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Joint User Association and Power Allocation in Heterogeneous NOMA Networks With Imperfect CSI

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ABSTRACT Non-orthogonal multiple access (NOMA)-enabled heterogeneous networks (N-HetNets) can provide higher system capacity and spectrum efficiency by allowing multiple users cooperating in the same channel. However, traditional resource allocation algorithms are achieved under perfect channel state information (CSI) which is difficult to obtain in practical systems. In this paper, we study the total energy efficiency maximization problem in a downlink multicell N-HetNets under imperfect CSI, where the constraints of the quality of service of small-cell users, the maximum transmit power, and the cross-tier interference are considered. With the help of the ellipsoidal uncertainty sets, an iterative-based power allocation and user association algorithm is proposed based on the worst-case approach and the Lagrange dual method. Moreover, computational complexity, robust sensitivity and the impact of imperfect CSI error on user's outage probability are provided to insight the performance of the proposed algorithm. Simulation results demonstrate the robustness of the proposed algorithm.

INDEX TERMS Heterogeneous network, non-orthogonal multiple access, resource allocation, energy efficiency.

I. INTRODUCTION

With the increasing requirements of high-speed data rates, it is obvious that the conventional wireless network architecture cannot support the requirements of future communication systems [1], [2]. To achieve the growth of mobile data and deploy the task offloading of macro base stations (MBSs), heterogeneous network (HetNet) has been regarded as a promising technology [3], where multiple small cells (SCs) are overlaid with the traditional macrocell network to improve system capacity and spectrum efficiency [4]–[7].

However, the additional deployment of SCs overlaid the existing macrocell inevitably brings more challenges to resource allocation (RA) problems due to the multi-user interference and the cross-tier interference. Moreover, with the increasing number of access users, the limited spectrum resource and multiple access interference can prevent further increase in system capacity. Thus, non-orthogonal multiple access (NOMA) as another promising technology has been proposed to enhance the spectrum reuse and connectivity density [8]–[14]. In NOMA systems, it is possible to further provide higher spectrum efficiency by allowing multiple users coexisting on the same resource block (e.g., subchannel), where successive interference cancellation (SIC) technology is used at the receiver for signal separation and co-channel interference reduction. Therefore, NOMA-enabled HetNets have been concerned in recent years [15]–[17].

A. RELATED WORKS

RA is an effective technology in NOMA-enabled HetNets to achieve interference management, power allocation and user association. Currently, RA problems have been

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concerned from perfect channel state information (CSI) and imperfect CSI.

Perfect CSI: The RA problems with the overall throughput maximization have been studied in [18]–[20]. For instance, in [18], a joint user scheduling and power control algorithm was proposed to maximize the overall throughput for downlink NOMA-based HetNets. But only one SC is considered. For a downlink NOMA-based heterogeneous cognitive network with one macrocell and multiple SCs [19], the sum throughput of multiple SCs was maximized by jointly optimizing bandwidth allocation, power allocation, and user clustering. In [20], a compressive sensing based spectrum allocation and power control algorithm was proposed to achieve the overall sum-rate maximization while considering both co-tier and cross-tier interferences. To improve data rate and reduce power consumption, EE-based RA problems have been studied in [21]–[24] according to different network scenarios. In [21], the authors aimed to maximize the sum EE of one macrocell and SCs by jointly optimizing subchannel and power allocation. For simultaneous wireless information and power transfer (SWIPT) based HetNets [22], an EE maximization problem of SCs was studied by decoupling the original problem into the suchannel allocation problem and the power control problem, respectively. For energy harvesting-enabled-power domain (PD)-NOMA-based HetNets [23], the authors proposed joint subcarrier assignment and power allocation algorithms to achieve fair energy-efficient RA under the constraints of SIC ordering and user's fairness. Moreover, the joint subcarrier assignment and global energy-efficient power allocation problem was investigated for energy-harvesting two-tier downlink NOMA HetNets. However, these aforementioned works about RA problems have been achieved under the assumption of perfect CSI, and lack of the discussion on imperfect CSI. It is difficult to obtain exact CSI due to the effects of estimation errors, feedback delays, and quantization errors of actual physical channels.

Imperfect CSI: Since it is difficult to obtain perfect CSI due to quantization errors and parametric estimation errors, robust RA schemes with imperfect CSI in NOMA-based HetNets have been considered in [25]–[28]. In [25], a bisection search algorithm via a gradient value was proposed to achieve robust EE maximization of SCs under the outage probability constraints of users. Moreover, consider the non-ideal SIC, the distributed cluster formation and power-bandwidth allocation problem was studied for downlink NOMA-based HetNets [26]. Robust RA problems have been also extended to heterogeneous vehicular networks [27] and heterogeneous Internet of Things (IoT) networks [28].

B. MOTIVATION AND CONTRIBUTIONS

Most of the existing works have not considered imperfect CSI. Although some RA algorithms with imperfect CSI have been studied from the perspective of stochastic optimization [25], [26], robust sensitivity (i.e., the impact of uncertain parameter on system performance) and computational complexity have not been analyzed, which is helpful to understand the impact of uncertainty on system performance, such as performance gap (i.e., the performance difference between the optimal RA design and the robust RA design). Therefore, it is necessary to insight the robust RA algorithm for multicell NOMA-based HetNets to obtain good system capacity and robustness. To the best of our knowledge, this topic has not been studied. In this paper, we address a robust EE maximization problem of multiple SCs under bounded channel uncertainties for downlink NOMA-based HetNets with one macrocell and multiple SCs. This work differs from previous works [25] and [26] in its imperfect CSI model and system models, where imperfect CSI errors follow certain statical models. The main contributions are summarized

- We formulate a robust EE maximization problem for a downlink NOMA-based multicell HetNet with bounded channel uncertainties. At the same time, we consider the factors which can affect the overall EE, including the quality of service (QoS) requirement of each user, the maximum transmit power constraint of base station, and the cross-tier interference power constraints for macrocell users (MUs). Moreover, an EE maximization framework with user association and power allocation is proposed.
- Considering that the initial robust EE optimization problem is non-linear and non-convex, the worst-case approach is introduced, which can convert the constraints and the objective function with uncertain parameters into the deterministic ones. Additionally, the fractional programming (FP) problem is transformed into an equivalent subtractive form. Then, the problem is solved by using Karush-Kuhn-Tucker (KKT) conditions and Lagrange dual methods.
- To insight the robustness, we further analyze the computational complexity, robust sensitivity, and the impact of estimation error on outage probability. Finally, simulation results verify the effectiveness of the proposed algorithm.

The rest of this paper is organized as follows. In Section II, an EE-based RA problem with channel uncertainties is presented. In Section III, a robust RA algorithm is provided, which includes the uncertainty modeling, the transformation of optimization problem, and a robust RA algorithm design. Section IV gives performance analysis. Simulation results are shown in Section V. Finally, Section VI summarizes this paper. The abbreviations used in this paper are given in Table 1.

II. SYSTEM MODEL

We consider a downlink two-tier NOMA-based HetNet, as shown in Fig. 1, where M SCs are overlaid with one macrocell. In the macrocell, there are K MUs. Denote the index set of SC base station (SBS) by $\mathcal{M} = \{1, 2, \cdots, M\}$ $(\forall m \in \mathcal{M})$ and the index set of MUs by $\mathcal{K} = \{1, 2, \cdots, K\}$ $(\forall k \in \mathcal{K})$. Assume that there are N small-cell users (SUs) in each SC, denoted by $\mathcal{N} = \{1, 2, \cdots, N\}$ $(\forall i, j \in \mathcal{N})$. Under an underlay spectrum sharing mode, SUs are allowed to access

TABLE 1. Abbreviation.

Abb.	Meaning
CSI	channel state information
EE	energy efficiency
FP	fractional programming
HetNet	heterogeneous network
IoT	Internet of Things
KKT	Karush-Kuhn-Tucker
MBS	macro base station
MU	macrocell user
NOMA	non-orthogonal multiple access
OFDMA	orthogonal frequency division multiple access
PD	power domain
QoS	quality of service
RA	resource allocation
SC	small cell
SU	small-cell user
SBS	small-cell base station
SIC	successive interference cancellation
SINR	signal-to-interference-plus-noise ratio
SWIPT	simultaneous wireless information and power transfer

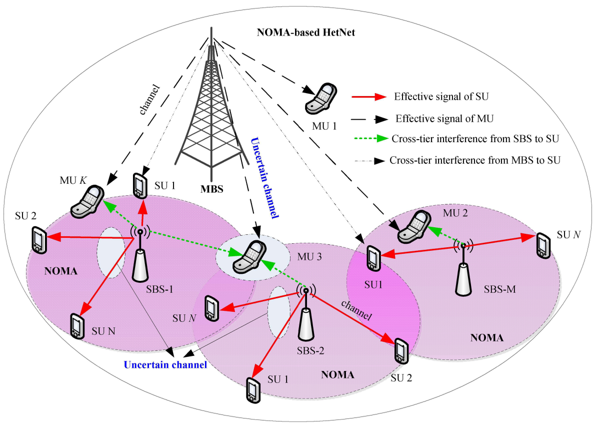

FIGURE 1. A downlink multicell NOMA-based HetNet with uncertain channels.

the spectrum owned by MUs when the cross-tier interference from SBSs to each MU receiver does not extend the interference limits of MUs. The interference power among different SCs can be ignored due to the low-power features and strong wall penetration loss [22], [29]. The symbols of system parameters are listed in Table 2.

Without loss of generality, the channel gains of all SUs in the m-th SC are sorted in descending order by $h_{m,1} \geq h_{m,2} \geq \cdots \geq h_{m,N}$ [30]–[32]. According to the downlink PD-based NOMA principle, the i-th SU can decode the signals of the j-th SU for $i > j$ and remove the weak interference item from its own signals, but treats the strong signals of the j'-th SU as the interference power for $j' > i$. The signal-to-interference-plus-noise ratio (SINR) received at the i-th SU of SC m is

$$\gamma_{m,i} = \frac{p_{m,i} h_{m,i}}{h_{m,i} \sum_{j=1}^{i-1} p_{m,j} + P_k g_{m,i}^k + \sigma^2}. \tag{1}$$

Since each SU can be served only by one SBS every time, we can define $\alpha_{m,i}$ as the user association factor. If the i-th SU associates with SBS m, $\alpha_{m,i} = 1$; otherwise $\alpha_{m,i} = 0$.

TABLE 2. System parameters.

Symbol	Meaning
$h_{m,i}$	channel gain from SBS m to SU i
$p_{m,i}$	transmit power from SBS m to SU i
$p_{m,j}$	transmit power from SBS m to SU j
σ^2	background noise power
$\alpha_{m,i}$	user-SBS association factor
I_k^{th}	interference power threshold of MU k
$g_{m,k}$	channel gain from SBS m to MU k
$\gamma_{m,i}$	SINR of SU i at SC m
$R_{m,i}$	data rate of SU i at SC m
$R_{m,i}^{\min}$	minimum rate threshold of SU i at SC m
P_c	total circuit power consumption of SCs
P_k	transmit power from MBS to MU k
$g_{m,i}^k$	channel gain from MBS to SU i in SC m
$p_m^{\max}$	maximum transmit power at SBS m
$\delta_{m,i}$	upper bound of channel estimation error on SU's link
ε_k	upper bound of channel estimation error on MU k
η	total EE of SUs

As a result, the user association constraint is

$$\alpha_{m,i} = \{0, 1\}, \quad \sum_{m=1}^{M} \alpha_{m,i} = 1. \tag{2}$$

Since the transmit power of each SBS cannot be infinite, we have the following transmit power constraint

$$\sum_{i=1}^{N} \alpha_{m,i} p_{m,i} \leq p_m^{\max}. \tag{3}$$

Additionally, it is necessary to consider the cross-tier interference power from SBSs to each MU receiver in our considered HetNets. Thus we have

$$\sum_{m=1}^{M} \sum_{i=1}^{N} \alpha_{m,i} p_{m,i} g_{m,k} \leq I_k^{th}. \tag{4}$$

It can guarantee the QoS requirement of MU k.

According to Shannon's capacity formula, the data rate of SU i in SC m is

$$R_{m,i} = \alpha_{m,i} \log_2(1 + \gamma_{m,i}). \tag{5}$$

Accordingly, considering the QoS constraint of each user, a joint user association and power allocation problem can be formulated to achieve total EE maximization of SUs

$$\begin{aligned}
&\max_{\alpha_{m,i}, p_{m,i}} \frac{\sum_{m=1}^{M} \sum_{i=1}^{N} R_{m,i}}{\sum_{m=1}^{M} \sum_{i=1}^{N} \alpha_{m,i} p_{m,i} + P_c} \\
&s.t.\ C_1: \sum_{m=1}^{M} \alpha_{m,i} = 1, \quad \alpha_{m,i} = \{0, 1\}, \\
&\quad C_2: \sum_{i=1}^{N} \alpha_{m,i} p_{m,i} \leq p_m^{\max}, \\
&\quad C_3: R_{m,i} \geq R_{m,i}^{\min}, \\
&\quad C_4: \sum_{m=1}^{M} \sum_{i=1}^{N} \alpha_{m,i} p_{m,i} g_{m,k} \leq I_k^{th},
\end{aligned} \tag{6}$$

where C_1 can guarantee that each SU has one associated SBS. C_2 denotes the overall transmit power of SUs in each SC below a maximum power threshold. C_3 denotes the minimum rate constraint of each SU. C_4 represents the cross-tier interference constraint to keep the QoS of each MU. Obviously, problem (6) is a mixed integer and non-convex FP problem with variables $\alpha_{m,i}$ and $p_{m,i}$, which is difficult to directly obtain the closed-form solutions. Moreover, problem (6) belongs to a non-robust optimization problem with perfect CSI since there is no channel estimation errors in variables $g_{m,k}$ and $h_{m,i}$ [5].

However, the assumption of perfect CSI is unreasonable for practical wireless communication systems. Because there is no cooperation between SUs and MUs so that channel gains of cross-tier links are difficult to exactly obtain due to the existence of channel estimation errors caused by quantization errors and channel delays.

Thus, the actual channel gains can be reformulated as

$$\begin{cases} h_{m,i} = \bar{h}_{m,i} + \Delta h_{m,i}, \\ g_{m,k} = \bar{g}_{m,k} + \Delta g_{m,k}, \end{cases} \tag{7}$$

where $\bar{h}_{m,i}$ and $\bar{g}_{m,k}$ denote the estimated channel gains, and $\Delta h_{m,i}$, $\Delta g_{m,k}$ are the corresponding channel estimation errors. For traditional existing RA algorithms with perfect CSI, they assume that there are no channel estimation errors, namely, $\Delta h_{m,i} = 0$, $\Delta g_{m,k} = 0$. In other words, the estimated channel gains are assumed to be equal to the true channel gains, such as $h_{m,i} = \bar{h}_{m,i}$ and $g_{m,k} = \bar{g}_{m,k}$ (problem (6)). But this may cause outage probabilities of users when the designed RA algorithms are employed in actual HetNets, which will be analyzed in the following Section.

If channel estimation errors formulated in (7) are considered in problem (6), we have a robust counterpart problem

$$\begin{aligned} \max_{\alpha_{m,i}, p_{m,i}} \; & \frac{\sum_{m=1}^{M}\sum_{i=1}^{N} R_{m,i}(\bar{h}_{m,i}, \Delta h_{m,i})}{\sum_{m=1}^{M}\sum_{i=1}^{N} \alpha_{m,i} p_{m,i} + P_c} \\ s.t.\; & C_1, C_2, \\ & C_3: R_{m,i}(\bar{h}_{m,i}, \Delta h_{m,i}) \geq R_{m,i}^{\min}, \\ & C_4: \sum_{m=1}^{M}\sum_{i=1}^{N} \alpha_{m,i} p_{m,i} (\bar{g}_{m,k} + \Delta g_{m,k}) \leq I_k^{th}, \\ & C_5: \Delta h_{m,i} \in \mathcal{R}_h, \quad \Delta g_{m,k} \in \mathcal{R}_g, \end{aligned} \tag{8}$$

where C_5 denotes uncertainty sets that can decide the impact of uncertainties on optimal solutions. For example, when $\Delta g_{m,k}$ is a very big value and bigger than zero, the maximum transmit power $p_{m,i}$ becomes small via C_4. Therefore, how to decide the uncertainty sets of channel estimation errors (e.g., $\Delta h_{m,i}$, $\Delta g_{m,k}$) is very important to balance between robustness and optimality. Moreover, problem (8) is non-convex and more complex due to the introduction of C_5, which is challenging to directly solve.

III. ROBUST RA ALGORITHM

A. UNCERTAINTY MODELING

According to the survey on robust RA [33], there are two methods to achieve uncertainty modeling: Bayesian approach and the worst-case approach. Bayesian approach formulates the estimation errors as statistical models. For instance, the estimation error belongs to a Gaussian distribution model [25]. On the other hand, the worst-case approach formulates the estimation errors as bounded uncertainty sets. For instance, the estimation error is bounded by a Euclidean norm [34], [35]. This method can overcome the impact of all possible channel estimation errors which are confined to certain ranges with upper bounds. Thus, it can provide more reliability without any outages.

In this paper, in order to well protect the QoS of MUs, we use the worst-case approach to model the uncertain parameters. The reason is that, under the underlay spectrum sharing mode, the MU has higher priority to use spectrum resource. The communication quality of MUs cannot be interrupted, such as user's outage. To provide user's robustness and improve system EE, we consider that the channel estimation errors are modeled by the bounded sets that represent the distances between the actual values and the estimated values. For instance, the distances are mathematically expressed by the general definition of norm [36]. In such case, the uncertainty set of channel estimation error on SU's link ($\Delta h_{m,i}$) is

$$\mathcal{R}_h = \left\{ \Delta h_{m,i} \mid \; |h_{m,i} - \bar{h}_{m,i}| \leq \delta_{m,i} \right\}, \tag{9}$$

where $|\cdot|$ denotes the absolute value operator. $\delta_{m,i}$ is the upper bound of imperfect CSI error from SBS m to SU i. When $\delta_{m,i}$ is a big value, it means that the estimated channel gain $\bar{h}_{m,i}$ is far from its true value $h_{m,i}$. Otherwise, the estimated channel gain is more accurate.

Similarly, the uncertainty set of the channel estimation errors between SBSs to the k-th MU is defined as

$$\mathcal{R}_g = \left\{ \boldsymbol{g}_k \mid \; \left\| \boldsymbol{g}_k - \bar{\boldsymbol{g}}_k \right\| \leq \varepsilon_k \right\}, \tag{10}$$

where $\|\cdot\|$ denotes 2-norm. $\boldsymbol{g}_k = [g_{1,k}, \cdots, g_{M,k}]^T$ and $\bar{\boldsymbol{g}}_k = [\bar{g}_{1,k}, \cdots, \bar{g}_{M,k}]^T$ are the actual and estimated channel gain vectors, respectively, and ε_k is the upper bound of overall imperfect CSI errors from SBSs to the k-th MU. Moreover, the values of $\delta_{m,i}$ and ε_k as well as the definition of norm are decided by the sizes of estimation errors and the sources of channel uncertainties. And the upper bounds of uncertainty sets can be obtained by the same approach in [34].

B. TRANSFORMATION OF ROBUST OPTIMIZATION PROBLEM

In this subsection, we will discuss how to transform the problem (8) with uncertainty sets (9) and (10) into a convex one. To do this, we have two challenges according to the structure of problem (8). The first one is how to covert the fractional objective function into a non-fractional one. The second one is how to transform the robust constraints C_3, C_4 and the objective function with uncertain parameters into the convex ones.

Since the integer optimal variable $\alpha_{m,i}$ makes the original problem be more challenging, hence, we introduce an auxiliary variable $\bar{p}_{m,i} = \alpha_{m,i}p_{m,i}$ to deal with this problem by a relaxation approach. As a result, problem (8) becomes

$$\begin{aligned}\max_{\alpha_{m,i},\bar{p}_{m,i}} & \frac{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{R}_{m,i}(\bar{h}_{m,i}, \Delta h_{m,i})}{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i} + P_c} \\ s.t.\ & \bar{C}_1: \ 0 \le \alpha_{m,i} \le 1, \\ & \bar{C}_2: \ \sum_{i=1}^{N}\bar{p}_{m,i} \le p_m^{\max}, \\ & \bar{C}_3: \ \bar{R}_{m,i}(\bar{h}_{m,i}, \Delta h_{m,i}) \ge R_{m,i}^{\min}, \\ & \bar{C}_4: \ \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k} + \Delta g_{m,k}) \le I_k^{th}, \\ & C_5: \ \Delta h_{m,i} \in \mathcal{R}_h, \quad \Delta g_{m,k} \in \mathcal{R}_g, \end{aligned} \tag{11}$$

where $\bar{R}_{m,i} = \alpha_{m,i}\log_2(1 + \frac{\bar{p}_{m,i}h_{m,i}}{h_{m,i}\sum_{j=1}^{i-1}\bar{p}_{m,j}+\alpha_{m,i}\bar{\sigma}^2})$ and $\bar{\sigma}^2 = P_k g_{m,i}^k + \sigma^2$.

To overcome the impact of channel uncertainties and achieve steady transmission without outage communication, problem (11) via the worst-case approach can be reformulated as

$$\begin{aligned}\max_{\alpha_{m,i},\bar{p}_{m,i}} \min & \left\{\frac{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{R}_{m,i}(\bar{h}_{m,i}, \Delta h_{m,i})}{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i} + P_c}\right\} \\ s.t.\ & \bar{C}_1, \bar{C}_2, C_5, \\ & \bar{C}_3: \ \min\left\{\bar{R}_{m,i}(\bar{h}_{m,i}, \Delta h_{m,i})\right\} \ge R_{m,i}^{\min}, \\ & \bar{C}_4: \ \max\left\{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k}+\Delta g_{m,k})\right\} \le I_k^{th}. \end{aligned} \tag{12}$$

The objective of the worst-case approach used in problem (12) is to keep the transmission quality without any outage under bounded channel uncertainty sets. This method can guarantee the minimal rate of SU i ($\bar{R}_{m,i}$) above the rate threshold $R_{m,i}^{\min}$ under the channel estimation error $\Delta h_{m,i}$. Moreover, it aims to keep the QoS of MU k under the maximum cross-tier interference power with channel estimation errors $\Delta g_{m,k}$. Obviously, the key problem is to transform the left items of constraints $\bar{C}_3$ and $\bar{C}_4$ into convex ones.

To deal with the uncertainties in problem (12), we decompose it into two subproblems with the uncertainty sets. For the rate constraint with estimation errors in $\bar{C}_3$, we need to determine the low bound of $\bar{R}_{m,i}$. Thus, combining (9) and $\bar{C}_3$, we have the following subproblem

$$\begin{aligned}&\min_{\Delta h_{m,i}} \bar{R}_{m,i} \\ &\quad s.t.\ h_{m,i} \in [\bar{h}_{m,i} - \delta_{m,i}, \bar{h}_{m,i} + \delta_{m,i}]. \end{aligned} \tag{13}$$

Then we have the following relationship

$$\begin{aligned}\bar{R}_{m,i} &= \alpha_{m,i}\log_2\left(1 + \frac{\bar{p}_{m,i}}{\sum_{j=1}^{i-1}\bar{p}_{m,j} + \frac{\alpha_{m,i}\bar{\sigma}^2}{h_{m,i}}}\right) \\ &\ge \alpha_{m,i}\log_2\left(1 + \frac{\bar{p}_{m,i}}{\sum_{j=1}^{i-1}\bar{p}_{m,j} + \sigma_{m,i}}\right), \end{aligned} \tag{14}$$

where $\sigma_{m,i} = \frac{\alpha_{m,i}\bar{\sigma}^2}{\bar{h}_{m,i}-\delta_{m,i}}$. Thus, the low bound of robust rate is

$$\bar{R}_{m,i}^{low} = \alpha_{m,i}\log_2\left(1 + \bar{\gamma}_{m,i}\right), \tag{15}$$

where $\bar{\gamma}_{m,i} = \frac{\bar{p}_{m,i}}{\sum_{j=1}^{i-1}\bar{p}_{m,j}+\sigma_{m,i}}$.

Similarly, we can get the subproblem of the robust interference constraint $\bar{C}_4$, i.e.,

$$\begin{aligned}&\max_{\Delta g_{m,k}} \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}g_{m,k} \\ &s.t.\ \Delta g_{m,k} \in \mathcal{R}_g. \end{aligned} \tag{16}$$

Based on Cauchy Schwartz inequality, the upper bound of the left side in $\bar{C}4$ can be rewritten as

$$\begin{aligned}\max_{\Delta g_{m,k}} & \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}g_{m,k} \\ &= \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}\bar{g}_{m,k} \\ &\quad + \max_{\Delta g_{m,k}}\left\{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}\Delta g_{m,k}\right\} \\ &\le \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}\bar{g}_{m,k} \\ &\quad + \sqrt{\sum_{m=1}^{M}\sum_{i=1}^{N}(\bar{p}_{m,i})^2}\sqrt{\sum_{m=1}^{M}\sum_{i=1}^{N}(\Delta g_{m,k})^2} \\ &\le \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}\bar{g}_{m,k} + \varepsilon_k\sqrt{\sum_{m=1}^{M}\sum_{i=1}^{N}(\bar{p}_{m,i})^2} \\ &\le \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k} + \varepsilon_k). \end{aligned} \tag{17}$$

Thus, we have the following relationship

$$\begin{aligned}\max_{\Delta g_{m,k}} & \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}g_{m,k} \\ &= \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k} + \varepsilon_k) \le I_k^{th}. \end{aligned} \tag{18}$$

As a result, based on (15) and (18), we have the deterministic optimization problem

$$\begin{aligned} \max_{\alpha_{m,i},\bar{p}_{m,i}} & \frac{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{R}_{m,i}^{low}}{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}+P_c} \\ s.t.\ & \bar{C}_1, \bar{C}_2, \\ & \tilde{C}_3: \bar{R}_{m,i}^{low} \geq R_{m,i}^{\min}, \\ & \tilde{C}_4: \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k}+\varepsilon_k) \leq I_k^{th}. \end{aligned} \tag{19}$$

The problem (19) is still non-convex, but it is a deterministic optimization problem with any uncertain parameter. Therefore, it is necessary to deal with the non-convexity problem caused by the fractional function and the coupled variables $\bar{p}_{m,i}$ and $\bar{p}_{m,j}$.

Based on Dinkelbach's method [38], we can transform the objective function in problem (19) into the equivalent form

$$\begin{aligned} &\max_{\alpha_{m,i},\bar{p}_{m,i}} \frac{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{R}_{m,i}^{low}}{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}+P_c} \\ &\triangleq \max_{\alpha_{m,i},\bar{p}_{m,i}} \left\{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{R}_{m,i}^{low} - \eta\left(\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}+P_c\right)\right\}, \end{aligned} \tag{20}$$

where $\eta > 0$ denotes an auxiliary variable which means the total EE. If we define the optimal solutions $\alpha_{m,i}{}^*$ and $\bar{p}_{m,i}^*$, we have the following optimal EE

$$\eta^* = \frac{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{R}_{m,i}^{low}(\alpha_{m,i}{}^*, \bar{p}_{m,i}^*)}{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}^*+P_c}. \tag{21}$$

Moreover, based on successive convex approximation [21], [39], we have the approximation relationship, such as

$$\bar{R}_{m,i}^{low} \geq \tilde{R}_{m,i} = a_{m,i}\alpha_{m,i}\log_2(\bar{\gamma}_{m,i}) + \alpha_{m,i}b_{m,i}, \tag{22}$$

where $a_{m,i} = \frac{\tilde{\gamma}_{m,i}}{1+\tilde{\gamma}_{m,i}}$ and $b_{m,i} = \log_2(1+\tilde{\gamma}_{m,i}) - a_{m,i}\log_2(\tilde{\gamma}_{m,i})$. And $\tilde{\gamma}_{m,i}$ denotes the value of the last iteration of $\bar{\gamma}_{m,i}$ [21], [22].

Therefore, according to (19)-(22), we have the following convex optimization problem

$$\begin{aligned} \max_{\alpha_{m,i},\bar{p}_{m,i}} & \sum_{m=1}^{M}\sum_{i=1}^{N}\tilde{R}_{m,i} - \eta\left(\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}+P_c\right) \\ s.t.\ & \bar{C}_1: 0 \leq \alpha_{m,i} \leq 1, \\ & \bar{C}_2: \sum_{i=1}^{N}\bar{p}_{m,i} \leq p_m^{\max}, \\ & \hat{C}_3: \tilde{R}_{m,i} \geq R_{m,i}^{\min}, \\ & \tilde{C}_4: \sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k}+\varepsilon_k) \leq I_k^{th}. \end{aligned} \tag{23}$$

C. ROBUST RA ALGORITHM DESIGN

To obtain the closed-form solutions, we can use Lagrange dual decomposition methods [37] to solve problem (23). Thus, the Lagrange function is given by

$$\begin{aligned} &L\left(\alpha_{m,i}, \bar{p}_{m,i}, \beta_{m,i}, \mu_{m,i}, \varpi_k, \chi_m\right) \\ &= \sum_{m=1}^{M}\sum_{i=1}^{N}\tilde{R}_{m,i} \\ &\quad - \eta\left(\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}+P_c\right) + \sum_{m=1}^{M}\sum_{i=1}^{N}\beta_{m,i}(1-\alpha_{m,i}) \\ &\quad + \sum_{m=1}^{M}\chi_m(p_m^{\max} - \sum_{i=1}^{N}\bar{p}_{m,i}) + \sum_{m=1}^{M}\sum_{i=1}^{N}\mu_{m,i}\left(\tilde{R}_{m,i} - R_{m,i}^{\min}\right) \\ &\quad + \sum_{k=1}^{K}\varpi_k\left(I_k^{th} - \left\{\sum_{m=1}^{M}\sum_{i=1}^{N}\bar{p}_{m,i}(\bar{g}_{m,k}+\varepsilon_k)\right\}\right), \end{aligned} \tag{24}$$

where $\beta_{m,i}$, $\mu_{m,i}$, ϖ_k, and χ_m are non-negative Lagrange multipliers. Define $X = \alpha_{m,i}, \bar{p}_{m,i}, \beta_{m,i}, \chi_m, \mu_{m,i}, \varpi_k$, the Eq. (24) can be rewritten as

$$\begin{aligned} L\left(X\right) &= \sum_{m=1}^{M}\sum_{i=1}^{N}L_{m,i}(X) + \sum_{m=1}^{M}\sum_{i=1}^{N}\beta_{m,i} - \eta P_c \\ &\quad + \sum_{k=1}^{K}\varpi_k I_k^{th} + \sum_{m=1}^{M}\chi_m p_m^{\max} - \sum_{m=1}^{M}\sum_{i=1}^{N}\mu_{m,i}R_{m,i}^{\min}, \end{aligned} \tag{25}$$

where

$$\begin{aligned} L_{m,i}(X) &= (1+\mu_{m,i})\tilde{R}_{m,i} - \eta\bar{p}_{m,i} - \beta_{m,i}\alpha_{m,i} \\ &\quad - \chi_m\bar{p}_{m,i} - \bar{p}_{m,i}\sum_{k=1}^{K}\varpi_k(\bar{g}_{m,k}+\varepsilon_k). \end{aligned} \tag{26}$$

As a result, the dual problem of (23) becomes

$$\begin{aligned} \min_{\beta_{m,i},\chi_m,\mu_{m,i},\varpi_k} & D\left(\beta_{m,i}, \chi_m, \mu_{m,i}, \varpi_k\right) \\ s.t.\ & \beta_{m,i} \geq 0, \quad \chi_m \geq 0,\ \mu_{m,i} \geq 0,\ \varpi_k \geq 0, \end{aligned} \tag{27}$$

where the dual function is defined as

$$D\left(\beta_{m,i}, \chi_m, \mu_{m,i}, \varpi_k\right) = \max_{\alpha_{m,i},\bar{p}_{m,i}} L(X). \tag{28}$$

From the structures of (27) and (28), it belongs to a two-layer optimization problem. Namely, the inter layer is to solve the optimal power allocation $p_{m,i}^*$ and the user association factor $\alpha_{m,i}^*$. The outer layer is to get the optimal Lagrange multipliers $(\beta_{m,i}^*, \chi_m^*, \mu_{m,i}^*, \varpi_k^*)$. Moreover, problem (24) can be considered as $M \times N$ subproblems for maximizing the utility function of each SU i in SC m.

As a result, based on KKT conditions [37], the optimal power allocation is

$$p_{m,i}^* = \left[\frac{a_{m,i}\log_2(e)(1+\mu_{m,i})}{(\eta+\chi_m+\sum_{k=1}^{K}\varpi_k(\bar{g}_{m,k}+\varepsilon_k)}\right]^+, \tag{29}$$

where $[x]^+ = \max(0, x)$.

Based on the same method, the partial derivative of $\frac{\partial L_{m,i}(X)}{\partial \alpha_{m,i}}$ is

$$\frac{\partial L_{m,i}(X)}{\partial \alpha_{m,i}} = \varphi_{m,i} - \beta_{m,i} \begin{cases} < 0, & \alpha_{m,i} = 0, \\ = 0, & 0 < \alpha_{m,i} < 1, \\ > 0, & \alpha_{m,i} = 1, \end{cases} \tag{30}$$

where

$$\begin{aligned} \varphi_{m,i} = {} & (1+\mu_{m,i}) a_{m,i} \log_2 \left(\frac{p_{m,i}(\bar{h}_{m,i} - \delta_{m,i})}{(\bar{h}_{m,i} - \delta_{m,i}) \sum_{j=1}^{i-1} p_{m,j} + \sigma^2} \right) \\ & - \chi_m p_{m,i} - p_{m,i} \sum_{k=1}^{K} \varpi_k (\bar{g}_{m,k} + \varepsilon_k) \\ & + (1+\mu_{m,i}) b_{m,i} - \eta p_{m,i}. \end{aligned} \tag{31}$$

Thus, the optimal user association policy is

$$\alpha_{m^*,i} = 1 \left| m^* = \max_{\forall m} \varphi_{m,i}. \right. \tag{32}$$

Additionally, based on subgradient methods [37], Lagrange multipliers can be updated by

$$\chi_m^{t+1} = \left[\chi_m^t - s_1 \cdot (p_m^{\max} - \sum_{i=1}^{N} \alpha_{m,i} p_{m,i}^t) \right]^+, \tag{33}$$

$$\varpi_k^{t+1} = \left[\varpi_k^t - s_2 \cdot (I_k^{th} - \sum_{m=1}^{M} \sum_{i=1}^{N} \alpha_{m,i} p_{m,i}^t (\bar{g}_{m,k} + \varepsilon_k)) \right]^+, \tag{34}$$

$$\mu_{m,i}^{t+1} = \left[\mu_{m,i}^t - s_3 \cdot \left(\tilde{R}_{m,i}(p_{m,i}^t) - R_{m,i}^{\min} \right) \right]^+, \tag{35}$$

where t denotes the iteration index. s_1, s_2, and s_3 are positive step sizes. When the step sizes are very small, it can guarantee the convergence of the proposed algorithm [37], [40], [41].

Although the above (29)-(35) give a solution for the joint power allocation and user association problem, it still remains to design an algorithm to indicate the execution structure and the executing entity for the equations. Therefore, we propose an iterative Algorithm 1, which gives the procedures of the implementation.

Specifically, the maximum iteration number and error tolerance are firstly set up, and the initial EE is also given. Since the original problem is transformed into an equivalent subtractive form by Dinkelbach's method. Thus, the optimal auxiliary variable η is also equivalent to the optimal value of the objective function in problem (19). Then the user association and power allocation are optimized by the Lagrangian dual decomposition method until the EE converges. The flow chart of the proposed algorithm is also presented in Fig. 2.

Algorithm 1 An Iterative Robust RA Algorithm

Initialize the maximum number of iterations f_{max} for the outer layer loop, set the initial iteration $f = 0$, EE $\eta(f)$, the maximum tolerance δ, the upper bounds of ε_k and $\delta_{m,i}$. Define the maximum number of SUs N, SCs M, and MUs K, set the circuit power consumption P_c, the maximum transmit power of the SBS $p_m^{\max}$, the maximum interference power threshold I_k^{th}, and the minimum rate requirement $R_{m,i}^{\min}$.
Initialize transmit power $p_{m,i}$ with a uniform power allocation for all SUs [42].
Initialize $\alpha_{m,i}$ with the user association method in [43].
while $\left| \sum_{m=1}^{M} \sum_{i=1}^{N} \tilde{R}_{m,i}(p_{m,i}^f) - \eta \left\{ \sum_{m=1}^{M} \sum_{i=1}^{N} \bar{p}_{m,i}^f + P_c \right\} \right| > \delta$ or $f < f_{max}$ **do**
    Initialize the maximum number of iterations T_{max} for the inter layer loop, set the initial iteration $t = 0$, and Lagrange multipliers χ_m, ϖ_k, and $\mu_{m,i}$;
    repeat
    for $k = 1$ to K **do**
        for $m = 1$ to M **do**
            for $n = 1$ to N **do**
                1) update $p_{m,i}^t$ using (29);
                2) calculate $\varphi_{m,i}^t$ using (31);
                3) update $\alpha_{m^*,i}^t$ using (32);
                4) update χ_m^t, ϖ_k^t, and $\mu_{m,i}^t$ using (33)-(35), respectively;
            end for
        end for
    end for
    $t = t + 1$;
    until Convergence or $t = T_{max}$;
    $f = f + 1$, update $\eta(f)$ using (21).
end while

IV. PERFORMANCE ANALYSIS

In this section, to insight system performance, we give the analysis results of computational complexity, robust sensitivity and the impact of CSI error on outage probability.

A. COMPLEXITY ANALYSIS

The asymptotic complexity of the proposed algorithm is analyzed in this subsection. In Algorithm 1, the worst-case calculation of (31) for each SU on user association is $\mathcal{O}(MN)$. According to (33)-(35), the updates of χ_m, ϖ_k and $\mu_{m,i}$ are $\mathcal{O}(M)$, $\mathcal{O}(K)$, and $\mathcal{O}(MN)$, respectively. Since the maximum iteration number of T_{max} for converging the subgradient method is a polynomial function of operations [42], thus the total complexity of the proposed algorithm is $\mathcal{O}(M^2N^2KT_{max})$.

B. ROBUST SENSITIVITY

To insight the impact of uncertain parameter on total EE, we analyze the robust sensitivity in this subsection, namely, the performance gap between the proposed robust algorithm via problem (23) and the non-robust algorithm based on problem (6).

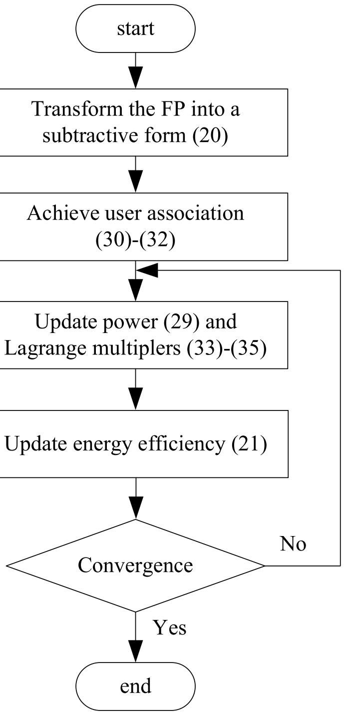

FIGURE 2. The flow chart of the proposed algorithm.

For the sake of problem analysis, define $F^{rob}(g_{m,k}, h_{m,i})$ and $F^{non}(\bar{g}_{m,k}, \bar{h}_{m,i})$ as the utility function with and without estimation errors, respectively, we have

$$\begin{aligned} &F^{rob}(\bar{g}_{m,k}+\varepsilon_k, \bar{h}_{m,i}-\delta_{m,i}) \\ &= \max\left\{\sum_{m=1}^{M}\sum_{i=1}^{N}\alpha_{m,i}\log_2\left(1+\frac{p_{m,i}}{\sum_{j=1}^{i-1}p_{m,j}+\frac{\sigma^2}{\bar{h}_{m,i}-\delta_{m,i}}}\right)\right. \\ &\quad \left. -\eta\left(\sum_{m=1}^{M}\sum_{i=1}^{N}\alpha_{m,i}p_{m,i}+P_c\right)\right\} \\ &\quad s.t.\ \bar{C}_1, \bar{C}_2, \hat{C}_3, \tilde{C}_4. \end{aligned} \tag{36}$$

$$\begin{aligned} &F^{non}(\bar{g}_{m,k}, \bar{h}_{m,i}) \\ &= \max\left\{\sum_{m=1}^{M}\sum_{i=1}^{N}\alpha_{m,i}\log_2\left(1+\frac{p_{m,i}}{\sum_{j=1}^{i-1}p_{m,j}+\frac{n_{m,i}}{\bar{h}_{m,i}}}\right)\right. \\ &\quad \left. -\eta\left(\sum_{m=1}^{M}\sum_{i=1}^{N}\alpha_{m,i}p_{m,i}+P_c\right)\right\} \\ &\quad s.t.\ \bar{C}_1, \bar{C}_2, \\ &\qquad C_6: R_{m,i}(\bar{h}_{m,i}) \geq R_{m,i}^{\min}, \\ &\qquad C_7: \sum_{m=1}^{M}\sum_{i=1}^{N}\alpha_{m,i}p_{m,i}\bar{g}_{m,i} \leq I_k^{th}. \end{aligned} \tag{37}$$

Thus, we can define performance gap as

$$P_{gap} = F^{rob}(g_{m,k}, h_{m,i}) - F^{non}(\bar{g}_{m,k}, \bar{h}_{m,i}). \tag{38}$$

According to the Lagrange function (24) and the dual function in (28), the impact of error on system performance can be derived from the distance of Lagrange function. Based on the robust sensitivity [44], when the uncertain parameter is very small, the optimal transmit power and Lagrange multipliers can be assumed to be same for the robust case and the non-robust case. Thus, define the optimal values $\alpha^*_{m,i}, p^*_{m,i}, \varpi^*_k, \mu^*_{m,i}$, we have the following relationship

$$P_{gap} \approx L^{rob}(X) - L^{non}(X), \tag{39}$$

where $L^{rob}(X)$ and $L^{non}(X)$ denote the Lagrange function of (36) and (37), respectively. Thus, the performance gap becomes

$$\begin{aligned} P_{gap} \approx &\sum_{m=1}^{M}\sum_{i=1}^{N}\alpha^*_{m,i}\Delta R_{m,i} \\ &-\sum_{m=1}^{M}\sum_{i=1}^{N}\mu^*_{m,i}R^{\min}_{m,i}\left(\frac{n_{m,i}}{\bar{h}_{m,i}-\delta_{m,i}}-\frac{n_{m,i}}{\bar{h}_{m,i}}\right) \\ &-\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{i=1}^{N}\varpi^*_k\alpha^*_{m,i}p^*_{m,i}\varepsilon_k, \end{aligned} \tag{40}$$

where

$$\begin{aligned} \Delta R_{m,i} = &\log_2\left(1+\frac{p^*_{m,i}}{\sum_{j=1}^{i-1}p^*_{m,j}+\frac{n_{m,i}}{\bar{h}_{m,i}-\delta_{m,i}}}\right) \\ &-\log_2\left(1+\frac{p^*_{m,i}}{\sum_{j=1}^{i-1}p^*_{m,j}+\frac{n_{m,i}}{\bar{h}_{m,i}}}\right) \leq 0. \end{aligned} \tag{41}$$

Because of $\delta_{m,i} \geq 0$ and $\varepsilon_k \geq 0$, combining (40) with (41), we have the conclusion as follows

$$P_{gap} \leq 0. \tag{42}$$

Thus, the total utility function under imperfect CSI is smaller than that under perfect CSI.

C. IMPACT OF IMPERFECT CSI ERROR ON USER's OUTAGE PROBABILITY

In this subsection, we will analyze how the upper bounds of channel estimation errors influence the user's outage probability. In other words, we try to explain why the designed algorithm has good robustness without any outage.

Since multiple SUs are coupled in C_4, it is difficult to directly analyze. Thus, to show the impact, we assume there is one SU in the network. The optimal transmit power becomes

$$p^*_{m,i} = \frac{I_k^{th}}{g_{m,i}}, \tag{43}$$

where $g_{m,i} = \bar{g}_{m,i} + \Delta g_{m,i}$. When we consider the case of perfect CSI (e.g., $\Delta g_{m,i} = 0$), the designed transmit power is

$$p_{m,i}^{non} = \frac{I_k^{th}}{\bar{g}_{m,i}}. \tag{44}$$

Accordingly, the robust transmit power is

$$p_{m,i}^{rob} = \frac{I_k^{th}}{\bar{g}_{m,i} + \varepsilon_k}, \tag{45}$$

where the upper bound of uncertainty is $\varepsilon_k \geq 0$. Therefore, we have

$$p_{m,i}^{rob} \leq p_{m,i}^{non}. \tag{46}$$

When this transmit power is applied in practical HetNets, the actual interference received at the MU under perfect CSI and imperfect CSI is $I^{non} = p_{m,i}^{non} g_{m,i}$ and $I^{rob} = p_{m,i}^{rob} g_{m,i}$, respectively. Thus, we have the following interference power

$$I^{non} = \frac{\bar{g}_{m,i} + \Delta g_{m,i}}{\bar{g}_{m,i}} I_k^{th} = I_k^{th} + \frac{\Delta g_{m,i}}{\bar{g}_{m,i}} I_k^{th}. \tag{47}$$

$$I^{rob} = \frac{\bar{g}_{m,i} + \Delta g_{m,i}}{\bar{g}_{m,i} + \varepsilon_k} I_k^{th}. \tag{48}$$

Remark: when the channel estimation error satisfies $\Delta g_{m,i} \leq 0$, namely, the channel gain is overestimated, such as $\bar{g}_{m,i} > g_{m,i}$. We have $I^{non} < I_k^{th}$ and $I^{rob} < I_k^{th}$. The QoS requirement of MU is well protected, there is no outage. When the channel estimation error satisfies $\Delta g_{m,i} > 0$ and $\Delta g_{m,i} \leq \varepsilon_k$, the channel gain is underestimated, such as $\bar{g}_{m,i} < g_{m,i}$. We have $I^{non} > I_k^{th}$ and $I^{rob} \leq I_k^{th}$. Therefore, the non-robust algorithm can cause user's outage. And the outage probability can be calculated by

$$Outage = \frac{\max(I^{non} - I_k^{th}, 0)}{I_k^{th}}. \tag{49}$$

Thus, our designed robust algorithm has good robustness.

V. SIMULATION RESULTS

In this section, the performance of the proposed algorithm under different parameter conditions will be discussed. In addition, the robustness and effectiveness of the proposed algorithm are compared with other algorithms. There is one macrocell and two SCs in the simulation, the coverage radius of MBS and SBS are 500 m and 20 m [15], [21], [22], respectively. The path-loss model is assumed to be $PL = (\beta/d^{\alpha})$, where d is the distance from the BS to user. The path-loss exponent α varies from 2 to 5, depending on the environment. And the attenuation parameter β is frequency-dependent [45]. Other simulation parameters are given in Table 3.

A. PERFORMANCE ANALYSIS OF THE PROPOSED ALGORITHM

In this subsection, we will show the performance of the proposed algorithm under different system parameters.

Fig. 3 shows the convergence of transmit power under the scenario where two SUs in each SC. Where p_1^{sum} and p_2^{sum} are the actual sum transmit power of 1-th and 2-th SBS, respectively. It can be seen that the algorithm can achieve convergence after about 13 iterations, and the sum transmit power of SBSs are under the transmit power threshold $p_m^{\max}$, which shows that the algorithm has good convergence performance.

Fig. 4 presents the actual interference power from SBSs to MU under different estimation error ϵ_k. It can be seen that the actual interference power can also quickly converge without exceeding the interference power threshold I_k^{th}. On the other hand, the actual interference power decreases

TABLE 3. Simulation parameters.

System parameters	Values
Number of MUs K	5
Number of SUs N	2
Transmit power threshold $p_m^{\max}$	1 W
Interference power threshold I_k^{th}	0.015 W
Minimum rate requirement $R_{m,i}^{\min}$	1 bps/Hz
Background noise power σ^2	10^{-8} W
Total circuit power consumption P_c	0.3 W
Upper bound of estimation error $\delta_{m,i}$	0.1
Upper bound of estimation error ϵ_k	0.01

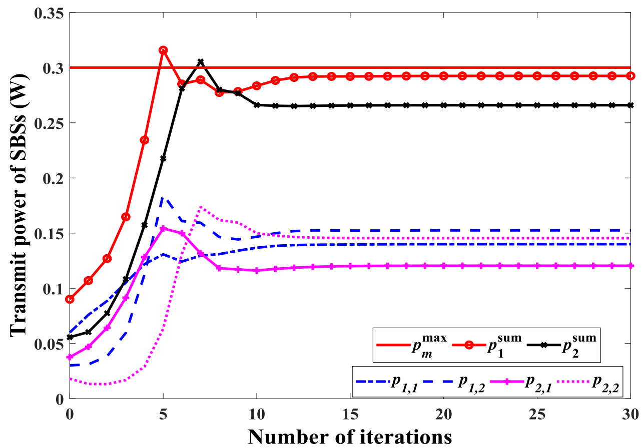

FIGURE 3. The convergence of transmit power. ($p_{1,1}$ and $p_{1,2}$: transmit power form SC 1 to SU 1 and SU 2; $p_{2,1}$ and $p_{2,2}$: transmit power from the 2-th SC to the corresponding SU 1 and SU 2).

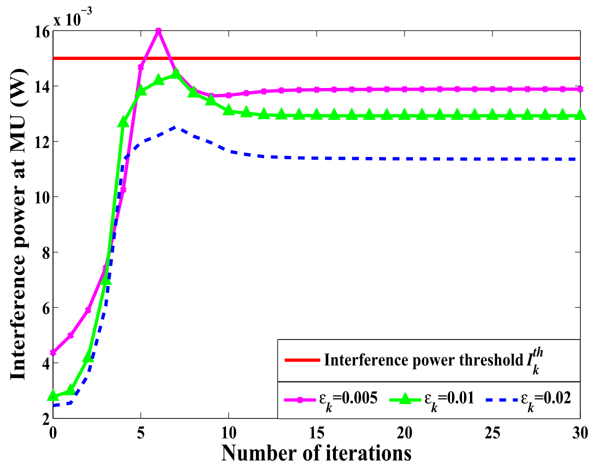

FIGURE 4. Actual interference power of MU under different ϵ_k.

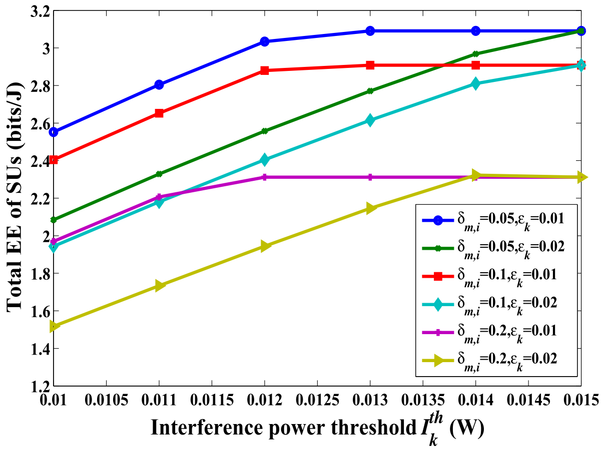

FIGURE 5. Total EE of SUs versus interference power threshold with different $\delta_{m,i}$ and ϵ_k.

as the increasing of channel estimation error ϵ_k. The reason is that the larger the channel estimation error is considered, the greater the uncertainty of the wireless channel environment is. And the SBS needs to reduce the transmit power to meet the interference constraints.

Fig. 5 depicts the relationship between total EE of SUs and interference power threshold I_k^{th} under the different estimation errors $\delta_{m,i}$ and ϵ_k. It can be seen that as the interference power threshold increases, the total EE of SUs first increases and then stabilizes. The reason is that the feasible range of the SBS's transmit power increases as the interference power threshold increases. Hence, the proposed algorithm can further achieve optimal RA to improve the total EE of the SUs. However, the transmit power of the SBS will be restricted by the transmit power threshold $p_m^{\max}$, so the total EE will eventually stabilize. Moreover, under the same estimation error of transmission link $\delta_{m,i}$, larger interference link estimation error ϵ_k will reduce total EE, and will cause the total EE stabilize at a higher interference threshold. These are because larger ϵ_k will reduce the feasible range of SBS's transmit power. Besides, under the same estimation error of interference link ϵ_k, lager estimation error of transmission link $\delta_{m,i}$ will reduce total EE. The reason is that the larger the $\delta_{m,i}$ is, the more unstable the channel conditions of the transmission link are, which will lead to a reduction on the total rate of SUs. Thus, the total EE of SUs will be affected accordingly.

Fig. 6 shows the total EE of SUs versus channel uncertainties of transmission link $\Delta h_{m,i}$ under different total circuit power consumption P_c. Where $\Delta h_{1,i}$ and $\Delta h_{2,i}$ denote the channel uncertainties of transmission link in 1-th and 2-th SCs, respectively. From Fig. 6, it can be seen that the total EE of SUs increase with the increasing of $\Delta h_{m,i}$. This can be interpreted as that as the channel uncertainties of transmission link $\Delta h_{m,i}$ increase, the SINR of the SUs also increases. Therefore, the data rate of SUs will increase accordingly, eventually leading to an improvement in total energy efficiency. Moreover, obviously, the total EE of SUs decrease with the increasing of total circuit power consumption.

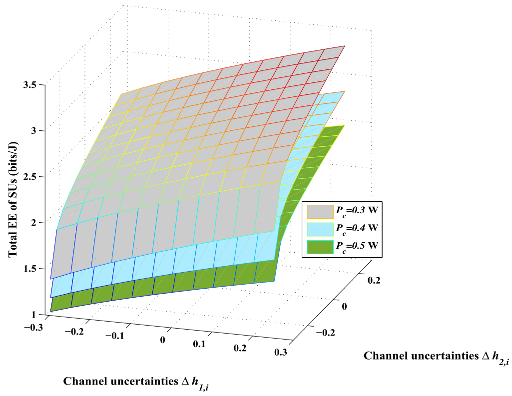

FIGURE 6. Total EE of SUs versus channel uncertainties $\Delta h_{m,i}$ under different P_c.

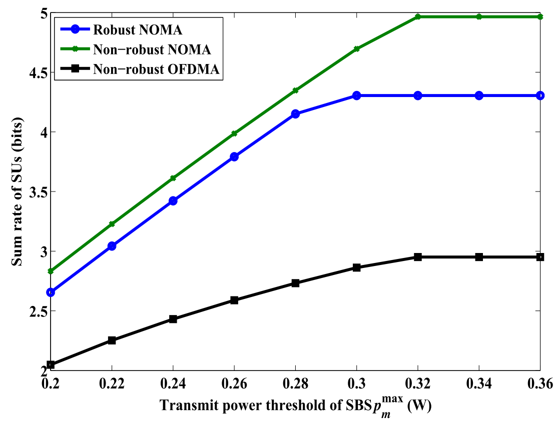

FIGURE 7. Sum rate of SUs versus $p_m^{\max}$ under different algorithms.

B. COMPARISONS WITH OTHER ALGORITHMS

In this subsection, the effectiveness of the proposed algorithm will be demonstrated by comparing with the existing algorithms. The proposed robust RA algorithm is defined as 'Robust NOMA'. The NOMA-based EE maximization algorithm under perfect CSI is defined as 'Non-robust NOMA'. And the orthogonal frequency division multiple access (OFDMA) based rate maximization algorithm under perfect CSI is defined as 'Non-robust OFDMA'.

Fig. 7 gives the sum rate of SUs versus transmit power threshold of SBS $p_m^{\max}$ under different algorithms. With the increasing of SBS's transmit power threshold, the sum rate of SUs first increases and then stabilizes. This is due to higher transmit power threshold makes the transmit power feasible region of SBS larger, which causes the sum rate of SUs improve. But the higher transmit power of SBSs will lead to greater actual interference power to MU, which will eventually be constrained by the interference power threshold. Besides, the sum rate of the two algorithms based on NOMA is higher than that based on OFDMA. The reason is that NOMA allows multiple users to share the same subchannel, which effectively improves the system rate. In addition, the sum rate of non-robust NOMA is higher than robust NOMA. The reason is that the proposed robust NOMA algorithm takes into account channel estimation errors in advance, which reduces the feasible range of transmit power.

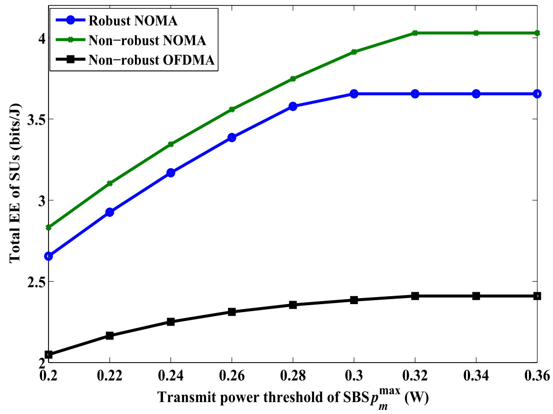

FIGURE 8. Total EE of SUs versus p_m^{max} under different algorithms.

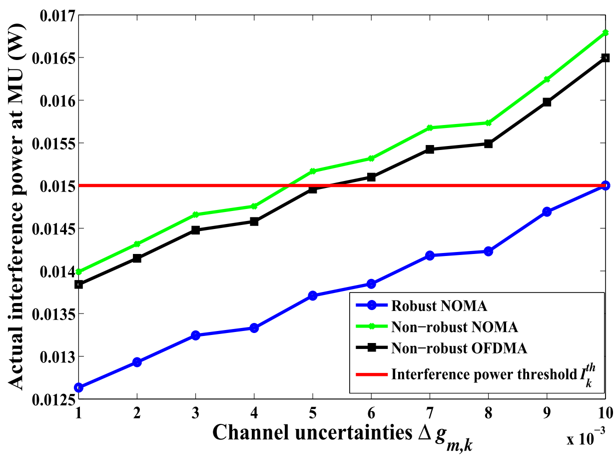

FIGURE 9. Actual interference power of MU versus channel uncertainties $\Delta g_{m,k}$ under different algorithms.

Fig. 8 depicts the total EE of SUs versus transmit power threshold of SBS p_m^{max} under different algorithms. Similarly, due to the influence of the feasible range of SBS's transmit power and the interference power threshold of MU, total EE of SUs first increases with transmit power threshold and then stabilizes. Moreover, the non-robust NOMA has the highest total EE, and the total EE of non-robust OFDMA is the lowest.

Fig. 9 gives the relationship of actual interference power at MU and channel uncertainties of interference link $\Delta g_{m,k}$ under different algorithms. It is clear that the actual interference power at MU under four algorithms increases with the bigger channel uncertainties, since the higher interference cannot be avoided when the channel uncertainties become greater. Moreover, with the increasing channel uncertainties, only the proposed robust NOMA algorithm can limit the actual interference power from SBS to the MU below the interference power threshold. It is clear that the robust algorithms have a good robustness to give a better protection for MUs as the cost of lower system performance.

VI. CONCLUSION

In this paper, we have studied the robust EE-based maximization problem in a two-tier heterogeneous NOMA network under bounded channel uncertainties by jointly optimizing transmit power and user association. Due to the complexity of the originally non-convex problem, we converted it into a deterministic and convex optimization problem by using the worst-case approach and Dinkelbach's method. Furthermore, we derived the closed-form solutions of power allocation and user association by introducing KKT conditions and Lagrange dual approach. To insight system performance, we further achieved the complexity analysis, robust sensitivity, and the impact of imperfect CSI error on user's outage probability. Simulation results demonstrated the proposed algorithm had good robustness and can reduce the outage probabilities of MUs.

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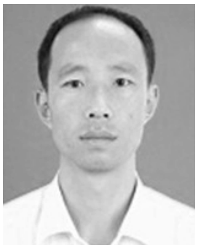

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