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Entropy Evaluation Method for Sandstone Uranium Reservoir Characteristics Based on Convex Hull Search

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ABSTRACT The continuity and homogeneity of sandstone uranium deposits are inherent characteristics that affect in situ leaching performance indexes and have important guiding effects on the optimization of mining design. In this work, we propose a spatial information entropy calculation method based on a convex hull search strategy to quantitatively evaluate the geological characteristics of uranium deposits. We use the 3-D block model of sandstone-hosted uranium deposits as hard data. First, we use the Alpha-Shapes algorithm to calculate the convex hull of the space area where various types of unit blocks are located. Second, we formulate a suitable multi-point configuration based on the convex hull size. Then, we use the multi-point density function. Scanning the convex hull point cloud establishes a probability density function to calculate the local information entropy of various sub-regions. Finally, the local spatial information entropy is aggregated using addition and multiplication to obtain the joint spatial information entropy of the entire evaluation area. A series of parameter sensitivity and performance analysis experiments prove the stability of the method and the accuracy of the quantitative and homogeneous evaluations of the complex geological conditions. In order to solve the problem of the difference between the designed production capacity and actual production capacity of a uranium deposit, we use a new method to evaluate the homogeneity and continuity of the lithology and grade of the deposit. The new method and evaluation conclusions can be used for the automation and understanding of future engineering optimization processes.

INDEX TERMS Sandstone-hosted uranium deposits, multiple-point density function, convex hull search, spatial information entropy, reservoir homogeneity and continuity.

I. INTRODUCTION

The in-situ leaching method is to inject the leaching solution into the orebody through the surface injection drilling, then the leaching solution chemically reacts with the target mineral, and the leaching solution with the target mineral is lifted to the surface through the pumping drilling, extraction-electrowinning to obtain the finished metal [1]. Compared with traditional open-pit and underground mining, in-situ leaching technology has three outstanding advantages. ① The security is relatively high. The in-situ leaching process is an integrated, fluidized working method. It eliminates the complicated roadway excavation and manual mining. Therefore, it is easier to realize intelligent remote control on the ground and unmanned mining underground, and has higher security [2]. ② Environmental pollution is small. In situ leaching does not produce waste rock, tailings and other wastes, not only can effectively prevent surface subsidence, but also the lowest carbon emissions [3]. ③ Mining costs are low. In situ leaching does not require the construction of large-scale development systems and hoisting systems, and capital construction and mining costs are only 50% of the average cost of traditional open-pit or underground mining [4]. In the 1960s, the in-situ leaching method was first successfully

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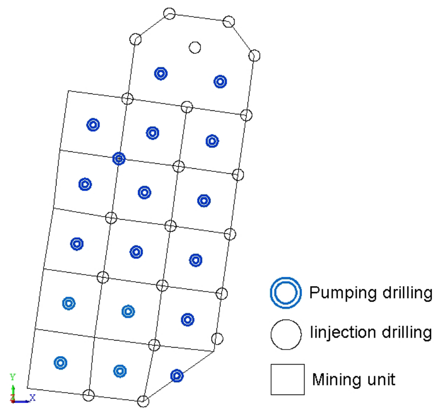


FIGURE 1. In-situ leaching well layout for sandstone-type uranium deposits.

applied to sandstone uranium deposit. At present, this method has been widely used in mines of strategic metal resources such as sandstone uranium deposit, ionic rare-earth ore, and sandstone copper oxide ore. In addition, it has achieved efficient leaching of 40% of uranium deposits worldwide [5]–[7].

When in-situ leaching of sandstone uranium deposits, its basic production units are mining areas and mining units (as shown in Fig. 1). For a mining area and a mining unit, the spatial continuity of lithology and the homogeneity of the mineralization degree have a great influence on its leaching performance and efficiency [8], [9]. The homogeneity of the grade will affect the chemical reaction performance of the mineral and the leaching solution, and the lithological heterogeneity will reduce the efficiency of the leaching solution and the solute transfer [10]–[12].

For a long time, in the well site design of sandstone uranium mine, people usually based on experience, consider the direction of groundwater flow, and arrange the well site according to the unified well pattern and well spacing. In addition, according to the lithology and grade information revealed by a single injection drilling or extraction drilling, the vertical borehole structure layout is completed [13]. This will undoubtedly lead to many problems such as increased construction and production costs, waste of equipment and materials, and reduced production efficiency and benefits during mining. Therefore, we need to provide a method and index that can quantitatively evaluate the continuity and homogeneity of any area of the reservoir, so as to achieve the optimal layout of the mining area and the mining unit.

Research on reservoir heterogeneity has been focused on the field of oil and gas reservoirs [14]. At present, the research methods of the reservoir heterogeneity can be roughly divided into two categories: qualitative description methods (such as geological genesis analysis, sequence stratigraphy research method, etc.) and quantitative research methods (such as geo-statistical methods, log interpretation methods, various mathematical calculation methods, etc.) [15]–[17]. In the characterization of reservoir heterogeneity, because the qualitative description method has been difficult to meet the needs of the mine, so the quantitative research methods (such as geo-statistical methods, log interpretation methods, various mathematical calculation methods, etc.) have become the mainstream direction of current research [18].

Geostatistics is one of the first methods to study reservoir heterogeneity quantitatively [15]. It can perform statistics and calculations on various geological properties of the reservoir (such as grade, lithology, porosity, etc.) to analyze the strength of the reservoir heterogeneity. Yan [19] used geostatistics to calculate the porosity, permeability, thickness and other parameters of a dissolved carbonate rock. The related parameters of variogram were used to reflect the heterogeneity of the reservoir. Coli *et al.* [20] proposed to use geostatistics approach to characterize the morphological and spatial variability of rock inclusions in bimrocks. Chen [21] applying geostatistics methods to calculate the ratio of vertical permeability and horizontal permeability in the first volcanic rock formation of the Yingcheng Formation in the Xudong area of the Songliao Basin to analyze reservoir heterogeneity. Ge [22] proposed to use magnetic resonance imaging and geostatistics methods to evaluate the heterogeneity of pore distribution in tight sandstone reservoirs. The geostatistics method has the advantages of simple algorithm thought and high solution efficiency. The disadvantage is that the variability function in the qualitative geostatistics method can only reflect the degree of correlation between the two point attributes in space, and cannot accurately quantitatively evaluate the degree of change in the value of the attribute points in space.

In order to better analyze the heterogeneity of various geological attributes of the reservoir, scholars have proposed log interpretation methods and various mathematical calculation methods [23]. The method of logging interpretation is to quantitatively study the heterogeneity of the reservoir by analyzing the data of the geophysical logging (such as permeability) and the degree of change of the core analysis data. For example, Yan *et al.* [24] proposed that the heterogeneity of mineralization could be reflected by the coefficient of variation, the coefficient of onrush and the permeability ratio of permeability. Zheng *et al.* [25], [26] proposed using the Lorenz coefficient method and Thiel index method to evaluate reservoir heterogeneity. However, because the evaluation parameters of the logging interpretation method are relatively single, the evaluation results are one-sidedness. To solve this problem, scholars have proposed some mathematical calculation methods that can integrate multiple evaluation parameters [27]. Wang *et al.* [28] proposed a hierarchical analysis and fuzzy logic method to integrate 9 parameters into a comprehensive index, thus overcoming the disadvantage of a single parameter. Xiao *et al.* [29] proposed a method of resale range analysis (R/S) and empirical model decomposition (EMD) to comprehensively measure reservoir heterogeneity.

The above studies are based on borehole data to analyze the heterogeneity of the reservoir. This method of sparse borehole data to evaluate the heterogeneity of the geological properties of the entire mining area is extremely one-sidedness. Therefore, how to accurately evaluate the heterogeneity of unknown areas between boreholes is a new challenge in reservoir heterogeneity evaluation. However, in the field of metal mine, based on geological exploration data, the establishment of geological big data system (3D geological model) which can reflect the spatial lithology and grade distribution of ore deposits by using multipoint geostatistics, radial basis function interpolation and geostatistics methods is an established workflow. on this basis, we need to establish a set of methods and indicators for the quantitative evaluation of reservoir heterogeneity and continuity of sandstone uranium deposits. In this way, we can optimize the layout of mining areas and mining units, reduce construction investment and production costs, and improve the recovery rate of uranium. Taking the 3D geological model as the research object, this paper proposes a method for evaluating the spatial information entropy of sandstone uranium deposits based on a convex hull search strategy. It can intelligently and quantitatively evaluate the homogeneity and continuity of the lithology and grade spatial distribution in the reservoir.

The remainder of this paper is organized as follows. Section 2 introduces the shortcomings of the two available methods for calculating information entropy and provides the background information used in the following sections. Section 3 introduces the main concepts of the spatial information entropy calculation method based on the convex hull search strategy and the detailed flow of the method. Section 4 analyses the parameter sensitivity of the new method and the performance comparison with the existing two methods, which reflects the applicability of the new method in complex geological data. In Section 5, in view of the low leaching rate in the existing high-grade area of sandstone-hosted uranium deposits in Northwest China, the homogeneity and continuity of grade and lithology of the deposit are evaluated using the new method. This illustrates the effectiveness of the new method in the evaluation of real geological data, and provides an evaluation index for the optimization of the mining design of the deposit. The final section contains some concluding remarks and ideas for future work.

A. BACKGROUND INFORMATION

B. INFORMATION ENTROPY

Aiming at massive information systems, Shannon [30] proposed measuring information amounts by applying information entropy based on the concept of "entropy" of thermodynamics and called the average information amount after eliminating redundancy in information "information entropy". Christakos [31] introduced information entropy in geostatistics to measure the uncertainty of probability density functions. André and Journel [32] first introduced the concept of spatial information entropy into the framework of spatial disorder. In this work, information entropy is used to evaluate the disorder of the simulated space. In practice, the difficulty of calculating information entropy lies in how to obtain an accurate probability density function. To solve this problem, Boisvert *et al.* [33] proposed the method of scanning the training image with a multi-point density function to obtain the probability density function of the model. Silva and Deutsch [34] used this method to evaluate the spatial disorder of multi-point geostatistics training images with spatial information entropy.

1) SHANNON ENTROPY

Shannon [30], the father of information theory, pointed out in his paper, *A Mathematical Theory of Communication* published in 1948, that any information has redundancy, and the size of redundancy is related to the probability or uncertainty of each symbol (number, letter or word) in the information. In the information source, instead of considering the uncertainty of the occurrence of a single symbol, consider the average uncertainty of all possible occurrences of the source. If the source symbol has n kinds of values: $U_1, \cdots, U_i, \cdots U_n$, the corresponding probability is $P_1, \cdots, P_i, \cdots P_n$, and the appearance of various symbols is independent of each other. At this point, the average uncertainty of the source can be expressed as follows:

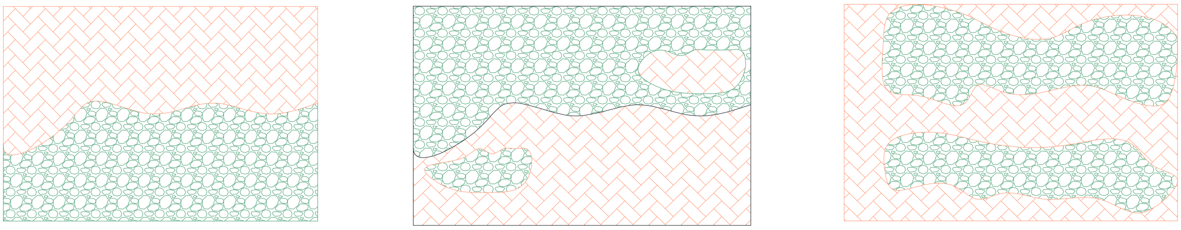
$$H(U) = -K \sum_{i=1}^{n} P_i \log P_i \tag{1}$$

P_i is the proportion of symbol U in the information source U_i, that is, the statistical information amount. K is a positive constant. In Eq. (1), we generally take 2 as the logarithmic base and the unit is bit. We can also take other logarithmic bases, of course, the calculation results of different logarithmic bases can be converted to each other.

In establishing a three-dimensional geological model of a sandstone uranium deposits in northwestern of China, we drew three geological profile maps, as shown in Fig. 2. The three pictures contain two lithologies: sandstone and mudstone, and the content of the two lithologies is the same. We use Eq. (1) to calculate the aroma information entropy of Fig. 2 (a) $\sim$ (c) as $H_a = H_b = H_c$ (the logarithmic base of Eq. (1) is 2). The Shannon entropy of the three geological profiles is the same, but their homogeneity and continuity are different. The reason is that the probability density function established by mathematical statistics does not consider the spatial distribution characteristics of attributes, so the calculated information entropy cannot measure the homogeneity and continuity of the spatial distribution of geological attributes.

2) SPATIAL INFORMATION ENTROPY

Spatial information entropy is introduced into geological statistics to evaluate the spatial disorder of the model. The difference from Shannon entropy is that the probability density function (PDF) P_i is obtained by scanning a 2D model or a 3D model with a multiple-point density function (MPDF) rather than counting the proportion of an attribute in the model. The calculation process of spatial information entropy is as



(a) (b) (c)

FIGURE 2. Geological profile. In figure (a)~(c), red indicates mudstone layers, with a ratio of approximately 50%; green indicates sandstone layers, with a ratio of approximately 50%.

follows: 1) The geological model is transformed into a block model, and a single basic block is represented by point data; 2) the specifications of the multiple-point template n ($n \geq 2$) are defined; usually, the number of regular templates generated by the MPDF is $NUM = X^n$, where X is the number of attributes included in the model; 3) MPDF scans the block model, counts the number of occurrences of each template, and constructs a PDF; 4) Eq. (1) calculates the spatial information entropy of the block model. Generally, the base of Eq. (1) is the number of effective templates generated by the MPDF. The calculation of the spatial information entropy has no specific requirements on the shape of the search template and can be arbitrary. Usually, a regular template configuration is used to facilitate the calculation.

According to the above steps, the spatial information entropy of the three geological model maps in Fig. 2 (a) $\sim$ (c) is calculated as $H_a = 0.3483$, $H_b = 0.4830$, $H_c = 0.5623$. Obviously, the calculation results of spatial information entropy can distinguish the differences between lithology homogeneity and continuity in the three geological profile maps. However, in actual engineering, the mudstone layer in Fig. 2 (a) does not affect the flow of the leaching solution in the sandstone layer, so we consider the model to be approximately homogeneous and continuous. However, the calculation method of spatial information entropy still assigns a relatively large value to Fig. 2 (a), which we do not want to see. The reason is that in practical applications, when there are multiple attributes in the 3D geological model, we are more used to analyze the homogeneity and continuity of the spatial distribution of each attribute one by one. For a block model containing multiple geological attributes, if each attribute is spatially independent of each other, has a clear distribution boundary, and has no canine staggered characteristics, we consider it to be approximately homogeneous and continuous. The PDF of the model of Fig. 2 (a) is shown in Fig. 3 (a). We find that both prob1 and prob5 are located near the peak of Eq. (1), which results in a larger space in Fig. 2 (a). If the spatial information entropy of sandstone and mudstone layers is calculated by partition, we can avoid the above problems.

C. ALPHA-SHAPES

When calculating the spatial information entropy of each attribute by partition, we need to know the spatial distribution range of point clouds of the same type of attribute. The alpha-shapes algorithm can extract edges from a set of unordered points. Gardiner *et al.* [35] proposed the alpha shapes algorithm to solve the point cloud boundary. The parameter "alpha" indicates the level of refinement. A larger alpha results in a coarser fit, and a smaller alpha results in a finer fit (Fig. 4). Therefore, when the value of α is small, every point is theoretically a boundary. If α is large ($\alpha \rightarrow \infty$), the boundary line obtained is the convex hull of the point cloud.

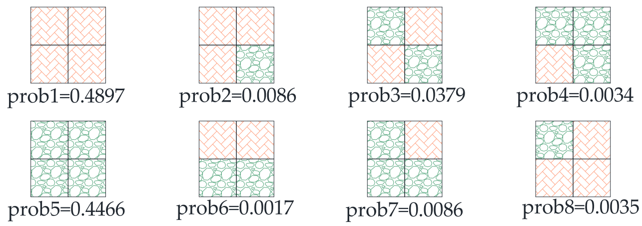


FIGURE 3. Section 2.1.1 Model (a) Distribution of probability density function (PDF).

Geological point cloud data of the same type of attribute are usually disorganized. Therefore, to delimit the boundary of all point clouds of the same attribute, we need a large α value, $\alpha = lnf$ (positive infinity) in this study.

D. SPATIAL INFORMATION ENTROPY AGGREGATION

The aggregation of spatial information entropy obtains a more stable, comprehensive and intuitive value to evaluate the chaos of a certain information set. Allard *et al.* [36] presented a comprehensive literature review for aggregating probability distributions. The linear pooling formula is a widely used probability-based addition method, and its main features are flexibility and simplicity [37]. Multiplicative methods include Bordley [38] and Tau models and log-linear pooling [39] (based on odd ratios). These two formulas have been proven to be equivalent for cases with only two attributes, but the latter has better performance for non-binary events [40]. In the field of geosciences, the concurrency of events is more emphasized; that is, two or more events act simultaneously and output a final result. Therefore, the log-linear pooling formula based on multiplication will have more advantages for concurrent operations in the field of geosciences.

The purpose of aggregation of spatial information entropy is very similar to that of probability aggregation. In this study, we use probability aggregation to aggregate spatial

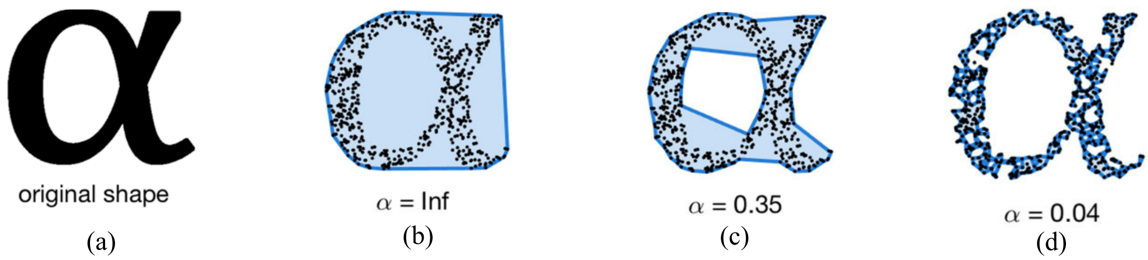


FIGURE 4. Calculation performance of the alpha-shape algorithm in a two-dimensional image [35]. (a) represents the original shape of n, a comparison chart for algorithm detection; (b) represents the convex hull formed by the α data set; (c)~(d) indicate that as the 'alpha' decreases, the boundary points are gradually refined.

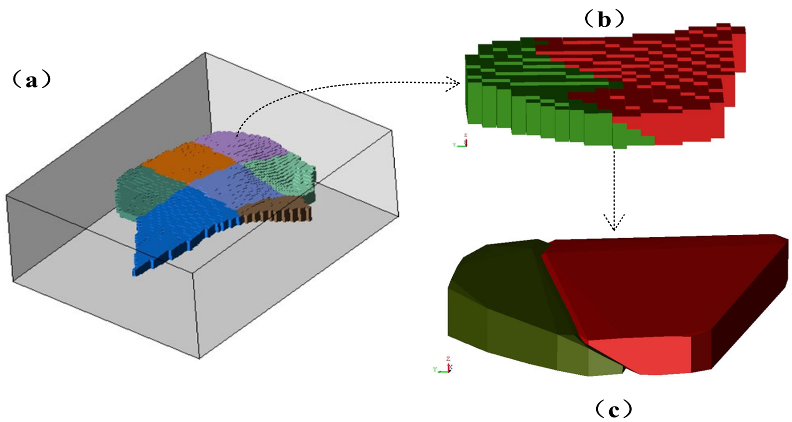


FIGURE 5. Schematic diagram of spatial information entropy search strategy. (a) is a block model diagram of the mining area, with different colours representing different mining units; (b) is a block model diagram of a mining unit block, with different colours representing different attributes; (c) is a point cloud convex hull model for each category of attributes.

information entropy. Our goal is to obtain a joint spatial information entropy that better reflects the chaos of the 3D point cloud.

1) LINEAR POOLING FORMULA

The linear pooling formula proposed by Stone is the most intuitive and simple method of probability fusion [37]. For the spatial information entropy distribution $H_1, \cdots, H_n$, the specific aggregation formula is as follows:

$$H_G(A) = \sum_{i=1}^{n} \omega_i H_i(A) \quad \text{with} : \omega_1, \cdots, \omega_n \in R^+ \tag{2}$$

In this formula, ω_i are positive weights, and their sum must equal 1 to obtain a global probability $H_G = [0, 1]$.

2) LOG-LINEAR POOLING FORMULA

The log-linear pooling formula is a linear operator of the logarithms of the probabilities [39]. For the spatial information entropy distribution $H_1, \cdots, H_n$, the specific aggregation formula is as follows:

$$H_G(A) \propto \prod_{i=1}^{n} H_i(A)^{\omega_i} \tag{3}$$

$\sum_{i=1}^{n} \omega_i = 1$ is needed to verify external Bayesian characteristics. There are no other constraints whatsoever on the weights ω_i, $i = 1, \cdots n$.

II. METHODOLOGY

A. CALCULATION METHOD OF SPATIAL INFORMATION ENTROPY BASED ON A CONVEX HULL SEARCH STRATEGY

Previous studies that analyzed reservoir heterogeneity based on borehole data cannot accurately evaluate the heterogeneity of unknown areas between boreholes. However, the 3D geological model established by applying radial basis function interpolation method and geostatistics method can help us to evaluate the homogeneity and continuity of reservoir lithology and grade spatial distribution in the entire mining area.

The traditional spatial information entropy calculation method uses the MPDF to search the PDF established by the geological model to calculate the spatial information entropy. When there are multiple attributes in the 3D geological model, we are more used to analyze the homogeneity and continuity of the spatial distribution of each attribute one by one. This makes it difficult for the traditional spatial information entropy method to distinguish the difference between the homogeneity and continuity of the geological model significantly. In view of the shortcomings of traditional

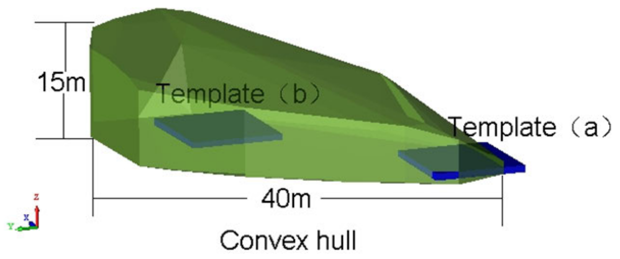


FIGURE 6. Schematic diagram of the convex hull search window.

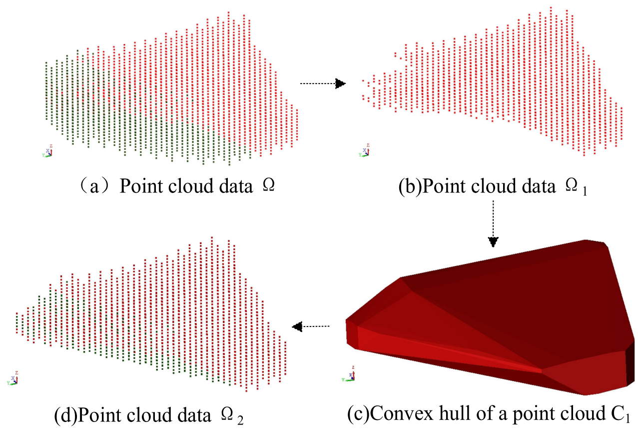


FIGURE 7. Circled hull convex hull flow chart.

spatial entropy calculation methods, we propose a new spatial information entropy calculation method based on a convex hull search strategy. Compared with existing spatial information entropy calculation methods, our new method only scans the convex hull of the area where each attribute unit block is located, not the entire unit space.

As illustrated in Fig. 5, the 3D block model is first divided into N small partitions according to the number of mining units (Fig. 5 (a)), and each partition contains n attributes (Fig. 5(b)). For any sub-block, we should delineate n convex hulls based on the number of attributes n it contains (Fig. 5 (c)). When calculating the entropy of each partition, the 3D template traverses each convex hull in turn rather than traversing the entire mining unit at once. At this time, n information entropies H_i will be calculated in a single mining unit, where $i = 1, \cdots, n$.

Another important point is the search window when scanning convex hulls. This strategy requires that when scanning the convex hull dataset, the search template must all be included in the convex hull before it can be counted as a valid search. In order to ensure that the search template can be completely included in the convex hull, we require that the length, width, and height of the template are far less than the convex hull. As shown in Fig 6, the convex hull has a length of about 40m, a width of about 20m $\sim$ 40m, and a height of about 15m. Therefore, we define the search template size as: 5 * 5 * 2 (as long as the template size is far smaller than that of the convex hull), the template (a) is not completely included in the convex hull, and the template (b) is completely included in the convex hull, so only the latter is an effective search template. There is no doubt that if the defined search template is too small, there are fewer types of theoretical templates generated by the multi-point density function, and the homogeneity and continuity of the convex hull cannot be accurately calculated. For example, for a convex hull containing two attributes, a template with a size of 2*2*1 has a maximum of 16 (2^4) configurations, and a template with a size of 4*2*2 has a maximum of 65536 (2^{16}) configurations. However, an overly large search template requires a convex hull to provide more search space. Therefore, the search template should be the right size.

B. CIRCLED SCATTER CONVEX HULL

A key point of the new method is to solve the convex hull of the dissolution point. In Section 2.2, we mentioned that the alpha-shape algorithm can extract edges from a bunch of disordered spatial points. In this section, we use the alpha shape algorithm to solve the convex hull of the dissolution point. The detailed algorithm flow is as follows:

1) POINT CLOUD DATA CLASSIFICATION

For a point cloud Ω of a mining unit that contains n types of attributes, it can be divided into Ω_i $(i \in 1, \cdots, n)$ according to the attributes. In Fig. 7 (a), the point cloud data set Ω has two attributes: red point cloud data represent high-grade (point set Ω_1); green point cloud data represent low-grade (point set Ω_2).

2) CALCULATING THE CONVEX HULL OF POINT CLOUD DATA

The purpose of this work is to calculate the convex hull of the point cloud Ω_i $(i \in 1, \cdots, n)$. We take 'Alpha' as Inf (positive infinity). Fig. 7 (b) shows the point cloud data for the convex hull calculation. Fig. 7 (c) shows the convex hull C_1 calculated by the alpha-shape algorithm.

3) FILLED CONVEX HULL

When α = Inf, there are multiple empty areas in the convex hull (Fig. 4 (c)). The convex hull shown in Fig. 7 (c) is obtained from the point cloud Ω_1, but some data of Ω_2 are still included in the convex hull C_1. To facilitate the search template to traverse the entire convex hull, we intersect the point set Ω in Fig. 5 (a) and convex hull C_1, and the convex hull point cloud is P_1, as shown in Fig. 7 (d).

4) REPEAT STEPS (2) AND (3) ABOVE UNTIL THE CALCULATION OF Ω_i $(i \in 1, \cdots, n)$ IS COMPLETE

When filling the convex hull, not all empty areas are filled, but the empty areas contained within the point cloud of the mining unit are filled. This is because the research object of this article is the point cloud dataset in the mining unit.

Algorithm 1 Calculation Process of Spatial Information Entropy Based on Convex Hull Search Strategy

```
Load data files. The dataset is divided into N types according to the number of mining units;
For cycle through each mining unit.
    The data set N is divided into n types according to the number of attributes contained in the mining unit.
    For loop through each type of property
        Extract the corresponding data set Ω_i, and save the number of datasets NUM.
        Alpha-Shapes algorithm calculates convex hull C_i of data set Ω_i.
        The in-Shape algorithm obtains the intersection of Ω and C_i, and saves the set of points in the convex hull.
        Define the multiple-point template size: dx * dy * dz.
        The multiple-point template scans the convex hull data set P_i to obtain the corresponding PDF.
        Calculate the space entropy of the PDF according to Eq. (1);
    End
    The spatial information entropy h_i of various attributes in the mining unit is aggregated to obtain the joint spatial
    information entropy H_j.
End
Aggregate the spatial information entropy H_j between each mining unit to obtain the joint spatial information entropy H
of the entire mining area.
```

C. AGGREGATION STRATEGY FOR CONVEX HULL SPATIAL INFORMATION ENTROPY

As an additive aggregation method, the linear pooling formula corresponds to a mixture model, which is related to the union of events and to the logical operator OR. This method is used to combine several independent spatial information entropy distributions to obtain a larger and more stable joint spatial information entropy distribution. The log-linear pooling formula, based on the multiplication of probabilities, is related to the intersection of events and to the logical operator AND. Therefore, we usually use such a method to aggregate the probabilities with significant correlation to acquire a conjunction probability.

In this study, N $(N \geq 1)$ spatial information entropy will be obtained from N mining units in a mining area. Usually, a mining unit contains n convex hulls, so a single mining unit will generate n spatial entropies. For n spatial entropies from the same mining unit, the purpose of our spatial information entropy aggregate is to obtain a more stable joint spatial information entropy. So we are more accustomed to use the Linear Pooling formula based on the addition operation to aggregate the n independent information entropy distributions. However, for N spatial entropies from the same mining area, the purpose of aggregation is to describe the heterogeneity of the entire mining area. We hope to obtain a total spatial information entropy which can reflect the homogeneity and continuity of the whole mining area. Therefore, the information entropy fusion method based on multiplication will be more suitable for our requirement of emphasizing concurrency. Based on the above analysis, we propose an optimal spatial information entropy aggregation strategy that fully considers the above two characteristics, which specifically includes the following two steps:

1. Aggregate n convex hull space entropies from the same mining unit using the linear pooling formula.

2. Use the log-linear pooling formula to aggregate the spatial information entropy from N mining units after the first step.

If there is only one attribute in a mining unit, there will be no first spatial information entropy aggregation operation. Otherwise, the first spatial information entropy aggregation operation will be triggered to complete the aggregation of n spatial entropies in a single mining unit. In this strategy, the weight of each spatial information entropy from the same mining unit is determined by the number of point clouds contained in the convex hull corresponding to the spatial information entropy. The specific calculation formula is as follows:

$$\omega_i = \frac{\mathrm{P}_i}{\sum\limits_{k=1}^{n} \mathrm{P}_k} \quad \text{with} : i \in 1, \cdots, n \tag{4}$$

This parameter setting method ensures that the entropy of the convex hull spaces is smoothly combined into a more realistic and reliable reflection of the uniformity and continuity of the mining unit. Similarly, the weight of each mining unit in the spatial information entropy aggregation in the second step is determined by the number of point clouds contained in the mining unit. The specific calculation formula is as follows:

$$W_j = \frac{\Omega_j}{\sum\limits_{k=1}^{N} \Omega_k} \quad \text{with} : j \in 1, \cdots, N \tag{5}$$

D. DETAILED ALGORITHM FLOW BASED ON A CONVEX HULL SEARCH STRATEGY

To sum up, on the basis of 3D geological model, we first use the alpha shapes algorithm to calculate the convex hull of lithology or grade attribute in each unit block, then calculate the spatial information entropy of each convex hull point cloud, and finally use the information entropy aggregation strategy to obtain the total spatial information entropy that

Algorithm 2 Scanning Process of a Convex Hull in a Mining Unit

Input: P_i: convex hull point cloud; $dx * dy * dz$: search sample size.
Output: PDF: probability density function obtained in the current convex hull.

```
Function [PDF] = Scan_ Convexhull (P_i, dx * dy * dz)
    Initialization pattern memory: temp = ones(1, dx × dy × dz + 1,'double'); Initialize PDF memory: pdf =
    zeros(1,1,'double').
    for Traverse convex hull point cloud P_i
        Grab the current simulation position P_{i_count}, and obtain the corresponding search area N_p according to the
        search template size dx * dy * dz;
        if  Search field N_p valid
            if N_p's attribute set [Pro]' ∈temp
                Find the line equal to [Pro]' in the pattern memory temp, add 1 to the corresponding end column
                and update the PDF of the corresponding line;
            else
                Add the attribute set [Pro]' to the pattern memory temp and update the PDF corresponding to it;
            end
        end
    end
end
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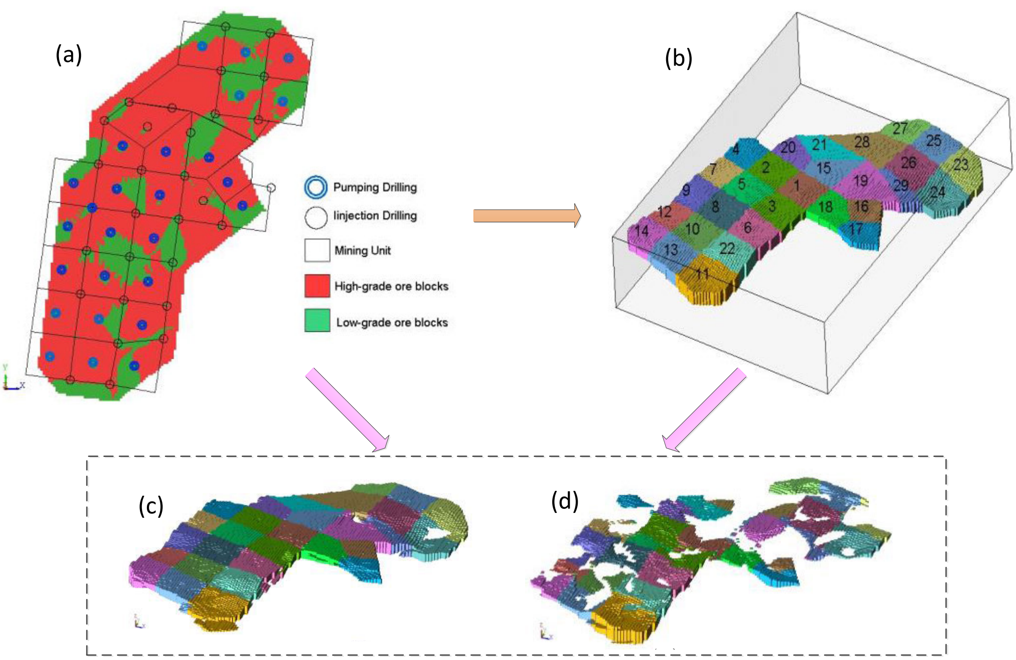

FIGURE 8. Geological model of a sandstone uranium deposit in northwestern China. (a) is the grade model diagram. (b) is the distribution map of mining units. There are a total of 29 mining units. The numbers in the figure are the numbers of each unit. (c) is high-grade ore blocks. (d) is low-grade ore blocks.

can evaluate the whole research mining area. We evaluate the homogeneity and continuity of each mining unit by comparing its spatial information entropy. Generally, the range of spatial information entropy after aggregation is $(0 \sim 1)$. The smaller the information entropy, the better the homogeneity and continuity of the mining area or the mining unit. On the contrary, the larger the information entropy, the poorer the homogeneity and continuity of the mining area or the mining unit. Based on the strategies proposed in the above sections, the detailed steps of our simulation algorithm proceed as illustrated in Algorithm 1.

The above algorithm flow introduces the implementation steps of the new method proposed in this paper. Step 1 is used to load the 3D geological model data and borehole coordinate data that need to be calculated. Then we divide the 3D model space into several mining units according to the plane layout of the borehole (as shown in Fig. 1). Step 2 is used to traverse each mining unit, and step 4 is used to traverse

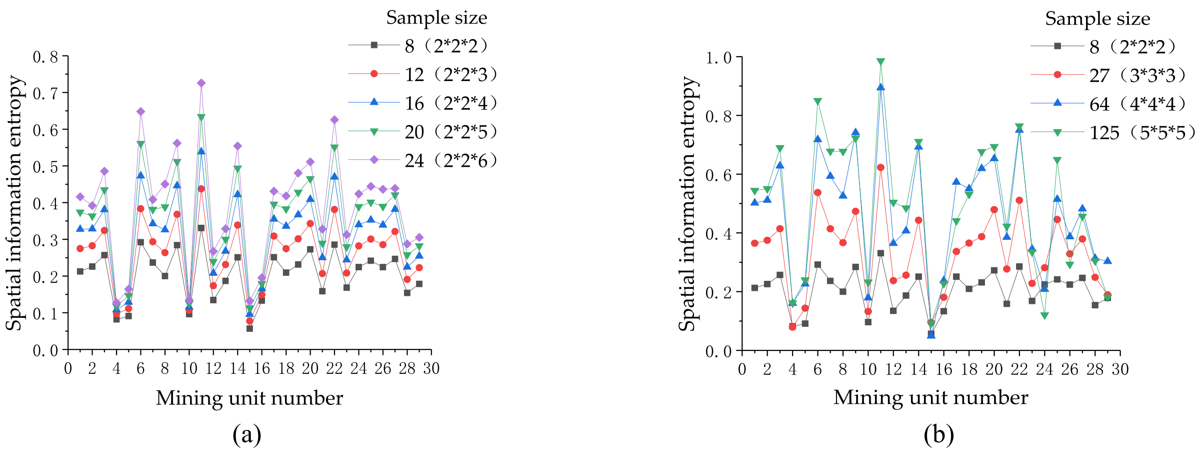

FIGURE 9. Relationship between spatial information entropy and template size. (a) Collection of sample sizes $2 * 2 * n$ $(2 \leq n \leq 6)$. (b) Collection of sample sizes $n * n * n$ $(2 \leq n \leq 5)$.

several attributes contained in the mining unit. Step 6 is achieved by calling the Alpha Shapes algorithm in MATLAB. In Section 3.2 we discussed that the value of 'Alpha' in this study is Inf (positive infinity). Step 7 is to fill the empty area of the convex hull, and the purpose is to facilitate the search template to traverse the entire convex hull. Use the point set Ω and the convex hull C_1 to find the intersection, and then add this part of the point cloud data to the convex hull C_1. Step 9 is to obtain the probability density function(PDF) of the convex hull. Obviously, step 9 is the most important procedure in our simulation algorithm. Our original idea was inspired by multi-point geostatistics modelling, and the detailed step algorithm is shown in Algorithm 2. Step 10 is to use formula (1) to calculate the spatial information entropy of the convex hull. On this basis, step 12 is used to aggregate the spatial information entropy of several convex hulls in a single mining unit. Step 14 is used to aggregate the spatial information entropy of several mining units in the mining area. The purpose is to obtain a total spatial information entropy that can characterize the homogeneity and continuity of the entire mining area.

III. PARAMETERIZATION AND PERFORMANCE ANALYSIS

In this section, we apply the above algorithm to several comprehensive experiments. Through comprehensive testing experiments, the sensitivity of the newly introduced parameters in the algorithm is fully verified and analysed. We compare the new method proposed in this study with the two existing methods in Section 2.1. The workflows and algorithms proposed in this work are developed in the MATLAB programming language. All experiments presented in this paper are implemented on a laptop computer with an Intel 4-Core i5-62000U Quad-core CPU, 16 GB RAM and 64 bit Windows 10.

A. SENSITIVITY ANALYSIS OF SEARCH TEMPLATE SCALE

Fig. 8 shows a model of a sandstone-hosted uranium deposit block in northwestern China. The relevant parameters of

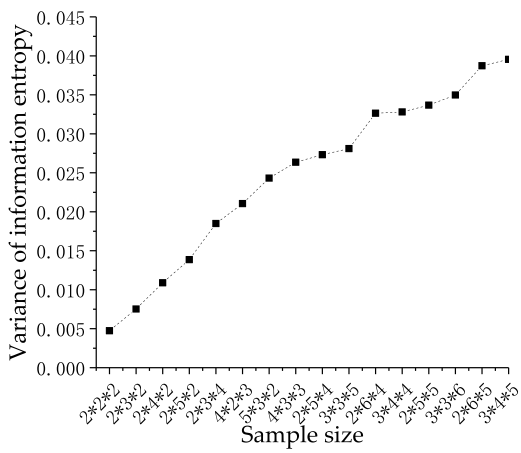

FIGURE 10. Variance of spatial information entropy.

the model are as follows: 1) The basic block size is 152685; 2) The basic block size is 2.5 m * 2.234375 m * 0.615234375 m. We divided the block model into 29 mining units according to the actual mining plan of the mine. The experiments in this section analyse the homogeneity and continuity of grade distribution in the model and divide them into high-grade ore blocks and low-grade ore blocks with 0.5‰ as the boundary.

To improve the calculation efficiency, the multiple-point templates selected in this study are all regular cuboid shapes. The size of the search template is one of the important parameters in the proposed algorithm. It affects the calculation speed and evaluation quality of the algorithm. Because of the spatial anisotropy of geological point cloud data, the convex hull we calculated is usually irregular. Selecting a template size that is too large will result in the template not obtaining enough search results. The size of the search template cannot be infinitely small. The specifications of the sample on a single plane (X-Y, Y-Z, X-Z direction) should be greater than or equal to 2 * 2. We do this to ensure that the homogeneity and continuity in the X, Y, and Z directions of the model are all characterized.

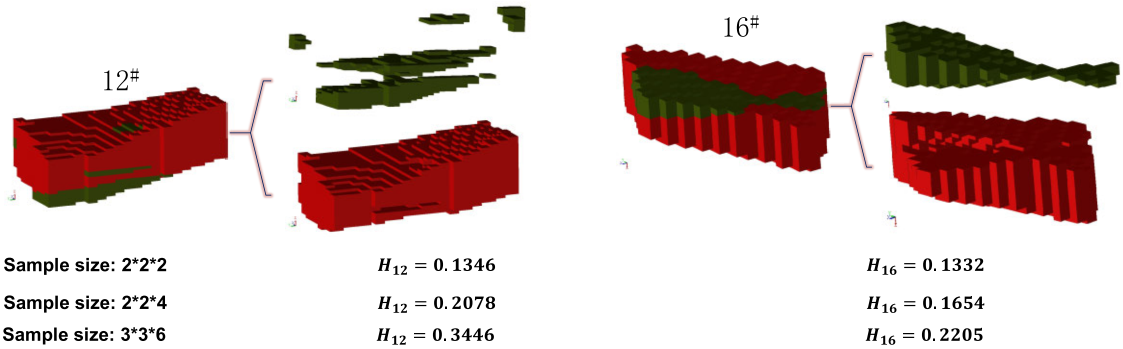

FIGURE 11. The relationship between the spatial information entropy of the $12^{\#}$ and $16^{\#}$ mining units and the search template.

Fig. 9 shows the change in the spatial information entropy of each mining unit as the search template size increases in the X, Y, and Z directions. In this test, the size of the template in each direction was increased from 2 to 6, while other parameters remained the same. With the increase of the sample size, the overall spatial information entropy of each mining unit has an increasing trend. Fig. 10 shows the variance of the spatial information entropy of the 29 elements in a single simulation result as the template size increases. In this test, as the sample size increases, the variance of the spatial information entropy of the 29 unit blocks also increases. With the increase of the size of the search template, the more theoretically generated configurations, the finer the homogeneity and continuity of the reflection. As shown in Fig. 11, the spatial information entropy of the $12^{\#}$ and $16^{\#}$ unit blocks in the 2 * 2 * 2 search template is almost equal, and the variance of the spatial information entropy of the two is approximately 0. With the increase in the size of the search template, the spatial information entropy of the $12^{\#}$ and $16^{\#}$ templates began to differ, and this change was in line with our expectations. However, the size of the search template can not be infinite. In this experiment, we found that when the sample size is 6 * 5 * 4, the size of the search template is larger than the convex hull size of the $24^{\#}$ mining unit, which makes it impossible for us to capture effective search results. Therefore, the information entropy of the mining unit can only be defined as 0. Obviously, the calculation results are biased. However, not every unit's spatial information entropy changes with the size of the template. For mining units with a more homogeneous and continuous distribution of properties, the spatial information entropy is independent of the size change of the template.

B. ALGORITHM PERFORMANCE

1) COMPARATIVE ANALYSIS OF EVALUATION ABILITY OF HOMOGENEITY AND CONTINUITY

To verify the ability of the new method proposed by this research to evaluate the homogeneity and continuity of the block model, we designed a comprehensive experiment in this section. We compared the new method with two existing methods (Shannon information entropy and spatial information entropy) to evaluate the homogeneity and continuity of the 29 mining units in Fig. 8. To ensure that the information entropy obtained by the three methods are comparable, the base of formula (1) is the same when calculating the information entropy in the experiments, and the search template size of the spatial information entropy and the new method is 4 * 3 * 2.

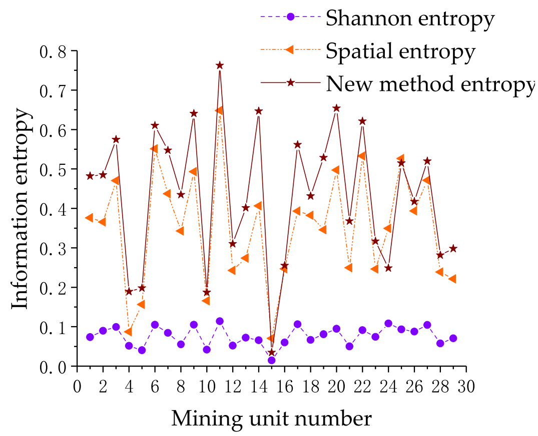

FIGURE 12. Information entropy distribution for three different methods.

Table 1 shows the evaluation results of the three methods, and the unit block numbers are arranged in ascending order according to the information entropy. As seen from the table, the evaluation results of the three methods on the completely homogeneous and continuous $15^{\#}$ units and the completely heterogeneous and discontinuous $11^{\#}$ units are basically the same. However, as the heterogeneity and discontinuity of grade attributes in mining units increase, the evaluation results of the three methods begin to differ. Fig. 12 shows the results of the information entropy distribution of the three methods. From the figure, it can be seen that the slope of the line chart of the new method is the largest.

Fig. 13 shows the mean and variance of the information entropy of the 29 mining units calculated by the three methods. The variance of the three methods' information entropy is: Shannon information entropy $Var_1 = 0.0006$, spatial information entropy $Var_2 = 0.0206$, and new method information entropy $Var_3 = 0.0309$. In summary, the new method proposed in this paper has better sensitivity and can better evaluate the homogeneity and continuity of each mining unit.

TABLE 1. Graphs of the evaluation results of the three methods. We sort the cell block numbers in ascending order according to the information entropy.

Numbering	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Shannon entropy	15#	5#	10#	21#	4#	12#	8#	28#	16#	14#	18#	29#	13#	1#	23#
Spatial entropy	15#	4#	5#	10#	29#	28#	12#	23#	16#	21#	13#	8#	19#	24#	2#
New method	15#	10#	4#	5#	24#	16#	28#	29#	12#	23#	21#	13#	26#	18#	8#
Numbering	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
Shannon entropy	19#	7#	26#	2#	22#	25#	20#	3#	27#	6#	9#	17#	24#	11#	
Spatial entropy	1#	18#	17#	26#	14#	7#	3#	27#	9#	20#	25#	22#	6#	11#	
New method	1#	2#	25#	27#	19#	7#	17#	3#	6#	22#	9#	14#	20#	11#	

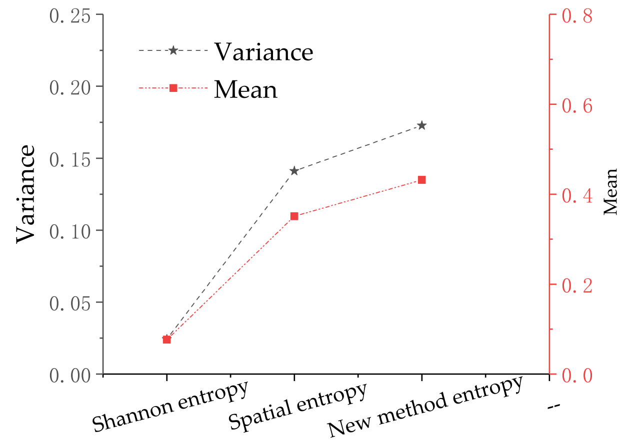

FIGURE 13. Means and variances of three information entropy calculation results.

Fig 14 shows the spatial distribution of 9#, 17#, and 25# unit blocks in the above model. It can be seen from the figure that the approximate order of the heterogeneity and discontinuity of the three mining units is $H_9 > H_{17} > H_{25}$. The evaluation results of the three methods of simulation experiments in this section are Shannon information entropy $H_9 = H_{17} = 0.1053$, $H_{25} = 0.0935$; spatial information entropy $H_9 = 0.4930, H_{17} = 0.3932, H_{25} = 0.5263$; new method information entropy $H_9 = 0.6407, H_{17} = 0.5613, H_{25} = 0.5148$. Obviously, the evaluation of the three maps by Shannon information entropy and space entropy has serious deviations, and the new method can well evaluate the heterogeneity and discontinuity of the grade attributes of the three mining units.

When optimizing the layout of mining areas and mining units according to the homogeneity and continuity of the 3D geological model, we focus on the sensitivity of the calculation results to each mining unit (that is, whether the calculation results can clearly distinguish the homogeneity and continuity of each mining unit). In the analysis of the sensitivity and evaluation ability of the new method, although we only know whether the results are consistent with our macroscopic realization, we do not use accurate values to measure the accuracy of the results. However, such a comparison method can still show the superiority of the new method.

2) COMPUTATIONAL PERFORMANCE ANALYSIS

The method proposed in this study is used to evaluate the continuity and homogeneity of the block model containing

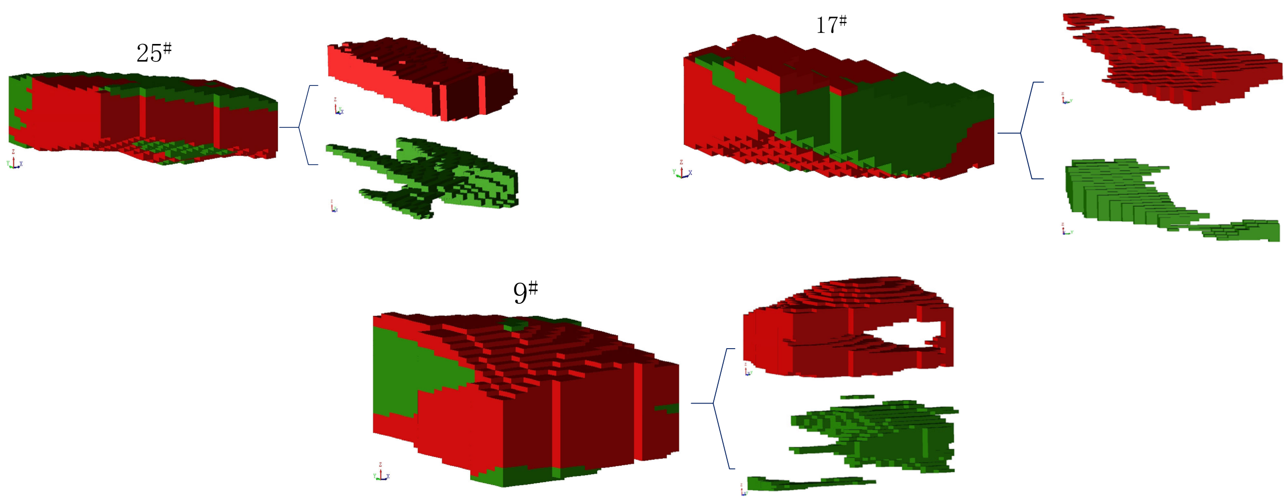

FIGURE 14. Fine-grained layout of grade models for 9#, 17#, 25# mining units.

152685 basic blocks in Fig. 8. In Section 4.1, the spatial information entropy calculations of different search templates are performed. As the size of the template in each direction increased from 2 to 6, the average calculation time achieved by each simulation was different, but the single simulation solution time was completed within 400 s.

IV. SYNTHETIC EXAMPLE

In order to further clarify the applicability of the method proposed in this paper, this section applies the new method to practical geological examples. The lower member of Xishanyao Formation of the mengqigur uranium deposit in Northwest China is a typical in-situ leachable sandstone uranium deposit. The exposed sand body in the exploration area is more than 1200 m in length and 1300-2500 m in width. The sand body has a net thickness of 7.60 to 40.75 m, an average thickness of 27.64 m, and a coefficient of variation of 23.57%. The sand body has a simple shape and a continuous distribution on the plane. The sand body is thick from south to east and gradually thins from north to west. The distribution of the internal characteristics (lithology, grade, etc.) of the sand body is relatively complex. Generally, the thickness of the mudstone interlayer of the ore bearing sand body increases gradually from the southeast to the northwest in the detailed investigation area. The ore bearing sand body is sandwiched with 1-3 layers of argillaceous interlayer, with a thickness of 0.20-16.20 m and an average of 3.08 M. the continuous length of the argillaceous interlayer varies from 50-400m on the section. Most of the sand bodies show the sedimentary characteristics of coarse and fine in the lower part, and coarse fine coarse in some places.

At present, the two mining areas that have been produced in the lower part of Xishanyao Formation are 16-1# mining area and 16-2# mining area. There are 55 boreholes in 16-1# mining area, 27 of which are liquid extraction drilling and 28 are liquid injection drilling, and 20 mining units are divided. There are 40 boreholes in 16-2# mining area, 15 of which are liquid extraction drilling, 25 are liquid injection drilling, and 15 mining areas are divided. The division of mining units in 16-1# and 16-2# mining areas is based on the experience of geologists and roughly in accordance with the unified shape (as shown in Fig. 15 (a)). The average grades of 16-1# mining area and 16-2# mining area (Fig. 15) are 9.63‰ and 3.61‰ respectively, but the leaching rate of uranium element in 16-1# mining area is significantly lower than that in 16-2# mining area when the production process (the same production system) and production conditions (such as leaching solution formula, chemical reaction time, pressure of extraction solution and injection solution) are the same. Here, the method proposed in this research is used to calculate the spatial information entropy of the grade and lithology of each mining unit in the two mining areas and evaluate the homogeneity and continuity of each mining unit, thereby revealing the geological reasons for the above phenomenon.

According to the actual engineering layout of the mine, the 16-1# mining area is divided into 21 mining units, and the 16-2# mining area is divided into 14 mining units. Fig. 15 shows two mining area block models, including grade attributes and lithology attributes. The basic block size is 2.5 * 2.234375 * 0.615234375. The size of the spatial information entropy search template is uniformly defined as 4 * 3 * 2. At the same time, to ensure the comparability of simulation results and all entropy values range from 0 to 1, the base value of Eq. (1) is the maximum value of the number of valid templates.

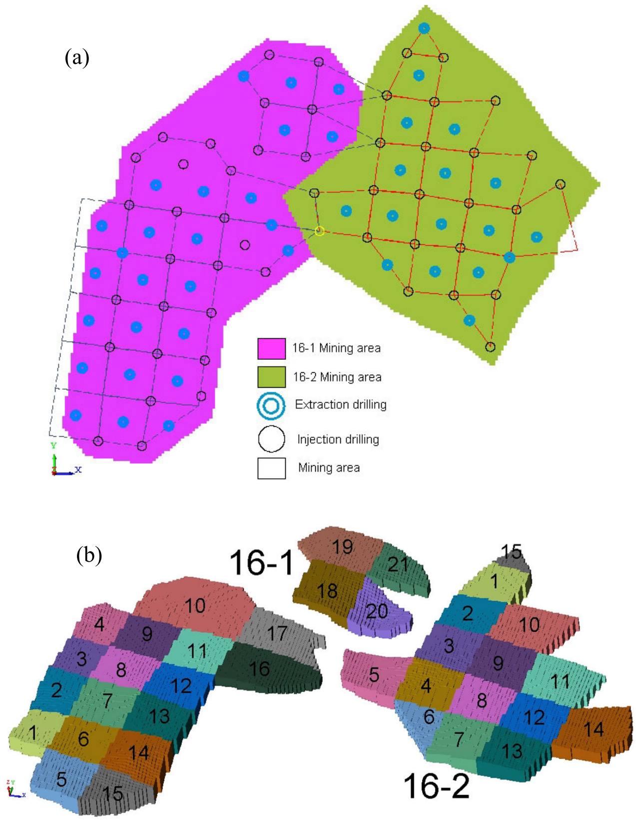

FIGURE 15. 16-1# and 16-2# mining area model. (a) Layout of boreholes and distribution of mining units in 16-1 and 16-2 mining areas. (b)We divided the model into 36 mining units according to the actual engineering layout of the mine, and the numbers in the figure are the number of each unit.

Fig. 16 (a) is the grade model of the 16-1# and 16-2# mining areas. With 0.5‰ as the threshold, the grade block model is divided into high-grade areas and low-grade areas. Fig. 16(b) shows the grade spatial information entropy of each mining unit in the 16-1# and 16-2# mining areas. We comprehensively analysed the correspondence between the continuity and homogeneity of 36 mining units and the spatial information entropy and divided them into four types: homogeneous and continuous regions (H/C) (H $\in$ 0 $\sim$ 0.2); weak heterogeneous and weak discontinuous regions (W. H/C) (H $\in$ 0.2 $\sim$ 0.4); heterogeneous and discontinuous regions (H/D)(H $\in$ 0.4 $\sim$ 0.6); strong heterogeneous and strongly discontinuous regions (S. H/D) (H $>$ 0.6). Figs. 16 (c)-(d) are typical diagrams of these four types of mining units in two mining areas. Based on a comprehensive analysis of Fig. 16, it can be seen that

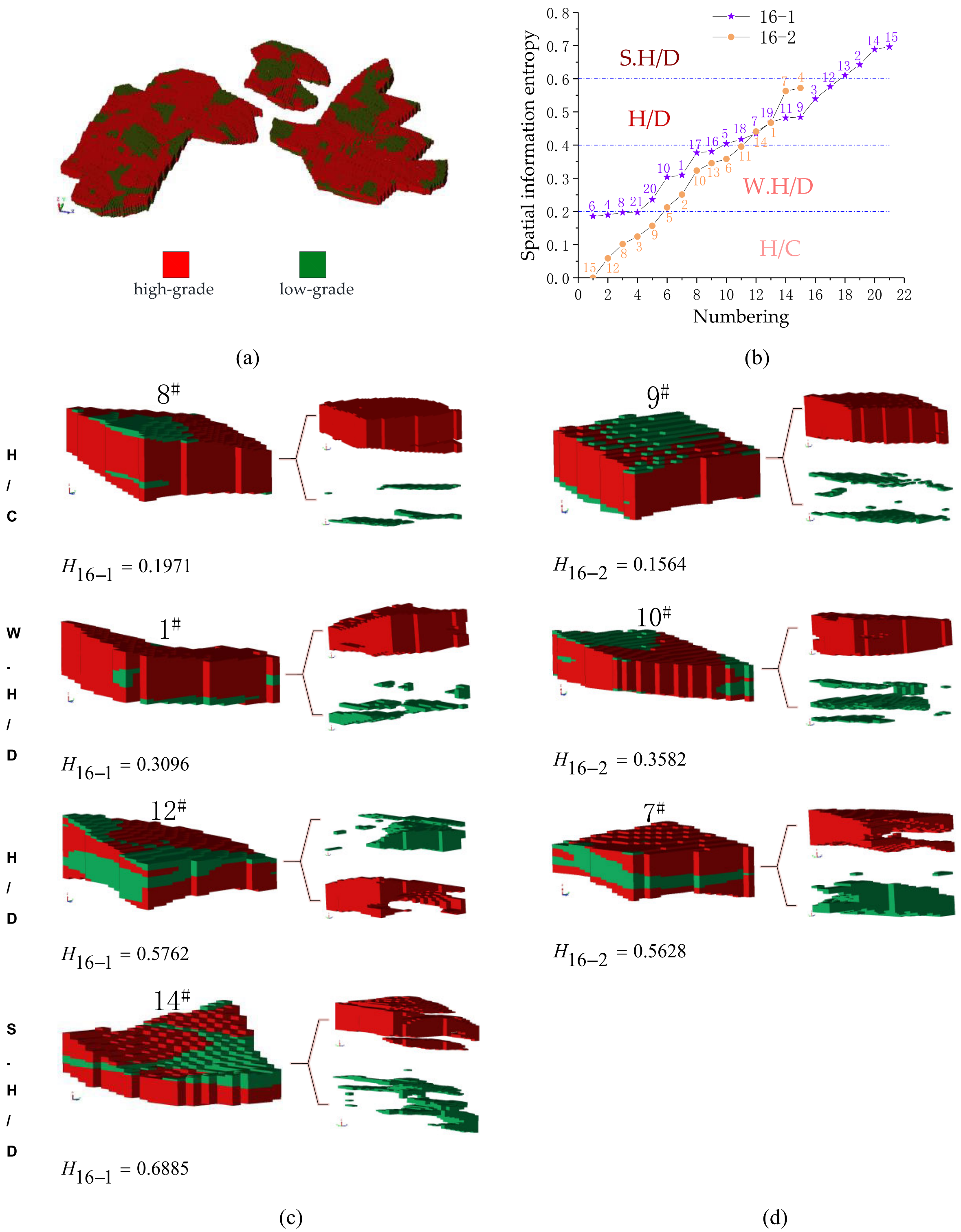

FIGURE 16. **Grade model spatial information entropy results. (a) Block model diagram of grade attributes of two mining areas. (b) The spatial information entropy distribution map of each mining unit. (c) Refined spatial layout of some mining units in 16-1# mining area. (d) Refined spatial layout of some mining units in 16-2# mining area.**

the spatial information entropy of the grade of each mining unit in the 16-1# mining area is concentrated in the weak heterogeneous area and the heterogeneous area, while the grade of the spatial information entropy of the 16-2# mining area is concentrated in the weak heterogeneous area homogeneous zone. According to Eq. (3), the comprehensive spatial information entropy of grades in 16-1 # and 16-2 # mining areas is calculated as follows: $H_{16-1} = 0.40$, $H_{16-2} = 0.26$.

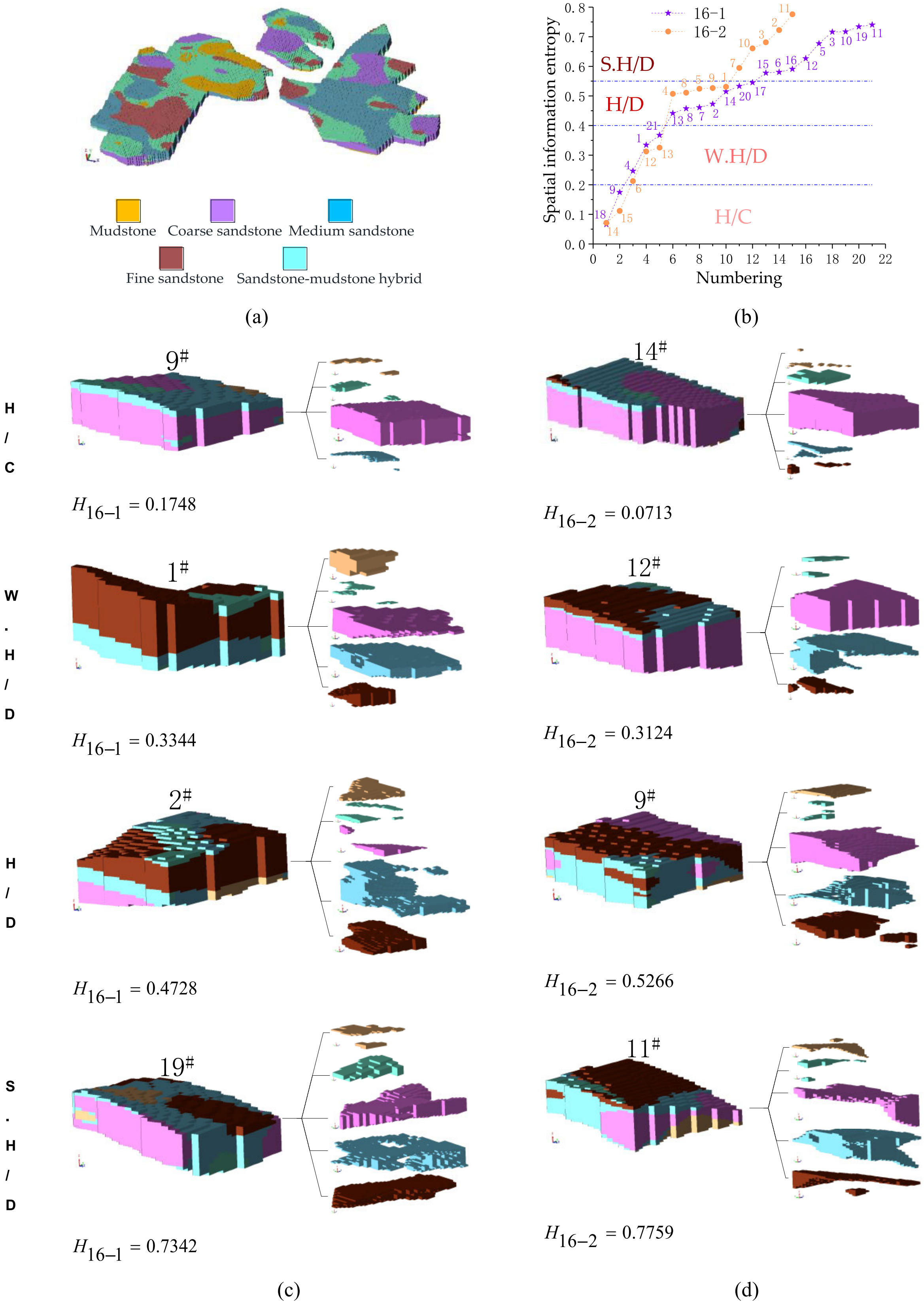

FIGURE 17. Spatial information entropy results for a lithology model. (a) Block model diagram of grade attributes of two mining areas. (b) The spatial information entropy distribution map of each mining unit. (c) Refined spatial layout of some mining units in 16-1# mining area. (d) Refined spatial layout of some mining units in 16-2# mining area.

TABLE 2. 16-1 # and 16-2 # mining area proportion of high-grade areas, coarse sandstone, medium sandstone, fine sandstone and sandstone-mudstone hybrid.

proportion / mining area	high-grade（%）	coarse sandstone（%）	medium sandstone（%）	Sandstone-mudstone hybrid（%）	fine sandstone（%）
$16\text{-}1^{\#}$ mining area	81.4	28	32	27	11
$16\text{-}2^{\#}$ mining area	87.3	49	20	23	4

There are 5 types of lithology in the mining area: mudstone, coarse sandstone, medium sandstone, fine sandstone, and sandstone-mudstone hybrid (Fig. 17 (a)). Fig 17 (b) shows the spatial information entropy distribution of the lithology of each mining unit in the study area. We comprehensively analysed the correspondence between the continuity and homogeneity of 36 mining units and the spatial information entropy and divided them into four types: homogeneous and continuous regions (H/C) ($H \in 0 \sim 0.2$); weak heterogeneous and weak discontinuous regions (W. H/C) ($H \in 0.2 \sim 0.4$); heterogeneous and discontinuous regions (H/D) ($H \in 0.4 \sim 0.55$); strong heterogeneous and strongly discontinuous regions (S. H/D) ($H > 0.55$). The lithology model contains five categories more than the grade model. When we use the same search template, the types of probability density functions theoretically generated by the lithology model are more abundant, so the sensitivity of the lithological model's spatial information entropy is higher than that of the grade model. Figs. 17 (c)-(d) are typical diagrams of these four types of mining units in two mining areas. By comprehensive analysis of Fig. 17, we can find that the lithological information entropy of the two mining areas is concentrated in the heterogeneous area and the strongly heterogeneous area. According to formula (3), the lithological comprehensive spatial information entropy of $16\text{-}1^{\#}$ and $16\text{-}2^{\#}$ mining areas are calculated as $H_{16-1} = 0.49$, $H_{16-2} = 0.41$.

According to statistics, the high-grade areas and various types of lithology in the two mining areas are shown in Table 2. From the table, we can see that 1) the $16\text{-}1^{\#}$ mining area has a high-grade area ratio of less than $16\text{-}2^{\#}$ mining area and 2) the $16\text{-}1^{\#}$ mining area has a lithological proportion outside the mudstone of 98% greater than the 16-2 # mining area. The area accounts for 96%, and the coarse sand portion of the $16\text{-}2^{\#}$ mining area accounts for nearly 50%.

The results suggest that the lithology and grade heterogeneity and discontinuity of the $16\text{-}1^{\#}$ mining area are relatively significant, and the lithology with low permeability accounts for a large proportion. In comparison, the homogeneity and continuity of lithology and grade of the $16\text{-}2^{\#}$ mining area are better than those of the $16\text{-}1^{\#}$ mining area, and coarse sandstone with good permeability accounts for a large proportion. The above conclusions reasonably explain why the leaching rate in the $16\text{-}1^{\#}$ mining area is significantly lower than in the $16\text{-}2^{\#}$ mining area, and we can use the information entropy to guide the optimization of the layout structure of the boreholes in $16\text{-}1^{\#}$ and $16\text{-}2^{\#}$ mining areas.

V. DISCUSSION AND CONCLUSION

In this paper, we presented a spatial information entropy calculation method based on a convex hull search strategy. The evaluation of the homogeneity and continuity of each attribute in the study area is a necessary work step in the mining design of sandstone uranium deposits. For a long time, due to the lack of mature evaluation indicators, the heterogeneity and discontinuity of the nature of the deposit have been ignored in previous engineering arrangements. Under the premise of constructing a 3-D block model, the new method proposed in this study can quantitatively evaluate the homogeneity and continuity of each natural property of the mineral layer. At the same time, comprehensive tests and application examples have verified the advantages of the new method in reservoir homogeneity and continuity evaluation.

Compared with the existing spatial entropy calculation methods, the newly proposed method requires pre-calculation that the properties of a single mining unit include different types of convex hulls and then scans each convex hull to obtain a probability density function by searching for a template instead of scanning the entire mining unit. This convex hull-based local search strategy can provide more reasonable multi-point statistical information because it can avoid interference with simulation results by other attributes unrelated to this attribute.

In addition, compared with the computational performance of Shannon information entropy and spatial information entropy, it is found that the new method proposed in this paper can compensate for their shortcomings. For example, for the models that are independent of each other and have no canine staggered characteristics, the existing methods cannot accurately evaluate its heterogeneity, and the method proposed in this study solved the above problems through the idea of partition solution and obtained the spatial information. At the same time, the calculated spatial information entropy can accurately evaluate the homogeneity and continuity of each mining unit.

Another important advantage of the proposed new method is a reasonable spatial information entropy fusion mechanism. The combination of two different information entropy fusion formulas is used to fuse the spatial information entropy of different attribute categories and different unit blocks. First, the spatial information entropy aggregation formula (Eq. (2)) based on addition is used to aggregate several independent spatial information entropies from the same mining unit to obtain a more stable information entropy.

Then, the information entropy aggregation formula based on multiplication (Eq. (3)) is used to aggregate the information entropy of the single mining unit obtained in the previous step. The weight of the above two steps is determined by the number of point sets contained in the corresponding convex hull.

The new method proposed in this paper is applied to two mining areas of sandstone-hosted uranium deposits in northwestern China. We calculated that the spatial information entropy of the 16-1# mining area grade model and lithology model is higher than the 16-2# mining area. This conclusion reasonably explains the problem that the leaching rate in mining area 16-1 is lower than that in mining area 16-2. At the same time, we can use the spatial information entropy to guide the optimization of drilling layout structure in 16-1 and 16-2 mining areas.

The new method proposed in this research can be further optimized to integrate the information entropy of various properties to comprehensively analyse the heterogeneity and discontinuity of the study area. In addition, further optimization of the implementation of the newly proposed method will further significantly improve the computing performance of the method.

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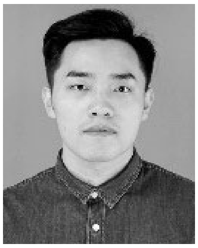

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