

Received December 24, 2019; accepted February 3, 2020; date of publication March 3, 2020; date of current version March 13, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2978006

# Iterative Sequential Estimation for Multiple Structured Signals

ZHIMEI HAO<sup>1</sup>, XIANXIANG YU<sup>2</sup>, (Student Member, IEEE), NA GAN<sup>2</sup>,  
AND GUOLONG CUI<sup>1</sup>, (Senior Member, IEEE)

<sup>1</sup>School of Electronic Information Engineering, Beihang University, Beijing 100083, China

<sup>2</sup>School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China

Corresponding author: Guolong Cui (cuiguolong@uestc.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61771109 and Grant 61501083.

**ABSTRACT** In this paper, we address the signal estimation problem for a linear combination of multiple structured models, which is widely employed in the passive and/or active sensing systems to characterize the behaviors, for example, jamming and multipath propagation, in radar and communication societies. An iterative sequential estimation (ISE) algorithm is presented to obtain simultaneously the multiple structured signals. At each iteration, employing the estimated signals at the previous step, the optimal linear filters, based on mean-squared error criteria, are designed to minimize the output average power for every element of each signal. Finally, we evaluate the performance of the proposed ISE method compared with the least-square and compressed sensing algorithms via numerical simulations. The results highlight the presented algorithm shows a better signal estimation performance at low SNR and plays a trade-off between the computational complexity and the signal estimation performance.

**INDEX TERMS** Signal estimation, radar and communication, iterative sequential estimation, multiple structured signals.

## I. INTRODUCTION

Detection and estimation of the structured signals have been widely emerged in a active/passive sensing system, e.g., radar and communication systems for the unknown multipath channel in multiple-input multiple-output (MIMO) communication, blind source separation, signal identification, the profiles of the Doppler frequencies and ranges in radars, as well as the spatial angles [1]–[20]. The classic least-squares (LS) approaches were presented in [21], [22], which were optimal under the mean-square error (MSE) criteria in additive white Gaussian noise. Nevertheless, the performance of the LS approach deteriorates significantly accounting for multiple targets or interferences in radar application. The signal estimation problem can also be settled by a multitude of effective algorithms, such as the maximum likelihood (ML) [23], amplitude and phase estimation (APES) [24] and the minimum-variance distortionless response (MVDR) [25]. Nevertheless, these algorithms would suffer from the performance deterioration under a small number of snapshots, even fail to estimate the locations of the coherent signal. Recently,

several well-known sparse methods based on Compressed Sensing (CS) theory were proposed to reconstruct signal [26], showing a good recovery performance at high Signal Noise Ratio (SNR). However, for low SNR, CS algorithm may fail to recover signal.

Some adaptive algorithms, e.g., adaptive pulse compression algorithms (APC) [27] and iterative adaptive algorithm (IAA) [13], were proposed to obtain the estimation by designing independent optimal receive filters for every elements of the structured signal, which share very low sidelobe levels in the range compression and spatial angle estimation.

In summary, the aforementioned work considers only limited structured signals, and the adaptive estimation of the multiple structured signals has never been considered. In this paper, we consider the signal estimation problem for a linear combination of multiple structured models. Exploiting the framework of the IAA, an iterative sequential estimation (ISE) algorithm is presented to obtain simultaneously the multiple structured signals. At each iteration, the optimal linear filters, based on mean-squared error (MSE) criteria, are derived to minimize the output average power for every elements of each signal by exploiting the estimated signals at the previous step.

The associate editor coordinating the review of this manuscript and approving it for publication was Hasan S. Mir.

The benefits of the proposed ISE method in comparison with the LS and CS methods are demonstrated via numerical simulations.

The remainder of the paper is organized as follows. In Section II, we formalize the signal model. In Section III, we present the ISE algorithm. In Section IV, we evaluate the performance of the proposed ISE method in comparison with the LS and CS estimation. Finally, in Section V, we provide concluding remarks.

## II. SIGNAL MODEL

Denote by  $\mathbf{y} = [y(1), y(2), \dots, y(N)]^T$  the  $N \times 1$  dimensional complex vector collecting by a sensing system in the spatial and/or temporal channels. The structured signals of interest  $\mathbf{x}_m$  are the  $K \times 1$  dimensional vectors in the known subspace spanned by  $\mathbf{H}_m$ , which are the  $N \times K$  dimensional for  $m = 1, \dots, M$ , e.g.,

$$\begin{aligned} \mathbf{H}_m &= [\mathbf{h}_{m,1}, \mathbf{h}_{m,2}, \dots, \mathbf{h}_{m,K}] \\ &= \begin{bmatrix} h_m(1,1) & h_m(1,2) & \dots & h_m(1,K) \\ h_m(2,1) & h_m(2,2) & \dots & h_m(2,K) \\ \vdots & \vdots & \ddots & \vdots \\ h_m(N,1) & h_m(N,2) & \dots & h_m(N,K) \end{bmatrix}. \end{aligned} \quad (1)$$

Hence, the collected data vector  $\mathbf{y}$  can be expressed as

$$\mathbf{y} = \sum_{m=1}^M \mathbf{H}_m \mathbf{x}_m + \mathbf{v}, \quad (2)$$

where  $\mathbf{v} = [v(1), v(2), \dots, v(N)]^T$  is the  $N \times 1$  dimensional complex circular zero-mean Gaussian random vector with identity covariance matrix  $\sigma^2 \mathbf{I}$ , while  $\mathbf{x}_m = [x_m(1), x_m(2), \dots, x_m(K)]^T$  is the  $K \times 1$  dimensional deterministic and unknown vector. In particular, we here assume that signal vector  $\mathbf{x}_m$  is sparse (i.e., most of its elements are 0).

## III. ISE ALGORITHM

This section is devoted to estimating the unknown signals  $\mathbf{x}_m$  for  $m = 1, \dots, M$ . Inspecting on  $\mathbf{y}$  in (2), one could observe that there is not an analytical solution due to the coupling among the unknown signal  $\mathbf{x}_m$ . In the following, we present an iterative sequential estimation algorithm based on minimum mean-squared error criteria.

### A. ISE PROCEDURE

To obtain the MSE estimation of  $x_m(k)$ , we pass the data vector  $\mathbf{y}$  through a  $N \times 1$  dimensional FIR filter  $\mathbf{w}_m[k]$ , e.g.,

$$\hat{x}_m(k) = \mathbf{w}_m^\dagger[k] \mathbf{y}, \quad (3)$$

where  $(\cdot)^\dagger$  denotes the conjugate transpose operation. Hence, the linear filter  $\mathbf{w}_m[k]$  is the solution of the following minimization problem

$$\begin{cases} \min_{\mathbf{w}_m[k]} E[|\mathbf{w}_m^\dagger[k] \mathbf{y}|^2] \\ \text{s.t.} \quad \mathbf{w}_m^\dagger[k] \mathbf{h}_{m,k} = 1, \end{cases} \quad (4)$$

where  $\mathbf{h}_{m,k}$  denotes the  $k$ th column of the matrix  $\mathbf{H}_m$ . In addition, the object function in (4) can be derived as

$$E[|\mathbf{w}_m^\dagger[k] \mathbf{y}|^2] = \mathbf{w}_m^\dagger[k] \Gamma \mathbf{w}_m[k], \quad (5)$$

where  $\Gamma$  denotes the covariance matrix of  $\mathbf{y}$ , computed as

$$\Gamma = E[\mathbf{y} \mathbf{y}^\dagger] = \sum_{m=1}^M \mathbf{H}_m \Pi_m \mathbf{H}_m^\dagger + \sigma^2 \mathbf{I}, \quad (6)$$

where  $\Pi_m = E[\mathbf{x}_m \mathbf{x}_m^\dagger]$ , for  $m = 1, \dots, M$ , are the covariance matrices of  $\mathbf{x}_m$ . Using lagrangian multiplier method, we can construct an object function as follows

$$J(\mathbf{w}_m[k]) = \mathbf{w}_m^\dagger[k] \Gamma \mathbf{w}_m[k] + \lambda (1 - \mathbf{w}_m^\dagger[k] \mathbf{h}_{m,k}). \quad (7)$$

Then the gradient of (7) can be calculated as  $2\Gamma \mathbf{w}_m[k] - 2\lambda \mathbf{h}_{m,k}$ . Considering that the covariance matrix  $\Gamma$  is nonsingular, letting the gradient equal to 0, we can get

$$\mathbf{w}_m[k] = \lambda \Gamma^{-1} \mathbf{h}_{m,k}. \quad (8)$$

Substituting (8) into the constraint of (4), we can get the expression of  $\lambda$ ,

$$\lambda = \frac{1}{\mathbf{h}_{m,k}^\dagger \Gamma^{-1} \mathbf{h}_{m,k}}. \quad (9)$$

Substituting (9) into (8), the optimal solution to problem (4) can be derived as

$$\mathbf{w}_m[k] = \frac{\Gamma^{-1} \mathbf{h}_{m,k}}{\mathbf{h}_{m,k}^\dagger \Gamma^{-1} \mathbf{h}_{m,k}}. \quad (10)$$

Finally, substituting (10) into (3), the estimate of  $x_m(k)$  can be computed as

$$\hat{x}_m(k) = \mathbf{w}_m^\dagger[k] \mathbf{y} = \frac{\mathbf{h}_{m,k}^\dagger \Gamma^{-1} \mathbf{y}}{\mathbf{h}_{m,k}^\dagger \Gamma^{-1} \mathbf{h}_{m,k}}. \quad (11)$$

*Remark:* The right side of expression (11) depends on the correlation matrices  $\Pi_m$  of  $\mathbf{x}_m$ ,  $m = 1, \dots, M$  through  $\Gamma$  which are of course unknown and required to be estimated. In particular, the matrix  $\Pi_m$  involved in  $\Gamma$  will become the diagonal matrices when the elements of  $\mathbf{x}_m$  are zero-mean and independent random variables. To this end, in practice, we just use the diagonal matrix

$$\hat{\Pi}_m(\hat{\mathbf{x}}_m) = \text{diag}(|\hat{x}_m(1)|^2, \dots, |\hat{x}_m(K)|^2) \quad (12)$$

to approximate  $\Pi_m = E[\mathbf{x}_m \mathbf{x}_m^\dagger]$ , where  $\hat{\mathbf{x}}_m = [\hat{x}_m(1), \dots, \hat{x}_m(K)]^T$  and  $\text{diag}(\cdot)$  is a diagonal operator.

Therefore, based on (11), we can obtain

$$\hat{x}_m(k) = \mathbf{w}_m^\dagger[k] \mathbf{y} = \frac{\mathbf{h}_{m,k}^\dagger \hat{\Gamma}(\hat{\mathbf{x}}_m)^{-1} \mathbf{y}}{\mathbf{h}_{m,k}^\dagger \hat{\Gamma}(\hat{\mathbf{x}}_m)^{-1} \mathbf{h}_{m,k}}, \quad (13)$$

where

$$\hat{\Gamma}(\hat{\mathbf{x}}_m) = \sum_{m=1}^M \mathbf{H}_m \hat{\Pi}_m(\hat{\mathbf{x}}_m) \mathbf{H}_m^\dagger + \sigma^2 \mathbf{I}. \quad (14)$$

An interesting observation is that the right side of expression (13) is also related to  $\hat{x}_m(k)$  thus leading to a coupled relationship. To this end, we present an iterative sequential estimation procedure to estimate  $x_m(k)$ . More specifically, we assume  $\hat{x}_m^{(u)}(k)$  denotes the  $u$ th iteration solution of  $x_m(k)$ . For the  $u$ th iteration, we first construct the covariance matrix

$$\hat{\Gamma}(\{\hat{\mathbf{x}}_m^{(u-1)}\}) = \sum_{m=1}^M \mathbf{H}_m \hat{\Pi}_m(\hat{\mathbf{x}}_m^{(u-1)}) \mathbf{H}_m^\dagger + \sigma^2 \mathbf{I} \quad (15)$$

by using  $\hat{x}_m^{(u-1)}(k)$ , where

$$\hat{\Pi}_m(\hat{\mathbf{x}}_m^{(u-1)}) = \text{diag}(|\hat{x}_m^{(u-1)}(1)|^2, \dots, |\hat{x}_m^{(u-1)}(K)|^2), \quad (16)$$

with  $\hat{\mathbf{x}}_m^{(u-1)} = [\hat{x}_m^{(u-1)}(1), \dots, \hat{x}_m^{(u-1)}(K)]^T$ .

We then estimate  $\hat{x}_m^{(u)}(k)$  by the following expression

$$\hat{x}_m^{(u)}(k) = \frac{\mathbf{h}_{m,k}^\dagger \hat{\Gamma}(\{\hat{\mathbf{x}}_m^{(u-1)}\})^{-1} \mathbf{y}}{\mathbf{h}_{m,k}^\dagger \hat{\Gamma}(\{\hat{\mathbf{x}}_m^{(u-1)}\})^{-1} \mathbf{h}_{m,k}}. \quad (17)$$

Next, we increase  $u$  and repeat the above procedure until convergence. The proposed ISE procedure is summarized as follows.

**Algorithm 1** : The ISE Procedure for  $\mathbf{x}_m, m = 1, 2, \dots, M$

**Require:**  $\mathbf{y}, \mathbf{H}_m$ , and  $\sigma^2$

**Ensure:**  $\mathbf{x}_m^*$  for  $m = 1, 2, \dots, M$ ;

- 1: initialize  $u = 0$ , and  $\hat{\mathbf{x}}_m^{(0)} = \mathbf{0}_{K \times 1}, m = 1, \dots, M$ ;
- 2:  $u := u + 1$ ;
- 3: Compute  $\hat{\Pi}_m(\hat{\mathbf{x}}_m^{(u-1)})$  via (16);
- 4: Construct  $\hat{\Gamma}(\{\hat{\mathbf{x}}_m^{(u-1)}\})$  via (15);
- 5: Estimate  $\hat{\mathbf{x}}_m^{(u)}(k), k = 1, \dots, K, m = 1, \dots, M$  by (17);
- 6: Compute  $\rho_m^{(u)} = \|\hat{\mathbf{x}}_m^{(u)} - \hat{\mathbf{x}}_m^{(u-1)}\|^2$ , if  $\rho_m^{(u)} \leq \epsilon_m, m = 1, 2, \dots, M$ , where  $\epsilon_m$  are the user selected parameters to control convergence, output  $\mathbf{x}_m^* = \hat{\mathbf{x}}_m^{(u)}$ . Otherwise, repeat step 2 until convergence.

**B. COMPUTATIONAL COMPLEXITY AND CONVERGENCE ANALYSIS**

In each iteration, the main computational complexity is connected to the computation of  $\hat{\Gamma}(\{\hat{\mathbf{x}}_m^{(u-1)}\})$  and the estimation of  $\hat{\mathbf{x}}_m^{(u)}(k), k = 1, \dots, K, m = 1, \dots, M$ . The former requires to perform (15) with the order of  $O(MKN^2)$ . The latter needs to perform the inversion of  $\hat{\Gamma}(\{\hat{\mathbf{x}}_m^{(u-1)}\})$  in (17) with the computational complexity of  $O(N^3)$ , while executing  $MK$  times to estimate  $\hat{\mathbf{x}}_m^{(u)}(k)$  on the order of  $O(MKN^2)$ . Thus, the total computational complexity in each iteration of the proposed algorithm is  $O(MKN^2 + N^3)$ .

It is worth highlighting that the estimation of  $\hat{\mathbf{x}}_m^{(u)}(k), k = 1, \dots, K, m = 1, \dots, M$  can be performed in parallel. Besides, for most practical applications, the proposed algorithm converges with typically no more than 10 iterations [13]. Finally, it is worth pointing out that a similar local convergence analysis of ISE can be found in [28].

**IV. NUMERICAL RESULTS**

In this section, we evaluate the performance of the proposed ISE via numerical simulations. For comparison, the LS and CS methods are provided. In particular, letting  $\mathbf{x} = \text{vec}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M)$ , and  $\mathbf{H} = [\mathbf{H}_1, \dots, \mathbf{H}_M]$ , where  $\text{vec}(\mathbf{A})$  denotes the vectorization of a matrix  $\mathbf{A}$ , the expression  $\mathbf{y}$  in (2) can be recast as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}. \quad (18)$$

Hence, the LS estimation of  $\mathbf{x}$  is

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{y}. \quad (19)$$

Note that we here assume that  $\mathbf{H}_m$  are the  $N \times K$  dimensional with full column rank matrices (i.e.,  $N \geq K$ ) for  $m = 1, \dots, M$ .

Since the signal vectors  $\mathbf{x}$  is sparse, we can obtain sparse solution  $\hat{\mathbf{x}}$  of (18) by resorting to the compressed sensing theory. The “ $l_0$ -norm” based optimization corresponding to (18) can be written as

$$\begin{aligned} \min_{\mathbf{x}} \|\mathbf{x}\|_0 \\ \text{s.t. } \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2 \leq \epsilon_1, \end{aligned} \quad (20)$$

where  $\|\mathbf{x}\|_0$  represents the number of nonzero elements in the vector in  $\mathbf{x}$  and  $\epsilon_1$  is a constant that controls the error. However, (20) is a NP-hard problem which is rarely used in practical applications. Hence, we convert (20) to be a “ $l_1$ -norm” based optimization as follows [26],

$$\begin{aligned} \min_{\mathbf{x}} \|\mathbf{x}\|_1 \\ \text{s.t. } \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2 \leq \epsilon_2, \end{aligned} \quad (21)$$

where  $\epsilon_2$  is a user parameter. (21) is a convex problem that can be solved via resorting to CVX toolbox [29].

Without loss of generality, we generate  $\mathbf{H}_m$ , for  $m = 1, \dots, M$ , from the  $N \times K$  dimensional complex circular zero-mean Gaussian random matrices. In addition, we consider the sparse signal vectors  $\mathbf{x}_m$ , containing  $Q_m$  non-zero elements and  $(K - Q_m)$  zero elements. The cells of the non-zero elements for all  $\mathbf{x}_m$  are generated uniformly with the same unit amplitudes and  $Q_m = Q, m = 1, \dots, M$ . Namely, we generate  $\mathbf{x}_m^2$  as follows:

$$x_m(k) = \begin{cases} 1, & k = l_{q_m}, q = 1, \dots, Q \\ 0, & \text{otherwise,} \end{cases}$$

where  $l_{q_m} (l_{q_m} \leq K)$  denotes the index corresponding the unit amplitude for  $\mathbf{x}_m^3$  generated at random by the computer.

<sup>1</sup>For this case  $N < K$ , the LS approach fails to estimate signal since the inverse of  $\mathbf{H}^T \mathbf{H}$  does not exist, but ISE and CS algorithms are still able to estimate  $\mathbf{x}$ . In particular, in this paper, we only focus on the case  $N \geq K$ . For this case  $N < K$ , the similar conclusion can be made in the signal estimation performance.

<sup>2</sup>The non-zero elements may not be equal and they can be complex numbers.

<sup>3</sup>Without loss of generality, in this paper, we consider a general model. Thus, we generate  $\mathbf{H}_m$  and  $\mathbf{x}_m$  through using a random matrix and a non special vector. For practical applications,  $\mathbf{H}_m$  and  $\mathbf{x}_m$  have specific meanings. For example, in radar signal processing,  $\mathbf{x}_m$  may denote the target reflection coefficients needed to estimate.

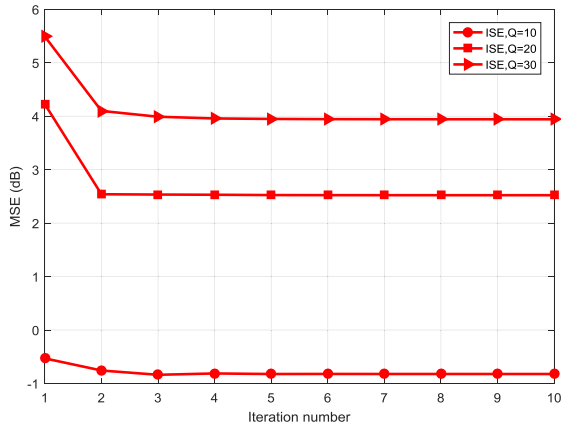


FIGURE 1. MSE (dB) versus the iteration index  $u$  for the ISE method considering  $Q = 10, 20$  and  $30$ .

In the following simulations, unless otherwise stated, we set  $M = 3, N = 1024, K = 256$ . The simulations are executed via using Matlab 2016b version on a standard PC (with a 2.8GHz Core i5 CPU and 8GB RAM).

A. CONVERGENCE ANALYSIS

In Fig. 1, we assume SNR in dB =  $10 \log_{10}(1/\sigma^2) = -3$  dB, and analyze the performance of the proposed ISE method in terms of convergence rate and the achievable estimation error for different values of  $Q$  considering as figure of merit

$$MSE^{(u)} = \sum_{q=1}^Q (\hat{x}_1^{(u)}(l_{q_1}) - x_1(l_{q_1}))^2. \quad (22)$$

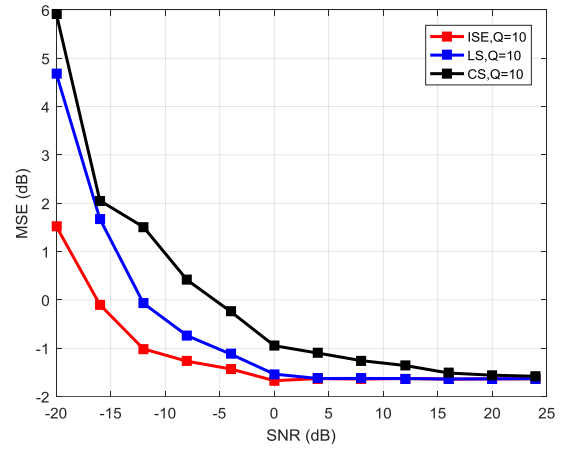
The results show that MSE decreases by increasing the iteration number and decreasing the values of  $Q$  for the ISE method. The convergence rate is quite quickly with a given  $Q$ . For instance, considering  $Q = 10$  and  $Q = 30$ , they require respectively only about 5 and 4 iterations. It is worth pointing out that the similar observations can be concluded for the signals  $\mathbf{x}_2$  and  $\mathbf{x}_3$ .

B. MSE FOR DIFFERENT SNR VALUES

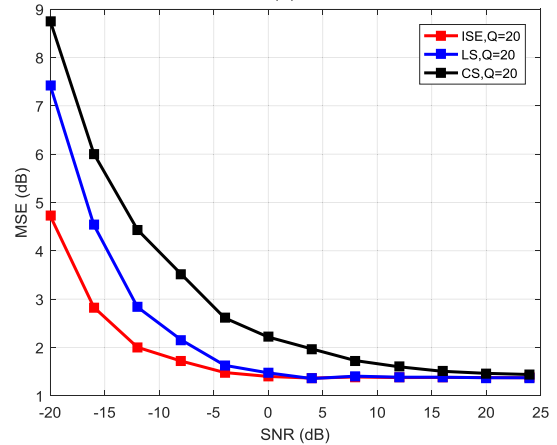
In Fig. 2, we study the MSE (dB) behaviours of  $\mathbf{x}_1$  versus SNR for different values of  $Q$ . The other parameters are the same as in Fig. 1. The curves illustrate that by increasing the SNR and decreasing the values of  $Q$ , the performance improves, and the MSE values of the ISE are smaller than these of the LS and CS methods. The smaller the SNR values, the larger the performance gaps. For example, for SNR =  $-20$  dB and  $Q = 10$ , the gap between ISE and LS methods is about 2 dB, while it is about 4dB between ISE and CS methods. Nevertheless, ISE and CS methods share very close performance for SNR > 10 dB.

C. ESTIMATES OF  $\mathbf{x}_M$  FOR LOW SNR VALUES

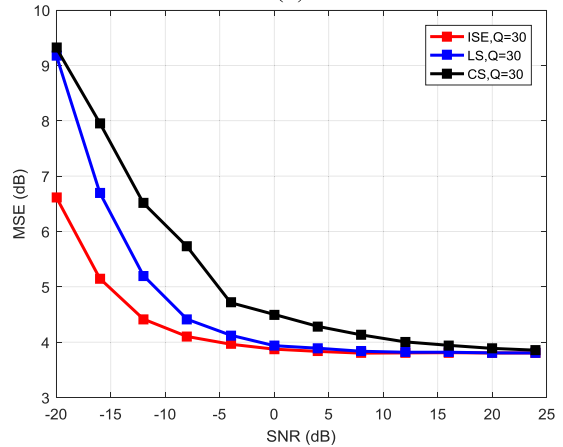
Figs. 3 and 4 show the estimates of  $\mathbf{x}_1, \mathbf{x}_2$  and  $\mathbf{x}_3$  obtained via ISE, LS and CS methods versus the index number for  $Q = 10$  considering SNR =  $-20$  dB and SNR =  $-28$  dB,



(a)



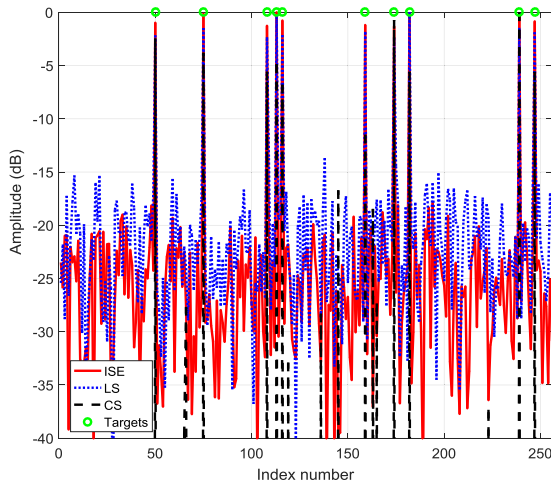
(b)



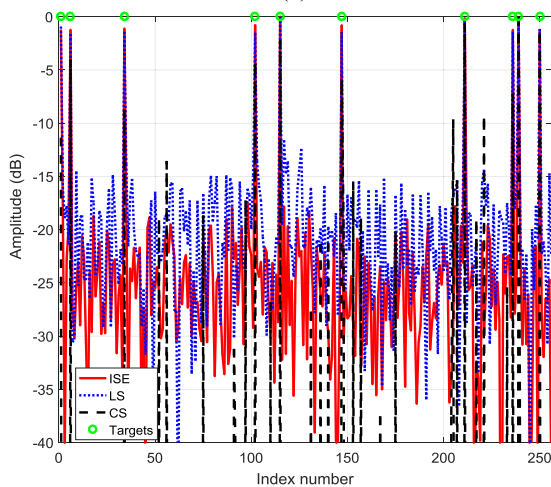
(c)

FIGURE 2. MSE (dB) versus the SNR (dB) for the ISE, LS and CS methods considering (a)  $Q = 10$ , (b)  $Q = 20$  and (c)  $Q = 30$ .

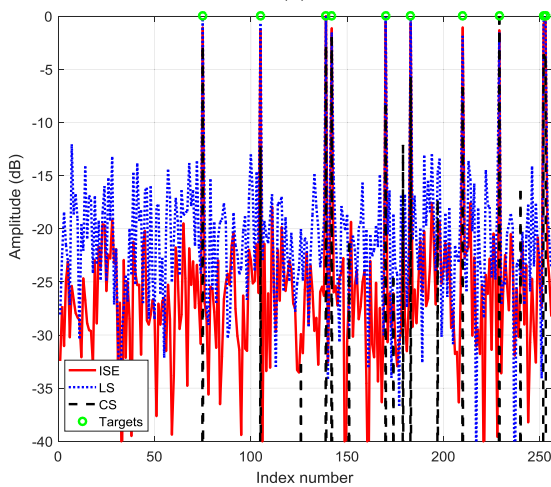
respectively. The other parameters are the same as in Fig. 1. The results exhibit that three methods can derive the non-zero components correctly for SNR =  $-20$  dB, whereas only ISE method is effective for SNR =  $-28$  dB. Letting the estimates of the zero components of the  $\mathbf{x}_1$  be the sidelobes, we observe that LS and CS method has higher sidelobe levels than these of ISE algorithm for all the estimates of  $\mathbf{x}_m, m = 1, 2, 3$ .



(a)



(b)

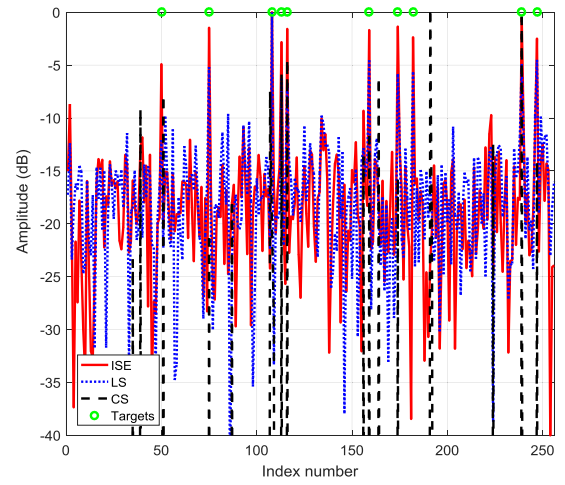


(c)

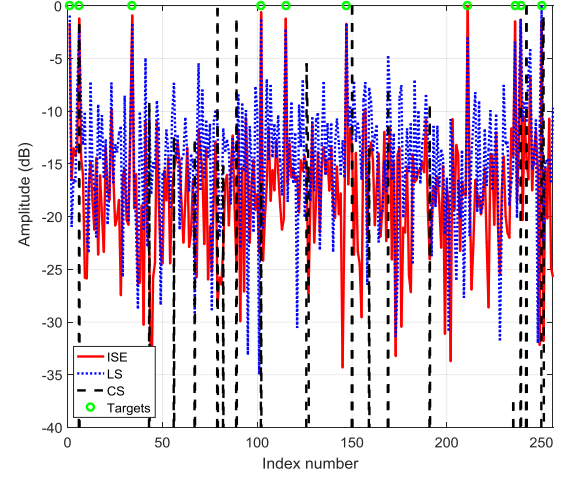
**FIGURE 3.** Estimates of  $x_m$ ; (a)  $m = 1$ ; (b)  $m = 2$ ; (c)  $m = 3$  versus the index number for  $Q = 10$ ,  $SNR = -20$  dB.

These behaviors imply ISE algorithm is more robust than the LS and CS methods for low SNR values.

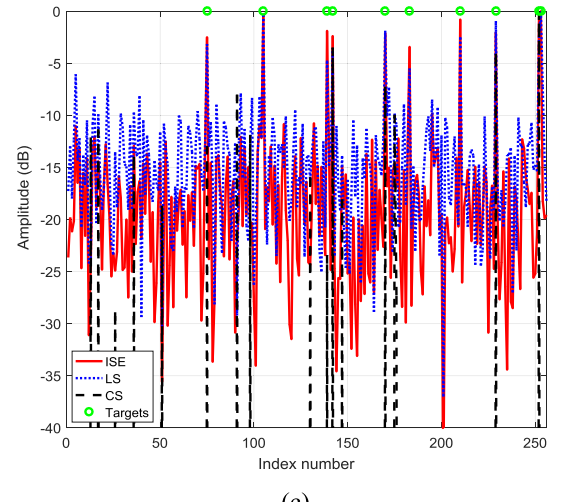
To further illustrate the effectiveness of ISE algorithm, we also consider more channels. For example, in Fig. 5,



(a)



(b)



(c)

**FIGURE 4.** Estimates of  $x_m$ ; (a)  $m = 1$ ; (b)  $m = 2$ ; (c)  $m = 3$  versus the index number for  $Q = 10$ ,  $SNR = -28$  dB.

the estimates of  $x_1, x_2, x_3, x_4$  (i.e.,  $M = 4$ ) are provided for  $SNR = -28$  dB. The results again confirm that ISE algorithm is still able to estimate the non-zero components correctly, while CS and LS methods are disabled.

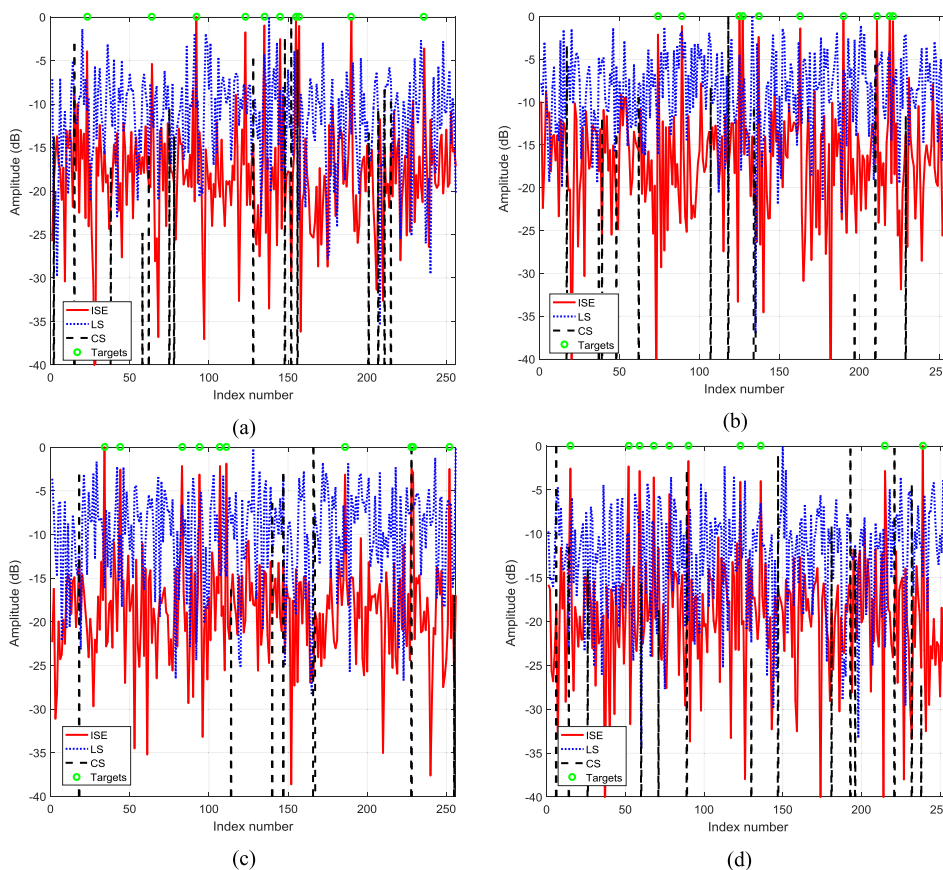


FIGURE 5. Estimates of  $x_m$ ; (a)  $m = 1$ ; (b)  $m = 2$ ; (c)  $m = 3$  and (d)  $m = 4$  versus the index number for  $Q = 10$ ,  $SNR = -28$  dB.

TABLE 1. Computational time (in seconds).

Algorithms	ISE	LS	CS
M=3, SNR=-20 dB	1.2275	0.1248	76.8510
M=3, SNR=-28 dB	1.1013	0.1067	71.0400
M=4, SNR=-28 dB	1.4913	0.1955	497.6213

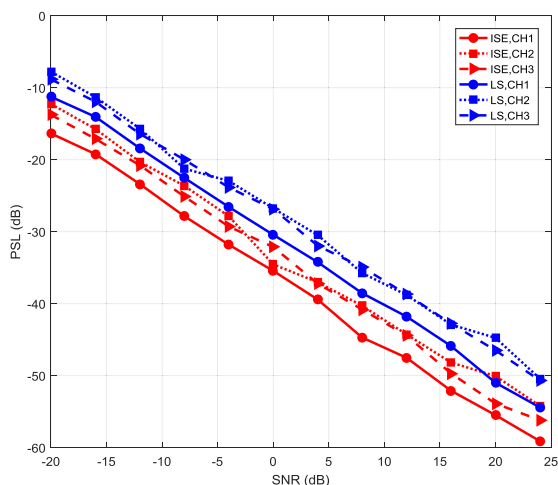


FIGURE 6. PSL values versus the SNR for  $Q = 10$ .

In Table 1, we also show the corresponding computational time for Figs. 3, 4 and 5. The results show that LS method outperforms ISE and CS algorithms. This is due to the facts that LS method can get a closed-form solution, while ISE

is an iteration procedure and CS method requires to solve a convex problem. Besides, we can see that ISE approach costs less time than CS method. The results show that ISE method plays a trade-off between the computational complexity and the signal estimation performance compared with LS method.

#### D. PSL FOR DIFFERENT SNR VALUES

Denote by PSL the ratio between the peak estimate of the non-zero of  $x_m$ ,  $m = 1, 2, 3$  and the maximum estimates of sidelobe levels. In Fig. 4, we plot the average PSL of the estimate  $x_1$ ,  $x_2$  and  $x_3$  versus the SNR for  $Q = 10$  via 20 independent trials. The results illustrate that ISE algorithm outperforms LS method by about 5dB for each channel.

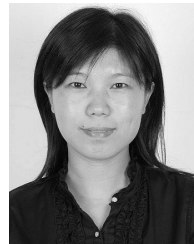
#### V. CONCLUSION

This paper has addressed the signal estimation problem for a linear combination of multiple structured models, and presented the ISE method, which derives the multiple structured signals in a sequential iterative way. At each iteration, the optimal linear filters, based on the MSE criteria, are designed by exploiting the estimated signals at the previous step. Finally, we have analyzed the performance of the ISE method. Results illustrate that the proposed ISE method converges fast only for several iterations and has lower MSE and PSL values especially for the lower SNR region and larger

number of the structured signals, in comparison with LS and CS algorithms.

## REFERENCES

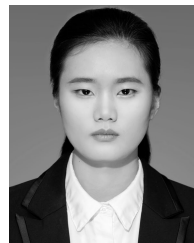
- [1] S. L. Primak and V. Kontorovich, *Wireless Multi-Antenna Channels: Modeling and Simulation*. Hoboken, NJ, USA: Wiley, 2012.
- [2] Y. G. Li, J. H. Winters, and N. R. Sollenberger, "MIMO-OFDM for wireless communications: Signal detection with enhanced channel estimation," *IEEE Trans. Commun.*, vol. 50, no. 9, pp. 1471–1477, Sep. 2002.
- [3] S. Zhang, Y. Yang, G. Cui, B. Wang, H. Ji, and S. Iommelli, "Range-velocity jamming suppression algorithm based on adaptive iterative filtering," in *Proc. IEEE Radar Conf.*, Philadelphia, PA, USA, May 2016, pp. 1–6.
- [4] G. Cui, H. Ji, V. Carotenuto, S. Iommelli, and X. Yu, "An adaptive sequential estimation algorithm for velocity jamming suppression," *Signal Process.*, vol. 134, pp. 70–75, May 2017.
- [5] A. Khwaja and M. Cetin, "Compressed sensing ISAR reconstruction considering highly maneuvering motion," *Electronics*, vol. 6, no. 1, p. 21, Mar. 2017.
- [6] A. De Maio and D. Orlando, "Adaptive radar detection of a subspace signal embedded in subspace structured plus Gaussian interference via invariance," *IEEE Trans. Signal Process.*, vol. 64, no. 8, pp. 2156–2167, Apr. 2016.
- [7] A. Aubry, V. Carotenuto, A. De Maio, and M. A. Govoni, "Multi-snapshot spectrum sensing for cognitive radar via block-sparsity exploitation," *IEEE Trans. Signal Process.*, vol. 67, no. 6, pp. 1396–1406, Mar. 2019.
- [8] S. Bidon, O. Besson, and J.-Y. Tourneret, "The adaptive coherence estimator is the generalized likelihood ratio test for a class of heterogeneous environments," *IEEE Signal Process. Lett.*, vol. 15, pp. 281–284, Feb. 2008.
- [9] D. Ciuonzo, A. De Maio, and D. Orlando, "A unifying framework for adaptive radar detection in homogeneous plus structured Interference—Part II: Detectors design," *IEEE Trans. Signal Process.*, vol. 64, no. 11, pp. 2907–2919, Jun. 2016.
- [10] W. Liu, W. Xie, J. Liu, and Y. Wang, "Adaptive double subspace signal detection in Gaussian Background—Part II: Partially homogeneous environments," *IEEE Trans. Signal Process.*, vol. 62, no. 9, pp. 2358–2369, May 2014.
- [11] N. Bon, A. Khenchaf, and R. Garello, "GLRT subspace detection for range and Doppler distributed targets," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 2, pp. 678–696, Apr. 2008.
- [12] R. C. de Lamare, L. Wang, and R. Fa, "Adaptive reduced-rank LCMV beamforming algorithms based on joint iterative optimization of filters: Design and analysis," *Signal Process.*, vol. 90, no. 2, pp. 640–652, Feb. 2010.
- [13] T. Yardibi, J. Li, P. Stoica, M. Xue, and A. B. Baggeroer, "Source localization and sensing: A nonparametric iterative adaptive approach based on weighted least squares," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 46, no. 1, pp. 425–443, Jan. 2010.
- [14] M. Ghogho, A. Swami, and T. Durrani, "Approximate maximum likelihood blind source separation with arbitrary source PDFs," in *Proc. 10th IEEE Workshop Stat. Signal Array Process.*, Aug. 2000, pp. 368–372.
- [15] X. Liu, N. D. Sidiropoulos, and A. Swami, "Blind high-resolution localization and tracking of multiple frequency hopped signals," *IEEE Trans. Signal Process.*, vol. 50, no. 4, pp. 889–901, Apr. 2002.
- [16] B. M. Sadler, R. J. Kozick, and T. J. Moorel, "On the performance of source separation with constant modulus signals," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, 2002, pp. 2373–2376.
- [17] A. Swami, S. Barbarossa, and B. M. Sadler, "Blind source separation and signal classification," in *Proc. 34th Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, vol. 2, 2000, pp. 1187–1191.
- [18] O. Dobre, "Signal identification for emerging intelligent radios: Classical problems and new challenges," *IEEE Instrum. Meas. Mag.*, vol. 18, no. 2, pp. 11–18, Apr. 2015.
- [19] Y. A. Eldemerdash, O. A. Dobre, and M. Oner, "Signal identification for multiple-antenna wireless systems: Achievements and challenges," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 3, pp. 1524–1551, 3rd Quart., 2016.
- [20] K. Sun, H. Meng, Y. Wang, and X. Wang, "Direct data domain STAP using sparse representation of clutter spectrum," *Signal Process.*, vol. 91, no. 9, pp. 2222–2236, Sep. 2011.
- [21] T. Felhauer, "Digital signal processing for optimum wideband channel estimation in the presence of noise," *IEE Proc. Radar Signal Process.*, vol. 140, no. 3, pp. 179–186, 1993.
- [22] B. Zmic, A. Zejak, A. Petrovic, and I. Simic, "Range sidelobe suppression for pulse compression radars utilizing modified RLS algorithm," in *Proc. IEEE 5th Int. Symp. Spread Spectr. Techn. Appl.*, vol. 3, Sep. 1998, pp. 1008–1011.
- [23] H. Krim and M. Viberg, "Two decades of array signal processing research: The parametric approach," *IEEE Signal Process. Mag.*, vol. 13, no. 4, pp. 67–94, Jul. 1996.
- [24] J. Li and P. Stoica, "An adaptive filtering approach to spectral estimation and SAR imaging," *IEEE Trans. Signal Process.*, vol. 44, no. 6, pp. 1469–1484, Jun. 1996.
- [25] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, no. 8, pp. 1408–1418, Aug. 1969.
- [26] D. Needell and J. A. Tropp, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," *Appl. Comput. Harmon. Anal.*, vol. 26, no. 3, pp. 301–321, May 2009.
- [27] S. D. Blunt and K. Gerlach, "Adaptive pulse compression via MMSE estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 42, no. 2, pp. 572–584, Apr. 2006.
- [28] W. Roberts, P. Stoica, J. Li, T. Yardibi, and F. A. Sadjadi, "Iterative adaptive approaches to MIMO radar imaging," *IEEE J. Sel. Topics Signal Process.*, vol. 4, no. 1, pp. 5–20, Feb. 2010.
- [29] S. Boyd, and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.



**ZHIMEI HAO** was born in 1979. She received the M.S. degree in communication and information system from the Department of Electronic Engineering, Nanjing University of Aeronautics and Astronautics, in 2004. She is currently pursuing the Ph.D. degree with Beihang University. Her current research interest includes radar signal processing.



**XIANXIANG YU** (Student Member, IEEE) was born in Sichuan, China, in 1991. He received the B.S. degree from the School of Electronic Engineering, University of Electronic Science and Technology of China (UESTC), in 2014. He is currently pursuing the Ph.D. degree with the School of Information and Communication Engineering, UESTC. His current research interests include array signal processing and waveform optimization.



**NA GAN** was born in Sichuan, China, in 1996. She received the B.S. degree from the School of Information Science and Technology, Chengdu University of Technology, in 2018. She is currently pursuing the M.S. degree with the School of Information and Communication Engineering, University of Electronic Science and Technology of China. Her current research interest includes waveform optimization.



**GUOLONG CUI** (Senior Member, IEEE) received the B.S., M.S., and Ph.D. degrees from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2005, 2008, and 2012, respectively. From January 2011 to April 2011, he was a Visiting Researcher with the University of Naples Federico II, Naples, Italy. From June 2012 to August 2013, he was a Postdoctoral Researcher with the Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ, USA. Since September 2013, he has been a Professor with UESTC. His current research interests include statistical signal processing in the field of statistical signal processing with an emphasis on radars, waveform optimization, and passive sensing.

• • •