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Homogeneous Domination Control for Uncertain Nonlinear Systems via Interval Homogeneity With Monotone Degrees

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ABSTRACT It is challenging and interesting to globally stabilize the p–normal form nonlinear system with unknown power integrators. The most difficulty arising from the unknown power integrators is that the power integrator drift causes uncertain homogeneity as well as unknown parameters in Lyapunov function. This paper revamps the tool of adding a power integrator to recursively construct a state-dependent homogeneous domination stabilizer for the p–normal form system with interval power integrators based on the new concept of interval homogeneity with monotone degrees. To judge the existence of interval homogeneity with monotone degrees, a so-called admissible index is proposed. We show that if the system has positive admissible index(es) then it has interval homogeneity with monotone degrees and the interval homogeneity as well as homogeneous weights can be calculated by a rule. Both theoretical analysis and simulations validate our method and conclusions.

INDEX TERMS Global stabilization, unknown power drift, interval homogeneity, homogeneous domination approach, adding a power integrator.

I. INTRODUCTION

One of the research focuses in the field of nonlinear control is the global stabilization problem for the p–normal form nonlinear system [6]–[9], [12], [16], defined by

$$\dot{x}_i = x_{i+1}^{p_i} + \phi_i(x_1, x_2, \cdots, x_i), \quad i = 1, 2, \cdots, n-1,$$
$$\dot{x}_n = u + \phi_n(x), \tag{1}$$

where $x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$ is system state ($\mathbb{R}$ is real number set), $u \in \mathbb{R}$ is control input, uncertain functions $\phi_i(\cdot) : \mathbb{R}^i \to \mathbb{R}$ are C^1 and vanish at the origin, and power integrators $p_i \in \mathbb{R}^+_{odd}$, $i = 1, 2, \cdots, n$ with $p_n = 1$ are ratios of positive odd integers.

The p–normal form system (1) represents general nonlinear systems. When $p_1 = p_2 = \cdots = p_n = 1$, it encompasses the well-known feedback linearizable systems [2], [4], [15]. In the case when one or more than one of the powers $p_i > 1$ ($i = 1, 2, \cdots, n$), system (1) is known as the power integrator system [3], [7], [13]. From theoretical point of view, it has been known that the p-normal form system is very challenging to be stabilized due to the singularity around the origin. Over the past two decades, there has been a surge of interests on stabilization of system (1), for example, see [5]–[7], [17] without claim of completeness. Among them, the *adding a power integrator* technique [7] is possibly the most important one. Recently, the work [6] introduced a homogeneity called *homogeneity with monotone degrees* (HWMD), which generalizes the tool of adding a power integrator to encompass different degrees of the system.

Up to present, almost of all interests focus on stabilizing system (1) in the case when its power integrators are precisely known as ratios of odd integers but not including the case when power integrators are just known to be intervals, i.e., $p_i \in [a_i, b_i]$ with $b_i \geq a_i > 0$ for all i. In other words, system (1) is now an inherently uncertain nonlinear system with unknown power integrators. The most difficulty emerged with the unknown power integrators is that the HWMD are now uncertain and thus lead to Lyapunov function with unknown parameters. To deal with this difficulty, we defined the concept of interval homogeneity of monotone

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degrees and stabilized system (1) in a special case when its power integrators drift in narrow ranges around unit one [11], i.e., $a_i = 1 - \bar{\delta}_i$, $b_i = 1 + \bar{\delta}_i$, where $\bar{\delta}_i$ is a constant upper bound. Nevertheless, it is too strictly to require all power integrators varying around the unit one. In fact, the p–normal form system should have arbitrary power integrators rather than those around the unit one. For instance, the system

$$\dot{x}_1 = x_2^{p_1} + x_1, \quad \dot{x}_2 = u, \tag{2}$$

with $p_1 = 3$, is such a simple one [3], and its power is reasonable to drift around the cubic, e.g., $p_1 \in [\frac{13}{5}, \frac{17}{5}]$, if parameter perturbation happens in practice.

It is not a trivial way to extend the existing method in [11] to stabilize the p-normal form system when its power integrators drift around more general powers. The difficulties lie in the following three aspects. Firstly, we need an index used to judge whether the p-normal form system can possess interval homogeneity or not. Then, a new rule should be explored based on such index to find homogeneous weights that can be used to calculate the interval homogeneity with monotone degrees. Finally, the stabilization analysis becomes more complex due to the considerations of more general interval power integrators as well as a general growth condition on unknown functions ϕ_i.

Motivated by above statements, this paper aims to revamp the tool of adding a power integrator to recursively construct a state-dependent homogeneous domination stabilizer for the p–normal form nonlinear system when its power integrators drift around more general powers and unknown functions are under a state-dependent homogeneous growth condition. The main contributions of this paper are in threefold:

1) A so-called *admissible index* is defined to identify the possible existence of system (1) with interval power integrators that possesses interval HWMD.
2) A rule is explored to find homogeneous weights that can be used to calculate the interval HWMD, playing important role in constructing Lyapunov functions with unknown parameters.
3) An axiomatic stabilization analysis is presented for system (1) with more general interval power integrators and a general growth condition on unknown functions.

It will reveal that under some conditions the p–normal form nonlinear system can be globally asymptotically stabilized when its powers drift in appropriate interval bounds.

The rest of this paper is organized as follows. Section 2 introduces some mathematical preliminaries. Section 3 presents the proposed method, and then two simulations are used to validate our method in the Section 4. The last section concludes this paper.

II. MATHEMATIC PRELIMINARIES

In this section, some important definitions and lemmas are introduced for the consequent work after recalling some basic definitions of homogeneous system theory.

Definition 1: [3] For real numbers $r_i > 0$, $i = 1, \cdots, n$ and fixed coordinates $(x_1, \cdots, x_n) \in \mathbb{R}^n$, the dilation $\Delta_\varepsilon^r(x)$ is defined by $\Delta_\varepsilon^r(x) = (\varepsilon^{r_1}x_1, \cdots, \varepsilon^{r_n}x_n), \forall \varepsilon > 0$, with r_i being called as the homogeneous weights of the coordinates.

Definition 2: (Interval HWMD) [11] A continuous vector field $f(x) = (f_1(x), \cdots, f_n(x))^T, x \in \mathbb{R}^n$ is said to satisfy interval homogeneity with monotone degrees, if we can find positive real numbers $(r_1, \cdots, r_n)$ and a series of intervals $[\underline{\tau}_i, \bar{\tau}_i]$, $i = 1, \cdots, n$ satisfying $\underline{\tau}_i \geq \bar{\tau}_{i+1}$, $i = 1, \cdots, n-1$ such that for $i = 1, \cdots, n$, the following holds

$$f_i(\epsilon^{r_1}x_1, \cdots, \epsilon^{r_n}x_n) = \epsilon^{r_i+\tau_i}f_i(x), \quad \forall \epsilon > 0, \ x \in \mathbb{R}^n \setminus \{0\}$$

for a possibly unknown constant $\tau_i \in [\underline{\tau}_i, \bar{\tau}_i]$.

It is evident that the interval HWMD will degenerate to the classical one if power integrators p_i are precisely known. To judge whether the system (1) has interval HWMD or not, the so-called *admissible index*, is defined as follow.

Definition 3: For a chain of unknown power integrators in form of $p_i \in [a_i, b_i], i = 1, 2, \cdots, n-1$, a series of admissible indexes, D_k, can be recursively defined by

$$\begin{aligned} D_0 &:= 1, \\ D_1 &:= a_{n-1} + 1, \\ D_k &:= (a_{n-k} + 1)D_{k-1} - b_{n-k+1}D_{k-2} \end{aligned} \tag{3}$$

for $k = 2, 3, \cdots, n-1$.

Based on the above Definition 3, we can explore a way to define homogeneous weights to guarantee the possible existence of interval HWMD for the system (1) when its power integrators drift in appropriate bounds that guarantee positive admissible index. We have the following lemma:

Lemma 1: If the bounds $[a_i, b_i]$ of power integrators of the system (1) satisfy $D_k > 0$ for all k, there exists a set of homogeneous weights recursively defined by

$$r_i = \tfrac{1}{D_{n-i+1}}(D_{n-i}r_{i-1} + b_i \cdots b_{n-1}b_n r_{n+1}) \tag{4}$$

with any real $r_1 > 0$ and $r_{n+1} := 1$ that can guarantee the existence of interval HWMD $\tau_i \in [\underline{\tau}_i, \bar{\tau}_i]$ with $\underline{\tau}_i := a_i r_{i+1} - r_i$ and $\bar{\tau}_i := b_i r_{i+1} - r_i$ for $i = 1, 2, \cdots, n-1$.

Proof: It can be concluded easily that $r_i > 0$ because of $r_1 > 0$, $r_{n+1} > 0$ and $D_k > 0$. To prove the existence of interval HWMD, we need to validate $\underline{\tau}_i \geq \bar{\tau}_{i+1}$, $i = 1, 2, \cdots, n-1$.

With (4) in mind, for $i = 1, 2, \cdots, n-2$, we have

$$\begin{aligned} \underline{\tau}_i - \bar{\tau}_{i+1} &= \min\{p_i r_{i+1} - r_i\} - \max\{p_{i+1}r_{i+2} - r_{i+1}\} \\ &= (a_i + 1)r_{i+1} - r_i - b_{i+1}r_{i+2}. \end{aligned} \tag{5}$$

Submitting

$$\begin{aligned} r_i &= \frac{D_{n-i}}{D_{n-i-1}}r_{i+1} - \frac{b_{i+1}\cdots b_n}{D_{n-i-1}}, \\ r_{i+2} &= \frac{1}{D_{n-i-1}}(D_{n-i-2}r_{i+1} + b_{i+2}\cdots b_n), \end{aligned}$$

into (5) results in

$$\begin{aligned}\underline{\tau}_i - \bar{\tau}_{i+1} &= (a_i+1)r_{i+1} - \frac{D_{n-i}}{D_{n-i-1}}r_{i+1} - \frac{b_{i+1}\cdots b_n}{D_{n-i-1}} \\ &\quad - b_{i+1}\frac{1}{D_{n-i-1}}(D_{n-i-2}r_{i+1} + b_{i+2}\cdots b_n) \\ &= \frac{1}{D_{n-i-1}}\Big\{\Big((a_i+1)D_{n-i-1} - b_{i+1}D_{n-i-2}\Big)r_{i+1} \\ &\quad - D_{n-i}r_{i+1}\Big\} \\ &= \frac{1}{D_{n-i-1}}(D_{n-i}r_{i+1} - D_{n-i}r_{i+1}) \\ &= 0. \end{aligned} \tag{6}$$

For $i = n-1$, with definitions of D_0 and D_1 in mind, we have

$$\begin{aligned}\underline{\tau}_{n-1} - \bar{\tau}_n &= \min\{p_{n-1}r_n - r_{n-1}\} - (r_{n+1} - r_n) \\ &= (a_{n-1}+1)r_n - r_{n-1} - 1 \\ &= (a_{n-1}+1)\frac{1}{D_1}(D_0 r_{n-1} + 1) - r_{n-1} - 1 \\ &= 0. \end{aligned} \tag{7}$$

Therefore, under the homogeneous weights r_i defined by (4), the system (1) satisfies interval HWMD $\tau_i \in [\underline{\tau}_i, \bar{\tau}_i]$, where $\underline{\tau}_i := a_i r_{i+1} - r_i$, $\bar{\tau}_i := b_i r_{i+1} - r_i$ for all $i = 1, 2, \cdots, n$. ■

Remark 1: The Lemma 1 also shows one way to find homogeneous weights in case of precise powers. For instance, when precisely knowing $p_i \geq 1$ and initializing $r_{n+1} = \frac{1}{p_1 \cdots p_n}$, we can recursively get

$$r_1 = 1, r_2 = \frac{1}{p_1}, r_3 = \frac{1}{p_1 p_2}, \cdots, r_n = \frac{1}{p_1 \cdots p_{n-1}}$$

and $\tau_i = 0$. In another case when $p_i = 1$ with initializing $r_{n+1} = \frac{1}{2n+1}$, we have

$$r_1 = 1, r_2 = 1 + \tau, \cdots, r_n = 1 + (n-1)\tau$$

and $\tau_i = \tau := -\frac{2}{2n+1}$. Both cases are the same as those done in [7] and [1]. With this viewpoint, Lemma 1 presents a general way to explore homogeneous weights. ■

In the rest of this section, we list the following four inequality lemmas to be used frequently throughout the paper, and their proofs can be found in the literature [6], [14].

Lemma 2: [6] For a ratio of positive odd integers $p \geq 1$, the following inequality holds for any $x, y \in \mathbb{R}$:

$$|x+y|^p \leq 2^{p-1}|x^p + y^p|, \tag{8}$$

$$|x^{1/p} - y^{1/p}| \leq 2^{1-1/p}|x-y|^{1/p}. \tag{9}$$

Lemma 3: [6] Let c, d be positive constants. Then, for any real-valued function $\eta(x, y) > 0$, the following inequality holds:

$$|x|^c|y|^d \leq \frac{c}{c+d}\eta(x,y)|x|^{c+d} + \frac{d}{c+d}\eta^{-\frac{c}{d}}(x,y)|y|^{c+d}. \tag{10}$$

Lemma 4: [14] For $x, y \in \mathbb{R}$ and a ratio of positive odd integers $p \geq 1$, the following inequalities hold:

$$|x^p - y^p| \leq p|x-y|(|x|^{p-1} + |y|^{p-1}). \tag{11}$$

Lemma 5: [14] For any $x_i \in \mathbb{R}$, $i = 1, \cdots, n$, and positive real number p,

$$(\sum_{i=1}^{n}|x_i|)^p \leq \max\{n^{p-1}, 1\}\sum_{i=1}^{n}|x_i|^p. \tag{12}$$

III. MAIN RESULTS

In this section, we stabilize system (1) under following Assumptions 1 and 2 when its power integrators are intervals, i.e., $p_i \in [a_i, b_i]$, $b_i \geq a_i > 0$, $i = 1, 2, \cdots, n-1$ and $p_n := 1$. Notice that the power integrators can not be guaranteed to be ratios of odd integers. With the help of a power sign function $[\cdot]^\alpha = \mathrm{sign}(\cdot)|\cdot|^\alpha$, system (1) can be reformulated as

$$\begin{aligned}\dot{x}_i &= [x_{i+1}]^{p_i} + \phi_i(x_1, \cdots, x_i), \\ \dot{x}_n &= u + \phi_n(x), \quad i = 1, \cdots, n-1. \end{aligned} \tag{13}$$

Assumption 1: The unknown power integrators of system (1) or (13) satisfy $a_i \leq p_i \leq b_i$ such that $D_k > 0$ for $i = 1, 2, \cdots, n-1$, $k = 0, 1, \cdots, n-1$.

Assumption 2: For $i = 1, 2, \cdots, n$, if unknown functions $\phi_i(\cdot) \neq 0$, the following inequalities

$$b_i r_{i+1} \leq r_j, \quad j = 1, 2, \cdots, i \tag{14}$$

hold for the homogeneous weights r_i defined by Lemma 1.

Remark 2: Assumption 1 imposes some restrictions on the uncertain power integrators to guarantee the possible existence of interval HWMD. In other words, the uncertain power integrators should drift in appropriate intervals around a power. This makes sense because it is impossible to globally stabilize system (13) when its power integrators drift in arbitrary width of intervals. In contrast, Assumption 2 plays two roles. On the one hand, it imposes an additional restriction on selecting more appropriate homogeneous weights r_i. On the other hand, Assumption 2 states in an implicit way that the triangular unknown C^1 functions should satisfy a state-dependent homogeneous growth condition with interval HWMD τ_i in order to achieve global stabilization, which is explained as follows: with $r_i + \tau_i = p_i r_{i+1} \leq b_i r_{i+1} \leq r_j$, $j = 1, 2, \cdots, i$, we have

$$\begin{aligned}|\phi_i(\cdot)| &\leq \eta_i(x_1, \cdots, x_i)(|x_1| + |x_2| + \cdots + |x_i|) \\ &\leq \tilde{\eta}_i(x_1, \cdots, x_i)(|x_1|^{\frac{b_i r_{i+1}}{r_1}} + \cdots + |x_i|^{\frac{b_i r_{i+1}}{r_i}}) \\ &\leq \bar{\eta}_i(x_1, \cdots, x_i)(|x_1|^{\frac{r_i+\tau_i}{r_1}} + \cdots + |x_i|^{\frac{r_i+\tau_i}{r_i}}) \end{aligned} \tag{15}$$

for C^0 functions $\eta_i(\cdot)$ and positive smooth functions $\tilde{\eta}_i(\cdot)$ and $\bar{\eta}_i(\cdot)$. Note that we cannot present the growth condition (15) directly, like done in existing literature, because we do not know the interval HWMD τ_i in advance. ■

The following example is presented to understand Assumptions 1 and 2 in a more explicit manner.

Example 1: By taking the system (2) when p_1 drifts in interval $[\frac{13}{5}, \frac{17}{5}]$ as an example, we have $D_0 = 1, D_1 = a_1 + 1 = \frac{18}{5} > 0$, which satisfies Assumption 1. According to Lemma 1, we have $r_2 = \frac{5}{18}(r_1+1), r_3 = 1$. There are a lot of choices for r_1. We can choose $r_1 = 17$ and thus get $r_2 = 5$ and interval HWMD $\tau_1 \in [-4, 0]$, $\tau_2 = -4$. In this way, Assumption 2 is satisfied because of $b_1 r_2 = \frac{17}{5} \times 5 \leq r_1$ and $b_2 r_3 = 1 \leq r_j, j = 1, 2$. Furthermore, the function $\phi_1 = x_1$ is C^1 and implies $|\phi_1| \leq \bar{\eta}_1(x_1)|x_1|^{\frac{17+\tau_1}{17}}$ with positive smooth function $\bar{\eta}_1(x_1) = 1 + x_1^2$. ■

With above preliminaries, we are now ready to construct a Lyapunov function with unknown parameters and a state-dependent feedback controller by revamping the tool of adding a power integrator. We have the following main theorem.

Theorem 1: Under Assumptions 1 and 2, the system (1) can be globally asymptotically stabilized by the following state feedback controller

$$u = -\beta_n(\cdot)\Big([x_n]^{\frac{\sigma}{r_n}} + \beta_{n-1}^{\frac{\sigma}{r_n}}(\cdot)\Big([x_{n-1}]^{\frac{\sigma}{r_{n-1}}} + \cdots + \beta_2^{\frac{\sigma}{r_3}}(\cdot)\Big([x_2]^{\frac{\sigma}{r_2}} + \beta_1^{\frac{\sigma}{r_2}}(\cdot)[x_1]^{\frac{\sigma}{r_1}}\Big)\cdots\Big)\Big)^{\frac{r_{n+1}}{\sigma}}, \quad (16)$$

where homogeneous weights r_i are defined recursively using Lemma 1; σ is a constant such that $\sigma \geq \max_{1\leq i\leq n}\{r_i\}$, and gains $\beta_i(x_1, \cdots, x_i)$ are positive smooth functions to be determined.

Proof: The proof consists of the following four parts.

Initial Step. Select one constant

$$\rho \geq \max_{1\leq i\leq n}\{r_i + \bar{\tau}_i, \sigma\},$$

where $\bar{\tau}_i$ are upper bounds of interval HWMD.

Construct

$$V_1(x_1) = \int_0^{x_1} \left([s]^{\frac{\sigma}{r_1}} - 0\right)^{\frac{2\rho - r_1 - \tau_1}{\sigma}} ds. \quad (17)$$

The derivative of V_1 along the trajectory of system (1) is

$$\begin{aligned}\dot{V}_1 &= [\xi_1]^{\frac{2\rho-r_1-\tau_1}{\sigma}}\dot{x}_1 \\ &= [\xi_1]^{\frac{2\rho-r_1-\tau_1}{\sigma}}([x_2^*]^{p_i} + \phi_1) + [\xi_1]^{\frac{2\rho-r_1-\tau_1}{\sigma}}\left([x_2]^{p_i} - [x_2^*]^{p_i}\right)\end{aligned} \quad (18)$$

with $\xi_1 = [x_1]^{\frac{\sigma}{r_1}}$.

With $[\xi_1]^{\frac{2\rho-r_1-\tau_1}{\sigma}}\phi_1 \leq |\xi_1|^{\frac{2\rho-r_1-\tau_1}{\sigma}}\bar{\eta}_1(x_1)|\xi_1|^{\frac{r_1+\tau_1}{\sigma}} := \gamma_1(x_1)|\xi_1|^{\frac{2\rho}{\sigma}}$ in mind and give a nonnegative smooth function $\alpha_1(x_1)$, the virtual controller x_2^* defined by

$$x_2^* := -\beta_1(x_1)[\xi_1]^{\frac{r_2}{\sigma}} \quad (19)$$

with $\beta_1(x_1) = (n + \alpha_1(x_1) + \gamma_1(x_1))^{\frac{1}{a_1}}$. With $r_1 + \tau_1 = p_1 r_2$ in mind, we have

$$\dot{V}_1 \leq -(n + \alpha_1(x_1))\xi_1^{\frac{2\rho}{\sigma}} + [\xi_1]^{\frac{2\rho-r_1-\tau_1}{\sigma}}\left([x_2]^{p_1} - [x_2^*]^{p_1}\right). \quad (20)$$

Inductive Step. Suppose at the $(k-1)th$ step for $k < n$, there exists a C^1 Lyapunov function $V_{k-1} : \mathbb{R}^{k-1} \to \mathbb{R}$, which is positive definite and proper, and a set of C^0 virtual controllers $x_1^*, x_2^*, \cdots, x_k^*$, defined by

$$\begin{aligned} x_1^* &:= 0, \quad \xi_1 = [x_1]^{\frac{\sigma}{r_1}} - [x_1^*]^{\frac{\sigma}{r_1}}, \\ x_2^* &:= -\beta_1(\bar{x}_1)[\xi_1]^{\frac{r_2}{\sigma}}, \quad \xi_2 = [x_2]^{\frac{\sigma}{r_2}} - [x_2^*]^{\frac{\sigma}{r_2}}, \\ &\vdots \quad \vdots \\ x_k^* &:= -\beta_{k-1}(\bar{x}_{k-1})[\xi_{k-1}]^{\frac{r_k}{\sigma}}, \quad \xi_k = [x_k]^{\frac{\sigma}{r_k}} - [x_k^*]^{\frac{\sigma}{r_k}}, \end{aligned} \quad (21)$$

with $\bar{x}_i := (x_1, \cdots, x_i)$ and smooth functions $\alpha_i(\bar{x}_i) \geq 0$ and $\beta_i(\bar{x}_i) > 0, i = 1, \cdots, k-1$, such that

$$\dot{V}_{k-1} \leq -\sum_{i=1}^{k-1}(n - k + 2 + \alpha_i(\bar{x}_i))\xi_i^{\frac{2\rho}{\sigma}} + |\xi_{k-1}|^{\frac{2\rho-r_{k-1}-\tau_{k-1}}{\sigma}}\left|[x_k]^{p_{k-1}} - [x_k^*]^{p_{k-1}}\right|. \quad (22)$$

It is evident that (22) reduces to (52) when $k = 2$ under the definitions of (21). In what follows, we show (22) can also be achieved at the kth step ($k = 2, \cdots, n-1$). To prove this, we define $V_k : \mathbb{R}^k \to \mathbb{R}$, as follow

$$V_k(\bar{x}_k) = V_{k-1}(\bar{x}_{k-1}) + W_k(\bar{x}_k), \quad (23)$$

with a C^1 Lyapunov function [7]

$$W_k = \int_{x_k^*}^{x_k}\left[[s]^{\frac{\sigma}{r_k}} - [x_k^*]^{\frac{\sigma}{r_k}}\right]^{\frac{2\rho-r_k-\tau_k}{\sigma}} ds. \quad (24)$$

The derivative of V_k along system (1) is

$$\begin{aligned}\dot{V}_k &= \dot{V}_{k-1} + \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}\dot{x}_i + [\xi_k]^{\frac{2\rho-r_k-\tau_k}{\sigma}}\dot{x}_k \\ &\leq -\sum_{i=1}^{k-1}(n-k+2+\alpha_i(\bar{x}_i))\xi_i^{\frac{2\rho}{\sigma}} \\ &\quad + |\xi_{k-1}|^{\frac{2\rho-r_{k-1}-\tau_{k-1}}{\sigma}}\left|[x_k]^{p_{k-1}} - [x_k^*]^{p_{k-1}}\right| \\ &\quad + [\xi_k]^{\frac{2\rho-r_k-\tau_k}{\sigma}}[x_{k+1}^*]^{p_k} \\ &\quad + |\xi_k|^{\frac{2\rho-r_k-\tau_k}{\sigma}}\left|[x_{k+1}]^{p_k} - [x_{k+1}^*]^{p_k}\right| \\ &\quad + \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}[x_{i+1}]^{p_i} + \sum_{i=1}^{k}\frac{\partial W_k}{\partial x_i}\phi_i \end{aligned} \quad (25)$$

for a virtual controller x_{k+1}^* to be determined later.

According to Propositions 1 and 3 in Appendix, the three terms in the right hand of (25) can be estimated respectively as

$$|\xi_{k-1}|^{\frac{2\rho-r_{k-1}-\tau_{k-1}}{\sigma}}\left|[x_k]^{p_{k-1}} - [x_k^*]^{p_{k-1}}\right| \leq \frac{1}{4}\xi_{k-1}^{\frac{2\rho}{\sigma}} + c_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}}, \quad (26)$$

$$\left|\sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}[x_{i+1}]^{p_i}\right| \leq \frac{1}{2}\sum_{i=1}^{k-2}\xi_i^{\frac{2\rho}{\sigma}} + \frac{1}{4}\xi_{k-1}^{\frac{2\rho}{\sigma}} + d_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}}, \quad (27)$$

$$\left|\sum_{i=1}^{k}\frac{\partial W_k}{\partial x_i}\phi_i\right| \leq \frac{1}{2}\sum_{i=1}^{k-1}\xi_i^{\frac{2\rho}{\sigma}} + \gamma_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}} \quad (28)$$

with three positive C^{∞} functions $c_k(\bar{x}_k)$, $d_k(\bar{x}_k)$ and $\gamma_k(\bar{x}_k)$. Substituting (26), (27) and (28) into (25) yields

$$\begin{aligned}\dot{V}_k \leq& -\sum_{i=1}^{k-1}(n-k+1+\alpha_i(\bar{x}_i))\,\xi_i^{\frac{2\rho}{\sigma}}\\&+(c_k(\bar{x}_k)+d_k(\bar{x}_k)+\gamma_k(\bar{x}_k))\xi_k^{\frac{2\rho}{\sigma}}+[\xi_k]^{\frac{2\rho-r_k-\tau_k}{\sigma}}[x_{k+1}^*]^{p_k}\\&+|\xi_k|^{\frac{2\rho-r_k-\tau_k}{\sigma}}\left|[x_{k+1}]^{p_k}-[x_{k+1}^*]^{p_k}\right|. \qquad (29)\end{aligned}$$

Define

$$\begin{aligned}\beta_k(\bar{x}_k)=\Big[&n-k+1+\alpha_k(\bar{x}_k)+c_k(\bar{x}_k)\\&+d_k(\bar{x}_k)+\gamma_k(\bar{x}_k)\Big]^{1/a_k} \qquad (30)\end{aligned}$$

with a nonnegative smooth function $\alpha_k(\bar{x}_k)$. Under the following virtual controller

$$x_{k+1}^* := -\beta_k\,(\bar{x}_k)\,[\xi_k]^{\frac{r_{k+1}}{\sigma}}, \qquad (31)$$

we have

$$\begin{aligned}&[\xi_k]^{\frac{2\rho-r_k-\tau_k}{\sigma}}[x_{k+1}^*]^{p_k}\\&\leq -\Big(n-k+1+\alpha_k(\bar{x}_k)+c_k(\bar{x}_k)+d_k(\bar{x}_k)\\&\quad+\gamma_k(\bar{x}_k)\Big)^{p_k/a_k}\xi_k^{\frac{2\rho}{\sigma}}\\&\leq -\Big(n-k+1+\alpha_k(\bar{x}_k)+c_k(\bar{x}_k)+d_k(\bar{x}_k)\\&\quad+\gamma_k(\bar{x}_k)\Big)\xi_k^{\frac{2\rho}{\sigma}}. \qquad (32)\end{aligned}$$

Substituting (32) into (29) arrives at

$$\begin{aligned}\dot{V}_k \leq& -\sum_{i=1}^{k}(n-k+1+\alpha_i(\bar{x}_i))\,\xi_i^{\frac{2\rho}{\sigma}}\\&+|\xi_k|^{\frac{2\rho-r_k-\tau_k}{\sigma}}\left|[x_{k+1}]^{p_k}-[x_{k+1}^*]^{p_k}\right|. \qquad (33)\end{aligned}$$

This completes the inductive proof.

The inductive argument reveals that (22) holds for $k=n+1$ with a set of virtual controllers (21). Thus, choosing the final virtual controller at the last step

$$u=x_{n+1}=x_{n+1}^* := -\beta_n(\bar{x}_n)[\xi_n]^{\frac{r_{n+1}}{\sigma}} \qquad (34)$$

with the positive smooth function $\beta_n(\cdot) = 1+\alpha_n(\bar{x}_n)+c_n(\bar{x}_n)+d_n(\bar{x}_n)+\gamma_n(\bar{x}_n)$ with a nonnegative smooth function $\alpha_n(\bar{x}_n)$, yields

$$\dot{V}_n \leq -\sum_{i=1}^{n}(\alpha_i(\bar{x}_i)+1)\,\xi_i^{\frac{2\rho}{\sigma}} \leq -\sum_{i=1}^{n}\xi_i^{\frac{2\rho}{\sigma}}. \qquad (35)$$

It can be seen that $\dot{V}_n < 0, \forall x \neq 0$ under virtual controllers (21), and V_n of the form (23) is positive definite and proper. Thus, the closed-loop system (1)-(16) is globally asymptotically stable. ■

From above proof, we can see that the interval HWMD provides us two new insights on the basic construction of the controller. On the one hand, under the given homogeneous weights, the controller (16) is homogeneous in the new variables ξ_i defined by (21) but not in the original states due to the presence of nonlinear functions $\beta_i(\cdot)$. On the other hand, the homogeneous weights give us a guidance to choose appropriate Lyapunov function candidates with unknown parameters (i.e., τ_i). The new design procedure in Theorems 1 is more advanced than the generalized adding a power integrator method presented in [6] and can now effectively handle the p-normal form with unknown powers drifting in more general form of interval bounds rather than those around unit one in [11]. Furthermore, due to the application of adding a power integrator method, the gain functions $\beta_i(\cdot)$ are state-dependent and thus the constructed controller in this paper is state-dependent homogeneous domination controller.

Remark 3: In practice, functions $\phi_i(\cdot)$ are usually known when modelling. In this precise case, we do not need the Assumption 2, and just redefine the virtual controller at the $k-th$ step in Theorem 1

$$x_{k+1}^* := -\beta_k\,(\bar{x}_k)\,[\xi_k]^{\frac{r_{k+1}}{\sigma}} - \phi_i(x_1,\cdots,x_k). \qquad (36)$$

with gain $\beta_k(\bar{x}_k) = [n-k+1+\alpha_k(\bar{x}_k)+c_k(\bar{x}_k)+d_k(\bar{x}_k)]^{1/a_k}$. ■

Remark 4: The performance of the controller (16) depends on gains $\beta_i(\bar{x}_i)$. It can be seen from the Theorem 1 that the construction of β_i is inseparable from uncertain power integrators and unknown functions ϕ_i. When constructing β_i, we should always remember that β_i is used to dominate nonlinearities of states $(x_1,\cdots,x_i)$. For instance, in inequalities (27)-(28), the components $d_i(\bar{x}_i)$, $\gamma_i(\bar{x}_i)$ in β_i are designed to dominate nonlinearities caused by powers p_i and functions ϕ_i respectively. Nevertheless, we can see from Theorem 1 that lots of tedious partial derivative calculations and gains should be calculated. From the theoretical point of view, Theorem 1 provides a tool to stabilize the p-normal form system with arbitrary order. However, when the system order is greater than three, it is really too heavy and tedious to compute the gains β_i. In this way, we will consider two examples in which system order is less than four to illustrate the application of Theorem 1 as well as how to design β_i in an explicit way. ■

IV. SIMULATIONS

In this section, two examples were used to illustrate how to apply Theorem 1 to design state feedback stabilizer for the p–normal form system with interval power integrators.

Example 2: Consider the following third order system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \mathrm{sign}(x_3)|x_3|^{p_2} + \frac{1}{4}\sin(x_1), \; \dot{x}_3 = u \qquad (37)$$

with $p_2 \in [a_2, b_2] = [\frac{13}{5}, 3]$, $p_1 \in [a_1, b_1] = [1, 1]$, $p_3 \in [a_3, b_3] = [1, 1]$. This system is a simplified one from the benchmark nonlinear system in [10] by replacing $p_2 = 3$ with $p_2 \in [\frac{13}{5}, 3]$.

According to Definition 3, we get $D_0 = 1$, $D_1 = a_2+1 = \frac{18}{5}$, $D_2 = (a_1+1)D_1 - b_2D_0 = \frac{21}{5}$. Hence, (37) satisfies Assumptions 1. With the help of Lemma 1 and initializing $r_4 = 1$ and $r_1 = 5$, we have

$$r_2 = \frac{1}{D_2}(D_1 r_1 + b_2 b_3 r_4) = 5,$$
$$r_3 = \frac{1}{D_1}(D_0 r_2 + b_3 r_4) = \frac{5}{3},$$

$\tau_1 \in [0, 0]$, $\tau_2 \in [-\frac{2}{3}, 0]$, $\tau_3 = -\frac{2}{3}$ and $\rho = \sigma = 5$. Meanwhile, $|\phi_2(x_1)| = \frac{1}{4}|\sin(x_1)| \le \frac{1}{4}|x_1|$ and $b_2 r_3 \le 3 \times \frac{5}{3} \le r_j, j = 1, 2$. It indicates that Assumption 2 can be guaranteed.

Define $V_1 = \int_0^{x_1}\left([s]^{\frac{\sigma}{r_1}} - 0\right) ds = \frac{1}{2}x_1^2$. $\dot V_1$ is

$$\dot V_1 = \xi_1 x_2 = \xi_1 x_2^* + \xi_1(x_2 - x_2^*). \quad (38)$$

Constructing virtual controller $x_2^* = -\beta_1(\cdot)\xi_1^{\frac{r_2}{\sigma}} = -\xi_1$, (38) becomes

$$\dot V_1 \le -\xi_1^2 + \xi_1(x_2 - x_2^*) \le -\xi_1^2 + \frac{1}{8}\xi_1^2 + 2\xi_2^2. \quad (39)$$

Defining $V_2 = V_1 + W_2$ with $W_2 = \int_{x_2^*}^{x_2}(s - x_2^*)^{1-\frac{\tau_2}{5}} ds$ and with (39) in mind, we have

$$\begin{aligned}\dot V_2 &= \dot V_1 + \frac{\partial W_2}{\partial x_1}\dot x_1 + \xi_2^{1-\frac{\tau_2}{5}}\dot x_2 \\ &\le -\frac{7}{8}\xi_1^2 + 2\xi_2^2 + \left|\frac{\partial W_2}{\partial x_1}\dot x_1\right| + \xi_2^{1-\frac{\tau_2}{5}}\dot x_2 \end{aligned} \quad (40)$$

in which

$$\begin{aligned}\left|\frac{\partial W_2}{\partial x_1}\dot x_1\right| &\le (1 - \frac{\tau_2}{5})|x_2 - x_2^*||\xi_2|^{-\frac{\tau_2}{5}}\left|\frac{\partial [x_2^*]}{\partial x_1}\dot x_1\right| \\ &\le c_1(\bar x_2)\xi_2^2 + \frac{1}{8}\xi_1^2(\bar x_2)\xi_2^2, \end{aligned} \quad (41)$$

where $c_1(\bar x_2) = 2(1 + \xi_2^2)$, $c_2(\bar x_2) = 8(1 + \xi_1^2 + \xi_2^2)$ and the last inequality is got by Lemma 3.

Noticing that $\frac{1}{4}\xi_2^{1-\frac{\tau_2}{5}}\sin(x_1) \le \frac{1}{4}|\xi_2|^{1-\frac{\tau_2}{5}}|\xi_1| \le \frac{1}{8}\xi_1^2 + c_3(\bar x_2)\xi_2^2$ with $c_3(\bar x_2) = 1 + \xi_1^2 + \xi_2^2$. Hence, $\dot V_2$ in (40) can be simplified as

$$\begin{aligned}\dot V_2 \le& -\frac{7}{8}\xi_1^2 + 2\xi_2^2 + \frac{1}{8}\xi_1^2 + c_1(\bar x_2)\xi_2^2 + c_2(\bar x_2)\xi_2^2 \\ &+ \frac{1}{4}\xi_2^{1-\frac{\tau_2}{5}}|x_1| + \xi_2^{1-\frac{\tau_2}{5}}([x_3]^{p_2} - [x_3^*]^{p_2}) + \xi_2^{1-\frac{\tau_2}{5}}[x_3^*]^{p_2} \\ \le& -\frac{5}{8}\xi_1^2 + (2 + c_1(\bar x_2) + c_2(\bar x_2) + c_3(\bar x_2))\xi_2^2 \\ &+ \xi_2^{1-\frac{\tau_2}{5}}([x_3]^{p_2} - [x_3^*]^{p_2}) + \xi_2^{1-\frac{\tau_2}{5}}[x_3^*]^{p_2}. \end{aligned} \quad (42)$$

Construct a virtual controller

$$x_3^* = -\beta_2(\cdot)[\xi_2]^{\frac{r_3}{\sigma}} = -\beta_2(\cdot)[\xi_2]^{\frac{1}{3}} \quad (43)$$

with $\beta_2(\cdot) = (2\frac{5}{8} + c_1(\bar x_2) + c_2(\bar x_2) + c_3(\bar x_2))^{\frac{5}{13}}$. With $p_2 r_3 = r_2 + \tau_2 = 5 + \tau_2$ in mind, $\dot V_2$ in (42) becomes

$$\dot V_2 \le -\frac{5}{8}\xi_1^2 - \frac{5}{8}\xi_2^2 + \xi_2^{1-\frac{\tau_2}{5}}([x_3]^{p_2} - [x_3^*]^{p_2}). \quad (44)$$

Define $V_3 = V_2 + W_3$ with $W_3 = \int_{x_3^*}^{x_3}([s]^3 - [x_3^*]^3)^{\frac{9}{5}} ds$. The derivative of V_3 is

$$\begin{aligned}V_3 &= \dot V_2 + \sum_{i=1}^{2}\frac{\partial W_3}{\partial x_i}\dot x_i + \xi_3^{\frac{9}{5}}\dot x_3 \\ &\le -\frac{5}{8}\xi_1^2 - \frac{5}{8}\xi_2^2 + \xi_2^{1-\frac{\tau_2}{5}}([x_3]^{p_2} - [x_3^*]^{p_2}) \\ &\quad + \sum_{i=1}^{2}\left|\frac{\partial W_3}{\partial x_i}\dot x_i\right| + \xi_3^{\frac{9}{5}}\dot x_3, \end{aligned} \quad (45)$$

in which the two terms in the right hand can be estimated respectively as

$$\begin{aligned}\xi_2^{1-\frac{\tau_2}{5}}([x_3]^{p_2} - [x_3^*]^{p_2}) &\le 2^{1-\frac{p_2}{3}}|\xi_2|^{1-\frac{\tau_2}{5}}|\xi_3|^{1+\frac{\tau_2}{5}} \\ &\le \frac{1}{16}\xi_2^2 + 9\xi_3^2, \end{aligned} \quad (46)$$

$$\begin{aligned}\sum_{i=1}^{2}\left|\frac{\partial W_3}{\partial x_i}\dot x_i\right| &\le \frac{9}{5}|x_3 - x_3^*||\xi_3|^{\frac{4}{5}}\sum_{i=1}^{2}\left|\frac{\partial [x_3^*]^3}{\partial x_i}\dot x_i\right| \\ &\le 3h(\bar x_2)|\xi_3|^{\frac{17}{15}}\Big(|\xi_2| + \frac{5}{4}|\xi_1| + |\xi_3|^{1+\frac{\tau_2}{17}} \\ &\quad + \beta_2^{1+\frac{\tau_2}{5}}(\cdot)|\xi_2|^{1+\frac{\tau_2}{5}}\Big) \\ &\le \frac{1}{8}\xi_1^2 + \frac{1}{16}\xi_2^2 + \sum_{k=4}^{7}c_k(\cdot)\xi_3^2, \end{aligned} \quad (47)$$

where $h(\bar x_2) := \frac{15}{13}\left(13\frac{5}{8} + 9\xi_1^2 + 11\xi_2^2\right)^{\frac{2}{13}}(9\xi_1^2 + 53\xi_2^2) + 2\left(13\frac{5}{8} + 9\xi_1^2 + 11\xi_2^2\right)^{\frac{15}{13}}$, $c_4(\bar x_3) = 46h^{\frac{32}{17}}(\bar x_3)(1 + \xi_2^2 + \xi_3^2)$, $c_5(\bar x_3) = 21h^{\frac{32}{17}}(\bar x_3)(1 + \xi_1^2 + \xi_3^2)$, $c_6(\bar x_3) = 29(1 + h^2(\bar x_3))(1 + \beta_2^2(\cdot))(1 + \xi_2^2 + \xi_3^2)$, and $c_7(\bar x_3) := 3\,h(\bar x_2)(1 + \xi_3^2)$.

In this way, (45) can be simplified as

$$\dot V_3 \le -\frac{1}{2}\xi_1^2 - \frac{1}{2}\xi_2^2 + \Big(9 + \sum_{k=4}^{7}c_k(\cdot)\Big)\xi_3^2 + \xi_3^{\frac{9}{5}}u. \quad (48)$$

To achieve $\dot V_3 \le -\frac{1}{2}\xi_1^2 - \frac{1}{2}\xi_2^2 - \frac{1}{2}\xi_3^2$, we can finally define the controller as

$$u = x_4^* = -\beta_3(\cdot)\left(x_3^3 + \beta_2^3(\cdot)\left(x_2 + \beta_1(\cdot)x_1\right)\right)^{\frac{1}{5}} \quad (49)$$

with gains $\beta_1(\cdot) = 1$, $\beta_2(\cdot) = (13\frac{5}{8} + 9\xi_1^2 + 11\xi_2^2)^{\frac{5}{13}}$ and $\beta_3(\cdot) = 9.5 + c_4(\bar x_2) + c_5(\bar x_3) + c_6(\bar x_3) + c_7(\bar x_3)$ in which smooth functions $c_i(\cdot)$ have been defined in previous equations.

The state trajectories of x_1, x_2 and x_3 are illustrated in Fig. 1, which indicates that the states can be rendered to the origin under the proposed controller even when uncertainty p_2 drifts within interval $[\frac{13}{5}, 3]$. ∎

Example 3: This example considers the following second order system in thermal power engineering [18]

$$\begin{cases}\dot x_1 = x_2^{[\frac{3}{5}, \frac{7}{5}]}, \\ \dot x_2 = -\frac{8}{225}u - \frac{30}{225}x_2 - \frac{1}{225}x_1.\end{cases} \quad (50)$$

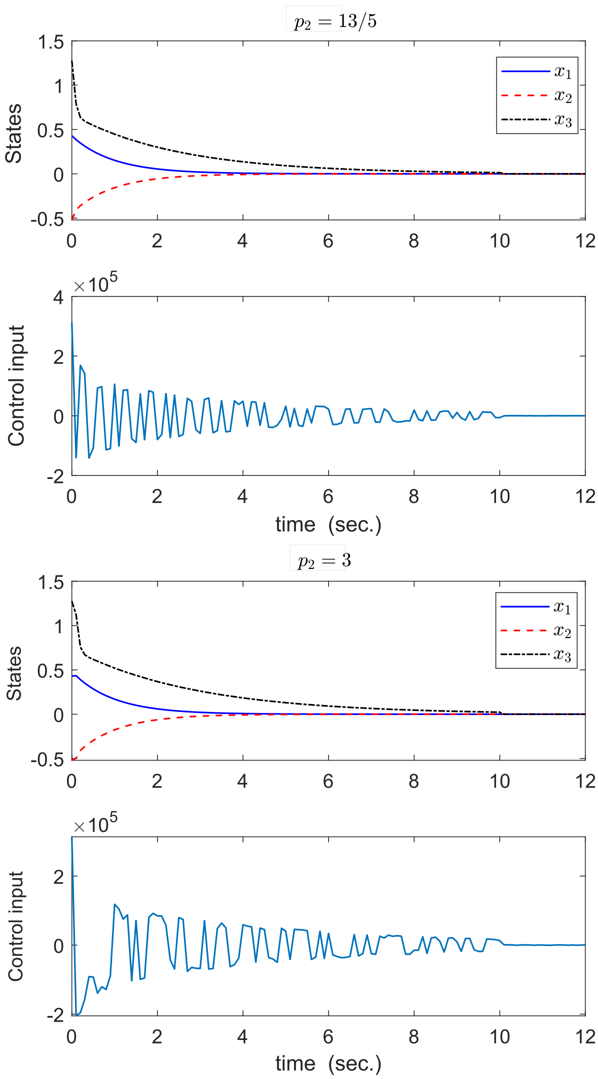

FIGURE 1. States of system (37) with initial condition. $(x_1(0), x_2(0), x_3(0)) = (0.4303, -0.5197, 1.2775)$.

Noticing that $p_2 = 1$, p_1 drifts in interval $[\frac{3}{5}, \frac{7}{5}]$ and we suppose $p_1 = 1+0.4\sin(0.1\pi t)$. According to Definition 2.3, we have $D_0 = 1$, $D_1 = \frac{8}{5}$, $b_2 = 1$. With the help of Lemma 3.1, we obtain $r_2 = 1$ by initializing $r_1 = 1$ and $r_3 = \frac{3}{5}$. In this case, $\sigma = \rho = 1$, $\tau_1 = [-\frac{2}{5}, \frac{2}{5}]$, $\tau_2 = -\frac{2}{5}$.

Define

$$V_1 = \int_0^{x_1}([s]^{\frac{\sigma}{r_1}} - 0)^{\frac{2\rho - r_1 - \tau_1}{\sigma}} ds = \int_0^{x_1}[s]^{1-\tau} ds. \quad (51)$$

We can calculate the derivative of (51) as

$$\begin{aligned}\dot{V}_1 &= [x_1]^{1-\tau_1}[x_2]^{p_1} \\ &= \xi_1^{1-\tau_1}([x_2]^{p_1} - [x_2^*]^{p_1}) + \xi_1^{1-\tau_1}[x_2^*]^{p_1}. \quad (52)\end{aligned}$$

Constructing the virtual controller $x_2^* = -\beta_1[\xi_1]$ with $\beta_1 = 0.5^{\frac{5}{3}}$, we have

$$\dot{V}_1 \le 2^{1-p_1}|\xi_2|^{p_1}|\xi_1|^{1-\tau_1} - \beta_1\xi_1^2, \quad (53)$$

where

$$\begin{aligned}2^{1-p_1}|\xi_2|^{p_1}|\xi_1|^{1-\tau_1} &\le 2^{1-p_1}\left(\frac{1-\tau_1}{2}\eta|\xi_1|^2 + \frac{p_1}{2}\eta^{-\frac{1-\tau_1}{p_1}}\xi_2^2\right) \\ &\le \frac{1}{2}\xi_1^2 + c_1\xi_2^2, \quad (54)\end{aligned}$$

in which the second inequality is induced according to Lemma 2.4 and the last inequality is obtained by selecting $\eta = \frac{1}{2}(2^{1-p_1}\frac{1-r_1}{2})^{-1}$ and $c_1 = \frac{p_1}{2}(2^{2-p_1}\frac{1-\tau_1}{2})^{\frac{1-\tau_1}{p_1}}$.

Then, the inequality (53) is rewritten as

$$\dot{V}_1 \le \frac{1}{2}\xi_1^2 + c_1\xi_2^2 - \beta_1\xi_1^2. \quad (55)$$

Next, define

$$\begin{aligned}V_2 &= V_1 + W_2, \\ W_2 &= \int_{x_2^*}^{x_2}([s] - [x_2^*])^{1-\tau_2} ds. \quad (56)\end{aligned}$$

Taking the derivative of V_2, we have

$$\dot{V}_2 = \dot{V}_1 + \dot{W}_2 = \dot{V}_1 + \frac{\partial W_2}{\partial x_1}\dot{x}_1 + \xi_2^{\frac{7}{5}}, \quad (57)$$

in which the term $\frac{\partial W_2}{\partial x_1}\dot{x}_1$ can be estimated as

$$\begin{aligned}\frac{\partial W_2}{\partial x_1}\dot{x}_1 &= \frac{7}{5}\int_{x_2^*}^{x_2}([s] - [x_2^*])^{\frac{2}{5}} ds \frac{\partial [x_2^*]}{\partial x_1}\dot{x}_1 \\ &\le \frac{7}{5}|\xi_2|^{\frac{2}{5}}|x_2 - x_2^*|\frac{\partial [x_2^*]}{\partial x_1}\dot{x}_1, \quad (58)\end{aligned}$$

with $\frac{\partial [x_2^*]}{x_1} = -\beta_1, x_2^{p_1} \le |\xi_2 - \beta_1\xi_1|^{p_1} \le |\xi_2|^{p_1} + |\beta_1|^{p_1}|\xi_1|^{p_1}$. The inequality (58) can be simplified by

$$\begin{aligned}\frac{\partial W_2}{\partial x_1}\dot{x}_1 &\le \frac{7}{5}\beta_1(|\xi_2|^{\frac{7}{5}+p_1} + \beta_1^{p_1}\xi_1^{p_1}|\xi_2|^{\frac{7}{5}}) \\ &\le \frac{7}{5}\beta_1(\xi_2^2(1+\xi_2^2)^{\frac{4}{5}} + \beta_1^{p_1}\xi_1^{p_1}|\xi_2|^{\frac{7}{5}}). \quad (59)\end{aligned}$$

Now, we estimate the rest term $\frac{7}{5}\beta_1^{1+p_1}\xi_1^{p_1}|\xi_2|^{\frac{7}{5}}$ as

$$\begin{aligned}\frac{7}{5}\beta_1^{1+p_1}\xi_1^{p_1}|\xi_2|^{\frac{7}{5}} &\le \frac{7}{5}\beta_1^{1+p_1}\Bigg(\frac{p_1}{\frac{5}{7}+p_1}\eta\xi_1^{p_1+\frac{7}{5}} \\ &\quad + \frac{\frac{5}{7}}{\frac{5}{7}+p_1}\eta^{-\frac{5p_1}{7}}\xi_2^{p_1+\frac{7}{5}}\Bigg) \\ &\le \frac{1}{2}\xi_1^2 + d_1\xi_2^2 \quad (60)\end{aligned}$$

with $d_1 = \beta_1^{p_1}\frac{\frac{7}{5}}{\frac{7}{5}+p_1}(2\frac{7}{5}\frac{p_1}{\frac{7}{5}+p_1}\beta_1^{1+p_1}(1+\xi_1^2)^{\frac{4}{5}})^{\frac{5p_1}{7}}(1+\xi_2^2)$. Then, (57) can be written as

$$\begin{aligned}\dot{V}_2 &\le \frac{1}{2}\xi_1^2 + c_1\xi_2^2 - \beta_1\xi_1^2 + \frac{7}{5}\beta_1(1+\xi_2^2)^{\frac{4}{5}}\xi_2^2 + (\frac{1}{2}\xi_1^2 + d_1\xi_2^2) \\ &\quad + \xi_2^{\frac{7}{5}} - (\frac{8}{225}u - \frac{30}{225}x_2 - \frac{1}{225}x_1). \quad (61)\end{aligned}$$

To achieve $\dot{V}_2 \le 0$, as stated in the Remark 3, we can design

$$u = \frac{225}{8}\beta_2[\xi_2]^{\frac{3}{5}} - \frac{30}{8}x_2 - \frac{1}{8}x_1 \quad (62)$$

with gains $\beta_2 = \frac{7}{5}\beta_1(1+\xi_2^2)^{\frac{4}{5}} + c_1 + d_1 + \alpha_2$ and $\alpha_2 = 0.2$.

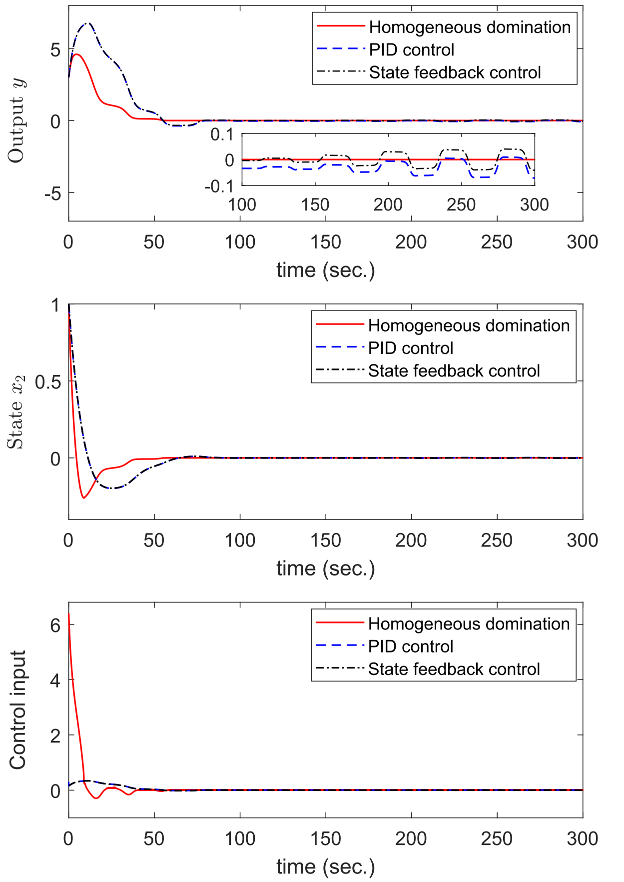

FIGURE 2. State trajectories and control input of system (50) with initial condition $(x_1(0), x_2(0)) = (3, 1)$.

To perform well, we compare our method with traditional state feedback method and PID. The gains of PID control are $k_p = 0.05$, $k_i = 1 \times 10^{-4}$, $k_d = 1.5 \times 10^{-3}$. The state feedback control law is $u = k_1x_1 + k_2x_2$ with $k_1 = 0.05$ and $k_2 = 1.5 \times 10^{-3}$. The results are shown in Fig. 2, from which we can see that the proposed homogeneous method has well robustness and can converge to zero more rapidly and precisely than other two methods, whereas the other two methods can just stabilize the output to zero with a bounded offset. ■

V. CONCLUSION

This paper develops a new tool to solve the problem of global stabilization for a class of p–normal form nonlinear systems with interval power integrators via state feedback controller. Based on the new concept of interval homogeneity with monotone degree, the new tool can be explicitly separated into two associated parts: the former aims to find the homogeneous weights using a rule that is based on a new definition of admissible index, whereas the latter focuses on recursively constructing state feedback stabilizer by revamping the adding a power integrator technique. It reveals that the proposed tool employs a new flexible Lyapunov function with unknown parameters which enables us to construct global state-dependent homogeneous domination stabilizers for the p–normal form nonlinear systems with interval power integrators and uncertain functions. Some simulations conducted on a numerical example and a practical example show the application of this new tool.

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APPENDIX A

Proposition 1: There exists a C^∞ function $c_k(\bar{x}_k) > 0$ such that

$$|\xi_{k-1}|^{\frac{2\rho-r_{k-1}-\tau_{k-1}}{\sigma}}\left|[x_k]^{p_{k-1}}-[x_k^*]^{p_{k-1}}\right| \le \frac{1}{4}\xi_{k-1}^{\frac{2\rho}{\sigma}} + c_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}}.$$

Proof: First, by the definitions (21)

$$\left|[x_i]^{p_{i-1}}-[x_i^*]^{p_{i-1}}\right| = \left|\left[[x_i]^{\frac{\sigma}{r_i}}\right]^{\frac{p_{i-1}r_i}{\sigma}} - \left[[x_i^*]^{\frac{\sigma}{r_i}}\right]^{\frac{p_{i-1}r_i}{\sigma}}\right|. \tag{A-1}$$

There are two different cases based on the value of $\frac{p_{i-1}r_i}{\sigma}$.

Case 1: When $\frac{p_{i-1}r_i}{\sigma} \le 1$, it follows from Lemma 2 that

$$\left|[x_i]^{p_{i-1}}-[x_i^*]^{p_{i-1}}\right| \le 2^{1-\frac{a_{i-1}r_i}{\sigma}}|\xi_i|^{\frac{p_{i-1}r_i}{\sigma}}.$$

By Lemma 3 and the fact that $p_{i-1}r_i = r_{i-1} + \tau_{i-1}$, we can conclude there is a positive smooth function $\hat{c}_i(\bar{x}_i)$ such that

$$[\xi_{i-1}]^{\frac{2\rho-r_{i-1}-\tau_{i-1}}{\sigma}}\left([x_i]^{p_{i-1}}-[x_i^*]^{p_{i-1}}\right) \le \frac{1}{4}\xi_{i-1}^{\frac{2\rho}{\sigma}} + \hat{c}_i(\cdot)\xi_i^{\frac{2\rho}{\sigma}}.$$

Case 2: When $\frac{p_{i-1}r_i}{\sigma} \ge 1$, by Lemma 4 it follows from (A-1) that

$$\begin{aligned}&\left|[x_i]^{p_{i-1}}-[x_i^*]^{p_{i-1}}\right| \\ &\quad \le \bar{c}_i(\bar{x}_{i-1})|\xi_i|\left(|\xi_i|^{\frac{p_{i-1}r_i}{\sigma}-1} + |\xi_{i-1}|^{\frac{p_{i-1}r_i}{\sigma}-1}\right),\end{aligned} \tag{A-2}$$

with a positive smooth function $\bar{c}_i(\cdot)$.

Applying Lemmas 3 (with $p_{i-1}r_i = r_{i-1} + \tau_{i-1}$) to the terms below leads to

$$\begin{aligned}&[\xi_{i-1}]^{\frac{2\rho-r_{i-1}-\tau_{i-1}}{\sigma}}\left([x_i]^{p_{i-1}}-[x_i^*]^{p_{i-1}}\right) \\ &\quad \le \bar{c}_i(\bar{x}_{i-1})\left(|\xi_{i-1}|^{\frac{2\rho-r_{i-1}-\tau_{i-1}}{\sigma}}|\xi_i|^{\frac{p_{i-1}r_i}{\sigma}} + |\xi_i||\xi_{i-1}|^{\frac{2\rho}{\sigma}-1}\right) \\ &\quad \le \frac{1}{4}\xi_{i-1}^{\frac{2\rho}{\sigma}} + \tilde{c}_i(\cdot)\xi_i^{\frac{2\rho}{\sigma}},\end{aligned}$$

where $\tilde{c}_i(\bar{x}_i)$ is a positive smooth function. Choosing $c_i(\bar{x}_i) = \hat{c}_i(\bar{x}_i) + \tilde{c}_i(\bar{x}_i)$ yields Proposition 1. ■

Proposition 2: There exists a C^∞ function $d_k(\bar{x}_k) > 0$ such that

$$\left|\sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}[x_{i+1}]^{p_i}\right| \le \frac{1}{2}\sum_{i=1}^{k-2}\xi_i^{\frac{2\rho}{\sigma}} + \frac{1}{4}\xi_{k-1}^{\frac{2\rho}{\sigma}} + d_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}}.$$

Proof: For simplicity, we denote by $\frac{2\rho-r_i-\tau_i}{\sigma} := q_i$ for all i. First, for $i \in [1, k-1]$, we have

$$\begin{aligned}
&\left|\frac{\partial W_k}{\partial x_i}[x_{i+1}]^{p_i}\right| \\
&= \left| q_k \int_{x_k^*}^{x_k} \left[[s]^{\frac{\sigma}{r_k}} - [x_k^*]^{\frac{\sigma}{r_k}}\right]^{q_k-1} ds \frac{\partial [x_k^*]^{\frac{\sigma}{r_k}}}{\partial x_i}[x_{i+1}]^{p_i}\right| \\
&\le \bar{a}_k \left|x_k - x_k^*\right| |\xi_k|^{q_k-1} \left|\frac{\partial [x_k^*]^{\frac{\sigma}{r_k}}}{\partial x_i}[x_{i+1}]^{p_i}\right| \\
&\le a_k |\xi_k|^{\frac{r_k}{\sigma}} |\xi_k|^{q_k-1} \left|\frac{\partial [x_k^*]^{\frac{\sigma}{r_k}}}{\partial x_i}[x_{i+1}]^{p_i}\right| \qquad \text{(A-3)}
\end{aligned}$$

with positive constants $\bar{a}_k$ and a_k.

Noting that

$$\begin{aligned}
[x_k^*]^{\frac{\sigma}{r_k}} &= -[\beta_{k-1}(\bar{x}_{k-1})[\xi_{k-1}]^{\frac{r_k}{\sigma}}]^{\frac{\sigma}{r_k}} \\
&:= -\bar{\beta}_{k-1}(\bar{x}_{k-1})\xi_{k-1}, \\
|x_{i+1}^{p_i}| &= \left|[[x_{i+1}]^{\frac{\sigma}{r_{i+1}}}]^{\frac{p_i r_{i+1}}{\sigma}}\right| \\
&\le |\xi_{i+1}|^{\frac{p_i r_{i+1}}{\sigma}} + \bar{\beta}_i^{\frac{p_i r_{i+1}}{\sigma}} |\xi_i|^{\frac{p_i r_{i+1}}{\sigma}},
\end{aligned}$$

the last term in (A-3) can be estimated as

$$\begin{aligned}
&\left|\frac{\partial [x_k^*]^{\frac{\sigma}{r_k}}}{\partial x_i}[x_{i+1}]^{p_i}\right| \\
&= \left|\left(\frac{\partial \bar{\beta}_{k-1}}{\partial x_i}\xi_{k-1} + \bar{\beta}_{k-1}\frac{\partial \xi_{k-1}}{\partial x_i}\right) x_{i+1}^{p_i}\right| \\
&\le \bar{h}_{k-1}(\bar{x}_{k-1})\Big(|x_i|^{\frac{\sigma-r_i}{r_i}} + |\xi_{k-1}|\Big)|x_{i+1}^{p_i}| \\
&\le \tilde{h}_{k-1}(\bar{x}_{k-1})\Big(|\xi_i - \bar{\beta}_{i-1}\xi_{i-1}|^{\frac{\sigma-r_i}{\sigma}} + |\xi_{k-1}|\Big) \cdot \\
&\quad \Big(|\xi_{i+1}|^{\frac{p_i r_{i+1}}{\sigma}} + |\xi_i|^{\frac{p_i r_{i+1}}{\sigma}}\Big) \\
&\le \widehat{h}_{k-1}(\bar{x}_{k-1}) \left(|\xi_{i-1}|^{\frac{\sigma-r_i}{\sigma}} + |\xi_i|^{\frac{\sigma-r_i}{\sigma}} + |\xi_{k-1}|^{\frac{\sigma-r_i}{\sigma}}\right) \\
&\quad \cdot\left(|\xi_{i+1}|^{\frac{r_i+\tau_i}{\sigma}} + |\xi_i|^{\frac{r_i+\tau_i}{\sigma}}\right) \qquad \text{(A-4)}
\end{aligned}$$

where $\bar{h}_{k-1}(\cdot), \tilde{h}_{k-1}(\cdot)$, and $\widehat{h}_{k-1}(\cdot)$ are positive smooth functions.

By applying Lemma 3 to each term in the last line of (A-4), there is a positive smooth function $h_{k-1}(\cdot)$ such that

$$\left|\frac{\partial [x_k^*]^{\frac{\sigma}{r_k}}}{\partial x_i}[x_{i+1}]^{p_i}\right| \le h_{k-1}(\bar{x}_{k-1})\Big(\sum_{l=i-1}^{i+1} |\xi_l|^{\frac{\sigma+\tau_i}{\sigma}} + |\xi_{k-1}|^{\frac{\sigma+\tau_i}{\sigma}}\Big).$$

Putting the above back to (A-3), by applying Lemmas 3 with the fact $\tau_k \le \tau_l$ for $k \ge l$, we have

$$\begin{aligned}
\sum_{i=1}^{k-1}\left|\frac{\partial W_k}{\partial x_i}[x_{i+1}]^{p_i}\right| &\le a_k h_{k-1}(\bar{x}_{k-1})|\xi_k|^{\frac{2\rho-\tau_k-\sigma}{\sigma}} \\
&\quad \cdot \sum_{i=1}^{k-1}\Big(\sum_{l=i-1}^{i+1} |\xi_l|^{\frac{\sigma+\tau_i}{\sigma}} + |\xi_{k-1}|^{\frac{\sigma+\tau_i}{\sigma}}\Big) \\
&\le \frac{1}{2}\sum_{i=1}^{k-2}\xi_i^{\frac{2\rho}{\sigma}} + \frac{1}{4}\xi_{k-1}^{\frac{2\rho}{\sigma}} + d_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}},
\end{aligned} \qquad \text{(A-5)}$$

for a smooth function $d_k(\cdot) > 0$. ∎

Proposition 3: There exists a C^∞ function $\gamma_k(\bar{x}_k) > 0$ such that

$$\left|\sum_{i=1}^{k}\frac{\partial W_k}{\partial x_i}\phi_i\right| \le \frac{1}{2}\sum_{i=1}^{k-1}\xi_i^{\frac{2\rho}{\sigma}} + \gamma_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}},$$

Proof: Substituting the coordinates (21) into (14), we can obtain the following relation

$$|\phi_i(\cdot)| \le \tilde{\eta}_i(\bar{x}_i)(|\xi_1|^{\frac{r_i+\tau_i}{\sigma}} + \cdots + |\xi_i|^{\frac{r_i+\tau_i}{\sigma}}) \qquad \text{(A-6)}$$

for a positive smooth function $\tilde{\eta}_i(\bar{x}_i)$.

With (A-6) in mind, we can clarity that

$$\begin{aligned}
\left|\frac{\partial W_k}{\partial x_i}\phi_i\right| &\le \tilde{g}_{k-1}(x_{k-1})|\xi_k|^{\frac{2\rho-\tau_k-\sigma}{\sigma}} \\
&\quad \cdot\left(\sum_{l=i-1}^{i}|\xi_l|^{\frac{\sigma-r_i}{\sigma}}\right)\tilde{\eta}_i(\bar{x}_i)\left(\sum_{j=1}^{i}|\xi_j|^{\frac{r_i+\tau_i}{\sigma}}\right) \\
&\le \hat{g}_{k-1}(x_{k-1})|\xi_k|^{\frac{2\rho-\tau_k-\sigma}{\sigma}}\sum_{j=1}^{i}|\xi_j|^{\frac{\sigma+\tau_i}{\sigma}}, \qquad \text{(A-7)}
\end{aligned}$$

with positive smooth functions $\tilde{g}_{k-1}(\cdot)$ and $\hat{g}_{k-1}(\cdot)$.

Hence, by combining Lemmas 3, we finally have

$$\begin{aligned}
\left|\sum_{i=1}^{k}\frac{\partial W_k}{\partial x_i}\phi_i\right| &\le \gamma_{0,k-1}(x_{k-1})|\xi_k|^{\frac{2\rho-\tau_k-\sigma}{\sigma}}\sum_{i=1}^{k}\sum_{j=1}^{i}|\xi_j|^{\frac{r_i+\tau_i}{\sigma}} \\
&\le \gamma_{1,k-1}(x_{k-1})|\xi_k|^{\frac{2\rho-\tau_k-\sigma}{\sigma}}\sum_{i=1}^{k}|\xi_i|^{\frac{r_i+\tau_i}{\sigma}} \\
&\le \frac{1}{2}\sum_{i=1}^{k-1}\xi_i^{\frac{2\rho}{\sigma}} + \gamma_k(\bar{x}_k)\xi_k^{\frac{2\rho}{\sigma}}, \qquad \text{(A-8)}
\end{aligned}$$

with the positive smooth function $\gamma_k(\cdot)$. ∎

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