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# Finite-Time Stability of Switched Linear Time-Delay Systems Based on Time-Dependent Lyapunov Functions

**TIANTIAN HUANG AND YUANGONG SUN**

School of Mathematical Sciences, University of Jinan, Jinan 250022, China

Corresponding author: Yuangong Sun (sunyuangong@163.com)

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**ABSTRACT** In this paper, finite-time stability of switched linear time-delay systems has been addressed. By constructing a class of time-dependent common (multiple) Lyapunov functions, new explicit conditions for finite-time stability of the system under arbitrary switching and average dwell-time switching are established respectively. Compared with most of existing results in the literature, our results are easily verifiable by solving several linear matrix inequalities rather than complex matrix Riccati differential equations. The effectiveness of the proposed method is demonstrated by numerical examples.

**INDEX TERMS** Finite-time stability, switched linear time-delay system, average dwell-time switching, time-dependent Lyapunov function.

## I. INTRODUCTION

Switched system contains a number of subsystems described by continuous or discrete dynamics, as well as the switching signal regulating the switching between subsystems at each switching time. Switched systems can be used to describe many practical systems with wide applications. In recent decades, switched system has attracted extensive attention in control theory and engineering practice. So far, there are many achievements on Lyapunov stability of switched systems [1]–[3].

Lyapunov stability reflects the state estimation of dynamical systems in infinite time. However, in many practical applications, the state estimation in a short period of time usually needs to be concerned. As a result, the concept of finite-time stability (FTS) was put forward. That is, when the initial state of a dynamic system is within a certain boundary, its state does not exceed a certain threshold in a limited time interval. On the other hand, time delay is an inherent feature of the system, which usually has a positive or negative impact on the performance of the system. Therefore, the research on FTS of time-delay systems is also a field of great concern.

Since Kamenkov first proposed the concept of finite-time stability, a large number of results have been obtained in the study of FTS of various of systems [4]–[15]. For FTS of switched systems, there are also many important results [9], [16]–[19]. It is worth noting that the chosen Lyapunov function in [9], [16]–[19] is independent of time, which usually leads to conservative results of FTS.

Finite-time stability based on time-dependent Lyapunov functions was studied in [20]–[27]. The concept of FTS was extended to interconnected pulse switching systems in [20], where sufficient conditions for FTS of the interconnected pulse switching system were proposed for the first time. FTS criteria for a class of switched linear systems were given by constructing a multiple Lyapunov function in [22]. Based on the state transition matrix of the system, sufficient conditions for FTS and uniform FTS of switched linear systems were established in [23]. It was shown that FTS of switched linear systems is related not only to subsystems, but also to switching signals in [24]. Although time-dependent Lyapunov functions may lead to less conservative FTS criteria, they also result in unsolvable matrix Riccati differential equations, which is not convenient for the application of the results.

In this paper, we further consider FTS of switched linear systems with time delay by constructing a class of time-dependent Lyapunov functions. New FTS criteria for the

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system under arbitrary switching and average dwell-time switching will be established. Compared with the results obtained in the literature, the conditions given in this paper do not contain the unsolvable matrix Riccati differential equation, and hence they are easily verifiable by solving several LMIs.

The structure of this paper is as follows. In Section 2, necessary preparations and problem statements are given. Section 3 establishes the main results of this paper. Numerical examples are presented in Section 4. Finally, we present a summary of this paper in Section 5.

## II. PROBLEM DESCRIPTION AND PRELIMINARIES

Throughout this paper, $\mathbb{R}^n$ stands for the vector space of all $n$-tuples of real numbers, $\mathbb{R}^{n\times n}$ is the space of $n\times n$ matrices with real entries. For a vector $x\in\mathbb{R}^n$, denote $x\succ 0$ ($x\prec 0$) if its each entry $x_i>0$ ($x_i<0$) for $i\in\{1,2,\cdots,n\}$. For a matrix $A\in\mathbb{R}^{n\times n}$, $A^\top$ is the transpose of $A$. For a symmetric matrix $P\in\mathbb{R}^{n\times n}$, $P>0$ ($P<0$) means that $P$ is positive definite (negative definite).

Consider the following switched linear time-delay system

$$\begin{cases}\dot{x}(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}x(t-\tau), & t\in[0,T],\\ x(t)=\phi(t), & t\in[-\tau,0],\end{cases}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, the switching signal $\sigma(t):[0,\infty)\to\{1,2,\ldots,N\}$ is a piecewise constant function, $A_i\in\mathbb{R}^{n\times n}$ and $B_i\in\mathbb{R}^{n\times n}$ are system matrices of the $i$th subsystem for $i\in\{1,2,\ldots,N\}$, $\tau>0$ is a constant delay, $\phi(t):[-\tau,0]\to\mathbb{R}^n$ is the continuous vector valued initial function.

In the sequel, denote the switching times by $0<t_1<t_2<\cdots<t_{m-1}<T$, $t_0=0$ and $t_m=T$, where $m$ is a positive integer. The following two well-known definitions are derived from [15] and [27].

*Definition 1:* Given positive scalars $T$, $c_1<c_2$ and a positive definite matrix $U$, system (1) is said to be finite-time stable with respect to $(T,U,c_1,c_2)$ if

$$\sup_{t\in[-\tau,0]} x^\top(t)Ux(t)\le c_1 \Rightarrow x^\top(t)Ux(t)\le c_2,\quad t\in[0,T].\tag{2}$$

*Definition 2:* For $0<t\le T$, let $N_\sigma(0,t)$ denote the number of switching of $\sigma(t)$ over $(0,t)$. If there exists a constant $\tau_a>0$ such that $N_\sigma(0,t)\le\frac{T-t}{\tau_a}$ holds, $\tau_a$ is called an average dwell time (ADT) of the switching signal $\sigma(t)$ over the time interval $[0,T]$.

## III. MAIN RESULTS

In this section, the main results of this paper are given. We first present an explicit FTS criterion for system(1) under arbitrary switching.

*Theorem 1:* Given positive scalars $T$, $c_1<c_2$, and a positive definite matrix $U$, system (1) is finite-time stable with respect to $(T,U,c_1,c_2)$ under arbitrary switching, if there exist positive definite matrices $P$ and $R$, symmetric matrices $Q$ and $W$ and real scalars $\alpha>1$ and $\beta>0$ such that for $i\in\{1,2,\cdots,N\}$,

$$\begin{pmatrix}A_i^\top P+PA_i+R-Q & PB_i\\ B_i^\top P & -R-\tau W\end{pmatrix}<0,\tag{3}$$

$$\begin{pmatrix}\Gamma_i & (P-TQ)B_i\\ B_i^\top(P-TQ) & -R+(T-\tau)W\end{pmatrix}<0,\tag{4}$$

$$P<\alpha U,\quad R<\beta U,\quad R+\tau W<\beta U,\tag{5}$$

$$U<P,\quad U<P-TQ,\tag{6}$$

$$0<R+\tau W,\quad 0<R-TW,\tag{7}$$

$$(\alpha+\tau\beta)c_1\le c_2,\tag{8}$$

where

$$\Gamma_i=A_i^\top(P-TQ)+(P-TQ)A_i+(R-TW)-Q.$$

*Proof:* Define a special time-dependent Lyapunov function of the form

$$V(t,x(t))=x^\top(t)G(t)x(t)+\int_{t-\tau}^{t}x^\top(s)F(s)x(s)ds,$$

where

$$G(t)=P-tQ,\quad t\in[0,T],$$

and

$$F(t)=R-tW,\quad t\in[-\tau,T].$$

For any $t\in[0,T)$, denote $\sigma(t)=i\in\{1,2,\cdots,N\}$ which is dependent of $t$. The derivative of $V(t,x(t))$ with respect to $t$ along the trajectory of system (1) yields

$$\begin{aligned}\dot V(t,x(t)) &= \dot x^\top(t)G(t)x(t)+x^\top(t)\dot G(t)x(t)\\ &\quad +x^\top(t)G(t)\dot x(t)+x^\top(t)F(t)x(t)\\ &\quad -x^\top(t-\tau)F(t-\tau)x(t-\tau)\\ &\le x^\top(t)(A_i^\top G(t)+G(t)A_i-Q)x(t)\\ &\quad +2x^\top(t)G(t)B_ix(t-\tau)\\ &\quad +x^\top(t)F(t)x(t)\\ &\quad -x^\top(t-\tau)F(t-\tau)x(t-\tau)\\ &= y^\top(t)\Omega_i(t)y(t),\end{aligned}$$

where $y(t)=(x^\top(t),x^\top(t-\tau))^\top$ and

$$\Omega_i(t)=\begin{pmatrix}A_i^\top G(t)+G(t)A_i+F(t)-Q & G(t)B_i\\ B_i^\top G(t) & -F(t-\tau)\end{pmatrix}.$$

Next we show that $\Omega_i(t)<0$ for $i\in\{1,2,\cdots,N\}$ and $t\in[0,T]$. Note that

$$\dot\Omega_i(t)=\begin{pmatrix}-A_i^\top Q-QA_i-W & -QB_i\\ -B_i^\top Q & W\end{pmatrix},$$

which is independent of time $t$. Therefore, for any $z\in\mathbb{R}^n$, $z^\top\Omega_i(t)z$ is monotone with respect to $t$ on $[0,T]$. It implies that $\Omega_i(t)<0$ for $i\in\{1,2,\cdots,N\}$ and $t\in[0,T]$ if $\Omega_i(0)<0$ and $\Omega_i(T)<0$, which is an immediate result of conditions (3) and (4). Consequently, we have that $\dot V(t,x)\le 0$ for $t\in[0,T]$. Since the last two inequalities of (5) imply that

$R - tW < \beta U$ for $t \in [-\tau, 0]$, we further conclude from (5) that

$$\begin{aligned} V(t, x(t)) &\leq V(0, x(0)) \\ &= x^\top(0)Px(0) \\ &\quad + \int_{-\tau}^{0} x^\top(s)(R - sW)x(s)ds \\ &\leq \alpha x^\top(0)Ux(0) + \beta \int_{-\tau}^{0} x^\top(s)Ux(s)ds, \end{aligned} \tag{9}$$

where $t \in [0, T]$.

Based on the same analysis mentioned above, (6) and (7) imply that $G(t) > U$ for $t \in [0, T]$ and $F(t) > 0$ for $t \in [-\tau, T]$. Therefore, if $\sup_{t\in[-\tau,0]} x^\top(t)Ux(t) \leq c_1$, we can get from (8) and (9) that

$$\begin{aligned} x^\top(t)Ux(t) &\leq V(t, x(t)) \\ &\leq (\alpha + \tau\beta)c_1 \\ &\leq c_2, \quad t \in [0, T]. \end{aligned}$$

That is, system (1) is finite-time stable with respect to $(T, U, c_1, c_2)$. The proof of Theorem 1 is finished. □

*Remark 1:* In Theorem 1, we do not assume that $R$ and $W$ are positive definite matrices, which reduces the conservativeness of the given result. On the other hand, conditions (3) and (4) are dependent of time delay $\tau$, which also leads to a less conservative result.

*Remark 2:* Conditions (3)-(7) are guaranteed by the existence of common positive definite matrices $P$ and $Q$ and symmetric matrices $R$ and $W$. For the case when it does not exist such common matrices $P$, $Q$, $R$ and $W$, we will design a time-dependent multiple Lyapunov function to derive another explicit FTS criterion for system (1).

Next, we consider FTS of system (1) under the average dwell time switching.

*Theorem 2:* Given positive scalars $T, c_1 < c_2$ and a positive definite matrix $U$, system (1) is finite-time stable with respect to $(T, U, c_1, c_2)$ under the switching with ADT $\tau_a > 0$, if there exist positive definite matrices $P_i$ and $R_i$, symmetric matrices $Q_i$ and $W_i$, constants $\mu > 1$, $a \geq 0$, $\alpha > 1$ and $\beta > 0$ such that for $i, j \in \{1, 2, \cdots, N\}$,

$$\begin{pmatrix} \Phi_i & P_iB_i \\ B_i^\top P_i & -R_j - \tau W_j \end{pmatrix} < 0, \tag{10}$$

$$\begin{pmatrix} \Psi_i & (P_i - TQ_i)B_i \\ B_i^\top (P_i - TQ_i) & -R_j - (T - \tau)W_j \end{pmatrix} < 0, \tag{11}$$

$$P_i < \mu P_j, \quad (P_i - Q_iT) < \mu(P_j - Q_jT), \tag{12}$$

$$U < P_i, \quad U < P_i - TQ_i, \tag{13}$$

$$0 < R_i + \tau W_i, \quad 0 < R_i - TW_i, \tag{14}$$

$$P_i < \alpha U, \quad R_i < \beta U, \quad R_i + \tau W_i < \beta U, \tag{15}$$

$$e^{aT} \mu^{\frac{T}{\tau_a}} \left(\alpha + \beta \frac{1 - e^{-a\tau}}{a}\right) c_1 \leq c_2, \tag{16}$$

where

$$\Phi_i = A_i^\top P_i + P_iA_i - aP_i + e^{-a\tau}R_i - Q_i,$$

$$\begin{aligned} \Psi_i = &A_i^\top (P_i - TQ_i) + (P_i - TQ_i)A_i - a(P_i + TQ_i) \\ &+ e^{-a\tau}(R_i - TW_i) - Q_i. \end{aligned}$$

*Proof:* Choose the following time-dependent Lyapunov function

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) = &x^\top(t)G_{\sigma(t)}(t)x(t) \\ &+ \int_{t-\tau}^{t} e^{a(t-s-\tau)}x^\top(s)F_{\sigma(s)}(s)x(s)ds, \end{aligned}$$

where

$$\begin{aligned} G_{\sigma(t)}(t) &= P_{\sigma(t)} - tQ_{\sigma(t)}, \quad t \in [0, T], \\ F_{\sigma(t)}(t) &= R_{\sigma(t)} - tW_{\sigma(t)}, \quad t \in [-\tau, T], \end{aligned}$$

and $\sigma(t) \equiv \sigma(0)$ for $t \in [-\tau, 0]$. The proof will be divided into the following three steps.

*Step I:* For any $t \in [t_k, t_{k+1})$ with $k = 0, 1, \cdots, m - 1$, where $t_0 = 0$, $t_m = T$, and $t_k$ $(1 \leq k \leq m - 1)$ is the switching time, we first prove that

$$V_{\sigma(t)}(t, x(t)) \leq e^{a(t-t_k)}V_{\sigma(t_k)}(t_k, x(t_k)). \tag{17}$$

Without loss of generality, denote $\sigma(t) = i$ and $\sigma(t - \tau) = j$. Then, the derivative of $V_i(t, x(t))$ with respect to $t \in [t_k, t_{k+1})$ along the trajectory of system (1) yields

$$\begin{aligned} \dot{V}_i(t, x(t)) \leq\ &x^\top(t)(A_i^\top G_i(t) + G_i(t)A_i - Q_i)x(t) \\ &+ 2x^\top(t)G_i(t)B_ix(t - \tau) \\ &+ a\int_{t-\tau}^{t} e^{a(t-s-\tau)}x^\top(s)F_{\sigma(s)}(s)x(s)ds \\ &+ e^{-a\tau}x^\top(t)F_i(t)x(t) \\ &- x^\top(t - \tau)F_j(t - \tau)x(t - \tau). \end{aligned}$$

Then, we obtain

$$\begin{aligned} &\dot{V}_i(t, x(t)) - aV_i(t, x(t)) \\ &\quad\leq x^\top(t)(A_i^\top G_i(t) \\ &\qquad + G_i(t)A_i - Q_i)x(t) \\ &\qquad + 2x^\top(t)G_i(t)B_ix(t - \tau) \\ &\qquad - ax^\top(t)G_i(t)x(t) + e^{-a\tau}x^\top(t)F_i(t)x(t) \\ &\qquad - x^\top(t - \tau)F_j(t - \tau)x(t - \tau) \\ &\quad= y^\top(t)\Omega_i(t)y(t), \end{aligned}$$

where $y(t) = (x^\top(t), x^\top(t - \tau))^\top$,

$$\Omega_{ij}(t) = \begin{pmatrix} \Theta_i(t) & G_i(t)B_i \\ B_i^\top G_i(t) & -F_j(t - \tau) \end{pmatrix},$$

and

$$\Theta_i(t) = A_i^\top G_i(t) + G_i(t)A_i - aG_i(t) + e^{-a\tau}F_i(t) - Q_i.$$

Following the same discussion given in the proof of Theorem 1, we have that $\Omega_{ij}(t) < 0$ for $t \in [t_k, t_{k+1}]$ if $\Omega_{ij}(0) < 0$ and $\Omega_{ij}(T) < 0$. Therefore, conditions (10) and (11) imply that

$$\dot{V}_i(t, x(t)) - aV_i(t, x(t)) \leq 0, \quad t \in [t_k, t_{k+1}).$$

By solving the above differential inequality, it is not difficult to derive (17).

*Step II:* Next, we show that for any switching signal $\sigma(t)$, it holds

$$V_{\sigma(t_k)}(t_k, x(t_k)) \leq \mu V_{\sigma(t_{k-1})}(t_k, x(t_k)), \tag{18}$$

where $k = 1, 2, \cdots, m-1$.

According to the definition of the Lyapunov function $V_{\sigma(t)}(t, x(t))$, it is sufficient to verify

$$G_i(t) \leq \mu G_j(t), \quad i, j \in \{1, 2, \cdots, N\}, \ t \in [0, T],$$

which can be derived from (12) based on the same analysis mentioned above.

*Step III:* Combining (17) and (18), for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} V_{\sigma(t)}(t, x(t)) &\leq e^{a(t-t_k)} V_{\sigma(t_k)}(t_k, x(t_k)) \\ &\leq \mu e^{a(t-t_k)} V_{\sigma(t_{k-1})}(t_k, x(t_k)) \\ &\leq \mu e^{a(t-t_{k-1})} V_{\sigma(t_{k-1})}(t_{k-1}, x(t_{k-1})) \\ &\leq \cdots \\ &\leq e^{at} \mu^{N_\sigma(0,t)} V_{\sigma(0)}(0, x(0)). \end{aligned}$$

Noting that $a \geq 0$, $N_\sigma(0, t) \leq N_\sigma(0, T)$, $\mu > 1$ and $N_\sigma(0, T) \leq \frac{T}{\tau_a}$, it implies that

$$V_{\sigma(t)}(t, x(t)) \leq e^{aT} \mu^{\frac{T}{\tau_a}} V_{\sigma(0)}(0, x(0)), \quad t \in [0, T]. \tag{19}$$

Following the same discussion as that given in Theorem 1, conditions (13) and (14) yield

$$V_{\sigma(t)}(t, x(t)) \geq x^\top(t) U x(t), \quad t \in [0, T], \tag{20}$$

and condition (15) implies that

$$\begin{aligned} V_{\sigma(0)}(0, x(0)) &\leq \alpha x^\top(0) U x(0) \\ &\quad + \beta \int_{-\tau}^{0} e^{a(-s-\tau)} x^\top U x(s) ds \\ &\leq \left(\alpha + \beta \frac{1 - e^{-a\tau}}{a}\right) c_1. \end{aligned} \tag{21}$$

This together with (16) and (19)-(21) yields that $x^\top(t) U x(t) \leq c_2$ for $t \in [0, T]$. Consequently, system (1) is finite-time stable with respect to $(T, U, c_1, c_2)$. This completes the proof of Theorem 2. □

## IV. NUMERICAL EXAMPLES

In this section, two illustrative examples are presented.

*Example 1:* Consider system (1) with $n = N = 2$, $\tau = 1.5$,

$$A_1 = \begin{pmatrix} 0 & 0.003 & 0.001 \\ 0.002 & 0 & 0.003 \\ 0.002 & 0.002 & 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 0.003 & -0.003 \\ 0.004 & 0 & 0.003 \\ 0.003 & -0.004 & 0 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0.003 & 0.001 & -0.002 \\ -0.004 & 0.003 & 0.001 \\ 0.001 & -0.002 & 0.001 \end{pmatrix},$$

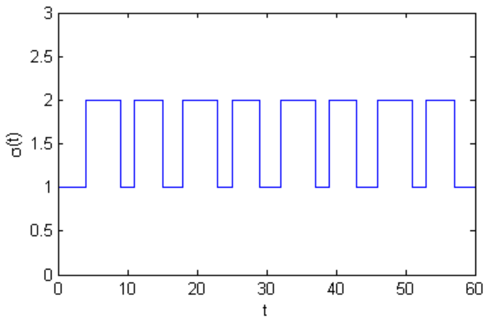


**FIGURE 1.** The chosen switching signal.

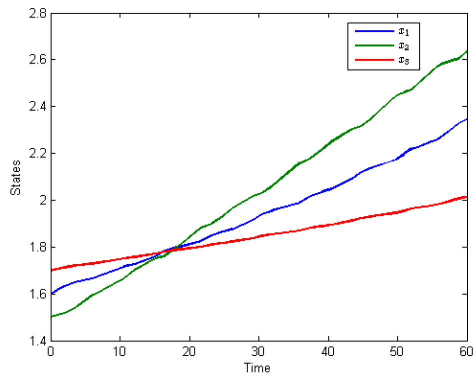


**FIGURE 2.** The state trajectory of system (1).

$$B_2 = \begin{pmatrix} 0.001 & 0.002 & 0.001 \\ 0.003 & -0.001 & 0.004 \\ 0.002 & 0.003 & -0.002 \end{pmatrix}.$$

For given $U = I$, $T = 60$, $c_1 = 7.7$ and $c_2 = 21.7$, solving inequalities (3)-(8), we get $\alpha = 2.9$, $\beta = 0.028$, and

$$P = \begin{pmatrix} 2.8960 & -0.1491 & 0.1619 \\ -0.1491 & 2.7227 & -0.0612 \\ 0.1619 & -0.0612 & 2.8992 \end{pmatrix},$$

$$Q = 10^{-2} \begin{pmatrix} 4.5951 & -0.1649 & 0.2871 \\ -0.1649 & 4.4116 & -0.0955 \\ 0.2871 & -0.0955 & -4.7396 \end{pmatrix},$$

$$R = 10^{-2} \begin{pmatrix} 2.7835 & -1.2940 & 0.0005 \\ -1.2940 & 2.2403 & -0.2437 \\ 0.0005 & -0.2437 & 2.5687 \end{pmatrix},$$

$$W = 10^{-4} \begin{pmatrix} 0.0652 & -1.9103 & -0.2759 \\ -1.9103 & -0.5690 & -0.2880 \\ -0.2759 & -0.2880 & -0.2816 \end{pmatrix}.$$

Therefore, by using Theorem 1, system (1) is finite-time stable with respect to $(T, U, c_1, c_2)$. Choose the switching signal shown in Fig. 1 and the initial condition $x(0) = (1.6, 1.5, 1.7)^\top$. The state trajectory of system (1) and the system response from 0 to 60s are shown in Fig. 2 and Fig. 3, respectively.

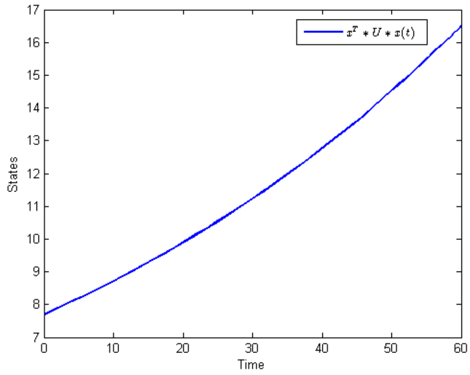


**FIGURE 3.** The system response from 0 to 60s.

*Example 2:* Consider system (1) with $n = N = 2$, $\tau = 2$,

$$A_1 = \begin{pmatrix} 0 & 0.005 & 0.001 \\ 0.004 & 0 & 0.003 \\ 0.002 & 0.005 & 0.001 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 0.003 & 0.003 \\ 0.004 & 0 & 0.003 \\ 0.003 & 0.004 & 0 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0.03 & 0.001 & 0.002 \\ -0.04 & 0.001 & 0.001 \\ 0.001 & -0.002 & 0.01 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0.001 & 0.004 & 0.01 \\ 0.003 & -0.001 & -0.004 \\ 0.002 & 0.03 & -0.002 \end{pmatrix}.$$

For given $U = I$, $T = 50s$, $a = 1.5$, $\mu = 1.29$, $c_1 = 7.1$ and $c_2 = 59.5$, solving inequalities (10)-(16), we get $\alpha = 1.47$, $\beta = 0.5$, and

$$P_1 = \begin{pmatrix} 0.54648 & 0.00286 & -0.00055 \\ 0.00286 & 0.54252 & -0.00072 \\ -0.00055 & -0.00072 & 0.54945 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} -0.08050 & 0.00154 & 0.00005 \\ 0.00154 & -0.08045 & 0.00021 \\ 0.00005 & 0.00021 & -0.07931 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 0.38936 & 0.00013 & -0.00104 \\ 0.00013 & 0.38207 & 0.00014 \\ -0.00104 & 0.00014 & 0.38876 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} -0.37921 & 0.00083 & 0.00036 \\ 0.00083 & -0.38620 & -0.00045 \\ 0.00036 & -0.00045 & -0.38389 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 0.98817 & 0.03231 & -0.00284 \\ 0.03231 & 0.98820 & 0.00048 \\ -0.00284 & 0.00048 & 1.03184 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} -0.09264 & -0.00016 & 0.00029 \\ -0.00016 & -0.09068 & 0.00028 \\ 0.00029 & 0.00028 & -0.09206 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 0.70449 & 0.01848 & -0.00328 \\ 0.01848 & 0.70437 & -0.00041 \\ -0.00328 & -0.00041 & 0.73725 \end{pmatrix},$$

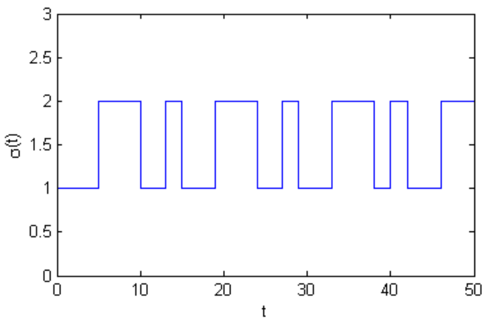


**FIGURE 4.** The chosen switching signal.

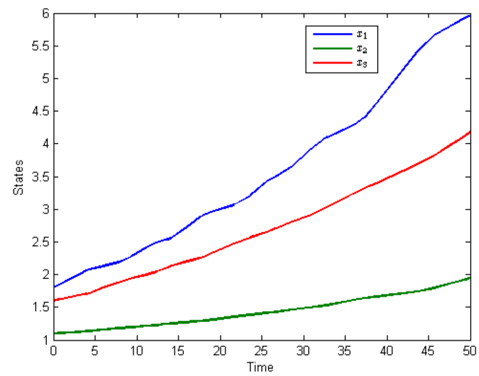


**FIGURE 5.** The state trajectory of system (1).

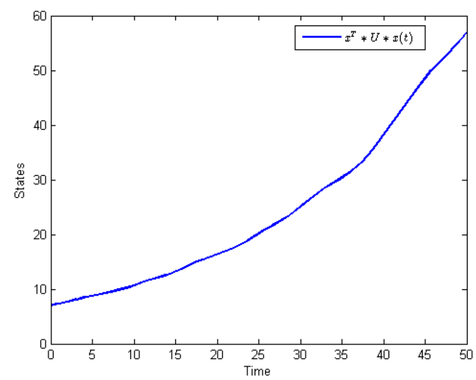


**FIGURE 6.** The system response from 0 to 50s.

$$W_2 = \begin{pmatrix} -0.24829 & 0.00272 & 0.00072 \\ 0.00272 & -0.32478 & -0.00060 \\ 0.00072 & -0.00060 & -0.31705 \end{pmatrix}.$$

Therefore, by using Theorem 1, for any switching signal with average dwell time $\tau_a = 5$, system (1) is finite-time stable with respect to $(T, U, c_1, c_2)$. Choosing the switching signal shown in Fig. 4 and the initial condition $x(0) = (1.8, 1.1, 1.6)^\top$, the state trajectory of system (1) and the system response from 0 to 50s are shown in Fig. 5 and Fig. 6, respectively.

## V. CONCLUSION

In this paper, finite-time stability of switched linear time-delay systems is studied by constructing a time-dependent

Lyapunov function. New explicit conditions for finite-time stability of the system under arbitrary switching and average dwell time switching are presented in terms of LMIs, which are easily verifiable and less conservative. For finite-time stability of the switched system with time-varying delay, it will remain for further study.

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