}{2\sigma^2\hat{\tau}^2}\left[\hat{\tau}^2s^2(n-1)+2\hat{b}\hat{\tau}^2+n\hat{\tau}^2\bar{x}^2+\hat{\theta}^2\sigma^2\right]}$$
$$\times e^{-\frac{1}{2\sigma^2\hat{\tau}^2}\left[\left(n\hat{\tau}^2+\sigma^2\right)\left[\mu-\mu_n\right]^2-\frac{\left(n\bar{x}\hat{\tau}^2+\hat{\theta}\sigma^2\right)^2}{\left(n\hat{\tau}^2+\sigma^2\right)}\right]}d\mu d\sigma^2$$
$$\propto\int_0^\infty\frac{\left(\sigma^2\right)^{-\left(\hat{a}+\frac{n}{2}+1\right)}}{\left[(n+1)\hat{\tau}^2+\sigma^2\right]^{1/2}}\cdot$$
$$\times e^{-\frac{1}{2\sigma^2}\left(s^2(n-1)+2\hat{b}+n\bar{x}^2+\mu^2-\frac{\left(x_{n+1}+n\mu_n\hat{\tau}^2+\mu_n\sigma^2\right)^2}{(n+1)\hat{\tau}^4+\sigma^2\hat{\tau}^2}\right)}d\sigma^2. \quad (24)$$

where $\mu_n=\frac{n\bar{x}\hat{\tau}^2+\hat{\theta}\sigma^2}{n\hat{\tau}^2+\sigma^2}$. Similarly, the posterior predictive distribution function $x_{n+1}|\underline{x}$ does not have a closed form which can be generated from the conditional posterior distribution via Gibbs sampling technique, that is $f\left(x_{n+1}|\mu,\sigma^2,\underline{x}\right)=f\left(x_{n+1}|\mu,\sigma^2\right)$ [32].

VI. GIBBS SAMPLING PROCEDURES

1) Draw $x_{n+1}^{(t+1)}$ from $x_{n+1}^{(t+1)}|\underline{x}\sim N(\mu,\sigma^2)$ where $\mu\sim N(\hat{\theta},\hat{\tau}^2)$, $\sigma^2\sim IG(\hat{a},\hat{b})$ and $\underline{x}=(x_1,\ldots,x_n)$.

2) Draw $\theta^{(t+1)}$ from $\theta^{(t+1)}|\tau^{2(t)}, a^{(t)}, b^{(t)}, \underline{x} \sim \hat{f}\left(\theta|\tau^2, a, b, \underline{x}\right)$, where $\hat{f}\left(\theta|\tau^2, a, b, \underline{x}\right) = \frac{1}{M}\sum_{t=1}^{M} h\left(\theta|\tau^{2(t)}, a^{(t)}, b^{(t)}, \underline{x}\right)$ that is to estimate $h\left(\theta|\tau^2, a, b, \underline{x}\right)$.
3) Draw $\tau^{2(t+1)}$ from $\tau^{2(t+1)}|\theta^{(t+1)}, a^{(t)}, b^{(t)}, \underline{x} \sim \hat{f}\left(\tau^2|\theta, a, b, \underline{x}\right)$, where $\hat{f}\left(\tau^2|\theta, a, b, \underline{x}\right) = \frac{1}{M}\sum_{t=1}^{M} h\left(\tau^2|\theta^{(t+1)}, a^{(t)}, b^{(t)}, \underline{x}\right)$ that is to estimate $h\left(\tau^2|\theta, a, b, \underline{x}\right)$.
4) Draw $a^{(t+1)}$ from $a^{(t+1)}|\theta^{(t+1)}, \tau^{2(t+1)}, b^{(t)}, \underline{x} \sim \hat{f}\left(a|\theta, \tau^2, b, \underline{x}\right)$, where $\hat{f}\left(a|\theta, \tau^2, b, \underline{x}\right) = \frac{1}{M}\sum_{t=1}^{M} h\left(a|\theta^{(t+1)}, \tau^{2(t+1)}, b^{(t)}, \underline{x}\right)$ that is to estimate $h\left(a|\theta, \tau^2, b, \underline{x}\right)$.
5) Draw $b^{(t+1)}$ from $b^{(t+1)}|\theta^{(t+1)}, \tau^{2(t+1)}, a^{(t+1)}, \underline{x} \sim \hat{f}\left(b|\theta, \tau^2, a, \underline{x}\right)$, where $\hat{f}\left(b|\theta, \tau^2, a, \underline{x}\right) = \frac{1}{M}\sum_{t=1}^{M} h\left(b|\theta^{(t+1)}, \tau^{2(t+1)}, a^{(t+1)}, \underline{x}\right)$ that is to estimate $h\left(b|\theta, \tau^2, a, \underline{x}\right)$.

VII. NUMERICAL AND RESULTS

Data are generated from standard normal distribution. The lot size is specified by N = 1,000, the sample size is defined by $n = 50$, and the number of iterations is given by $t = 1,000$. The producer's risk (α) is 0.05 and the consumer's risk (β) is 0.10. The proportion of defective units is determined by a six-sigma process level in which the proportion of defective units at APL is $p = 0.00034$. In this study, we consider the EB in SSP in case of unknown mean μ and unknown variance σ^2, assuming informative priors on μ and σ^2: $\mu \sim N(\theta, \tau^2), \sigma^2 \sim IG(a, b)$. The estimation of the parameters μ and σ^2 are determined from SEL and PL functions. The hyper-parameters θ, τ^2, a and b can be obtained using Gibbs sampling technique. The proposed plans are compared with traditional approaches according the single sampling plan and the sequential sampling plan by variables (SSP by variables). The P_a and ASN are considered as the criteria for comparison. The result of simulation can be illustrated in Table 1 and 2.

The simulation result for variables process mean testing under $H_1 : \mu_2 > \mu_1$ provide that $\mu_1 = 1.50$ and $\mu_2 = 1.92$ which is determined from referring to (1). Results for single sampling plan can be expressed by the sample size (n) and acceptance limits K, which are $n = 50$ and K = 1.6857. Also, the decision-making process for this plan is to accept the lot if $\bar{x} \geq 1.6857$ or to reject the lot if $\bar{x} < 1.6857$. Figure 1 to 3 shows that are compared with the SSP by variables, EB in SSP with SEL and PL functions, respectively. The cumulative of sample mean of the SSP by variables is compared with Y_1 and Y_2 where the intercept of acceptance line is $h_1 = 5.4387$ and the intercept of rejection line is $h_2 = 6.9826$. For EB in SSP with SEL and PL, the cumulative of the mean of $E(x_{n+1}|\underline{x})$ are compared with Y_1 and Y_2 where the intercept of acceptance line and rejection line of the EB in SSP for SEL and PL are $h_1 = 7.8318$, $h_2 = 10.0549$, and $h_1 = 8.4980$ and $h_2 = 10.9104$, respectively. It can see that the SSP by

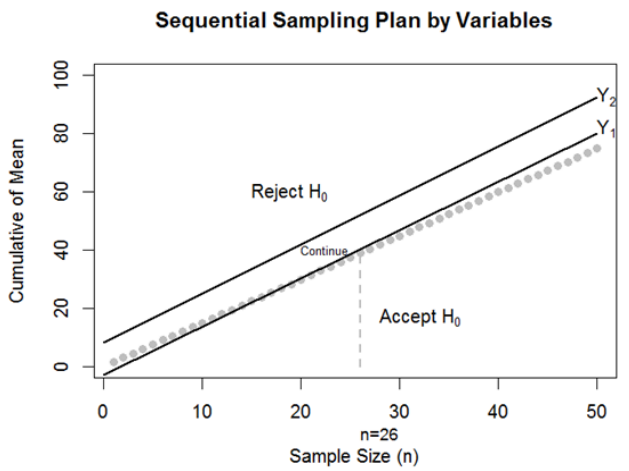

FIGURE 1. The cumulative of sample mean with the SSP by variables versus sample size (n).

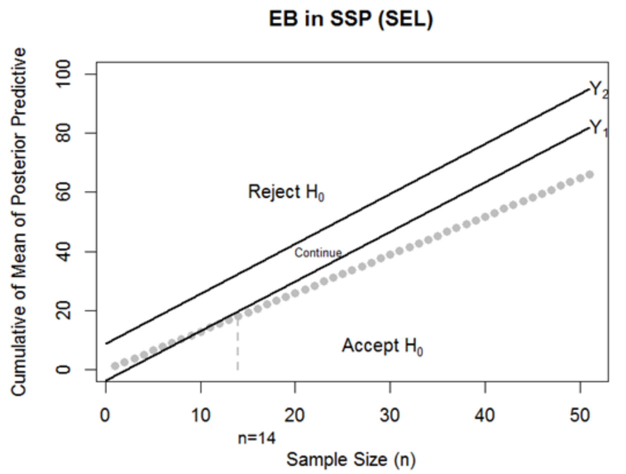

FIGURE 2. The cumulative of $E(x_{n+1}|\underline{x})$ with EB in SSP for SEL at various sample sizes.

$$\hat{\sigma}^2_{PL} = \sqrt{\hat{\sigma}^4 - \left(\frac{2\hat{\sigma}^6}{D}\right)\left\{1 + \hat{\sigma}^2 - 2\left(\hat{a}+1\right) + \frac{2\hat{b}}{\hat{\sigma}^2} + \left(\frac{2\hat{\sigma}^6}{D}\right)\left[\frac{3\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}{\hat{\sigma}^6} - \frac{n}{\hat{\sigma}^4}\right]\right\}}. \quad (23)$$

TABLE 1. Comparison P_a of single SP, SSP by variables, EB in SSP for SEL and PL.

P_a			
Single SP	SSP	EB in SSP (SEL)	EB in SSP (PL)
0.0674	0.9662	0.9971*	0.9969*
0.0805	0.9598	0.9970*	0.9968*
0.0906	0.9548	0.9970*	0.9968*
0.2056	0.8883	0.9971*	0.9969*
0.0865	0.9568	0.9972*	0.9970*
0.1098	0.9448	0.9970*	0.9968*
0.0709	0.9645	0.9970*	0.9968*
0.0589	0.9703	0.9973*	0.9971*
0.1484	0.9235	0.9967*	0.9965*
0.0716	0.9642	0.9971*	0.9969*
0.1175	0.9407	0.9968*	0.9966*
0.0698	0.9651	0.9973*	0.9971*
0.0452	0.9767	0.9970*	0.9969*
0.1877	0.8998	0.9970*	0.9968*
0.0958	0.9521	0.9972*	0.9970*
0.0520	0.9735	0.9969*	0.9968*
0.0829	0.9586	0.9969*	0.9967*
0.0613	0.9691	0.9971*	0.9969*
0.1119	0.9437	0.9969*	0.9967*
0.0848	0.9577	0.9968*	0.9966*
0.1125	0.9434	0.9972*	0.9970*
0.0913	0.9544	0.9972*	0.9970*
0.0830	0.9586	0.9971*	0.9969*
0.2107	0.8849	0.9969*	0.9967*
0.1070	0.9463	0.9970*	0.9968*
0.1034	0.9482	0.9971*	0.9969*
0.1214	0.9386	0.9969*	0.9967*
0.1621	0.9154	0.9971*	0.9970*
0.1106	0.9444	0.9969*	0.9968*
0.0494	0.9748	0.9970*	0.9968*
0.0755	0.9623	0.9972*	0.9970*
0.1346	0.9313	0.9972*	0.9970*
0.0724	0.9638	0.9970*	0.9968*
0.1053	0.9472	0.9970*	0.9968*
0.1421	0.9271	0.9970*	0.9968*
0.0727	0.9637	0.9971*	0.9969*
0.1506	0.9222	0.9969*	0.9967*
0.1050	0.9473	0.9971*	0.9969*
0.0993	0.9503	0.9970*	0.9968*
0.0915	0.9543	0.9972*	0.9971*
0.1584	0.9177	0.9971*	0.9969*
0.0794	0.9604	0.9972*	0.9970*
0.0890	0.9556	0.9972*	0.9970*
0.0627	0.9685	0.9970*	0.9968*
0.0988	0.9506	0.9970*	0.9968*
0.0905	0.9548	0.9970*	0.9968*
0.0946	0.9527	0.9972*	0.9970*
0.0927	0.9537	0.9970*	0.9968*
0.0941	0.9530	0.9971*	0.9968*
0.1883	0.8994	0.9969*	0.9967*
0.0999	-	0.9968*	0.9967*

Note : * is the estimated value difference at the 5th decimals position.

TABLE 2. Comparison ASN of single SP, SSP by variables, EB in SSP for SEL and PL.

ASN			
Single SP	SSP	EB in SSP (SEL)	EB in SSP (PL)
50	21.1673	12.9083	13.0517
50	22.0605	12.9836	13.0954
50	22.7102	12.9861	13.1219
50	28.5539	12.8893	13.0236
50	22.4496	12.8342	12.9705
50	23.8553	12.9746	13.1128
50	21.4148	12.9460	13.0976
50	20.5323	12.7495	12.9031
50	25.9091	13.2061	13.3501
50	21.4582	12.9359	13.0375
50	24.2880	13.1353	13.2502
50	21.3328	12.7518	12.9048
50	19.4190	12.9788	13.0941
50	27.7680	13.0278	13.1432
50	23.0272	12.8843	13.0052
50	19.9923	13.0185	13.1460
50	22.2191	13.0676	13.2008
50	20.7170	12.8990	13.0350
50	23.9723	13.0650	13.2175
50	22.3429	13.1324	13.2620
50	24.0095	12.8559	12.9981
50	22.7502	12.8512	12.9992
50	22.2226	12.9391	13.0763
50	28.7711	13.0212	13.1680
50	23.6941	13.0054	13.1246
50	23.4808	12.9269	13.0344
50	24.5044	13.0768	13.1634
50	26.5839	12.8555	12.9982
50	23.8986	13.0375	13.1445
50	19.7757	12.9665	13.0900
50	21.7248	12.8381	12.9613
50	25.2059	12.8513	12.9723
50	21.5153	12.9751	13.1208
50	23.5936	12.9464	13.0908
50	25.5931	13.0123	13.1525
50	21.5356	12.9255	13.0184
50	26.0231	13.0749	13.2105
50	23.5789	12.9306	13.0756
50	23.2380	12.9713	13.1110
50	22.7621	12.8041	12.9218
50	26.4014	12.9273	13.0499
50	21.9923	12.8562	12.9767
50	22.6079	12.8485	12.9807
50	20.8212	12.9566	13.0878
50	23.2112	12.9943	13.1117
50	22.7027	12.9627	13.1179
50	22.9562	12.8589	12.9832
50	22.8379	13.0088	13.1249
50	22.9239	12.9504	13.1064
50	27.7936	13.0405	13.1984
50	-	13.0922	13.1999

Note : * is the estimated value difference at the 5th decimals position.

variables provides the highest sample size for accepting the lot which is $n = 26$. However, EB in SSP with SEL and PL give smaller sample size than single sampling plan and SSP by variables for accepting the lot.

Table 1 shows that the P_a and ASN of EB in SSP with SEL and PL are compared with single sampling plan and SSP by variables. The P_a for SSP by variables and EB in SSP can be calculated from (5). It is apparent that much of the P_a for the single sampling plan falls in the range of 0.0452 and 0.2107 and shows a generally upward trend in cases where the average is high. In the case of the P_a for the SSP by variables, this is much greater than the single sampling plan and lies in the range of 0.8849 to 0.9767. The trend shows a declining tendency when the average is large. However, the P_a of EB

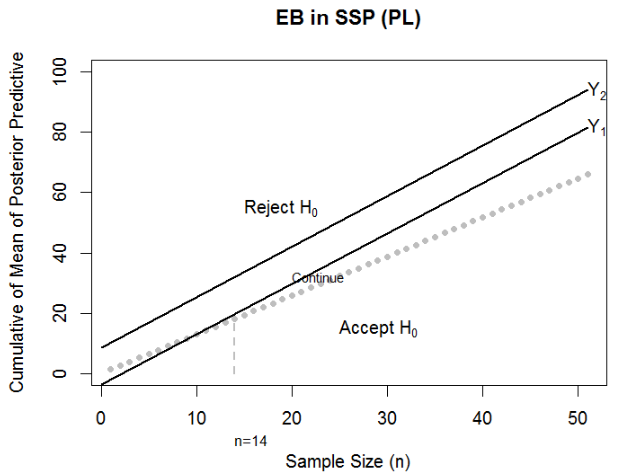

FIGURE 3. The cumulative of $E(x_{n+1}|\underline{x})$ with EB in SSP for PL at various sample sizes.

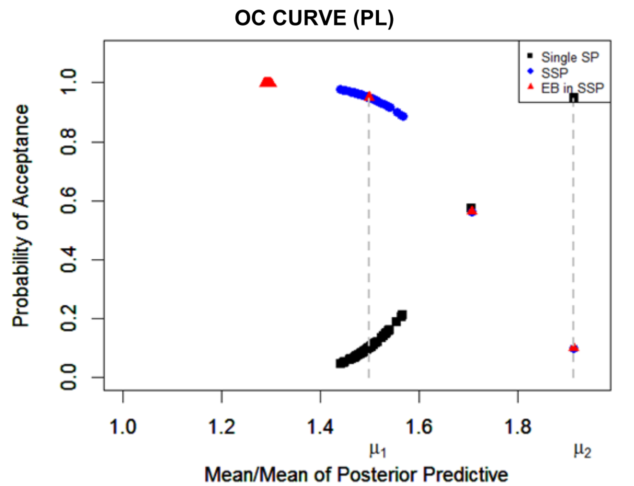

FIGURE 5. The comparison P_a of single SP, SSP by variables and EB in SSP for PL.

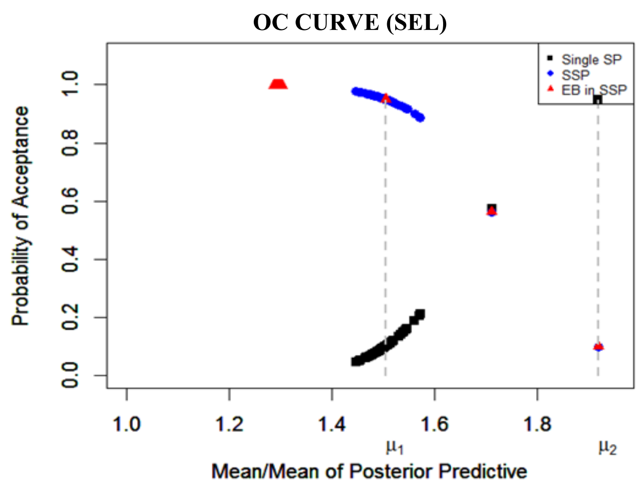

FIGURE 4. The comparison P_a of single SP, SSP by variables and EB in SSP for SEL.

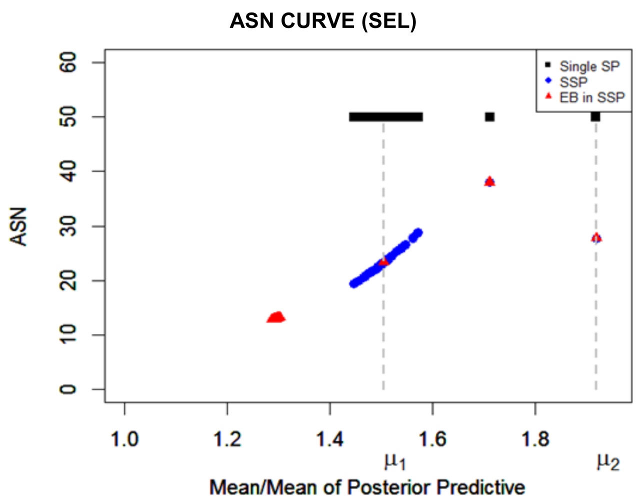

FIGURE 6. The comparison ASN of the single SP, SSP by variables and EB in SSP for SEL.

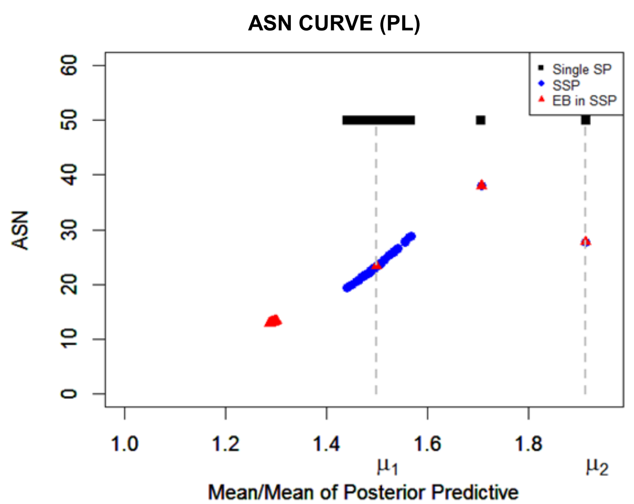

FIGURE 7. The comparison ASN of the single SP, SSP by variables and EB in SSP for PL.

in SSP for SEL and PL yield the highest P_a level at almost 0.99. For these four methods, the OC curves are presented in Figure 4 and Figure 5, respectively. Table 2 illustrates ASN for the single sampling plan amounts to 50 per lot. The ASN for SSP by variables and EB in SSP can be calculated refer to (6), the ASN in the range of 19 to 29 per lot is not as large as the single sampling plan, but if the average begins to increase, the ASN for SSP by variables also trends upwards. The ASN for EB in SSP with SEL and PL are the smallest, from 12 to 14 per lot which was clearly the lowest among all types. It can see in Figure 6 and Figure 7.

VIII. AN EXAMPLE

The proposed method is applied to real data, Flash Memory 128-bit EEPROM chip, as studied by Ntzoufras [33]. The data considered of $n = 142$, where USL $= 5$ μA, $\alpha = 0.05$ and $\beta = 0.10$.

4.140, 3.914, 3.993, 3.390, 3.200, 4.201, 4.066, 4.049, 4.210, 4.247, 4.106, 4.650, 3.470, 4.420, 4.216, 3.746, 4.590,

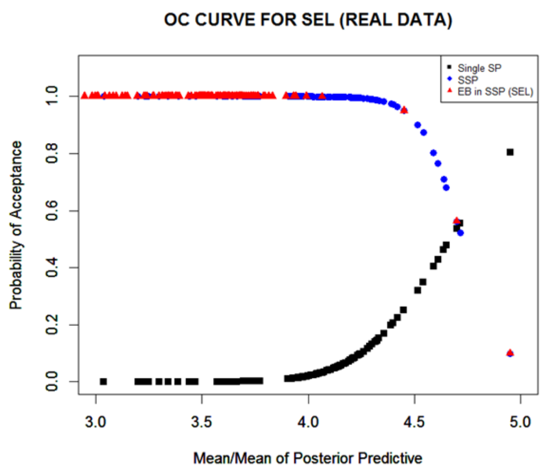

FIGURE 8. The comparison P_a of single SP, SSP by variables and EB in SSP for SEL in real data.

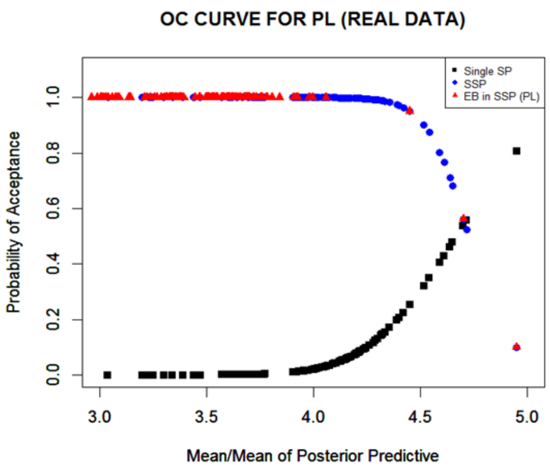

FIGURE 9. The comparison P_a of single SP, SSP by variables and EB in SSP for PL in real data.

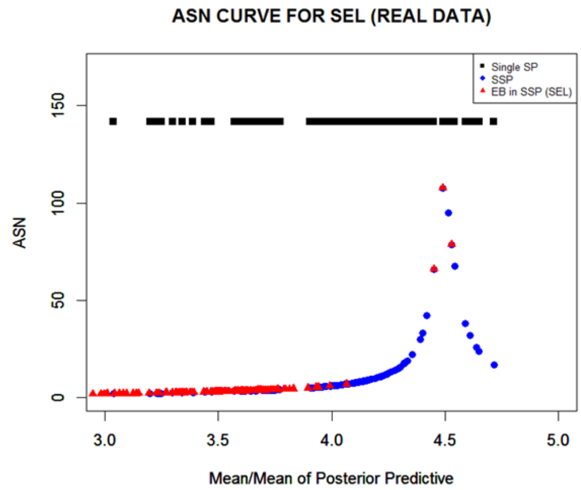

FIGURE 10. The comparison ASN of the single SP, SSP by variables and EB in SSP for SEL in real data.

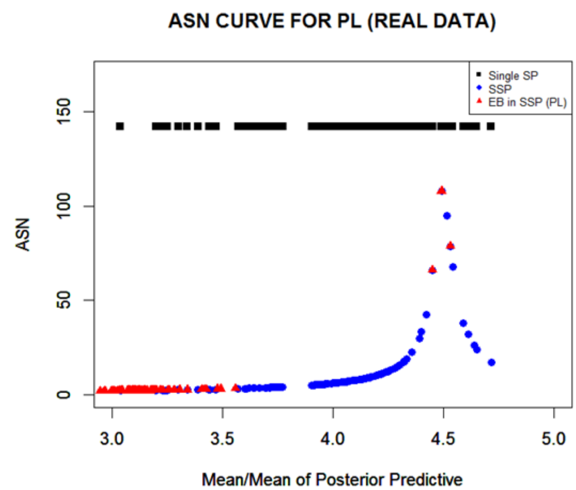

FIGURE 11. The comparison ASN of the single SP, SSP by variables and EB in SSP for PL in real data.

3.945, 3.390, 3.342, 4.175, 4.100, 3.644, 3.946, 4.090, 3.696, 3.729, 4.024, 3.975, 3.720, 4.211, 3.440, 3.931, 4.091, 4.057, 3.761, 3.965, 3.976, 3.940, 4.154, 4.156, 4.316, 3.700, 3.917, 3.953, 4.145, 3.910, 4.000, 4.040, 4.170, 4.042, 3.906, 4.260, 4.241, 4.153, 3.620, 4.139, 3.200, 3.240, 3.752, 4.610, 4.020, 3.571, 4.015, 3.300, 3.230, 4.233, 3.905, 4.290, 3.761, 4.059, 4.333, 3.921, 3.300, 3.250, 4.040, 4.715, 4.123, 3.640, 4.103, 3.957, 4.400, 3.717, 3.921, 4.515, 3.666, 3.740, 3.695, 4.146, 4.025, 3.740, 4.100, 4.320, 4.127, 3.740, 4.191, 4.120, 4.045, 4.220, 3.730, 4.245, 4.279, 4.301, 3.713, 4.046, 3.619, 4.356, 3.250, 3.763, 3.610, 4.130, 4.075, 3.040, 3.700, 3.960, 3.943, 4.637, 3.745, 4.199, 4.139, 3.730, 4.390, 3.442, 3.965, 4.025, 4.166, 4.123, 3.955, 3.773, 4.060, 4.191, 3.950, 3.994, 4.005, 4.541, 4.147, 3.767, 3.970, 3.770, 4.324, 3.600, 4.140.

The sample mean and SD of this data are 3.96 and 0.33, respectively. Where APL(μ_1) is 4.45 mm, RPL(μ_2) is 4.54. Results show that P_a values of EB in SSP for SEL and PL are mostly higher than single SP and SSP by variables which are about 0.99, which shown in Figure 8 and Figure 9, respectively. In addition, the ASN of the proposed plans provided smaller ASN than classical methods, as displayed in Figure 10 and Figure 11.

IX. CONCLUSION

In this paper, we considered variables process mean testing when data follow normal distribution under situation unknown mean and variance. The lot size is $N = 1,000$, the sample size is $n = 50$, and the number of iterations is $t = 1{,}000$. The proportion of defective units is determined by a six-sigma process level in which the proportion of defective units at APL is $p = 0.00034$, $\alpha = 0.05$ and $\beta = 0.10$. The objective of this study is to use the EB in SSP with SEL and PL functions, and compare with the classical approaches: single sampling plan and SSP by variables. Result shows that the EB in SSP with SEL and PL provide the highest P_a and the smallest the ASN. Therefore, the proposed plans are more efficient than traditional approaches.

The value of P_a and ASN of EB in SSP for SEL and PL are similar because the estimated values of μ and σ^2 under SEL are close to those under PL. In addition, we applied the proposed plans to real data, Flash Memory 128-bit EEPROM chip, which yielded consistent results with those in simulation.

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