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Identification of Multiple First-Order Continuous-Time Dynamic Models From Special Segments in Historical Data

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ABSTRACT Continuous-time dynamic models are often required for controller design, process monitoring, and operation optimization. This paper proposes an approach to estimate unknown parameters of multiple first-order continuous-time dynamic models from special segments in historical data. The approach is composed by two main steps. First, special segments are defined as the ones with two ends in steady states and the middle part having significant amplitude variations in transient states; the special segments are found automatically by exploiting a piece-wise linear representation technique from a large amount of historical data samples. Second, static gains are estimated by solving a set of linear equations based on steady-state values of inputs and outputs; sums of time constants and delays are obtained by solving another set of linear equations based on integrals of model output errors from data samples in transient states; the sums are used as an optimization constraint for maximizing the fitness value between measured and simulated outputs, from which separated estimates of time constants and delays are yielded. Numerical examples are provided to illustrate the proposed approach and compare with existing ones.

INDEX TERMS Continuous-time model identification, steady states, transient states, model validation.

I. INTRODUCTION

Continuous-time dynamic models are often required for controller design, process monitoring, and operation optimization in a systematic manner [1], [2]. As a frequently-used technique, system identification builds mathematical models for dynamic systems based on observed input and output data [3]. Continuous-time models can be either identified directly from observed data and indirectly from discrete-time counterparts; the direct way has certain advantages in practical applications [4].

Identification of continuous-time dynamic models has received many attentions in recent years; see, e.g., survey papers [5]–[8] and books [9]–[11]. There are quite a few existing identification approaches in the literature. One group of identification approaches has been developed for general types of inputs, e.g., the prediction err approach [12], the subspace approach [13], the maximum likelihood (ML) approach [14], and the instrumental variable (IV) approach [15]. Among them, the ML approach and the IV approach have been well developed and widely accepted [16]–[20]; Matlab System Identification Toolbox [21] and CONTSID Toolbox [22] are the representative algorithms to implement the ML and IV approaches, respectively. Another group of identification approaches is based on designed experiments using particular inputs, such as step tests [23], relay feedback tests [24] and impulse excitations [25]. Relevant identification approaches include the integral equation approach [26], the frequency domain approach [27], the variational Bayesian inference [28], the robust identification approach [29], the recursive instrumental variable approach [30], and the multimodel approach [31]. However, specially-designed identification experiments may perturb normal operations of industrial plants.

This paper is inspired by a common practice that is often adopted by industrial engineers to estimate parameters of continuous-time dynamic models based on some special data segments. The main idea is to read static gains of dynamic models from steady-state values of inputs and outputs, as well

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as time constants and delays from transient-state values. As a major advantage, these model parameters are known to be correct by looking at steady- and transient-state values. However, there are two major disadvantages: (i) the common practice is usually limited to single-input and single-output systems, (ii) it is time-consuming to manually find these special data segments. Our motivation is to improve the common practice in terms of removing its major disadvantages by extending to multiple-input and single-output systems and finding special data segments automatically.

This paper proposes an approach to estimate parameters of continuous-time dynamic systems based on special data segments extracted from historical data samples. The approach is composed by two main steps:

i Special data segments are defined as the ones with two ends in steady states and the middle part having significant amplitude variations in transient states. These segments are found automatically by exploiting a piece-wise linear representation technique from a large amount of historical data samples.

ii Static gains are estimated by solving a set of linear equations based on steady-state values of inputs and outputs. Sums of time constants and delays are obtained by solving another set of linear equations based on integrals of model output errors from data samples in transient states. The sums are used as an optimization constraint for maximizing the fitness value between measured and simulated outputs, from which separated estimates of time constants and delays are yielded.

The innovation of the proposed method is mainly from the separated estimations of different model parameters by exploiting special data segments. Doing so is able to yield accurate estimates of static gains from steady-state data samples, as well as time constants and delays from transient-state data samples; in addition, estimated model parameters can be validated in a convincing manner by looking at the measured and simulated outputs in the special data segments. By contrast, many continuous-time model identification approaches do not exploit data samples in steady and transient states separately; as a result, it is not easy to validate estimated model parameters in such a convincing manner.

The rest of this paper is organized as follows. Section II describes the problem to be solved. Section III presents the detailed steps of the proposed approach. Section IV provides numerical examples for illustration. Some concluding remarks are given in Section V.

II. PROBLEM DESCRIPTION

Consider a linear time-invariant, multiple-input single-output (MISO), continuous-time dynamic system,

$$y(n) = \sum_{i=1}^{I} G_i u_i(n) + C,$$

where u_i is the i-th input, y is the output, G_i is the i-th subsystem between u_i and y, and C is a constant from a fact that u_i and y are often not equal to zeros at the same time. The symbol $n \in \mathbb{Z}^+$ is the sampling time index associated with a real-valued sampling period h ($\mathbb{Z}^+$ is the set of non-negative integers).

The first-order plus dead time model is a good approximation to many industrial processes [33],

$$G_i(s) = \frac{K_i e^{-\theta_i s}}{(T_i s + 1)},$$

where K_i, T_i and θ_i respectively are the static gain, time constant and time delay of G_i. Given historical data samples $\{y(n), u_1(n), \cdots, u_I(n)\}_{n=1}^{N}$, the following conditions are assumed:

Steady-state

A1 segments in $\{y(n), u_1(n), \cdots, u_I(n)\}_{n=1}^{N}$ take more than $I+1$ different steady-state values.

A2 Changes of multiple inputs in $\{y(n), u_1(n), \cdots, u_I(n)\}_{n=1}^{N}$ have certain time differences rather than always occurring simultaneously.

The objective is to find special data segments satisfying the above conditions from historical data samples $\{y(n), u_1(n), \cdots, u_I(n)\}_{n=1}^{N}$, and estimate the model parameters K_i's, T_i's, θ_i's of the MISO system based on these special data segments.

The main idea of the proposed approach is to estimate static gains K_i's from data samples in steady states, as well as time constants T_i's and delays θ_i's from data samples in transient states. Without loss of generality, a system with two inputs and one output is illustrated, four data segments in steady states are found in Figure 1. Static gains K_1 and K_2 are estimated by solving a set of linear equations based on a relationship between steady-state values ($u_{1,ss}$, $u_{2,ss}$, y_{ss}) of $u_1(n)$, $u_2(n)$ and $y(n)$. Sums of time constants and delays, $T_1 + \theta_1$ and $T_2 + \theta_2$, are calculated by solving another set of linear equations based on integrals of model output errors in transient states. The model output error is the shaded part in Figure 1-(a), which is the difference between $y(n)$ and $K_1u_1(n)+K_2u_2(n)$. Given the estimated static gains, as well as the sums of time constants and delays, the separated estimates of time constants and time delays can be accurately obtained afterwards.

The maximum likelihood (ML) approach and the instrumental variable (IV) approach are the standard ones in the literature to be applied in this context. Numerical comparisons between the two approaches and the proposed one will be provided later in Section IV. It will be observed that the ML and IV approaches are very effective in fitting measured data samples by identifying models. However, the two approaches do not perform well in estimating model parameters, especially for the time constants and delays. This is due to a fact that the two approaches estimate static gains, time constants and delays simultaneously in one optimization involving all data samples, and do not treat steady- and transient-state data samples separately. As a result, accuracies of some

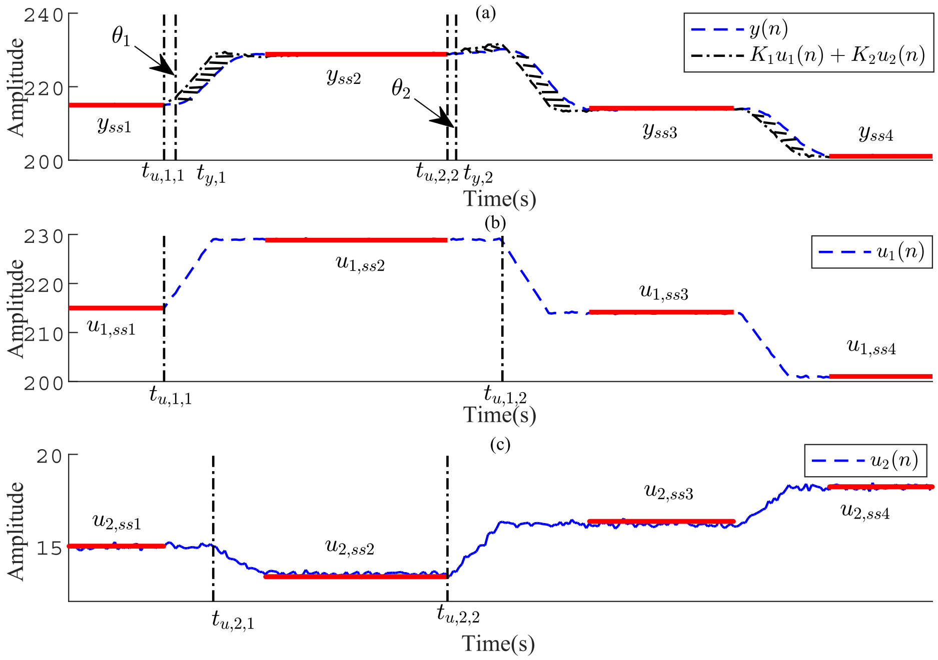

FIGURE 1. A schematic illustration of the proposed approach: (a) $y(n)$, (b) $u_1(n)$, (c) $u_2(n)$.

TABLE 1. Estimated model parameters from three approaches in one typical simulation with $\sigma_e^2 = 0.01$ in Section IV.

Approaches	$\hat{K}_1$	$\hat{K}_2$	$\hat{T}_1$	$\hat{T}_2$	$\hat{\theta}_1$	$\hat{\theta}_2$	R
ML	3.038	1.557	81.43	196.69	0	5	98.20%
IV	3.036	1.542	77.94	150.44	0.5	39	98.27%
Proposed	**3.000**	**1.498**	**48.45**	**119.55**	**31.15**	**69.75**	**99.96%**
True values	3	1.5	50	120	30	70	

model parameters are sacrificed in an optimization balance. By contrast, the proposed approach estimates static gains from steady-state data samples, and time constants and delays from transient-state data samples, in a separated manner. Hence, the proposed approach yields more accurate estimates of model parameters than the ML and IV approaches (to be shown later in Table 1 and Figures 4-6).

III. THE PROPOSED APPROACH

This section presents the detailed steps of the proposed approach.

A. ESTIMATION OF MODEL PARAMETERS

This subsection estimates model parameters based on data samples in steady and transient states, in three steps.

The first step is to estimate steady-state gains $K_1, K_2, \cdots, K_I$ based on the set V_F of steady-state data samples. The set $V_F = \{v_1, \cdots, v_F\}$ is found from historical data samples, where F is the number of steady states segments. The detailed steps to obtain V_F will be introduced later in Section III-B. Steady state values of $u_1, \cdots, u_I, y$ are denoted as $\{u_{1,ss}[f], \cdots, u_{I,ss}[f], y_{ss}[f]\}_{f=1}^{F}$, being obtained later in (16). The MISO system in steady states becomes

$$y_{ss}[f] = \sum_{i=1}^{I} K_i u_{i,ss}[f] + C.$$

Static gains are estimated simultaneously by solving a set of linear equations,

$$\underbrace{\begin{bmatrix} y_{ss}[1] \\ y_{ss}[2] \\ \vdots \\ y_{ss}[F] \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} u_{1,ss}[1] & \cdots & u_{I,ss}[1] & 1 \\ u_{1,ss}[2] & \cdots & u_{I,ss}[2] & 1 \\ \vdots & \ddots & \vdots & \vdots \\ u_{1,ss}[F] & \cdots & u_{I,ss}[F] & 1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} K_1 \\ \vdots \\ K_I \\ C \end{bmatrix}}_{K}$$

via the least-squares method as

$$\hat{K} = \left(U^T U\right)^{-1} U^T Y. \tag{1}$$

The second step is to estimate sums of time constants and delays $T_1 + \theta_1, T_2 + \theta_2 \cdots, T_I + \theta_I$ based on the set Φ_D of transient-state data segments. The set $\Phi_D = \{\phi_1, \phi_2, \cdots, \phi_D\}$ is found from the historical data samples, where $\phi_d = \{y(n), u_1(n), \cdots, u_I(n)\}_{n=n_{f,s}}^{n_{f+1,e}}$ with $d \in [1, D]$, D is the number of the transient-state data segments, $n_{f,s}$ and $n_{f+1,e}$ are respectively the starting and the ending points of ϕ_d. The steps to find Φ_D will be introduced in Section III-B. Let the variable γ_i represent the sum $T_i + \theta_i$ of G_i. The sums $\gamma_1, \gamma_2, \cdots, \gamma_I$ will be determined from a so-called model output error,

$$e_u(n) = \sum_{i=1}^{I} K_i u_i(n) - y(n).$$

There is a relationship between the integral of e_u and γ_i.

Propostion 1: The integral of e_u and γ_i are connected as,

$$\lim_{t\to+\infty}\int_0^t e_u(t)\,\mathrm{d}t=\sum_{i=1}^{I}A_{u_i}K_i\gamma_i, \tag{2}$$

where A_{u_i} is the amplitude change of u_i in steady-state segments at the two ends of ϕ_d.

Proof: The i-th input u_i of ϕ_d can be approximated by a series of ramp signals [34],

$$u_i(n)=u_{i,1}(n)+u_{i,2}(n)+\cdots+u_{i,Z}(n),$$

where

$$u_{i,z}(n)=\begin{cases}0, & 0\le n<n_{z-1}\\ \alpha_z(n-n_{z-1}), & n_{z-1}\le n<n_z\\ \alpha_z(n_z-n_{z-1}), & n_z\le n<\infty\end{cases}.$$

Here α_z is the slope of $u_{i,z}$ for $z\in[1,Z]$. Applying the final value theorem to the integral of e_{u_i} yields,

$$\begin{aligned}\lim_{t\to+\infty}\int_0^t e_{u_i}(t)\,\mathrm{d}t&=K_i\sum\nolimits_{z=1}^{Z}|\alpha_i(n_z-n_{z-1})|\gamma_i\\&=K_iA_{u_i}\gamma_i.\end{aligned} \tag{3}$$

Here $e_{u_i}=K_iu_i(n)-y(n)$; $A_{u_i}=\sum_{z=1}^{Z}|\alpha_i(n_z-n_{z-1})|=|a(n_z)-a(n_1)|$; $a(n_1)$ and $a(n_Z)$ are the initial and final steady-state values of $u_i(n)$, respectively.

Due to the Taylor expansion formula, the time delay term can be approximated as $e^{-\theta_i s}=1-\theta_i s$. From the superposition principle and the final value theorem, it is ready to obtain the integral of e_u as

$$\begin{aligned}\lim_{t\to+\infty}\int_0^t e_u(t)\mathrm{d}t&=\sum_{i=1}^{I}\lim_{t\to+\infty}\int_0^t e_{u_i}(t)\,\mathrm{d}t\\&=A_{u_i}\sum_{i=1}^{I}\lim_{s\to0}\frac{K_i(T_is+1)-e^{-\theta_is}}{(T_is+1)s}\\&=A_{u_i}K_i\sum_{i=1}^{I}\lim_{s\to0}\frac{T_i+\theta_i}{T_is+1}\\&=\sum_{i=1}^{I}A_{u_i}K_i\gamma_i.\end{aligned}$$

Remark 1: A preliminary form of (2) was developed in [35] for single-input and single-output systems subject to step signals. Here (2) is extended for MISO systems subject to general types of signals.

Let φ_d denote the numerical integration of $e_u(n)$ in ϕ_d, i.e., $\varphi_d=h\sum_{n=1}^{N_d}e_u(n)$, where N_d is the number of data samples in ϕ_d, and h is the sampling period. Given the set Φ_D and $\hat{K}$ in (1), γ_i can be estimated simultaneously by solving a set of linear equations,

$$\underbrace{\begin{bmatrix}\varphi_1\\ \varphi_2\\ \vdots\\ \varphi_D\end{bmatrix}}_{\Theta}=\underbrace{\begin{bmatrix}A_{1,u_1}\hat{K}_1 & \cdots & A_{1,u_I}\hat{K}_I & 1\\ A_{2,u_1}\hat{K}_1 & \cdots & A_{2,u_I}\hat{K}_I & 1\\ \vdots & \vdots & \vdots & \vdots\\ A_{D,u_1}\hat{K}_1 & \cdots & A_{D,u_I}\hat{K}_I & 1\end{bmatrix}}_{\Psi}\underbrace{\begin{bmatrix}\gamma_1\\ \gamma_2\\ \vdots\\ \gamma_I\\ C_\gamma\end{bmatrix}}_{\Gamma}$$

via the least-squares method as

$$\hat{\Gamma}=\left(\Psi^T\Psi\right)^{-1}\Psi^T\Theta. \tag{4}$$

The third step is to estimate the time delay θ_i of G_i based on the time difference between change points of $u_i(n)$ and $y(n)$. Time sequences of ϕ_d are separated into piece-wise linear representations (PLR) to obtain the sequences $P_{u_i}(n)$ and $P_y(n)$ as the trends $u_i(n)$ and $y(n)$, respectively. The detailed steps will be introduced later in Section III-B. A variable $H_{y,l}$ takes the value '1' for increasing or decreasing trends of the l-th segment of $y(n)$ in ϕ_d, and the value '0' for no-changing trends, i.e.,

$$H_{y,l}=\begin{cases}0, & \{P_y(n)\}_{n_l}^{n_l+N_l+1}=1\\ 1, & \{P_y(n)\}_{n_l}^{n_l+N_l+1}=0\end{cases},$$

where n_l and N_l for $l\in[1,L]$ are respectively the sampling index and the number of data samples in the l-th segment, L is the number of segments in ϕ_d, and $P_y(n)$ is given later in (13). The trend combination is $H_{y,d}=\{H_{y,1},H_{y,2},\cdots,H_{y,L}\}_{l=1}^{L}$. Analogously, the trend combination $H_{u_i,d}$ of $u_i(n)$ in ϕ_d is obtained.

A changing point $t_{y,d}$ of $y(n)$ in ϕ_d is defined as the time index when $H_{y,d}$ makes a trend change from 0 to 1 for the first time, i.e.,

$$t_{y,d}=n_l,\ \text{for } H_{y,l}=1 \text{ and } \sum_{a=1}^{l-1}H_{y,a}=0. \tag{5}$$

Note that $H_{y,1}$ is not equal to 1 because both ends of ϕ_d are in steady states. Analogously to (5), the changing point $t_{u_i,d}$ in $[n_{l-1},n_{l+1}]$ of u_i in ϕ_d is found.

A new variable $R_{i,d}$ is introduced to take the value '1' for the change of $y(n)$ caused by a single input $u_i(n)$, i.e.,

$$R_{i,d}=\begin{cases}1, & t_{u_i,d}<t_{u_j,d}+T_{tp},\ i\neq j \text{ and } j=1,2,\cdots,I\\ 0, & \text{otherwise},\end{cases} \tag{6}$$

where T_{tp} is the largest threshold value of all the time delays. The estimate $\hat{\theta}_{i,d}$ is determined from the time difference between $y(n)$ and $u_i(n)$ corresponding to $R_{i,d}=1$, i.e.,

$$\hat{\theta}_{i,d}=t_{y,d}-t_{u_{i,d}},\ \text{where } R_{i,d}=1. \tag{7}$$

By repeating the steps in (6) and (7), all variables $R_{i,s}$'s with $s\in[1,L_i]$ are found, and the estimated initial value of delay $\hat{\theta}_{i,0}$ can be determined as the average of all time differences $\hat{\theta}_{i,s}$'s related to $R_{i,s}$'s equal to 1, i.e.,

$$\hat{\theta}_{i,0}=\frac{1}{L_i}\sum\nolimits_{s=1}^{L_i}\hat{\theta}_{i,s},\ \text{where } R_{i,s}=1. \tag{8}$$

Given the estimates $\hat{\gamma}_i$ in (4) and $\hat{\theta}_{i,0}$ in (8), it is ready to calculate the initial value of time constant $T_{i,0}$ as

$$\hat{T}_{i,0}=\hat{\gamma}_i-\hat{\theta}_{i,0}. \tag{9}$$

By using $\hat{\theta}_{i,0}$ in (8) and $\hat{T}_{i,0}$ in (9) as initial values, as well as $\hat{K}_i$ in (1) and $\hat{\gamma}_i$ in (4), T_i's can be estimated by solving an optimization problem:

$$\hat{T}_i = \arg\max_{\tilde{T}_i} R\left(\hat{K}_i, \tilde{T}_i, \hat{\gamma}_i - \tilde{T}_i\right), \quad (10)$$

where $R\left(\hat{K}_i, \tilde{T}_i, \hat{\gamma}_i - \tilde{T}_i\right)$ is the fitness between the measured output $y(n)$ and the simulated output $\hat{y}(n)$,

$$R\left(\hat{K}_i, \tilde{T}_i, \hat{\gamma}_i - T_{i,g}\right) = \left(1 - \frac{\sum_{n=1}^{N}\left(y(n) - \hat{y}(n)\right)^2}{\sum_{n=1}^{N}\left(y(n) - \bar{y}\right)^2}\right).$$

Here $\bar{y} = \frac{1}{N}\sum_{n=1}^{N} y(n)$, and $\hat{y}(n)$ is obtained by passing u_i's through $\hat{G}_i\left(s; \hat{K}_i, \tilde{T}_i, \hat{\gamma}_i - \tilde{T}_i\right)$'s with model parameters $\hat{K}_i, \tilde{T}_i, \hat{\gamma}_i - \tilde{T}_i$ by using the Matlab function 'lsim' as

$$\hat{y}(n) = \text{lsim}\left(\hat{G}_i\left(s; \hat{K}_i, \tilde{T}_i, \hat{\gamma}_i - \tilde{T}_i\right), u_i(n)\right).$$

Given the estimates $\hat{\gamma}_i$ in (4) and $\hat{T}_i$ in (10), it is ready to recalculate the delay $\hat{\theta}_i$ as

$$\hat{\theta}_i = \hat{\gamma}_i - \hat{T}_i. \quad (11)$$

In summary, the parameters K_i, T_i, and θ_i in the MISO system have been determined.

B. ACQUISITION OF SPECIAL DATA SEGMENTS

This subsection exploits a piece-wise linear representation (PLR) technique in [36] to find special data segments for estimating dynamic model parameters.

The PLR separates a long time sequence $\{y(n)\}_{n=1}^{N}$ into M short ones $\{y(n)\}_{n=n_1}^{n_2+N_2-1}, \cdots, \{y(n)\}_{n=n_m}^{n_m+N_m-1}$, where $n_1 = 1$, $n_m + N_m - 1 = N$, n_m and N_m for $m \in [1, M]$ are respectively the sampling index and the number of data samples in the m-th segment. The m-th data segment $\{y(n)\}_{n=n_m}^{n_m+N_m-1}$ is described by a simple linear regression model, i.e.,

$$y(n) = a_m + b_m n + e(n),$$

where $e(n)$ is white noise with zero mean and variance σ_e^2. The unknown parameters a_m and b_m are estimated from

$$\hat{a}_m = \bar{y} - \hat{b}_m n,$$
$$\hat{b}_m = \frac{\sum_{n=n_m}^{n_{m+1}-1}\left(y(n) - \bar{y}\right)(n - \bar{n})}{\sum_{n=n_m}^{n_{m+1}-1}(n - \bar{n})^2}, \quad (12)$$

where $\bar{y} = \frac{\sum_{n=n_m}^{n_{m+1}-1} y(n)}{n_{m+1}-n_m}$ and $\bar{n} = \frac{\sum_{n_m}^{n_{m+1}-1} n}{n_{m+1}-n_m}$. Thus, the m-th data segment is represented by a straight line,

$$\hat{y}(n) = \hat{a}_m + \hat{b}_m n.$$

The loss function $L(M)$ between $y(n)$ and $\hat{y}(n)$ is

$$L(M) = \sum_{m=1}^{M}\sum_{n=n_m}^{n_{m+1}-1}\left(y(n) - \hat{y}(n)\right)^2.$$

The number M of data segments can be determined as the smallest choice of M's that $L(M-1)$, $L(M)$ and $L(M+1)$ are close to each other [37].

Two state sequences $\{P_y(n)\}_{n=n_m}^{n_m+N_m-1}$ and $\{Q_y(n)\}_{n=n_m}^{n_m+N_m-1}$ are introduced to characterize steady and transient states. $\{P_y(n)\}_{n=n_m}^{n_m+N_m-1}$ takes the value '1' for $\{y(n)\}_{n=n_m}^{n_m+N_m-1}$ in steady states and otherwise the value '0', i.e.,

$$\{P_y(n)\}_{n=n_m}^{n_m+N_m-1} = \begin{cases} 1, & \hat{A}_{y,m} \le A_{y,p},\ N_m \ge N_y \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

Here, $\hat{A}_{y,m}$ is the amplitude change in the m-th data segment $\{y(n)\}_{n=n_m}^{n_m+N_m-1}$, i.e.,

$$\hat{A}_{y,m} = \left|\hat{y}(n_m + N_m - 1) - \hat{y}(n_m)\right|. \quad (14)$$

Symbol N_y is the minimum time threshold, and $A_{y,p}$ is the maximum amplitude change threshold for $y(n)$ in steady states. Similarly, $\{Q_y(n)\}_{n=n_m}^{n_m+N_m-1}$ takes the value '1' in transient states and otherwise the value '0', i.e.,

$$\{Q_y(n)\}_{n=n_m}^{n_m+N_m-1} = \begin{cases} 1, & \hat{A}_{y,m} \ge A_{y,q},\ N_m \ge N_y \\ 0, & \text{otherwise,} \end{cases} \quad (15)$$

where $A_{y,q}$ is the minimum amplitude change threshold for $y(n)$ to be in transient states. The threshold parameters N_y, $A_{y,p}$ and $A_{y,q}$ will be decided later at the end of this subsection.

Analogously to (13), (14) and (15), amplitude changes of $u_i(n)$'s and state sequences $P_{u_i}(n)$ and $Q_{u_i}(n)$ are obtained. The overall state sequences $P_o(n)$ and $Q_o(n)$ in steady and transient states are

$$P_o(n) = \prod_{i=1,\cdots,I} P_{u_i}(n) \cdot P_y(n),$$
$$Q_o(n) = \prod_{i=1,\cdots,I} Q_{u_i}(n) \cdot Q_y(n).$$

The starting and ending positions of $P_o(n)$ taking consecutive values of '1' are located as $n_{f,s}$ and $n_{f,e}$,

$$\begin{cases} P_o\left(n_{f,s} - 1\right) = 0 \\ P_o\left(n_{f,e} + 1\right) = 0 \\ \sum_{n=n_{f,s}}^{n_{f,e}} P_o(n) = n_{f,e} - n_{f,s} + 1, \quad \text{for } n_{f,s} < n_{f,e}. \end{cases}$$

Data segments of $y(n)$ and $u_i(n)$ between $n_{f,s}$ and $n_{f,e}$ are in steady states, to be embedded into a data set v_f, i.e.,

$$v_f = \{y(n), u_1(n), \cdots, u_I(n)\}_{n=n_{f,s}}^{n_{f,e}}.$$

Steady state values $y_{ss}[f]$ and $u_{i,ss}[f]$ are respectively calculated as the sample means of v_f, i.e.,

$$y_{ss}[f] = \frac{\sum_{n=n_{f,s}}^{n_{f,e}} y(n)}{n_{f,e} - n_{f,s} + 1}, \quad u_{i,ss}[f] = \frac{\sum_{n=n_{f,s}}^{n_{f,e}} u_i(n)}{n_{f,e} - n_{f,s} + 1}. \quad (16)$$

A set V_F is obtained to enclose steady-state segments, i.e.,

$$V_F = \{v_1, \cdots, v_F\}, \quad (17)$$

where F is the number of all steady-states segments. The starting and ending positions of $Q_o(n)$ taking consecutive values of '1' are located as $n_{b,s}$ and $n_{b,e}$,

$$\begin{cases} Q_o\left(n_{b,s}-1\right)=0 \\ Q_o\left(n_{b,e}+1\right)=0 \\ \sum_{n=n_{b,s}}^{n_{b,e}} Q_o\left(n\right)=n_{b,e}-n_{b,s}+1, \text{ for } n_{b,s}<n_{b,e}. \end{cases}$$

Data segments of $y(n)$ and $u_i(n)$ between $n_{b,s}$ and $n_{b,e}$ are in transient states, to be embedded into a data set w_b, i.e.,

$$w_b=\{y\left(n\right), u_1\left(n\right), \cdots, u_I\left(n\right)\}_{n=n_{b,s}}^{n_{b,e}}.$$

A set W_B is obtained to enclose transient-state segments, i.e.,

$$W_B=\{w_1, \cdots, w_B\}, \tag{18}$$

where B is the number of all transient states segments. For the data segment w_b with interval $\left[n_{b,s}, n_{b,e}\right]$, the nearest v_f and v_{f+1} in front and behind of w_b are found, so that a special data segment is obtained as

$$\phi_d=\{y\left(n\right), u_1\left(n\right), \cdots, u_I\left(n\right)\}_{n=n_{f,s}}^{n_{f+1,e}},$$

where $n_{f,s}$ and $n_{f+1,e}$ are respectively the starting index of v_f and the ending index of v_{f+1}. A set Φ_D is the one enclosing all the special data segments, i.e.,

$$\Phi_D=\{\phi_1, \phi_2, \cdots, \phi_D\}, \tag{19}$$

where D is the number of all special data segments.

The threshold parameters N_y, $A_{y,q}$ and $A_{y,p}$ are determined here. The introduction of N_y is based on a common sense that a data segment in the steady state should last for a while before telling its operating condition. The rationale of N_y is similar to the alarm delay timer [32], whose role is to raise (clear) an alarm when several consecutive data points are larger (smaller) than an alarm limit. A default value $N_y=60$ is used to represent 60 sec if the sampling period is $h=1$ sec [32]. The parameter $A_{y,q}$ has been investigated in our early work [39],

$$A_{y,q}=6.9282\hat{\sigma}_e,$$

where $\hat{\sigma}_e$ is an unbiased estimate of σ_e [38],

$$\hat{\sigma}_e=\sqrt{\frac{\sum_{n=n_m}^{n_m+N_y-1}(y(n)-\hat{y}(n))^2}{N_y-1}}.$$

The threshold parameter $A_{y,p}$ is decided based on hypothesis tests as follows. Consider the segment $\{y(n)\}_{n=n_m}^{n_m+N_y-1}$ as independent realizations of a random variable y with mean μ_y and variance σ_e^2. The estimated slope parameter $\hat{b}_m$ in (12) is known to be with the Gaussian distribution with mean b_m and variance $\sigma_e^2/\sum_{n=n_m}^{n_m+N_y-1}(n-\bar{n})^2$ [38]. A hypothesis test can be formulated with the null hypothesis $H_0: b_m=0$ and the alternative hypothesis $H_1: b_m\neq 0$, based on the Students's t-statistic [38],

$$t=\frac{\hat{b}_m-0}{\hat{\sigma}_e/\sqrt{\sum_{n=n_m}^{n_m+N_y-1}(n-\bar{n})^2}}=\frac{\hat{b}_m}{S_{\hat{b}_m}}.$$

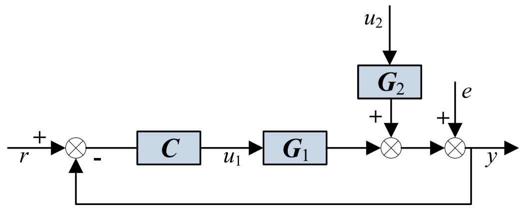

FIGURE 2. A schematic diagram of the feedback control system in Section IV.

The interval estimate of b_m with the confidence level $100(1-\alpha)\%$ is $[\hat{b}_m - t_{(N_y-1,\alpha/2)}\cdot S_{\hat{b}_m}, \hat{b}_m + t_{(N_y-1,\alpha/2)}\cdot S_{\hat{b}_m}]$, where $t_{(N_y-1,\alpha/2)}$ is the critical value of Student's t-distribution with N_y-1 degrees of freedom. The test is carried out by comparing $\hat{b}_m$ in (12) with the appropriate critical value $t_{(N_y-1,\alpha/2)}$ [38], that is, H_0 is accepted at the significance level α if

$$|t|<t_{(N_y-1,\alpha/2)}.$$

Let us choose $t_{(N_y-1,\alpha/2)}S_{\hat{b}_m}$ as the true value of b_m for a worst case scenario, being associated with $\alpha=0.01$, $N_y=60$, and $t_{(N_y-1,\alpha/2)}=2.39$, i.e.,

$$b_m=t_{(N_y-1,\alpha/2)}S_{\hat{b}_m}=0.0178\hat{\sigma}_e.$$

Thus, $A_{y,p}$ can be determined from b_m as

$$A_{y,p}=(N_y-1)b_m=1.0502\hat{\sigma}_e.$$

C. STEPS OF THE PROPOSED APPROACH

In the previous two subsections, four steps are presented to estimate model parameters from historical data samples. They are summarized as follows:

Step 1: Extract the set V_F in (17) of steady-state data segments, the set W_B in (18) of transient-state data segments, and the set Φ_D in (19) of special data segments from historical data samples.

Step 2: Calculate the static gains $\hat{K}_i$'s from (1) of multiple dynamic systems.

Step 3: Obtain $\hat{\gamma}_i$'s from (4) as the sum of time constants and delays of multiple dynamic systems.

Step 4: Determine the time constant $\hat{T}_i$ in (10), and the time delay $\hat{\theta}_i$ in (11).

IV. EXAMPLES

This section provides numerical examples to illustrate the proposed approach and compare with existing ones.

A typical feedback control system is depicted in Figure 2, where process variables r, y, u_1, u_2 are respectively the reference, output, controller output and disturbance; e is the Gaussian white noise with zero mean and variance σ_e^2. Symbols C, G_1 and G_2 are respectively the proportional-integral controller $C(s)=0.13\left(1+\frac{1}{40s}\right)$, the dynamic system between u_1 and y, and the disturbance dynamics between

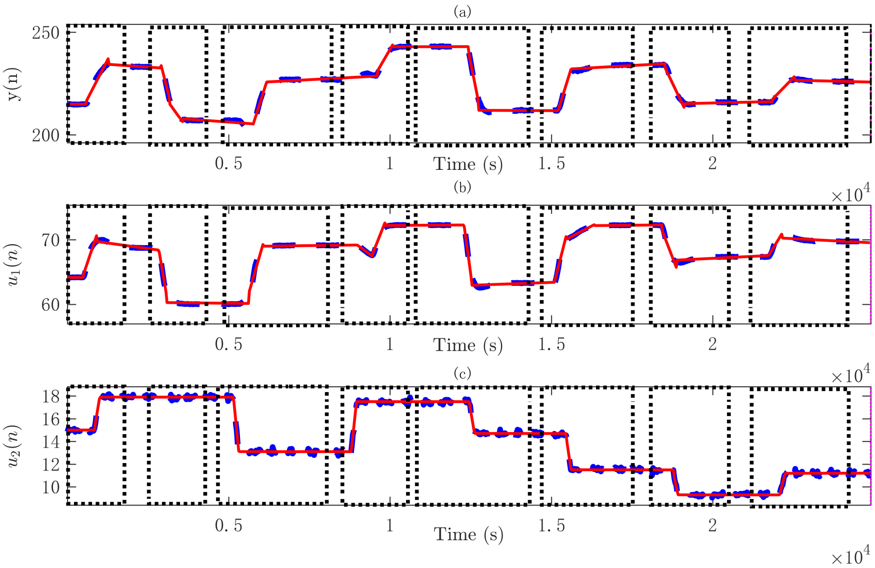

FIGURE 3. Time sequences in a typical simulation: (a) $y(n)$ (blue dash) and its PLR $\hat{y}(n)$ (red solid), special data segments (black dash line frame), (b) the counterparts for $u_1(n)$, (c) the counterparts for $u_2(n)$.

u_2 and y,

$$G_1(s) = \frac{3}{50s+1}e^{-30s}, \quad G_2(s) = \frac{1.5}{120s+1}e^{-70s}.$$

The proposed approach is applied to data samples of r, y, u_1 and u_2, where the sampling period is $h = 0.5$ sec. Figure 3 presents the time sequences and the PLRs results of r, y, u_1 and u_2 in a typical simulation with $\sigma_e^2 = 0.01$. Eight special data segments are found, marked by black dash line frames in Figure 3; their starting and ending sampling indices are [1, 2515], [2605, 4825], [4927, 7942], [8220, 11476], [11792, 14463], [14820, 17855], [18142, 21252] and [21327, 24265]. As expected, two ends of these data segments are in steady states, and the middle parts are with significant amplitude changes of u_1, u_2 and y.

Based on steady-state values of u_1, u_2 and y, the estimated static gains $\hat{K}_1 = 3.000$ and $\hat{K}_2 = 1.498$ are obtained from (1), and are very close to the true values $K_1 = 3$ and $K_2 = 1.5$. Based on transient data samples, the estimated sums of the time constants and delays $\hat{\gamma}_1 = \hat{T}_1 + \hat{\theta}_1 = 79.6$ and $\hat{\gamma}_2 = \hat{T}_2 + \hat{\theta}_2 = 189.3$ are obtained from (4), and are very close to the true values $\gamma_1 = T_1 + \theta_1 = 80$ and $\gamma_2 = T_2 + \theta_2 = 190$. Eq. (10) leads to the estimated time constants $\hat{T}_1 = 48.45$ and $\hat{T}_2 = 119.55$, so that (11) gives the estimated time delays $\hat{\theta}_1 = \hat{\gamma}_1 - \hat{T}_1 = 79.6 - 48.45 = 31.15$ and $\hat{\theta}_2 = 189.3 - 119.55 = 69.75$. The estimated time constants and delays are quite close to the true values.

As a comparison, the ML and IV approaches are deployed here. The time delays are determined by the function *delayest* [21] as the initial estimates, and the structure of the identification model is given in advance. The function *tfest* in Matlab System Identification Toolbox [21] and the

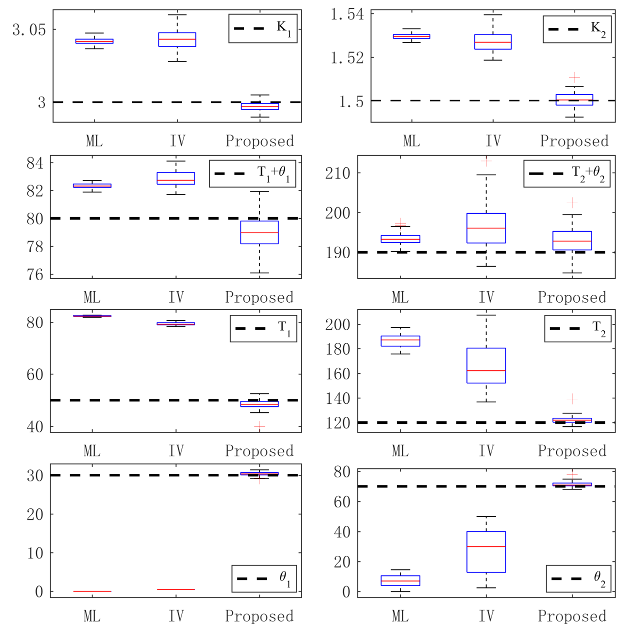

FIGURE 4. Boxplots of the estimated parameters in 100 Monte Carlo simulations with $\sigma_e^2 = 0.1$, where black dash lines are the true values of model parameters.

function *srivc* in CONTSID Toolbox [22] are used to obtain the identification parameter results of the ML and IV approaches, respectively.

Table 1 lists the results from the three approaches in one typical simulation. The model quality is measured by a fitness R between the measured output $y(n)$ and the simulated output $\hat{y}(n)$, i.e.,

$$R = \left(1 - \frac{\sum_{n=1}^{N}\left(y(n) - \hat{y}(n)\right)^2}{\sum_{n=1}^{N}\left(y(n) - \bar{y}\right)^2}\right), \tag{20}$$

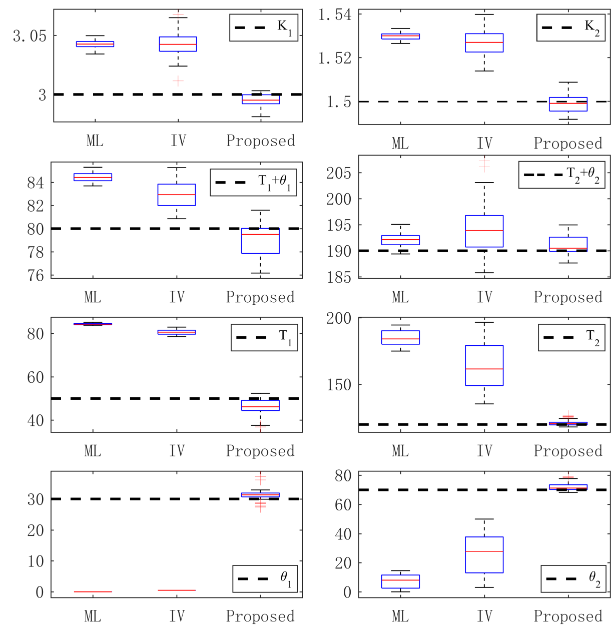

FIGURE 5. **Boxplots of the estimated parameters in 100 Monte Carlo simulations with $\sigma_e^2 = 0.5$, where black dash lines are the true values of model parameters.**

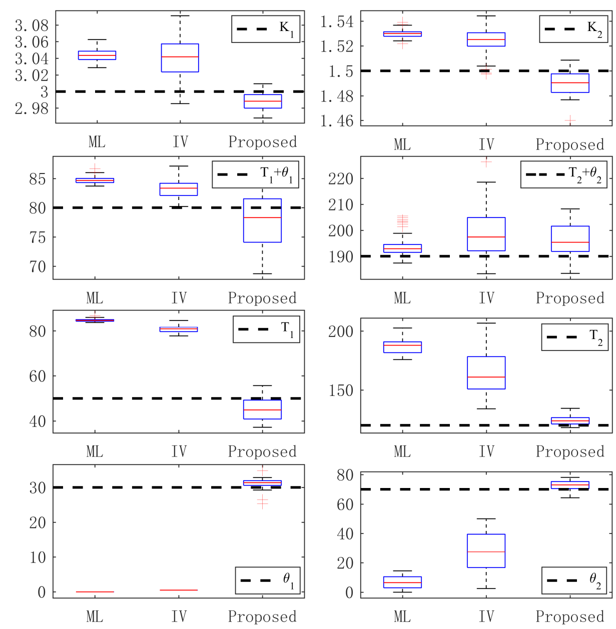

FIGURE 6. **Boxplots of the estimated parameters in 100 Monte Carlo simulations with $\sigma_e^2 = 1$, where black dash lines are the true values of model parameters.**

where $\bar{y} = \frac{1}{N}\sum_{n=1}^{N} y(n)$, and $\hat{y}(n)$ is obtained by passing u_i's through the identified models $\hat{G}_i(s)$'s by using the Matlab function 'lsim' as

$$\hat{y}(n) = \text{lsim}\left(\hat{G}_i(s), u_i(n)\right).$$

All fitness values in (20) of three approaches are above 98%, saying that the three approaches have achieved satisfactory output fitting results. However, estimated model parameters from the proposed approach are more accurate than the counterparts from the ML and IV approaches. In particular, the estimated time constants and delays from the ML and IV approaches have large errors away from the true values.

Figures 4, 5 and 6 present the distributions of estimated model parameters from the three approaches in Monte Carlo simulations with different noise levels. Similar to the results in Table 1, the proposed approach yields more accurate estimates of model parameters than the ML and IV approaches. The estimated static gains, time constants and delays from the proposed approach are very accurate at different noise levels. By contrast, estimated model parameters from the ML and IV approaches are often deviated away from the true values. This is under expectation, because the ML and IV approaches do not treat steady- and transient-state data samples separately, and estimate static gains, time constants and delays simultaneously in one optimization. As a result, there is an optimization balance among three estimated model parameters; in other words, different combinations of static gains, time constants and delays can lead to similar optimization results. This can be revealed from the estimates of $T_i+\theta_i$ in the second top subplots of Figures 4, 5 and 6: the true values of $T_i + \theta_i$ are often covered by the distributions of estimates, but the true values of T_i (or θ_i) are not.

Besides the accuracies of estimated model parameters, there is another major difference in term of model validation between the proposed approach and two existing ones. For the proposed approach, the accuracies of estimated static gains (time constants and delays) can be clearly revealed by comparing the estimated and measured outputs in steady (transient) states. However, for the ML and IV approaches, there is not such a transparent way for validating the estimated model parameters.

V. CONCLUSION

The paper proposed an approach to estimate model parameters of multiple continuous-time dynamic systems based on special data segments extracted from historical data samples. First, special data segments in steady- and transient-states were found automatically by exploiting piece-wise linear representations from historical data samples. Second, static gains, time constants and delays were respectively estimated by solving two sets of multiple linear equations based on steady- and transient-state values of inputs and outputs. Numerical examples were provided to show that the proposed approach yielded more accurate estimates of model parameters than two existing continuous-time model identification approaches; moreover, estimated static gains (time constants and delays) could be validated by comparing the measured and simulated outputs in steady (transient) states.

The proposed approach can be extended to nonlinear or linear time-varying dynamic systems, by classifying steady-state and transient-state data samples being associated with different static gains, time constants and delays into corresponding groups. Estimated model parameters can be validated by looking at the measured and simulated outputs in steady- and transient-states. Such an extension will be a feasible and practical way to resolve these challenging identification problems.

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