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Optimal Mobile Relays Positions and Resource Allocation for Multi-Relay Multi-Destination Wireless Networks

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ABSTRACT Deploying relays is a major means to improve the performance of wireless networks. In this paper, multiple mobile relays carried by smart drones are sent by a source to assist the communication to multiple destinations for the Public Safety Wireless Broadband Network, where the sole source is not sufficient to satisfy the quality of service (QoS) requirement of all destinations. With Rician fading between source and relays, and relays and destinations, we study two related questions: The first is where to deploy the relays for minimizing the maximum outage probability of all destinations; the second is where to deploy the relays and how to allocate the resource for maximizing the minimum throughput of all destinations. Efficient algorithms are proposed to compute the optimal solutions, which are shown to be highly effective via simulations. The maximum outage probability decreases and the minimum throughput increases as the number of relays increases. But beyond a certain number, more relays do not provide further improvement in the minimum throughput. Meanwhile, large Rician factor could decrease the maximum outage probability and increase the minimum throughput, except for the cases with very low transmit power.

INDEX TERMS Relays positions, resource allocation, maximum outage probability, minimum throughput.

I. INTRODUCTION

Relays have been proposed as a major means to improve the performance of wireless networks in Long-Term-Evolution Advanced (LTE-A) and 802.16 standards [1]. The primary purpose is either increasing the data rate or extending the wireless coverage. However, the challenging work about how to optimally deploy the relays subject to the quality of service (QoS) requirement is a NP hard problem [2].

Most literatures have studied the deployment problem, but assuming the coverage of each device is independent. With identical coverage of each device, it was shown in [3] that the triangular deployment pattern on a plane obtains the minimum number of nodes to guarantee full coverage, which is used to deploy the base stations for cellular networks. The optimal deployment patterns for full coverage and k-connectivity were studied in [4], [5], where each nodes is connected with at least k neighboring nodes. But in many situations, deterministic deployment is neither feasible nor practical, and numerical method is another option [6]–[10]. The algorithms update the relays positions, add and reduce relays step by step to achieve full coverage with a minimum number of relays.

In practical relay networks, the coverage of each device is determined by the relays positions, allocated resources and QoS requirement, which should be jointly considered. When the QoS is the outage probability, references [11]–[13] studied the optimal relays positions for minimizing the outage probability. Reference [14] discussed the optimal relays positions for maximizing the coverage subject to the outage probability requirement. When the QoS is the throughput, reference [15] studied the optimal relays positions for maximizing the throughput, but only considering the large scale fading. In [16], we discussed the optimal relays positions for maximizing the coverage subject to the throughput requirement. In [17], we considered the situations where it is impossible or too risky to let human beings deploy relays, so that a mobile relay carried by a smart drone is deployed to assist

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the communication from a source to a destination. An important use case of the results is in the Public Safety Wireless Broadband Network,[1] where deploying relays is often a major means to improve the QoS for search, rescue and reconstruction at incident scenes and disaster areas [18]–[21].

All the previous scenarios consider the communication from a source to a destination, which is assisted by one or multiple relays locating along the line segment connecting the source and the destination. In this paper, we consider the communication from a source to multiple randomly distributed destinations, which is somewhat more general. The sole source is not sufficient to satisfy the QoS requirement of all destinations, so that multiple mobile relays carried by smart drones are deployed to assist the communication. Then there exist one direct link and multiple relay links from the source to each destination, which share the total spectrum. In order to establish reliable relay networks, we optimize the spectrum and power allocation in [22] after the relays are placed. This paper studies where to place the relays along with the resource allocation for minimizing the maximum outage probability or maximizing the minimum throughput of all destinations. The major contributions are summarized as follows:

(1): The Public Safety Wireless Broadband Network should guarantee the successful delivery of short packages regardless of delay, such as rescue and sign of life information. For this scenario, the QoS metric is the outage probability, which is dominated by the relays positions. However, achieving the optimal relays positions for minimizing the maximum outage probability of all destinations is NP hard [2]. After analyzing the outage probability of each destination, an efficient iterative algorithm is proposed to compute the optimal solutions.The maximum outage probability decreases as the number of relays increases.

(2): The Public Safety Wireless Broadband Network should also guarantee the successful delivery of stream packages considering delay, such as voice and video information. For this scenario, the QoS metric is the throughput, which is dominated by the relays positions and resource allocation. However, achieving the optimal relays positions and spectrum allocation for maximizing the minimum throughput of all destinations is somewhat more complicated. After analyzing the throughput of each destination, an efficient greedy algorithm is proposed to compute the optimal solutions. The minimum throughput increases as the number of relays increases. But beyond a certain number, more relays do not provide further improvement.

Meanwhile, carried by drones, there may exist line of sight (LOS) component for the channels between source and relays, and relays and destinations. Hence our analysis is based on the Rician fading channel, and the system performance with different Rician factor is also studied. The maximum outage probability decreases and the minimum throughput increases as the Rician factor increases, when the transmit power is large. However, with very low transmit power, the maximum outage probability may increase and the minimum throughput my decrease as the Rician factor increases.

[1] The Public Safety Wireless Broadband Network, conceived to be a single, connected, universal network for all public safety purposes, is currently being planned and tested in the U.S.

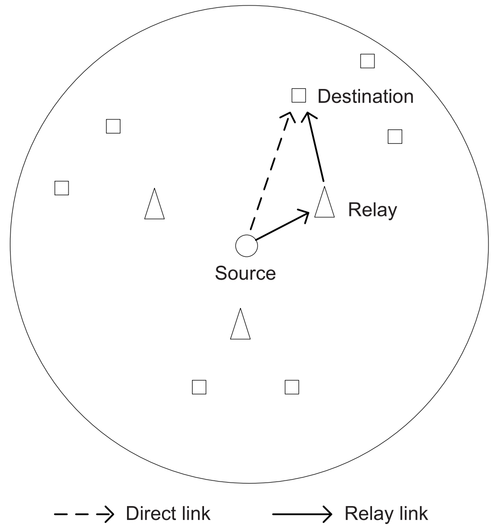

FIGURE 1. Network model with a source, M relays and N destinations.

The rest of this paper is organized as follows. Section II introduces the network model and channel model. Section III studies the optimal relays positions for minimizing the maximum outage probability of all destinations. Section IV studies the optimal relays positions and resource allocation for maximizing the minimum throughput of all destinations. Section V discusses the optimal solutions. Section VI presents numerical simulations and Section VII concludes the paper.

II. SYSTEM MODEL

In this section, we introduce the network model where multiple relays carried by smart drones are sent by a source to assist the communication to multiple destinations. Then the Rician fading model is described, along with the outage probability and the outage capacity.

A. NETWORK MODEL

The network model with a source, M relays and N destinations is shown in Fig. 1. The positions of the source, i-th relay and j-th destination are (x_0, y_0), $(\tilde{x}_i, \tilde{y}_i)$ and (x_j, y_j) for $i \in \{1, \ldots, M\}, j \in \{1, \ldots, N\}$. The total spectrum bandwidth is W.

The source could communicate with the destinations directly or assisted by the relays, while the communication between relays is prohibited. Then there exist one single-hop direct link and M two-hop relay links from the source to each destination. The $N(M+1)$ links share the resource. The relays positions influence the large scale fading of the two-hop relay links. The resource allocation affects the throughput of both the single-hop direct link and the two-hop relay

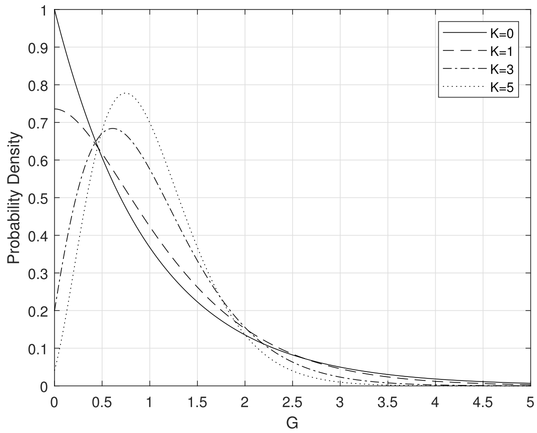

FIGURE 2. The probability density of the power gain of Rician fading channel with unit mean, when the Rician factor is 0, 1, 3 and 5.

links. In this paper, we study the optimal relays positions and resource allocation for minimizing the maximum outage probability or maximizing the minimum throughput of all destinations.

B. CHANNEL MODEL

The propagation path loss is:

$$h = Ad^{-\alpha} \tag{1}$$

where A is a constant value that considers frequency and antenna gain, d is the distance from the transmitter to the receiver, α is the path loss exponent.

With Rician fading channel, the probability density function (PDF) of the power gain G with unit mean is [25]:

$$\Pr(G) = (1+K)e^{-K}\exp\left(-(1+K)G\right) \cdot I_o\left(2\sqrt{K(1+K)G}\right) \tag{2}$$

where $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind, and K is the Rician factor which is the ratio of the power of the LOS component to the power of the non-LOS multipath components. When $K = 0$, the channel reduces to Rayleigh fading channel. When K increases, the probability of G to be allocated close to the mean value increases. With $K = 0, 1, 3, 5$, the probability density of the power gain is shown in Fig.2

At the receiver, in the absence of interference, the signal-to-noise ratio (SNR) is:

$$\text{SNR} = \frac{PAG}{\sigma^2 d^\alpha W} \tag{3}$$

where P is the transmit power, σ^2 is the power spectral density of the white Gaussian noise, and W is the bandwidth.

Let γ be a prescribed SNR threshold, and the corresponding modulation scheme is used at the transmitter [24]. If the SNR at the receiver is equal or greater than the threshold, the signal could be decoded. Otherwise, the signal could not be decoded and an outage event is declared. The outage probability is [25], [26]:

$$\mathcal{P} = \Pr(\text{SNR} < \gamma) = \int_{G=0}^{\frac{\gamma\sigma^2 d^\alpha W}{PA}} \Pr(G)dG = 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(1+K)\gamma\sigma^2 d^\alpha W}{PA}}\right) \tag{4}$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q-function.

Let t be the allocated time. The outage capacity is [27]:

$$\mathcal{C} = tW\log_2(1+\gamma)\Pr(\text{SNR} \geq \gamma) = tW\log_2(1+\gamma)Q_1\left(\sqrt{2K}, \sqrt{\frac{2(1+K)\gamma\sigma^2 d^\alpha W}{PA}}\right) \tag{5}$$

which is the expected throughput.

The resource is shared using time division scheme, where every hop uses the entire frequency band and all hops are orthogonal in time. This scheme is simple and guarantees the robustness much needed by the Public Safety Wireless Broadband Network. Meanwhile, the scheme could be directly implemented in the real system, without the modification of standards and hardware [28]. Understanding of this simple scheme is also the first step toward more sophisticated solutions involving spectrum sharing. The analysis in this paper considers the downlink channel from the source to the destinations, and the uplink channel from the destinations to the source follows the same way.

III. THE OPTIMAL RELAYS POSITIONS FOR MINIMIZING THE MAXIMUM OUTAGE PROBABILITY

When the QoS metric is the outage probability, the optimal relays positions minimize the maximum outage probability of all destinations. For the Public Safety Wireless Broadband Network, this is the scenario to guarantee the successful delivery of short packages regardless of delay, such as rescue and sign of life information.

A. THE OPTIMIZATION PROBLEM

There exist one single-hop direct link and M two-hop relay links from the source to each destination. An outage event is declared when the SNR of all the links fall below the prescribed SNR threshold. Thus the outage probability of the destination is the product of that of the single-hop direct link and M two-hop relay links.

Let P_0 be the transmit power of the source, the outage probability of the single-hop direct link from the source to the j-th destination is:

$$\mathcal{P}_{j0} = \Pr\left(\text{SNR}_{j0} < \gamma\right) = 1 - Q_1\left(\sqrt{2K_{j0}}, \sqrt{\frac{2(1+K_{j0})\gamma\sigma^2 d_{j0}^\alpha W}{P_0 A}}\right) \tag{6}$$

where SNR_{j0}, K_{j0} and d_{j0} are the received SNR, Rician factor and distance of the direct link, i.e.,

$$d_{j0} = \sqrt{(x_0 - x_j)^2 + (y_0 - y_j)^2}. \tag{7}$$

For the two-hop relay link, an outage event is declared when either the received SNR of the link from the source to the relay or that of the link from the relay to the destination falls below the prescribed SNR threshold. Let P_i be the transmit power of the i-th relay, the outage probability of the two-hop relay link from the source to the j-th destination assisted by the i-th relay is:

$$\begin{aligned}\mathcal{P}_{ji0} &= \Pr\left(\min\{\text{SNR}_{ji0,1}, \text{SNR}_{ji0,2}\} < \gamma\right)\\ &= 1 - Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)\\ &\quad \cdot Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)\end{aligned} \tag{8}$$

where $\text{SNR}_{ji0,1}$, K_{i0} and l_{i0} are the received SNR, Rician factor and distance of the first hop from the source to the i-th relay, and $\text{SNR}_{ji0,2}$, K_{ji} and l_{ji} are the received SNR, Rician factor and distance of the second hop from the i-th relay to the j-th destination, i.e.,

$$l_{i0} = \sqrt{(\tilde{x}_i - x_0)^2 + (\tilde{y}_i - y_0)^2} \tag{9a}$$

$$l_{ji} = \sqrt{(\tilde{x}_i - x_j)^2 + (\tilde{y}_i - y_j)^2}. \tag{9b}$$

Then the outage probability of the j-th destination is:

$$\mathcal{P}_j = \mathcal{P}_{j0} \times \prod_{i=1}^{N} \mathcal{P}_{ji0} \tag{10}$$

which is determined by the relays positions and means all the links fail to deliver the packets.

The optimal relays positions minimize the maximum outage probability of all destinations. Assuming $\mathcal{P}_{\max}$ is the maximum outage probability, the optimization problem is:

$$\text{minimize: } \mathcal{P}_{\max} \tag{11a}$$

$$\text{subject to: } \mathcal{P}_{\max} \geq \mathcal{P}_j \quad \forall j \in \{1, \ldots, N\} \tag{11b}$$

$$\text{variables: } (\tilde{x}_i, \tilde{y}_i) \quad \forall i \in \{1, \ldots, M\} \tag{11c}$$

where constraint (11b) means that the maximum outage probability is no less than that of all destinations. The optimization problem (11) is non-convex in general, which has $2M$ variables. Assuming the exhaustive searching method quantizes each variable into L values, the complexity is the comparison of L^{2M} quantized tuples. For a large L, solving the optimization problem lay on exhaustive searching method is challenging. In the following, we study the structure of the optimization problem to develop an efficient greedy algorithm to compute the optimal solutions.

B. THE OPTIMAL SOLUTIONS

For the optimization problem (11), the $2M$ variables should be optimized for minimizing the maximum outage probability of all destinations. With arbitrary relays positions, the greedy algorithm updates the relays positions to reduce the maximum outage probability step by step.

Specifically, with fixed relays positions, the outage probability of all destinations are computed by (11b). Let k be the index of the destination with the maximum outage probability, i.e.,

$$k = \arg \max_{j\in\{1,\ldots,N\}} \mathcal{P}_j. \tag{12}$$

The next step is to select the relay whose position will be updated to reduce the outage probability of the k-th destination. Let s be the index of the selected relay, which benefits the k-th destination most after the update of its position. In other words, the first derivative of the outage probability of the k-th destination with respect to the position of the s-th relay is the largest, i.e.,

$$s = \arg \max_{i\in\{1,\ldots,M\}} \left(\frac{\partial \mathcal{P}_k}{\partial \tilde{x}_i}\right)^2 + \left(\frac{\partial \mathcal{P}_k}{\partial \tilde{y}_i}\right)^2 \tag{13}$$

where $\frac{\partial \mathcal{P}_k}{\partial \tilde{x}_i}$ and $\frac{\partial \mathcal{P}_k}{\partial \tilde{y}_i}$ are calculated using equations (14), as shown at the bottom of the next page.

The update direction is:

$$h_{\tilde{x}_s} = -\frac{\frac{\partial \mathcal{P}_k}{\partial \tilde{x}_s}}{\sqrt{\left(\frac{\partial \mathcal{P}_k}{\partial \tilde{x}_s}\right)^2 + \left(\frac{\partial \mathcal{P}_k}{\partial \tilde{y}_s}\right)^2}} \tag{15a}$$

$$h_{\tilde{y}_s} = -\frac{\frac{\partial \mathcal{P}_k}{\partial \tilde{y}_s}}{\sqrt{\left(\frac{\partial \mathcal{P}_k}{\partial \tilde{x}_s}\right)^2 + \left(\frac{\partial \mathcal{P}_k}{\partial \tilde{y}_s}\right)^2}} \tag{15b}$$

The position of the s-th relay moves a short distance in the update direction. Then the outage probabilities of all destinations are updated, and another iteration is triggered until the maximum outage probability does not decrease.

The numerical method for computing the optimal relays positions for minimizing the maximum outage probability of all destinations is summarized as Algorithm 1, where Δ represents the update distance of the relays positions during each step. Meanwhile, the number of relays and destinations are predetermined. The positions of the destinations are known by the source, and the relays carried by smart drones are sent to the optimal positions right after the convergence of Algorithm 1. In reality, the positions of destinations may not be known exactly, but there are many possibilities for the destinations to signal their positions to the source, such as the use of location based services supported by Long Term Evolution (LTE) and Global Positioning System (GPS) [29].

In Algorithm 1, the maximum outage probability decreases during each step and has a lower bound, which guarantees the convergence. Recall that with L quantized values of each variable in the optimization problem (11), the complexity of the exhaustive searching method is the comparison of L^{2M} quantized tuples. In Algorithm 1, assuming each relay moves L steps during the iteration, the complexity is the comparison of LM quantized tuples. Generally, we have $L \gg M$ and the complexity of Algorithm 1 is much smaller than that of the exhaustive searching method.

Algorithm 1 Computing the Optimal Relays Positions for Minimizing the Maximum Outage Probability of All Destinations

1: **Input:** $W, \gamma, \sigma^2, \alpha, A, \Delta, P_0, P_i, (x_0, y_0), (x_j, y_j), \forall i \in \{1, \ldots, M\}, \forall j \in \{1, \ldots, N\}$.
2: **Output:** $(\tilde{x}_i, \tilde{y}_i), \forall i \in \{1, \ldots, M\}$.
3: Initialize $(\tilde{x}_i, \tilde{y}_i) = (x_0, y_0), \forall i \in \{1, \ldots, M\}$.
4: Begin iteration:
5: Compute the outage probability of all destinations by (10).
6: Find the k-th destination with the maximum outage probability by (12).
7: Find the s-th relay which benefits the k-th destination most after the update of its position by (13).
8: Compute the update direction $h_{\tilde{x}_s}$ and $h_{\tilde{y}_s}$ by (15).
9: Update the position of the s-th relay:

$$\tilde{x}_s = \tilde{x}_s + h_{\tilde{x}_s}\Delta$$
$$\tilde{y}_s = \tilde{y}_s + h_{\tilde{y}_s}\Delta$$

10: Go to 5 until the maximum outage probability does not decrease.
11: End iteration.
12: **return** $(\tilde{x}_i, \tilde{y}_i), \forall i \in \{1, \ldots, M\}$.

We note that the optimal relays positions computed by Algorithm 1 may be local optimal. The reason is that the outage probability of each destination is non-convex with respect to multiple relays positions as shown by (10), hence also the maximum outage probability of all destinations. Therefore, Algorithm 1 which reduces the maximum outage probability step by step may converge to one local optimal solutions. The numerical simulations will show that the system performance of the local optimal solutions is almost the same as that of the global optimal solutions obtained by exhaustive searching method.

IV. THE OPTIMAL RELAYS POSITIONS AND SPECTRUM ALLOCATION FOR MAXIMIZING THE MINIMUM THROUGHPUT

When the QoS metric is the throughput, the optimal relays positions and resource allocation maximize the minimum throughput of all destinations. For the Public Safety Wireless Broadband Network, this is the scenario to guarantee the successful delivery of stream packages, such as voice and video information.

A. THE OPTIMIZATION PROBLEM

There exist one single-hop direct link and M two-hop relay links from the source to each destination. The data streams received by the relays are fully decoded and forwarded to the destinations [30], [31]. With perfect synchronization at the destination, the data streams of different links could be jointly decoded, which is difficult to be implemented in practical systems [32], [33]. In this paper, we consider the realistic cases without the modification of standards and hardware, where the data stream of each link is independently decoded. Then the throughput of each destination is the sum throughput of the single-hop direct link and M two-hop relay links.

The throughput of the single-hop direct link from the source to the j-th destination is:

$$\mathcal{C}_{j0} = s_{j0} W \log_2(1+\gamma) \cdot Q_1\left(\sqrt{2K_{j0}}, \sqrt{\frac{2(1+K_{j0})\gamma\sigma^2 d_{j0}^{\alpha} W}{P_0 A}}\right) \quad (16)$$

where s_{j0} is the allocated time.

$$\frac{\partial \mathcal{P}_k}{\partial \tilde{x}_i} = \frac{\mathcal{P}_k \alpha\gamma\sigma^2 W}{\mathcal{P}_{ki0} A}\Bigg[\frac{l_{i0}^{\alpha-2}(\tilde{x}_i - x_0)}{P_0} Q_1\left(\sqrt{2K_{ki}}, \sqrt{\frac{2(1+K_{ki})\gamma\sigma^2 l_{ki}^{\alpha} W}{P_i A}}\right)(1+K_{i0})e^{-K_{i0}} \exp\left(-(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}\right) \cdot I_o\left(2\sqrt{K_{i0}(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right) + \frac{l_{ki}^{\alpha-2}(\tilde{x}_i - x_k)}{P_i} Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)(1+K_{ki})e^{-K_{ki}} \exp\left(-(1+K_{ki})\frac{\gamma\sigma^2 l_{ki}^{\alpha} W}{P_i A}\right) \cdot I_o\left(2\sqrt{K_{ki}(1+K_{ki})\frac{\gamma\sigma^2 l_{ki}^{\alpha} W}{P_i A}}\right)\Bigg] \quad (14a)$$

$$\frac{\partial \mathcal{P}_k}{\partial \tilde{y}_i} = \frac{\mathcal{P}_k \alpha\gamma\sigma^2 W}{\mathcal{P}_{ki0} A}\Bigg[\frac{l_{i0}^{\alpha-2}(\tilde{y}_i - y_0)}{P_0} Q_1\left(\sqrt{2K_{ki}}, \sqrt{\frac{2(1+K_{ki})\gamma\sigma^2 l_{ki}^{\alpha} W}{P_i A}}\right)(1+K_{i0})e^{-K_{i0}} \exp\left(-(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}\right) \cdot I_o\left(2\sqrt{K_{i0}(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right) + \frac{l_{ki}^{\alpha-2}(\tilde{y}_i - y_k)}{P_i} Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)(1+K_{ki})e^{-K_{ki}} \exp\left(-(1+K_{ki})\frac{\gamma\sigma^2 l_{ki}^{\alpha} W}{P_i A}\right) \cdot I_o\left(2\sqrt{K_{ki}(1+K_{ki})\frac{\gamma\sigma^2 l_{ki}^{\alpha} W}{P_i A}}\right)\Bigg] \quad (14b)$$

For the two-hop relay link, most literatures allocate identical resources to the link from the source to the relay and the link from the relay to the destination, which leads to low resources efficiency [34]. In order to increase the resources efficiency, we optimally allocate the resources to each link according to its channel condition. Then the throughput of the link from the source to the i-th relay is:

$$\mathcal{D}_{i0} = t_{i0} W \log_2(1+\gamma) \cdot Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right) \tag{17}$$

where t_{i0} is the allocated time. The throughput of the link from the i-th relay to the j-th destination is:

$$\mathcal{D}_{ji} = t_{ji} W \log_2(1+\gamma) \cdot Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right) \tag{18}$$

where t_{ji} is the allocated time.

Since all the data streams forwarded by the i-th relay should be correctly decoded by itself, the throughput of the link from the source to the i-th relay should be no less than the total throughput of the links from the i-th relay to all destinations, i.e.,

$$\mathcal{D}_{i0} \geq \sum_{j=1}^{N} \mathcal{D}_{ji}. \tag{19}$$

Then the throughput of the j-th destination is:

$$\mathcal{C}_j = \mathcal{C}_{j0} + \sum_{i=1}^{M} \mathcal{D}_{ji} \tag{20}$$

which is determined by the relays positions and spectrum allocation.

The optimal relays positions and resource allocation maximize the minimum throughput of all destinations. Assuming $\mathcal{C}_{\min}$ is the minimum throughput, the optimization problem is:

$$\begin{aligned}
&\text{maximize: } \mathcal{C}_{\min} && (21a)\\
&\text{subject to: } \mathcal{C}_{\min} \leq \mathcal{C}_j \quad \forall j \in \{1,\ldots,N\} && (21b)\\
&\mathcal{D}_{i0} \geq \sum_{j=1}^{N} \mathcal{D}_{ji} \quad \forall i \in \{1,\ldots,M\} && (21c)\\
&\sum_{j=1}^{N} s_{j0} + \sum_{i=1}^{M} t_{i0} + \sum_{j=1}^{N}\sum_{i=1}^{M} t_{ji} = 1 && (21d)\\
&\text{variables: } (\tilde{x}_i, \tilde{y}_i), s_{j0}, t_{i0}, t_{ji} \\
&\qquad \forall i \in \{1,\ldots,M\}, \forall j \in \{1,\ldots,N\} && (21e)
\end{aligned}$$

where constraint (21b) means that the minimum throughput is no greater than that of all destinations, constraint (21c) means that the throughput of the link from the source to each relay is no less than the total throughput of the links from the relay to all destinations, constraint (21d) is the resources constraint. The optimization problem (21) is non-convex in general, which has $MN + 3M + N$ variables. Assuming the exhaustive searching method quantizes each variable into L values, the complexity is the comparison of $L^{(MN+3M+N)}$ quantized tuples. For a large L, solving the optimization problem lay on exhaustive searching method is challenging. In the following, we study the structure of the optimization problem to develop an efficient distributed algorithm to compute the optimal solutions.

B. THE OPTIMAL SOLUTIONS

For the optimization problem (21), the $MN + 3M + N$ variables should be optimized for maximizing the minimum throughput of all destinations. There exist one single-hop direct link and M two-hop relay links from the source to each destination. But with the optimal solutions, we have *Proposition 1*.

Proposition 1: For each destination, with the optimal solutions of the optimization problem (21), only the link with the highest resource efficiency will be active and other links are inactive.

Proof: For each destination, the allocated time should be optimally allocated to its single-hop direct link or two-hop relay links for maximizing its throughput. The resource efficiency is defined as the throughput divided by the allocated time. For the single-hop direct link, the resource efficiency is:

$$\mathcal{E}_{j0} = W \log_2(1+\gamma) \cdot Q_1\left(\sqrt{2K_{j0}}, \sqrt{\frac{2(1+K_{j0})\gamma\sigma^2 d_{j0}^{\alpha} W}{P_0 A}}\right) \tag{22}$$

For the two-hop relay links, the resource efficiency is:

$$\mathcal{E}_{ji0} = W \log_2(1+\gamma) \Bigg/ \left[\frac{1}{Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)} + \frac{1}{Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)}\right] \tag{23}$$

With fixed relays positions, the resource efficiency of each link is a constant. The optimal time allocation scheme will only allocate time to the link with the highest resource efficiency.

Otherwise, we shift the time from other links to the one with the highest resource efficiency. Then the throughput of the destination increases which contradicts the assumption. Hence only the link with the highest resource efficiency will be active and others are inactive, and the proposition is proved. □

The optimal active links of all destinations depend on the optimal relays positions, which in turn depend on the optimal active links of all destinations. In the following, we compute the candidate optimal relays positions and resource allocation for the cases with different active links, the one with the maximum minimum throughput are the optimal solutions.

Specifically, for the j-th destination, f_{j0} is defined as the flag of the single-hop direct link. If the single-hop direct link is active, f_{j0} equals to one. Otherwise, f_{j0} equals to zero. Similarly, f_{ji0} is defined as the flag of the two-hop relay link assisted by the i-th relay. We have:

$$f_{j0} + \sum_{i=1}^{M} f_{ji0} = 1 \tag{24}$$

which means that each destination only has one active link.

With fixed flags, the optimal candidate relays positions and resource allocation maximize the minimum throughput of all destinations. It is easy to note the throughput of each destination is identical. The reason is that the throughput of each destination is linearly increasing in the amount of allocated spectrum. If the throughput of each destination is not identical, we shift some time from the one with the maximum throughput to the one with the minimum throughput. Then the minimum throughput increases which contradicts the assumption. In other words, the inequality of (21b) must be tight and the time allocated to each link is:

$$s_{j0} = \frac{\mathcal{C}_{\min} f_{j0}}{W \log_2(1+\gamma) Q_1\left(\sqrt{2K_{j0}}, \sqrt{\frac{2(1+K_{j0})\gamma\sigma^2 d_{j0}^{\alpha} W}{P_0 A}}\right)} \tag{25a}$$

$$t_{i0} = \frac{\mathcal{C}_{\min} \sum_{j=1}^{N} f_{ji0}}{W \log_2(1+\gamma) Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)} \tag{25b}$$

$$t_{ji} = \frac{\mathcal{C}_{\min} f_{ji0}}{W \log_2(1+\gamma) Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)} \tag{25c}$$

where (25b) means that the throughput of the link from the source to the i-th relay is $\sum_{j=1}^{N} f_{ji0}$ times the minimum throughput $\mathcal{C}_{\min}$.

Substituting (25) into (21d), the minimum throughput is:

$$\mathcal{C}_{\min} = \frac{W \log_2(1+\gamma)}{\sum_{j=1}^{N} g_{j0} + \sum_{i=1}^{M} \sum_{j=1}^{N} g_{ji0}} \tag{26}$$

where

$$g_{j0} = \frac{f_{j0}}{Q_1\left(\sqrt{2K_{j0}}, \sqrt{\frac{2(1+K_{j0})\gamma\sigma^2 d_{j0}^{\alpha} W}{P_0 A}}\right)} \tag{27a}$$

$$g_{ji0} = \frac{f_{ji0}}{Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)} + \frac{f_{ji0}}{Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)} \tag{27b}$$

According to (26), the minimum throughput $\mathcal{C}_{\min}$ is a function of the relays positions. The candidate optimal relays positions maximize the minimum throughput, which is equivalent to minimize the denominator of equality (26). The candidate optimal position of the i-th relay satisfies:

$$(\tilde{x}_i, \tilde{y}_i) = \arg\min_{(\tilde{x}_i, \tilde{y}_i)} \sum_{j=1}^{N} g_{ji0} \tag{28}$$

Then the candidate optimal position of each relay could be computed by the gradient descent method.

The gradient descent method updates the position of the i-th relay in the direction of the negative gradient of the function $\sum_{j=1}^{N} g_{ji0}$, whose first derivative with respect to $\tilde{x}_i$ and $\tilde{y}_i$ are calculated as equations (29), as shown at the bottom of the next page.

Then the update direction is:

$$\tilde{h}_{\tilde{x}_i} = -\frac{\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{x}_i}}{\sqrt{\left(\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{x}_i}\right)^2 + \left(\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{y}_i}\right)^2}} \tag{30a}$$

$$\tilde{h}_{\tilde{y}_i} = -\frac{\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{y}_i}}{\sqrt{\left(\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{x}_i}\right)^2 + \left(\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{y}_i}\right)^2}}. \tag{30b}$$

The position of the i-th relay moves in the update direction step by step to find the optimal one for minimizing $\sum_{j=1}^{N} g_{ji0}$.

After achieving the candidate optimal relays positions, the candidate optimal resource allocation is computed by (25). Comparing all the candidate optimal relays positions and resource allocation, the one with the maximum minimum throughput are the optimal solutions. The numerical method for computing the optimal relays positions and spectrum allocation for maximizing the minimum throughput of all destinations is summarized as Algorithm 2, where Δ represents the update distance of the relays positions during each step.

In Algorithm 2, the minimum throughput increases during each step and has a upper bound, which guarantees the convergence. Recall that with L quantized values of each variable in the optimization problem (21), the complexity of the exhaustive searching method is the comparison of $L^{(MN+3M+N)}$ quantized tuples. In Algorithm 2, the source and each relay serves a group of destinations and there is no difference between relays. Then there exist $\frac{(M+1)^N}{M!}$ different groups, where each group indicates a searching path of the optimal solutions. Assuming each relay moves L steps during the iteration, the complexity is the comparison of no more than $\frac{(M+1)^N}{M!} LM$ candidate tuples. Generally, we have $L \gg \max\{N, M\}$ and the complexity of Algorithm 2 is much smaller than that of the exhaustive searching method.

V. DISCUSSIONS

When the QoS metric is the outage probability, the optimal relays positions computed by Algorithm 1 minimize the

Algorithm 2 Computing the Optimal Relays Positions and Resource Allocation for Maximizing the Minimum Throughput of All Destinations

1: **Input:** W, γ, σ^2, W, α, A, Δ, P_0, P_i, (x_0, y_0), (x_j, y_j), $\forall i \in \{1, \ldots, M\}, \forall j \in \{1, \ldots, N\}$.
2: **Output:** $(\tilde{x}_i, \tilde{y}_i)$, w_{j0}, v_{i0}, v_{ji}, $\forall i \in \{1, \ldots, M\}$, $\forall j \in \{1, \ldots, N\}$.
3: **for all** $f_{j0}, f_{ji0} \in [0, 1]$ such that $f_{j0} + \sum_{i=1}^{M} f_{ji0} = 1$ **do**
4:   **for all** $i \in \{1, \ldots, M\}$ **do**
5:     Initialize $(\tilde{x}_i, \tilde{y}_i) = (x_0, y_0)$.
6:     Begin iteration:
7:     Compute the update direction $\tilde{h}_{\tilde{x}_s}$ and $\tilde{h}_{\tilde{y}_s}$ by (30).
8:     Update the position of the i-th relay:

$$\tilde{x}_i = \tilde{x}_i + \tilde{h}_{\tilde{x}_i}\Delta$$
$$\tilde{y}_i = \tilde{y}_i + \tilde{h}_{\tilde{y}_s}\Delta$$

9:     Go to 7 until the minimum throughput does not increase.
10:     End iteration.
11:   **end for**
12:   Compute the minimum throughput $\mathcal{C}_{\min}$ by (26).
13:   Compute the resource allocation t_{j0}, s_{i0}, s_{ji}, $\forall i \in \{1, \ldots, M\}, \forall j \in \{1, \ldots, N\}$ by (25).
14: **end for**
15: **return** the $(\tilde{x}_i, \tilde{y}_i)$, w_{j0}, v_{i0}, v_{ji}, $\forall i \in \{1, \ldots, M\}$, $\forall j \in \{1, \ldots, N\}$ with the maximum $\mathcal{C}_{\min}$.

maximum outage probability of all destinations. When the QoS metric is the throughput, the optimal relays positions and resource allocation computed by Algorithm 2 maximize the minimum throughput of all destinations. We have *Proposition 2*.

Proposition 2: The maximum outage probability decreases and the minimum throughput increases as the number of relays increases. But beyond a certain number, more relays do not provide further improvement in the minimum throughput.

Proof: The outage probability of each destination computed by (11a) is the product of the outage probability of different links. The outage probability of each link is smaller than one, so that the outage probability of each destination decreases as the number of relays increases, hence also the maximum outage probability of all destinations, i.e.,

$$\mathcal{P}_{\max}(M, N) < \mathcal{P}_{\max}(M', N) \quad \forall M < M' \tag{31}$$

where $\mathcal{P}_{\max}(M, N)$ is the maximum outage probability with M relays and N destinations. Moreover, when the number of relays goes to infinity, the maximum outage probability of all destinations tends to be zero.

The throughput of each destination computed by (21) is the sum of the throughput of different links. However, as discussed in *Proposition 1*, only the link with the highest spectrum efficiency will be active for each destination. When the number of relays exceeds the number of destinations, deploying one exclusive relay for each destination achieves the upper bound of the minimum throughput, and more relays do not provide further improvement, i.e.,

$$\mathcal{C}_{\min}(M, N) = \mathcal{C}_{\min}(N, N) \quad \forall M \geq N \tag{32}$$

where $\mathcal{C}_{\min}(M, N)$ is the minimum throughput with M relays and N destinations. We note that for the destinations close

$$\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{x}_i} = \sum_{j=1}^{N} \frac{f_{ji0}\alpha\gamma\sigma^2 W}{A}\Bigg[\frac{l_{i0}^{\alpha-2}(\tilde{x}_i - x_0)}{P_0}\frac{(1+K_{i0})e^{-K_{i0}}\exp\left(-(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}\right)\cdot I_o\left(2\sqrt{K_{i0}(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)}{Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)} + \frac{l_{ji}^{\alpha-2}(\tilde{x}_i - x_j)}{P_i}\frac{(1+K_{ji})e^{-K_{ji}}\exp\left(-(1+K_{ji})\frac{\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}\right)\cdot I_o\left(2\sqrt{K_{ji}(1+K_{ji})\frac{\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)}{Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)}\Bigg] \tag{29a}$$

$$\frac{\partial \sum_{j=1}^{N} g_{ji0}}{\partial \tilde{y}_i} = \sum_{j=1}^{N} \frac{f_{ji0}\alpha\gamma\sigma^2 W}{A}\Bigg[\frac{l_{i0}^{\alpha-2}(\tilde{y}_i - y_0)}{P_0}\frac{(1+K_{i0})e^{-K_{i0}}\exp\left(-(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}\right)\cdot I_o\left(2\sqrt{K_{i0}(1+K_{i0})\frac{\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)}{Q_1\left(\sqrt{2K_{i0}}, \sqrt{\frac{2(1+K_{i0})\gamma\sigma^2 l_{i0}^{\alpha} W}{P_0 A}}\right)} + \frac{l_{ji}^{\alpha-2}(\tilde{y}_i - y_j)}{P_i}\frac{(1+K_{ji})e^{-K_{ji}}\exp\left(-(1+K_{ji})\frac{\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}\right)\cdot I_o\left(2\sqrt{K_{ji}(1+K_{ji})\frac{\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)}{Q_1\left(\sqrt{2K_{ji}}, \sqrt{\frac{2(1+K_{ji})\gamma\sigma^2 l_{ji}^{\alpha} W}{P_i A}}\right)}\Bigg] \tag{29b}$$

TABLE 1. Main simulation parameters.

Parameter	Value
Total bandwidth W	20MHz
Rician factor from the source to the destination K_{j0}	0
Rician factor from the source to the relay $K_{sr} = K_{i0}$	varies
Rician factor from the relay to the destination $K_{rd} = K_{ji}$	varies
Transmit power of the source P_0	P dBm
Transmit power of the relay P_i	$P - 3$ dBm
Large scale fading $Ad^{-\alpha}$	$-15.3 - 37.6 \log_{10}(d)$ dB, d in meters
Power spectral density of the white Gaussian noise σ^2	-174dBm/Hz

to the source, the link with the highest spectrum efficiency may be the single-hop direct link. Thus the number of needed relays to achieve the upper bound may be smaller than the number of destinations. Hence the proposition is proved. □

VI. NUMERICAL SIMULATIONS

In this section, the system performance is shown by numerical simulations to evaluate the performance of the proposed algorithms, which compute the optimal relays positions and resource allocation for minimizing the maximum outage probability or maximizing the minimum throughput of all destinations. Throughout the simulation, the source is located at the origin and the destinations are randomly distributed in a cell with radius 750m. The main parameters according to the LTE standards [35] are listed in TABLE 1.

A. MINIMIZING THE MAXIMUM OUTAGE PROBABILITY

When the QoS metric is the outage probability, the optimal relays positions minimize the maximum outage probability of all destinations. As discussed in Section III, for the Public Safety Wireless Broadband Network, this is the scenario to guarantee the successful delivery of short packages regardless of delay, such as rescue and sign of life information. Then the packets could be transmitted using low order modulation schemes to decrease the outage probability [24], and the simulations use the SNR threshold $\gamma = 5$ dB.

Using Monte Carlo simulations, the destinations' positions are realized 40 times and the average maximum outage probability of all destinations is computed. When the number of destinations is three, the average maximum outage probability without relays, with one and two relays as the transmit power increases with the Rician factor $K_{sr} = K_{rd} = 1$ are shown in Fig. 3. It could be seen that, the average maximum outage probability decreases as the transmit power and the number of relays increase. For example, when the transmit power is 26 dBm, the average maximum outage probability

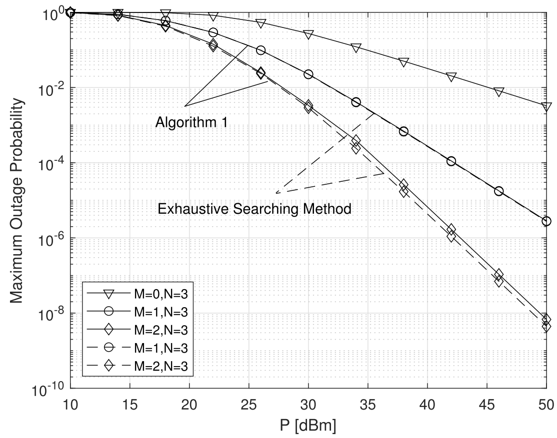

FIGURE 3. When the number of destinations is three, the average maximum outage probability without relays, with one and two relays as the transmit power increases with the SNR threshold $\gamma = 5$ dB and the Rician factor $K_{sr} = K_{rd} = 1$.

is 0.54, 0.10 and 0.02 for the cases without relay, with one and two relays. As discussed in Section V, the maximum outage probability tends to be zero when the number of relays goes to infinity. Moreover, the maximum outage probability computed by Algorithm 1 and that of the exhaustive searching method are almost the same. Meanwhile, the complexity of Algorithm 1 is much smaller than that of the exhaustive searching method. Using the simulations environment with i7-4700MQ CPU @ 2.4GHz (8CPU) and 8G RAM, the Algorithm 1 takes 28 seconds and the exhaustive searching method consumes 40 minutes.

Since the relays are carried by drones, the outage probability performances with different Rician factors are also simulated. When the number of destinations is three, the average maximum outage probability with one relay as the transmit power increases with the Rician factor $K_{sr} = K_{rd} = 0, 1, 3, 5$ are shown in Fig. 4. It could be seen that the average maximum outage probability decreases as the transmit power increases. With large transmit power, the average maximum outage probability decreases as the Rician factor increases. For example, when the transmit power is 26 dBm, the average maximum outage probability is 0.12, 0.10, 0.07 and 0.06 with Rician fator $K_{sr} = K_{rd} = 0, 1, 3, 5$. However with very low transmit power, the average maximum outage probability may increase as the Rician factor increases, which could be seen in the enlarged subfigure. The reason is that, with low transmit power, the successful delivery happens with large path gain G. As shown in Fig.2, with large G, its probability density may decrease as the Rician factor increases. Then the successful delivery probability and the outage probability decreases and increases as the Rician factor increases.

B. MAXIMIZING THE MINIMUM THROUGHPUT

When the QoS metric is the throughput, the optimal relays positions and resource allocation maximize the minimum throughput of all destinations. As discussed in Section IV, for the Public Safety Wireless Broadband Network, this is

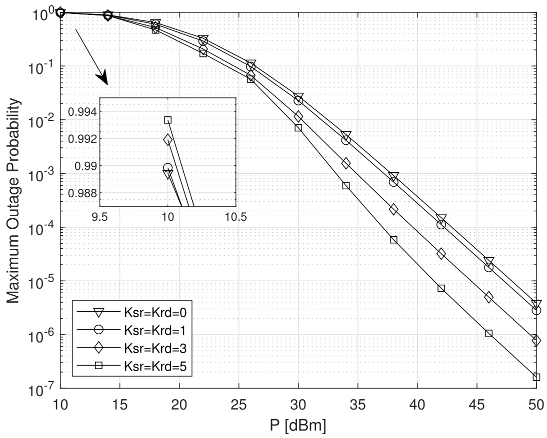

FIGURE 4. When the number of destinations is three, the average maximum outage probability with one relay as the transmit power increases with the SNR threshold $\gamma = 5$ dB and the Rician factor $K_{sr} = K_{rd} = 0, 1, 3, 5$.

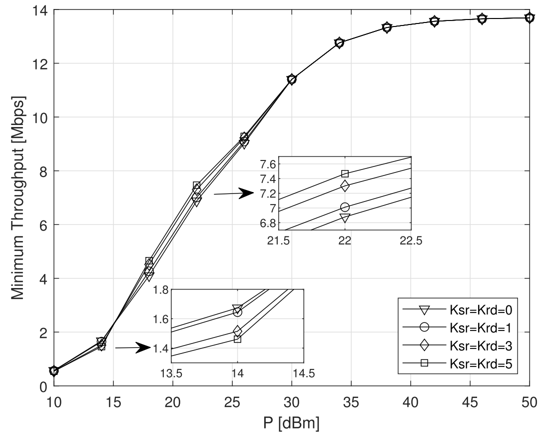

FIGURE 6. When the number of destinations is three, the average minimum throughput with one relay as the transmit power increases with the SNR threshold $\gamma = 5$ dB and the Rician factor $K_{sr} = K_{rd} = 0, 1, 3, 5$.

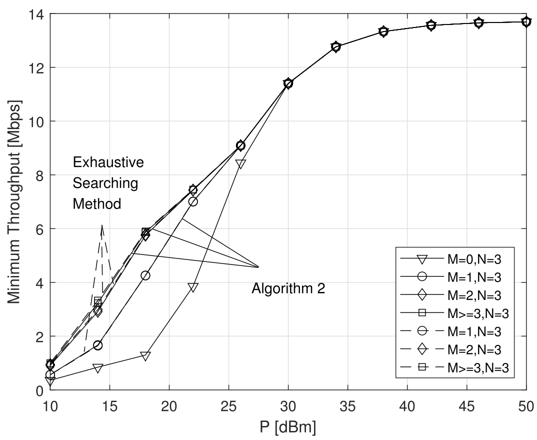

FIGURE 5. When the number of destinations is three, the average minimum throughput without relays, with one, two and three relays as the transmit power increases with the SNR threshold $\gamma = 5$ dB and the Rician factor $K_{sr} = K_{rd} = 1$.

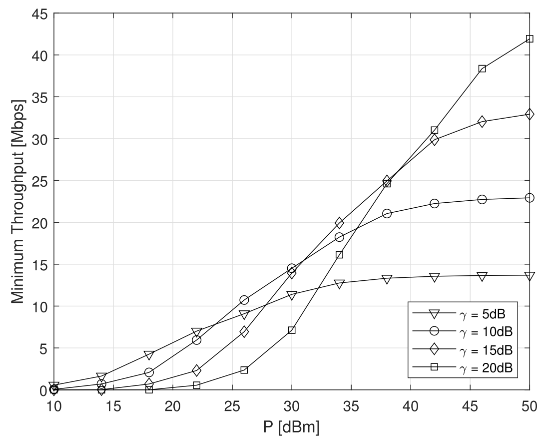

FIGURE 7. When the number of destinations is three, the average minimum throughput with one relay as the transmit power increases with the SNR threshold $\gamma = 5, 10, 15, 20$ dB and the Rician factor $K_{sr} = K_{rd} = 1$.

the scenario to guarantee the successful delivery of stream packages, such as voice and video information. Then the modulation scheme may vary with different SNR threshold to increase the throughput [24].

Using Monte Carlo simulations, the destinations' positions are realized 40 times and the average minimum throughput of all destinations is computed. When the number of destinations is three, the average minimum throughput without relays, with one, two and three relays as the transmit power increases with the SNR threshold $\gamma = 5$ dB and the Rician factor $K_{sr} = K_{rd} = 1$ are shown in Fig. 5. It could be seen that, the average minimum throughput increases as the transmit power increases. For example, when the transmit power is 18 dBm, the average minimum throughput is 1.30, 4.26, 5.78 and 5.90 Mbps for the cases without relay, with one, two and three relays. But beyond 30 dBm, more relays do not increase the minimum throughput. The reason is that the sole source is powerful enough to provide wireless service to the destinations, so that the relays are not necessary. As discussed in Section V, even with arbitrary transmit power, the minimum throughput does not increases when the number of relays exceeds three. Moreover, the minimum throughput computed by Algorithm 2 and that of the exhaustive searching method are almost the same. Meanwhile, the complexity of Algorithm 2 is much smaller than that of the exhaustive searching method. Using the simulations environment with i7-4700MQ CPU @ 2.4GHz (8CPU) and 8G RAM, the Algorithm 2 takes 2 minutes and the exhaustive searching method consumes 88 minutes.

Since the relays are carried by drones, the minimum throughput performances with different Rician factors are also simulated. When the number of destinations is three, the average minimum throughput with one relay as the transmit power increases with the SNR threshold $\gamma = 5$ dB and the Rician factor $K_{sr} = K_{rd} = 0, 1, 3, 5$ are shown in Fig. 6. It could be seen that the average minimum throughput increases as the transmit power increases. With medium transmit power, the average minimum throughput increases as the Rician factor increases. For example, when the transmit power is 18 dBm, the average minimum throughput is 4.10, 4.26, 4.52 and 4.66 Mbps with Rician factor $K_{sr} = K_{rd} = 0, 1, 3, 5$. But beyond 26 dBm, the average minimum throughput with different Rician factor overlap, since their successful probabilities are almost the same. However with very low transmit power, the average minimum throughput may decrease as the Rician factor increases, which could be seen in the left bottom enlarged subfigure. The reason is that, with low transmit power, the successful delivery happens with large path gain G. As shown in Fig.2, with large G, its probability density may decrease as the Rician factor increases, hence also the minimum throughput.

Because the modulation scheme may vary with different SNR threshold to increase the throughput, the minimum throughput performances with different SNR thresholds are also simulated. When the number of destinations is three, the average minimum throughput with one relay as the transmit power increases with the SNR threshold $\gamma = 5, 10, 15, 20$ dB and the Rician factor $K_{sr} = K_{rd} = 1$ are shown in Fig. 7. It could be seen that the average minimum throughput increases as the transmit power increases. Meanwhile, for the transmit power in the simulation range [10, 23.6], [23.6, 31], [31, 38.9], [38.9, 50] dBm, the optimal SNR thresholds are 5, 10, 15, and 20 dB. In the real system, the SNR threshold and related modulation scheme should be optimized according to the transmit power.

VII. CONCLUSION

In this paper, multiple relays carried by smart drones are sent by a source to assist the communication to multiple destinations for the Public Safety Wireless Broadband Network. To guarantee the successful delivery of short and stream packages, efficient algorithms are proposed to compute the optimal relays positions and resource allocation to minimize the maximum outage probability and maximize the minimum throughput of all destinations. The maximum outage probability decreases as the number of relays increases. The minimum throughput increases as the number of relays increases but below a certain number. Beyond that, more relays do not further increase the minimum throughput. Meanwhile, carried by drones with Rician fading between source and relays, and relays and destinations, the maximum outage probability decreases and the minimum throughput increases as the Rician factor increases, when the transmit power is large. However, with very low transmit power, the maximum outage probability may increase and the minimum throughput may decrease as the Rician factor increases. Without the modification of standards and hardware, the analysis could be directly implemented in the real system. Maximizing the minimum throughput of all destinations using spectrum sharing scheme is left for future work, where interference management is crucial to the system performance.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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