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# Comprehensive QoS Analysis of Urban Public Charging Network in Topology and Capacity With Its Application in Optimization

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**ABSTRACT** Booming electric vehicles (EVs) lead to degraded quality-of-service (QoS) of urban public charging network (UPCN) consisting of public charging stations (PCSs) and traffic network, such as prolonged waiting time at popular PCSs. UPCN QoS analysis creates broad prospects in QoS optimization by providing reliable tools to improve UPCN performance. Conventional methods focusing on the impact of network topology on QoS underestimates effects of network capacity and its distribution among PCSs. In this paper, we proposed a heterogeneous UPCN model with limited PCS service capacity, and constructed a QoS analysis method according to PCS and EV characteristics from classic method for traffic assignment model, in which we analyzed waiting time at PCSs by queueing theory and calculated feasible routes for EVs recharging half-way by screening from deep-first-search results. Finally, we demonstrated an application of the proposed model and method in QoS optimization. We carried out experiments on popular Sioux Falls network with randomly-added PCSs. Results indicate that proportion of traditionally neglected waiting time at PCSs may reach 22% in EVs' total travel time if fast charging piles serve 2 vehicles/h, and the gap between our method and conventional method in average waiting time and spatial throughput distribution is growing as EV amount increases. These results support our method's advantages in simulating EVs' charging choice trade-off between farther routes and popular PCSs as well as higher performance in UPCN QoS optimization.

**INDEX TERMS** Charging network, electric vehicles, queueing analysis, traffic control.

## I. INTRODUCTION

Urban public charging network (UPCN) is forming upon the original urban traffic network along with broadly constructed public charging stations (PCSs) [1], in order to satisfy huge charging demand generated by quickly promoted electric vehicles (EVs) whose global stock reached 5.1 million in 2018 because of their advantages in green transportation and energy efficiency [2]. Meanwhile, UPCN plays an increasingly significant role in providing charging service for EVs suffering from unavailable private charging piles or urgent midway recharging. However, as the number of EV has been growing rapidly, apart from power distribution network [3], UPCN is under huge pressure leading to

deterioration in quality-of-service (QoS) [4] such as longer delay and higher blocking probability.

In order to improve UPCN QoS, it would be more promising to increase UPCN service capability than to schedule EVs whose performance depends largely on the full cooperation of private EVs. Increasing UPCN service capability by changing its topology and capacity, such as establishing new PCSs and expanding PCS capacity, is intuitively feasible [5], since UPCN is a heterogeneous network made up of roads and various PCSs. Therefore, for more reliable performance in QoS optimization, it is necessary to establish a method analyzing the impact of UPCN topology and capacity on network QoS. Fortunately, QoS optimization will be easier with an analysis method to evaluate a specific UPCN providing the best QoS among several candidate solutions. Classified as the mixed-integer nonlinear problem (MINP), this procedure can be completed by heuristic algorithms such as the

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genetic algorithm [6], [7]. However, in UPCN, highly random EV behaviors and spatially correlated PCSs are both difficulties confront with network QoS analysis.

In literature, there are three classes of methods analyzing EV charging network QoS. References [7]–[10] model EVs as individuals with independent destination, state-of-charge (SoC) and charging preference, estimate their possible routes, and obtain QoS such as delay and throughput. References [11], [12] divide the city into several zones according to PCS location, quantify zonal EV by traffic flow, and apply queueing model to each charging station for EV waiting time analysis. References [13]–[16] describe EV travel demand by origin-destination (OD) matrix, study EV flows distributed around the city based on traffic assignment model, and finally estimate network QoS metrics. Comparatively, methods based on traffic assignment model show advantages in describing statistics of EV behavior patterns by traffic flow, discussing non-cooperativity of private EV by user equilibrium (UE) principle, and studying spatial correlation among PCSs through establishing a complete network passing interactions of PCSs. Several modifications considering EV’s nonnegligible charging time and limited state-of-charge (SoC) have been made in the traffic assignment model originally constructed for the fuel vehicle. In [17]–[19], charging time is analyzed by building the flow-dependent energy consumption function and SoC-dependent charging time function. Limited SoC and its influence on EV flow distribution are studied with or without recharging, respectively. Without recharging, [20] researches the distribution of EVs with determined initial SoC by involving out-of-energy penalty into travel cost functions, and [21] discusses EV flow distribution on trip chain level by constructing a distance-constrained traffic assignment model for EVs with different ranges. With recharging, [19] allows EV to visit one or more battery swapping charging stations in trips.

However, focusing on the impact of UPCN topology on QoS, these traffic assignment models ignoring waiting time at PCSs actually overestimate UPCN capacity, for such an ideal situation of unlimited PCS capacity. As a consequence, UPCN QoS may be optimistically estimated. This ideal assumption also overlooks difference between PCSs, and then obstructs effective capacity expansion in QoS optimization. Other than this, influence of limited SoC on EV flow distribution hasn’t been researched with time-costly midway recharging yet.

The main contributions of this paper are summarized as follows:

- 1) We construct a novel UPCN model consisting of traffic network and heterogeneous PCS nodes allowing independent capacity quantification and adjustment. We view these PCSs with limited capacity as different  $M/G/N/K$  queueing systems for waiting time estimation.
- 2) We establish a UPCN QoS analysis method based on traffic assignment model. We estimate travel cost including time in driving, waiting and charging, search

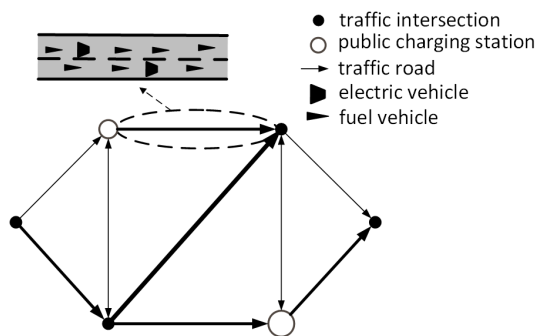


FIGURE 1. Urban public charging network.

feasible routes for EVs with limited SoC under recharging condition, analyze EV flow distribution according to UE principle by translating original problem into an equivalent optimization problem with non-negative linear constraints that could be solved by an iterative method, and finally analyze charging network QoS from EV flow distribution.

- 3) We demonstrate an application of the UPCN model and its analysis method in QoS optimization. Based on PCS-extended Sioux Falls network, we optimize average waiting time through genetic algorithm (GA) by expanding charging piles in existing PCSs. As a result, we verify the importance of UPCN capacity in QoS analysis and optimization.

The remainder of this paper is organized as following: Section II illustrates UPCN model, explains its influence on QoS from topology and capacity, and describes QoS analysis by a toy example. Section III discusses system models including driving time model of roads, queueing model of PCSs and energy consumption model of EVs. Section IV represents the QoS analysis algorithm. Section V presents the application in QoS optimization. Section VI presents the simulation settings, case study and results analysis. Finally, Section VII concludes the paper and provides directions for future studies.

## II. PROBLEM FORMULATION

In this section, we describe UPCN as a weighted directed graph, reveal the impact of UPCN topology and capacity on network QoS through EV flow distribution, and explain the process of charging network QoS analysis by a toy example.

### A. URBAN PUBLIC CHARGING NETWORK

Illustrated in Fig. 1, the UPCN made up of traffic network and PCSs is described as a weighted directed graph  $G = (V, E)$ , where  $V$  is the set of nodes and  $E$  is the set of traffic roads. Nodes are further categorized into the traffic intersection and the PCS in set  $S$  ( $S \subseteq V$ ). PCS in  $S$  is denoted as  $a$ . Traffic road in  $E$  is denoted as  $arc(n_1, n_2)$ ,  $n_1, n_2 \in V$ , which is also simplified as  $e$  if there is no need of location information.

Distinguished by node size or line width, weight of a node or edge indicates its service capacity. PCS service capacity

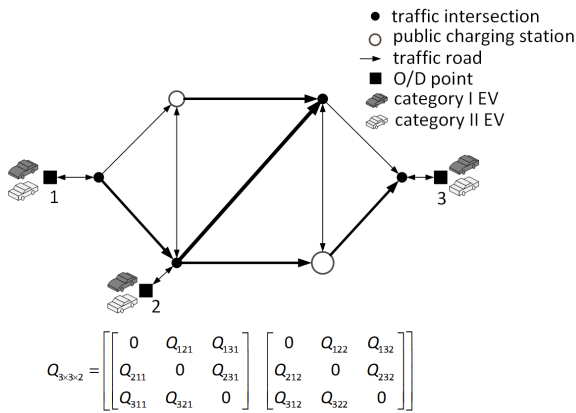


FIGURE 2. Urban public EV charging network with O/D points and OD matrix.

is determined by scale and charging pile performance, which can be independently expanded by increasing charging piles, enhancing charging power and reducing charging time. Road service capacity is affected by parameters like lane amount and maximum speed limit. Models to quantify their service capacities are detailed discussed in section III.

Towards fuel vehicle and EV in UPCN, traffic flow describes statistics of their highly random behavior: flow rate reflects their density and average speed; flow direction reflects their trips. Besides, Poisson flow model is further chosen to simulate EV movements for reasons that: 1) mixture of Poisson flows still forms a Poisson flow whose rate is the sum of original flows', making EV classification possible; 2) Poisson flow is widely used in queueing system for waiting time estimation. In this paper,  $q$  denotes the EV flow rate, and  $q^F$  is the rate of fuel vehicle flow which is the background flow in EV flow analysis.

### B. EV FLOW DISTRIBUTION

EV flow distribution among nodes and edges establishes a bridge from UPCN topology and capacity to QoS. EV flow distribution is the integration of all EV trips, mainly influenced by UPCN topology and capacity through limiting feasible routes and affecting travel cost. What's more, it models the burden of gigantic EVs around the city under which UPCN performance is quantified as QoS metrics. Therefore, EV flow distribution is the key in UPCN QoS analysis.

EV travel demand described by OD matrix should be introduced before formulating the EV flow distribution problem. As shown in Fig. 2, travel demand is categorized by EV parameters such as initial SoC and mass. OD matrix  $Q_{I \times I \times C}$ 's element  $Q_{ijc}$  is the flow rate of category  $c$  EVs travelling from origin point  $i$  to destination point  $j$ . O/D points are embedded into the UPCN according to their position, forming a new weighted directed graph  $G = (\tilde{V}, \tilde{E})$ . In this new graph,  $\tilde{V} = V \cup D$  where  $D$  indicates the set of O/D points, and set  $\tilde{E}$  is extended by edges connecting O/D point in set  $D$  and nodes in set  $V$ . In remaining paper, we default

that the graph has been expanded and no longer distinguish their symbols.

EVs schedule routes between O/D points, form traffic flow on nodes and edges, and finally contribute to EV flow distribution. Their trips are influenced by UPCN topology and capacity. On one hand, network topology prevents EVs from routes with unconnectable nodes or unbearable energy consumption. On the other hand, network capacity motivates non-cooperative EVs to inexpensive routes with less traffic and higher service capacity. These mechanisms are formulated by (1)-(4).

In UPCN, a route is expressed as vector  $r_{1 \times m}$  whose element  $r_k$  is the index of the  $k$ th node passed by: for a route between O/D point  $i$  and  $j$ ,  $r_1 = i$ ,  $r_m = j$ . Feasible routes meet these two conditions for all  $k$  for 2 to  $m$ :

$$arc(r_{k-1}, r_k) \in E \quad (1)$$

$$\sum_{l=1}^k [C(r_l) + C(arc(r_{l-1}, r_l))] > \epsilon \quad (2)$$

where function  $C$  discussed in section III calculates energy consumption, constant  $\epsilon$  is the threshold to judge whether the EV has broken down, and  $C(r_1)$  is the initial SoC. Apparently, equation (1) ensures all nodes are connectable, and (2) requests EVs to finish the route without breaking down due to insufficient energy. Routes satisfying (1) and (2) form the feasible route set denoted as  $R^{ijc}$ . It is common that EVs of different categories own different feasible route sets even if they may travel between the same O/D points. In some extreme case, e.g. too low initial SoC, no route is feasible and  $R^{ijc}$  is empty.

Since non-cooperative EVs select feasible routes freely, each feasible route owns the same chance of being assigned EV flow to:

$$\begin{cases} \sum_r q_r^{ijc} = Q_{ijc}, r \in R^{ijc} \\ q_r^{ijc} \geq 0, \forall r \in R^{ijc} \end{cases} \quad \forall i, j \in D, \forall c \quad (3)$$

where  $q_r^{ijc}$  is the flow rate of EVs choosing feasible route  $r$  from  $R^{ijc}$ . Besides, EVs are supposed to choose routes with the least cost, expressed as the first Wardrop principle [22]:

$$\begin{cases} q_r^{ijc} (f_r^{ijc} - f^{ijc}) = 0 \\ f_r^{ijc} - f^{ijc} \geq 0 \end{cases} \quad \forall r \in R^{ijc}, \quad \forall i, j \in D, \forall c \quad (4)$$

where  $f^{ijc}$  is the least cost between O/D points  $i$  and  $j$ , and  $f_r^{ijc}$  is the travel cost of route  $r$ . Travel cost is calculated by EV-flow-dependent functions whose parameters are determined by service capacity of nodes and edges along the way. Briefly, less traffic and higher capacity lead to lower cost.

EV flow distribution satisfying (1)-(4) shows statistics of EVs random behavior in the UPCN, and reveals the impact of UPCN topology and capacity on network QoS.

### C. A TOY EXAMPLE OF NETWORK QoS ANALYSIS

Shown in Fig. 3, a toy example is constructed to explain the process of analyzing EV flow distribution and network QoS,

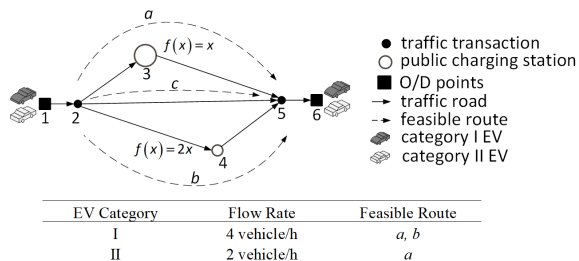


FIGURE 3. A toy example of UPCN QoS analysis.

in the case of limited network capacity and EV SoC. The toy UPCN includes several traffic intersections and roads, a pair of O/D points and 2 PCSs. Its connectivity determines 3 routes from O/D points 1 to 6: route *a* connects 2 and 5 through PCS 3, route *b* via PCS 4, and a non-stop route *c* directly reaches node 5. PCS service capacity is quantified as parameters of waiting time function in Fig. 3: compared with PCS 4, PCS 3 has higher service capacity because of shorter waiting time under the same flow rate condition. To simplify the example, only waiting time at PCSs contributes to travel cost.

EVs are classified into three categories with different feasible route sets. EVs in category I have sufficient energy for all the three routes, whereas other EVs unable to afford route *c* have to recharge half-way. EVs in category II and category III are differentiated according to whether their initial SoC may afford the trip from node 2 to 4. Their detailed feasible route sets and flow rate are presented in the table of Fig. 3.

EV flow distribution is analyzed based on (3) and (4), then UPCN QoS is researched. Taking waiting time as travel cost, category I EVs undoubtedly choose route *c* with zero cost. Category III EVs have no choice but route *a* to complete their trips. Then, category II EVs need to trade off route *a* and route *b*: route *b* with zero traffic is a prior choice, but route *a* with higher capacity is also competitive. Actually, category II EVs may select both routes, and form equal waiting time in PCS 3 and PCS 4. The result is as following: EV flow on route *a* is 4 vehicles/h consisting half category II EVs and half category III EVs; EV flow on route *b* is 2 vehicles/h of category II EVs only; EV flow on route *c* is 4 vehicles/h of category I EVs only. This result is stable because any EV changing route suffers from longer travel cost. Further, UPCN QoS including waiting time and throughput is calculated: waiting time in both PCS 3 and 4 is 0.4 hours, and throughput of PCS 3 and 4 is 4 vehicles/h and 2 vehicles/h, respectively.

With comprehensive information about network topology and capacity, it is possible to obtain more accurate EV flow distribution as well as more detailed QoS like driving time and blocking probability. What's more, accompanying with the adjustments on UPCN capacity, EV flow distribution may change, so does the UPCN QoS, which is the theoretic basis of QoS optimization by capacity expansion.

In reality, large-scale UPCN is too complex to be analyzed by such simple operations. So, we investigate on an

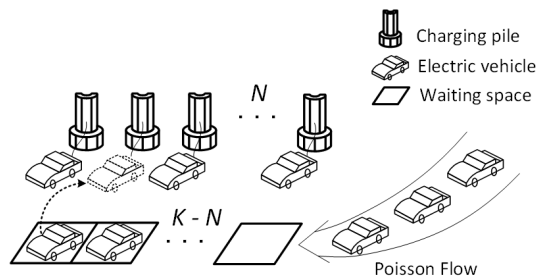


FIGURE 4. Charging station model.

efficient method for EV flow distribution analysis, shown in section IV.

### III. SYSTEM MODELING

#### A. PCS SERVICE CAPACITY AND QUEUEING MODEL

A PCS is typically divided into two sections: charging section with charging piles and waiting section with parking lots. Charging section provides charging service for EVs with the same count of charging piles simultaneously. Waiting section is prepared for queueing in case all charging piles are occupied. PCS service capacity is determined by scale and charging pile performance expressed by the probability distribution function of charging time.

Queueing model is utilized to quantify PCS service capacity: shorter waiting time in serving the same EV flow means better service capacity. Illustrated in Fig. 4, we describe the queueing process at a PCS by a  $M/G/N/K$  queueing model, where  $M$  represents that EVs reach the place according to exponential distribution with arrival rate  $\lambda$ ,  $G$  represents that charging time follows a general distribution,  $N$  is the charging pile amount,  $K$  is PCS scale, and  $K - N$  is the parking lot amount.

Based on this model, waiting time at the PCS is estimated by method in [23].

$$t_a^w(\lambda) = \frac{1}{\lambda} \times \frac{(N\rho)^N}{N!} \times \frac{\xi}{(1-\rho)(1-\xi)} \times P_0 \times \left[ 1 - \xi^{K-N} - (K-N)(1-\xi)\rho\xi^{K-N-1} \right] \quad (5)$$

where

$$P_0 = \left[ \sum_{l=0}^{N-1} \frac{(N\rho)^l}{l!} + \frac{(N\rho)^N}{N!} \frac{1-\rho\xi^K}{1-\rho} \right]^{-1}, \quad (6)$$

$\rho = \lambda/NE[G]$ ,  $\xi = \rho R_G / (1 - \rho + \rho R_G)$ , and

$$R_G = \frac{(1 + c_s^2)R_D}{(2R_D - 1)c_s^2 + 1} \quad (7)$$

where  $c_s^2$  is the coefficient of variance of charging time distribution and  $R_D = EW(M/M/S)/EW(M/D/S)$  is approximated in [24]. It's worth to note that waiting time approximation in [23], [24] is suitable for  $\rho < 1$ , and in case of  $\rho > 1$  we use  $M/M/N/K$  model [25] for rough approximation. Moreover, we estimate charging time according to general

distribution

$$t_a^c = E[G] \quad (8)$$

### B. ROAD CAPACITY AND DRIVING TIME MODEL

Powered by electric motors, EV has different travelling patterns comparing fuel vehicle such as excelling accelerate performance and better braking effect. However, considering the vast majority of fuel vehicles on roads, driving time of EV is approximated by road capacity according to the most widely used Bureau of Public Roads (BPR) function originally established for fuel vehicle

$$t_e(q_e) = t_e^0 \left[ 1 + 0.15 \left( \frac{q_e + q_e^F}{c_e} \right)^4 \right], \quad \forall e \in E \quad (9)$$

where variable  $q_e$  is the EV flow rate on road  $e$ ,  $q_e^F$  is the background flow rate of fuel vehicle,  $t_e^0$  is free-flow travel time, and  $c_e$  is road capacity.

### C. EV ENERGY CONSUMPTION MODEL

EV energy consumption during driving is mainly affected by three factors: EV parameters such as mass and frontal area, road condition such as grade, and driving process such as speed and acceleration. Towards traffic flow, it is impractical to collect precise data about each EV's driving process especially acceleration and deceleration operations. Therefore, energy consumption is calculated by supposing that EVs travel along the roads at a constant speed, and coefficient  $\eta^c$  is used to approximate energy cost due to non-uniform motion and energy conversion efficiency. Energy consumption function is as follows:

$$C_e(q_e) = \eta^c l_e [M^c g (\sin \alpha_e + f_r^c \cos \alpha_e) + \frac{1}{2} \rho_a C_D^c A_f^c v_e^2] \quad (10)$$

where  $l_e$  is length of road  $e$ ,  $M^c$  is mass of EVs belonging to category  $c$ ,  $g$  is the gravitational acceleration,  $\alpha_e$  is road grade,  $f_r^c$  is rolling resistance coefficient,  $\rho_a$  is air mass density,  $C_D^c$  is aerodynamic drag coefficient,  $A_f^c$  is the frontal area of the vehicle, and  $v_e$  is average speed. In order to highlight the impact of EV parameters on energy consumption and simplify calculation, we estimate energy consumption based on average speed of background flows:

$$v_e = \frac{l_e}{t_e(0)} \quad (11)$$

What's more, the total energy that EVs gain in charging station  $a$  is computed by charging power  $P_a$  and average charging time  $t_a^c$ :

$$C_a(q_a) = P_a t_a^c \quad (12)$$

## IV. UPCN QoS ANALYSIS METHOD

### A. PRINCIPLE

In order to analyze UPCN QoS, we should model spatial correlation among PCSs and then figure out statistics of highly-random EV behavior under the consideration of UPCN topology and capacity. In our method, spatial

correlation among PCSs is researched by establishing an UPCN model, whose topology spreads interactions between PCSs by traffic roads, and whose capacity influences strength of these interactions. Meanwhile, statistics of highly-random EV behavior described as the EV flow distribution among UPCN is analyzed according to UE principle.

EV flow distribution is the integration of EVs selecting feasible routes between O/D points to satisfy travelling demands. Naturally, feasible route search and EV traffic assignment among these feasible routes are two main procedures during EV flow distribution analysis. Feasible route search reflects spatial correlation among PCSs from UPCN topology because close PCSs are more likely to appear in the same feasible route set. EV traffic assignment implemented under UE principle also embodies spatial correlation among PCSs from UPCN capacity because EVs prefer PCS with higher service capacity.

UPCN QoS including delay, blocking probability and throughput is obtained after EV flow distribution analysis by combining queueing model and driving model.

### B. FEASIBLE ROUTE SEARCH

Feasible routes satisfying constraints formulated in (1) and (2) are searched according to pseudocode shown in Algorithm 1. In the first procedure, all routes between each pair of O/D points meeting (1) are investigated by deep-first-search (DFS). There is no limitation on the amount or order of PCSs along the routes. In reality, these routes cover all possible recharging choices and greatly exceeds the scope of finally chosen routes. So, based on the number of passed nodes  $|r|$ , coefficient  $\alpha$  is applied for a rough filtration among preliminary results.

Then, considering SoC limitation in (2), routes with unbearable energy consumption are deleted. As shown in the second procedure, each route is examined, and travel cost of detected route is estimated based on background flow. For simplification, only routes with the first  $\beta$  minimum travel cost before EV flow assignment are remained in the final feasible route sets.

Time complexity of this algorithm is the product of graph size, O/D points number and EV categories. Whereas fine division of EVs leads to accurate and complete results, the number of EV categories should be set based on actual needs for the sake of proper running time.

### C. EV TRAFFIC ASSIGNMENT

Faced with feasible routes, EVs are assumed to prefer those with the least travel cost: routes taken the shortest total time in trips including driving time on the road, waiting and charging time at the charging stations. Travel cost function of a PCS is the sum of waiting time and charging time at the station:

$$f_a(q_a) = t_a^w(q_a) + t_a^c \quad (13)$$

and travel cost function of a road is calculated according to driving time model:

$$f_e(q_e) = t_e(q_e) \quad (14)$$

In section II, expressions (1)-(4) just formulate rules that EV flow distribution follows, but they offer no information of a specific method. In this part, we construct an equivalent optimization problem whose optimal solution is exactly the expected EV flow distribution satisfying (1)-(4). EV flow distribution is obtained by solving the equivalent optimization problem:

$$\text{Min } z(x) = \sum_{a \in S} \int_0^{x_a} f_a(x_a) dx_a + \sum_{e \in E} \int_0^{x_e} f_e(x_e) dx_e \quad (15)$$

$$\text{s.t. } \sum_r q_r^{ijc} = Q_{ijc} \quad \forall i, j \in D, c \quad (16)$$

$$q_r^{ijc} \geq 0 \quad \forall r \in R^{ijc}, i, j \in D, c \quad (17)$$

$$x_e = \sum_{i,j,c,r} q_r^{ijc} \delta_e^{ijcr} \quad (18)$$

$$x_a = \sum_{i,j,c,r} q_r^{ijc} \delta_a^{ijcr} \quad (19)$$

where  $x_a$  and  $x_e$  are EV flow rate on PCS or road respectively,  $f_a$  and  $f_e$  are travel cost functions, coefficients  $\delta_a^{ijcr}$  and  $\delta_e^{ijcr}$  indicate whether a route  $r \in R^{ijc}$  crosses through corresponding PCS and road, if so,  $\delta_a^{ijcr} = 1$  and  $\delta_e^{ijcr} = 1$ , or else  $\delta_a^{ijcr} = 0$  and  $\delta_e^{ijcr} = 0$ . Apparently, equations (18) and (19) define the relationship between edge flow and route flow.

Equivalence between the optimal solution and expected EV flow distribution is established by proving the solution meeting (1)-(4). Expressions (1) and (2) regulating routes feasibility implicit in the definition of feasible route set  $R^{ijc}$ . Formulation (3) is the combination of (16) and (17). Therefore, the optimal solution is the expected EV flow distribution once proofing the optimal solution satisfying (4).

First of all, we explore solution by defining Lagrange function according to original problem

$$L(\mathbf{q}, \mathbf{u}) = z(x(\mathbf{q})) + \sum_{ijc} u^{ijc} \left( Q_{ijc} - \sum_r q_r^{ijc} \right) \quad (20)$$

whose optimal conditions are represented as follows according to Kuhn-Tucker conditions

$$\begin{cases} q_r^{ijc} \frac{\partial L(\mathbf{q}, \mathbf{u})}{\partial q_r^{ijc}} = 0 \\ \frac{\partial L(\mathbf{q}, \mathbf{u})}{\partial q_r^{ijc}} \geq 0 \\ q_r^{ijc} \geq 0, \end{cases} \quad \forall i, j, c, r \quad (21)$$

$$\frac{\partial L(\mathbf{q}, \mathbf{u})}{\partial u^{ijc}} = 0, \quad \forall i, j, c \quad (22)$$

Expand (21) as

$$\begin{aligned} \frac{\partial L(\mathbf{q}, \mathbf{u})}{\partial q_r^{ijc}} &= \sum_{a \in S} \frac{\partial}{\partial q_r^{ijc}} \int_0^{x_a} f_a(x_a) dx_a \times \frac{\partial q_r^{ijc}}{\partial x_a} \\ &+ \sum_{e \in E} \frac{\partial}{\partial q_r^{ijc}} \int_0^{x_e} f_e(x_e) dx_e \times \frac{\partial q_r^{ijc}}{\partial x_e} - u^{ijc} \end{aligned} \quad (23)$$

### Algorithm 1 Feasible Route Search

**Input:** graph  $G=(V, E)$ , OD matrix  $Q_{I \times I \times C}$

**Output:**  $\tilde{R}_{N \times N}$

```

1: for all  $Q_{ijc}$  in  $Q_{I \times I \times C}$  do
2:   if  $i \neq j$  then
3:     Search route set  $\tilde{R}_{ij}$  between  $i$  and  $j$  by DFS
4:     for all route  $r$  in  $\tilde{R}_{ij}$  do
5:       if  $|r| > \alpha \times |V|$  then
6:         Delete route  $r$  from  $\tilde{R}_{ij}$ 
7:       end if
8:     end for
9:   end if
10: end for
11: return  $\tilde{R}_{N \times N}$ 

Input: graph  $G=(V, E)$ , energy function  $P$ , OD matrix  $Q_{I \times I \times C}$ ,  $\tilde{R}_{N \times N}$ 
Output:  $R_{N \times N \times C}$ 
1: for all  $\tilde{R}_{ij}$  in  $\tilde{R}_{N \times N}$  do
2:   for all  $c \in C$  do
3:     if  $Q_{ijc} > 0$  then
4:        $R_{ijc} \leftarrow \tilde{R}_{ij}$ 
5:       for all route  $r$  in  $R_{ijc}$  do
6:         if EV break down in route  $r$  then
7:           delete route  $r$  from  $R_{ijc}$ 
8:         end if
9:       Estimate travel cost based on background flow.
10:      end for
11:      Save  $\beta$  routes with least travelling cost from  $R_{ijc}$ .
12:    end if
13:  end for
14: end for
15: return  $R_{N \times N \times C}$ 

```

where

$$\frac{\partial q_r^{ijc}}{\partial x_a} = \delta_a^{ijcr}, \quad \frac{\partial q_r^{ijc}}{\partial x_e} = \delta_e^{ijcr} \quad (24)$$

Then, combine (23) and (24) into

$$\frac{\partial L(\mathbf{q}, \mathbf{u})}{\partial q_r^{ijc}} = \sum_{a \in S} f_a(x_a) \delta_a^{ijcr} + \sum_{e \in E} f_e(x_e) \delta_e^{ijcr} - u^{ijc} \quad (25)$$

Setting

$$f_r^{ijc} = \sum_{a \in S} f_a(x_a) \delta_a^{ijcr} + \sum_{e \in E} f_e(x_e) \delta_e^{ijcr} \quad (26)$$

that represents total travel cost of one feasible route, formulation (21) is finally simplified as

$$\begin{cases} q_r^{ijc} (f_r^{ijc} - u^{ijc}) = 0 \\ f_r^{ijc} - u^{ijc} \geq 0 \\ q_r^{ijc} \geq 0, \end{cases} \quad \forall i, j, c, r \quad (27)$$

Regarding  $u^{ijc}$  as the least travelling cost, equation (27) is equivalent to (4), proving the equivalence between this optimization problem's solution and EV flow distribution.

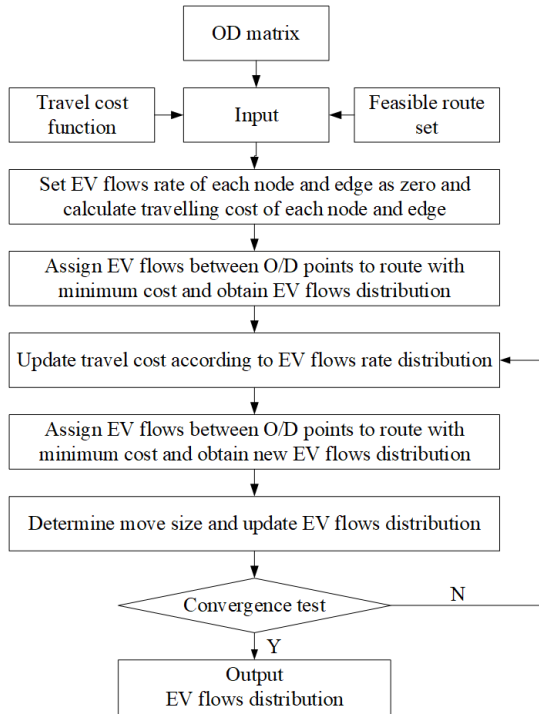


FIGURE 5. EV flow distribution analysis.

As a result, EV flow distribution is obtained by researching optimal solution of the problem expressed in (15)-(19). The iterative method [22] for solution is shown in Fig. 5. As initialization, travel cost of each node or edge is calculated based on its travel cost function by setting EV flow to zero, and EV flows are totally assigned to feasible route with minimal cost. Then start the iterative process. Based on the old assignment of EV flow  $x$ , update travel cost, assign all EV flows to feasible routes with the least cost, and further gain new assignment of EV flow  $x'$ . Search move size for candidate  $\tau^*$  by solving equation

$$\sum_{a \in S} (x'_a - x_a) f_a(x_a + \tau^*(x'_a - x_a)) + \sum_{e \in E} (x'_e - x_e) f_e(x_e + \tau^*(x'_e - x_e)) = 0, \quad (28)$$

select final move size  $\tau$  minimizing following function from  $\tau^*$ , 0 and 1

$$\sum_{a \in S} \int_0^{x_a + \tau(x'_a - x_a)} f_a(\omega) d\omega + \sum_{e \in E} \int_0^{x_e + \tau(x'_e - x_e)} f_e(\omega) d\omega, \quad (29)$$

and update assignment of EV flow by calculating the convex combination of old and new assignments according to move size. Repeat the iterative process until the difference between input and output assignment of EV flow satisfies convergence test. The final assignment of EV flow is the expected EV flow distribution.

#### D. NETWORK QoS ANALYSIS

EV flow distribution provides adequate information for QoS analysis. Network delay from one node to another can be analyzed by calculating driving time, waiting time and charging time spending on corresponding nodes and edges. Network throughput is the sum of EV flow rates captured by PCSs in the charging network. Network blocking probability, i.e. probability that the PCS has been full when one EV arrives, is obtained according to the queueing model of each PCS. In summary, from EV flow distribution as well as nodes and edges capacity model, we obtain a macro perspective of UPCN performance by network QoS analysis.

#### V. APPLICATION IN QoS OPTIMIZATION

As scarce land resource in metropolis, it may be impractical to serve high-density EVs with increasing charging demand by keeping establishing new PCSs. Expanding service capacity of existing PCSs is feasible in UPCN QoS optimization. In this situation, it is the impact of UPCN capacity, rather than UPCN topology, on network QoS that plays a significant role in QoS optimization. Therefore, the disadvantage of neglecting PCSs' difference in service capacity prevents the traditional model from high QoS optimization performance. In this section, we formulate an UPCN QoS optimization problem focusing on PCS service capacity, and explain a feasible way combining our QoS analysis method and genetic algorithm.

##### A. QoS ANALYSIS PROBLEM

Several spare charging piles should be assigned to existing PCSs for shorter average waiting time, and the objective function defining average waiting time is expressed as

$$\text{Min } \bar{r}^w(\mathbf{y}) = \sum_{a \in S} q_a t_a^w / \sum_{a \in S} q_a \quad (30)$$

where  $\mathbf{y}$  is the vector indicating charging pile amount of all PCSs,  $q_a$  is the rate of EV flow captured by PCS  $a$ , and  $t_a^w$  is waiting time obtained by queueing model.

Constraints simply includes total amount of charging piles for expansion as well as maximum charging pile amount of each PCS: 1) Total amount of new charging piles is restricted by  $M$

$$\sum_{a \in S} y_a \leq M, \quad a \in S \quad (31)$$

2) Maximum charging pile amount in PCS  $a$  is limited by  $M_a$

$$0 \leq y_a < M_a, \quad \forall a \in S \quad (32)$$

##### B. GENETIC ALGORITHM

Considering the QoS optimization problem in section A, average waiting time can be calculated by our QoS analysis method once updating charging pile amount  $\mathbf{y}$ :  $q_a$  is the result of EV flow distribution analysis, and  $t_a^w$  is the approximation based on queueing model. Further, classifying the optimal

problem as a mixed-integer nonlinear problem, genetic algorithm (GA) is applied for solution, which is decomposed into the following steps.

- 1) Chromosome Coding: Convert the amount of new charging pile in PCS a into binary number whose length is set by  $M_a$ . Then, code a chromosome representing one solution by splicing binary numbers.
- 2) Fitness Calculation: Filter out individuals dissatisfying (29) and (30) before analyzing average waiting time of remaining individuals. Process one generation  $(\bar{t}_1^w, \bar{t}_2^w, \dots, \bar{t}_n^w)$  by the sigmoid function instead of directly utilizing average waiting time as fitness, in order to improve distinction of individuals. Individual fitness is calculated as following

$$\bar{f}_k = \left(1 + e^{\sigma[\bar{t}_k - E(\bar{t}_n)]}\right)^{-1}, \quad E(\bar{t}_n) = \frac{1}{n} \sum_{k=1}^n \bar{t}_k \quad (33)$$

where coefficient  $\sigma$  is responsible for distinction adjustment.

- 3) Selection: Choose next generation by fitness proportionate selection, where fitness level is used to associate a probability of selection with each individual.
- 4) Crossover: Generate offspring according to two-point crossover. Pick two random crossover points from the parent chromosomes, and swap digits between the two points to combine genetic information of two parent chromosomes.
- 5) Mutation: Negate random bits of chromosomes who face the same rate for mutation, aiming to maintain genetic diversity from one generation to the next and avoid local optimum.
- 6) Elitist preservation: Allow the best individual from the current generation to carry over to the next, which avoids deterioration in solution quality during iterations.

## VI. EXPERIMENTS

### A. SIMULATION SETTINGS

This section exhibits the UPCN, OD matrix and EV parameters in simulation. As illustrated in Fig. 6, the topology of charging network is constructed by randomly facilitating PCSs in conventional Sioux Falls traffic network, a popular benchmark network in numerous traffic assignment problems, consisting 24 nodes, 76 links and 552 OD pairs. PCSs are introduced by establishing additional edges connecting adjacent nodes to prevent original traffic network topology. It's worth noting that this structure doesn't build any new roads but only enhance the traffic network with roadside PCSs. So, EV is free to decide whether to charge or keep driving at the roadside PCS. Taking PCS 25 for example, EVs recharging at this station travel along with road 4-25 and 25-5, and others simply travel through road 4-5. In detailed analysis, road 4-25, 25-5 and 4-5 have the same flow including fuel vehicles and EVs when calculate driving

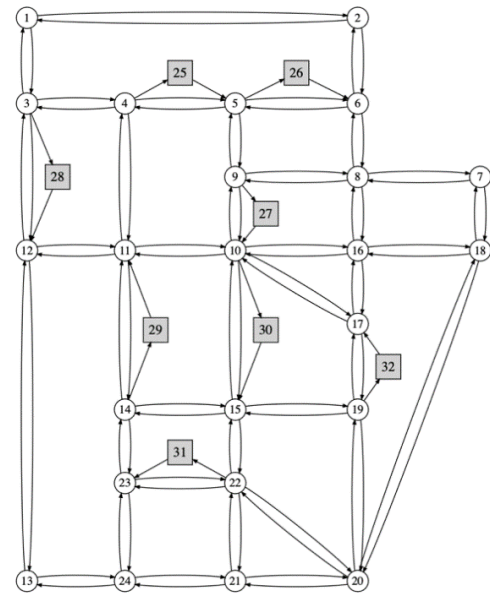


FIGURE 6. Urban Public Charging network for simulation.

TABLE 1. Public charging station scale.

Node Number	25	26	27	28	29	30	31	32
Scale	6	7	9	8	7	6	7	10

time and energy consumption, and node 25 only have the EV flow for recharging.

In order to quantify UPCN capacity intuitively, service capacity of every PCS in the experiment is independently adjusted in the charging pile amount instead of charging rate. Parameters of the charging pile are set according to [26]: charging power  $P_a$  is 44kW, probability distribution function of charging time is regulated by its mean and coefficient of variance  $c_s^2$  that are 0.5 h and 0.6 respectively. Therefore, upper limit of a PCS's service capacity is restricted by its scale which is set in TABLE 1. Additionally, traffic road capacity referred from [27] is listed in TABLE 2. Every road's grade is 0, free-flow speed is 60 km/h, and background flow is set according to the equilibrium state of fuel vehicle flows in [27].

OD matrix describing EV travel demand is 1% of the OD matrix downloaded from [27] since the market share of EVs among all vehicles is 1% in China as of 2018.

As shown in TABLE 3, five categories of EVs with various low initial SoC but the same other parameters are chosen for simulation because of more possibility in recharging half-way. Other parameters referred from [28] include the following: battery energy capacity is 24.3 kwh, aerodynamic drag coefficient  $C_D$  is 0.6, vehicle frontal area  $A_f$  is  $3.504 m^2$ , rolling resistance coefficient  $f_r$  is 0.01, vehicle mass composed of curb weight (including battery weight) and vehicle load capacity is 2800 kg, coefficient  $\eta$  for energy consumption estimation is 1.5, and energy threshold  $\epsilon$  is 25%.



TABLE 2. Edge parameters for experiments.

Edge	Length (km)	Capacity (10 <sup>3</sup> veh/h)	Bg Flow (10 <sup>3</sup> veh/h)	Edge	Length (km)	Capacity (10 <sup>3</sup> veh/h)	Bg Flow (10 <sup>3</sup> veh/h)	Edge	Length (km)	Capacity (10 <sup>3</sup> veh/h)	Bg Flow (10 <sup>3</sup> veh/h)
1-2	6	25.90	4.49	11-10	5	10.00	17.60	20-22	5	5.08	7.00
1-3	4	23.40	8.12	11-12	6	4.91	8.37	21-20	6	5.06	6.24
2-1	6	25.90	4.52	11-14	4	4.88	9.78	21-22	2	5.23	8.62
2-6	5	4.96	5.97	12-3	4	23.40	9.97	21-24	3	4.89	10.31
3-1	4	23.40	8.09	12-11	6	4.91	8.40	22-15	3	9.60	18.39
3-4	4	17.11	14.01	12-13	3	25.90	12.29	22-20	5	5.08	7.00
3-12	4	23.40	10.02	13-12	3	25.90	12.38	22-21	2	5.23	8.61
4-3	4	17.11	14.03	13-24	4	5.09	11.12	22-23	4	5.00	9.66
4-5	2	17.78	18.01	14-11	4	4.88	9.81	23-14	4	4.92	8.39
4-11	6	4.91	5.20	14-15	5	5.13	9.04	23-22	4	5.00	9.63
5-4	2	17.78	18.03	14-23	4	4.92	8.40	23-24	2	5.08	7.90
5-6	4	4.95	8.80	15-10	6	13.51	23.19	24-13	4	5.09	11.11
5-9	5	10.00	15.78	15-14	5	5.13	9.08	24-21	3	4.89	10.26
6-2	5	4.96	5.99	15-19	3	14.56	19.08	24-23	2	5.08	7.86
6-5	4	4.95	8.80	15-22	3	9.60	18.41	4-25	1	17.78	18.01
6-8	2	4.90	12.49	16-8	5	5.05	8.41	25-4	1	17.78	18.01
7-8	3	7.84	12.10	16-10	4	4.85	11.07	25-4	1	17.78	18.01
7-18	2	23.40	15.79	16-17	2	5.23	11.70	26-6	2	4.95	8.80
8-6	2	4.90	12.53	16-18	3	19.68	15.28	9-27	1.5	13.92	21.74
8-7	3	7.84	12.04	17-10	8	4.99	8.10	27-10	1.5	13.92	21.74
8-9	10	5.05	6.88	17-16	2	5.23	11.68	3-28	2	23.40	10.02
8-16	5	5.05	8.38	17-19	2	4.82	9.95	28-12	2	23.40	10.02
9-5	5	10.00	15.80	18-7	2	23.40	15.85	14-29	2	4.88	9.81
9-8	10	5.05	6.84	18-16	3	19.68	15.33	29-11	2	4.88	9.81
9-10	3	13.92	21.74	18-20	4	23.40	18.98	10-30	3	13.51	23.13
10-9	3	13.92	21.81	19-15	3	14.56	19.12	30-15	3	13.51	23.13
10-11	5	10.00	17.73	19-17	2	4.82	9.94	22-31	2	5.00	9.66
10-15	6	13.51	23.13	19-20	4	5.00	8.69	31-23	2	5.00	9.66
10-16	4	4.85	11.05	20-18	4	23.40	18.99	19-32	1	4.82	9.94
10-17	8	4.99	8.10	20-19	4	5.00	8.71	32-17	1	4.82	9.94
11-4	6	4.91	5.30	20-21	6	5.06	6.30				

TABLE 3. EV parameters for experiments.

Category	Initial SoC	Percent
I	30% - 35%	6%
II	35% - 40%	6%
III	40% - 45%	7%
IV	45% - 50%	8%
V	50% - 55%	9%

**B. CASE STUDY**

We design two cases for comparison in UPCN QoS analysis and optimization between our method and the traditional method with unlimited PCS capacity assumption.

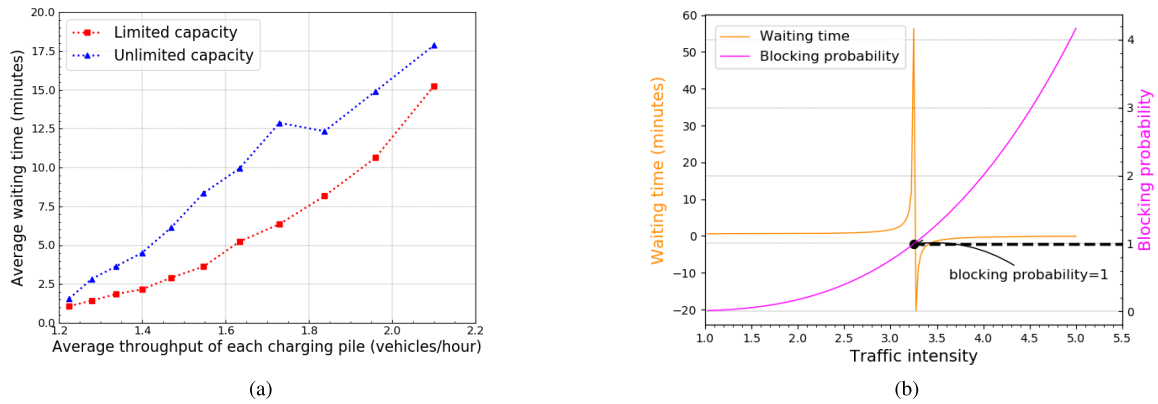
In QoS analysis, UPCNs with different capacities are researched under the same topology shown in Fig. 6. UPCN capacity is classified by the amount of all charging piles in the network, changing from 28 to 48 in the step of 2. For each category, 30 randomly selected UPCNs with various amount of charging piles among PCSs are researched, since it is careless to neglect charging pile’s distribution. What’s more, two analytical methods share the same feasible route searching algorithm whose coefficients  $\alpha$  and  $\beta$  are set as 0.5 and 20.

In QoS optimization, UPCN capacity expansion problem formulated in section V is worked out: for each PCS, original

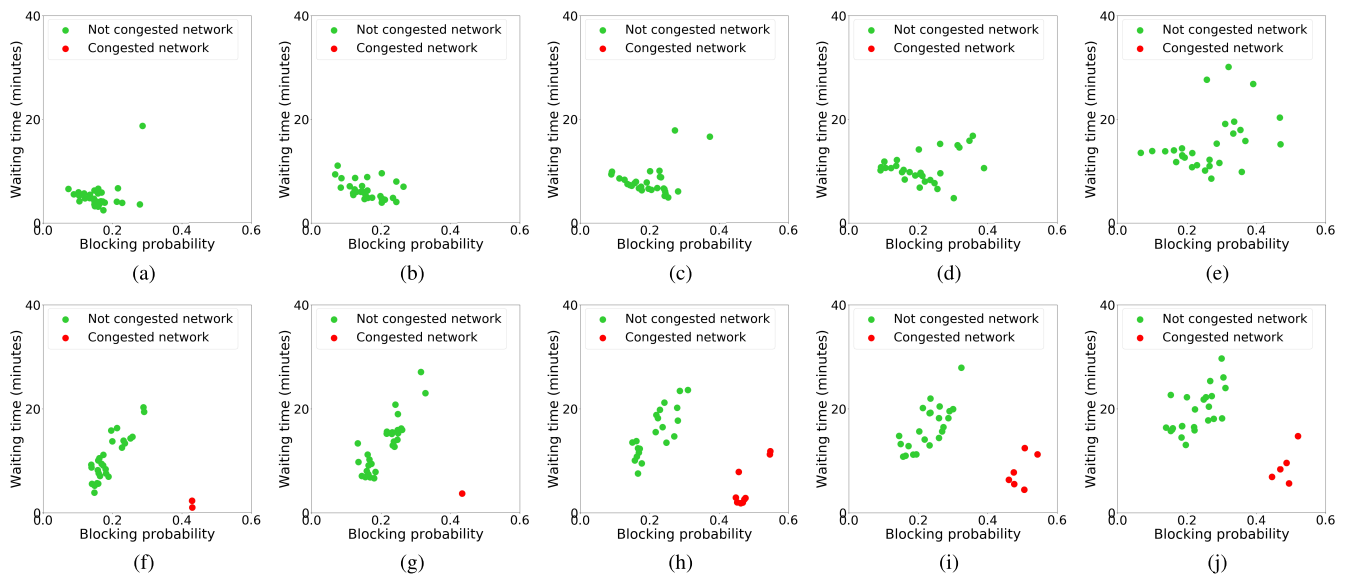
charging pile amount is setting as 2, and charging pile amount after optimization is limited by PCS’s scale. Two methods based on different QoS analytical tools independently search for optimal solutions with  $M$  varying from 1 to 13 in the step of 2. Based on our method, GA is utilized for optimal solutions as explained in section V. Based on the traditional method, changes in the amount of charging piles among PCSs make no difference in the travel cost excluding waiting time and further the EV flow distribution. Therefore, greedy algorithm assigning new charging pile to PCS with the heaviest congestion sequentially is applied for solution, which is sufficient for optimizing charging network with fixed EV flow.

**C. RESULT ANALYSIS**

In UPCN QoS analysis, we compare two methods’ results from waiting time, blocking probability and throughput. Fig. 7a depicts the average waiting time of UPCNs with various capacities. While average waiting time basically extents as average throughput of each charging pile increases, results obtained under the unlimited capacity assumption has a paradox marked in Fig. 7a: average waiting time decreases even if charging piles need to serve more EVs. Fig. 7b explains the paradox by showing the average waiting time and blocking probability analyzed by a queueing model with 1 charging



**FIGURE 7. (a) Average waiting time of EVs in UPCNs with different capacities. (b) Average waiting time and blocking probability in a PCS with 1 charging piles and 3 parking lots.**



**FIGURE 8. UPCN average waiting time and blocking probability when network capacity is 36, 34, 32, 30 and 28 respectively. (a)-(e) Results of our proposed method. (f)-(j) Results of the traditional method.**

pile and 3 parking lots. According to the queueing model, average waiting time prolongs as the traffic intensity (ratio of arrival rate to service rate) increases, and then plunges to negative when blocking probability exceeds 1. In reality, blocking probability exceeding 1 means extreme congestion in the PCS that no EV accept charging service, leading to meaningless negative waiting time.

Fig. 8 reveals average waiting time as well as blocking probability of UPCNs whose capacity varies from 36 to 28 in the step of  $-2$ . Red points in Fig. 8 illustrate congested networks with high blocking probability but low average waiting time. As explained by Fig. 7b, these unreasonable results imply unbearable traffic intensity in several PCSs, blaming for unlimited capacity assumption where EVs lack of motivation to avoid congested PCSs.

Fig. 9a depicts modified average waiting time of UPCNs by removing the results of congested networks shown in Fig. 8, and corrects the paradox represented in the Fig. 7a.

These results are close as expected when average throughput of each charging pile is low, because unlimited capacity assumption is rational when UPCN capacity is sufficient. However, the impact of different assumptions on PCS service capacity appears with average throughput of charging pile increasing. Fig. 9b shows the ratio of neglected waiting time under the unlimited capacity assumption to travel time. It is reckless to not consider the effect of waiting time with high proportion especially under high average throughput condition. However, results in Fig. 9a seems to be strange that average waiting time is actually shorter under limited capacity assumption, as network throughput is unchanged according to our results. Supposing discrepancy between each PCS's throughput to be the reason, we conducted further research.

For a queueing system, traffic intensity is an intuitive indicator to estimate system busyness, and variance of several queueing systems' traffic intensity naturally indicates differences between systems' busyness. In UPCN with fixed

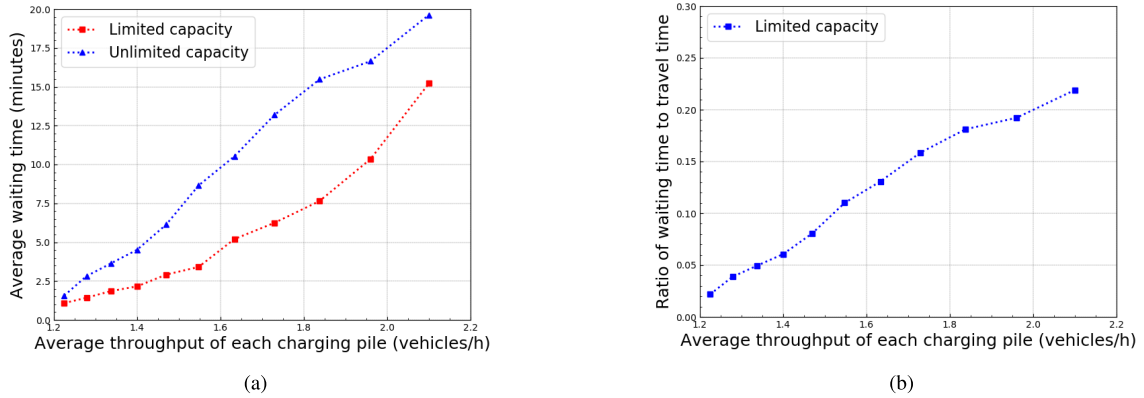


FIGURE 9. (a) Modified average waiting time of UPCNs. (b) Ratio of neglected waiting time to travel time.

TABLE 4. Optimization result.

Node	CPNum (before)	CPNum (after)	Flow rate (veh/h) (before)	Flow rate (veh/h) (after)	Waiting time (min) (before)	Waiting time (min) (after)
25	2	5	8.32	15.21	40.26	3.36
26	2	2	10.03	8.06	106.50	38.52
27	2	2	4.31	3.85	48.00	42.66
28	2	2	5.60	2.17	43.92	15.96
29	2	2	8.67	5.70	43.02	35.40
30	2	2	7.34	6.63	31.68	29.28
31	2	4	8.91	13.59	46.08	23.88
32	2	2	5.61	3.40	62.64	42.72

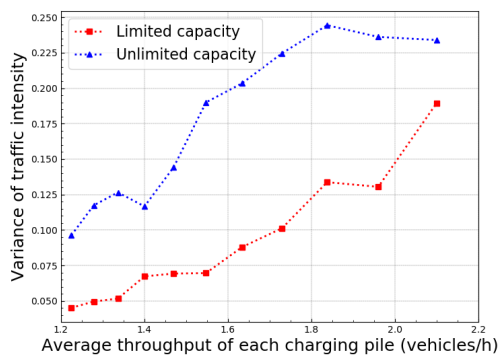


FIGURE 10. Variance of traffic intensity.

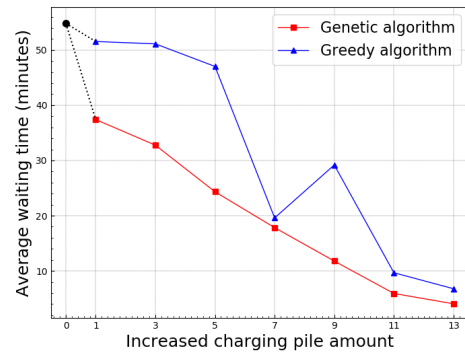


FIGURE 11. Average waiting time after QoS optimization.

throughput, lower variance of traffic intensity means more balanced EV flow distributed among PCSs. Fig. 10 represents the variance of traffic intensity calculated from each category of UPCNs. Combining Fig. 10 and Fig. 9(a), it is reasonable to conclude that limited capacity assumption leads to more balanced load among PCSs and further the shorter average waiting time. In addition, more balanced load among PCSs is actually the macro reflection that EVs avoid popular PCSs spontaneously by rescheduling routes, which is possible under limited capacity assumption.

In QoS optimization, GA parameters are set as following: generation number is 10, population number is 30, efficient  $\sigma$

is 1.67, crossover rate is 0.7 and mutation rate is 0.2. Final results are illustrated in Fig. 11. Whereas average waiting time after optimization basically decreases as increased charging pile amount grows, our method shows higher efficiency, especially in low network capacity condition. What's more, traditional method obtains an unsatisfactory result: when amount of increased charging pile grows from 7 to 9, average waiting time even prolongs. It seems that traditional method fails to grab the law behind UPCN QoS optimization.

Detailed results of our method in the case that  $M$  is 5 are shown in TABLE 4. The optimal solution decreases average waiting time from 54.8 minutes to 24.33 minutes by adding

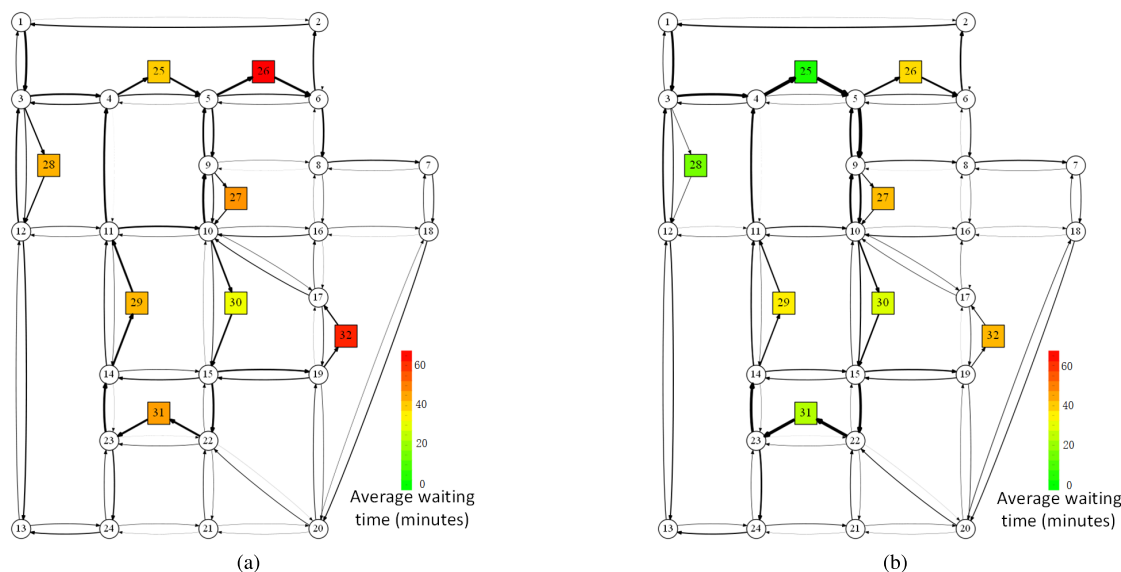


FIGURE 12. EV flow distribution. (a) Before optimization. (b) After optimization.

3 charging pile in node 25 as well as 2 charging piles in node 31. In addition to simply reducing the average waiting time, we notice significant change in EV flow distribution among PCSs which further influences average waiting time.

For more comprehensive presentation of difference in EV flow distribution before and after optimization, Fig. 12 uses size of edges to describe EV flows rate and color of nodes to describe average waiting time.

In summary, our method shows advantages in simulating EVs spontaneous behavior of avoiding popular PCSs by adjusting routes, which is important to QoS analysis and optimization of UPCN with low capacity. Traditional method ignoring waiting time at PCSs under unlimited capacity assumption is unreasonable especially in congested networks, and exposed limitations in reflecting impact of UPCN capacity on EV flow distribution which leads to optimistic estimation in QoS analysis and low performance in QoS optimization.

## VII. CONCLUSION

In this paper, we construct a novel UPCN model with independent quantification and adjustment of PCS capacity in order to explore the impact of network capacity on QoS analysis and optimization. We design a QoS analysis method for this model by creatively introducing queueing process at PCSs into the conventional traffic assignment model and searching feasible routes for EVs recharging half-way. Finally, we demonstrate an application of our model and its QoS analysis method in UPCN capacity expansion for superior QoS.

We simulated the urban public charging network for EV on the classic Sioux Falls traffic network enhanced by randomly added public charging stations. Results indicate that realizing the importance of conventionally neglected queueing delays at the stations, the proposed method manage to simulate the

trade-off in EV charging station selection between nearby hotspots and distant idle ones to save trip time. In this case, EVs prefer travelling further to seemingly distant but relatively idle charging stations to rushing into nearby crowded hotspots. Such reasonable practice reshapes EV distribution in the UPCN, network QoS, and it becomes essential to QoS optimization as the EV population booms or network capacity shrinks. Moreover, we have shown the fact that the queueing delay at the stations account for a considerable portion in the EV's total trip time, and provoked the conventional assumption of zero queueing delay.

In the future, we are going to analyze UPCN QoS by dynamic OD matrix to simulate change of EV travel demands at different time, and simplify the feasible route search algorithm to promote this method to more complex network. We will also explore more realistic mechanism of UPCN topology and capacity on EV behavior patterns from travel cost function.

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