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An Optimal Fuzzy Control Method for Nonlinear Time-Delayed Batch Processes

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ABSTRACT An optimal fuzzy controller design scheme is proposed to address the influence of time delay and disturbance on the control performance of nonlinear batch processes. First, a two-dimensional (2D) equivalent Takagi-Sugano (T-S) fuzzy error model is formulated. By introducing a quadratic performance index function and adopting 2D Lyapunov-Krasovskii theory, the existence condition of the optimal fuzzy control law is given. Furthermore, its solvable condition, which depends on time-delay bounds, is constructed in terms of linear matrix inequalities, and its gain is obtained by using an optimization algorithm. This design has the advantages of faster tracking and better tracking performance. Finally, two different algorithms (with and without optimization) are used to control the water level of a triple-capacity water tank. The results show that the presented strategy is more effective and feasible.

INDEX TERMS Nonlinear time-delayed batch processes, 2D-T-S fuzzy model, fuzzy guaranteed cost control, 2D Lyapunov-Krasovskii functional approach, linear matrix inequalities.

I. INTRODUCTION

To improve production efficiency and save production costs, there is an urgent need for advanced control strategies and optimization methods in industrial production mode [1]. In many advanced control algorithms [2]–[11], feedback control and iterative learning control (ILC) has attracted much attention. For batch processes, ILC takes into account the batch and time direction characteristics of the batch process and has a better control effect than iterative learning control or feedback control. Since this algorithm was proposed [5], many achievements have been made. With the description of system models, the algorithm has been extended from linear systems [5], [6] to nonlinear systems [9]. Regarding the characteristic of phases, it has developed from single-phase [5], [6] to multiphase [7], [8], [11]. Even fault-tolerant control of systems has been achieved [3], [5], [9]–[11]. Because time delay and disturbances are the main factors of system performance degradation, the control algorithms designed for these two aspects are still a popular research topic.

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Time delays exist in many aspects of industrial processes. They can lead to system instability or the degradation of control performance. Studies on time-delayed systems have emerged in recent decades. In general, there are two methods to address the delay problem with Lyapunov stability theory: one is based on the Lyapunov-Rajomijin function (LRF) [12]–[14] method and the other is based on the Lyapunov-Krasuski function (LKF) method. The LKF method analyzes the stability of high-dimensional augmented systems in which the state variables include all delayed states [15], [16]. Although there is some difficulty in designing the V function, it is widely used due to its small degree of conservatism. Since it is impossible to avoid time delay in the batch process, it is essential to study the stability of the batch process with time delay. There are some research results at present [17]–[23]. For practical systems with disturbances, to devise a control law ensuring the stable operation of the system is only one of the goals; the other is to maintain the system with optimal control performance. Therefore, the guaranteed cost control method proposed by Chang and Peng has been applied [24]. Recently, some results have been obtained. Wang et al. proposed a delay-range-dependent method for iterative learning guaranteed cost control for batch processes [25] and then extended the results to optimal

fault-tolerant control [26]. At present, these research results have been extended to optimal control and optimal fault-tolerant control of multi-phase batch processes [27], [28].

All of the above research results are directly dependent on linear models. Batch processes have strong nonlinearity, and it is difficult to establish a suitable linear model [29], [30]. In this case, the famous Takagi-Sugano (T-S) fuzzy models [31] proposed by Japanese scholars have been applied, which provide a basic framework for the analysis and synthesis of fuzzy control and profoundly affect the study and application of fuzzy control theory [32]. Fuzzy control has been proven to be an effective method to deal with nonlinear systems [33]–[35], and various fuzzy control methods are emerging [36]–[40]. At present, the results are also reflected in time-delayed nonlinear systems [41], [42]. Moreover, the fault detection problem can be studied by fuzzy methods [43], [44]. With the development of research technology, some results on nonlinear control theory for batch processes have become available recently [45]–[50]. For nonlinear batch processes, the current solutions are mainly the following: (1) using model predictive control and feedback control to solve nonlinearity and constraint problems [45]; (2) treating nonlinearity as uncertainty, such as in [48], [49]; and (3) developing 2D fuzzy iterative learning control for batch processes with disturbances and time delays based on T-S fuzzy control [50].

As mentioned earlier, steady operation is the basic demand of the production process. It is the lifelong goal of all enterprises and researchers that systems have better control performance to achieve the goal of energy conservation and emission reduction. Based on this objective, the optimal control of batch processes with time delay has attracted much attention. However, all the models considered in these results are linear system models.

For this reason, fuzzy iterative learning optimal control for nonlinear time-delayed batch processes is discussed in the paper. The specific strategies are as follows. First, a fuzzy controller is designed, and an error function is introduced to establish the equivalent fuzzy 2D Rosser model. Then, a fuzzy update law preserving optimal performance is designed, and solvable conditions are given, where the upper bound of the quadratic performance index is not more than a certain bounded value. The scheme of the designed control law is given in terms of linear matrix inequalities (LMIs). Furthermore, an algorithm analysis of the optimal performance index is proposed, and the controller gain and optimal performance index are obtained by certain constraint conditions. Compared with a controller without optimization algorithms, the advantage of this kind of controller is that the actual output of the system tracks the given output faster and the tracking performance is better. In the long run, the goals of energy conservation and consumption reduction are achieved. Taking controlling the water level of the three tanks as an example, two different algorithms (with or without optimization) are used for comparison, and the results show that the algorithm presented in this article is more effective and feasible.

Certain notation is used throughout the paper. \mathbb{R}^n is an n -dimensional Euclidean space and $\mathbb{R}^{(n+l) \times (n+l)}$ is a set of $(n+l) \times (n+l)$ real matrices. $\bar{x}_{i,k}^T$ denotes the transpose of $\bar{x}_{i,k}$. # represents a transposed element in the symmetric position.

II. PROBLEM FORMULATION

Omitting the step of transforming the nonlinear batch process model into a fuzzy model as in [39], the following model with unknown internal uncertainties and external disturbance is given directly:

$$\left\{ \begin{array}{l} x_k(t+1) = \sum_{i=1}^r h_i(x_k(t)) A_i(t, k) x_k(t) \\ \quad + \sum_{i=1}^r h_i(x_k(t)) A_{id} x_k(t-d(t)) \\ \quad + \sum_{i=1}^r h_i(x_k(t)) B_i u_k(t) + w_k(t) \\ y_k(t) = C_i x_k(t) \\ x_k(0) = x_{0,k}, \quad i = 1, 2, \dots, r, \quad 0 \leq t \leq T; \quad k = 1, 2, \dots \end{array} \right. \quad (1)$$

where $x_k(t) \in \mathbb{R}^n$, $x_k(t-d(t)) \in \mathbb{R}^n$, $y_k(t) \in \mathbb{R}^l$ and $u_k(t) \in \mathbb{R}^m$ denote the state, time delay state, output and control input, respectively; $A_i(t, k) = A_i + \Delta A_i(t, k) = A_i + D_i \Delta_i(t, k) E_i$, $A_i, A_{id}, B_i, C_i, D_i$ and E_i are constant matrices under the corresponding rules such that $\Delta_i^T(t, k) \Delta_i(t, k) \leq I$; r is the fuzzy rule number; $x_k(0)$ denotes the initial value of the k th batch; $w_k(t)$ is the external disturbance; the time-varying delay $d(t)$ satisfies $\underline{d} \leq d(t) \leq \bar{d}$, where \underline{d} and \bar{d} represent the lower and upper bounds of time delay; t is time; k is the batch; and $h_i(x_k(t))$ satisfies $\sum_{i=1}^r h_i(x_k(t)) = 1$, $h_i(x_k(t)) \geq 0$.

For system (1), the objective of this paper is to propose an optimal control strategy to guarantee that the actual output of the system can follow the set trajectory of the given output and preserve the optimal performance J^* .

III. THE DESIGN OF THE FUZZY OPTIMAL CONTROLLER

This section is divided into two parts: the first part is the construction of the equivalent model. The second part is the control law design based on this equivalent model. In this part, the existence condition and solvable condition of the controller are given, and the upper bound of the performance index that the designed controller must satisfy is also given.

A. EQUIVALENT SYSTEM FORMULATION

To accomplish the above control target, we use the iterative learning control method, and the law is designed as follows:

$$\left\{ \begin{array}{l} u_k(t) = u_{k-1}(t) + r_k(t) \\ u(t, 0) = 0 \end{array} \right. , \quad t = 0, 1, 2, \dots, T \quad (2)$$

where $r_k(t)$ is the update law designed at time t in batch k and $u(t, 0)$ is the initial iterative value, which is set as 0. Thus,

if we want to design $u_k(t)$, we only need to design the update law $r_k(t)$ to guarantee that the system output $y_k(t)$ tracks the desired output y_r .

The system state error and output tracking error are defined as (3) and (4), respectively:

$$\Delta(x_k(t)) = x_k(t) - x_{k-1}(t) \tag{3}$$

$$e_k(t) = y_r - y_k(t) \tag{4}$$

We can obtain from (1) - (3) that

$$\begin{aligned} \Delta(x_k(t+1)) &= \sum_{i=1}^r h_i(x_k(t))A_i(t, k)\Delta(x_k(t)) \\ &+ \sum_{i=1}^r h_i(x_k(t))A_{id}\Delta(x_k(t-d(t))) \\ &+ \sum_{i=1}^r h_i(x_k(t))B_i r_k(t) + \bar{w}_k(t) \end{aligned} \tag{5}$$

where

$$\bar{w}_k(t) = \omega_k(t) + \Delta(w_k(t)), \Delta(w_k(t)) = w_k(t) - w_{k-1}(t),$$

$$\begin{aligned} \omega_k(t) &= \sum_{i=1}^r \Delta(h_i(x_k(t)))A_i x_{k-1}(t) \\ &+ \sum_{i=1}^r h_i(x_k(t))\Delta A_i(t, k)x_{k-1}(t) \\ &- \sum_{i=1}^r h_i(x_{k-1}(t))\Delta A_i(t, k-1)x_{k-1}(t) \\ &+ \sum_{i=1}^r \Delta(h_i(x_k(t)))B_i u_{k-1}(t) \\ &+ \sum_{i=1}^r \Delta(h_i(x_k(t)))A_{id}x_{k-1}(t-d(t)) \end{aligned}$$

For $y_k(t) = C_i x_k(t)$, taking into account the special circumstance that $C_i = C (i = 1, 2, \dots, r)$, we can conclude from (1) - (5) that

$$\begin{aligned} e_k(t+1) &= e_{k-1}(t+1) - C \left(\sum_{i=1}^r h_i A_i(t, k)\Delta(x_k(t)) + \bar{w}_k(t) \right) \\ &+ \sum_{i=1}^r h_i B_i r_k(t) + \sum_{i=1}^r h_i A_{id}\Delta(x_k(t-d(t))) \end{aligned} \tag{6}$$

We design the global 2D T-S fuzzy law of the system:

$$r_k(t) = \sum_{i=1}^r h_i K_i \begin{bmatrix} \Delta(x_k(t)) \\ e_{k-1}(t+1) \end{bmatrix}. \tag{7}$$

Let

$$\begin{aligned} \bar{x}'_{t,k} &= \begin{bmatrix} \Delta(x_k(t+1)) \\ e_k(t+1) \end{bmatrix} = \begin{bmatrix} x_k^h(t+1) \\ x_{k+1}^v(t) \end{bmatrix}, \\ \bar{x}_{t,k} &= \begin{bmatrix} \Delta(x_k(t)) \\ e_{k-1}(t+1) \end{bmatrix} = \begin{bmatrix} x_k^h(t) \\ x_k^v(t) \end{bmatrix}, \end{aligned}$$

$$\bar{x}_{d,h} = \begin{bmatrix} \Delta_k(x(t-d(t))) \\ e_{k-1-h(k-1)}(t+1) \end{bmatrix} = \begin{bmatrix} x_d^h \\ x_h^v \end{bmatrix},$$

$$h_i = h_i(x_k(t)), \quad \bar{w}_k(t) = \bar{w}_{t,k}.$$

From formulas (5)-(7), the equivalent 2D model is represented as follows:

$$\begin{aligned} \bar{x}'_{t,k} &= \sum_{i=1}^r \sum_{i=1}^r h_i^2 \hat{A}_{ii}(t, k)\bar{x}_{t,k} + \sum_{i=1}^r h_i \bar{A}_{id}\bar{x}_{d,h} + \bar{C}\bar{w}_{t,k} \\ &+ 2 \sum_{i=1}^r \sum_{i<j} h_i h_j \left(\frac{\hat{A}_{ij}(t, k) + \hat{A}_{ji}(t, k)}{2} \right) \bar{x}_{t,k} \\ z_{t,k} &= e_{k-1}(t+1) = \bar{D}\bar{x}_{t,k} \end{aligned} \tag{8}$$

where

$$\hat{A}_{ij}(t, k) = \bar{A}_i(t, k) + \bar{B}_i K_j, \hat{A}_{ji} = \bar{A}_j(t, k) + \bar{B}_j K_i (i, j \leq r).$$

Suppose $h(k-1)$ satisfies $h \leq h(k-1) \leq \bar{h}$, $\bar{A}_i(t, k) = \begin{bmatrix} A_i(t, k) & \bar{0} \\ -CA_i(t, k) & I \end{bmatrix}$, $\bar{A}_{id} = \begin{bmatrix} A_{id} & \bar{0} \\ -CA_{id} & \bar{0} \end{bmatrix}$, $\bar{C} = \begin{bmatrix} I \\ -C \end{bmatrix}$, $\bar{D} = [\bar{0} \ I]$, where I is a unit matrix and $\bar{0}$ is a zero matrix.

System (8) is the equivalent model of the original model (1). Under this model, the design of the fuzzy control law $u_k(t)$ is transformed into the design of the fuzzy update law $r_k(t)$. Next, we need to design the update law (7).

Remark 1: At present, there are two main methods for designing controllers. One is to regard the batch process as a one-dimensional system only related to time, such as tracking control [42]. In this kind of method, the controller is composed of an extended dimension state formed by combining the state error of adjacent time and the tracking error. It is mainly aimed at reducing information mismatch between batches. If information between batches is repeated, the two-time-dimension (2D) control method that we propose in this paper is a good choice. This method adopts an iterative learning control strategy. By introducing the state error along with batches and tracking error, the original model is transformed into an equivalent 2D Roessor model, and a control law including time and batch direction is designed, which has the merits of fast tracking and good tracking performance. In particular, optimal control is considered in this paper. The control effect will therefore be better.

B. THE DESIGN OF THE FUZZY OPTIMAL CONTROLLER

For the fuzzy time-delay system (8), we define the performance index function:

$$\begin{aligned} J &= \sum_{t=0}^{N_1} \sum_{k=1}^{N_2} \left[\bar{x}_{t,k}^T Q_1 \bar{x}_{t,k} + r_{t,k}^T Q_2 r_{t,k} \right] \\ &= \sum_{t=0}^{N_1} \sum_{k=1}^{N_2} \bar{x}_{t,k}^T \left[Q_1 + \sum_{i=1}^r \sum_{j=1}^r h_i h_j K_i^T Q_2 K_j \right] \bar{x}_{t,k} \end{aligned} \tag{9}$$

where $Q_1, Q_2 > 0$.

For a two-dimensional system, assume there is a set of finite initial conditions and two positive integers

satisfying

$$\|\bar{x}_{(t,-h)}\| = \max\{\|\bar{x}(t, 0)\|, \|\bar{x}(t, -h(0))\|, \|\Delta(t, -h(0))\|\} \\ t \geq r_1 \geq 0, \quad -\bar{h} \leq k \leq 0 \quad (10)$$

$$\|\bar{x}_{(-d,k)}\| = \max\{\|\bar{x}(0, k)\|, \|\bar{x}(-d(0), k)\|, \|\Delta(-d(0), k)\|\} \\ k \geq r_2 \geq 0, \quad -\bar{d} \leq t \leq 0 \quad (11)$$

where $r_1 < \infty$ and $r_2 < \infty$ are positive integers. The initial boundary condition is:

$$S = \left\{ \begin{aligned} &\|\bar{x}_{(t,-h)}\|, \|\bar{x}_{(-d,k)}\| \in R^n : \|\bar{x}_{(t,-h)}\| = \Phi \vartheta_1, \\ &\|\bar{x}_{(-d,k)}\| = \Phi \vartheta_2, \vartheta_m^T \vartheta_m \leq I, (m = 1, 2) \end{aligned} \right\} \quad (12)$$

where Φ is a given matrix.

Remark 2: In contrast to existing research results [33], to realize fast tracking control of the system and maintain a certain control performance, (9)-(12) are introduced here. Therefore, general fuzzy control is transformed into fuzzy optimal control. In addition, because the system state is affected by time delay, the initial conditions are defined to be related to time delay. Such consideration may reduce the conservatism of the system.

Definition 1: For any scalar $\gamma > 0$, $\bar{w}_{t,k} \neq 0$, and zero boundary conditions (12), take any disturbances as $\bar{w}_{t,k} \in I$, if the system output satisfies

$$\|z_{t,k}\| < \gamma \|\bar{w}_{t,k}\|$$

then system (8) is stable and has H ∞ performance γ .

Definition 2: For all admissible uncertain parameters and $\gamma > 0$, $r_{t,k}^*$ is called the robust H ∞ optimal control law for system (8) if the following conditions are satisfied:

- (1) when $\bar{w}_{t,k} = 0$, system (8) is asymptotically stable,
- (2) under the zero initial condition, the controller output $z(t, k)$ satisfies $\|z_{t,k}\| < \gamma \|\bar{w}_{t,k}\|$,
- (3) the cost function of system (8) satisfies $J \leq J^*$.

Lemma 1 [51]: For any vector $\psi(t) \in R^n$, there are positive values κ_0, κ_1 and a matrix $0 < R \in R^{n \times n}$ such that the following inequality holds:

$$-(\kappa_1 - \kappa_0 + 1) \sum_{t=\kappa_0}^{\kappa_1} \psi^T(t) R \psi(t) < - \sum_{t=\kappa_0}^{\kappa_1} \psi^T(t) R \sum_{t=\kappa_0}^{\kappa_1} \psi(t)$$

Based on the above definition and lemma, the existence condition of the fuzzy update law is given. The results are as follows:

Theorem 1: For system (8) with (10-12), for any given scalars $0 < \underline{d} < \bar{d}$, $0 < \underline{h} < \bar{h}$ and matrices $Q_1, Q_2 > 0$, when $\bar{w}_{t,k} = 0$ holds, if the matrices to be solved $P, Q, M, G = \text{diag}(\bullet_h, \bullet_v) > 0$ and the controller gains K_i, K_j satisfy the following inequalities:

$$\begin{bmatrix} \psi_1 & \bar{0} & G & \Gamma_{14} & \Gamma_{15} & Q_1^{\frac{1}{2}} & K_1^T Q_2^{\frac{1}{2}} \\ \# & -Q & \bar{0} & \bar{A}_{id}^T P & \bar{A}_{id}^T M & \bar{0} & \bar{0} \\ \# & \# & -G - M & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \# & \# & \# & -P & \bar{0} & \bar{0} & \bar{0} \\ \# & \# & \# & \# & -T^{-2} M & \bar{0} & \bar{0} \\ \# & \# & \# & \# & \# & -I & \bar{0} \\ \# & \# & \# & \# & \# & \# & -I \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} \psi_1 & \bar{0} & G & \Gamma_{24} & \Gamma_{25} & Q_1^{\frac{1}{2}} & K_{ij}^T Q_2^{\frac{1}{2}} \\ \# & -Q & \bar{0} & \bar{A}_{id}^T P & \bar{A}_{id}^T M & \bar{0} & \bar{0} \\ \# & \# & -G - M & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\ \# & \# & \# & -P & \bar{0} & \bar{0} & \bar{0} \\ \# & \# & \# & \# & -T^{-2} M & \bar{0} & \bar{0} \\ \# & \# & \# & \# & \# & -I & \bar{0} \\ \# & \# & \# & \# & \# & \# & -I \end{bmatrix} < 0 \quad (14)$$

then system (8) is asymptotically stable. Additionally, the cost function (9) satisfies the following upper bound:

$$\begin{aligned} J \leq & \sum_{t=0}^{N_1} \left[\bar{x}_{t,0}^{hT} P_h \bar{x}_{t,0}^h + \bar{x}_{t,0}^{vT} P_v \bar{x}_{t,0}^v + \sum_{r=t-d(t)}^{t-1} \bar{x}_{r,0}^{hT} Q_h \bar{x}_{r,0}^h \right. \\ & + \sum_{r=-h(0)}^{-1} \bar{x}_{t,r}^{vT} Q_v \bar{x}_{t,r}^v \\ & + \sum_{r=t-\bar{d}}^{t-1} \bar{x}_{r,0}^{hT} M_h \bar{x}_{r,0}^h + \sum_{r=-\bar{h}}^{-1} \bar{x}_{t,r}^{vT} M_v \bar{x}_{t,r}^v \\ & + \sum_{s=-\bar{d}}^{-\bar{d}} \sum_{r=t+s}^{t-1} \bar{x}_{r,0}^{hT} Q_h \bar{x}_{r,0}^h \\ & + \sum_{s=-\bar{h}}^{-\bar{h}} \sum_{r=s}^{-1} \bar{x}_{t,r}^{vT} Q_v \bar{x}_{t,r}^v + \bar{d} \sum_{s=-\bar{d}}^{-\bar{d}} \sum_{r=t+s}^{t-1} \Delta_{r,0}^{hT} G_h \Delta_{r,0}^h \\ & \left. + \bar{h} \sum_{s=-\bar{h}}^{-\bar{h}} \sum_{r=s}^{-1} \Delta_{t,r}^{vT} G_v \Delta_{t,r}^v \right] \\ & + \sum_{k=0}^{N_2} \left[\bar{x}_{0,k}^{hT} P_h \bar{x}_{0,k}^h + \bar{x}_{0,k}^{vT} P_v \bar{x}_{0,k}^v + \sum_{r=d(0)}^{-1} \bar{x}_{r,k}^{hT} Q_h \bar{x}_{r,k}^h \right. \\ & + \sum_{r=k-h(k)}^{k-1} \bar{x}_{0,r}^{vT} Q_v \bar{x}_{0,r}^v + \sum_{r=-\bar{d}}^{-1} \bar{x}_{r,k}^{hT} M_h \bar{x}_{r,k}^h \\ & + \sum_{r=k-\bar{h}}^{k-1} \bar{x}_{0,r}^{vT} M_v \bar{x}_{0,r}^v + \sum_{s=-\bar{d}}^{-\bar{d}} \sum_{r=s}^{-1} \bar{x}_{r,k}^{hT} Q_h \bar{x}_{r,k}^h \\ & + \sum_{s=-\bar{h}}^{-\bar{h}} \sum_{r=k+s}^{k-1} \bar{x}_{0,r}^{vT} Q_v \bar{x}_{0,r}^v + \bar{d} \sum_{s=-\bar{d}}^{-\bar{d}} \sum_{r=s}^{-1} \Delta_{r,k}^{hT} G_h \Delta_{r,k}^h \\ & \left. + \bar{h} \sum_{s=-\bar{h}}^{-\bar{h}} \sum_{r=k+s}^{k-1} \Delta_{0,r}^{vT} G_v \Delta_{0,r}^v \right] \\ & = J^* \end{aligned} \quad (15)$$

where $\Gamma_{14} = \hat{A}_{ii}^T(t, k)P$, $\Gamma_{15} = (\hat{A}_{ii}^T(t, k) - I)M$,

$$\Gamma_{24} = \hat{H}_{ij}^T(t, k)P, \Gamma_{25} = (\hat{H}_{ij}^T(t, k) - I)M,$$

$$T = \text{diag}(\bar{d}I_h, \bar{h}I_v), D = \text{diag}((\bar{d} - d)I_h, (\bar{h} - h)I_v),$$

$$\hat{A}_{ii}(t, k) = \bar{A}_i(t, k) + \bar{B}_i K_i, K_{ij} = \frac{K_i + K_j}{2},$$

$$\hat{H}_{ij} = \frac{\hat{A}_{ij}^T(t, k) + \hat{A}_{ji}^T(t, k)}{2}, \text{ and } \psi_1 = -P + G + DQ + Q - M.$$

Proof: Define: $\Delta_{r,k}^h = \bar{x}_{r+1,k}^h - \bar{x}_{r,k}^h$, $\Delta_{t,r}^v = \bar{x}_{t,r+1}^v - \bar{x}_{t,r}^v$, $\bar{x}_{\bar{d},\bar{h}} = \begin{bmatrix} \bar{x}_{t-\bar{d},k}^h \\ \bar{x}_{t,k-\bar{h}}^v \end{bmatrix}$, and $\phi_{t,k} = \begin{bmatrix} \bar{x}_{t,k}^T & \bar{x}_{d,h}^T & \bar{x}_{\bar{d},\bar{h}}^T \end{bmatrix}^T$. Select the following Lyapunov function:

$$V(\bar{x}_{t,k}) = V_h(\bar{x}_{t,k}^h) + V_v(\bar{x}_{t,k}^v) \quad (16)$$

where

$$V_h(\bar{x}_{t,k}^h) = \sum_{n=1}^5 V_{hn}(\bar{x}_{t,k}^h), \quad V_v(\bar{x}_{t,k}^v) = \sum_{n=1}^5 V_{vn}(\bar{x}_{t,k}^v),$$

$$V_{h1}(\bar{x}_{t,k}^h) = \bar{x}_{t,k}^{hT} P_h \bar{x}_{t,k}^h, \quad V_{v1}(\bar{x}_{t,k}^v) = \bar{x}_{t,k}^{vT} P_v \bar{x}_{t,k}^v,$$

$$V_{h2}(\bar{x}_{t,k}^h) = \sum_{r=t-\bar{d}(t)}^{t-1} \bar{x}_{r,k}^{hT} Q_h \bar{x}_{r,k}^h,$$

$$V_{v2}(\bar{x}_{t,k}^v) = \sum_{r=k-\bar{h}(k)}^{k-1} \bar{x}_{t,r}^{vT} Q_v \bar{x}_{t,r}^v,$$

$$V_{h3}(\bar{x}_{t,k}^h) = \sum_{r=t-\bar{d}}^{t-1} \bar{x}_{r,k}^{hT} G_h \bar{x}_{r,k}^h,$$

$$V_{v3}(\bar{x}_{t,k}^v) = \sum_{r=k-\bar{h}}^{k-1} \bar{x}_{t,r}^{vT} G_v \bar{x}_{t,r}^v,$$

$$V_{h4}(\bar{x}_{t,k}^h) = \sum_{s=-\bar{d}}^{-\bar{d}+t+s} \sum_{r=t+s}^{t-1} \bar{x}_{r,k}^{hT} Q_h \bar{x}_{r,k}^h,$$

$$V_{v4}(\bar{x}_{t,k}^v) = \sum_{s=-\bar{h}}^{-\bar{h}+r+k+s} \sum_{r=k+s}^{k-1} \bar{x}_{t,r}^{vT} Q_v \bar{x}_{t,r}^v,$$

$$V_{h5}(\bar{x}_{t,k}^h) = \bar{d} \sum_{s=-\bar{d}}^{-\bar{d}+r=t+s} \sum_{r=t+s}^{t-1} \Delta_{r,k}^{hT} M_h \Delta_{r,k}^h,$$

$$V_{v5}(\bar{x}_{t,k}^v) = \bar{h} \sum_{s=-\bar{h}}^{-\bar{h}+r=k+s} \sum_{r=k+s}^{k-1} \Delta_{t,r}^{vT} M_v \Delta_{t,r}^v,$$

and $P_h, P_v, Q_h, Q_v, M_h, M_v, G_h$ and G_v are unknown positive definite matrices. The increment of $V(\bar{x}_{t,k})$ is expressed as

$$\Delta V(\bar{x}_{t,k}) = V_h(\bar{x}_{t+1,k}^h) - V_h(\bar{x}_{t,k}^h) + V_v(\bar{x}_{t,k+1}^v) - V_v(\bar{x}_{t,k}^v) \quad (17)$$

With Lemma 1, we can obtain

$$\bar{x}_{t,k}^T Q_1 \bar{x}_{t,k} + r_{t,k}^T Q_2 r_{t,k} + \Delta V(\bar{x}_{t,k}) = \phi_{t,k}^T \Pi \phi_{t,k} \quad (18)$$

where $\Pi = \psi + \Lambda_1^T P \Lambda_1 + \Lambda_2^T H^2 R \Lambda_2$,

$$\psi = \begin{bmatrix} \psi_1 + Q_1 + K_i^T Q_2 K_j & \bar{0} & M \\ \# & -Q & \bar{0} \\ \# & \# & -G - M \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r h_i h_j \hat{A}_{ij}(t, k) & \sum_{i=1}^r h_i \bar{A}_{id} & \bar{0} \end{bmatrix}, \text{ and}$$

$$\Lambda_2 = \begin{bmatrix} \sum_{i=1}^r \sum_{j=1}^r h_i h_j \hat{A}_{ij}(t, k) - I & \sum_{i=1}^r h_i \bar{A}_{id} & \bar{0} \end{bmatrix}.$$

From (13)-(14), we can obtain $\phi_{t,k}^T \Pi \phi_{t,k} < 0$.

For $\bar{x}_{t,k}^T Q_1 \bar{x}_{t,k} + r_{t,k}^T Q_2 r_{t,k} \geq 0$, $\Delta V(\bar{x}_{t,k}) \leq 0$. Thus,

$$V_h(\bar{x}_{t+1,k}^h) + V_v(\bar{x}_{t,k+1}^v) \leq V_h(\bar{x}_{t,k}^h) + V_v(\bar{x}_{t,k}^v).$$

For the above inequality, t is added from N to 0 and k is added from 0 to N . For any nonnegative integer N and from Lemma 1 in [6], we conclude that

$$\begin{aligned} & V(\bar{x}_{1,N}^h) + V(\bar{x}_{0,N+1}^v) + V(\bar{x}_{2,N-1}^h) + V(\bar{x}_{1,N-1}^v) \\ & + \dots + V(\bar{x}_{N,0}^h) + V(\bar{x}_{N,1}^v) \\ & = \sum_{t+k=N+1} V \left(\begin{bmatrix} \bar{x}_{t,k}^h \\ \bar{x}_{t,k}^v \end{bmatrix} \right) \leq V \left(\begin{bmatrix} \bar{x}_{0,k}^h \\ \bar{x}_{0,k}^v \end{bmatrix} \right) \\ & + \dots + V \left(\begin{bmatrix} \bar{x}_{N,0}^h \\ \bar{x}_{N,0}^v \end{bmatrix} \right) = \sum_{t+k=N} V \left(\begin{bmatrix} \bar{x}_{t,k}^h \\ \bar{x}_{t,k}^v \end{bmatrix} \right) \end{aligned} \quad (19)$$

Obviously, the Lyapunov function decreases. Then,

$$\lim_{t+k \rightarrow \infty} \bar{x}_{t,k} \rightarrow 0 \quad (20)$$

According to the initial conditions (10-12) and theorem 1, we obtain

$$\sum_{t=0}^{N_1} \sum_{k=0}^{N_2} \bar{x}_{t,k}^T \left(Q_1 + \sum_{i=1}^r \sum_{j=1}^r h_i h_j K_i^T Q_2 K_j \right) \bar{x}_{t,k} + \Delta V(\bar{x}_{t,k}) \leq 0 \quad (21)$$

Then,

$$\begin{aligned} & \sum_{t=0}^{N_1} \sum_{k=1}^{N_2} \bar{x}_{t,k}^T \left[\sum_{i=1}^r \sum_{j=1}^r h_i h_j (Q_1 + K_i^T Q_2 K_j) \right] \bar{x}_{t,k} \\ & \leq - \sum_{t=0}^{N_1} \sum_{k=1}^{N_2} [\Delta V(\bar{x}_{t,k})] = \sum_{t=0}^{N_1} V(t, 0) + \sum_{k=0}^{N_2} V(0, k) \end{aligned} \quad (22)$$

Thus,

$$\begin{aligned} J \leq & \sum_{t=0}^{N_1} \left[\bar{x}_{t,0}^{hT} P_h \bar{x}_{t,0}^h + \bar{x}_{t,0}^{vT} P_v \bar{x}_{t,0}^v + \sum_{r=t-\bar{d}(t)}^{t-1} \bar{x}_{r,0}^{hT} Q_h \bar{x}_{r,0}^h \right. \\ & + \sum_{r=-\bar{h}(0)}^{-1} \bar{x}_{t,r}^{vT} Q_v \bar{x}_{t,r}^v + \sum_{r=t-\bar{d}}^{t-1} \bar{x}_{r,0}^{hT} G_h \bar{x}_{r,0}^h \\ & + \sum_{r=-\bar{h}}^{-1} \bar{x}_{t,r}^{vT} G_v \bar{x}_{t,r}^v \\ & + \sum_{s=-\bar{d}}^{-\bar{d}+r=t+s} \sum_{r=t+s}^{t-1} \bar{x}_{r,0}^{hT} Q_h \bar{x}_{r,0}^h + \sum_{s=-\bar{h}}^{-\bar{h}+r=s} \sum_{r=s}^{-1} \bar{x}_{t,r}^{vT} Q_v \bar{x}_{t,r}^v \\ & \left. + \bar{d} \sum_{s=-\bar{d}}^{-\bar{d}+r=t+s} \sum_{r=t+s}^{t-1} \Delta_{r,0}^{hT} M_h \Delta_{r,0}^h + \bar{h} \sum_{s=-\bar{h}}^{-\bar{h}+r=s} \sum_{r=s}^{-1} \Delta_{t,r}^{vT} M_v \Delta_{t,r}^v \right] \\ & + \sum_{k=0}^{N_2} \left[\bar{x}_{0,k}^{hT} P_h \bar{x}_{0,k}^h + \bar{x}_{0,k}^{vT} P_v \bar{x}_{0,k}^v + \sum_{r=-\bar{d}(0)}^{-1} \bar{x}_{r,k}^{hT} Q_h \bar{x}_{r,k}^h \right] \end{aligned}$$

$$\begin{aligned}
 & + \sum_{r=k-h(k)}^{k-1} \bar{x}_{0,r}^{vT} Q_v \bar{x}_{0,r}^v + \sum_{r=-\bar{d}}^{-1} \bar{x}_{r,k}^{hT} G_h \bar{x}_{r,k}^h \\
 & + \sum_{r=k-\bar{h}}^{k-1} \bar{x}_{0,r}^{vT} G_v \bar{x}_{0,r}^v \\
 & + \sum_{s=-\bar{d}}^{-1} \sum_{r=s}^{-1} \bar{x}_{r,k}^{hT} Q_h \bar{x}_{r,k}^h + \sum_{s=-\bar{h}}^{-1} \sum_{r=k+s}^{k-1} \bar{x}_{0,r}^{vT} Q_v \bar{x}_{0,r}^v \\
 & + \bar{d} \sum_{s=-\bar{d}}^{-1} \sum_{r=s}^{-1} \Delta_{r,k}^{hT} M_h \Delta_{r,k}^h + \bar{h} \sum_{s=-\bar{h}}^{-1} \sum_{r=k+s}^{k-1} \Delta_{0,r}^{vT} M_v \Delta_{0,r}^v \Big] \\
 & = J^* \tag{23}
 \end{aligned}$$

It can be seen from the equations (19-20) that the state converges. Because the state $\bar{x}_{t,k}$ includes the output error $e_{k-1}(t+1)$, the output error converges. This conclusion will be verified in the simulation. Theorem 1 gives the existence condition of the control law K_i, K_j . Under the condition of the above theorem, we look for the solvable condition of K_i, K_j under repetitive disturbances and nonrepetitive disturbances. Then, the following two theorems hold.

Theorem 2: For model (8) with (10-12), any scalar $0 < \underline{d} < \bar{d}$, $0 < \underline{h} < \bar{h}$ and matrices $Q_1, Q_2 > 0$, when $\bar{w}_{t,k} = 0$ holds, if a scalar $\varepsilon > 0$ and the matrices to be solved $L, S, N = \text{diag}(\bullet_h, \bullet_v) > 0$, $G_1, G_2, G_3 = \text{diag}(\bullet_{h1}, \bullet_{v1}) > 0$, and Y_i, Y_j , satisfy the matrix inequalities

$$\begin{bmatrix}
 \bar{\psi}_1 & \bar{0} & L & \tilde{\Gamma}_{14} & \tilde{\Gamma}_{15} & LQ_1^{\frac{1}{2}} & Y_i^T Q_2^{\frac{1}{2}} & L\tilde{E}_i^T & \bar{0} \\
 \# & -S & \bar{0} & L\tilde{A}_{id}^T & L\tilde{A}_{id}^T & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \tilde{\Gamma}_{33} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & -L & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \varepsilon\tilde{D}_i^T \\
 \# & \# & \# & \# & -T^{-2}N & \bar{0} & \bar{0} & \bar{0} & \varepsilon\tilde{D}_i^T \\
 \# & \# & \# & \# & \# & -I & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & \# & -I & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & \# & \# & -2\varepsilon & \bar{0} \\
 \# & \# & \# & \# & \# & \# & \# & \# & -\varepsilon
 \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix}
 \bar{\psi}_1 & \bar{0} & L & \check{\Gamma}_{14} & \check{\Gamma}_{15} & LQ_1^{\frac{1}{2}} & Y_{ij}^T Q_2^{\frac{1}{2}} & L\check{E}_i^T & L\check{E}_j^T \\
 \# & -S & \bar{0} & L\check{A}_{id}^T & L\check{A}_{id}^T & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \tilde{\Gamma}_{33} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \check{\Gamma}_{44} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \check{\Gamma}_{55} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & -I & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & \# & -I & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & \# & \# & -2\varepsilon & \bar{0} \\
 \# & \# & \# & \# & \# & \# & \# & \# & -2\varepsilon
 \end{bmatrix} < 0 \tag{25}$$

then the system (8) is optimally controllable, and the controller's gains are $K_i = Y_i L^{-1}, K_j = Y_j L^{-1}$. In addition, the optimal performance index satisfies formula (15), where

$$\tilde{\Gamma}_{14} = L\tilde{A}_i^T + Y_i^T \tilde{B}_i, \tilde{\Gamma}_{15} = L\tilde{A}_i^T + Y_i^T \tilde{B}_i - L, \check{\Gamma}_{14} = L\check{G}_{ij} + \check{H}_{ij},$$

$$\begin{aligned}
 \check{\Gamma}_{15} &= L\check{G}_{ij} + \check{H}_{ij} - L, \tilde{\Gamma}_{33} = -G_2 - N, \check{\Gamma}_{44} = -L + \tilde{\Gamma}_{\bar{D}}, \\
 \check{\Gamma}_{55} &= -T^{-2}N + \tilde{\Gamma}_{\bar{D}}, \bar{\psi}_1 = -L + G_1 + DS + S - G_3, \\
 \tilde{\Gamma}_{\bar{D}} &= \frac{\varepsilon(\tilde{D}_i \tilde{D}_i^T + \tilde{D}_j \tilde{D}_j^T)}{2}, \tilde{D}_i = \begin{bmatrix} D_i \\ -CD_i \end{bmatrix}, \tilde{E}_i = [E_i \ \bar{0}], \\
 \check{H}_{ij} &= \frac{Y_j^T \tilde{B}_i^T + Y_i^T \tilde{B}_j^T}{2}, Y_{ij} = \frac{Y_i + Y_j}{2}, \check{G}_{ij} = \frac{\check{A}_i^T + \check{A}_j^T}{2}, \text{ and} \\
 \tilde{D}_j \text{ and } \tilde{E}_j. \tilde{D}_i \text{ and } \tilde{E}_i &\text{ have the same meaning, but the symbols are different.}
 \end{aligned}$$

Proof: Based on theorem 1, we left-multiply and right-multiply with $\text{diag}[P^{-1}, P^{-1}, M^{-1}, P^{-1}, M^{-1}, I, I]$ of formulas (13) and (14), and we let $L = P^{-1}, P = \text{diag}[P_h, P_v], G_1 = P^{-1}GP^{-1}, X = M^{-1}, S = P^{-1}QP^{-1}, G_2 = M^{-1}GM^{-1}, G_3 = P^{-1}MP^{-1}, Y_i = K_i L, Y_j = K_j L$. We obtain formulas (24) and (25). Theorem 2 is proven.

Theorem 2 gives the control law design under repetitive disturbances, but for batch production processes, disturbances cannot be completely repetitive in a batch. In this case, it is necessary to design a control algorithm to reduce non-repetitive disturbances. The H_∞ control method is widely used because of its strong anti-interference. Next, we will give the stability conditions of the system under nonrepetitive disturbances.

Theorem 3: For system (8) with (10-12), any scalars $0 < \underline{d} < \bar{d}$, $0 < \underline{h} < \bar{h}$, and $\gamma > 0$ and matrices $Q_1, Q_2 > 0$, when $\bar{w}(t, k) \neq 0$ holds, if there exist a scalar $\varepsilon > 0$ and matrices to be solved $L, S, N = \text{diag}(\bullet_h, \bullet_v) > 0$, $G_1, G_2, G_3 = \text{diag}(\bullet_{h1}, \bullet_{v1}) > 0$, and matrices Y_i, Y_j satisfy the matrix inequalities

$$\begin{bmatrix}
 \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \bar{0} \\
 \# & -L & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \varepsilon\tilde{D}_i^T \\
 \# & \# & -T^{-2}N & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \varepsilon\tilde{D}_i^T \\
 \# & \# & \# & -\gamma I & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & -I & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & -I & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & \# & -2\varepsilon & \bar{0} \\
 \# & \# & \# & \# & \# & \# & \# & -\varepsilon
 \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix}
 \hat{\Pi}_{11} & \tilde{\Pi}_{12} & \tilde{\Pi}_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} \\
 \# & \check{\Gamma}_{44} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \check{\Gamma}_{55} & \bar{0} & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & -\gamma I & \bar{0} & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & -I & \bar{0} & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & -I & \bar{0} & \bar{0} \\
 \# & \# & \# & \# & \# & \# & -2\varepsilon & \bar{0} \\
 \# & \# & \# & \# & \# & \# & \# & -2\varepsilon
 \end{bmatrix} < 0 \tag{27}$$

then system (8) is considered robust optimal control, the robust performance index is γ , and the controllers' gains are $K_i = Y_i L^{-1}, K_j = Y_j L^{-1}, (i, j \in \{1, 2, \dots, r\})$. Additionally, the optimal performance index's upper bound satisfies formula (15). $r * (t, k) = \sum_{i=1}^r h_i K_i \bar{x}_{t,k}$ is the 2D optimal

fuzzy time-delayed state feedback controller, where $\hat{\Pi}_{11} =$

$$\begin{aligned} \begin{bmatrix} \Pi_{11} & \bar{0} \\ \bar{0} & -\gamma I \end{bmatrix}, \Pi_{11} &= \begin{bmatrix} \bar{\psi}_1 & \bar{0} & L \\ \# & -S & \bar{0} \\ \# & \# & -G_2 - N \end{bmatrix}, \\ \hat{\Pi}_{12} &= \begin{bmatrix} L\bar{A}_i^T + Y_i^T \bar{B}_i^T \\ L\bar{A}_{id}^T \\ \bar{0} \\ \bar{C}^T \end{bmatrix}, \hat{\Pi}_{13} = \begin{bmatrix} L\bar{A}_i^T + Y_i^T \bar{B}_i^T - L \\ L\bar{A}_{id}^T \\ \bar{0} \\ \bar{C}^T \end{bmatrix}, \\ \Pi_{15} &= \begin{bmatrix} LQ_1^{\frac{1}{2}} \\ \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}, \Pi_{16} = \begin{bmatrix} Y_i^T Q_2^{\frac{1}{2}} \\ \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}, \Pi_{17} = \begin{bmatrix} L\bar{E}_i^T \\ \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}, \\ \Pi_{18} &= \begin{bmatrix} L\bar{E}_j^T \\ \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}, \tilde{\Pi}_{12} = \begin{bmatrix} L\bar{G}_{ij} + \bar{H}_{ij} \\ L\bar{A}_{id}^T \\ \bar{0} \\ \bar{C}^T \end{bmatrix}, \\ \tilde{\Pi}_{13} &= \begin{bmatrix} L\bar{G}_{ij} + \bar{H}_{ij} - L \\ L\bar{A}_{id}^T \\ \bar{0} \\ \bar{C}^T \end{bmatrix}, \text{ and } \Pi_{14} = [\bar{D}L \ \bar{0} \ \bar{0} \ \bar{0}]^T. \end{aligned}$$

Proof: We introduce

$$J_\infty = \sum_{t=0}^{\infty} \sum_{k=0}^{\infty} \left[\gamma^{-1} z_{t,k}^T z_{t,k} - \gamma \bar{w}_{t,k}^T \bar{w}_{t,k} \right] \quad (28)$$

$$J = \sum_{t=0}^{\infty} \sum_{k=0}^{\infty} \left\{ \bar{x}_{t,k}^T \left(Q_1 + \sum_{i=1}^r \sum_{j=1}^r h_i h_j K_i^T Q_2 K_j \right) \bar{x}_{t,k} \right\}. \quad (29)$$

Then, for $\bar{w}_{t,k} \in l_2 \{[0, \infty], [0, \infty]\}$, we can obtain

$$\begin{aligned} J \leq & \sum_{t=0}^{\infty} \sum_{k=0}^{\infty} \left[\gamma^{-1} z_{t,k}^T z_{t,k} - \gamma \bar{w}_{t,k}^T \bar{w}_{t,k} \right] \\ & + \bar{x}_{t,k}^T \left(Q_1 + \sum_{i=1}^r \sum_{j=1}^r h_i h_j K_i^T Q_2 K_j \right) \bar{x}_{t,k} + \Delta V(\bar{x}_{t,k}) \end{aligned} \quad (30)$$

$$\begin{aligned} & \gamma^{-1} z_{t,k}^T z_{t,k} - \gamma \bar{w}_{t,k}^T \bar{w}_{t,k} \\ & + \bar{x}_{t,k}^T \left(Q_1 + \sum_{i=1}^r \sum_{j=1}^r h_i h_j K_i^T Q_2 K_j \right) \bar{x}_{t,k} + \Delta V(\bar{x}_{t,k}) \\ & = \begin{bmatrix} \phi_{t,k} \\ \bar{w}_{t,k} \end{bmatrix}^T \left(\hat{\psi}_1 + \begin{bmatrix} \Lambda_1^T \\ \bar{C}^T \end{bmatrix} P \begin{bmatrix} \Lambda_1^T \\ \bar{C}^T \end{bmatrix} \right. \\ & \quad \left. + H^2 \begin{bmatrix} \Lambda_2^T \\ \bar{C}^T \end{bmatrix} G \begin{bmatrix} \Lambda_2^T \\ \bar{C}^T \end{bmatrix} \right) \begin{bmatrix} \phi_{t,k} \\ \bar{w}_{t,k} \end{bmatrix} \end{aligned} \quad (31)$$

where $\hat{\psi}_1 = \begin{bmatrix} \bar{\psi}_1 & \bar{0} \\ \bar{0} & -\gamma I \end{bmatrix}$ and $\tilde{\psi}_1 = \begin{bmatrix} \bar{\psi}_1 & \bar{0} & G \\ \# & -Q & \bar{0} \\ \# & \# & -M - G \end{bmatrix}$,

with $\psi_1 + \gamma^{-1} \bar{D}^T \bar{D} + Q_1 + K_i^T Q_2 K_j = \bar{\psi}_1$. Using formulas (28) and (29), the proof method is the same as for theorem

1 and theorem 2. We obtain

$$\hat{\psi}_1 + \begin{bmatrix} \Lambda_1^T \\ \bar{C}^T \end{bmatrix} P \begin{bmatrix} \Lambda_1^T \\ \bar{C}^T \end{bmatrix} + H^2 \begin{bmatrix} \Lambda_2^T \\ \bar{C}^T \end{bmatrix} G \begin{bmatrix} \Lambda_2^T \\ \bar{C}^T \end{bmatrix} < 0 \quad (32)$$

Then, $\|z_{t,k}\| < \gamma \|\bar{w}_{t,k}\|$ is established.

IV. ANALYSIS OF THE OPTIMAL CONTROL ALGORITHM

To obtain the optimal update law $r^*(t, k) = \sum_{i=1}^r h_i K_i \bar{x}_{t,k}$ and the minimum of the upper bound of the performance index, we must optimize formula (15).

From the upper bound of inequality (15) based on the initial condition provided by the system, for $N_1 \geq r_1$ and $N_2 \geq r_2$, the upper bound of performance satisfying inequalities (11-12) and (15) is represented as follows:

$$\begin{aligned} J & \leq \sum_{t=0}^{N_1} V(t, 0) + \sum_{k=0}^{N_2} V(0, k) \\ & = r_1 \left(\bar{x}_{t,0}^T P \bar{x}_{t,0} + \bar{x}_{d(t,0)}^T E_M Q \bar{x}_{d(t,0)} + \bar{x}_{d(t,0)}^T E_M G \bar{x}_{d(t,0)} \right. \\ & \quad \left. + \bar{x}_{d(t,0)}^T F_M Q \bar{x}_{d(t,0)} + \bar{x}_{M(t,0)}^T G_M G \bar{x}_{M(t,0)} \right) \\ & \quad + r_2 \left(\bar{x}_{0,k}^T P \bar{x}_{0,k} + \bar{x}_{d(0,k)}^T E_M G \bar{x}_{d(0,k)} + \bar{x}_{d(0,k)}^T E_M G \bar{x}_{d(0,k)} \right. \\ & \quad \left. + \bar{x}_{d(0,k)}^T F_M Q \bar{x}_{d(0,k)} + \bar{x}_{M(0,k)}^T G_M G \bar{x}_{M(0,k)} \right) \\ & \leq J^* = (r_1 + r_2) (\beta I + E_M \gamma_1 + E_M \gamma_2 + F_M \gamma_1 + G_M \gamma_3) \end{aligned} \quad (33)$$

where $\bar{x}_{t,0} = \begin{bmatrix} \bar{x}_{t,0}^h \\ \bar{x}_{t,0}^v \end{bmatrix}$, $\bar{x}_{0,k} = \begin{bmatrix} \bar{x}_{0,k}^h \\ \bar{x}_{0,k}^v \end{bmatrix}$, $\bar{x}_{d(t,0)} = \begin{bmatrix} \bar{x}_{t-d(t),0}^h \\ \bar{x}_{t-d(t),0}^v \end{bmatrix}$, $\bar{x}_{M(t,0)} = \begin{bmatrix} \Delta_{t-d(t),0}^h \\ \Delta_{t-d(t),0}^v \end{bmatrix}$, $\bar{x}_{d(0,k)} = \begin{bmatrix} \bar{x}_{0,k-d(0),k}^h \\ \bar{x}_{0,k-d(0),k}^v \end{bmatrix}$, $\bar{x}_{M(0,k)} = \begin{bmatrix} \Delta_{0,k-h(k)}^h \\ \Delta_{0,k-h(k)}^v \end{bmatrix}$, $E_M = \text{diag}(\bar{d}I_h, hI_v)$, $G_M = \text{diag} \left[\frac{\bar{d}(\bar{d}+d)(\bar{d}-d+1)}{2} I_h, \frac{\bar{h}(\bar{h}+h)(\bar{h}-h+1)}{2} I_v \right]$, and $F_M = \text{diag} \left[\frac{(d+\bar{d})(d-d+1)}{2} I_h, \frac{(h+\bar{h})(h-h+1)}{2} I_v \right]$.

$$\begin{aligned} \begin{bmatrix} -\beta I & \Phi^T \\ \Phi & -P^{-1} \end{bmatrix} < 0, & \begin{bmatrix} -\gamma_1 I & \Phi^T \\ \Phi & -Q^{-1} \end{bmatrix} < 0, \\ \begin{bmatrix} -\gamma_2 I & \Phi^T \\ \Phi & -G^{-1} \end{bmatrix} < 0, & \text{ and } \begin{bmatrix} -\gamma_3 I & \Phi^T \\ \Phi & -M^{-1} \end{bmatrix} < 0. \end{aligned} \quad (34)$$

To realize optimal control and have the smallest upper bound of the performance index, the following optimization issues must be resolved:

$$\begin{aligned} & \min (r_1 + r_2) (\beta + E_M \gamma_1 + E_M \gamma_2 + F_M \gamma_1 + G_M \gamma_3) \\ & \text{subject to (26), (27), and (34)}. \end{aligned} \quad (35)$$

Obviously, in order to obtain the optimal controller based on the above optimization algorithm, the first step is to address certain parameters and matrices in the inequality. Here, some parameters need to be given or calculated when

designing the controller gain. For example, $0 < \underline{d} < \bar{d}$, $0 < \underline{h} < \bar{h}$, r_1, r_2 , and the anti-interference value $\gamma > 0$ are given in advance, and $\varepsilon > 0$ is obtained by directly solving inequalities. Some matrices, such as $Q_1, Q_2 > 0$, are also given. Thus, the matrices $Q_1^{\frac{1}{2}}, Q_2^{\frac{1}{2}}$ in the inequalities are known and do not affect the solution of the linear inequality. Of course, the constant value or matrix given here is adjustable until we find the optimal controller gain. The steps of the optimization algorithm are as follows:

Step 1: Construct the upper bound of the performance indexes J^* ;

Step 2: According to the given initial conditions, the variables to be solved in J^* are transformed into inequalities (34);

Step 3: Minimize J^* , which satisfies constraints (26), (27), and (34);

Step 4: Adjust the known matrices in the constraint conditions (26), (27) and (34). If they do not have the optimal values, return to step 3 until the controller gain matrices $K_i = Y_i L^{-1}, K_j = Y_j L^{-1}, (i, j \in \{1, 2 \dots, r\})$ are obtained. Then, the algorithm ends.

V. SIMULATION CASE

The simulation case analysis in this chapter is based on a 3-capacity water tank as an example. The type is TTS20, and it is shown in Fig. 1.



FIGURE 1. The three water tanks of TTS20.

In general, for a test, only one injection process is used for observing the controller's result. This chapter regards each water injection process as a batch. To show the effect of control, the chosen model is similar to that in [50], and the design results in [50] are compared with those of the method in this paper. As described in [50], the fuzzy rule is given in Fig. 2. The 2D fuzzy time-delayed system transformed is shown through formula (1) given in this paper, in which the system matrices are:

$$A_1 = \begin{bmatrix} 0.9951 & 0.0035 \\ 0.0025 & 0.9930 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9944 & 0.0040 \\ 0.0029 & 0.9919 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.9863 & 0.0098 \\ 0.0071 & 0.9804 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0.9807 & 0.0137 \\ 0.0100 & 0.9724 \end{bmatrix}$$

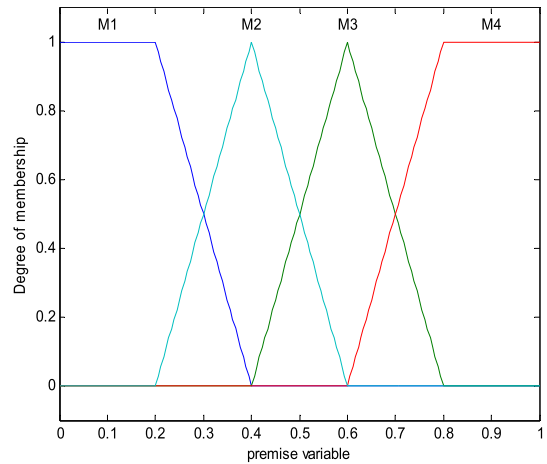


FIGURE 2. Membership function.

$$B_1 = \begin{bmatrix} 32.3885 \\ 0.0414 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 32.3763 \\ 0.0478 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 64.4900 \\ 0.2327 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 64.3072 \\ 0.3276 \end{bmatrix}$$

$$A_{1d} = \begin{bmatrix} 0.1057 & 0.0004 \\ 0.0002 & 0.0807 \end{bmatrix}, \quad A_{2d} = \begin{bmatrix} 0.1072 & 0.0005 \\ 0.0003 & 0.0807 \end{bmatrix}$$

$$A_{3d} = \begin{bmatrix} 0.0962 & 0.0011 \\ 0.007 & 0.1096 \end{bmatrix}, \quad A_{4d} = \begin{bmatrix} 0.0858 & 0.0013 \\ 0.0010 & 0.1029 \end{bmatrix}$$

The fuzzy control law is considered in [50]. Clearly, a control law that is designed depending on repeated disturbances only analyzes the stability of the system and does not consider its control performance. As we have explained, it is not sufficient to only consider stability in an actual production project. Control performance must be considered to achieve the goal of energy saving and consumption reduction. This is also the main research goal of this article. To achieve this goal, using the above optimization algorithm and solving the constraint condition (35), we can obtain the optimal fuzzy controller gain under repetitive disturbance as shown in formula (36).

$$K_1^o = [-0.0056 \quad -0.0003 \quad 0.00109]$$

$$K_2^o = [-0.0056 \quad -0.0003 \quad 0.00109]$$

$$K_3^o = [-0.0045 \quad -0.0003 \quad 0.0070]$$

$$K_4^o = [-0.0046 \quad -0.0003 \quad 0.0070] \quad (36)$$

$$K_1^w = [-0.0045 \quad -0.0001 \quad 0.0098]$$

$$K_2^w = [-0.0047 \quad -0.0002 \quad 0.0089]$$

$$K_3^w = [-0.0033 \quad -0.0001 \quad 0.0059]$$

$$K_4^w = [-0.0031 \quad -0.0001 \quad 0.0049] \quad (37)$$

K_i^o is designed by the optimal control algorithm, and K_i^w is designed without optimal control as in [50]. Simulation runs of 50 batches are chosen, and 300-step runs are assumed in each batch. To evaluate the control effect, the evaluation index RSSE (root-sum-squared-error) $DT(k) = \sqrt{\sum_{t=1}^{T_p} e^2(t, k)}$ is

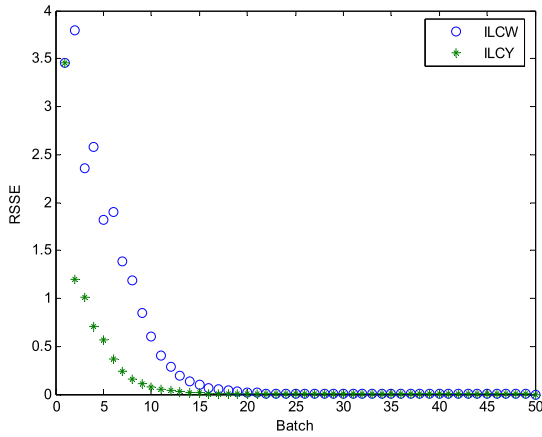


FIGURE 3. RSSE comparison with and without optimization algorithms under repetitive disturbances.

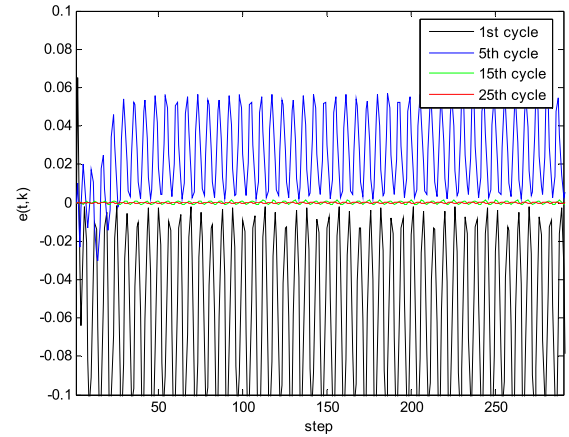


FIGURE 5. Tracking error in different batches ($e(t, k)$).

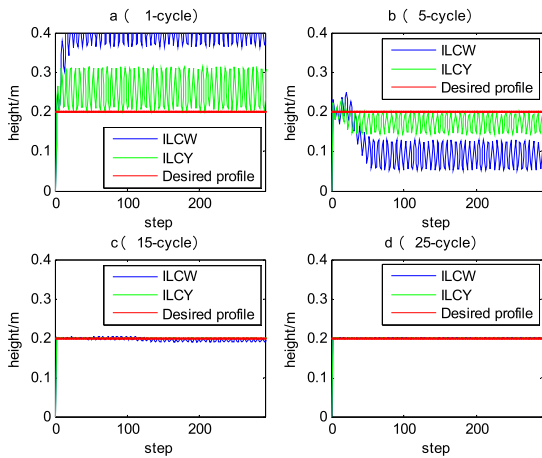


FIGURE 4. Output comparison with and without optimization algorithms under repetitive disturbances.

adopted. The benefits of the optimization algorithm will be shown as follows:

A. CASE 1: REPETITIVE DISTURBANCE

The repetitive disturbance between batches is chosen as $\omega_{t,k} = 0.05 * [\sin(t) \cos(t)]$. Figs. 3 and 4 show the comparison results between the non-optimization algorithm and the optimization algorithm. It is shown in Fig. 3 that the tracking performance of the initial batch systems is better than it is without the optimization algorithm. It can also be seen in Fig. 4 that the output of this algorithm tracks quickly and can ultimately achieve zero-error tracking. In general, the optimal control algorithm presented in this paper has faster convergence speed and better control performance. To show the convergence of output errors of different batches, Fig. 5 shows several simulation graphs of different batches. As seen from the graphs, under repeated disturbances, although the convergence effect of tracking errors of the first few batches is poor, the errors converge to zero in the 25th batch. This point proves that the system is asymptotically stable.

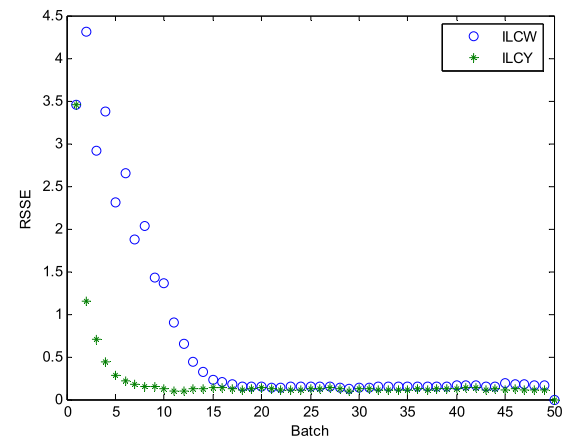


FIGURE 6. RSSE under nonrepetitive disturbances.

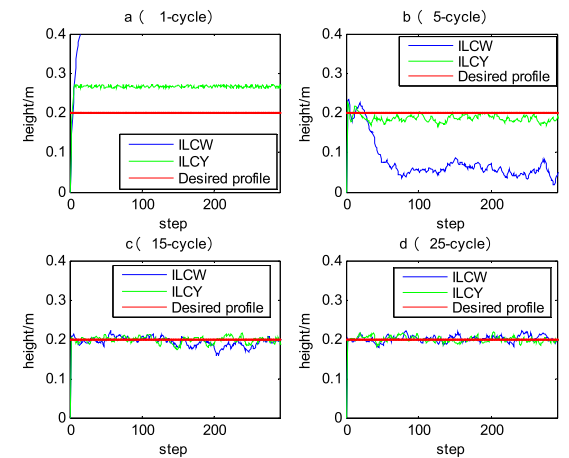


FIGURE 7. Output curve under nonrepetitive disturbances.

B. CASE 2: NONREPETITIVE DISTURBANCE

In this case, the nonrepetitive disturbance is taken to be $\omega = [0.01 \ 0.01] * \Delta$, where Δ is a random number in $[0, 1]$. Despite the effect of the time delay and nonrepetitive disturbance, the algorithm proposed in this paper still has better tracking performance except that the control performance is decreased to some extent, as shown in Fig. 6. Fig. 7 shows

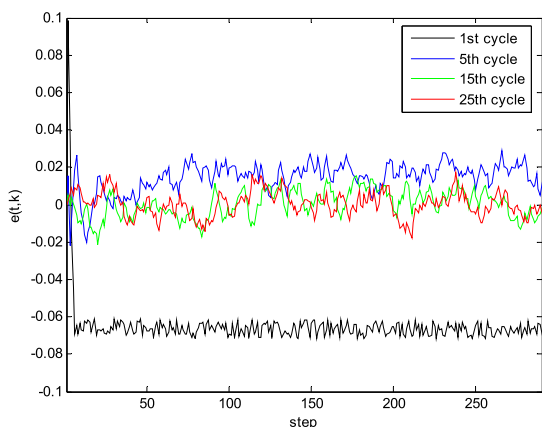


FIGURE 8. Tracking error under nonrepetitive disturbances ($e(t, k)$).

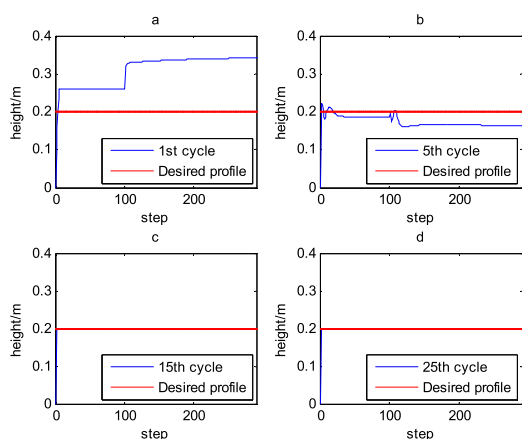


FIGURE 9. Output curve when the step signal is the disturbance signal.

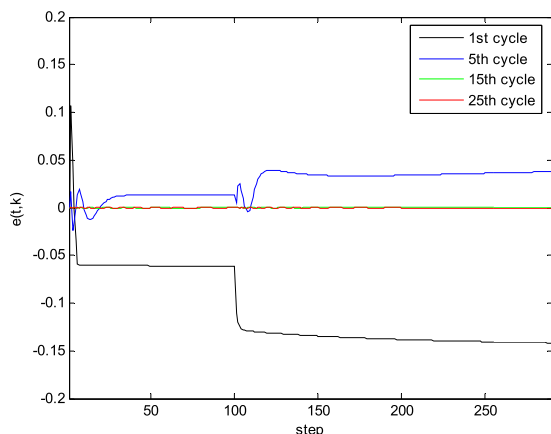


FIGURE 10. Tracking error when the step signal is the disturbance signal.

the same result. Clearly, because it is affected by the above factors, the output curve fluctuates continuously around a given point but does not deviate too much. Fig. 8 shows the tracking error curves of different batches of the system under nonrepeated disturbances. It can be seen from the figure that under nonrepeated disturbance, the convergence effect of the tracking error is obviously poor, especially for the first few batches. Although it cannot converge to zero, it still fluctuates around zero.

C. CASE 3: THE STEP SIGNAL IS THE DISTURBANCE SIGNAL

To show its control performance, we simulate the step signal as an interference signal. The step signal is taken as 0.05, and the occurrence time is 100 steps in the first batch. As seen from Figs. 9 and 10, the tracking effect is poor. However, since the designed iterative learning control method can resist the influence of disturbances, the control effect is very good after several batches, and even zero-error tracking is realized.

VI. CONCLUSION

A strong nonlinear time-delayed batch process is addressed by using a 2D T-S fuzzy model including unknown internal uncertainties and external disturbance. A 2D fuzzy optimal control strategy based on time-delay upper and lower bounds is proposed. The optimal design algorithm of the optimal fuzzy control law is also given in terms of LMI constraints. The simulation result of a 3-capacity water tank shows that the designed controller in this article is more practical than that without optimization. At the same time, it also has great robustness.

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