IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

SPECIAL SECTION ON GREEN INTERNET OF THINGS

Received January 30, 2020, accepted February 22, 2020, date of publication March 6, 2020, date of current version March 18, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2979055

DOA Estimation of Strictly Noncircular Sources in Wireless Sensor Array Network via Block Sparse Representation

LIANGLIANG LI[1], TINGTING FU[1], XIANPENG WANG[1], (Member, IEEE), MENGXING HUANG[1], LIANGTIAN WAN[2], (Member, IEEE), AND YONGQIN YANG[1]

[1]State Key Laboratory of Marine Resource Utilization in South China Sea, School of Information and Communication Engineering, Hainan University, Haikou 570228, China

[2]Key Laboratory for Ubiquitous Network and Service Software of Liaoning Province, School of Software, Dalian University of Technology, Dalian 116620, China

Corresponding authors: Xianpeng Wang (wxpeng1986@126.com) and Mengxing Huang (huangmx09@163.com)

This work was supported in part by the Key Research and Development Program of Hainan Province under Grant ZDYF2019011, in part by the National Natural Science Foundation of China under Grant 61701144, Grant 61861015, and Grant 61961013, in part by the Scientific Research Projects of University in Hainan Province under Grant Hnky2018ZD-4, in part by the Young Elite Scientists Sponsorship Program by the China Association for Science and Technology (CAST) under Grant 2018QNRC001, in part by the Scientific Research Setup Fund of Hainan University under Grant KYQD (ZR) 1731, and in part by the Program of Hainan Association for Science and Technology Plans to Youth Research and Development Innovation under Grant QCXM201706.

ABSTRACT In this article, the direction of arrival estimation (DOA) of strictly noncircular sources with unknown mutual coupling is considered for wireless sensor array network (WSAN), and then a joint weighted block sparse recovery algorithm based on weighted subspace fitting (WSF) is proposed for DOA estimation.. In the proposed method, two block sparse representation models associated with the WSF principle are firstly constructed to remove the influence of unknown mutual coupling. Then, combining the advantages of noncircularity, a joint weighted block sparse recovery scheme is proposed to estimate DOA, in which the noncircular MUSIC-like (NC MUSIC-Like) spectrum function is utilized to form a weighted matrix for enhancing the solutions sparsity. Finally, the desired DOAs can be achieved with the help of the reconstructed block sparse matrix. Extensive experiments are simulated to verify that the proposed method can achieve superior estimation performance under the condition of unknown mutual coupling.

INDEX TERMS DOA estimation, noncircular sources, unknown mutual coupling, weighted block sparse recovery, weighted subspace fitting.

I. INTRODUCTION

Wireless sensor array network (WSAN) technique is widely applied in the field of communications and radar during the last decades [1]. As a vital research branch and hot spot, direction-of-arrival (DOA) estimation for WSAN has drawn remarkable attention. It gets well utilized in many fields, such as radar, medical diagnosis, sources location, etc. [2], [3]. It's evident that much work has been done on DOA estimation, accompanied by a series of corresponding high-resolution estimation algorithms [4]–[7]. According to distinct principles of estimating DOA, it can be deemed that there are two main categories for existing methods, which are subspace-based algorithms and sparse signal recovery (SSR) algorithms, respectively. The conventional subspace-based algorithms are famous for its high-resolution capability in the past few years. Among them, multiple signal classification (MUSIC) algorithm [4] and estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [5] can be regarded as classical representatives of the subspace-based methods. However, they are all rely on the eigenvalue decomposition (EVD) of covariance matrix, which indicates that those subspace-based methods may be unable to work properly in the challenging environment, such as limited snapshots or/and low signal-to-noise ratio (SNR) [6], [7]. After that, SSR technique are considerably applied in DOA estimation [8]–[13], which including l_1-SVD (singular value decomposition) algorithm [8], sparse

The associate editor coordinating the review of this manuscript and approving it for publication was Zhenyu Zhou.

Bayesian learning (SBL) based algorithm [9] and their corresponding variants [10], [11]. Compared with the traditional subspace-based algorithms, many experiment results have illustrated that SSR-based methods do much better than traditional subspace-based methods [12], [13].

Obviously, these methods mentioned above presuppose that the radiation signals are complex circular. Nevertheless, with the further study of signal inherent properties, noncircular sources have emerged and occupied an increasingly important position in angular estimation region [14], [15]. And amplitude modulation (AM), binary phase shift keying (BPSK) are commonly noncircular sources. The main difference between noncircular sources and circular sources is weather the corresponding elliptic covariance matrix would tend to be zero. For noncircular sources, its elliptic covariance matrix is non-zero, which indicates that the information utilization can be significantly improved via exploiting the elliptic covariance matrix information as much as possible. Then, based on noncircular sources, much related studies for DOA estimation have been implemented [16]–[21]. On the one hand, a novel noncircular MUSIC-like (NC MUSIC-like) algorithm is proposed [16], in which the augmented date model is established by the received data and its corresponding conjugate data form. Simulation results are well declared that the noncircularity can be adopted to improve detection ability. Following this idea, an improved root NC MUSIC-like method [17] and conjugate ESPRIT-like methods [18] are further presented to address the problem of large-scale spectral peak search of MUSIC-based methods. Furthermore, as shown in [19], the most challenged scenario where the circular and strictly NC sources (the noncircularity rate ρ of signals is maximum. i.e. $\rho = 1$) coexist is considered. On the other hand, the problem of DOA estimation for noncircular sources is well solved from the perspective of sparse recovery [20], [21]. In [20], a joint sparsity-aware scheme is structured by taking the noncircularity into account. The similar work is also done in [21], while the weighted constraint measure is further adopted by the latter to enhance the sparsity of the solutions.

It's worth noting that these methods mentioned above are all based on the ideal steering matrix. Whereas, in practice, the distance between sensors will be decreased as the number of sensors increases if the array aperture is fixed, which means that the closely spaced antennas would appear the unknown mutual coupling effect. Then the performance of those aforementioned algorithms would be considerably compromised due to disturbed array manifold. In order to deal with this problem, many calibrated methods have been designed [22]–[28]. In [22], with the help of auxiliary arrays, an ESPRIT-like method is exploited via considering the particular banded complex symmetric Toeplitz structure of mutual coupling matrix (MCM). While the additional array compensation is needed, which leads to great inconvenience in reality. Then by applying a parameterized operation to the actual disturbed array manifold, a novel block-structure steering matrix is deduced to avoid the unknown mutual coupling effect [23]. Unfortunately, this subspace-based method is still limited by snapshots and SNRs although without losing the array aperture. Moreover, from the sparse recovery aspect, the particular selection matrix is put forward to eliminate the interference of unknown mutual coupling [24], [25]. But they all suffer from array aperture loss. Furthermore, the parameterized operation utilizing the entire array aperture is re-applied to SSR region [26]. Meanwhile, based on the above parameterization idea, two weighted block sparse recovery algorithms are further presented in data domain and correlation domain, respectively [27], [28]. While it can be observed that above methods only consider noncircular sources or unknown mutual coupling.

In this article, considering both noncircular sources and unknown mutual coupling in WSAN, a weighted subspace fitting (WSF) framework via block sparse representation is presented to estimate DOA. Firstly, based on the principle of WSF, two block-structured sparse data models are structured to deal with the influence of unknown mutual coupling without losing the array aperture. Then, combining the noncircularity, a joint weighted block sparse recovery framework is established for DOA estimation. The extensive experiment results illustrate that the proposed method not only works normally with unknown mutual coupling but also is superior to others.

Notation: $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^*$, $diag\{\cdot\}$ and $blkdiag\{\cdot\}$ denote conjugate-transpose, transpose, conjugate, diagonalization operation and block diagonalization operation, respectively. $\boldsymbol{I}_M$ is an unit matrix with $M \times M$ dimension. $\boldsymbol{\Sigma}_M$ stands for an $M \times M$ dimensional exchange matrix with ones on the anti-diagonal and zeros elsewhere. $||\cdot||_0$, $||\cdot||_1$, $||\cdot||_2$ and $||\cdot||_F$ represent l_0-norm, l_1-norm, l_2-norm and Frobenius norm, respectively. $det|\cdot|$, $E[\cdot]$ and $tr\{\cdot\}$ mean determinant operations, mathematical expectation and trace operation, respectively.

II. DATA MODEL

In this section, the data model for strictly noncircular signals in WSAN under the condition of unknown mutual coupling is formulated. Combining noncircular sources, the augmented signal model is firstly formulated by the original received data and its corresponding conjugate form. Then the effect of unknown mutual coupling is well avoided via parameterizing the steering vector.

A. NONCIRCULAR AUGMENTED DATA MODEL WITH UNKNOWN MUTUAL COUPLING

Suppose that there is a uniform linear array (ULA) composed of M sensors in WASN, and each of sensors is separated by half-wavelength. K far -field narrowband and independent NC signals $\{s_k(t)\}_{k=1}^K$ with distinct direction $\{\theta_k\}_{k=1}^K$ incident to the ULA. t represents the sample snapshot. And the first sensor is selected as the reference standard. Then the received data at the t-th snapshot can be depicted as

$$z(t) = \boldsymbol{A}\boldsymbol{s}_d(t) + \boldsymbol{e}(t) \tag{1}$$

where $\boldsymbol{z}(t) = [z_1(t), z_2(t), \ldots, z_M(t)]^T$ denotes the received data from the array. $\boldsymbol{s}_d(t) = [s_{d,1}(t), s_{d,2}(t), \ldots, s_{d,K}(t)]^T$ means the noncircular source vector. $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \ldots, \boldsymbol{a}(\theta_K)] \in \mathbb{C}^{M\times K}$ represents the ideal array manifold matrix which is made up of distinct steering vectors $\boldsymbol{a}(\theta_k)$. And $\boldsymbol{a}(\theta_k) = [1, \eta(\theta_k), \ldots, \eta^{M-1}(\theta_k)]^T \in \mathbb{C}^{M\times 1}$ in which $\eta(\theta_k) = e^{-j2\pi d/\lambda \sin(\theta_k)}$. λ stands for the signal wavelength and d means the distance between adjacent sensors. $\boldsymbol{e}(t) = [e_1(t), e_2(t), \ldots, e_M(t)]^T \in \mathbb{C}^{M\times 1}$ denotes the complex Gaussian white noise, whose mean and noise power are zero and σ_e^2, respectively.

As described in [21], the second-order statistical properties of noncircular signals satisfy the following expression

$$\boldsymbol{E}\{\boldsymbol{Y}\boldsymbol{Y}^T\} = \rho e^{j\psi}\delta^2 = \rho e^{j\psi}\boldsymbol{E}\{\boldsymbol{Y}\boldsymbol{Y}^H\} \tag{2}$$

where ψ is the noncircular phase. Noncircular rate ρ satisfies $0 < \rho \leq 1$ and relies on the signal modulation type. In addition, strictly noncircular source with $\rho = 1$ is considered in this article, such as AM, BPSK modulation signals. Then strictly noncircular source vector $\boldsymbol{s}_d(t)$ can be further represented as

$$\boldsymbol{s}_d(t) = \boldsymbol{\psi}^{1/2}\boldsymbol{s}(t) \tag{3}$$

where $\boldsymbol{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T \in \mathbb{R}^{K\times 1}$ denotes the real-valued version of the incident signal $\boldsymbol{s}_d(t)$. The diagonal matrix $\boldsymbol{\Psi}^{1/2} = diag(e^{j\psi_1/2}, e^{j\psi_2/2}, \ldots, e^{j\psi_K/2})$ is composed of the phase shift of the impinging sources.

It's known that unknown mutual coupling may appear between sensors in reality when spatial electromagnetic field of the closely spaced sensors interacts with each other. According to the introduction of [28], a banded complex symmetric Toeplitz matrix can be utilized to model the mutual coupling matrix (MCM). i.e.

$$\boldsymbol{G} = Toeplitz([1, g_1, \ldots, g_{H-1}, 0_{1\times(M-H)}]) \tag{4}$$

where $\{g_i\}_{i=0}^{H-1}$ mean the non-zero mutual coupling coefficients which satisfy $g_0 = 1 > |g_1| >, \ldots, > |g_{H-1}| > 0$. And H denotes the maximum distance between sensors over which the influence of mutual coupling cannot be neglected. In other words, electromagnetic disturbance may fail to work when the sensors are far away, so that the residual elements in MCM can be deemed as zeroes.

Thus, the received data in Eq. (1) in the environment of unknown mutual coupling can be revised as follows

$$\boldsymbol{z}(t) = \boldsymbol{G}\boldsymbol{A}\boldsymbol{s}_d(t) + \boldsymbol{e}(t) = \widehat{\boldsymbol{A}}\boldsymbol{s}_d(t) + \boldsymbol{e}(t) \tag{5}$$

where $\widehat{\boldsymbol{A}} = \boldsymbol{G}\boldsymbol{A} = [\widehat{\boldsymbol{a}}(\theta_1), \widehat{\boldsymbol{a}}(\theta_2), \ldots, \widehat{\boldsymbol{a}}(\theta_K)]$ is the actual steering matrix with $\widehat{\boldsymbol{a}}(\theta_k) = \boldsymbol{G}\boldsymbol{a}(\theta_k)$. It can be found that compared with the ideal steering vector $\boldsymbol{a}(\theta_k)$, the actual steering vector $\widehat{\boldsymbol{a}}(\theta_k)$ isn't of the great structure because of the existence of non-zero mutual coupling coefficients.

Based on the noncircularity of strictly NC signals, the corresponding conjugate form of $\boldsymbol{z}(t)$ in Eq. (5) can be further written as

$$\boldsymbol{z}^*(t) = (\widehat{\boldsymbol{A}}\boldsymbol{s}_d(t) + \boldsymbol{e}(t))^* = \boldsymbol{G}^*\boldsymbol{A}^*\boldsymbol{\psi}^*\boldsymbol{s}_d(t) + \boldsymbol{e}^*(t) \tag{6}$$

The array aperture of noncircular sources can be virtually doubled by constructing an extended data model, which indicates that the DOA estimation performance can be further improved. Based on the above analysis, taking a combination of the noncircular signal model in Eq. (5) and its conjugate model in Eq. (6), a novel augmented model of array output can be constructed as

$$\begin{aligned}\hat{z}(t) &= \begin{bmatrix} z(t) \\ \boldsymbol{\Sigma}_M z^*(t) \end{bmatrix} = \begin{bmatrix} \widehat{\boldsymbol{A}}\boldsymbol{s}_d(t) + \boldsymbol{e}(t) \\ \boldsymbol{\Sigma}_M(\widehat{\boldsymbol{A}}\boldsymbol{s}_d(t) + \boldsymbol{e}(t))^* \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{G}\boldsymbol{A} \\ \boldsymbol{\Sigma}_M\boldsymbol{G}^*\boldsymbol{A}^*\boldsymbol{\psi}^* \end{bmatrix} s_d(t) + \begin{bmatrix} \boldsymbol{e}(t) \\ \boldsymbol{\Sigma}_M\boldsymbol{e}^*(t) \end{bmatrix} \\ &= \hat{\boldsymbol{D}}s_d(t) + \hat{\boldsymbol{e}}(t) \end{aligned} \tag{7}$$

where $\hat{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{G}\boldsymbol{A} \\ \boldsymbol{\Sigma}_M\boldsymbol{G}^*\boldsymbol{A}^*\boldsymbol{\psi}^* \end{bmatrix}$ means a $2M \times K$ dimensional extended steering matrix, and $\hat{\boldsymbol{e}}(t) = \begin{bmatrix} \boldsymbol{e}(t) \\ \boldsymbol{\Sigma}_M\boldsymbol{e}^*(t) \end{bmatrix} \in \mathbb{C}^{2M\times 1}$ stands for an augmented additive white noise matrix.

Then, the covariance matrix of the latest augmented array output in Eq. (7) can be expressed as

$$\hat{\boldsymbol{R}} = E\{\hat{z}(t)\hat{z}^H(t)\} = \hat{\boldsymbol{D}}\hat{R}_s\hat{\boldsymbol{D}}^H + \sigma_e^2\boldsymbol{I}_{2M} \tag{8}$$

where $\hat{\boldsymbol{R}}_s = E\{\boldsymbol{s}_d(t)\boldsymbol{s}_d^H(t)\}$, whose rank is deemed as K. Obviously, $\hat{\boldsymbol{R}}$ just stays at the theoretical level which is not suitable for the actual case of finite snapshots number. Hence considering T snapshots, the sample covariance matrix $\tilde{R}$ can be obtained to substitute $\hat{\boldsymbol{R}}$. i.e. Then, the covariance of the latest extended array output model in Eq. (9) can be

$$\tilde{\boldsymbol{R}} = \frac{1}{T}\sum\nolimits_{t=1}^{T}\hat{z}(t)\hat{z}^H(t) \tag{9}$$

Then applying the eigenvalue decomposition to $\hat{\boldsymbol{R}}$ in Eq. (9), yields

$$\tilde{\boldsymbol{R}} = \sum\nolimits_{m=1}^{2M}\gamma_m\boldsymbol{v}_m\boldsymbol{v}_m^H = \boldsymbol{E}_s\boldsymbol{\Lambda}_s\boldsymbol{E}_s^H + \boldsymbol{E}_e\boldsymbol{\Lambda}_e\boldsymbol{E}_e^H \tag{10}$$

where $\{\gamma_m\}_{m=1}^{2M}$, $\{\boldsymbol{v}_m\}_{m=1}^{2M}$ are the eigenvalues and its corresponding eigenvectors, respectively. In addition, these eigenvalues satisfy $\gamma_1 \geq \gamma_2 \geq, \ldots, \geq \gamma_K > \gamma_{K+1} =, \ldots, = \gamma_{2M}$. $\boldsymbol{\Lambda}_s = diag\{\gamma_1, \gamma_2, \ldots, \gamma_K\}$, $\boldsymbol{\Lambda}_e = diag\{\gamma_{K+1}, \gamma_{K+2}, \ldots, \gamma_{2M}\}$. $\boldsymbol{E}_s = [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_K]$ represents the signal subspace and relates to the K largest eigenvalues. Similarly, $\boldsymbol{E}_e = [\boldsymbol{v}_{K+1}, \boldsymbol{v}_{K+2}, \ldots, \boldsymbol{v}_{2M}]$ is regard as noise subspace [29].

B. PARAMETERIZATION OF THE ARRAY MANIFOLD UNDER THE CONDITION OF UNKNOWN MUTUAL COUPLING

It has been pointed that the signal subspace $\boldsymbol{E}_s$ spans the same subspace as that of array manifold matrix $\hat{\boldsymbol{D}}$ [30], which means that the expression can be obtained as follows

$$\boldsymbol{E}_s = \hat{\boldsymbol{D}}U = \begin{bmatrix} \boldsymbol{G}\boldsymbol{A} \\ \boldsymbol{\Sigma}_M\boldsymbol{G}^*\boldsymbol{A}^*\boldsymbol{\psi}^* \end{bmatrix}\boldsymbol{U} \tag{11}$$

where $\boldsymbol{U}$ is a $K \times K$ dimensional column full rank matrix. Whereas it's difficult to build an over-complete dictionary for sparse representation model because of the coexistence of unknown noncircular phases and non-zero mutual coupling coefficients, which indicates that these negative effects must be dealt with. Thus, following the parameterized operation in [23], the actual array manifold $\widehat{\boldsymbol{a}}(\theta)$ can be turned into a calibrated block-structure vector to get rid of the influence of unknown mutual coupling. i.e.

$$\widehat{\boldsymbol{a}}(\theta) = \boldsymbol{G}\boldsymbol{a}(\theta) = \boldsymbol{\Omega}(\theta)\boldsymbol{\Gamma}(\theta) \tag{12}$$

where

$$\boldsymbol{\Omega}(\theta) = blkdiag\{\boldsymbol{\Omega}_1, \boldsymbol{\Omega}_2, \boldsymbol{\Omega}_3\} \in \mathbb{C}^{M\times F} \tag{13}$$

$$\boldsymbol{\Gamma}(\theta) = [\lambda_1(\theta), \ldots, \lambda_{H-1}(\theta), \tau(\theta), \sigma_1(\theta), \ldots, \sigma_{H-1}(\theta)]^T \tag{14}$$

with $\boldsymbol{\Omega}_1 = diag\{1, \eta(\theta), \ldots, \eta^{H-2}(\theta)\} \in \mathbb{C}^{(H-1)\times(H-1)}$, $\boldsymbol{\Omega}_2 = \{\eta^{H-1}(\theta), \eta^{H}(\theta), \ldots, \eta^{M-H}(\theta)\}^T \in \mathbb{C}^{(M-2H+2)}$, $\boldsymbol{\Omega}_3 = diag\{\eta^{M-H+1}(\theta), \eta^{M-H+2}(\theta), \ldots, \eta^{M-1}(\theta)\} \in \mathbb{C}^{(H-1)\times(H-1)}$ with $F = 2H-1$. $\lambda_d(\theta) = 1+\sum_{i=1}^{H-1} g_i\eta^i(\theta)+\sum_{i=1}^{d-1} g_i\eta^{-i}(\theta)$, $\sigma_d(\theta) = 1 + \sum_{i=1}^{H-1} g_i\eta^{-i}(\theta) + \sum_{i=1}^{H-d-1} g_i\eta^{i}(\theta)$, $\tau(\theta) = \sum_{i=1-H}^{H-1} g_{|i|}\eta^i(\theta)$. Besides, $\tau(\theta)$ is deemed as non-zero since the occurrence probability of the particular event $\tau(\theta) = 0$ is very low [23]. Hence, applying Eq. (12) to Eq. (7), the augmented model in Eq. (7) can be re-expressed as

$$\hat{z}(t) = \begin{bmatrix} z(t) \\ \boldsymbol{\Sigma}_M z^*(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{\Delta} \\ \boldsymbol{\Sigma}_M\boldsymbol{B}^*\boldsymbol{\Delta}^*\boldsymbol{\psi}^* \end{bmatrix} s_d(t) + \begin{bmatrix} \boldsymbol{e}(t) \\ \boldsymbol{\Sigma}_M\boldsymbol{e}^*(t) \end{bmatrix} = \boldsymbol{D}\boldsymbol{s}(t) + \hat{\boldsymbol{e}}(t) \tag{15}$$

where $\boldsymbol{D} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{\Delta} \\ \boldsymbol{\Sigma}_M\boldsymbol{B}^*\boldsymbol{\Delta}^*\boldsymbol{\psi}^* \end{bmatrix} \in \mathbb{C}^{2M\times K}$ denotes the novel calibrated augmented array manifold matrix. $\boldsymbol{B} = [\boldsymbol{\Omega}(\theta_1), \boldsymbol{\Omega}(\theta_2), \ldots, \boldsymbol{\Omega}(\theta_K)] \in \mathbb{C}^{M\times KF}$ is the block steering matrix with respect to DOA information and free of the unknown mutual coupling coefficients. In addition, it has a distinct difference between the new calibrated steering vector $\boldsymbol{\Omega}(\theta)$ and the actual array manifold $\widehat{\boldsymbol{a}}(\theta)$. After taking this parameterized operation, the non-zero unknown mutual coupling coefficients are transferred to the latter block diagonal matrix $\boldsymbol{\Delta} = blkdiag\{\boldsymbol{\Gamma}(\theta_1), \boldsymbol{\Gamma}(\theta_2), \ldots, \boldsymbol{\Gamma}(\theta_K)\} \in \mathbb{C}^{KF\times K}$.

III. JOINT WEIGHTED BLOCK SPARSE RECOVERY BASED ON WSF

In this section, a joint weighted block sparse recovery algorithm is put forward to estimate DOA, where the WSF principle is utilized to compose the constraint of block sparse recovery. Moreover, a weighted matrix is further imposed on the joint block sparse recovery scheme to obtain the higher sparsity.

A. THE FRAMEWORK OF WSF

Based on the above analysis, it can be concluded that the signal subspace $\boldsymbol{E}_s$ still stays the same subspace as the novel augmented steering matrix $\boldsymbol{D}$, which indicates that Eq. (11) can be revised as

$$\boldsymbol{E}_s = \begin{bmatrix} \boldsymbol{G}\boldsymbol{A} \\ \boldsymbol{\Sigma}_M\boldsymbol{G}^*\boldsymbol{A}^*\boldsymbol{\psi}^* \end{bmatrix}\boldsymbol{U} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{\Delta} \\ \boldsymbol{\Sigma}_M\boldsymbol{B}^*\boldsymbol{\Delta}^*\boldsymbol{\psi}^* \end{bmatrix}\boldsymbol{U} = \boldsymbol{D}\boldsymbol{U} \tag{16}$$

However, Eq. (16) may fail to work when noise presents, since the spanned space of the signal subspace $\boldsymbol{E}_s$ is not a same space range as those of the new calibrated augmented array manifold matrix $\boldsymbol{D}$. Based on [30], a WSF framework can be constructed to tackle this problem as follows

$$[\vec{\theta}, \vec{U}] = \underset{\theta,U}{argmin}\, ||E_sW^{1/2} - D(\theta)U||_F^2 \tag{17}$$

where $\boldsymbol{D}(\theta)$ is a calibrated augmented steering matrix parameterized by θ. $\boldsymbol{W} \in \mathbb{C}^{K\times K}$ represents a weighted matrix, which is positive definite and varies with distinct algorithms. What's more, it effects the asymptotic characteristics of the fitting estimation error. Thus, based on the above analysis, many previous DOA estimation algorithms can be treated as variants of the general WSF method. In [30], it has been shown that for $\boldsymbol{D}(\theta)$ and $\boldsymbol{E}_s\boldsymbol{W}^{1/2}$, the matching degree of their space ranges can be measured by minimizing $\boldsymbol{U}$ when $\boldsymbol{D}(\theta)$ is fixed. And the optimal weighted matrix is further given as $\boldsymbol{W}_{opt} = (\boldsymbol{\Lambda}_s - \tilde{\sigma}_e^2\boldsymbol{I}_K)^2\boldsymbol{\Lambda}_s^{-1}$, which can achieve the lowest asymptotic fitting estimation error variance. $\tilde{\sigma}_e^2$ represents the estimated noise variance and can be calculated by averaging the $2M-K$ smallest eigenvalues corresponding to the noise subspace. Then, applying the optimal weighted matrix $\boldsymbol{W}_{opt}$ to Eq. (17), yields

$$[\vec{\theta}, \vec{U}] = \underset{\theta,U}{argmin}\, ||E_sW_{opt}^{1/2} - D(\theta)U||_F^2 \tag{18}$$

As shown in [31], one of the key points of the subspace fitting issue is to separate $\boldsymbol{D}(\theta)$ and $\boldsymbol{U}$. And $\boldsymbol{U}$ can be treated as an auxiliary parameter because the new extended steering matrix $\boldsymbol{D}(\theta)$ is parameterized by the desired DOA information. When the $\boldsymbol{D}(\theta)$ is fixed, the least squares solution of $\boldsymbol{U}$ can be depicted as

$$\boldsymbol{U} = (\boldsymbol{D}(\theta))^{\dagger}\boldsymbol{E}_s\boldsymbol{W}_{opt}^{1/2} \tag{19}$$

Bringing Eq. (19) back into Eq. (18), yields

$$\vec{\theta} = \underset{\theta}{argmin}\, tr\{P_{D(\theta)}^{\perp} E_sW_{opt}E_s^H\} = \underset{\theta}{argmin}\; N(\theta) \tag{20}$$

where $\boldsymbol{P}_{\boldsymbol{D}(\theta)}^{\perp} = \boldsymbol{I}_{2M} - \boldsymbol{P}_{\boldsymbol{D}(\theta)} = \boldsymbol{I}_{2M} - \boldsymbol{D}(\theta)(\boldsymbol{D}^H(\theta)\boldsymbol{D}(\theta))^{-1}\boldsymbol{D}^H(\theta)$ means the orthogonal projection matrix of $\boldsymbol{D}(\theta)$.

B. JOINT BLOCK SPARSE RECOVERY

Based on the optimal weighted matrix $\boldsymbol{W}_{opt}$, multiplying $\boldsymbol{W}_{opt}^{1/2}$ with Eq. (16), yields

$$\boldsymbol{E}_s\boldsymbol{W}_{opt}^{1/2} = \boldsymbol{D}\boldsymbol{U}\boldsymbol{W}_{opt}^{1/2} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{\Delta} \\ \boldsymbol{\Sigma}_M\boldsymbol{B}^*\boldsymbol{\Delta}^*\boldsymbol{\psi}^* \end{bmatrix}\boldsymbol{U}\boldsymbol{W}_{opt}^{1/2} \tag{21}$$

It's known that over-complete basis is crucial if the above subspace fitting issue is solved from sparse recovery perspective. Whereas, it's impossible for Eq. (21) to successfully establish an over-complete basis because the unknown mutual coupling coefficients and noncircular phases coexist. In order to address this problem, Eq. (21) can be regarded as two independent parts. i.e.

$$\begin{bmatrix} \boldsymbol{E}_{s1}\boldsymbol{W}_{opt}^{1/2} \\ \boldsymbol{E}_{s2}\boldsymbol{W}_{opt}^{1/2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{\Delta}\boldsymbol{U}\boldsymbol{W}_{opt}^{1/2} \\ \boldsymbol{\Sigma}_M\boldsymbol{B}^*\boldsymbol{\Delta}^*\boldsymbol{\psi}^*\boldsymbol{U}\boldsymbol{W}_{opt}^{1/2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{U}_{1w} \\ \boldsymbol{B}_2\boldsymbol{U}_{2w} \end{bmatrix} \quad (22)$$

where $\boldsymbol{E}_{s1}$ and $\boldsymbol{E}_{s2}$ are made up of the first and last K rows of $\boldsymbol{E}_s$, respectively. $\boldsymbol{B}_2 = \boldsymbol{\Sigma}_M\boldsymbol{B}^*$ and $\boldsymbol{U}_{1w} = \boldsymbol{\Delta}\boldsymbol{U}\boldsymbol{W}_{opt}^{1/2}$, $\boldsymbol{U}_{2w} = \boldsymbol{\Delta}^*\boldsymbol{\psi}^*\boldsymbol{U}\boldsymbol{W}_{opt}^{1/2}$. It's obvious that $\boldsymbol{U}_{1w}$ and $\boldsymbol{U}_{2w}$ are all block matrices. On the other hand, since Eq. (21) is split into two independent parts of Eq. (22), the fitting framework in Eq. (20) also needs to be revised as

$$\begin{aligned} \vec{\theta}_1 &= \underset{\theta}{argmin}\, tr\{P_{B(\theta)}^{\perp} E_{s1} W_{opt} E_{s1}^H\} = argmin\, N_1(\theta) \\ \vec{\theta}_2 &= \underset{\theta}{argmin}\, tr\{P_{B_2(\theta)}^{\perp} E_{s2} W_{opt} E_{s2}^H\} = argmin\, N_2(\theta) \end{aligned} \quad (23)$$

where $\boldsymbol{P}_{\boldsymbol{B}(\theta)}^{\perp} = I_M - \boldsymbol{P}_{\boldsymbol{B}(\theta)} = I_M - \boldsymbol{B}(\theta)(\boldsymbol{B}^H(\theta)\boldsymbol{B}(\theta))^{-1}\boldsymbol{B}^H(\theta)$, $\boldsymbol{P}_{\boldsymbol{B}_2(\theta)}^{\perp} = I_M - \boldsymbol{P}_{\boldsymbol{B}_2(\theta)} = I_M - \boldsymbol{B}_2(\theta)(\boldsymbol{B}_2^H(\theta)\boldsymbol{B}_2(\theta))^{-1}\boldsymbol{B}_2^H(\theta)$. $\boldsymbol{P}_{\boldsymbol{B}(\theta)}^{\perp}$ and $\boldsymbol{P}_{\boldsymbol{B}_2(\theta)}^{\perp}$ are the orthogonal projection matrices of $\boldsymbol{P}_{\boldsymbol{B}(\theta)}$ and $\boldsymbol{P}_{\boldsymbol{B}_2(\theta)}$, respectively.

Then the entire spatial range from $-90°$ to $90°$ can be discretized into L parts. Let $\bar{\theta} = \{\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_L\}$ be a sampling grids set which covers all the possible targets. And L satisfies $L \gg M > K$, which means that the noncircular sources own the spatial sparsity. Then, Eq. (22) can be sparsely expressed as

$$\begin{bmatrix} \boldsymbol{E}_{s1}\boldsymbol{W}_{opt}^{1/2} \\ \boldsymbol{E}_{s2}\boldsymbol{W}_{opt}^{1/2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}^{\bar{\theta}}\boldsymbol{U}_{1w}^{\bar{\theta}} \\ \boldsymbol{B}_2^{\bar{\theta}}\boldsymbol{U}_{2w}^{\bar{\theta}} \end{bmatrix} \quad (24)$$

where $\boldsymbol{B}^{\bar{\theta}} = [\boldsymbol{\Omega}(\bar{\theta}_1), \boldsymbol{\Omega}(\bar{\theta}_2), \ldots, \boldsymbol{\Omega}(\bar{\theta}_L)] \in \mathbb{C}^{M\times LF}$ and $\boldsymbol{B}_2^{\bar{\theta}} = \boldsymbol{\Sigma}_M\boldsymbol{B}^{\bar{\theta}} \in \mathbb{C}^{M\times LF}$ are two over-complete bases, respectively. And the corresponding block sparse matrices $\boldsymbol{U}_{1w}^{\bar{\theta}} = [\boldsymbol{U}_{1w}^{\bar{\theta}_1}, \boldsymbol{U}_{1w}^{\bar{\theta}_2}, \ldots, \boldsymbol{U}_{1w}^{\bar{\theta}_L}] \in \mathbb{C}^{LF\times K}$, $\boldsymbol{U}_{2w}^{\bar{\theta}} = [\boldsymbol{U}_{2w}^{\bar{\theta}_1}, \boldsymbol{U}_{2w}^{\bar{\theta}_2}, \ldots, \boldsymbol{U}_{2w}^{\bar{\theta}_L}] \in \mathbb{C}^{LF\times K}$ are all made up of L sub-blocks with $F \times K$ dimension. As the l-th sub-block of $\boldsymbol{U}_{1w}^{\bar{\theta}}$ and $\boldsymbol{U}_{2w}^{\bar{\theta}}$, $\boldsymbol{U}_{1w}^{\bar{\theta}_l}$ and $\boldsymbol{U}_{2w}^{\bar{\theta}_l}$ are equal to the $(Fl - F + 1)$th to (Fl)th rows of $\boldsymbol{U}_{1w}^{\bar{\theta}}$ and $\boldsymbol{U}_{2w}^{\bar{\theta}}$, respectively. Then, two new column vectors are introduced for conveniently expressing, that are $\boldsymbol{t}_1 = [t_1^{\bar{\theta}_1}, t_1^{\bar{\theta}_2}, \ldots, t_1^{\bar{\theta}_L}]^T$ and $\boldsymbol{t}_2 = [t_2^{\bar{\theta}_1}, t_2^{\bar{\theta}_2}, \ldots, t_2^{\bar{\theta}_L}]^T$. $t_1^{\bar{\theta}_l}$ and $t_2^{\bar{\theta}_l}$ represent the l_2-norm of $\boldsymbol{U}_{1w}^{\bar{\theta}_l}$ and $\boldsymbol{U}_{2w}^{\bar{\theta}_l}$, respectively. And both $\boldsymbol{U}_{1w}^{\bar{\theta}_l}$ and $\boldsymbol{U}_{2w}^{\bar{\theta}_l}$ are non-zero sub-blocks if $\bar{\theta}_l = \theta_k (l = 1, 2, \ldots, L;$ $k = 1, 2, \ldots, K)$, which results in $t_1^{\bar{\theta}_l}$ and $t_2^{\bar{\theta}_l}$ being non-zero as well. Evidently, DOA estimation can eventually be solved as a problem of block sparse recovery. Thus, for exploiting the noncircularity of the signals, a joint WSF framework based on l_0-norm penalty via combining the principle of block sparse recovery can be constructed as

$$\begin{aligned} &min\, ||\boldsymbol{T}||_0 \\ &s.t.\, \boldsymbol{T}(l) \geq \sqrt{(t_1^{\bar{\theta}_l})^2 + (t_2^{\bar{\theta}_l})^2} \quad l = 1, 2, \ldots, L \\ &\quad \boldsymbol{E}_{s1}\boldsymbol{W}_{opt}^{1/2} = \boldsymbol{B}^{\bar{\theta}}\boldsymbol{U}_{1w}^{\bar{\theta}}, \ \boldsymbol{E}_{s2}\boldsymbol{W}_{opt}^{1/2} = \boldsymbol{B}_2^{\bar{\theta}}\boldsymbol{U}_{2w}^{\bar{\theta}} \end{aligned} \quad (25)$$

However, the Eq. (25) is a NP-hard problem [8], [26]. Taking the practical feasibility of problem into consideration, l_1-norm penalty with lower computational complexity is universally adopted since it is the optimal convex approximation of l_0-norm constraint. Hence the l_0-norm optimization can eventually be replaced by l_1-norm penalty. i.e.

$$\begin{aligned} &min\, ||\boldsymbol{T}||_1 \\ &s.t.\, \boldsymbol{T}(l) \geq \sqrt{(t_1^{\bar{\theta}_l})^2 + (t_2^{\bar{\theta}_l})^2} \quad l = 1, 2, \ldots, L \\ &\quad ||\boldsymbol{E}_{s1}\boldsymbol{W}_{opt}^{1/2} - \boldsymbol{B}^{\bar{\theta}}\boldsymbol{U}_{1w}^{\bar{\theta}}||_F \leq \varepsilon_1, \\ &\quad ||\boldsymbol{E}_{s2}\boldsymbol{W}_{opt}^{1/2} - \boldsymbol{B}_2^{\bar{\theta}}\boldsymbol{U}_{2w}^{\bar{\theta}}||_F \leq \varepsilon_2 \end{aligned} \quad (26)$$

where ε_1 and ε_2 are the regularized parameters which associate with the upper value of fitting estimation error. Based on Eq. (23), it can be concluded that the estimation error of subspace fitting are $\sqrt{\boldsymbol{N}_1(\theta)}$ and $\sqrt{\boldsymbol{N}_2(\theta)}$, respectively. And as shown in [29], both functions $(2T/\tilde{\sigma}_e^2)\boldsymbol{N}_1(\theta)$ and $(2T/\tilde{\sigma}_e^2)\boldsymbol{N}_2(\theta)$ asymptotically follow the chi-square distribution with $2K(M-K)$ degrees of freedom when $\theta = \theta_k$ $(k = 1, 2\ldots, K)$. Thus, the parameter ε_1 and ε_2 can be calculated by $\sqrt{\boldsymbol{N}_1(\theta)} \leq \varepsilon_1$ and $\sqrt{\boldsymbol{N}_2(\theta)} \leq \varepsilon_2$with a high confidence interval of $1 - \rho$, respectively. In this article, $\rho = 0.01$ is selected to carry out the simulation experiments.

C. JOINT WEIGHTED BLOCK SPARSE RECOVERY

It can be concluded that larger coefficients are heavier penalized by l_1-norm constraint than those smaller ones, which indicates that the recovery performance of l_1-norm penalty is limited to some extent. Then according to [21], a NC MUSIC-like weighted constraint can be further imposed on Eq. (26) for enforcing the solutions sparsity. Considering the orthogonality between the new calibrated augmented over-complete basis and extended noise subspace, yields

$$\begin{aligned} \boldsymbol{J}(\bar{\theta}_l, g, \psi_l) &= \left[\boldsymbol{\Gamma}^H(\bar{\theta}_l)e^{-j\psi_l/2} \ \ \boldsymbol{\Gamma}^T(\bar{\theta}_l)e^{j\psi_l/2}\right]\boldsymbol{Q}(\bar{\theta}_l) \\ &\begin{bmatrix} \boldsymbol{\Gamma}(\bar{\theta}_l)e^{j\psi_l/2} \\ \boldsymbol{\Gamma}^*(\bar{\theta}_l)e^{-j\psi_l/2} \end{bmatrix} = \boldsymbol{d}(\bar{\theta}_l, g, \psi_l)\boldsymbol{Q}(\bar{\theta}_l) \\ &\boldsymbol{d}^H(\bar{\theta}_l, g, \psi_l) \to 0 \end{aligned} \quad (27)$$

where

$$\boldsymbol{d}(\bar{\theta}_l, g, \psi_l) = \left[\boldsymbol{\Gamma}^H(\bar{\theta}_l)e^{-j\psi_l/2} \ \ \boldsymbol{\Gamma}^T(\bar{\theta}_l)e^{j\psi_l/2}\right] \quad (28)$$

$$\begin{aligned} \boldsymbol{Q}(\bar{\theta}_l) &= \begin{bmatrix} \boldsymbol{\Omega}(\bar{\theta}_l) & 0 \\ 0 & \boldsymbol{\Sigma}_M\boldsymbol{\Omega}^*(\bar{\theta}_l) \end{bmatrix}^H \boldsymbol{E}_e\boldsymbol{E}_e^H \\ &\times \begin{bmatrix} \boldsymbol{\Omega}(\bar{\theta}_l) & 0 \\ 0 & \boldsymbol{\Sigma}_M\boldsymbol{\Omega}^*(\bar{\theta}_l) \end{bmatrix} \quad l = 1, 2, \ldots, L \end{aligned} \quad (29)$$

Based on the analysis in the above section, it can be easily summarized that $\boldsymbol{\Gamma}(\bar{\theta}_l) \neq 0$, which means that $\boldsymbol{d}(\bar{\theta}_l, g, \psi_l) \neq 0$. And $\boldsymbol{Q}(\bar{\theta}_l) \in \mathbb{C}^{2F\times 2F}$ is positive definite and consistent estimate of the rank deficient matrix. Hence, for the sake of noncircular sources, the spectrum function of NC MUSIC-like can be adopted to structure the weights as follows

$$\bar{q}_l = det\{\boldsymbol{Q}(\bar{\theta}_l)\} \quad l = 1, 2, \ldots, L \quad (30)$$

Then, exploiting the weights in Eq. (30), the weighted matrix can be depicted as

$$\bar{\boldsymbol{Q}} = diag\,\{\boldsymbol{Q}\} \tag{31}$$

where $\boldsymbol{Q} = [\boldsymbol{Q}_1, \boldsymbol{Q}_2] = [q_1, q_2, \ldots, q_L]$ with $q_l = \bar{q}_l / \max\{\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_L\}$. The weights in $\boldsymbol{Q}_1$ associate with the true DOAs which are smaller than those in $\boldsymbol{Q}_2$ and more likely to be zero when snapshots is infinite. After taking this weighted measure, whatever larger or smaller coefficients can be further equivalently punished. Thus, the joint weighted block sparse recovery scheme can be formulated as

$$\begin{aligned} &min\ ||\bar{\boldsymbol{Q}}\boldsymbol{T}||_1 \\ &s.t.\ \boldsymbol{T}(l) \geq \sqrt{(t_1^{\tilde{\theta}_l})^2 + (t_2^{\tilde{\theta}_l})^2} \quad l = 1, 2, \ldots, L \\ &\quad ||\boldsymbol{E}_{s1}\boldsymbol{W}_{opt}^{1/2} - \boldsymbol{B}^{\tilde{\theta}}\boldsymbol{U}_{1w}^{\tilde{\theta}}||_F \leq \varepsilon_1, \\ &\quad ||\boldsymbol{E}_{s2}\boldsymbol{W}_{opt}^{1/2} - \boldsymbol{B}_2^{\tilde{\theta}}\boldsymbol{U}_{2w}^{\tilde{\theta}}||_F \leq \varepsilon_2 \end{aligned} \tag{32}$$

Apparently, as a convex optimization problem, Eq. (32) can be settled by software packages in MATLAB, like CVX [32]. In this way, DOA estimation can eventually be obtained by searching the spatial spectrum of the recovered block sparse matrix.

Up to now, a joint weighted block sparse recovery scheme for strictly noncircular signals with unknown mutual coupling in WSAN has been achieved. The main steps of the proposed algorithm can be summarized as follows:

Step.1 Estimate the covariance matrix of the augmented data in Eq. (7) via Eq. (9).

Step.2 Achieve the signal subspace $\boldsymbol{E}_s$ and the noise subspace $\boldsymbol{E}_e$ by applying the eigenvalue decomposition to $\hat{\boldsymbol{R}}$ in Eq. (9), respectively.

Step.3 Establish the new calibrated augmented steering matrix $\boldsymbol{D}_{\text{in}}$ Eq. (15) via Eq. (12).

Step.4 Formulate Eq. (22) via splitting Eq. (21) based on WSF principle

Step.5 Construct NC MUSIC-like weighted matrix $\bar{\boldsymbol{Q}}$ via Eq. (29), Eq. (30) and Eq. (31).

Step.6 Estimate DOA through the joint weighted block sparse recovery scheme in Eq. (32).

IV. SIMULATION RESULTS

In this section, various simulation trails are carried out simultaneously to prove the superiority of the proposed method. Three methods are selected to compare with our proposed method, which including l_1-SVD methods in [24], BSR method in [26] and weighted BSR method in [27]. And referring to [21], the corresponding Cramer-Rao bound (CRB) for noncircular sources in unknown mutual coupling is re-deduced and selected as the performance evaluation indicator for all algorithms. Suppose that there are $K = 2$ strictly NC sources impinging on a half-wavelength spacing ULA in WASN. The sensors number of ULA is assumed to be $M = 10$ and the desired DOAs can be denoted as $\theta_1 = -2.8°$ and $\theta_2 = 6.9°$. Furthermore, the non-zero coefficients of MCM can be deemed as $\boldsymbol{c} = [1, 0.5725 - \mathrm{j} * 0.1024]$ with $H = 2$. The whole space $[-90°, 90°]$ is discretized into multiple grid points with a step size of 0.02°. The root means square error (RMSE) is selected to calculate the fitting estimation error, it takes the following specific form:

$$RMSE = \sqrt{\frac{1}{100K}\sum_{i=1}^{100}\sum_{k=1}^{K}(\tilde{\theta}_{i,k} - \theta_k)^2} \tag{33}$$

where $\bar{\theta}_{i,k}$ represents the estimated value of the k-th signal θ_k at the i-th Monte Carlo trial. And in this paper, Monte Carlo trials are implemented 100 times for comparative methods and proposed method, respectively.

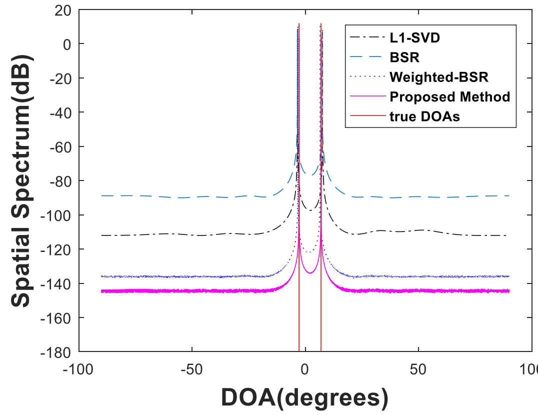


FIGURE 1. The Spatial spectrum of various algorithms.

Fig. 1 depicts the spatial spectrum of various algorithms in which SNR is set as −5dB and snapshots number is selected as $T = 100$. It can be observed that the proposed method has much sharper peaks and lower sidelobe, which illustrates that our method performs better in spatial resolution than other methods. What's more, compared with the residual methods, the proposed method is much closer to the desired DOAs, which indicates that it also outperforms other comparative methods in terms of accuracy.

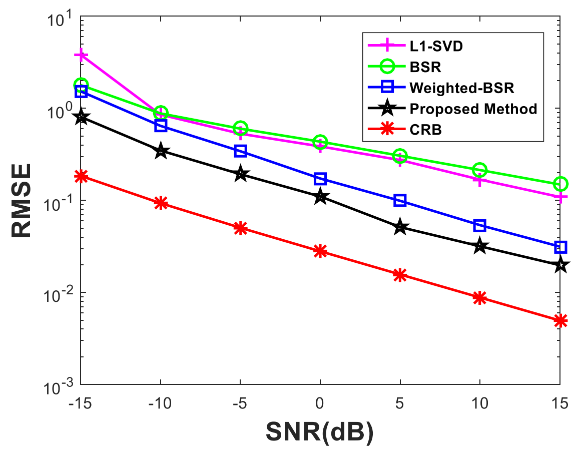


FIGURE 2. RMSE versus SNR for various methods.

Fig. 2 and Fig. 3 show the RMSE and PSD versus SNR, respectively. The corresponding snapshots number is fixed

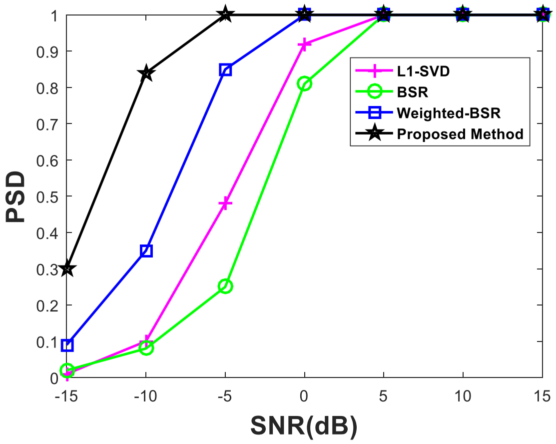


FIGURE 3. PSD versus SNR for various methods.

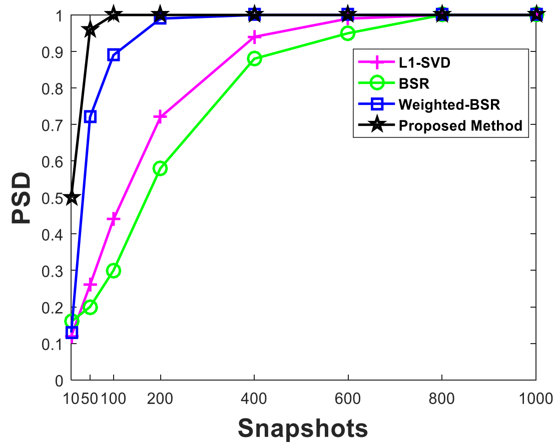


FIGURE 5. PSD versus snapshots for various methods.

at $T = 100$. Besides, the probability of successful detection (PSD) can be calculated by $|\theta_k - \widehat{\theta}_k| < 0.6°$, which means that the estimated error between the desired DOA θ_k and the estimated DOA $\widehat{\theta}_k$ is less than 0.6°. Based on Fig. 2 and Fig. 3, it can be seen that the corresponding RMSE decreases and PSD increases when the SNR increases, which indicates that the estimation performance of all algorithms gradually tends to be ideal. What's more, the proposed method behaves better than the residual algorithms since it shares a much lower RMSE and higher PSD for the entire SNR range. It is mainly because that the proposed method not only exploits the noncircularity of the signals, but also uses the optimal subspace fitting scheme and the weighted measure, which ensures that the solutions sparsity can be further strengthened. Besides, the proposed method can firstly achieve 100% PSD at a relatively low SNR.

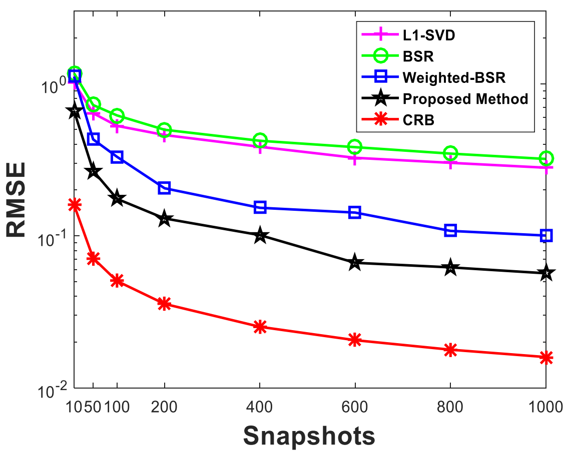


FIGURE 4. RMSE versus snapshots for various methods.

Fig. 4 and Fig. 5 express the comparison of RMSE versus snapshots and PSD versus snapshots, respectively. The corresponding SNR is selected as -5dB. As can be seen from Fig. 4 and Fig. 5, RMSE of all methods remains declining and PSD stays rising when the number of snapshots continuous to increase. Furthermore, RMSE and PSD of the proposed method are smaller and higher than those of the comparative algorithms in the whole selected snapshots range, respectively. Based on the above analysis, it can be easily concluded that the proposed method has outstanding advantage in estimation accuracy than other comparative algorithms.

V. CONCLUSION

In this paper, inspiring by the WSF principle, a joint weighted block sparse recovery scheme for strictly noncircular signals with unknown mutual coupling is constructed for estimating DOA in WSAN. In the proposed method, the unknown mutual coupling effect is firstly tackled without losing the array aperture. Then, aiming at noncircular sources, the joint weighted block sparse recovery scheme is structured by the WSF principle to estimate DOA. Moreover, the subspace fitting error upper value is also presented. Simulation results have confirmed that the proposed method owns the remarkable estimation performance for strictly noncircular sources in unknown mutual coupling.

REFERENCES

[1] H. Krim and M. Viberg, "Two decades of array signal processing research: The parametric approach," *IEEE Signal Process. Mag.*, vol. 13, no. 4, pp. 67–94, Jul. 1996.

[2] D. Zhao, T. Jin, Y. Dai, Y. Song, and X. Su, "A three-dimensional enhanced imaging method on human body for ultra-wideband multiple-input multiple-output radar," *Electronics*, vol. 7, no. 7, pp. 101–120, 2018.

[3] H. Wang, L. Wan, M. Dong, K. Ota, and X. Wang, "Assistant vehicle localization based on three collaborative base stations via SBL-based robust DOA estimation," *IEEE Internet Things J.*, vol. 6, no. 3, pp. 5766–5777, Jun. 2019.

[4] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.

[5] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust. Speech Signal Process.*, vol. 37, no. 7, pp. 984–995, Jul. 1989.

[6] H. Huang, Y. Peng, J. Yang, W. Xia, and G. Gui, "Fast beamforming design via deep learning," *IEEE Trans. Veh. Technol.*, vol. 69, no. 1, pp. 1065–1069, Jan. 2020, doi: 10.1109/TVT.2019.2949122.

[7] X. Wang, L. Wan, M. Huang, C. Shen, and K. Zhang, "Polarization channel estimation for circular and non-circular signals in massive MIMO systems," *IEEE J. Sel. Topics Signal Process.*, vol. 13, no. 5, pp. 1001–1016, Sep. 2019.

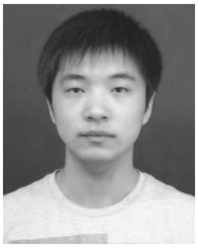
[8] D. Malioutov, M. Cetin, and A. S. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010–3022, Aug. 2005.
[9] M. E. Tipping, "Sparse Bayesian learning and the relevance vector machine," *J. Mach. Learn. Res.*, vol. 1, pp. 211–244, Sep. 2001.
[10] X. Wang, W. Wang, J. Liu, X. Li, and J. Wang, "A sparse representation scheme for angle estimation in monostatic MIMO radar," *Signal Process.*, vol. 104, pp. 258–263, Nov. 2014.
[11] H. Wang, X. Wang, L. Wan, and M. Huang, "Robust sparse Bayesian learning for off-grid DOA estimation with non-uniform noise," *IEEE Access*, vol. 6, pp. 64688–64697, 2018.
[12] X. Wang, L. Wang, X. Li, and G. Bi, "Nuclear norm minimization framework for DOA estimation in MIMO radar," *Signal Process.*, vol. 135, pp. 147–152, Jun. 2017.
[13] X. Wang, W. Wang, X. Li, and J. Liu, "Real-valued covariance vector sparsity-inducing DOA estimation for monostatic MIMO radar," *Sensors*, vol. 15, no. 11, pp. 28271–28286, Nov. 2015.
[14] B. Picinbono, "On circularity," *IEEE Trans. Signal Process.*, vol. 42, no. 12, pp. 3473–3482, Dec. 1994.
[15] W. Xie, C. Wang, F. Wen, J. Liu, and Q. Wan, "DOA and gain-phase errors estimation for noncircular sources with central symmetric array," *IEEE Sensors J.*, vol. 17, no. 10, pp. 3068–3078, May 2017.
[16] H. Abeida and J.-P. Delmas, "MUSIC-like estimation of direction of arrival for noncircular sources," *IEEE Trans. Signal Process.*, vol. 54, no. 7, pp. 2678–2690, Jul. 2006.
[17] P. Chargé, Y. Wang, and J. Saillard, "A non-circular sources direction finding method using polynomial rooting," *Signal Process.*, vol. 81, no. 8, pp. 1765–1770, Aug. 2001.
[18] W. Wang, X. Wang, H. Song, and Y. Ma, "Conjugate ESPRIT for DOA estimation in monostatic MIMO radar," *Signal Process.*, vol. 93, no. 7, pp. 2070–2075, Jul. 2013.
[19] X. Wang, L. Wan, M. Huang, C. Shen, Z. Han, and T. Zhu, "Low-complexity channel estimation for circular and noncircular signals in virtual MIMO vehicle communication systems," *IEEE Trans. Veh. Technol.*, to be published, doi: 10.1109/TVT.2020.2970967.
[20] Z.-M. Liu, Z.-T. Huang, Y.-Y. Zhou, and J. Liu, "Direction-of-arrival estimation of noncircular signals via sparse representation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 3, pp. 2690–2698, Jul. 2012.
[21] X. Wang, W. Wang, X. Li, Q. Liu, and J. Liu, "Sparsity-aware DOA estimation scheme for noncircular source in MIMO radar," *Sensors*, vol. 16, no. 4, pp. 539–551, Apr. 2016.
[22] Z. Ye and C. Liu, "On the resiliency of MUSIC direction finding against antenna sensor coupling," *IEEE Trans. Antennas Propag.*, vol. 56, no. 2, pp. 371–380, Feb. 2008.
[23] B. Liao, Z.-G. Zhang, and S.-C. Chan, "DOA estimation and tracking of ULAs with mutual coupling," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 891–905, Jan. 2012.
[24] J. Dai, D. Zhao, and X. Ji, "A sparse representation method for DOA estimation with unknown mutual coupling," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 1210–1213, 2012.
[25] Z. Zheng, J. Zhang, and J. Zhang, "Joint DOD and DOA estimation of bistatic MIMO radar in the presence of unknown mutual coupling," *Signal Process.*, vol. 92, no. 12, pp. 3039–3048, Dec. 2012.
[26] Q. Wang, T. Dou, H. Chen, W. Yan, and W. Liu, "Effective block sparse representation algorithm for DOA estimation with unknown mutual coupling," *IEEE Commun. Lett.*, vol. 21, no. 12, pp. 2622–2625, Dec. 2017.
[27] X. Wang, D. Meng, M. Huang, and L. Wan, "Reweighted regularized sparse recovery for DOA estimation with unknown mutual coupling," *IEEE Commun. Lett.*, vol. 23, no. 2, pp. 290–293, Feb. 2019.
[28] D. Meng, X. Wang, M. Huang, C. Shen, and G. Bi, "Weighted block sparse recovery algorithm for high resolution DOA estimation with unknown mutual coupling," *Electronics*, vol. 7, no. 10, pp. 217–289, 2018.
[29] M. Viberg, B. Ottersten, and T. Kailath, "Detection and estimation in sensor arrays using weighted subspace fitting," *IEEE Trans. Signal Process.*, vol. 39, no. 11, pp. 2436–2449, Nov. 1991.
[30] M. Viberg and B. Ottersten, "Sensor array processing based on subspace fitting," *IEEE Trans. Signal Process.*, vol. 39, no. 5, pp. 1110–1121, May 1991.
[31] G. H. Golub and V. Pereyra, "The differentiation of pseudo-inverses and nonlinear least squares problems whose variables separate," *SIAM J. Numer. Anal.*, vol. 10, no. 2, pp. 413–432, Apr. 1973.
[32] M. Grant, S. Boyd, and Y. Ye. *CVX: MATLAB Software for Disciplined Convex Programming*. Accessed: 2009. [Online]. Available: http://www.stanford.edu/boyd/cvx

LIANGLIANG LI was born in 1996. She received the B.S. degree from Fuyang Normal University, in 2018. She is currently pursuing the M.S. degree in information and communication engineering with Hainan University, Haikou, China. Her research interests include array signal processing and noncircular sources.

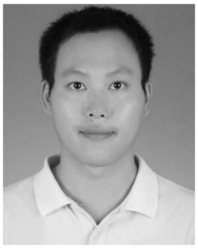
TINGTING FU was born in 1999. She is currently pursuing the B.S. degree in information and communication engineering with Hainan University, Haikou, China. Her research interests include array signal processing and source localization.

XIANPENG WANG (Member, IEEE) was born in 1986. He received the M.S. and Ph.D. degrees with the College of Automation, Harbin Engineering University (HEU), Harbin, China, in 2012 and 2015, respectively. He was a full-time Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, from 2015 to 2016. He is currently a Professor with the School of Information and Communication Engineering, Hainan University. He is the author of over 80 articles published in related journals and international conference proceedings. His major research interests include communication systems, array signal processing, radar signal processing, and compressed sensing and its applications. He has served as a Reviewer of over 20 journals.

MENGXING HUANG received the Ph.D. degree from Northwestern Polytechnical University, in 2007. He then joined as a staff with the Research Institute of Information Technology, Tsinghua University as a Postdoctoral Researcher. In 2009, he joined Hainan University. He is currently a Professor, a Ph.D. Supervisor of computer science and technology, and the Dean of the School of Information and Communication Engineering. He is also the Executive Vice-President of Hainan Province Institute of Smart City and the Leader of the Service Science and Technology Team, Hainan University. He has published more than 60 academic articles as the first or corresponding author. He has reported 12 patents of invention, owns three software copyright, and published two monographs and two translations. His current research interests include signal processing for sensor systems, big data, and intelligent information processing. He has been awarded one Second Class and one Third Class Prizes of The Hainan Provincial Scientific and Technological Progress.



LIANGTIAN WAN (Member, IEEE) received the B.S. and Ph.D. degrees from the College of Information and Communication Engineering, Harbin Engineering University, Harbin, China, in 2011 and 2015, respectively. From 2015 to 2017, he was a Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He is currently an Associate Professor with the School of Software, Dalian University of Technology, China. He has published over 40 scientific articles in international journals and conferences. His research interests include social network analysis and mining, big data, array signal processing, wireless sensor networks, and compressive sensing and its application. He is an Associate Editor of IEEE ACCESS.



YONGQIN YANG was born in 1984. He received the Ph.D. degree from the school of physics, Sun Yat-sen University, Guangdong, China. He is currently a Lecturer with the College of Information Science and Technology, Hainan University, Haikou, China. He is the author of over 100 articles published in related journals. His major research interests include signal processing, wireless communications, and sensor networks.