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Modeling and Analysis for Uncertainty in Logistic Chains Based on Logical Time Petri Nets

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ABSTRACT A present logistic chain is a cooperative system with multiple partners. Batch processing function and passing value uncertainty are two important properties, and time-related schedulability analysis is key for improving efficiency of business operations in a cooperative logistic chain system. A logical time Petri net is proposed in order to model and analyze time-related property, batch processing function and passing value uncertainty in logistic chains. The analysis methods are presented for calculating time-related property for different types of transitions such as logical input transitions, logical output transitions and traditional transitions. An example of a logistic chain is given to illustrate the use of the proposed methods.

INDEX TERMS Passing value uncertainty, logistic chains, firing time interval, logical transitions.

I. INTRODUCTION

In a modern logistic chain, more and more partners are involved. Logistic chains are a cooperative system [1]–[3]. One partner may communicate with a batch of other partners. For example, one supplier interacts with multiple buyers. A supplier may send product information to many potential buyers at one time. A supplier may receive orders from different buyers at one time. A shipping company ships various items to several customers in the meantime. In these cases, the batch processing functions emerge. However, one partner cannot terminate a business process because of lack of data of partial partners. And it needs work with other partners who have submitted requests. For example, even though some orders of partial buyers have not been received before a deadline, a supplier still needs to go on dealing with orders from other buyers and provides goods for them on time. Even if some items of some customers from providers have not been obtained before a deadline, a shipping company also needs to ship other items to other customers in time based on contracts. In above cases, passing value uncertainty occurs [4]. In addition, logistic chains are not only a cooperative system, but also a system associated with time. In general, when one partner executes a task or an activity, both the function and execution time of a task are concerned. For example, if a buyer submits a purchase request to a producer, the producer should receive this purchase request within time limit and replies in a certain timeframe. Moreover, the workflow is also subject to time constraint, i.e. a workflow needs to be finished before a deadline. To avoid a partner waiting for a long time and improve efficiency of business operations, a task needs to be subject to time constraint. In conclusion, batch processing function, passing value uncertainty and temporal constraint are real and important for an logistic chain system. Thus, modeling and analysis for them are vital in practical.

Petri nets are an important tool for modeling and analyzing distributed systems such as Web services [5]–[8], concurrent programs [9], [10] and process mining [11], [12]. They have a solid mathematical foundation and formal semantic and can describe concurrency and conflict conveniently [13]–[16]. On the basis of basic Petri nets, Petri nets are extended in different aspects. Time Petri nets are given and used for modeling computer systems and communication protocols [17]. Different real time systems are modeled and analyzed based on time Petri nets [18]. Timed Petri nets [19] and stochastic Petri nets [20] are presented afterwards. Time is added to places, transitions and directed arcs between places and transitions to form various Petri nets. Timing constraint Petri nets are given in [21]. In view of occurrence conditions of activities sometimes constraint with time, timing constraint Petri nets add time limit in places and transitions to analyze schedulability. Timing constraint Petri nets can be used to model a real-time system specification and determine whether the specification is schedulable with respect to imposed timing constraints [22]. Stochastic Petri nets are useful modeling tools for analyzing the performance

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and reliability of systems. Stochastic Petri nets utilize the continuous-time stochastic processes.

Petri nets are extended not only with time in places and transitions, but also with transition types. Logical Petri nets extend Petri nets with logical input transitions and logical output transitions to describe batch processing function and passing value uncertainty. Some work related to logical Petri nets has been conducted, such as its modeling capability [23], analysis [24]–[27], the soundness preservation in composition [28], transformation [29] and substitution [30]. Logical Petri nets can be used to model Web Service, E-commerce systems and so on [31]. Besides, logical Petri nets can be used to represent and analyze process workflow in process discovery [32].

The issues related to extending logical Petri nets with time are, however, unexplored in the existing studies. This work proposes logical time Petri nets (*LTPN*s) for modeling and analyzing uncertainty in logistic chains. *LTPN*s are helpful for time-related and schedulability analysis in logistic chains with batch processing function and passing value uncertainty. *LTPN*s can deal with batch processing function and passing value uncertainty different from previous Petri nets. And time-related property analysis in an *LTPN* is also different from previous Petri nets with time.

The rest of this paper is organized as follows. Section 2 presents logical time Petri nets. Section 3 is devoted to analyzing firing time interval of transitions. Section 4 describes an logistic chain system and constructs its *LTPN*. The use of firing time interval of transitions for scheduling analysis is illustrated. Concluding remarks are made in Section 5.

II. LOGICAL TIME PETRI NETS AND ANALYSIS TECHNIQUES

Logical Time Petri nets are constructed. The graphical representation is given. Time-related analysis methods of Logical Time Petri nets are put forward in this section.

A. LOGICAL TIME PETRI NETS

Definition 1: $N = (P, T, F)$ is a net where

(1) P is a finite set of places;
(2) T is a finite set of transitions with $P \cup T \neq \varnothing$ and $P \cap T = \varnothing$; and
(3) $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs.

Definition 2: Let $N = (P, T, F)$ be a net and $x \in P \cup T$ be a node in N. ${}^\bullet x = \{y | (y, x) \in F\}$ is called a pre-set of x; and $x^\bullet = \{y | (x, y) \in F\}$ is called a post-set of x. If $X \subseteq P \cup T$, its pre-set and post-set are as follows ${}^\bullet X = \cup_{x \in X} {}^\bullet x$ and $X^\bullet = \cup_{x \in X} x^\bullet$

Definition 3: A matrix with two columns is called a transaction time matrix if the first column denotes the transaction number and the second column denotes the processing time corresponding to the transaction number.

A transaction time matrix is shown above the white rectangle in Figure 1. The first column denotes transaction number and the second column denotes the processing time corresponding to the transaction number. If the transaction number is 1, the processing time is 5 time units (*TU*). If the transaction number is 2, the processing time is 8 time units. If the number of transaction is 3, the processing time is 11 time units.

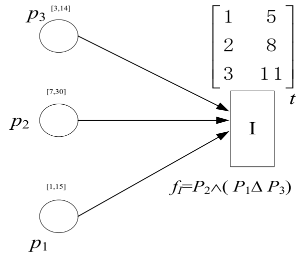

FIGURE 1. The example of a logical input transition.

Definition 4: $LTN = (P, T, F, I, O, UT, DU, LU)$ is an logical time net where

(1) P is a finite set of places;

(2) $T = T_G \cup T_I \cup T_O$ is a finite set of transitions, $T \cup P \neq \varnothing$, $\forall t \in T_I \cup T_O$: ${}^\bullet t \cap t^\bullet = \varnothing$, where

(a) T_G denotes a set of traditional transitions as defined in Petri nets;

(b) T_I denotes a set of logical input transitions, where $\forall t \in T_I$, the input places of t are restricted by a logical expression $f_I(t)$ on ${}^\bullet t$;

(c) T_O denotes a set of logical output transitions, where $\forall t \in T_O$, the output places of t are restricted by a logical expression $f_O(t)$ on $t^\bullet$;

(3) $F \subseteq (P \times T) \cup (T \times P)$ is a finite set of directed arcs;

(4) I is a mapping from a logical input transition to a logical input expression, i.e., $\forall t \in T_I$, $I(t) = f_I(t)$;

(5) O is a mapping from a logical output transition to a logical output expression, i.e., $\forall t \in T_O$, $O(t) = f_O(t)$;

(6) UT is a *mapping* from P to time interval set, $UT(p) = [T_{EU}(p), T_{LU}(p)]$, $T_{EU}(p) \leq T_{LU}(p)$, where $T_{EU}(p)$ denotes the earliest time at which the token in p can be used after it reaches p, and $T_{LU}(p)$ denotes the latest time at which the token in p can be used after it reaches p;

(7) DU is a mapping from T_G to time set, $DU(t) = T$, $t \in T_G$, where T denotes the firing duration of t;

(8) TU is a mapping from $T_I \cup T_O$ to a set of transaction time matrix;

For a transition $t \in T_G \cup T_I \cup T_O$ in an *LTPN*, there exists a firing time interval $FTI\ (t) = [T_{EF}(t), T_{LF}(t)]$, where $T_{EF}(t)$ represent the earliest firing time and $T_{LF}(t)$ denotes the latest firing time. They can be obtained by ${}^\bullet t$ based on modeled system behaviors. For $t \in T_G \cup T_I \cup T_O$, $FTI\ (t) = [T_{EF}(t), T_{LF}(t)]$ can be calculated and the method is given in Section 3.

In a graph representation of an *LTPN*, a traditional transition $t \in T_G$ is drawn as a black rectangle as shown in Figure 2, a logical input transition $t \in T_I$ is drawn as a white rectangle marked with a letter I as shown in Figure 1 and a logical output transition $t \in T_I$ is drawn as a white rectangle marked with a letter O as shown in Figure 3.

In Figure 1, t is a logical input transition. A transaction time matrix is shown above the logical input transition. The logical expression $f_I(t)$ is shown below the logical input transition.

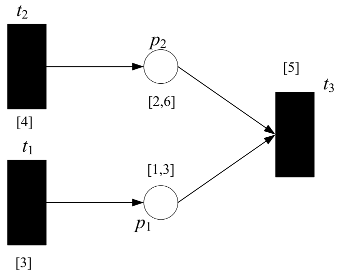

FIGURE 2. The example of a traditional transition.

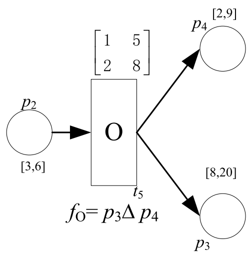

FIGURE 3. The example of a logical output transition.

There are three places and each place have a time interval.

In Figure 2, t_1, t_2 and t_3 are three traditional transitions. Each traditional transition have a firing duration adjacent to it.

In Figure 3, t_5 is a logical output transition. A transaction time matrix is shown above the logical output transition. The logical expression is shown below the logical output transition.

In an *LTPN*, $\forall t \in T_I \cup T_O$, t can deal with multiple different transactions based on the practical run as shown in Figure 1 and Figure 3, and $f_I(t)$ or $f_O(t)$ expresses the batch processing functions and passing value uncertainty in such systems. In a practical logistic chains system, based on different transaction numbers such as different order numbers of buyers received by a supplier, the processing time of the task is different. The matrix for describing processing time of transactions of a task is adopted to show the difference. The matrix includes two columns, the first column denotes transaction numbers in a task, and the second column denotes the processing time of the corresponding transaction number in a task.

Definition 5: Let $LTN=(P, T, F, I, O, UT, DU, LU)$ be a logical time net. $LTPN=(P, T, F, I, O, UT, DU, LU, M_0)$ is a logical time Petri net where M is a marking mapping from P to a natural number set expressing the state of *LTPN* and M_0 denotes the initial marking. $\forall p \in P$, $M(p)=k \in \mathbb{N}$ denotes the number of tokens in a place p.

Definition 6: Let $LTPN=(P, T, F, I, O, UT, DU, LU, M_0)$ be a logical time Petri net and $FTI(t)=[T_{EF}(t), T_{LF}(t)]$ be a firing time interval of a transition $t \in T_G \cup T_I \cup T_O$, the firing rules of *LTPN* are as follows:

(1) $\forall t \in T_G$, t is enabled if $T_{LF}(t)-T_{EF}(t) \geq 0$ and $\forall p \in {}^\bullet t$; $M(p)=1$. Firing enabled t results in a new marking M': $\forall p \in {}^\bullet t$: $M'(p) = 0$; $\forall p \in t^\bullet$: $M'(p) = M(p) + 1$;

(2) $\forall t \in T_I$, $I(t)=f_I(t)$, t is enabled if $T_{LF}(t) - T_{EF}(t) \geq 0$ and $f_I(t)|_M = {}_\bullet T_\bullet$, i.e., all input places of t satisfy the logical input expression $f_I(t)$ at M. Firing enabled t generates a new marking M': $\forall p \in {}^\bullet t$: $M'(p) = 0$; $\forall p \in t^\bullet$: $M'(p) = M(p) + 1$; and

(3) $\forall t \in T_O$, $O(t)=f_O(t)$, t is enabled if $T_{LF}(t) - T_{EF}(t) \geq 0$ and $\forall p \in {}^\bullet t$, $M(p)=1$. Firing enabled t generates a new marking M': $\forall p \in {}^\bullet t$: $M'(p)=M(p)-1$; for all $p \in t^\bullet$, $f_O(t)|_{M'} = {}_\bullet T_\bullet$, i.e., all output places of t satisfy the logical output expression $f_O(t)$ at M'.

In an *LTPN*, a transition $t \in T_G \cup T_I \cup T_O$ is enabled at M, which is denoted by $M[t >$; If t is enabled, it can fire, and a new marking M' is generated from M, which is represented by $M[t > M'$.

B. TIME-RELATED ANALYSIS OF LOGICAL TIME PETRI NETS

In the following, we suppose $\forall p \in P$, $|p^\bullet| = |{}^\bullet p| = 1$ to simplify the analysis complexity.

In an *LTPN*, the firing time interval of a transition $t \in T_G \cup T_O$ is $FTI(t) = [T_{EF}(t), T_{LF}(t)]$. $\forall p \in {}^\bullet t$, $UT(p) = [T_{EU}(p), T_{LU}(p)]$. Only when $\forall p \in {}^\bullet t$ can be used, t can be fired, thus, $T_{EF}(t) \geq \max\{T_{EU}(p) |\ p \in {}^\bullet t\}$ and $T_{LF}(t) \leq \min\{T_{LU}(p) |\ p \in {}^\bullet t\}$.

Definition 7: Let $LTPN = (P, T, F, I, O, UT, DU, LU, M_0)$ be a logical time Petri net and $t \in T$. If $p \in {}^\bullet t$, and $t' \in {}^\bullet p$ then t' is called a direct pre-transition of t. $Dpt(t) = {}^{\bullet\bullet} t$ is the set of all direct pre-transitions of t.

For example, in Figure 2, $Dpt(t_3) = \{t_1, t_2\}$.

In logistic chains, every task or activity takes some time to execute. Accordingly, a transition t representing a task or activity is associated with a duration $DU(t)$. The start time of firing of a transition t is denoted by $ST(t)$ and the finish time of firing of a transition t is denoted by $FT(t)$. The firing time interval of a transition t is $FTI(t)=[T_{EF}(t), T_{LF}(t)]$. Obviously, some inequations and equations are obtained as follows.

$$FT(t) \geq ST(t)$$
$$T_{EF}(t) \leq ST(t), FT(t) \leq T_{LF}(t)$$
$$FT(t) = ST(t) + DU(t)$$
$$DU(t) = FT(t) - ST(t)$$

In an *LTPN*, suppose that there is only one input place p of a transition t, i.e. $|{}^\bullet t|=1$ and ${}^\bullet t=\{p\}$. $UT(p)=[T_{EU}(p), T_{LU}(p)]$ and $T_{arr}(p)$ denote the time at which a token reaches p. Some equations are as follows:

$$T_{EF}(t) = T_{arr}(p) + T_{EU}(p)$$
$$T_{LF}(t) = T_{arr}(p) + T_{LU}(p) - DU(t)$$

Because $T_{arr}(p)$ is associated with $t' \in {}^\bullet p = {}^{\bullet\bullet} t$ and $FTI(t') = [T_{EF}(t'), T_{LF}(t')]$, the minimum of $T_{arr}(p)$ is $T_{EF}(t')$ and the maximum is $T_{LF}(t') + DU(t')$, i.e., $T_{EF}(t') \leq T_{arr}(p) \leq T_{LF}(t') + DU(t')$.

Theorem 1: Let $LTPN = (P, T, F, I, O, UT, DU, LU, M_0)$ be a logical time Petri net and a transition $t \in T_G$, $|{}^\bullet t|=1$, ${}^\bullet t = \{p\}$ and ${}^\bullet p = \{t'\}$. $FTI(t)=[T_{EF}(t), T_{LF}(t)]$ can be obtained by t', where $T_{EF}(t)=T_{EF}(t') + DU(t') + T_{EU}(p)$ and $T_{LF}(t) = T_{LF}(t') + DU(t') + T_{LU}(p) - DU(t)$.

TABLE 1. The meaning of places and transitions.

Places or Transitions	Meaning (1≤j≤2)	Places or Transitions	Meaning(1≤j≤2)
i_0	the state of a supplier ready to send product information	t_0	a supplier is ready to send product information
i_2	the state of Buyer_1 receiving productinformation	i_3	the state of Buyer_2 receiving product information
t_{j1}	Buyer_j sends the order	B_j_order	the transmission state of the order from Buyer_j
i_1	the state of a supplier waiting for orders from buyers	t_1	a supplier receives orders from buyers
p_{j2}	the state of Buyer_j waiting for the processing result from a supplier	B_j_refuse	the transmission state of the refusal message to Buyer_j
p_2	the state of a supplier checking orders from buyers	t_2	a supplier determines to accept or refuse orders from buyers based on stock and their credit
p_3	the state of a supplier choosing the refused buyers	t_3	a supplier sends refusal messages to the refused buyers
t_{j2}	Buyer_j receives the refusal message	p_6	the state of a supplier recording the processing result
p_4	the state of a supplier preparing goods for accepted buyers	t_4	a supplier sends the goods to the accepted buyers
B_j_goods	the transmission state of the goods for Buyer_j	t_{j3}	Buyer_j receives the goods
p_{j3}	the state of Buyer_j preparing the payment	t_{j4}	Buyer_j pays for the orders
B_j_payment	the state of Buyer_j processing payment	p_5	the state of a supplier waiting for payments from buyers
t_5	a supplier receives payments from buyers	p_7	the state of a supplier recording the processing result
t_6	a supplier archives the orders from buyers	o_1	the end of one workflow run of a supplier
o_2	the end of one workflow run of Buyer_1	o_3	the end of one workflow run of Buyer_2

In an *LTPN*, suppose that there is more than one input place of a transition t, i.e. $|{}^\bullet t|>1$ and $\forall p_i \in {}^\bullet t$, $UT(p_i)=[T_{EU}(p_i), T_{LU}(p_i)]$ and $T_{arr}(p_i)$ denote the time at which a token reaches p_i. Some equations are as follows.

$T_{EF}(t)=\max\{T_{arr}(p_i)+T_{EU}(p_i)|1\le i\le|{}^\bullet t|\}$.

$T_{LF}(t)=\min\{T_{arr}(p_i)+T_{LU}(p_i)-DU(t)|1\le i\le|{}^\bullet t|\}$

Theorem 2: Let $LTPN=(P, T, F, I, O, UT, DU, LU, M_0)$ be a logical time Petri net and a transition $t \in T_G$, $|{}^\bullet t|>1$, $Dpt(t)$ is the set of all direct pre-transitions of t. $FTI(t)=[T_{EF}(t), T_{LF}(t)]$ can be obtained by $Dpt(t)$, where $T_{EF}(t)=\max\{T_{EF}(t_i)+DU(t_i)+T_{EU}(p_i)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 1\le i\le|Dpt(t)|\}$ and $T_{LF}(t)=\min\{T_{LF}(t_i)+DU(t_i)+T_{LU}(p_i)-DU(t)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 1\le i\le|Dpt(t)|\}$.

In an *LTPN*, a logical input transition $t\in T_I$ sometimes need not contain a token in each place of ${}^\bullet t$. If $T_{LF}(t)-T_{EF}(t)\ge 0$ and $f_I(t)|_M={}_\bullet T_\bullet$, t is enabled and can be fired based the firing rules in an *LTPN*. Under such condition $f_I(t)|_M={}_\bullet T_\bullet$, sometimes some place of ${}^\bullet t$ may be empty. For example, in Figure 1, ${}^\bullet t_1=\{p_1, p_2, p_3\}$, if $M(p_2)=M(p_1)=1$, even if $M(p_3)=0$, it can be derived $f_I(t)|_M={}_\bullet T_\bullet$., and as long as $T_{LF}(t)-T_{EF}(t)\ge 0$, t can be enabled and fired. In this case, the firing time interval $FTI(t)=[T_{EF}(t), T_{LF}(t)]$ are obtained only based on $UT(p_1)$ and $UT(p_2)$ independent of $UT(p_3)$. Because $M(p_2)=M(p_1)=1$, there are two transaction and $DU(t)=8$ based on transaction time matrix of t. And $T_{EF}(t)=\max\{T_{EF}(t_i)+DU(t_i)+T_{EU}(p_i)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 1\le i\le 2\}$, $T_{LF}(t)=\min\{T_{LF}(t_i)+DU(t_i)+T_{LU}(p_i)-DU(t)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 1\le i\le 2\}$ and if $T_{LF}(t)-T_{EF}(t)\ge 0$, t can be enabled and fired. In the second case of $M(p_2)=M(p_3)=1$ and $M(p_1)=0$, $f_I(t)|_M={}_\bullet T_\bullet$ Because $M(p_2)=M(p_3)=1$, there are two transaction and $DU(t)=8$ based on transaction time matrix of t. And $T_{EF}(t)=\max\{T_{EF}(t_i)+DU(t_i)+T_{EU}(p_i)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 2\le i\le 3\}$ and $T_{LF}(t)=\min\{T_{LF}(t_i)+DU(t_i)+T_{LU}(p_i)-DU(t)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 2\le i\le 3\}$. And if $T_{LF}(t)-T_{EF}(t)\ge 0$, t can be enabled and fired. Likewise, in the third case of $M(p_1)=M(p_2)=M(p_3)=1$, $f_I(t)|_M={}_\bullet T_\bullet$ Because $M(p_2)=M(p_1)=M(p_3)=1$, there are three transaction and $DU(t)=11$ based on transaction time matrix of t. and $T_{EF}(t)=\max\{T_{EF}(t_i)+DU(t_i)+T_{EU}(p_i)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 1\le i\le 3\}$ and $T_{LF}(t)=\min\{T_{LF}(t_i)+DU(t_i)+T_{LU}(p_i)-DU(t)|t_i\in Dpt(t)\wedge p_i\in t_i^\bullet, 1\le i\le 3\}$. And if $T_{LF}(t)-T_{EF}(t)\ge 0$, t can be enabled and fired.

Theorem 3: Let $LTPN = (P, T, F, I, O, UT, DU, LU, M_0)$ be a logical time Petri net and a transition $t \in T_I$, $|{}^\bullet t|>1$, $Dpt(t)$ is the set of all direct pre-transitions of t, $Dpt'(t)$ is the set of direct pre-transitions associated with places satisfying $f_I(t)={}_\bullet T_\bullet$ at M of t in a practical run. $DU(t)$ can be gotten based on the transaction time matrix of t and the number of places satisfying $f_I(t)={}_\bullet T_\bullet$ at M. $FTI(t)=[T_{EF}(t), T_{LF}(t)]$ can be obtained by $Dpt'(t)$, where $T_{EF}(t)=\max\{T_{EF}(t_i)+DU(t_i)+T_{EU}(p_i)|t_i\in Dpt'(t)\wedge p_i\in t_i^\bullet, 1\le i\le|Dpt'(t)|\}$

and $T_{LF}(t)$=min{$T_{LF}(t_i)$ + $DU(t_i)$ + $T_{LU}(p_i)$ $-DU(t)|t_i \in Dpt'(t) \wedge p_i \in t_i^{\bullet}, 1 \le i \le |Dpt'(t)|$}.

Theorem 4: Let $LTPN=(P, T, F, C, D, I, O, M_0)$ be a logical time Petri net and a transition $t \in T_O$, $|^{\bullet}t| \ge 1$, $Dpt(t)$ is the set of all direct pre-transitions of t, $DU(t)$ can be gotten based on the transaction time matrix of t and the number of places satisfying $f_O(t)={}_{\bullet}T_{\bullet}$. in a practical run. $FTI(t)=[T_{EF}(t), T_{LF}(t)]$ can be obtained by $Dpt(t)$, where $T_{EF}(t)=\max\{T_{EF}(t_i)+DU(t_i)+T_{EU}(p_i)|t_i \in Dpt(t) \wedge p_i \in t_i^{\bullet}, 1 \le i \le |Dpt(t)|\}$ and $T_{LF}(t)=\min\{T_{LF}(t_i)+DU(t_i)+T_{LU}(p_i)-DU(t)|t_i \in Dpt(t) \wedge p_i \in t_i^{\bullet}, 1 \le i \le |Dpt(t)|\}$.

III. AN EXAMPLE OF A LOGISTIC CHAIN SYSTEM

In a logistic chain system, suppose there are a supplier and two buyers. First, the supplier sends product information to two buyers. Two buyers receive product information from the supplier and then they submit orders. The supplier receives orders from buyers and determines whether an order is accepted or refused based on stock or credit of a buyer. If an order is refused, the supplier sends refuse message to a buyer. A buyer receive refuse message and end purchase from the supplier. If an order is accepted, the supplier prepares products and ships them to a buyer. A buyer receives products and then pays for them. The supplier receives payment and file information about this trade.

The *LTPN* model of the system is shown in Figure 4. The implication of most main places and transitions is illustrated in Table 1. Time interval and transaction time matrix associated with places and transition is marked in Figure 4.

IV. EXPERIMENTAL EVALUATIONS

Suppose a supplier sends product information to two buyers at time T_0, i.e., the transition t_0 is fired at time T_0. The earliest and latest firing intervals of three transitions are calculated in detail as follows. Other transitions can be calculated similarly.

(1) Based on Theorem 1, the earliest and latest firing interval of a transition t_{11} $FTI(t_{11})=[T_{EF}(t_{11}), T_{LF}(t_{11})]$ can be obtained by t_0.

$T_{EF}(t_{11})=T_{EF}(t_0)+DU(t_0)+T_{EU}(i_2)=T_0+7$.

$T_{LF}(t_{11})=T_{LF}(t_0)+DU(t_0)+T_{LU}(i_2)-DU(t_{11})=T_0+7$.

Because $T_{LF}(t_{11})-T_{EF}(t_{11}) \ge 0$, t_{11} can be fired.

(2) Based on Theorem 3, if logical input expression of a logical input transition t $f_I(t)={}_{\bullet}T_{\bullet}$, t maybe fired. If $p \in {}^{\bullet}t$ and $M(p)=0$, $UT(p)$ is omitted in computing the earliest and latest firing interval of a transition t.

For example, If logical input expression of a logical input transition t_1, $f_I(t_1)={}_{\bullet}T_{\bullet}$, three exist three cases: $M(\text{B}_1\text{\_order})=M(i_1)=M(\text{B}_2\text{\_order})=1$; $M(\text{B}_1\text{\_order})=M(i_1)=1$, $M(\text{B}_2\text{\_order})=0$; $M(\text{B}_2\text{\_order})=M(i_1)=1$, $M(\text{B}_1\text{\_order})=0$

In the case of $M(\text{B}_1\text{\_order})=M(i_1)=M(\text{B}_2\text{\_order})=1$, $DU(t_1)=8$

$T_{EF}(t_1)=\max\{T_{EF}(t_{11})+DU(t_{11})+T_{EU}(\text{B}_1\text{\_order}), T_{EF}(t_0)+DU(t_0)+T_{EU}(i_1), T_{EF}(t_{21})+DU(t_{21})+T_{EU}(\text{B}_2\text{\_order})\}$

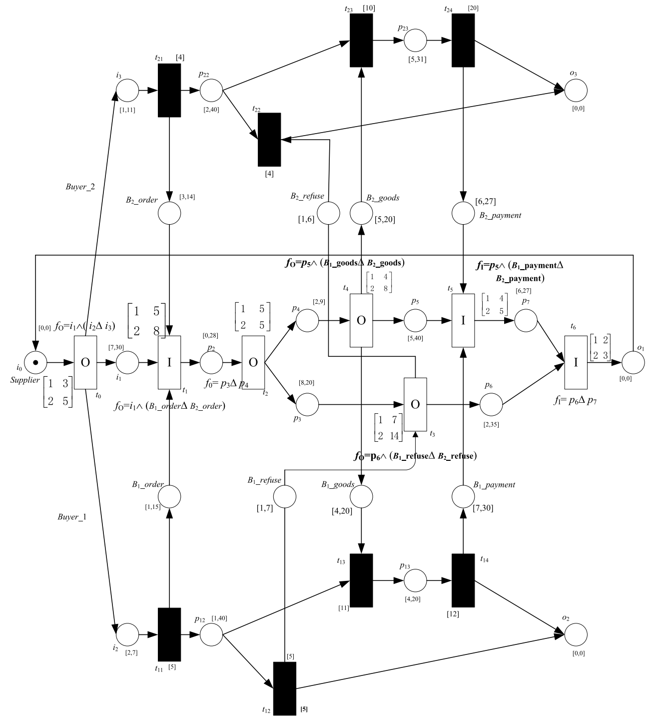

FIGURE 4. The example of a logistic chain system with a supplier and two buyers.

$=T_0+13$ $T_{LF}(t_1)=\min\{T_{LF}(t_{11})+DU(t_{11})+T_{LU}(\text{B}_1\text{\_order})-DU(t_1), T_{LF}(t_0)+DU(t_0)+T_{LU}(i_1)-DU(t_1), T_{LF}(t_{21})+DU(t_{21})+T_{LU}(\text{B}_2\text{\_order})-DU(t_1)\}=T_0+19$.

Because $T_{LF}(t_1)-T_{EF}(t_1) \ge 0$, t_1 can be fired.

In the case of $M(\text{B}_1\text{\_order})=M(i_1)=1$ and $M(\text{B}_2\text{\_order})=0$; $DU(t_1)=5$.

$T_{EF}(t_1)=\max\{T_{EF}(t_{11})+DU(t_{11})+T_{EU}(\text{B}_1\text{\_order}), T_{EF}(t_0)+DU(t_0)+T_{EU}(i_1)\}=T_0+13$ $T_{LF}(t_1)=\min\{T_{LF}(t_{11})+DU(t_{11})+T_{LU}(\text{B}_1\text{\_order})-DU(t_1), T_{LF}(t_0)+DU(t_0)+T_{LU}(i_1)-DU(t_1)\}=T_0+22$

Because $T_{LF}(t_1)-T_{EF}(t_1) \ge 0$, t_1 can be fired.

In the case of $M(\text{B}_2\text{\_order})=M(i_1)=1$, $M(\text{B}_1\text{\_order})=0$; $DU(t_1)=5$.

$T_{EF}(t_1)=\max\{T_{EF}(t_{21})+DU(t_{21})+T_{EU}(\text{B}_2\text{\_order}), T_{EF}(t_0)+DU(t_0)+T_{EU}(i_1)\}=T_0+13$.

$T_{LF}(t_1)=\min\{T_{LF}(t_{21})+DU(t_{21})+T_{LU}(\text{B}_2\text{\_order})-DU(t_1), T_{LF}(t_{11})+DU(t_{11})+T_{LU}(\text{B}_1\text{\_order})-DU(t_1)\}=T_0+25$.

Because $T_{LF}(t_1)-T_{EF}(t_1) \ge 0$, t_1 can be fired.

(3) Based on Theorem 4, for a logical output transition t_1, the output of t_4 should satisfy logical output expression $f_o(t)$. In a practical system, the output of t is based on practical cases.

In the case of $M(\text{B}_1\text{\_order})=M(i_1)=M(\text{B}_2\text{\_order})=1$, $DU(t_4)=6$.

$T_{EF}(t_4)=T_{EF}(t_2)+DU(t_2)+T_{EU}(p_4)=T_0+28$.

$T_{LF}(t_4)=T_{LF}(t_2)+DU(t_2)+T_{LU}(p_4)-DU(t_4)=T_0+58$.

Because $T_{LF}(t_4)-T_{EF}(t_4) \ge 0$, t_4 can be fired. After firing t_4, $M(\text{B}_2\text{\_goods})=M(\text{B}_1\text{\_goods})=M(p_5)=1$.

TABLE 2. The case of a supplier accepting two orders from two buyers.

Transitions	T_{EF}	T_{LF}	Action	Transitions	T_{EF}	T_{LF}	Action
t_{11}	T_0+7	T_0+7	Yes	t_{21}	T_0+6	T_0+12	Yes
t_1	T_0+13	T_0+19	Yes	t_2	T_0+21	T_0+50	Yes
t_3			No	t_4	T_0+28	T_0+58	Yes
t_{12}			No	t_{22}			No
t_{13}	T_0+38	T_0+41	Yes	t_{23}	T_0+12	T_0+56	Yes
t_{14}	T_0+53	T_0+60	Yes	t_{24}	T_0+27	T_0+77	Yes
t_5	T_0+74	T_0+97	Yes	t_6	T_0+85	T_0+126	Yes

TABLE 3. The firing time intervals of transitions in the second case.

Transitions	T_{EF}	T_{LF}	Action	Transitions	T_{EF}	T_{LF}	Action
t_{11}	T_0+7	T_0+7	Yes	t_{21}	T_0+6	T_0+12	Yes
t_1	T_0+13	T_0+19	Yes	t_2	T_0+21	T_0+50	Yes
t_3	T_0+34	T_0+68	Yes	t_4	T_0+28	T_0+60	Yes
t_{12}			No	t_{22}	T_0+42	T_0+52	Yes
t_{13}	T_0+36	T_0+41	Yes	t_{23}			No
t_{14}	T_0+43	T_0+60	Yes	t_{24}			No
t_5	T_0+62	T_0+98	Yes	t_6	T_0+72	T_0+107	Yes

In the case of $M(\mathrm{B}_1\_\mathrm{order})=M(i_1)=1$, $M(\mathrm{B}_2\_\mathrm{order})=0$, $DU(t_4)=4$, and in the case of $M(\mathrm{B}_2\_\mathrm{order})=M(i_1)=1$, $M(\mathrm{B}_1\_\mathrm{order})=0$, $DU(t_4)=4$, the calculation is similar.

In the above logistic chain system, many business processes can be formed based on accepting or refusing orders. Because of the length and space constraints, firing time intervals of transitions are given in two cases and other cases can be calculated similarly. The first case is to accept two orders from two buyers and provide products for two buyers as shown in Table 2.

Based on Table 2, $T_{EF}(t_6)=T_0+85$ and $T_{LF}(t_6)=T_0+126$, the earliest end time of t_6 is $T_{EF}(t_6)=T_0+85$ and the latest end time is $T_{LF}(t_6)=T_0+126$.

The second case is to accept an order from Buyer_1 and refuse an order from Buyer_2. The firing time intervals of transitions are given in Table 3.

Based on Table 3, $T_{EF}(t_6)=T_0+72$ and $T_{LF}(t_6)=T_0+107$, the earliest end time of t_6 is $T_{EF}(t_6)=T_0+72$ and the latest end time $T_{LF}(t_6)=T_0+107$. See the Action column in tables 2 and 3 for details of whether the transition can be fired. If yes, the transition can be fired. Otherwise, it cannot be fired.

V. CONCLUSION

*LTPN*s are presented to model and analyze time efficiency of a logistic chain system with batch processing function and passing value uncertainty in this paper. Different computation methods of the firing time interval corresponding to an ordinary transition and two kinds of logical transitions are proposed. An example of logistic chain systems is given to illustrate the use of methods. Further research work is to develop a tool for visually constructing a *LTPN* model of for different practical systems, automatically analyzing time-related properties of models and providing analysis result.

REFERENCES

[1] Y. Han, J. Wang, and P. Zhang, "Business-oriented service modeling: A case study," *Simul. Model. Pract. Theory*, vol. 17, no. 8, pp. 1413–1429, Sep. 2009.
[2] X. Lu, M. Zhou, A. C. Ammari, and J. Ji, "Hybrid Petri nets for modeling and analysis of microgrid systems," *IEEE/CAA J. Automatica Sinica*, vol. 3, no. 4, pp. 349–356, Oct. 2016.
[3] M. Silva, "Modeling, analysis, and control with Petri nets," in *Encyclopedia of Systems and Control*. London, U.K.: Springer, 2015.
[4] S. Reveliotis and Z. Fei, "Robust deadlock avoidance for sequential resource allocation systems with resource outages," *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 4, pp. 1695–1711, Oct. 2017.

[5] Q. Hu, Y. Y. Du, and S. X. Yu, "Service net algebra based on logic Petri nets," *Inf. Sci.*, vol. 268, no. 6, pp. 271–289, 2014.
[6] H. Qi, Y. Du, and W. Liu, "Process model repairing method based on reachable markings," *J. Shandong Univ. Sci. Technol., Nature Sci.*, vol. 36, no. 1, pp. 118–124, Feb. 2017.
[7] C. Yan, H. Sun, and W. Liu, "Study of fuzzy association rules and cross-selling toward property insurance customers based on FARMA," *J. Intell. Fuzzy Syst.*, vol. 31, no. 6, pp. 2789–2794, Dec. 2016.
[8] N. Ran, H. Su, and S. Wang, "An improved approach to test diagnosability of bounded Petri nets," *IEEE/CAA J. Automatica Sinica*, vol. 4, no. 2, pp. 297–303, Apr. 2017.
[9] W. Duo, X. Jiang, O. Karoui, X. Guo, D. You, S. Wang, and Y. Ruan, "A deadlock prevention policy for a class of multithreaded software," *IEEE Access*, vol. 8, pp. 16676–16688, 2020.
[10] W. Liu, L. Wang, Y. Du, and M. Li, "Deadlock property analysis of concurrent programs based on Petri net structure," *Int. J. Parallel Program.*, vol. 45, no. 4, pp. 879–898, Aug. 2017.
[11] H. X. Li and Y. Y. Du, "A survey of research issues and key technology for business process management," *J. Shandong Univ. Sci. Technol, Nature Sci.*, vol. 34, no. 1, pp. 22–28, 2015.
[12] Y. H. Tian and Y. Y. Du, "A grouping algorithm of optimal alignments," *J. Shandong Univ. Sci. Technol.*, vol. 34, no. 1, pp. 29–34, 2015.
[13] S. Wang, D. You, M. Zhou, and C. Seatzu, "Characterization of admissible marking sets in Petri nets with uncontrollable transitions," *IEEE Trans. Autom. Control*, vol. 61, no. 7, pp. 1953–1958, Jul. 2016.
[14] S. Wang, D. You, and M. Zhou, "A necessary and sufficient condition for a resource subset to generate a strict minimal siphon in s 4PR," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 4173–4179, Aug. 2017.
[15] Y. Y. Wang and Y. Y. Du, "Conformance checking based on extended footprint matrix," *J. Shandong Univ. Sci. Technol., Nature Sci.*, vol. 37, no. 2, pp. 9–15, 2018.
[16] X. Guo, S. Wang, D. You, Z. Li, and X. Jiang, "A siphon-based deadlock prevention strategy for S^3PR," *IEEE Access*, vol. 7, pp. 86863–86873, 2019.
[17] P. A. Bourdil, B. Berthomieu, and S. D. Zilio, "Symmetry reduced state classes for time Petri nets," *ACM Symp. Appl. Comput.*, vol. 132, no. 2, pp. 209–225, 2016.
[18] M. Sogbohossou and A. Vianou, "Formal modeling of grafcets with time Petri nets," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 5, pp. 1978–1985, Sep. 2015.
[19] Y.-S. Huang, Y.-S. Weng, and M. Zhou, "Design of traffic safety control systems for emergency vehicle preemption using timed Petri nets," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 4, pp. 2113–2120, Aug. 2015.
[20] A. Rogge-Solti and M. Weske, "Prediction of business process durations using non-Markovian stochastic Petri nets," *Inf. Syst.*, vol. 54, pp. 1–14, Dec. 2015.
[21] J. J. P. Tsai, S. Jennhwa Yang, and Y.-H. Chang, "Timing constraint Petri nets and their application to schedulability analysis of real-time system specifications," *IEEE Trans. Softw. Eng.*, vol. 21, no. 1, pp. 32–49, Jan. 1995.
[22] W. Song, "Timing constraint Petri nets and their schedulability analysis and verification," *J. Softw.*, vol. 18, no. 1, pp. 11–21, 2007.
[23] Y. Y. Du and B. Q. Guo, "Logic Petri nets and equivalency," *Inf. Technol. J.*, vol. 8, no. 1, pp. 95–100, Jan. 2009.
[24] W. Liu, M. Lin, and C. Yan, "Formal interoperability models of sensor networks based on logical workflow nets," *Int. J. Softw. Eng. Knowl. Eng.*, vol. 29, no. 5, pp. 671–691, May 2019.
[25] Y. Du, L. Qi, and M. Zhou, "A vector matching method for analysing logic Petri nets," *Enterprise Inf. Syst.*, vol. 5, no. 4, pp. 449–468, Nov. 2011.
[26] Y. Du and Y. Ning, "Property analysis of logic Petri nets by marking reachability graphs," *Frontiers Comput. Sci.*, vol. 8, no. 4, pp. 684–692, Aug. 2014.
[27] J. Wang, Y. Du, and S. Yu, "Coloured logic Petri nets and analysis of their reachable trees," *Enterprise Inf. Syst.*, vol. 9, no. 8, pp. 900–919, 2015.
[28] W. Liu, Y. Y. Du, and C. Yan, "Soundness preservation in composed logical time workflflow nets," *Enterprise Inf. Syst.*, vol. 6, no. 1, pp. 95–113, 2012.
[29] W. Liu, Y. Du, M. Zhou, and C. Yan, "Transformation of logical workflow nets," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 44, no. 10, pp. 1401–1412, Oct. 2014.
[30] W. Liu, L. Wang, X. Feng, M. Qi, C. Yan, and M. Li, "Soundness analytics of composed logical workflow nets," *Int. J. Parallel Program.*, vol. 47, no. 4, pp. 709–724, Aug. 2019.
[31] J. Sha, Y. Du, and L. Qi, "A user requirement oriented Web service discovery approach based on logic and threshold Petri net," *IEEE/CAA J. Automatica Sinica*, vol. 6, no. 6, pp. 1528–1542, Nov. 2019.
[32] L. Wang, Y. Du, and L. Qi, "Efficient deviation detection between a process model and event logs," *IEEE/CAA J. Automatica Sinica*, vol. 6, no. 6, pp. 1352–1364, Nov. 2019.

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