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Incomplete and Inconsistent Information Analysis Method Considering Time Factors: Dynamic Paraconsistent Soft Sets and Their Application to Decision Making

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ABSTRACT Paraconsistent soft sets can address issues involving incomplete and inconsistent information simultaneously. In this paper, paraconsistent soft sets are furtherly extended to dynamic paraconsistent soft sets by introducing time factors. We define some basic operations, such as dynamic paraconsistent soft subsets, complement, "AND", restricted intersection, relaxed intersection, restricted cross and relaxed cross. Subsequently, we also propose the definitions of dynamic paraconsistent soft decision system, compound time choice value, compound time decision value, compound time weighting vector and final decision value. Additionally, we construct a decision making algorithm for dynamic paraconsistent soft sets, which can address issues involving both dynamic incomplete and inconsistent information. Furthermore, we apply the proposed algorithm to a practical loan problem for small and micro enterprises. Finally, we perform a sensitivity analysis and a comparative analysis to prove the effectiveness and feasibility of the proposed algorithm.

INDEX TERMS Dynamic paraconsistent soft sets, incomplete information, inconsistent information, compound time, decision making.

I. INTRODUCTION

Many complex issues in social science, economics, medical science and engineering involve uncertainties. Scholars have proposed some theories such as the theory of probability, fuzzy set [1] and interval mathematics [2] to address these complex issues. However, Molodtsov [3] pointed out that these theories have inherent limitations in insufficient parameterization tools, and he proposed soft set theory as a newly mathematical tool to cope with uncertainties. Currently, researches on soft sets have been growing rapidly, such as extended theories [4]–[6], algebraic structure [7]–[9], medical diagnosis [10], normal parameter reduction [11]–[14], combination forecast [15], data mining [16] and decision making [17]–[22].

In complex issues, incomplete and inconsistent information exists widely. Incomplete information appears when the information is not collected or lost. And inconsistent information is likely to be appeared because of the different sources they are collected from. As an effective tool for handling uncertainties, soft set theory has been playing an important role in incomplete and inconsistent information analysis.

In terms of the analysis of incomplete information, Zou and Xiao [23] initiated the incomplete information analysis method based on soft sets, by constructing a weighted average method for standard soft sets under incomplete information.

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Kong *et al.* [24] simplified the weighted average method of Zou and Xiao [23] by directly replacing incomplete information with simplified probability. Qin and Ma [25] proposed an incomplete information analysis method based on interval-valued fuzzy soft sets. Wang and Qin [26] proposed weighted incomplete fuzzy soft sets and incomplete weighted fuzzy soft sets, and put forward corresponding decision making methods.

In terms of the analysis of inconsistent information, Maji [27] originally researched inconsistent information based on neutrosophic soft sets by integrating neutrosophic sets into soft set theory. Karaaslan [28] proposed the correlation coefficient of single-valued neutrosophic refined soft sets and applied it to cluster analysis. Guan *et al.* [29] established neutrosophic soft sets forecasting model based on multi-attribute time series. Abu Qamar and Hassan [30] proposed Q-neutrosophic soft sets, and put forward the decision making algorithm based on Q-neutrosophic soft relation [30] and Q-neutrosophic-set aggregation operator [31], respectively.

However, the existing studies cannot address issues involving both incomplete and inconsistent information. In order to make up for the defect, Dong and Hou [32] put forward the concept of paraconsistent soft sets by combining paraconsistent reasoning with soft set theory. Paraconsistent soft sets employ four-valued structure [33] in paraconsistent reasoning. Four-valued structure expresses not only the parameters of classical soft sets, but also three other parameters, namely approximate opposite, incomplete information and inconsistent information. Four-valued structure effectively extends the parameter expression ability, and enables paraconsistent soft sets to address complex issues involving both incomplete and inconsistent information.

In real world, information changes with time and presents dynamicity. Nevertheless, paraconsistent soft sets can only describe information at a certain time, and do not consider the dynamicity of information. The defect limits the modeling of some issues involving dynamic incomplete and inconsistent information. Therefore, it is of great significance to introduce the time factor into paraconsistent soft sets. In this paper, we propose the definition of dynamic paraconsistent soft sets, as a more general perspective than paraconsistent soft sets, to describe issues changing with time and involving both incomplete and inconsistent information. Then, we define basic operations, such as complement, "And", restricted intersection, relaxed intersection, restricted cross and relaxed cross. Also, this study introduces the concepts of dynamic paraconsistent soft decision system, compound time choice value, compound time decision value, compound time weighting vector and final decision value. Moreover, we construct the corresponding decision making method, and employ it to a practical loan problem. Finally, a sensitivity analysis and a comparative analysis with the previous method are performed.

The remainder of this paper is organized as follows: Section II reviews the definitions of soft set and paraconsistent soft set, and the time weighting vector. Section III introduces the concept of dynamic paraconsistent soft set and defines the basic operations. Section IV proposes a dynamic paraconsistent soft decision system and a decision making method of dynamic paraconsistent soft sets. Furthermore, a sensitivity analysis and a comparative analysis are implemented to demonstrate the validity of the proposed method in Section V. Some research conclusions are presented in Section VI.

II. PRELIMINARIES

In this section, the basic concepts of soft set and paraconsistent soft set are reviewed. Besides, the method of obtaining the time weight vector is also introduced briefly.

A. SOFT SET

Molodtsov [3] originally introduced the concept of soft set which is free from the inadequacy of the parameterized tools of in the existing methods. A pair (F, E) is called a soft set over U, where F is a mapping on the parameter set E given by

$$F : E \to P(U).$$

B. PARACONSISTENT SOFT SET

In order to address issues involving both incomplete and inconsistent information in reality, Dong and Hou [32] proposed paraconsistent soft sets. Let (F, P) be a soft set over universe of discourse U, P be a family of parameters set and F be a mapping defined by $F : P \to P(U)$. (F, P) is said to be a paraconsistent soft set, if it satisfies the following conditions:

(i) $\varepsilon = \left\{\varepsilon^{+}, \varepsilon^{-}, \varepsilon^{\perp}, \varepsilon^{\mathrm{T}}\right\}, \varepsilon \in P$.

$\varepsilon^{*}(* = +, -, \perp, \mathrm{T})$ is called a cell parameter. Among them, ε^{+} and ε^{-} respectively represent "approximately belonging to ε" and "approximately not belonging to ε". $\varepsilon^{\perp}$ indicates "lacking in information on the parameter ε", and ε^{T} stands for "inconsistency of the parameter ε".

(ii) $\cup_{\varepsilon^{*}\in\varepsilon\in P} F(\varepsilon) = U$.

(iii) For any two cell parameters $\varepsilon^{i}, \varepsilon^{j} \in \varepsilon$, $\varepsilon^{i} \neq \varepsilon^{j}$, $F(\varepsilon^{i}) \cap \mathrm{F}(\varepsilon^{j}) = \phi$.

C. OBTAINING THE TIME WEIGHTING VECTOR

Considering that the information in real world changes with time, we need to comprehensively consider the information at different times in the decision making process. However, there are differences in the importance of information at different times, then Guo *et al.* [34] proposed a method to determine the time weighting vector, which employs $W = \{\omega_1, \omega_2, \ldots, \omega_p\}^{\mathrm{T}}$ to express the importance degree of each single time. Before giving the mathematical programming method for determining $W = \{\omega_1, \omega_2, \ldots, \omega_p\}^{\mathrm{T}}$, we present the definition of the entropy I and the time-degree λ successively.

The entropy I reveals average information in information theory and is defined as $I = -\sum_{k=1}^{p} \omega_k \ln\omega_k$. The higher the entropy is, the less information it carries.

The time-degree λ reflects the importance degree of each single time. The more λ leans towards 0, the more attention decision makers pay to the recent information. On the contrary, the more λ leans towards 1, the more attention decision makers pay to the previous information. And $\lambda = 0.5$ indicates that decision makers attach equal importance to information under each single time. And the time-degree λ is defined as $\lambda = \sum_{k=1}^{p} \frac{p-k}{p-1}\omega_k$.

Then the time weighting vector $W = \{\omega_1, \omega_2, \ldots, \omega_p\}^{\mathrm{T}}$ is obtained by solving the following nonlinear programming problem based on the given time-degree λ.

$$\max I = -\sum_{k=1}^{p} \omega_k \ln\omega_k$$

$$\begin{cases} \lambda = \sum_{k=1}^{p} \frac{p-k}{p-1}\omega_k \\ \sum_{k=1}^{p} \omega_k = 1, \omega_k \in [0,1] \\ k = 1, 2, \ldots, p \end{cases}$$

III. DYNAMIC PARACONSISTENT SOFT SETS

A. CONCEPT OF DYNAMIC PARACONSISTENT SOFT SET

In this part, we introduce the concept of dynamic paraconsistent soft set and give an example.

In order to address the issues which involve both dynamic incomplete and inconsistent information, we propose the concept of dynamic paraconsistent soft set. Let (F, P) be a soft set over U and P be a family of parameter set, and F be a mapping defined by $F : P \to P(U)$. Considering that $T = \{t_1, t_2, \ldots, t_n\}$ is a time set, if $(F, P)_t$ satisfies the following conditions, we call $(F, P)_t$ a single time dynamic paraconsistent soft set.

(i) $\varepsilon = \{\varepsilon^+, \varepsilon^-, \varepsilon^\perp, \varepsilon^{\mathrm{T}}\}, \varepsilon \in P$.

(ii) $\cup_{\varepsilon^* \in \varepsilon \in P} F_{t_i}(\varepsilon) = U, t_i \in T$.

(iii) For any two cell parameters $\varepsilon^i, \varepsilon^j \in \varepsilon, \varepsilon^i \neq \varepsilon^j, F_{t_i}(\varepsilon^i) \cap F_{t_i}(\varepsilon^j) = \phi, t_i \in T$.

Further, we call $(F, P)_{t_m t_n}$ a compound time dynamic paraconsistent soft set, where $t_m t_n (t_m, t_n \in T)$ is said to be a compound time.

Example 1: Suppose that the domain $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is a set of six small and micro enterprises under the time set $T = \{t_1, t_2\}$, $P = \{e_1, e_2, e_3\}$ is a family of parameters set, where e_1, e_2, e_3 represent the capacity of production and operation, profit and applied innovation.

In this case, we define the single time dynamic paraconsistent soft set $(F, P)_t$ to describe the capability of small and micro enterprises. The mapping of $(F, P)_t$ is as follows:

$F_{t_1}(e_1^+) = \{u_1, u_2\}$, $F_{t_1}(e_1^-) = \{u_4\}$,
$F_{t_1}(e_1^\perp) = \{u_3, u_5\}$, $F_{t_1}(e_1^{\mathrm{T}}) = \{u_6\}$;
$F_{t_2}(e_1^+) = \{u_2\}$, $F_{t_2}(e_1^-) = \{u_3, u_4\}$, $F_{t_2}(e_1^\perp) = \{u_1, u_6\}$,
$F_{t_2}(e_1^{\mathrm{T}}) = \{u_5\}$;
$F_{t_1}(e_2^+) = \{u_1\}$, $F_{t_1}(e_2^-) = \{u_6\}$,
$F_{t_1}(e_2^\perp) = \{u_3\}$, $F_{t_1}(e_2^{\mathrm{T}}) = \{u_2, u_4, u_5\}$;
$F_{t_2}(e_2^+) = \{u_4\}$, $F_{t_2}(e_2^-) = \{u_5, u_6\}$, $F_{t_2}(e_2^\perp) = \{u_1\}$,
$F_{t_2}(e_2^{\mathrm{T}}) = \{u_2, u_3\}$;
$F_{t_1}(e_3^+) = \{u_1\}$, $F_{t_1}(e_3^-) = \{u_3\}$, $F_{t_1}(e_3^\perp) = \{u_2, u_5\}$,
$F_{t_1}(e_3^{\mathrm{T}}) = \{u_6\}$;
$F_{t_2}(e_3^+) = \{u_1, u_3\}$, $F_{t_2}(e_3^-) = \{u_2, u_4, u_6\}$,
$F_{t_2}(e_3^\perp) = \{u_5\}$, $F_{t_2}(e_3^{\mathrm{T}}) = \{u_3\}$.

$(F, \{e_1, e_2\})_t$ is a single time dynamic paraconsistent soft set, whereas $(F, \{e_1, e_3\})_t$ and $(F, \{e_2, e_3\})_t$ are not single time dynamic paraconsistent soft sets. Further, $F_{t_1}(e_1^+) = \{u_1, u_2\}$ approximately means that u_1, u_2 have a good capacity of production and operation in t_1, and $F_{t_1}(e_1^-) = \{u_4\}$ approximately indicates that the production and operation capacity of u_4 is poor in t_1. $F_{t_1}(e_1^\perp) = \{u_3, u_5\}$ approximately represents that u_3, u_5 lack in relevant information on the capacity of production and operation in t_1. $F_{t_1}(e_1^{\mathrm{T}}) = \{u_6\}$ approximately shows that there are contradictions in the production and operation capacity information of u_6 in t_1, which may be caused by the multiple sources of information.

The single time dynamic paraconsistent soft set $(F, \{e_1, e_2\})_t$ can be shown as Table 1.

TABLE 1. Tabular representation of $(F, \{e_1, e_2\})_t$.

t_1	e_1	e_2	t_2	e_1	e_2
u_1	+	+	u_1	⊥	⊥
u_2	+	T	u_2	+	T
u_3	⊥	⊥	u_3	−	T
u_4	−	T	u_4	−	+
u_5	⊥	T	u_5	T	−
u_6	T	−	u_6	⊥	−

B. SOME BASIC OPERATIONS OF DYNAMIC PARACONSISTENT SOFT SETS

Some basic operations and examples of dynamic paraconsistent soft sets can be defined as follows:

Definition 1: Let $(F, P)_t$ and $(G, Q)_t$ be two single time dynamic paraconsistent soft sets over U under time set $T = \{t_1, t_2, \ldots, t_n\}$. $(F, P)_t$ is said to be a single time dynamic paraconsistent soft subset of $(G, Q)_t$, if $P \subseteq Q$ and for $\forall t_i \in T, \varepsilon^* \in \varepsilon \in P, F_{t_i}(\varepsilon^*)$ and $G_{t_i}(\varepsilon^*)$ are approximately equal. We denote it as $(F, P)_t \tilde{\subseteq} (G, Q)_t$.

Similarly, if $(G, Q)_t$ is a single time dynamic paraconsistent soft subset of $(F, P)_t$, we call $(F, P)_t$ a single time dynamic paraconsistent soft superset of $(G, Q)_t$ and denote it as $(F, P)_t \tilde{\supseteq} (G, Q)_t$.

Definition 2: For two single time dynamic paraconsistent soft sets $(F, P)_t$ and $(G, Q)_t$ over U under the time set $T = \{t_1, t_2, \ldots, t_n\}$, if $(F, P)_t$ is a single time dynamic paraconsistent soft subset of $(G, Q)_t$ and $(G, Q)_t$ is a single time dynamic paraconsistent soft subset of $(F,P)_t$, we call that $(F,P)_t$ and $(G,Q)_t$ are equal.

Example 2: Suppose that $(F,P)_t$ and $(G,Q)_t$ are two single time dynamic paraconsistent soft sets over $U=\{u_1,u_2,u_3,u_4,u_5,u_6\}$ under the time set $T=\{t_1,t_2\}$, and P, Q be the parameter sets represented by $P=\{e_1\}$, $Q=\{e_1,e_2\}$.

Obviously, $P\subseteq Q$.

$$\begin{aligned}
&F_{t_1}(e_1^+)=\{u_1,u_2\},\ F_{t_1}(e_1^-)=\{u_4\},\ F_{t_1}(e_1^\perp)=\{u_3,u_5\},\\
&F_{t_1}(e_1^{\mathrm{T}})=\{u_6\};\\
&F_{t_2}(e_1^+)=\{u_2\},\ F_{t_2}(e_1^-)=\{u_3,u_4\},\ F_{t_2}(e_1^\perp)=\{u_1,u_6\},\\
&F_{t_2}(e_1^{\mathrm{T}})=\{u_5\};\\
&G_{t_1}(e_1^+)=\{u_1,u_2\},\ G_{t_1}(e_1^-)=\{u_4\},\ G_{t_1}(e_1^\perp)=\{u_3,u_5\},\\
&G_{t_1}(e_1^{\mathrm{T}})=\{u_6\};\\
&G_{t_2}(e_1^+)=\{u_2\},\ G_{t_2}(e_1^-)=\{u_3,u_4\},\\
&G_{t_2}(e_1^\perp)=\{u_1,u_6\},G_{t_2}(e_1^{\mathrm{T}})=\{u_5\};\\
&G_{t_1}(e_2^+)=\{u_3,u_5\},\ \mathrm{G}_{t_1}(e_2^-)=\{u_2\},\\
&\mathrm{G}_{t_1}(e_2^\perp)=\{u_1\},\mathrm{G}_{t_1}(e_2^{\mathrm{T}})=\{u_4,u_6\};\\
&\mathrm{G}_{t_2}(e_2^+)=\{u_4\},\ \mathrm{G}_{t_2}(e_2^-)=\{u_5,u_6\},\\
&\mathrm{G}_{t_2}(e_2^\perp)=\{u_2,u_3\},\mathrm{G}_{t_2}(e_2^{\mathrm{T}})=\{u_1\}.
\end{aligned}$$

Therefore, $(F,P)_t\tilde{\subseteq}(G,Q)_t$.

Definition 3: The complement of single time dynamic paraconsistent soft set $(F,P)_t$ is represented as $(F,P)_t^c$ and is defined by $(F,P)_t^c=(F_t^c,\rceil P)$, where $F_t^c:\rceil P\rightarrow P(U)$ is a mapping given by $F_t^c(\varepsilon^*)=F_t(\neg\varepsilon^*)$ for $\forall t_i\in T,\forall\varepsilon^*\in\varepsilon\in P$.

We call F_t^c the single time dynamic paraconsistent soft complement function of F_t. Obviously, $(F_t^c)^c=F_t$ and $((F,P)_t^c)^c=(F,P)_t$.

Example 3: For Example 1, the complement of single time dynamic paraconsistent soft set $(F,\{e_1,e_2\})_t$ is shown as follows:

$$\begin{aligned}
&F_{t_1}^c(e_1^+)=\{u_4\},\ F_{t_1}^c(e_1^-)=\{u_1,u_2\},\ F_{t_1}^c(e_1^\perp)=\{u_3,u_5\},\\
&F_{t_1}^c(e_1^{\mathrm{T}})=\{u_6\};\\
&F_{t_2}^c(e_1^+)=\{u_3,u_4\},\ F_{t_2}^c(e_1^-)=\{u_2\},\ F_{t_2}^c(e_1^\perp)=\{u_1,u_6\},\\
&F_{t_2}^c(e_1^{\mathrm{T}})=\{u_5\};\\
&F_{t_1}^c(e_2^+)=\{u_6\},\ F_{t_1}^c(e_2^-)=\{u_1\},\ F_{t_1}^c(e_2^\perp)=\{u_3\},\\
&F_{t_1}^c(e_2^{\mathrm{T}})=\{u_2,u_4,u_5\};\\
&F_{t_2}^c(e_2^+)=\{u_5,u_6\},\ F_{t_2}^c(e_2^-)=\{u_4\},\ F_{t_2}^c(e_2^\perp)=\{u_1\},\\
&F_{t_2}^c(e_2^{\mathrm{T}})=\{u_2,u_3\}.
\end{aligned}$$

Definition 4: Let $(F,P)_t$ and $(G,Q)_t$ be two single time dynamic paraconsistent soft sets over U under the time set $T=\{t_1,t_2,\ldots,t_n\}$. "$(F,P)_t$ AND $(G,Q)_t$" is represented by $(F,P)_t\wedge(G,Q)_t$ and is defined as $(F,P)_{t_m}\wedge(G,Q)_{t_n}=(H_{t_mt_n},P\times Q)$, where $H_{t_mt_n}(\alpha^i,\beta^j)=F_{t_m}(\alpha^i)\cap G_{t_n}(\beta^j)$, for $\forall(\alpha^i,\beta^j)\in(\alpha,\beta)\in P\times Q,(i,j\in\{+,-,\perp,\mathrm{T}\})$.

Example 4: Suppose that $(F,P)_t$ and $(G,Q)_t$ are two single time dynamic paraconsistent soft sets over $U=\{u_1,u_2,u_3,u_4,u_5,u_6\}$ under the time set $T=\{t_1,t_2\}$, and the parameter sets P and Q are represented as $P=\{e_1,e_2\}$, $Q=\{e_1,e_3\}$. Let

$$\begin{aligned}
&F_{t_1}(e_1^+)=\{u_1,u_2\},\ F_{t_1}(e_1^-)=\{u_4\},\ F_{t_1}(e_1^\perp)=\{u_3,u_5\},\\
&F_{t_1}(e_1^{\mathrm{T}})=\{u_6\};\\
&F_{t_2}(e_1^+)=\{u_2\},\ F_{t_2}(e_1^-)=\{u_3,u_4\},\ F_{t_2}(e_1^\perp)=\{u_1,u_6\},\\
&F_{t_2}(e_1^{\mathrm{T}})=\{u_5\};\\
&F_{t_1}(e_2^+)=\{u_1\},\ F_{t_1}(e_2^-)=\{u_6\},\ F_{t_1}(e_2^\perp)=\{u_3\},\\
&F_{t_1}(e_2^{\mathrm{T}})=\{u_2,u_4,u_5\};\\
&F_{t_2}(e_2^+)=\{u_4\},\ F_{t_2}(e_2^-)=\{u_5,u_6\},\ F_{t_2}(e_2^\perp)=\{u_1\},\\
&F_{t_2}(e_2^{\mathrm{T}})=\{u_2,u_3\}.
\end{aligned}$$

And $G_{t_1}(e_1^+)=\{u_1\}$, $G_{t_1}(e_1^-)=\{u_3\}$, $G_{t_1}(e_1^\perp)=\{u_2,u_5\}$, $G_{t_1}(e_1^{\mathrm{T}})=\{u_4,u_6\}$;

$$\begin{aligned}
&G_{t_2}(e_1^+)=\{u_5,u_6\},\ G_{t_2}(e_1^-)=\{u_1\},\\
&G_{t_2}(e_1^\perp)=\{u_3,u_4\},G_{t_2}(e_1^{\mathrm{T}})=\{u_2\};\\
&G_{t_1}(e_3^+)=\{u_3,u_5\},\ G_{t_1}(e_3^-)=\{u_2\},\\
&G_{t_1}(e_3^\perp)=\{u_1\},G_{t_1}(e_3^{\mathrm{T}})=\{u_4,u_6\};\\
&G_{t_2}(e_3^+)=\{u_4\},\ G_{t_2}(e_3^-)=\{u_5,u_6\},\\
&G_{t_2}(e_3^\perp)=\{u_2,u_3\},G_{t_2}(e_3^{\mathrm{T}})=\{u_1\}.
\end{aligned}$$

The AND operation between $F_{t_1}(e_1)$ and $G_{t_2}(e_3)$ is denoted by $F_{t_1}(e_1)\wedge G_{t_2}(e_3)$, and it can be obtained as follows:

$$\begin{aligned}
&H_{t_1t_2}(e_1^+,e_3^+)=\phi,\ H_{t_1t_2}(e_1^+,e_3^-)=\phi,\\
&H_{t_1t_2}(e_1^+,e_3^\perp)=\{u_2\},H_{t_1t_2}(e_1^+,e_3^{\mathrm{T}})=\{u_1\};\\
&H_{t_1t_2}(e_1^-,e_3^+)=\{u_4\},\ H_{t_1t_2}(e_1^-,e_3^-)=\phi,\\
&H_{t_1t_2}(e_1^-,e_3^\perp)=\phi,H_{t_1t_2}(e_1^-,e_3^{\mathrm{T}})=\phi;\ H_{t_1t_2}(e_1^\perp,e_3^+)=\phi,\\
&H_{t_1t_2}(e_1^\perp,e_3^-)=\{u_5\},\ H_{t_1t_2}(e_1^\perp,e_3^\perp)=\{u_3\},\\
&H_{t_1t_2}(e_1^\perp,e_3^{\mathrm{T}})=\phi;\\
&H_{t_1t_2}(e_1^{\mathrm{T}},e_3^+)=\phi,\ H_{t_1t_2}(e_1^{\mathrm{T}},e_3^-)=\{u_6\},\ H_{t_1t_2}(e_1^{\mathrm{T}},e_3^\perp)=\phi,\\
&H_{t_1t_2}(e_1^{\mathrm{T}},e_3^{\mathrm{T}})=\phi.
\end{aligned}$$

Definition 5: Let $(F,P)_t$ and $(G,Q)_t$ be two single time dynamic paraconsistent soft sets over U under the time set $T=\{t_1,t_2,\ldots,t_n\}$. The restricted intersection of $(F,P)_t$ and $(G,Q)_t$ is represented by $(F,P)_t\tilde{\cap}_{\mathcal{S}}(G,Q)_t$ and is defined as $(F,P)_{t_m}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_n}=(H_{t_mt_n},Y)$ where $Y=P\cap Q$. For $\forall\varepsilon^*\in\varepsilon\in Y,H_{t_mt_n}(\varepsilon^*)$ is given by

$$\begin{aligned}
H_{t_mt_n}(\varepsilon^+)=&F_{t_m}(\varepsilon^+)\cap G_{t_n}(\varepsilon^+);\\
H_{t_mt_n}(\varepsilon^-)=&\left(F_{t_m}(\varepsilon^-)\cap G_{t_n}(\varepsilon^-)\right)\cup\left(F_{t_m}(\varepsilon^+)\cap G_{t_n}(\varepsilon^-)\right)\\
&\cup\left(F_{t_m}(\varepsilon^-)\cap G_{t_n}(\varepsilon^+)\right)\cup\left(F_{t_m}(\varepsilon^\perp)\cap G_{t_n}(\varepsilon^-)\right)\\
&\cup\left(F_{t_m}(\varepsilon^-)\cap G_{t_n}(\varepsilon^\perp)\right)\cup\left(F_{t_m}(\varepsilon^{\mathrm{T}})\cap G_{t_n}(\varepsilon^-)\right)\\
&\cup\left(F_{t_m}(\varepsilon^-)\cap G_{t_n}(\varepsilon^{\mathrm{T}})\right)\cup\left(F_{t_m}(\varepsilon^\perp)\cap G_{t_n}(\varepsilon^{\mathrm{T}})\right)\\
&\cup\left(F_{t_m}(\varepsilon^{\mathrm{T}})\cap G_{t_n}(\varepsilon^\perp)\right);\\
H_{t_mt_n}(\varepsilon^\perp)=&\left(F_{t_m}(\varepsilon^+)\cap G_{t_n}(\varepsilon^\perp)\right)\cup\left(F_{t_m}(\varepsilon^\perp)\cap G_{t_n}(\varepsilon^+)\right)\\
&\cup\left(F_{t_m}(\varepsilon^\perp)\cap G_{t_n}(\varepsilon^\perp)\right);
\end{aligned}$$

$$H_{t_mt_n}(\varepsilon^{\mathrm{T}}) = \left(F_{t_m}(\varepsilon^{+}) \cap G_{t_n}(\varepsilon^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon^{\mathrm{T}}) \cap G_{t_n}(\varepsilon^{+})\right) \cup \left(F_{t_m}(\varepsilon^{\mathrm{T}}) \cap G_{t_n}(\varepsilon^{\mathrm{T}})\right).$$

Example 5: We perform restricted intersection operation on $(F, P)_t$ and $(G, Q)_t$ in Example 4, and the results are shown as follows:

$$H_{t_1t_1}(e_1^{+}) = F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{+}) = \{u_1\};$$
$$H_{t_1t_2}(e_1^{+}) = \phi;\ H_{t_2t_1}(e_1^{+}) = \phi;\ H_{t_2t_2}(e_1^{+}) = \phi.$$
$$H_{t_1t_1}(e_1^{-}) = \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_3, u_4\};$$
$$H_{t_1t_2}(e_1^{-}) = \{u_1, u_4\};\ H_{t_2t_1}(e_1^{-}) = \{u_3, u_4, u_5, u_6\};$$
$$H_{t_2t_2}(e_1^{-}) = \{u_1, u_3, u_4\}.$$
$$H_{t_1t_1}(e_1^{\perp}) = \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_2, u_5\};$$
$$H_{t_1t_2}(e_1^{\perp}) = \{u_3, u_5\};\ H_{t_2t_1}(e_1^{\perp}) = \{u_1, u_2\};\ H_{t_2t_2}(e_1^{\perp}) = \{u_6\}.$$
$$H_{t_1t_1}(e_1^{\mathrm{T}}) = \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) = \{u_6\};$$
$$H_{t_1t_2}(e_1^{\mathrm{T}}) = \{u_2, u_6\};\ H_{t_2t_1}(e_1^{\mathrm{T}}) = \phi;\ H_{t_2t_2}(e_1^{\mathrm{T}}) = \{u_2, u_5\}.$$

Definition 6: Let $(F, P)_t$ and $(G, Q)_t$ be two single time dynamic paraconsistent soft sets over U under the time set $T = \{t_1, t_2, \ldots, t_n\}$. The relaxed intersection of $(F, P)_t$ and $(G, Q)_t$ is represented by $(F, P)_{t_m} \tilde{\cap}_{\mathscr{L}} (G, Q)_{t_n}$ and is defined as $(F, P)_{t_m} \tilde{\cap}_{\mathscr{L}} (G, Q)_{t_n} = (H_{t_mt_n}, Y)$ where $Y = P \cap Q$. For $\forall \varepsilon^* \in \varepsilon \in Y$, $H_{t_mt_n}(\varepsilon^*)$ is given by

$$H_{t_mt_n}(\varepsilon^{+}) = \left(F_{t_m}(\varepsilon^{+}) \cap G_{t_n}(\varepsilon^{+})\right) \cup \left(F_{t_m}(\varepsilon^{+}) \cap G_{t_n}(\varepsilon^{-})\right) \cup \left(F_{t_m}(\varepsilon^{-}) \cap G_{t_n}(\varepsilon^{+})\right) \cup \left(F_{t_m}(\varepsilon^{+}) \cap G_{t_n}(\varepsilon^{\perp})\right) \cup \left(F_{t_m}(\varepsilon^{\perp}) \cap G_{t_n}(\varepsilon^{+})\right) \cup \left(F_{t_m}(\varepsilon^{+}) \cap G_{t_n}(\varepsilon^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon^{\mathrm{T}}) \cap G_{t_n}(\varepsilon^{+})\right) \cup \left(F_{t_m}(\varepsilon^{\perp}) \cap G_{t_n}(\varepsilon^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon^{\mathrm{T}}) \cap G_{t_n}(\varepsilon^{\perp})\right);$$
$$H_{t_mt_n}(\varepsilon^{-}) = \left(F_{t_m}(\varepsilon^{-}) \cap G_{t_n}(\varepsilon^{-})\right);$$
$$H_{t_mt_n}(\varepsilon^{\perp}) = \left(F_{t_m}(\varepsilon^{\perp}) \cap G_{t_n}(\varepsilon^{\perp})\right) \cup \left(F_{t_m}(\varepsilon^{\perp}) \cap G_{t_n}(\varepsilon^{-})\right) \cup \left(F_{t_m}(\varepsilon^{-}) \cap G_{t_n}(\varepsilon^{\perp})\right);$$
$$H_{t_mt_n}(\varepsilon^{\mathrm{T}}) = \left(F_{t_m}(\varepsilon^{\mathrm{T}}) \cap G_{t_n}(\varepsilon^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon^{\mathrm{T}}) \cap G_{t_n}(\varepsilon^{-})\right) \cup \left(F_{t_m}(\varepsilon^{-}) \cap G_{t_n}(\varepsilon^{\mathrm{T}})\right).$$

Example 6: We perform relaxed intersection operation on $(F, P)_t$ and $(G, Q)_t$ in Example 4, and the results are presented as follows:

$$H_{t_1t_1}(e_1^{+}) = \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_1, u_2\};$$
$$H_{t_1t_2}(e_1^{+}) = \{u_1, u_2, u_5, u_6\};\ H_{t_2t_1}(e_1^{+}) = \{u_1, u_2, u_5, u_6\};$$
$$H_{t_2t_2}(e_1^{+}) = \{u_2, u_5, u_6\}.$$
$$H_{t_1t_1}(e_1^{-}) = \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{-})\right) = \phi;\ H_{t_1t_2}(e_1^{-}) = \phi;$$
$$H_{t_2t_1}(e_1^{-}) = \{u_3\};\ H_{t_2t_2}(e_1^{-}) = \phi.$$
$$H_{t_1t_1}(e_1^{\perp}) = \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_3, u_5\};$$
$$H_{t_1t_2}(e_1^{\perp}) = \{u_3, u_4\};\ H_{t_2t_1}(e_1^{\perp}) = \phi;$$
$$H_{t_2t_2}(e_1^{\perp}) = \{u_1, u_3, u_4\}. H_{t_1t_1}(e_1^{\mathrm{T}}) = \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) = \{u_4, u_6\};$$
$$H_{t_1t_2}(e_1^{\perp}) = \phi;\ H_{t_2t_1}(e_1^{\perp}) = \{u_4\};\ H_{t_2t_2}(e_1^{\perp}) = \phi.$$

Definition 7: Let $(F, P)_t$ and $(G, Q)_t$ be two single time dynamic paraconsistent soft sets over U under the time set $T = \{t_1, t_2, \ldots, t_n\}$. The restricted cross of $(F, P)_t$ and $(G, Q)_t$ is represented by $(F, P)_{t_m} \sim_{\mathbb{S}} (G, Q)_{t_n}$ and is defined as $(F, P)_{t_m} \sim_{\mathbb{S}} (G, Q)_{t_n} = (C_{t_mt_n}, R)$ where $R = \cup_{\varepsilon_i \in P, \varepsilon_j \in Q, \varepsilon_m \in P \cap Q}\{\varepsilon_{mj}, \varepsilon_{im}; \varepsilon_{mj} = \{\varepsilon_m, \varepsilon_j\}, \varepsilon_{im} = \{\varepsilon_i, \varepsilon_m\}; m \neq i; m \neq j\}$. For $\forall \varepsilon_{ij}^* \in \varepsilon_{ij} \in R$, $C_{t_mt_n}(\varepsilon_{ij}^*)$ is given by

$$C_{t_mt_n}(\varepsilon_{ij}^{+}) = F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{+});$$
$$C_{t_mt_n}(\varepsilon_{ij}^{-}) = \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{-})\right) \cup \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{-})\right) \cup \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{+})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{-})\right) \cup \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{\perp})\right) \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{-})\right) \cup \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{\perp})\right);$$
$$C_{t_mt_n}(\varepsilon_{ij}^{\perp}) = \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{\perp})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{+})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{\perp})\right);$$
$$C_{t_mt_n}(\varepsilon_{ij}^{\mathrm{T}}) = \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{+})\right) \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right).$$

Example 7: We perform restricted cross operation on $(F, P)_t$ and $(G, Q)_t$ in Example 4, and the results are computed as follows:

$$\begin{aligned}
&C_{t_1t_1}(e_{13}^{+}) = F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{+}) = \phi;\ C_{t_1t_2}(e_{13}^{+}) = \phi;\\
&C_{t_2t_1}(e_{13}^{+}) = \phi;\ C_{t_2t_2}(e_{13}^{+}) = \phi.\\
&C_{t_1t_1}(e_{13}^{-}) = \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{-})\right) \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{+})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{\perp})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{\mathrm{T}})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{\perp})\right) = \{u_2, u_4\};\\
&C_{t_1t_2}(e_{13}^{-}) = \{u_4, u_5, u_6\};\ C_{t_2t_1}(e_{13}^{-}) = \{u_2, u_3, u_4, u_6\};\\
&C_{t_2t_2}(e_{13}^{-}) = \{u_1, u_3, u_4, u_5, u_6\}.\\
&C_{t_1t_1}(e_{13}^{\perp}) = \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{\perp})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{\perp})\right) = \{u_1, u_3, u_5\};\\
&C_{t_1t_2}(e_{13}^{\perp}) = \{u_2, u_3\};\ C_{t_2t_1}(e_{13}^{\perp}) = \{u_1\};\ C_{t_2t_2}(e_{13}^{\perp}) = \{u_2\}.\\
&C_{t_1t_1}(e_{13}^{\mathrm{T}}) = \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{\mathrm{T}})\right) = \{u_6\};\\
&C_{t_1t_2}(e_{13}^{\mathrm{T}}) = \{u_1\};\ C_{t_2t_1}(e_{13}^{\mathrm{T}}) = \{u_5\};\ C_{t_2t_2}(e_{13}^{\mathrm{T}}) = \phi.\\
&C_{t_1t_1}(e_{21}^{+}) = F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{+}) = \{u_1\};\\
&C_{t_1t_2}(e_{21}^{+}) = \phi;\ C_{t_2t_1}(e_{21}^{+}) = \phi;\ C_{t_2t_2}(e_{21}^{+}) = \phi.\\
&C_{t_1t_1}(e_{21}^{-}) = \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{\mathrm{T}})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_2, u_3, u_5, u_6\};\\
&C_{t_1t_2}(e_{21}^{-}) = \{u_1, u_4, u_6\};\ C_{t_2t_1}(e_{21}^{-}) = \{u_2, u_3, u_5, u_6\};\\
&C_{t_2t_2}(e_{21}^{-}) = \{u_1, u_3, u_5, u_6\}.\\
&C_{t_1t_1}(e_{21}^{\perp}) = \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{\perp})\right) = \phi;\\
&C_{t_1t_2}(e_{21}^{\perp}) = \{u_3\};\ C_{t_2t_1}(e_{21}^{\perp}) = \{u_1\};\ C_{t_2t_2}(e_{21}^{\perp}) = \{u_4\}.\\
&C_{t_1t_1}(e_{21}^{\mathrm{T}}) = \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) = \{u_4\};\\
&C_{t_1t_2}(e_{21}^{\mathrm{T}}) = \{u_2, u_5\};\ C_{t_2t_1}(e_{21}^{\mathrm{T}}) = \{u_4\};\ C_{t_2t_2}(e_{21}^{\mathrm{T}}) = \{u_2\}.
\end{aligned}$$

Definition 8: For two single time dynamic paraconsistent soft sets $(F, P)_t$ and $(G, Q)_t$ over U under the time set $T = \{t_1, t_2, \ldots, t_n\}$, the relaxed cross of $(F, P)_t$ and $(G, Q)_t$ is represented by $(F, P)_{t_m} \sim_{\mathscr{L}} (G, Q)_{t_n}$ and is defined as $(F, P)_{t_m} \sim_{\mathscr{L}} (G, Q)_{t_n} = (C_{t_mt_n}, R)$ where $R = \cup_{\varepsilon_i \in P, \varepsilon_j \in Q, \varepsilon_m \in P \cap Q}\{\varepsilon_{mj}, \varepsilon_{im}: \varepsilon_{mj} = \{\varepsilon_m, \varepsilon_j\}, \varepsilon_{im} = \{\varepsilon_i, \varepsilon_m\}; m \neq i; m \neq j\}$. For $\forall \varepsilon_{ij}^{*} \in \varepsilon_{ij} \in R$, $C_{t_mt_n}(\varepsilon_{ij}^{*})$ is given by

$$\begin{aligned}
&C_{t_mt_n}(\varepsilon_{ij}^{+}) = \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{+})\right) \cup \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{-})\right)\\
&\qquad \cup \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{+})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{+})\right)\\
&\qquad \cup \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{\perp})\right) \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{+})\right)\\
&\qquad \cup \left(F_{t_m}(\varepsilon_i^{+}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right)\\
&\qquad \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{\perp})\right);\\
&C_{t_mt_n}(\varepsilon_{ij}^{-}) = F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{-});\\
&C_{t_mt_n}(\varepsilon_{ij}^{\perp}) = \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{\perp})\right) \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{-})\right)\\
&\qquad \cup \left(F_{t_m}(\varepsilon_i^{\perp}) \cap G_{t_n}(\varepsilon_j^{\perp})\right);\\
&C_{t_mt_n}(\varepsilon_{ij}^{\mathrm{T}}) = \left(F_{t_m}(\varepsilon_i^{-}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right) \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{-})\right)\\
&\qquad \cup \left(F_{t_m}(\varepsilon_i^{\mathrm{T}}) \cap G_{t_n}(\varepsilon_j^{\mathrm{T}})\right).
\end{aligned}$$

Example 8: We perform relaxed cross operation on $(F, P)_t$ and $(G, Q)_t$ in Example 4, and the results are obtained as follows:

$$\begin{aligned}
&C_{t_1t_1}(e_{13}^{+}) = \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{+})\right) \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{+})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{\perp})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{+}) \cap G_{t_1}(e_3^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{\mathrm{T}})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{\perp})\right) = \{u_1, u_2, u_3, u_5\};\\
&C_{t_1t_2}(e_{13}^{+}) = \{u_1, u_2, u_4\};\ C_{t_2t_1}(e_{13}^{+}) = \{u_2, u_3, u_5, u_6\};\\
&C_{t_2t_2}(e_{13}^{+}) = \{u_1, u_2, u_4\}.\\
&C_{t_1t_1}(e_{13}^{-}) = F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{-}) = \phi;\ C_{t_1t_2}(e_{13}^{-}) = \phi;\\
&C_{t_2t_1}(e_{13}^{-}) = \phi;\ C_{t_2t_2}(e_{13}^{-}) = \phi.\\
&C_{t_1t_1}(e_{13}^{\perp}) = \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{\perp})\right) \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{\perp}) \cap G_{t_1}(e_3^{\perp})\right) = \phi;\\
&C_{t_1t_2}(e_{13}^{\perp}) = \{u_3, u_5\};\ C_{t_2t_1}(e_{13}^{\perp}) = \{u_1\};\\
&C_{t_2t_2}(e_{13}^{\perp}) = \{u_3, u_6\}.\\
&C_{t_1t_1}(e_{13}^{\mathrm{T}}) = \left(F_{t_1}(e_1^{-}) \cap G_{t_1}(e_3^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_1^{\mathrm{T}}) \cap G_{t_1}(e_3^{\mathrm{T}})\right) = \{u_4, u_6\};\\
&C_{t_1t_2}(e_{13}^{\mathrm{T}}) = \{u_6\};\ C_{t_2t_1}(e_{13}^{\mathrm{T}}) = \{u_4\};\\
&C_{t_2t_2}(e_{13}^{\mathrm{T}}) = \{u_5\}.\\
&C_{t_1t_1}(e_{21}^{+}) = \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{-})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{+})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{+})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{+}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{\mathrm{T}})\right)\\
&\qquad \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_1, u_2, u_5\};
\end{aligned}$$

$C_{t_1t_2}(e_{21}^{+}) = \{u_1, u_4, u_5, u_6\}$; $C_{t_2t_1}(e_{21}^{+}) = \{u_1, u_2, u_4\}$;
$C_{t_2t_2}(e_{21}^{+}) = \{u_3, u_4, u_5, u_6\}$.
$C_{t_1t_1}(e_{21}^{-}) = F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{-}) = \phi$; $C_{t_1t_2}(e_{21}^{-}) = \phi$;
$C_{t_2t_1}(e_{21}^{-}) = \phi$; $C_{t_2t_2}(e_{21}^{-}) = \phi$.

$$C_{t_1t_1}(e_{21}^{\perp}) = \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{\perp})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_2^{\perp}) \cap G_{t_1}(e_1^{\perp})\right) = \{u_3\};$$

$C_{t_1t_2}(e_{21}^{\perp}) = \{u_3\}$; $C_{t_2t_1}(e_{13}^{\perp}) = \{u_5\}$; $C_{t_2t_2}(e_{13}^{\perp}) = \{u_1\}$.

$$C_{t_1t_1}(e_{21}^{\mathrm{T}}) = \left(F_{t_1}(e_2^{-}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{-})\right) \cup \left(F_{t_1}(e_2^{\mathrm{T}}) \cap G_{t_1}(e_1^{\mathrm{T}})\right) = \{u_4, u_6\};$$

$C_{t_1t_2}(e_{21}^{\mathrm{T}}) = \{u_2\}$; $C_{t_2t_1}(e_{21}^{\mathrm{T}}) = \{u_3, u_6\}$; $C_{t_2t_2}(e_{21}^{\mathrm{T}}) = \{u_2\}$.

IV. DYNAMIC PARACONSISTENT SOFT DECISION SYSTEM

In this section, we propose a dynamic paraconsistent soft decision system and give a decision making algorithm.

A. DYNAMIC PARACONSISTENT SOFT DECISION SYSTEM AND RELATED DEFINITIONS

Let $(H_{t_mt_n}, Y)$ and $(C_{t_mt_n}, R)$ be the intersection and cross of two single time dynamic paraconsistent soft sets $(F, P)_{t_m}$ and $(G, Q)_{t_n}$, respectively. Assuming that n_+ and n_- respectively represent the number of elements which belongs to $C_{t_mt_n}(\varepsilon_{ij}^{+})$ and $C_{t_mt_n}(\varepsilon_{ij}^{-})$ in U, and the dynamic paraconsistent soft decision system is defined as $(H_{t_mt_n}^{d}, Y)$ where $T = \{t_1, t_2, \ldots, t_n\}$. For $\forall \varepsilon^* \in \varepsilon \in Y$, the dynamic paraconsistent soft decision rules under incomplete and inconsistent information are shown as follows:

If $u \in H_{t_mt_n}(\varepsilon^{+})$, then $u \in H_{t_mt_n}^{d}(\varepsilon^{+})$;
If $u \in H_{t_mt_n}(\varepsilon^{-})$, then $u \in H_{t_mt_n}^{d}(\varepsilon^{-})$;
If $n_+ > n_-$, $u \in H_{t_mt_n}(\varepsilon^{\perp}) \cup H_{t_mt_n}(\varepsilon^{\mathrm{T}})$, then $u \in H_{t_mt_n}^{d}(\varepsilon^{+})$;
If $n_+ < n_-$, $u \in H_{t_mt_n}(\varepsilon^{\perp}) \cup H_{t_mt_n}(\varepsilon^{\mathrm{T}})$, then $u \in H_{t_mt_n}^{d}(\varepsilon^{-})$;
If $n_+ = n_-$, $u \in H_{t_mt_n}(\varepsilon^{\perp})$, then $u \in H_{t_mt_n}^{d}(\varepsilon^{\perp})$;
If $n_+ = n_-$, $u \in H_{t_mt_n}(\varepsilon^{\mathrm{T}})$, then $u \in H_{t_mt_n}^{d}(\varepsilon^{\mathrm{T}})$.

If $(H_{t_mt_n}, Y)$ and $(C_{t_mt_n}, R)$ are restricted intersection and restricted cross of two single time dynamic paraconsistent soft sets $(F, P)_{t_m}$ and $(G, Q)_{t_n}$, we call $(H_{t_mt_n}^{d}, Y)$ a restricted dynamic paraconsistent soft decision system, denoted as $(H_{t_mt_n}^{d}, Y)_{\mathcal{S}}$. And decision based on restricted dynamic paraconsistent soft decision system is called restricted decision.

If $(H_{t_mt_n}, Y)$ and $(C_{t_mt_n}, R)$ are relaxed intersection and relaxed cross of two single time dynamic paraconsistent soft sets $(F, P)_{t_m}$ and $(G, Q)_{t_n}$, we call $(H_{t_mt_n}^{d}, Y)$ a relaxed dynamic paraconsistent soft decision system, defined by $(H_{t_mt_n}^{d}, Y)_{\mathcal{L}}$. And decision based on relaxed dynamic paraconsistent soft decision system is called relaxed decision.

In addition, we define compound time choice value, compound time decision value, compound time weighting vector and final decision value as shown below and apply them to decision problems.

Definition 9: $(H_{t_mt_n}^{d}, Y)$ is a dynamic paraconsistent soft decision system, and $(C_{t_mt_n}, R)$ is the intersection of $(F, P)_{t_m}$ and $(G, Q)_{t_n}$. Suppose that n_+, n_-, $n_\perp$ and n_{T} represent the number of elements belongs to $C_{t_mt_n}(\varepsilon_{ij}^{+})$, $C_{t_mt_n}(\varepsilon_{ij}^{-})$, $C_{t_mt_n}(\varepsilon_{ij}^{\perp})$ and $C_{t_mt_n}(\varepsilon_{ij}^{\mathrm{T}})$ in U. The choice values of elements u_p and $\varepsilon_q \in P \cap Q$ in U are defined as follows:

$$c_{t_mt_n}^{pq} = \begin{cases} \dfrac{n_+}{n_+ + n_- + n_\perp + n_{\mathrm{T}}}, & u_p \in H_{t_mt_n}^{d}(\varepsilon_q^{+}) \\ \dfrac{n_-}{n_+ + n_- + n_\perp + n_{\mathrm{T}}} \times (-1), & u_p \in H_{t_mt_n}^{d}(\varepsilon_q^{+}) \\ 0, & u_p \in H_{t_mt_n}^{d}(\varepsilon_q^{\perp}) \cup H_{t_mt_n}^{d}(\varepsilon_q^{\mathrm{T}}) \end{cases}$$

Definition 10: $(H_{t_mt_n}^{d}, Y)$ is a dynamic paraconsistent soft decision system, and the decision value of compound time of the element u_p in U is defined as $d_{t_mt_n}^{p} = \sum_q c_{t_mt_n}^{pq}$, where $\varepsilon_q = P \cap Q$.

Considering that recent information is more important than previous information in decision making, we propose compound time weighting vector based on the time weighting vector [34].

Definition 11: For the time set $T = \{t_1, t_2, \ldots, t_m, t_n, \ldots, t_p\}$,there are p^2 compound time t_mt_n to compose $T^c = \{t_1^c, \ldots, t_k^c, \ldots, t_{p^2}^c\}(t_k^c = t_mt_n, m \in [1, p], n \in [1, p])$. Then, we construct the compound time weighting vector $W = \{\omega_1, \omega_2, \ldots, \omega_{p^2}\}^{\mathrm{T}}$. The entropy of compound time is defined as $I = -\sum_{k=1}^{p^2} \omega_k \ln\omega_k$, and the time-degree is presented as $\lambda = \sum_{k=1}^{p^2} \frac{p^2-k}{p^2-1}\omega_k$.

Suppose that the weights of the compound times t_mt_n and t_lt_r are ω_i, ω_j. If $m + n > l + r$, then $\omega_i > \omega_j$; and if $m + n = l + r$, then $\omega_i = \omega_j$.

Given a time-degree λ, we can obtain the compound time weighting vector $W = \{\omega_1, \omega_2, \ldots, \omega_{p^2}\}^{\mathrm{T}}$ by addressing the nonlinear programming issue as follows:

$$\max I = -\sum_{k=1}^{p^2} \omega_k \ln\omega_k$$
$$\begin{cases} \lambda = \sum_{k=1}^{p^2} \frac{p^2-k}{p^2-1}\omega_k \\ \sum_{k=1}^{p^2} \omega_k = 1, \omega_k \in [0, 1] \\ \omega_i = \omega_j, m + n = l + r \\ k = 1, 2, \ldots, p^2 \end{cases}$$

Definition 12: $(H_{t_mt_n}^{d}, Y)$ is a dynamic paraconsistent soft decision system. The final decision value of the element u_p in U is defined as $d_p = \sum \omega_i d_{t_mt_n}^{p}$,where ω_i indicates the weight of the element u_p under the compound time t_mt_n.

Based on the comparison of the final decision value with 0, the decision makers can decide whether to choose or eliminate an object. If $d_p > 0$, then the object will be considered excellent and supposed to be chosen. And the set of these excellent objects is called the chosen set and is expressed by $\mathcal{S}$. If $d_p < 0$, then the object will be considered inferior and supposed to be deleted. The collection of these inferior

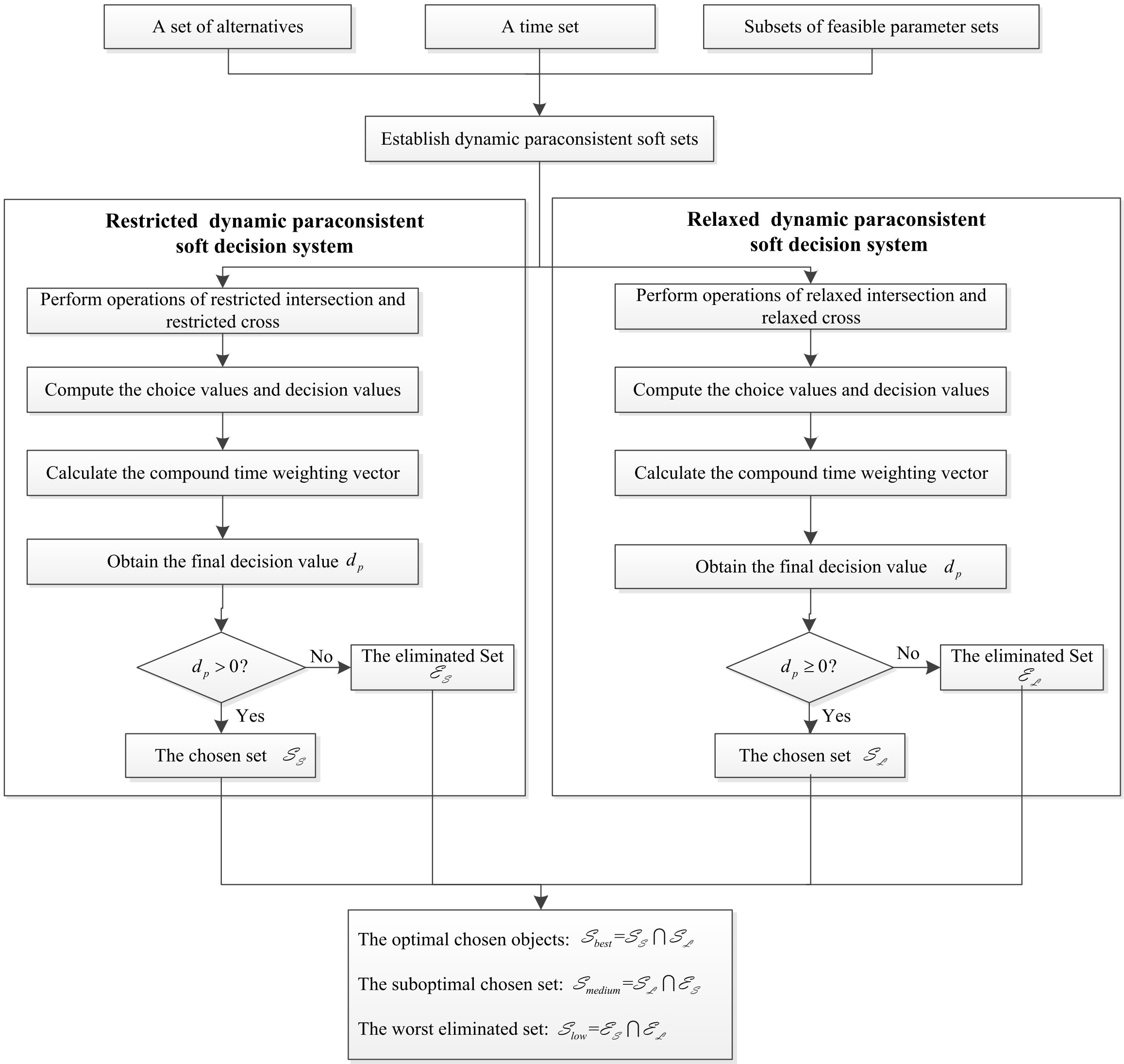

FIGURE 1. **The flowchart of the complete decision making model.**

objects is called the eliminated set and is represented by $\mathscr{E}$. Note that for restricted decision, $d_p = 0$ indicates that the objects will be eliminated, and for relaxed decision, $d_p = 0$ indicates that the objects will be chosen.

B. DECISION MAKING METHOD OF DYNAMIC PARACONSISTENT SOFT SETS

Next, we propose a dynamic paraconsistent soft set decision making method by the following algorithm, and draw a flowchart of the complete decision making model as shown in Figure 1.

Algorithm.

Step 1: Select the subsets of feasible parameter sets with respect to practical problems.

Step 2: Establish dynamic paraconsistent soft sets for each set of parameters under time set.

Step 3: Perform restricted intersection operation, relaxed intersection operation, restricted cross operation and relaxed cross operation on dynamic paraconsistent soft sets established above, based on Definitions 3.5-3.8.

Step 4: Construct a restricted dynamic paraconsistent soft decision system and a relaxed dynamic paraconsistent soft decision system.

Step 5: Compute the choice values and decision values at each compound time for the restricted and relaxed dynamic paraconsistent soft decision system by Definitions 4.1 and 4.2 respectively.

Step 6: Calculate the compound time weighting vector through Definition 11, based on the time-degree λ suggested by relevant experts.

Step 7: Obtain the final decision value by Definition 12.

Step 8: Determine the chosen set $\mathcal{S}_{\mathcal{S}}$ and $\mathcal{S}_{\mathscr{L}}$, and eliminated set $\mathscr{E}_{\mathcal{S}}$ and $\mathscr{E}_{\mathscr{L}}$ of the restricted and relaxed dynamic paraconsistent soft decision system, respectively.

Note that if a more subdivided information classification is needed to support complex decision making, then we can

TABLE 2. Tabular representation of $(F, P)_{t_1}$ and $(G, Q)_{t_1}$.

t_1	$(F,P)_{t_1}$				$(G,Q)_{t_1}$				
	e_1	e_2	e_3	e_4	e_3	e_4	e_5	e_6	e_7
u_1	+	−	T	+	+	+	−	−	T
u_2	−	+	−	⊥	+	⊥	+	+	⊥
u_3	+	+	−	−	−	−	⊥	+	−
u_4	+	+	+	+	+	+	+	−	+
u_5	−	⊥	⊥	T	T	−	T	+	+
u_6	⊥	T	+	+	+	+	+	⊥	+

TABLE 3. Tabular representation of $(F, P)_{t_2}$ and $(G, Q)_{t_2}$.

t_2	$(F,P)_{t_2}$				$(G,Q)_{t_2}$				
	e_1	e_2	e_3	e_4	e_3	e_4	e_5	e_6	e_7
u_1	−	⊥	−	T	−	+	+	−	+
u_2	⊥	−	+	+	+	+	−	+	−
u_3	+	+	−	−	−	−	+	⊥	−
u_4	+	+	+	+	+	+	−	+	+
u_5	T	+	+	−	T	⊥	+	+	−
u_6	+	T	+	+	+	+	⊥	−	+

calculate the optimal chosen set $\mathcal{S}_{best}$, the suboptimal chosen set $\mathcal{S}_{medium}$, and the worst eliminated set $\mathscr{E}_{low}$. Among them, $\mathcal{S}_{best} = \mathcal{S}_{\mathcal{S}} \cap \mathcal{S}_{\mathscr{L}}$ represents the set of optimal objects, $\mathcal{S}_{medium} = \mathcal{S}_{\mathscr{L}} \cap \mathscr{E}_{\mathcal{S}}$ indicates suboptimal objects set and $\mathcal{S}_{low} = \mathscr{E}_{\mathcal{S}} \cap \mathscr{E}_{\mathscr{L}}$ consists of the set of the eliminated objects.

V. AN APPLICATION OF THE PROPOSED ALGORITHM

In this sectionwe give an illustrative example of a loan problem to demonstrate the application of the proposed method, and the corresponding sensitivity analysis is also performed. Moreover, the existing method in [32] is also employed for a comparative analysis to prove the feasibility and superiority of the proposed method.

A. EXAMPLE ANALYSIS

A financial institution needs to select one of six small and micro enterprises to issue loans based on seven parameters. Suppose that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the enterprises set, and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ is the set of parameters. The parameters $e_1, e_2, e_3, e_4, e_5, e_6, e_7$ represent "production and operation capacity", "profit capacity", "applied innovation capacity", "loan repayment capacity", "enterprise strategic partnership", "manager credit" and "enterprise development potential", respectively. The parameter information is collected from two sources: the enterprises themselves and third-party rating agencies.

And the two sources both include incomplete and inconsistent information, due to information loss and the difference in collection time. Meanwhile, considering the dynamicity of information, it is necessary to employ dynamic paraconsistent soft decision making system to make decisions. Assume that

TABLE 4. Tabular representation of $(F, P)_{t_1} \tilde{\cap}_{\mathcal{S}} (G, Q)_{t_1}$ and $(F, P)_{t_1} \tilde{\cap}_{\mathscr{L}} (G, Q)_{t_1}$.

$t_1 t_1$	$(F,P)_{t_1} \tilde{\cap}_{\mathcal{S}} (G,Q)_{t_1}$		$(F,P)_{t_1} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_1}$	
	e_3	e_4	e_3	e_4
u_1	T	+	+	+
u_2	−	⊥	+	⊥
u_3	−	−	−	−
u_4	+	+	+	+
u_5	−	−	+	T
u_6	+	+	+	+

we consider the information under the time set $T = \{t_1, t_2\}$, where t_1, t_2 approximately represent the previous information and recent information. Then, two dynamic paraconsistent soft sets over U can be established.

Step 1: Small and micro enterprises and the third-party rating agency consider parameters sets $P = \{e_1, e_2, e_3, e_4\}$ and $Q = \{e_3, e_4, e_5, e_6, e_7\}$.

Step 2: Construct two single time dynamic paraconsistent soft sets $(F, P)_t$ and $(G, Q)_t$ over U based on the information obtained, as shown in Tables 2 and 3.

Step 3: Calculate the restricted intersection $(F, P)_{t_m} \tilde{\cap}_{\mathcal{S}} (G, Q)_{t_n}$ and relaxed intersection $(F, P)_{t_m} \tilde{\cap}_{\mathscr{L}} (G, Q)_{t_n}$ as shown in Tables 4-7, and the restricted cross $(F, P)_{t_m} \sim_{\mathcal{S}} (G, Q)_{t_n}$ and the relaxed cross $(F, P)_{t_m} \sim_{\mathscr{L}} (G, Q)_{t_n}$ as shown in Tables 8-15.

Step 4: Construct the restricted dynamic paraconsistent soft decision system $(H^d_{t_m t_n}, Y)_{\mathcal{S}}$ and the relaxed dynamic

TABLE 5. Tabular representation of $(F, P)_{t_1} \tilde{\cap}_{\mathcal{S}} (G, Q)_{t_2}$ and $(F, P)_{t_1} \tilde{\cap}_{\mathcal{L}} (G, Q)_{t_2}$.

t_1t_2	$(F,P)_{t_1} \tilde{\cap}_{\mathcal{S}} (G,Q)_{t_2}$		$(F,P)_{t_1} \tilde{\cap}_{\mathcal{L}} (G,Q)_{t_2}$	
	e_3	e_4	e_3	e_4
u_1	–	+	T	+
u_2	–	⊥	+	+
u_3	–	–	–	–
u_4	+	+	+	+
u_5	–	–	+	+
u_6	+	+	+	+

TABLE 6. Tabular representation of $(F, P)_{t_2} \tilde{\cap}_{\mathcal{S}} (G, Q)_{t_1}$ and $(F, P)_{t_2} \tilde{\cap}_{\mathcal{L}} (G, Q)_{t_1}$.

t_2t_1	$(F,P)_{t_2} \tilde{\cap}_{\mathcal{S}} (G,Q)_{t_1}$		$(F,P)_{t_2} \tilde{\cap}_{\mathcal{L}} (G,Q)_{t_1}$	
	e_3	e_4	e_3	e_4
u_1	–	T	+	+
u_2	+	⊥	+	+
u_3	–	–	–	–
u_4	+	+	+	+
u_5	T	–	+	–
u_6	+	+	+	+

TABLE 7. Tabular representation of $(F, P)_{t_2} \tilde{\cap}_{\mathcal{S}} (G, Q)_{t_2}$ and $(F, P)_{t_2} \tilde{\cap}_{\mathcal{L}} (G, Q)_{t_2}$.

t_2t_2	$(F,P)_{t_2} \tilde{\cap}_{\mathcal{S}} (G,Q)_{t_2}$		$(F,P)_{t_2} \tilde{\cap}_{\mathcal{L}} (G,Q)_{t_2}$	
	e_3	e_4	e_3	e_4
u_1	–	T	–	+
u_2	+	+	+	+
u_3	–	–	–	–
u_4	+	+	+	+
u_5	T	–	+	⊥
u_6	+	+	+	+

paraconsistent soft decision system $(H^d_{t_m t_n}, Y)_{\mathcal{L}}$, as shown in Tables 16-23.

Step 5: Calculate the choice values $c^{p3}_{t_m t_n}$, $c^{p4}_{t_m t_n}$ and the decision values $d^p_{t_m t_n}$ of decision system $(H^d_{t_m t_n}, Y)_{\mathcal{S}}$ and $(H^d_{t_m t_n}, Y)_{\mathcal{L}}$, as shown in Tables 24-25.

Step 6: Determine the compound time weighting vector based on the time-degree $\lambda = 0.1$ through Definition 11.

$$W = (0.01{,}0.09{,}0.09{,}0.81)^{\mathrm{T}}$$

Step 7: Calculate the final decision value as shown in Table 26.

Step 8: Determine the chosen sets $\mathcal{S}_{\mathcal{S}}$ and $\mathcal{S}_{\mathcal{L}}$, and the eliminated sets $\mathcal{E}_{\mathcal{S}}$ and $\mathcal{E}_{\mathcal{L}}$.

$\mathcal{S}_{\mathcal{S}} = \{u_4, u_6\}$; $\mathcal{E}_{\mathcal{S}} = \{u_1, u_2, u_3, u_5\}$; $\mathcal{S}_{\mathcal{L}} = \{u_1, u_2, u_4, u_5, u_6\}$; $\mathcal{E}_{\mathcal{L}} = \{u_3\}$.

Therefore, when the market is worse, financial institutions should adopt stricter standards to assess the small and micro enterprises. The financial institution only supposed to issue loans to u_4, u_6. When the market is better, financial institutions should adopt looser standards to assess the small and micro enterprises. Then the financial institution only supposed to issue loans to u_1, u_2, u_4, u_5, u_6.

In order to conduct more precise assessments, we further carry out the optimal chosen set $\mathcal{S}_{best}$, the suboptimal chosen set $\mathcal{S}_{medium}$ and the worst eliminated set $\mathcal{S}_{low}$ as follows.

$$\mathcal{S}_{best} = \mathcal{S}_{\mathcal{S}} \cap \mathcal{S}_{\mathcal{L}} = \{u_4, u_6\};$$
$$\mathcal{S}_{medium} = \mathcal{S}_{\mathcal{L}} \cap \mathcal{E}_{\mathcal{S}} = \{u_1, u_2, u_5\};$$
$$\mathcal{S}_{low} = \mathcal{E}_{\mathcal{S}} \cap \mathcal{E}_{\mathcal{L}} = \{u_3\}.$$

Obviously, u_4 and u_6 are supposed to be given priority to grant loans. u_1, u_2 and u_5 are the suboptimal choice for granting loans, and u_3 cannot be granted loans.

B. SENSITIVITY ANALYSIS

In the proposed decision framework, there is a parameter which may change in the dynamic environment, the time-degree λ. The reason is that decision makers may attach different importance degrees to the information at different compound times in the decision making process.

In order to observe the influence of the change of the time-degree λ on selection results and ranking results, we first calculate the final decision values of the restricted decision and the relaxed decision under different time-degrees λ, and show the calculation results in Tables 27 and 28 and Figures 2 and 3. Finally, the ranking results under different time-degrees λ are obtained, which are shown in Table 29.

As shown in Figures 2 and 3, either in the restricted decision or the relaxed decision, the chosen set and the eliminated set obtained under different time-degrees λ are the same, but the ranking results are changing with the time-degree λ as shown in Table 29. For the restricted decision, when the values of the time-degree λ are respectively 0, 0.1, 0.2, 0.3 and 0.4, the ranking results are roughly the same. When the value of the time-degree λ is 0.5, the ranking result is $u_4 \succ u_6 \succ u_2 \succ u_1 \succ u_5 \succ u_3$. And when the values of the time-degree λ are respectively 0.6, 0.7, 0.8, 0.9 and 1, the ranking results are roughly the same. And for the relaxed decision, when the values of the time-degree λ are respectively 0, 0.1, 0.2 and 0.3, the ranking results are roughly the same. When the values of the time-degree λ are respectively 0.4, 0.5, 0.6 and 0.7, the ranking results are the same. And when the values of the time-degree λ are respectively 0.8, 0.9 and 1, the ranking results are roughly the same.

From the above, we can see that either in the restricted decision or the relaxed decision, the ranking results can be classified into three situations based on the values of time-degree, which are respectively approaching 0, around 0.5, and approaching 1. As the time-degree λ is approaching 0, it means that decision makers pay more attention to recent information. As the time-degree λ is around 0.5, decision makers attach the same degree of importance to the information at each compound time. And as the time-degree λ is

TABLE 8. Tabular representation of $(F, P)_{t_1} \sim_S (G, Q)_{t_1}$.

t_1t_1	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	T	−	−	T	+	−	+	+	−	−	T	+	−	T
u_2	−	−	−	−	−	+	⊥	⊥	⊥	⊥	⊥	−	⊥	−
u_3	−	−	−	−	−	−	−	−	−	−	−	−	−	−
u_4	+	+	−	+	+	+	+	+	+	−	+	+	+	+
u_5	−	−	⊥	⊥	−	−	T	T	T	T	T	−	−	−
u_6	+	+	⊥	+	⊥	T	+	+	+	⊥	+	⊥	T	+

TABLE 9. Tabular representation of $(F, P)_{t_1} \sim_S (G, Q)_{t_2}$.

t_1t_2	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	T	T	−	T	−	−	−	−	+	−	+	+	−	T
u_2	−	−	−	−	−	+	⊥	⊥	−	⊥	−	−	−	−
u_3	−	−	−	−	−	−	−	−	−	−	−	−	−	−
u_4	+	−	+	+	+	+	+	+	−	+	+	+	+	+
u_5	⊥	⊥	⊥	−	−	−	T	T	T	T	−	−	⊥	⊥
u_6	+	⊥	−	+	⊥	T	+	+	⊥	−	+	⊥	T	+

TABLE 10. Tabular representation of $(F, P)_{t_2} \sim_S (G, Q)_{t_1}$.

t_2t_1	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	−	−	−	−	−	⊥	T	T	−	−	T	−	−	−
u_2	⊥	+	+	⊥	⊥	−	+	+	+	+	⊥	⊥	−	⊥
u_3	−	−	−	−	−	−	−	−	−	−	−	−	−	−
u_4	+	+	−	+	+	+	+	+	+	−	+	+	+	+
u_5	−	T	+	+	T	T	−	−	−	−	−	−	⊥	−
u_6	+	+	⊥	+	+	T	+	+	+	⊥	+	+	T	+

TABLE 11. Tabular representation of $(F, P)_{t_2} \sim_S (G, Q)_{t_2}$.

t_2t_2	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	T	−	−	−	−	−	−	−	T	−	T	−	⊥	T
u_2	+	−	+	−	⊥	−	+	+	−	+	−	⊥	−	+
u_3	−	−	−	−	−	−	−	−	−	−	−	−	−	−
u_4	+	−	+	+	+	+	+	+	−	+	+	+	+	+
u_5	−	+	+	−	T	T	−	−	−	−	−	−	⊥	−
u_6	+	⊥	−	+	+	T	+	+	⊥	−	+	+	T	+

approaching 1, it means that the decision makers pay more attention to the previous information. In summary, it can be seen that the difference in the emphasis on previous and recent information causes the ranking results to present three situations at different time-degree λ.

C. COMPARATIVE ANALYSIS

A comparative analysis with the previous method is performed to validate the effectiveness and superiority of the proposed method. Considering that there are few studies which can address issues involving both dynamic inconsistencies and incompleteness, this paper compares the proposed method with the method in [32] based on paraconsistent soft sets which only considers a single time. For the example in Section 4.1, since the method in [32] does not consider the dynamicity of information, that only considers the information under a single time, this study respectively employs the information in t_1 and t_2, which approximately represent the

TABLE 12. Tabular representation of $(F, P)_{t_1} \sim_{\mathscr{L}} (G, Q)_{t_1}$.

t_1t_1	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	+	–	T	T	+	+	+	+	+	+	+	+	+	+
u_2	⊥	+	+	⊥	+	+	+	+	+	+	⊥	⊥	+	⊥
u_3	–	⊥	+	–	+	+	–	–	⊥	+	–	+	+	–
u_4	+	+	+	+	+	+	+	+	+	+	+	+	+	+
u_5	⊥	+	+	+	T	+	+	+	T	+	+	–	⊥	⊥
u_6	+	+	+	+	+	+	T	T	+	+	+	+	+	+

TABLE 13. Tabular representation of $(F, P)_{t_1} \sim_{\mathscr{L}} (G, Q)_{t_2}$.

t_1t_2	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	+	+	T	+	+	–	+	+	+	+	+	+	+	+
u_2	+	–	+	–	+	+	+	+	⊥	+	⊥	+	+	+
u_3	–	+	⊥	–	+	+	–	–	+	⊥	–	+	+	–
u_4	+	+	+	+	+	+	+	+	+	+	+	+	+	+
u_5	⊥	+	+	⊥	T	+	T	T	+	+	T	⊥	⊥	⊥
u_6	+	+	+	+	+	+	+	+	+	+	+	+	+	+

TABLE 14. Tabular representation of $(F, P)_{t_2} \sim_{\mathscr{L}} (G, Q)_{t_1}$.

t_2t_1	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	+	–	–	T	+	+	+	+	T	T	T	+	+	+
u_2	+	+	+	+	+	+	+	+	+	+	+	⊥	⊥	+
u_3	–	⊥	+	–	+	+	–	–	⊥	+	–	+	+	–
u_4	+	+	+	+	+	+	+	+	+	+	+	+	+	+
u_5	+	+	+	+	T	+	T	T	T	+	+	T	+	+
u_6	+	+	+	+	+	+	+	+	+	+	+	+	+	+

TABLE 15. Tabular representation of $(F, P)_{t_2} \sim_{\mathscr{L}} (G, Q)_{t_2}$.

t_2t_2	e_{34}	e_{35}	e_{36}	e_{37}	e_{13}	e_{23}	e_{43}	e_{43}	e_{45}	e_{46}	e_{47}	e_{14}	e_{24}	e_{34}
u_1	+	+	–	+	–	⊥	T	T	+	T	+	+	+	+
u_2	+	+	+	+	+	+	+	+	+	+	+	+	+	+
u_3	–	+	⊥	–	+	+	–	–	+	⊥	–	+	+	–
u_4	+	+	+	+	+	+	+	+	+	+	+	+	+	+
u_5	+	+	+	+	T	+	T	T	+	+	–	+	+	+
u_6	+	+	+	+	+	+	+	+	+	+	+	+	+	+

previous information and recent information. Then, we compare the results obtained by the method in [32] based on the information under t_1 and t_2 separately and the proposed method which considers both information under t_1 and t_2.

For convenience, decision based on the restricted paraconsistent soft decision system [32] is called the restricted decision, and the decision based on the relaxed paraconsistent soft decision system [32] is called the relaxed decision.

Based on the method in [32], we can obtain the decision value $d^p_{\mathbb{S}}$ and $d^p_{\mathscr{L}}$ for the restricted decision and the relaxed decision under the information of the single times t_1 and t_2, as shown in Table 30. Then, we rank the six small and micro

TABLE 16. Tabular representation of $(H^d_{t_1t_1}, Y)_{\mathcal{S}}$.

t_1t_1	$(H_{t_1t_1},Y)=(F,P)_{t_1}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_1}$	e_3^d	$(H_{t_1t_1},Y)=(F,P)_{t_1}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_1}$	e_4^d
u_1	$n_->n_+$ $u_1\in H_{t_1t_1}(e_3^{\perp})\cup H_{t_1t_1}(e_3^{\mathrm{T}})$	$-$	$u_1\in H_{t_1t_1}(e_4^+)$	$+$
u_2	$u_2\in H_{t_1t_1}(e_3^-)$	$-$	$n_->n_+$ $u_2\in H_{t_1t_1}(e_4^{\perp})\cup H_{t_1t_1}(e_4^{\mathrm{T}})$	$-$
u_3	$u_3\in H_{t_1t_1}(e_3^-)$	$-$	$u_3\in H_{t_1t_1}(e_4^-)$	$-$
u_4	$u_4\in H_{t_1t_1}(e_3^+)$	$+$	$u_4\in H_{t_1t_1}(e_4^+)$	$+$
u_5	$u_5\in H_{t_1t_1}(e_3^-)$	$-$	$u_5\in H_{t_1t_1}(e_4^-)$	$-$
u_6	$u_6\in H_{t_1t_1}(e_3^+)$	$+$	$u_6\in H_{t_1t_1}(e_4^+)$	$+$

TABLE 17. Tabular representation of $(H^d_{t_1t_2}, Y)_{\mathcal{S}}$.

t_1t_2	$(H_{t_1t_2},Y)=(F,P)_{t_1}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_2}$	e_3^d	$(H_{t_1t_2},Y)=(F,P)_{t_1}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_2}$	e_4^d
u_1	$u_2\in H_{t_1t_2}(e_3^-)$	$-$	$u_1\in H_{t_1t_2}(e_4^+)$	$+$
u_2	$u_2\in H_{t_1t_2}(e_3^-)$	$-$	$n_->n_+$ $u_2\in H_{t_2t_1}(e_4^{\perp})\cup H_{t_2t_1}(e_4^{\mathrm{T}})$	$-$
u_3	$u_3\in H_{t_1t_2}(e_3^-)$	$-$	$u_3\in H_{t_1t_2}(e_4^-)$	$-$
u_4	$u_4\in H_{t_1t_2}(e_3^+)$	$+$	$u_3\in H_{t_1t_2}(e_4^+)$	$+$
u_5	$u_5\in H_{t_1t_2}(e_3^-)$	$-$	$u_5\in H_{t_1t_2}(e_4^-)$	$-$
u_6	$u_6\in H_{t_1t_2}(e_3^+)$	$+$	$u_6\in H_{t_1t_2}(e_4^+)$	$+$

TABLE 18. Tabular representation of $(H^d_{t_2t_1}, Y)_{\mathcal{S}}$.

t_2t_1	$(H_{t_2t_1},Y)=(F,P)_{t_2}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_1}$	e_3^d	$(H_{t_2t_1},Y)=(F,P)_{t_2}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_1}$	e_4^d
u_1	$u_1\in H_{t_2t_1}(e_3^-)$	$-$	$n_->n_+$ $u_1\in H_{t_2t_1}(e_4^{\perp})\cup H_{t_2t_1}(e_4^{\mathrm{T}})$	$-$
u_2	$u_1\in H_{t_2t_1}(e_3^+)$	$+$	$n_+>n_-$ $u_2\in H_{t_2t_1}(e_4^{\perp})\cup H_{t_2t_1}(e_4^{\mathrm{T}})$	$+$
u_3	$u_3\in H_{t_2t_1}(e_3^-)$	$-$	$u_3\in H_{t_2t_1}(e_4^-)$	$-$
u_4	$u_4\in H_{t_2t_1}(e_3^+)$	$+$	$u_4\in H_{t_2t_1}(e_4^+)$	$+$
u_5	$n_+=n_-$ $u_5\in H_{t_2t_1}(e_3^{\mathrm{T}})$	T	$u_5\in H_{t_2t_1}(e_4^-)$	$-$
u_6	$u_6\in H_{t_2t_1}(e_3^+)$	$+$	$u_6\in H_{t_2t_1}(e_4^+)$	$+$

TABLE 19. Tabular representation of $(H^d_{t_2t_2}, Y)_{\mathcal{S}}$.

t_2t_2	$(H_{t_2t_2},Y)=(F,P)_{t_2}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_2}$	e_3^d	$(H_{t_2t_2},Y)=(F,P)_{t_2}\tilde{\cap}_{\mathcal{S}}(G,Q)_{t_2}$	e_4^d
u_1	$u_1\in H_{t_2t_2}(e_3^-)$	$-$	$n_->n_+$ $u_1\in H_{t_2t_2}(e_4^-)$	$-$
u_2	$u_2\in H_{t_2t_2}(e_3^+)$	$+$	$u_2\in H_{t_2t_2}(e_4^+)$	$+$
u_3	$u_3\in H_{t_2t_2}(e_3^-)$	$-$	$u_3\in H_{t_2t_2}(e_4^-)$	$-$
u_4	$u_4\in H_{t_2t_2}(e_3^+)$	$+$	$u_4\in H_{t_2t_2}(e_4^+)$	$+$
u_5	$n_->n_+$ $u_5\in H_{t_2t_2}(e_3^{\perp})\cup H_{t_2t_2}(e_3^{\mathrm{T}})$	$-$	$u_5\in H_{t_2t_2}(e_4^-)$	$-$
u_6	$u_6\in H_{t_2t_2}(e_3^+)$	$+$	$u_6\in H_{t_2t_2}(e_4^+)$	$+$

enterprises, based on the decision values shown in Table 30 and the final decision values in Table 26 respectively. The ranking results are reflected in Table 31.

Next, we attempt to describe and analyze the ranking results in Table 31. For the restricted decision, the difference of the three ranking results is shown in the enterprises u_1, u_2 and u_5, which are $u_1 \succ u_2 = u_5$, $u_2 \succ u_5 = u_1$ and $u_2 \succ u_5 \succ u_1$, respectively. For the relaxed decision, the difference is reflected in the enterprises u_1, u_2, u_4, u_5 and u_6, which are $u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5$, $u_4 = u_6 = u_2 \succ u_5 \succ u_1$ and $u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1$, respectively.

As can be seen from Table 31, the differences between the ranking results of the method [32] and the proposed method is mainly reflected in two aspects, either in the restricted

TABLE 20. Tabular representation of $(H^d_{t_1t_1}, Y)_{\mathscr{L}}$.

t_1t_1	$(H_{t_1t_1}, Y) = (F,P)_{t_1} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_1}$	e_3^d	$(H_{t_1t_1}, Y) = (F,P)_{t_1} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_1}$	e_4^d
u_1	$u_1 \in H_{t_1t_1}(e_3^+)$	+	$u_1 \in H_{t_1t_1}(e_4^+)$	+
u_2	$u_2 \in H_{t_1t_1}(e_3^+)$	+	$n_+ > n_-$ $u_2 \in H_{t_1t_1}(e_4^{\perp}) \cup H_{t_1t_1}(e_4^{\mathrm{T}})$	+
u_3	$u_3 \in H_{t_1t_1}(e_3^-)$	−	$u_3 \in H_{t_1t_1}(e_4^-)$	−
u_4	$u_4 \in H_{t_1t_1}(e_3^+)$	+	$u_4 \in H_{t_1t_1}(e_4^+)$	+
u_5	$u_5 \in H_{t_1t_1}(e_3^+)$	+	$n_+ > n_-$ $u_5 \in H_{t_1t_1}(e_4^{\perp}) \cup H_{t_1t_1}(e_4^{\mathrm{T}})$	+
u_6	$u_6 \in H_{t_1t_1}(e_3^+)$	+	$u_6 \in H_{t_1t_1}(e_4^+)$	+

TABLE 21. Tabular representation of $(H^d_{t_1t_2}, Y)_{\mathscr{L}}$.

t_1t_2	$(H_{t_1t_2}, Y) = (F,P)_{t_1} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_2}$	e_3^d	$(H_{t_1t_2}, Y) = (F,P)_{t_1} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_2}$	e_4^d
u_1	$n_+ > n_-$ $u_1 \in H_{t_1t_2}(e_3^{\mathrm{T}}) \cup H_{t_1t_2}(e_3^{\mathrm{T}})$	+	$u_1 \in H_{t_1t_2}(e_4^+)$	+
u_2	$u_2 \in H_{t_1t_2}(e_3^+)$	+	$u_2 \in H_{t_1t_2}(e_4^+)$	+
u_3	$u_3 \in H_{t_1t_2}(e_3^-)$	−	$u_3 \in H_{t_1t_2}(e_4^-)$	−
u_4	$u_4 \in H_{t_1t_2}(e_3^+)$	+	$u_4 \in H_{t_1t_2}(e_4^+)$	+
u_5	$u_5 \in H_{t_1t_2}(e_3^+)$	+	$u_5 \in H_{t_1t_2}(e_4^+)$	+
u_6	$u_6 \in H_{t_1t_2}(e_3^+)$	+	$u_6 \in H_{t_1t_2}(e_4^+)$	+

TABLE 22. Tabular representation of $(H^d_{t_2t_1}, Y)_{\mathscr{L}}$.

t_2t_1	$(H_{t_2t_1}, Y) = (F,P)_{t_2} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_1}$	e_3^d	$(H_{t_2t_1}, Y) = (F,P)_{t_2} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_1}$	e_4^d
u_1	$u_1 \in H_{t_2t_1}(e_3^+)$	+	$u_1 \in H_{t_2t_1}(e_4^+)$	+
u_2	$u_2 \in H_{t_2t_1}(e_3^+)$	+	$u_2 \in H_{t_2t_1}(e_4^+)$	+
u_3	$u_3 \in H_{t_2t_1}(e_3^-)$	−	$u_3 \in H_{t_2t_1}(e_4^-)$	−
u_4	$u_4 \in H_{t_2t_1}(e_3^+)$	+	$u_4 \in H_{t_2t_1}(e_4^+)$	+
u_5	$u_5 \in H_{t_2t_1}(e_3^+)$	+	$u_5 \in H_{t_2t_1}(e_4^-)$	−
u_6	$u_6 \in H_{t_2t_1}(e_3^+)$	+	$u_6 \in H_{t_2t_1}(e_4^+)$	+

TABLE 23. Tabular representation of $(H^d_{t_2t_2}, Y)_{\mathscr{L}}$.

t_2t_2	$(H_{t_2t_2}, Y) = (F,P)_{t_2} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_2}$	e_3^d	$(H_{t_2t_2}, Y) = (F,P)_{t_2} \tilde{\cap}_{\mathscr{L}} (G,Q)_{t_2}$	e_4^d
u_1	$u_1 \in H_{t_2t_2}(e_3^-)$	−	$u_1 \in H_{t_2t_2}(e_4^+)$	+
u_2	$u_2 \in H_{t_2t_2}(e_3^+)$	+	$u_2 \in H_{t_2t_2}(e_4^+)$	+
u_3	$u_3 \in H_{t_2t_2}(e_3^-)$	−	$u_3 \in H_{t_2t_2}(e_4^-)$	−
u_4	$u_4 \in H_{t_2t_2}(e_3^+)$	+	$u_4 \in H_{t_2t_2}(e_4^+)$	+
u_5	$u_5 \in H_{t_2t_2}(e_3^+)$	+	$n_+ > n_-$ $u_5 \in H_{t_2t_2}(e_4^{\perp}) \cup H_{t_2t_2}(e_4^{\mathrm{T}})$	+
u_6	$u_6 \in H_{t_2t_2}(e_3^+)$	+	$u_6 \in H_{t_2t_2}(e_4^+)$	+

decision or the relaxed decision. On the one hand, the ranking result obtained by the method [32] under t_1 is significantly different from the other two. However, there is not much difference between ranking results obtained by the other two. For the restricted decision, the difference reflects in u_1 and u_5, and for the relaxed decision, the difference is shown in u_2, u_4 and u_6. On the other hand, compared with the proposed method, the method in [32] under a single time cannot well differentiate the enterprises, which is reflected in equivalence relation under the single times t_1 and t_2. For the restricted decision, the ranking result of u_2 and u_5 under t_1 and u_1 and u_5 under t_2 are equivalent. And for the relaxed decision, u_2, u_4 are good as u_6 under the single time t_2. In contrast, the proposed method has better distinguishing ability, which

TABLE 24. Choice values and decision values under t_1t_1 and t_1t_2.

t_1t_1	$(H^d_{t_1t_1},Y)_{\mathscr{S}}$			$(H^d_{t_1t_1},Y)_{\mathscr{L}}$			t_1t_2	$(H^d_{t_1t_2},Y)_{\mathscr{S}}$			$(H^d_{t_1t_2},Y)_{\mathscr{L}}$		
	$c^{p3}_{t_1t_1}$	$c^{p4}_{t_1t_1}$	$d^{p}_{t_1t_1}$	$c^{p3}_{t_1t_1}$	$c^{p4}_{t_1t_1}$	$d^{p}_{t_1t_1}$		$c^{p3}_{t_1t_2}$	$c^{p4}_{t_1t_2}$	$d^{p}_{t_1t_2}$	$c^{p3}_{t_1t_2}$	$c^{p4}_{t_1t_2}$	$d^{p}_{t_1t_2}$
u_1	-3/7	2/7	-1/7	4/7	1	11/7	u_1	-4/7	3/7	-1/7	5/7	1	12/7
u_2	-5/7	-2/7	-1	5/7	4/7	9/7	u_2	-5/7	-4/7	-9/7	5/7	5/7	10/7
u_3	-1	-1	-2	-3/7	-3/7	-6/7	u_3	-1	-1	-2	-3/7	-3/7	-6/7
u_4	6/7	6/7	12/7	1	1	2	u_4	5/7	6/7	11/7	1	1	2
u_5	-4/7	-3/7	-1	5/7	3/7	8/7	u_5	-3/7	2/7	-1/7	3/7	2/7	5/7
u_6	4/7	4/7	8/7	6/7	6/7	12/7	u_6	3/7	3/7	6/7	1	1	2

TABLE 25. Choice values and decision values under t_2t_1 and t_2t_2.

t_2t_1	$(H^d_{t_2t_1},Y)_{\mathscr{S}}$			$(H^d_{t_1t_1},Y)_{\mathscr{L}}$			t_2t_2	$(H^d_{t_2t_2},Y)_{\mathscr{S}}$			$(H^d_{t_2t_2},Y)_{\mathscr{L}}$		
	$c^{p3}_{t_2t_1}$	$c^{p4}_{t_2t_1}$	$d^{p}_{t_2t_1}$	$c^{p3}_{t_2t_1}$	$c^{p4}_{t_2t_1}$	$d^{p}_{t_2t_1}$		$c^{p3}_{t_2t_2}$	$c^{p4}_{t_2t_2}$	$d^{p}_{t_2t_2}$	$c^{p3}_{t_2t_2}$	$c^{p4}_{t_2t_2}$	$d^{p}_{t_2t_2}$
u_1	-5/7	-5/7	-10/7	4/7	4/7	8/7	u_1	-6/7	-3/7	-9/7	-2/7	5/7	3/7
u_2	3/7	3/7	6/7	1	5/7	12/7	u_2	3/7	3/7	-6/7	1	1	2
u_3	-1	-1	-2	-3/7	-3/7	-6/7	u_3	-1	-1	-2	-3/7	-3/7	-6/7
u_4	6/7	6/7	12/7	1	1	2	u_4	6/7	6/7	12/7	1	1	2
u_5	0	-6/7	-6/7	5/7	0	5/7	u_5	-3/7	-6/7	-9/7	5/7	5/7	10/7
u_6	5/7	5/7	10/7	1	1	2	u_6	4/7	4/7	8/7	1	1	2

TABLE 26. Tabular representation of final decision value d_p.

	$(H_d,Y)_{\mathscr{S}}$	$(H_d,Y)_{\mathscr{L}}$
	d_p	d_p
u_1	-8.29/7	4.34/7
u_2	-5.2/7	13.41/7
u_3	-2	-6/7
u_4	11.91/7	2
u_5	-7.99/7	9.08/7
u_6	8/7	13.98/7

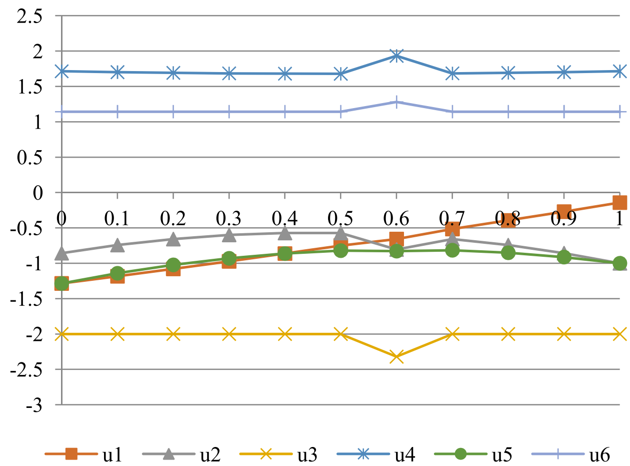

FIGURE 2. The final decision value under different λ for restricted decision.

is reflected in the absence of equivalence relation between enterprises. There are two main reasons for the differences.

(1) The method proposed in this paper makes a decision on the basis of the time set $T = \{t_1, t_2\}$, which fully considers the dynamicity of information. Meanwhile, the proposed method pays more attention to recent information under t_2. In terms of the real-time degree of information, the information under the single time t_1 is relatively lagging relative to the single time t_2. That's why the ranking result obtained by the method [32] under the single time t_1 are obviously different from the other two. Further, the information under the single time t_2 and the time set $T = \{t_1, t_2\}$ is biased towards the recent information, so the ranking result computed by the method [32] under the single time t_2 and the proposed method under the time set $T = \{t_1, t_2\}$ are slightly different.

(2) It can be seen that the ranking results based on the method in [32] only consider the information under the single time t_1 or t_2. However, the ranking result obtained by the proposed method not only considers the information at both the single times t_1 and t_2, also considers the information under the compound times t_1t_2 and t_2t_1. From the perspective of information coverage, the proposed method contains more

TABLE 27. **The final decision value under different λ for restricted decision.**

d_p	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$	$\lambda=0.4$	$\lambda=0.5$	$\lambda=0.6$	$\lambda=0.7$	$\lambda=0.8$	$\lambda=0.9$	$\lambda=1$
u_1	-9/7	-8.29/7	-7.56/7	-6.81/7	-6.04/7	-5.25/7	-4.6/7	-3.61/7	-2.76/7	-1.89/7	-1/7
u_2	-6/7	-5.2/7	-4.6/7	-4.2/7	-4/7	-4/7	-5.64/7	-4.6/7	-5.2/7	-6/7	-1
u_3	-14/7	-2	-2	-2	-2	-2	-16.24/7	-2	-2	-2	-2
u_4	12/7	11.91/7	11.84/7	11.79/7	11.76/7	11.75/7	13.52/7	11.79/7	11.84/7	11.91/7	12/7
u_5	-9/7	-7.99/7	-7.16/7	-6.51/7	-6.04/7	-5.75/7	-5.8/7	-5.71/7	-5.96/7	-6.39/7	-1
u_6	8/7	8/7	8/7	8/7	8/7	8/7	8.96/7	8/7	8/7	8/7	8/7

TABLE 28. **The final decision value under different λ for relaxed decision.**

d_p	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$	$\lambda=0.4$	$\lambda=0.5$	$\lambda=0.6$	$\lambda=0.7$	$\lambda=0.8$	$\lambda=0.9$	$\lambda=1$
u_1	3/7	4.34/7	5.56/7	6.66/7	7.64/7	8.5/7	11.16/7	9.86/7	10.36/7	10.74/7	11/7
u_2	2	13.41/7	12.84/7	12.29/7	11.76/7	11.25/7	12.36/7	10.29/7	9.84/7	9.41/7	9/7
u_3	-6/7	-6/7	-6/7	-6/7	-6/7	-6/7	-6.96/7	-6/7	-6/7	-6/7	-6/7
u_4	2	2	2	14/7	2	2	16.24/7	2	2	2	2
u_5	10/7	9.08/7	8.32/7	7.72/7	7.28/7	1	7.68/7	6.92/7	7.12/7	7.48/7	8/7
u_6	2	13.98/7	13.92/7	13.82/7	13.68/7	13.5/7	15.52/7	13.02/7	12.72/7	12.38/7	12/7

TABLE 29. **Ranking results under different λ.**

	Restricted decision	Relaxed decision
$\lambda=0$	$u_4 \succ u_6 \succ u_2 \succ u_5 = u_1 \succ u_3$	$u_4 = u_6 = u_2 \succ u_5 \succ u_1 \succ u_3$
$\lambda=0.1$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$
$\lambda=0.2$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$
$\lambda=0.3$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$
$\lambda=0.4$	$u_4 \succ u_6 \succ u_2 \succ u_5 = u_1 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_1 \succ u_5 \succ u_3$
$\lambda=0.5$	$u_4 \succ u_6 \succ u_2 \succ u_1 \succ u_5 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_1 \succ u_5 \succ u_3$
$\lambda=0.6$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_1 \succ u_5 \succ u_3$
$\lambda=0.7$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_1 \succ u_5 \succ u_3$
$\lambda=0.8$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$
$\lambda=0.9$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$
$\lambda=1$	$u_4 \succ u_6 \succ u_1 \succ u_2 = u_5 \succ u_3$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$

TABLE 30. **Decision value $d_{\mathcal{S}}^{p}$ and $d_{\mathcal{L}}^{p}$ under t_1 and t_2.**

	t_1			t_2	
	$d_{\mathcal{S}}^{p}$	$d_{\mathcal{L}}^{p}$		$d_{\mathcal{S}}^{p}$	$d_{\mathcal{L}}^{p}$
u_1	-1/7	11/7	u_1	-9/7	3/7
u_2	-1	9/7	u_2	-6/7	2
u_3	-2	-6/7	u_3	-2	-6/7
u_4	12/7	2	u_4	12/7	2
u_5	-1	8/7	u_5	-9/7	10/7
u_6	8/7	12/7	u_6	8/7	2

TABLE 31. A comparative study with the existing method.

	Restricted decision	Relaxed decision
The method in [32] under single time t_1	$u_4 \succ u_6 \succ u_1 \succ u_2 = u_5 \succ u_3$	$u_4 \succ u_6 \succ u_1 \succ u_2 \succ u_5 \succ u_3$
The method in [32] under single time t_2	$u_4 \succ u_6 \succ u_2 \succ u_5 = u_1 \succ u_3$	$u_4 = u_6 = u_2 \succ u_5 \succ u_1 \succ u_3$
The proposed method under time set $T = \{t_1, t_2\}$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$	$u_4 \succ u_6 \succ u_2 \succ u_5 \succ u_1 \succ u_3$

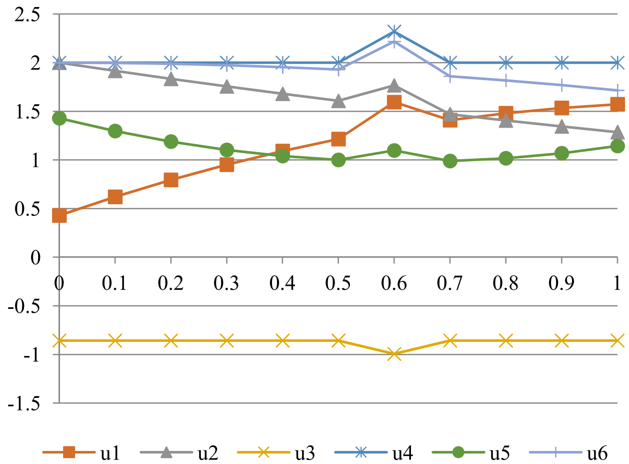

FIGURE 3. The final decision value under different λ for relaxed decision.

comprehensive information, so it has a stronger distinguishing ability than the previous method [32].

In general, compared with the method [32] which only considers a single time, the proposed method significantly improves their disadvantages and makes the results more scientific and consistent with facts. Therefore, the method proposed in this paper has obvious effectiveness and superiority.

VI. CONCLUSION

Paraconsistent soft sets are effective tools for addressing issues involving both incomplete and inconsistent information. However, they can only describe the information at a certain time, and cannot analyze information which changes with time and presents dynamic in the real word. Considering time factor, this paper extends the concept of paraconsistent soft sets to dynamic paraconsistent soft sets. Then, we define dynamic paraconsistent soft subsets, complement, "AND", restricted intersection, relaxed intersection, restricted cross, and relaxed cross. In order to apply dynamic paraconsistent soft sets to decision making, we also present the definitions of dynamic paraconsistent soft decision system, compound time choice value, compound time decision value, compound time weighting vector and final decision value. Moreover, we construct a dynamic paraconsistent soft set decision algorithm, to address issues changing with time and involving both incomplete and inconsistent information. Further, to prove the feasibility and effectiveness, this paper applies the algorithm to a loan problem for small and micro enterprises. Finally, a sensitivity analysis and a comparative analysis with the previous method are performed.

In reality, multi-attribute decision making is often influenced by the preferences of decision makers. Future work could learn from the concept of dominance relation [35] to realize the four-valued logic representation of preference relationship of decision makers, by expressing the preference model as logical statements in the decision rule. Further, a multi-attribute decision making algorithm based on dynamic paraconsistent soft set that considers the preference information of the decision maker could be constructed.

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965.
[2] M. B. Gorzałczany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," *Fuzzy Sets Syst.*, vol. 21, no. 1, pp. 1–17, Jan. 1987.
[3] D. Molodtsov, "Soft set theory—First results," *Comput. Math. Appl.*, vol. 37, nos. 4–5, pp. 19–31, 1999.
[4] S. Vijayabalaji and A. Ramesh, "Belief interval-valued soft set," *Expert Syst. Appl.*, vol. 119, pp. 262–271, Apr. 2019.
[5] M. B. Kandemir, "The concept of σ-algebraic soft set," *Soft Comput.*, vol. 22, no. 13, pp. 4353–4360, Jul. 2018.
[6] Z. Zhang, C. Wang, D. Tian, and K. Li, "A novel approach to interval-valued intuitionistic fuzzy soft set based decision making," *Appl. Math. Model.*, vol. 38, no. 4, pp. 1255–1270, Feb. 2014.
[7] A. Aygünoğlu and H. Aygün, "Introduction to fuzzy soft groups," *Comput. Math. Appl.*, vol. 58, no. 6, pp. 1279–1286, Sep. 2009.
[8] H. Aktaş and N. Çağman, "Soft sets and soft groups," *Inf. Sci.*, vol. 177, no. 13, pp. 2726–2735, Jul. 2007.
[9] U. Acar, F. Koyuncu, and B. Tanay, "Soft sets and soft rings," *Comput. Math. Appl.*, vol. 59, no. 11, pp. 3458–3463, Jun. 2010.
[10] S. Yuksel, T. Dizman, G. Yildizdan, and U. Sert, "Application of soft sets to diagnose the prostate cancer risk," *J. Inequalities Appl.*, vol. 2013, no. 1, p. 229, 2013.
[11] A. Khan and Y. Zhu, "A novel approach to parameter reduction of fuzzy soft set," *IEEE Access*, vol. 7, pp. 128956–128967, 2019.
[12] X. Ma and H. Qin, "Soft set based parameter value reduction for decision making application," *IEEE Access*, vol. 7, pp. 35499–35511, 2019.
[13] Z. Kong, J. Ai, L. Wang, P. Li, L. Ma, and F. Lu, "New normal parameter reduction method in fuzzy soft set theory," *IEEE Access*, vol. 7, pp. 2986–2998, 2019.
[14] Z. Kong, L. Gao, L. Wang, and S. Li, "The normal parameter reduction of soft sets and its algorithm," *Comput. Math. Appl.*, vol. 56, no. 12, pp. 3029–3037, Dec. 2008.
[15] Z. Xiao, K. Gong, and Y. Zou, "A combined forecasting approach based on fuzzy soft sets," *J. Comput. Appl. Math.*, vol. 228, no. 1, pp. 326–333, Jun. 2009.
[16] K. Gong, Y. Wang, M. Xu, and Z. Xiao, "BSSReduce an $O(|U|)$ incremental feature selection approach for large-scale and high-dimensional data," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3356–3367, Dec. 2018.
[17] M. Akram, G. Ali, and J. C. R. Alcantud, "New decision-making hybrid model: Intuitionistic fuzzy N-soft rough sets," *Soft Comput.*, vol. 23, no. 20, pp. 9853–9868, Oct. 2019.
[18] S. Das, D. Malakar, S. Kar, and T. Pal, "Correlation measure of hesitant fuzzy soft sets and their application in decision making," *Neural Comput. Appl.*, vol. 31, no. 4, pp. 1023–1039, Apr. 2019.

[19] F. Feng, H. Fujita, M. I. Ali, R. R. Yager, and X. Liu, "Another view on generalized intuitionistic fuzzy soft sets and related multiattribute decision making methods," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 3, pp. 474–488, Mar. 2019.
[20] F. Feng, Y. B. Jun, X. Liu, and L. Li, "An adjustable approach to fuzzy soft set based decision making," *J. Comput. Appl. Math.*, vol. 234, no. 1, pp. 10–20, May 2010.
[21] F. Feng, Y. Li, and N. Çağman, "Generalized uni–int decision making schemes based on choice value soft sets," *Eur. J. Oper. Res.*, vol. 220, no. 1, pp. 162–170, 2012.
[22] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Comput. Math. Appl.*, vol. 44, nos. 8–9, pp. 1077–1083, Oct. 2002.
[23] Y. Zou and Z. Xiao, "Data analysis approaches of soft sets under incomplete information," *Knowl.-Based Syst.*, vol. 21, no. 8, pp. 941–945, Dec. 2008.
[24] Z. Kong, G. Zhang, L. Wang, Z. Wu, S. Qi, and H. Wang, "An efficient decision making approach in incomplete soft set," *Appl. Math. Model.*, vol. 38, nos. 7–8, pp. 2141–2150, Apr. 2014.
[25] H. Qin and X. Ma, "Data analysis approaches of interval-valued fuzzy soft sets under incomplete information," *IEEE Access*, vol. 7, pp. 3561–3571, 2019.
[26] L. Wang and K. Qin, "Incomplete fuzzy soft sets and their application to decision-making," *Symmetry*, vol. 11, no. 4, p. 535, Apr. 2019.
[27] P. K. Maji, "Neutrosophic soft set," *Ann. Fuzzy Math. Inform.*, vol. 5, no. 1, pp. 157–168, Jan. 2013.
[28] F. Karaaslan, "Correlation coefficients of single-valued neutrosophic refined soft sets and their applications in clustering analysis," *Neural Comput. Appl.*, vol. 28, no. 9, pp. 2781–2793, Sep. 2017.
[29] H. Guan, J. He, S. Guan, and A. Zhao, "Neutrosophic soft sets forecasting model for multi-attribute time series," *IEEE Access*, vol. 7, pp. 25575–25588, 2019.
[30] M. Abu Qamar and N. Hassan, "Q-neutrosophic soft relation and its application in decision making," *Entropy*, vol. 20, no. 3, p. 172, Mar. 2018.
[31] M. Abu Qamar and N. Hassan, "An approach toward a Q-Neutrosophic soft set and its application in decision making," *Symmetry*, vol. 11, no. 2, p. 139, Feb. 2019.
[32] Y. Dong and C. Hou, "A useful method for analyzing incomplete and inconsistent information: Paraconsistent soft sets and corresponding decision making methods," *J. Intell. Fuzzy Syst.*, vol. 37, no. 1, pp. 901–912, Jul. 2019.
[33] N. D. Belnap, "A useful four-valued logic," in *Modern Uses of Multiple-Valued Logic*. Dordrecht, The Netherlands: Springer, 1977, pp. 5–37.
[34] Y.-J. Guo, Y. Yao, and P.-T. Yi, "Method and application of dynamic comprehensive evaluation," *Syst. Eng.-Theory Pract.*, vol. 27, no. 10, pp. 154–158, Oct. 2007.
[35] S. Greco, B. Matarazzo, and R. Slowinski, "Rough sets theory for multicriteria decision analysis," *Eur. J. Oper. Res.*, vol. 129, no. 1, pp. 1–47, Feb. 2001.

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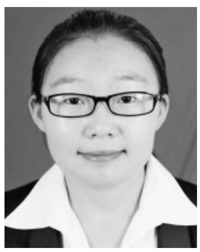

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