MPC-Based Offset-Free Tracking Control for Intermittent Transonic Wind Tunnel

BAOCANG DING[ID][1], **JUN WANG**[ID][2], **AND XIAOMING TANG**[ID][1]
[1]College of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[2]School of Automation Science and Engineering, Faculty of Electrical and Information Engineering, Xi'an Jiaotong University, Xi'an 710049, China

Corresponding author: Baocang Ding (baocangding@126.com)

ABSTRACT This paper addresses the offset-free model predictive control (MPC) for the intermittent transonic wind tunnel (ITWT). The offset-free property holds by introducing the controller state equation which introduces an integral action, while the controller state can keep as the decision variable. The infinite-horizon control moves are parameterized as a sequence of degrees of freedom followed by a control law. The terminal constraint set and terminal control law are designed based on the asymptotic invariance ellipsoid in which physical constraints are satisfied. The previous dual-mode MPC is appropriately modified withholding the closed-loop stability. Simulation results verify the effectiveness of the proposed methods.

INDEX TERMS Model predictive control, offset-free control, stability, wind tunnel.

I. INTRODUCTION

Wind tunnel is a kind of pipeline equipment being utilized to manually generate and control airflow in order to investigate flow behaviour of objects [1]–[3]. It is the most commonly used and effective tool for aerodynamic experiments. In the experiment, the object model is fixed in wind tunnel for blowing repeatedly, and test data are obtained by measuring and controlling instruments. The parameters, such as Mach number, total pressure and total temperature, are the key indexes to evaluate the quality of flow filed. With the development of aerospace technology and the requirement from modern military, the control system of wind tunnel plays an important role in the wind tunnel construction.

In view of control system design, the wind tunnel is a complicated system with nonlinearity, time-varying behaviour, time-delay, state coupling and uncertainties in the flow characteristics. The stability, robustness and offset-free property naturally become the key issues to be considered in the control strategies. Inspired by theses problems, a large number of researchers and institutes devote to investigating effective control strategies to guarantee the control performance (see.,e.g. [4], [5] and the reference therein). In the early design of control system for wind tunnel, the classical proportion-integration-differentiation (PID), linear quadratic regulation (LQR) were widely applied. For instance, based on

a linear model, the author in [6] designed a PID controller for a transonic blow down to regulate the Mach number. In [7], the authors made a performance comparison of a H_∞ controller with that of a LQR for regulating the pressure inside of the settling chamber of a hypersonic wind tunnel. However, the parameters of PID are determined through thousands of blower debugging, and it is difficult to achieve optimal control. In order to improve operation efficiency, other advanced modeling and control methods have been introduced, including but not limited to various neural networks, robust control, model predictive control (MPC). The recent works can be found in [8]–[10].

In practice, MPC is one of the most promising methods to a wide range of industrial processes, such as chemical, petrochemical, pulp, gas pipeline and metallurgical [11], [12]. At each sampling time, the controller computes an optimal control sequence and implements the first control input, then the entire optimization is repeated at subsequent sampling time [13]–[15]. Distinguishing from other conventional control methods, one of the most appealing features of the MPC is that it can drive the plant to the most profitable operating condition with constraint satisfaction [16]. For tracking problem, various MPC approaches for linear systems have emerged [17], [18]. The work of [19] studies the tracking of a piecewise constant reference for linear time-invariant (LTI) system subject to state constraint and bounded disturbance. In [20], a robust MPC tracking algorithm is proposed for uncertain linear time-varying systems.

The authors in [21] addressed an offset-free MPC algorithm for the constrained linear systems with unmeasured and bounded disturbances. In [22], the authors presented a novel MPC for constrained linear systems to track piecewise constant references, where the controller ensures feasibility by adding an artificial steady-state and input as decision variable of the optimization problem. The authors in [23] investigated the problem of offset-free MPC for linear system tracking an asymptotically constant reference. For a constrained linear system with additive uncertainties, the authors in [24] proposed a novel robust MPC method to track changing targets. The designed controller steers the uncertain system to (a neighborhood of) the desired steady-state and maintains these properties under any change of reference. Recently in [25], the author studied an MPC tracking algorithm for linear dynamic system subject to input constraint, where an integrator is inserted into the feedback loop to track the setpoint. Reference [26] considers the tracking problem of MPC for linear system under time-varying input constraints, and the proposed control law consists of a dual-mode MPC law and a target recalculation mechanism. In [27], the tracking problem for linear parameter-varying (LPV) systems, described by affine parameter-dependent state-space model with additive stochastic uncertainties, is addressed.

For a wind tunnel experiment, it usually requires the flow field parameters to maintain at a desired test condition, which implies that the flow, Mach number, pressures and temperature offset-freely track the steady-state targets. However, the practical features of wind tunnel limit the applications of other methods in modeling and controller design. In addition, the research on tracking MPC method for linear system is becoming mature, but approaches of MPC for tracking problem of wind tunnel are rare. To the author's best knowledge, an initial work of offset-free MPC for intermittent transonic wind tunnel (ITWT) was addressed in [10], which focused on handling the problem of variance of angle of attack in the presence of external disturbances. Therefore, this paper further contributes to investigate the offset-free MPC strategies for ITWT to guarantee flow filed control performance.

The aim of this paper is to study the offset-free tracking problem for ITWT based on MPC techniques. The main ideas can be summarized as follows:

1) an identification procedure is taken to obtain a linear model to describe the dynamics of wind tunnel around its steady-state working condition;

2) an MPC controller is designed by solving a finite-horizon tracking optimization problem, where the cost function is composed of a finite input and state horizon cost and a terminal cost;

3) the controller state not only introduces integral action, but also can consistently serve as a decision variable;

4) the infinite-horizon input constraints are explicitly handed by using terminal invariant set and Lyapunov approach.

In the controller design, we consider two methods for control move, which result in different offset-free MPC strategies.

TABLE 1. Notations.

| Symbol	Meaning
P_0	total pressure of the tunnel
Ma	Mach number at test section
D_{me}	displacement of the main exhaust valve
D_{cf}	displacement of the choke finger
$P_{0,eq}$	value of P_0 at the equilibrium point
Ma_{eq}	value of Ma at the equilibrium point
$D_{me,eq}$	value of D_{me} at the equilibrium point
$D_{cf,eq}$	value of D_{cf} at the equilibrium point
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathcal{E}(M, v)$	$\{x | (x-v)^T M (x-v) \leq 1\}$
x^{ss}	steady-state value of x
I_n	n-dimensional identity matrix
$\star$	a symmetric structure in the matrix inequalities
$*$	an optimal solution to the MPC optimization problem
$\|x\|_Q^2$	$x^T Q x$
$x(k+i|k)$	the value of vector x at time $k+i$, predicted at time k
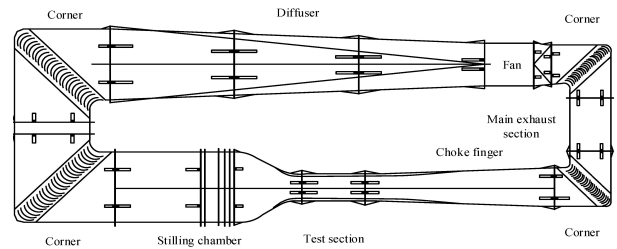
FIGURE 1. Intermittent transonic wind tunnel.

The distinguishing difference between two methods is that, the former incorporates controller state into the open-loop control sequence optimization in order to improve the control performance, while the latter considers a general formulation of closed-loop system in order to enlarge the feasibility range of the overall optimization problem.

This paper is organized as follows. Section 2 briefs the ITWT and its model description. The tracking MPC algorithm for linear system with controller optimization, and the general offset-free MPC tracking method, are detailed in Sections 3 and 4, respectively. In Section 5, a numerical simulation on ITWT is given to illustrate the effectiveness of the proposed algorithm. Section 6 concludes this paper. The notations used in this paper are shown in Table 1.

II. PROBLEM FORMULATION
A. FACILITY DESCRIPTION FOR INTERMITTENT TRANSONIC WIND TUNNEL

The schematic of ITWT is shown in Fig. 1, which consists of a cascade of several ducts with physical devices. The operation of ITWT and the function of its components are introduced briefly. First, air injected from the storage tank is accelerated by the fan. Then, air flows through diffuser to reach a lower velocity before it reaches the first two corners. The corner contains turning vanes in order to avoid rotation of the flow. The objective of diffuser is to convert dynamic pressure (kinetic energy) into static pressure (potential energy). In the stilling chamber, the potential energy of air is translated

to kinetic energy in a contraction cone with straightening elements so that air accelerates to a high speed and satisfy the flow quality requirement. Lastly, the flow rapidly passes over and around the tested object in the test section, and air flows over the choke finger and corner. A part of air is exhausted through the main exhaust valve and the rest circles in the tunnel.

B. MATHEMATICAL MODEL OF INTERMITTENT TRANSONIC WIND TUNNEL

For a wind tunnel experiment, the feature of the flow field can be characterized by total pressure P_0 and Mach number Ma, to a certain extent. The two parameters, in the test process, requires to keep at certain set-values within a certain precision range. Therefore, the main control goal of wind tunnel is how to guarantee the total pressure P_0 and Mach number Ma to arrive at steady-state without offset.

In the wind tunnel control system, the manipulate variables (MVs) are the displacement variables D_{me} and D_{cf}. In reality, the scale of wind tunnel determines and restricts the size of a tested model. The test process always works around the nominal state, which indicates that MVs vary with small amplitudes. Therefore, in a nominal working-point, it is reasonable to utilize a linear model to represent the wind tunnel model.

Considering the coupling relationship between the total pressure P_0 and Mach number Ma, the multivariable system is theoretically transformed into a dual-input dual-output systems, which is described by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{0.08}{3s+1}e^{-0.5s} & -\dfrac{0.02}{(0.8s+1)^3}e^{-0.4s} \\ -\dfrac{6.5\times10^{-4}}{(1.5s+1)^2}e^{-0.4s} & -\dfrac{6\times10^{-4}}{(0.7s+1)^2}e^{-0.4s} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{1}$$

where $y_1 = P_0 - P_{0,eq}, y_2 = Ma - Ma_{eq}, u_1 = D_{me} - D_{me,eq}$ and $u_2 = D_{cf} - D_{cf,eq}$.

C. STATE-SPACE MODEL DESCRIPTION

Consider the linear model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$, $y(k) \in \mathbb{R}^{n_y}$ are the state, the input and the output, respectively; A, B and C are known constant matrices of appropriate dimensions.

In order to achieve offset-free tracking, an effective approach is to introduce an integral action in the controller design. Hence, we define

$$x_c(k+1) = x_c(k) + y_s(k) - y(k), \tag{3}$$

where $x_c(k) \in \mathfrak{R}^{n_c}$ is the controller state, and $y_s(k) \in \mathfrak{R}^{n_y}$ is a reference signal.

In this way, the model (2)-(3) can be formulated as

$$\tilde{x}(k+1) = \mathcal{A}\tilde{x}(k) + \mathcal{B}u(k) + Dy_s(k), \tag{4}$$

$$y(k) = \mathcal{C}\tilde{x}(k), \tag{5}$$

where

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} A & 0 \\ -C & I \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$\mathcal{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

Without loss of generality, throughout this paper, the following assumptions are given.

Assumption 1: The system state x is measurable.
Assumption 2: The pair $(\mathcal{A}, \mathcal{B})$ is stabilizable.
Assumption 3: The model (2) is subject to input constraint, i.e.,

$$-\underline{u} \le u(k) \le \bar{u}, \quad k \ge 0, \tag{6}$$

where $\underline{u} := [\underline{u}_1, \underline{u}_2, \cdots, \underline{u}_{n_u}]^T$ and $\bar{u} := [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_{n_u}]^T$ with $\underline{u}_j > 0, \bar{u}_j > 0, i = 1, 2, \cdots, n_u$.

In this paper, the problem we consider is to design an MPC controller to track a changing reference and steer the tracking error to the origin while satisfying the input constraint (6).

At each time k, the controller solves an optimization problem that minimizes a given performance index, to yield an optimal control sequence.

The following of Section II is suitable for the case with $y_s(k+i|k) = y_s(k) = 0$. Solve the following optimization problem:

$$\min_{\tilde{u}(k)} J(k) = \sum_{i=0}^{N-1} \left[\|\tilde{x}(k+i|k)\|_Q^2 + \|u(k+i|k)\|_R^2 \right]$$
$$+ \|\tilde{x}(k+N|k)\|_P^2, \tag{7}$$
$$s.t.\, \tilde{x}(k+i+1|k) = \mathcal{A}\tilde{x}(k+i|k) + \mathcal{B}u(k+i|k)$$
$$+ Dy_s(k), \quad i = 0, \ldots, N-1, \tag{8}$$
$$-\underline{u} \le u(k+i|k) \le \bar{u}, \quad i = 0, \ldots, N-1, \tag{9}$$
$$\tilde{x}(k+N|k) \in \mathcal{E}_N(k), \tag{10}$$

where $\tilde{u}(k) = [u(k|k)^T, u(k+1|k)^T, \cdots, u(k+N-1|k)^T]^T$, $Q \ge 0$ and $R \ge 0$ are the weighting matrices. P is the positive-definite weighting matrix for the terminal cost. N denotes the prediction horizon and $\mathcal{E}_N(k)$ is the terminal set that needs to be designed as a controlled invariant set in the neighborhood of the origin. The terminal constraint guarantees that the predicted state reaches the terminal set after N steps, then converges towards the origin by applying the terminal control law.

In a standard MPC method, in order to guarantee the stability of closed-loop system, it should satisfy the following conditions:

$$\|\tilde{x}(k+i|k)\|_P^2 - \|\tilde{x}(k+i+1|k)\|_P^2$$
$$\ge \|\tilde{x}(k+i|k)\|_Q^2 + \|u(k+i|k)\|_R^2, \quad i \ge N, \tag{11}$$
$$u(k+i|k) = F\tilde{x}(k+i|k), \quad i \ge N, \tag{12}$$
$$\tilde{x}(k+N|k) \in \mathcal{E}_N(k), \quad i \ge N. \tag{13}$$

Lemma 1 [28]: Consider the system (4)-(5) under Assumptions 1-3. Using the terminal control law (12), suppose there exist a positive scalar γ_2, a positive-definite matrix $S = \gamma_2 P^{-1}$, and a matrix $Y = FS$, such that

$$\begin{bmatrix} S & \star & \star & \star \\ \mathcal{A}S + \mathcal{B}Y & S & \star & \star \\ R^{1/2}Y & 0 & \gamma_2 I & \star \\ Q^{1/2}S & 0 & 0 & \gamma_2 I \end{bmatrix} \geq 0, \quad (14)$$

$$\begin{bmatrix} S & \star \\ Y & Z \end{bmatrix} \geq 0, \quad Z_{jj} \leq \bar{u}_{j,inf}^2, \ j = 1, 2 \dots, n_u, \quad (15)$$

where $\bar{u}_{j,inf} = \min\{\underline{u}, \bar{u}\}$. Then (11) is satisfied, and

$$-\underline{u} \leq u(k + i|k) \leq \bar{u}, \ i \geq 0$$

i.e., the input constraint is satisfied in the terminal set.

The feedback gain matrix F can be determined by minimizing γ_2 subject to (14)-(15).

Based on Lemma 1, we can further conclude the following proposition.

Proposition 1: Consider the feedback gain matrix F and positive-definite matrix P satisfying (14) and (15). Then,
1) the set $\mathcal{E}_N(k) = \{\xi \in \Re^{n_x+n_c} | \|\xi\|_P^2 \leq \gamma_2\}$ is an invariant set;
2) $V(\tilde{x}(k + i|k)) = \tilde{x}(k + i|k)^T P \tilde{x}(k + i|k)$ serves as a Lyapunov function of closed-loop system (4)-(5) in the terminal set $\mathcal{E}(P, \gamma_2)$, i.e., $V(\tilde{x}(k + i + 1|k)) - V(\tilde{x}(k + i|k)) \leq -\tilde{x}(k + i|k)^T(Q + F^T RF)\tilde{x}(k + i|k)$ for any $i \geq N$.

In the following sections, we give details on the offset-free MPC tracking methods by handling the optimization problem (7)-(10) including $y_s(k + i|k) \neq 0$.

III. OFFSET-FREE MPC STRATEGY WITH CONTROLLER STATE OPTIMIZATION
In this section, based on the previous work [25], we design an offset-free MPC tracking method by optimizing both open-loop optimal control sequence and controller state.

Assume that $y_s(k + i|k) = y_s(k)$. For the system (4), defining $e(k + i|k) = y_s(k) - y(k + i|k)$, one has

$$\tilde{x}(k + i + 1|k) = \mathcal{A}\tilde{x}(k+i|k) + \mathcal{B}u(k+i|k) + Dy_s(k), \quad (16)$$

$$e(k + i|k) = \mathcal{C}\tilde{x}(k + i|k) + Ey_s(k), \quad (17)$$

with $\mathcal{C} = [-C \ 0]$ and $E = I$.

Before proceeding, in order to guarantee the solvability of output regulation problem, the following assumption is made.

Assumption 4: There exist matrices $\Phi \in \Re^{n \times n_y}$, $\Psi \in \Re^{n_u \times n_y}$ satisfying

$$\mathcal{A}\Phi + \mathcal{B}\Psi + D = \Phi, \quad (18)$$

$$\mathcal{C}\Phi + E = 0. \quad (19)$$

By defining $z(k+i|k) = \tilde{x}(k+i|k) - \Phi y_s(k)$ and $v(k+i|k) = u(k + i|k) - \Psi y_s(k)$, (16) and (17) can be rewritten as

$$z(k + i + 1|k) = \mathcal{A}z(k + i|k) + \mathcal{B}v(k + i|k), \quad (20)$$

$$e(k + i|k) = \mathcal{C}z(k + i|k). \quad (21)$$

A. OPEN-LOOP CONTROL SEQUENCE OPTIMIZATION
For the closed-loop system (20) and (21), the cost function for the control sequence $\tilde{v}(k) = [v(k|k)^T, v(k+1|k)^T, \cdots, v(k+ N - 1|k)^T]^T$ is given by

$$J_0^{N-1}(k) = \sum_{i=0}^{N-1}\{\|z(k + i|k)\|_Q^2 + \|v(k + i|k)\|_R^2\}, \quad (22)$$

where the predictions of $z(k + i|k)$, $i = 0, 1, \dots, N - 1$ is described by

$$\tilde{z}(k) = \mathbb{A}z(k) + \mathbb{A}_B\tilde{v}(k), \quad (23)$$

with

$$\tilde{z}(k) = \begin{bmatrix} z(k) \\ z(k + 1|k) \\ \vdots \\ z(k + N - 1|k) \end{bmatrix}, \quad \mathbb{A} = \begin{bmatrix} I \\ \mathcal{A} \\ \vdots \\ \mathcal{A}^{N-1} \end{bmatrix},$$

$$\mathbb{A}_B = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \mathcal{B} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \mathcal{A}^{N-2}\mathcal{B} & \cdots & \mathcal{B} & 0 \end{bmatrix}.$$

Let

$$\sum_{i=0}^{N-1}\{\|z(k + i|k)\|_Q^2 + \|v(k + i|k)\|_R^2\} \leq \gamma_1, \quad (24)$$

where γ_1 gives an upper bound of $J_0^{N-1}(k)$.

By using the Schur complement, (24) is guaranteed by

$$\begin{bmatrix} \gamma_1 & \star & \star \\ \mathbb{A}z(k) + \mathbb{A}_B\tilde{v}(k) & \mathbb{Q}^{-1} & \star \\ \tilde{v}(k) & 0 & \mathbb{R}^{-1} \end{bmatrix} \geq 0, \quad (25)$$

where $\mathbb{Q} = diag\{Q, \cdots, Q\}$ and $\mathbb{R} = diag\{R, \cdots, R\}$.

The input constraint (6) for $i = 0, 1, .., N - 1$ can be expressed as

$$-\underline{u} \leq \tilde{v}(k) + \tilde{\Psi}y_s(k) \leq \bar{u}, \quad (26)$$

where $\tilde{\Psi} = [\Psi^T, \Psi^T, \cdots, \Psi^T]^T$.

B. TERMINAL CONTROL LAW AND TERMINAL CONSTRAINT
For the system (20)-(21), we adopt the predictive control law for any $i \geq N$ in the following form:

$$v(k + i|k) = Fz(k + i|k). \quad (27)$$

Similarly to the conditions (11)-(13), the terminal control law $v(k + i|k) = Fz(k + i|k)$ for $i \geq N$ can be determined by Lemma 1 with the closed-loop system (20)-(21).

Based on Proposition 1, the terminal set $\mathcal{E}(P, \gamma_2)$ is positively invariant. Applying the Schur complement on $z(k + N|k) \in \mathcal{E}(P, \gamma_2)$ with $P = \gamma_2 S^{-1}$ yields

$$\begin{bmatrix} 1 & \star \\ \mathcal{A}^N z(k) + \tilde{\mathbb{A}}_B\tilde{v}(k) & S \end{bmatrix} \geq 0, \quad (28)$$

where $\tilde{\mathbb{A}}_B = [\mathcal{A}^{N-1}\mathcal{B}, \dots, \mathcal{A}\mathcal{B}, \mathcal{B}]$.

In summary, at each time k, the on-line offset-free MPC tracking problem is approximated as

$$\min_{\tilde{v}(k),\gamma_1,\gamma_2,Y,S,Z,x_c} \gamma_1 + \gamma_2,$$
$$s.t.\ (14), (15), (25), (26)\ and\ (28). \quad (29)$$

Note that the optimization problem (29) is composed of a set of LMIs, which can be easily solved by LMI toolbox.

Remark 1: In the problem (29), the controller state $x_c(k)$ is selected as a decision variable, which can accelerate the decreasing of cost function and improve the tracking performance. In [25], when the cost function becomes small, the controller state is not optimized but acts as an integral action to eliminate the offset. However, under Assumption 4, the output tracking problem essentially becomes a regulation problem. Therefore, compared with "Algorithm 1" of [25], in this paper the controller state is always optimized.

C. MPC PROPERTIES: FEASIBILITY AND CLOSED-LOOP STABILITY

In the following, the feasibility and stability by applying the problem (29) will be discussed.

Theorem 1: For the system (20)-(21), if the optimization problem (29) is feasible at time k, then it is feasible for any $t > k$, and $\lim_{k\to\infty} e(k|k) = 0$ holds.

Proof: Without loss of generality, it is assumed that there exists a feasible solution to (29) at time k. In the following we show that, at the next time $k + 1$, a feasible solution to the optimization problem is guaranteed to exist by invoking the optimal solution calculated at time k. Firstly, let

$$v(k+i+1|k+1) = \begin{cases} v^*(k+i+1|k), & 0 \le i \le N-2 \\ Fz^*(k+i+1|k), & i = N-1, \end{cases}$$
$$(30)$$

and let $x_c(k+1)$ be calculated by (3). Then, (26) is guaranteed when k shifts to $k + 1$.

Based on (30), it is shown that

$$\bar{J}(k + 1)$$
$$= \sum_{i=1}^{N-1}\{\|z(k+i+1|k+1)\|_Q^2 + \|v(k+i+1|k+1)\|_R^2\}$$
$$\quad + \|z(k+N+1|k+1)\|_{P(k+1)}^2$$
$$= \sum_{i=1}^{N-1}\{\|z^*(k+i|k)\|_Q^2 + \|v^*(k+i|k)\|_R^2\}$$
$$\quad + \|z^*(k+N|k)\|_Q^2 + \|F^*(k)z^*(k+N|k)\|_R^2$$
$$\quad + \|z^*(k+N+1|k)\|_{P^*(k)}^2. \quad (31)$$

Since (14) guarantees that $\|z(k+i+1|k)\|_{P(k)}^2 - \|z(k+i|k)\|_{P(k)}^2 \le -\|z(k+i|k)\|_Q^2 - \|v(k+i|k)\|_R^2$ for all $i \ge N$, the following relation is satisfied:

$$\|z^*(k+N+1|k)\|_{P^*(k)}^2 - \|z^*(k+N|k)\|_{P^*(k)}^2$$
$$\le -\|z^*(k+N|k)\|_Q^2 - \|F^*(k)z^*(k+N|k)\|_R^2. \quad (32)$$

Note that

$$\bar{J}^*(k) \le \eta^*(k) \triangleq \|z(k)\|_Q^2 + \gamma_1^*(k) + \gamma_2^*(k),$$
$$\bar{J}(k+1) \le \eta(k+1) \triangleq \|z^*(k+1|k)\|_Q^2 + \gamma_1(k+1)$$
$$\quad + \gamma_2(k+1).$$

Following from (31) and (32), it is concluded that

$$\gamma_2(k+1) = \gamma_2^*(k) - \|z^*(k+N|k)\|_Q^2$$
$$\quad - \|F^*(k)z^*(k+N|k)\|_R^2,$$
$$\gamma_1(k+1) = \gamma_1^*(k) - \|v^*(k)\|_R^2 - \|z^*(k+1|k)\|_Q^2$$
$$\quad + \|z^*(k+N|k)\|_Q^2 + \|F^*(k)z^*(k+N|k)\|_R^2,$$
$$\{Y,S,Z\}(k+1) = \frac{\gamma_2(k+1)}{\gamma_2^*(k)}\{Y,S,Z\}^*(k),$$

are the feasible choices to (29) when k shifts to $k + 1$.

The optimization problem is re-solved at $k + 1$, which results in $\gamma_1^*(k+1) + \gamma_2^*(k+1) \le \gamma_1(k+1) + \gamma_2(k+1)$. Hence, the relation $\eta^*(k+1) - \eta^*(k) \le -\|z(k)\|_Q^2 - \|v^*(k)\|_R^2$ holds, and $\eta^*(k)$ can serve as the Lyapunov function. It is concluded that $\lim_{k\to\infty} z(k) = 0$ and $\lim_{k\to\infty} e(k) = 0$. The proof is complete.

IV. GENERAL OFFSET-FREE MPC STRATEGY

In this section, we propose a variant general offset-free MPC method for system (2) subject to input constraint (6). The following assumption and definition are made.

Assumption 5: For system (2), the following condition holds:

$$rank\begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix} = n_x + n_y. \quad (33)$$

Proposition 2 (Controllability of Augmented System [21]): The augmented system (4)-(5) is said to be controllable if and only if the controllability of (A, B), and rank condition (33) in Assumption (5), hold.

It is easier to satisfy Assumption 5 than to Assumption 4. Hence, the following designed method is a more general offset-free method.

A. DYNAMIC PREDICTIVE CONTROLLER DESIGN

For the augmented system (4)-(5), the following control move is applied:

$$u(k) = F\tilde{x}(k) + v(k), \quad (34)$$

where F is a stabilizing controller gain, which will be designed later. $v(k)$ is a free perturbation item to be determined by a finite-horizon MPC optimization problem.

Under the feedback control law (34), the predictions of the augmented system (4)-(5), for any $i = 0, 1, \ldots, N - 1$, are

$$\tilde{x}(k+i+1|k) = (\mathcal{A}+\mathcal{B}F)\tilde{x}(k+i|k) + \mathcal{B}v(k+i|k)$$
$$\quad + \mathcal{D}y_s(k),$$
$$\tilde{x}(k+i|k) = \begin{bmatrix} x(k+i|k) \\ x_c(k+i|k) \end{bmatrix}. \quad (35)$$

The control objective of the proposed offset-free MPC method in this section is achieved by

1) designing a stabilizing linear time-invariant controller and computing an invariant set for the closed-loop system (4)-(5) with $v(k) = 0$, while guaranteeing the satisfaction of input constraint;

2) calculating the free perturbation items $v(k+i|k)$ for any $i = 0, 1, \ldots, N-1$ in a finite-horizon optimization problem with constraint satisfaction.

B. TERMINAL CONSTRAINT AND TERMINAL COST

Consider the optimization problem (7)-(10) with closed-loop system (35). Let $y_s(k) \equiv 0$. The terminal control law $u(k+i|k) = F\tilde{x}(k+i|k)$ can be determined by Lemma 1.

Hence, based on the predictions of $\tilde{x}(k+N|k)$, applying the Schur complement on the terminal constraint

$$\|\tilde{x}(k+N|k)\|_P^2 \leq \gamma_2$$

yields

$$\begin{bmatrix} 1 & \star \\ \mathcal{A}^N \tilde{x}(k|k) + \tilde{\mathbb{A}}_B \tilde{u}(k) & S \end{bmatrix} \geq 0. \quad (36)$$

where $\tilde{\mathbb{A}}_B = [\mathcal{A}^{N-1}\mathcal{B}, \ldots, \mathcal{A}\mathcal{B}, \mathcal{B}]$.

C. THE OVERALL OFFSET-FREE MPC OPTIMIZATION PROBLEM

Based on the above discussions, at each time k, the general offset-free MPC method can be summarized as

$$\min_{\tilde{v}(k)} J(k) = \sum_{i=0}^{N-1} \left[\|\tilde{x}(k+i|k)\|_Q^2 + \|v(k+i|k)\|_R^2 \right] + \|\tilde{x}(k+N|k)\|_P^2, \quad (37)$$

$$s.t. \ (35), (36), \underline{u} \leq u(k+i|k) \leq \bar{u}, \quad i = 0, 1, \ldots, N-1, \quad (38)$$

where $\tilde{v}(k) = \{v(k|k), v(k+1|k), \ldots, v(N-1|k)\}$. Therefore, at time k, the controller solves the optimization problem (37)-(38) and updates the optimal control sequence $\tilde{v}^*(k)$ in a receding horizon way. The real control move is calculated by $u^*(k) = F\tilde{x}(k) + v^*(k|k)$ and implemented to the system (2).

Remark 2: Compared with (29), the optimization problem (37)-(38) is a quadratic programming (QP) with respect to decision variable $\tilde{v}(k)$, which can be solved by the mature algorithms such as the interior point algorithm.

Theorem 2: If the closed-loop, to which the control input (34) is applied, is stable, then the steady-state output of the system (2) will converge to the equilibrium y_{eq} asymptotically.

Proof: The closed-loop system reaching to a steady-state means that $\tilde{x}^{ss}(k+1) = \tilde{x}^{ss}(k) = \tilde{x}^{ss}$ as $k \to \infty$. Therefore, at steady-state, (3) becomes

$$x_c^{ss} = x_c^{ss} - Cx^{ss} + y_s(\infty) = x_c^{ss} - y^{ss} + y_s(\infty),$$

which means $y^{ss} = y_s(\infty)$ as $k \to \infty$. Hence, $y^{ss} + y_{eq} = y_s(\infty) + y_{eq} = y_{eq}$, and the proof is complete.

Remark 3: The closed-loop stability mentioned in Theorem 2 can be proved by referring to [29].

V. SIMULATION RESULTS

This section will illustrate the effectiveness of the developed offset-free MPC methods for the considered ITWT model (1).

In order to obtain a state-space model, an identification experiment that transforms the transfer function model into the state-space formulation is performed. Here we introduce the experiment briefly. Select the subspace identification method N4SID to identify the model (1), with the relevant parameter settings: 1) the equilibrium point $y_{eq} = [0, 0]^T$, $u_{eq} = [0, 0]^T$; 2) the general binary noise (GBN) with magnitude 1; 3) the system order $n_x = 5$. By performing this identification process, the resulting system matrices are

$$A = \begin{bmatrix} 0.9247 & 0.0992 & 0.00557 & -0.0456 & 0.0036 \\ -0.0591 & 0.9414 & -0.3092 & -0.0162 & -0.1357 \\ -0.0617 & 0.0585 & 0.8691 & 0.3928 & 0.0549 \\ 0.0227 & 0.0551 & -0.0377 & 0.8867 & 0.5703 \\ -0.0107 & 0.0110 & -0.0377 & -0.1542 & 0.5703 \end{bmatrix},$$

$$B = \begin{bmatrix} -8.0905 & 0.0021 & -0.0012 & 0.0019 & 4.5048 \\ -0.0023 & -5.2663 & -7.2934 & 0.0035 & 1.4913 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} -1.0004 & 0.4508 & 0.4115 & -0.2115 & 0.1046 \\ -0.0113 & -0.0016 & -0.0029 & -0.0040 & 0.0109 \end{bmatrix}.$$

Based on the identified system matrices (A, B, C), the offset-free MPC tracking methods in Sections III and IV are verified. The input constraint is set as $|u_1(k+i|k)| \leq 1.2$ and $|u_2(k+i|k)| \leq 1$ for $i \geq 0$. The optimization problem (29) is infeasible since the condition (18) in Assumption 4 is violated. For the general offset-free MPC strategy, choose the weighting matrices $Q = I_6$ and $R = I_2$. Then, minimizing γ_2 subject to (14)-(15) obtains the feedback gain matrix F.

In the problem (37)-(38), the control horizon is chosen as $N = 3$ to calculate the optimal free perturbation sequence $\tilde{v}(k)$, where the initial state is $x(0) = [0\,0\,0\,0\,0\,0\,0]^T$
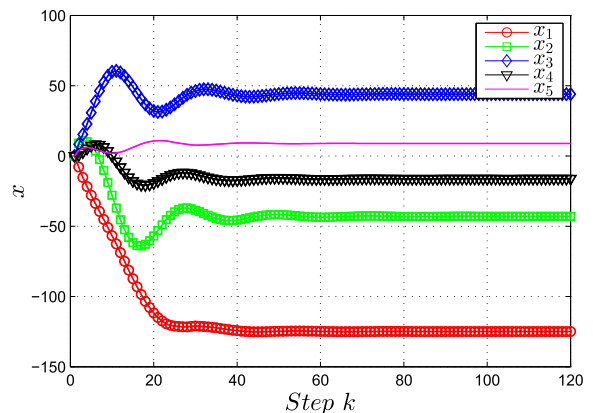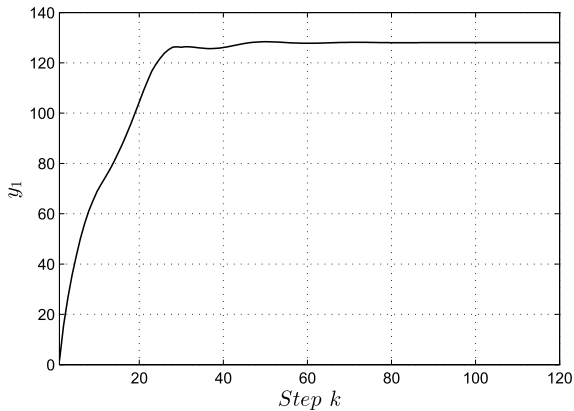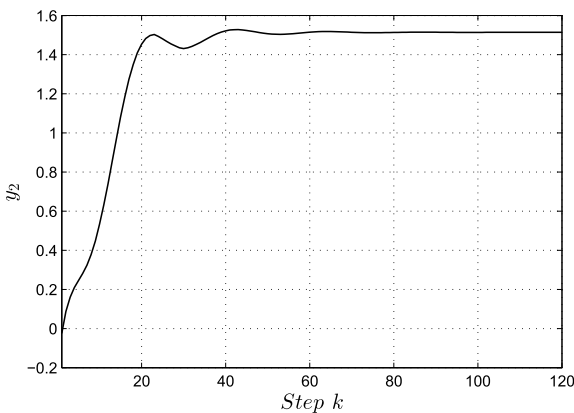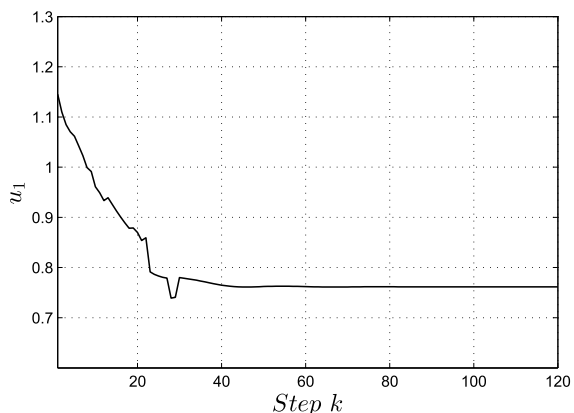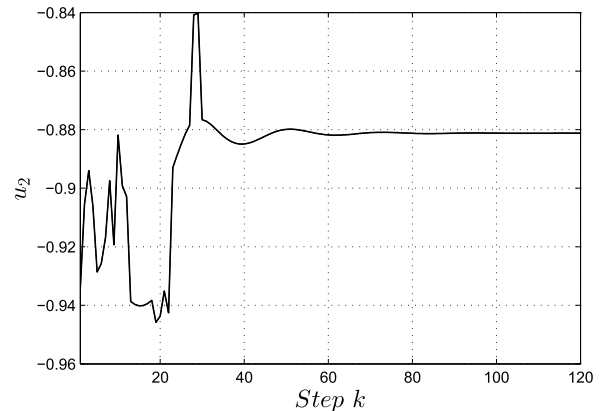
FIGURE 2. State trajectory of closed-loop system.

FIGURE 3. Output responses (y_1).

FIGURE 4. Output responses (y_2).

FIGURE 5. Control input of plant (2) (u_1).

FIGURE 6. Control input of plant (2) (u_2).

offset-free MPC method is more effective and applicable since Assumption 4 is a conservative condition, which is a harsh condition for an real process.

VI. CONCLUSION
In this paper, we have investigated offset-free MPC tracking methods for linear systems subject to input constraints and studied their effectiveness on an ITWT model. The controller consists of an open-loop optimal sequence and a terminal control law. The resulting MPC tracking method is treated as a finite horizon optimization problem with a terminal constraint. The terminal constraint and terminal control law are designed so that the input constraints are satisfied in the terminal set. It is necessary to incorporate the stochastic description and disturbance into the modeling and controller design (see, e.g. [30]–[32]), which will be our future topics.

REFERENCES
[1] R. Howell, N. Qin, J. Edwards, and N. Durrani, "Wind tunnel and numerical study of a small vertical axis wind turbine," *Renew. Energy*, vol. 35, no. 2, pp. 412–422, Feb. 2010.
[2] G. Ocokoljić, B. Rašuo, and M. Kozić, "Supporting system interference on aerodynamic characteristics of an aircraft model in a low-speed wind tunnel," *Aerosp. Sci. Technol.*, vol. 64, pp. 133–146, May 2017.
[3] K. Kanistras, P. C. Saka, K. P. Valavanis, and M. J. Rutherford, "Wind tunnel investigation of a circulation control wing with dual-radius flaps," *J. Aircr.*, vol. 55, no. 4, pp. 1731–1741, Jul. 2018.
[4] X. Wang, P. Yuan, and Z. Mao, "Ensemble fixed-size LS-SVMs applied for the mach number prediction in transonic wind tunnel," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 4, pp. 3167–3181, Oct. 2015.
[5] M. Sabanca, G. Brenner, and E. Durst, "Error control and adaptivity for low-Mach-number compressible flows," *AIAA J.*, vol. 40, no. 11, pp. 2234–2240, Jan. 2002.
[6] H. Miwa, M. Sato, S. Sakakibara, and K. Takashima, "Closedloop Mach number control and measurement," in *Proc. Int. Symp. Fluid Control Meas.*, Tokyo, Japan, 1985, pp. 2–5.
[7] S. Rajani and N. Usha, "Performance comparison of H-infinity and LQR controllers for the pressure regulation of a hypersonic wind tunnel," *Int. J. Mod. Eng. Re.*, vol. 3, no. 3, pp. 1623–1629, 2013.
[8] L. Ji and X. F. Lian, "Application of blow-off wind tunnel control based on genetic algorithm optimized BP-neural network PID neural network," *Appl. Mech. Mater.*, vol. 310, pp. 557–559, Feb. 2013.
[9] C. R. Nott, S. M. Ölçmen, D. R. Lewis, and K. Williams, "Supersonic, variable-throat, blow-down wind tunnel control using genetic algorithms, neural networks, and gain scheduled PID," *Int. J. Speech Technol.*, vol. 29, no. 1, pp. 79–89, Aug. 2007.

and the reference signal is $y_s(k) = [128\ 1.51]$. The simulation results are shown in Fig. 2-6. In Fig 2-4, both state and output reach to their steady-state setpoints, where the output quickly tracks. Fig 5-6 show the control inputs of the closed-loop system. It can be observed that the control signals always satisfy the constraints. Therefore, the simulation results demonstrate a reliable and desirable tracking performance by the offset-free control. Besides, we also conclude that the general

[10] J. Zhang, P. Yuan, and K.-S. Chin, "Model predictive control for the flow field in an intermittent transonic wind tunnel," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 1, pp. 324–338, Feb. 2018.

[11] R. Zhang, S. Wu, Z. Cao, J. Lu, and F. Gao, "A systematic Min-Max optimization design of constrained model predictive tracking control for industrial processes against uncertainty," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 6, pp. 2157–2164, Nov. 2018.

[12] R. Zhang, S. Wu, and F. Gao, "State space model predictive control for advanced process operation: A review of recent development, new results, and insight," *Ind. Eng. Chem. Res.*, vol. 56, no. 18, pp. 5360–5394, Apr. 2017.

[13] D. Li, Y. Xi, and F. Gao, "Synthesis of dynamic output feedback RMPC with saturated inputs," *Automatica*, vol. 49, no. 4, pp. 949–954, Apr. 2013.

[14] S. Yu, C. Böhm, H. Chen, and F. Allgöwer, "Model predictive control of constrained LPV systems," *Int. J. Control*, vol. 85, no. 6, pp. 671–683, Jun. 2012.

[15] L. Li, J. Song, X. Zhang, J. Ye, and S. Yang, "Model deficiency diagnosis and improvement via model residual assessment in model predictive control," *Ind. Eng. Chem. Res.*, vol. 56, no. 42, pp. 12151–12162, Oct. 2017.

[16] D. He, L. Wang, and J. Sun, "On stability of multiobjective NMPC with objective prioritization," *Automatica*, vol. 57, pp. 189–198, Jul. 2015.

[17] S. Aghaei, F. Sheikholeslam, M. Farina, and R. Scattolini, "An MPC-based reference governor approach for offset-free control of constrained linear systems," *Int. J. Control*, vol. 86, no. 9, pp. 1534–1539, Sep. 2013.

[18] T. L. M. Santos, "Modified reference tracking MPC for constrained linear systems," *Int. J. Syst. Sci.*, vol. 49, no. 8, pp. 1674–1684, May 2018.

[19] L. Chisci and G. Zappa, "Dual mode predictive tracking of piecewise constant references for constrained linear systems," *Int. J. Control*, vol. 76, no. 1, pp. 61–72, Jan. 2003.

[20] G. Pannocchia, "Robust model predictive control with guaranteed setpoint tracking," *J. Process Control*, vol. 14, no. 8, pp. 927–937, Dec. 2004.

[21] G. Pannocchia and E. C. Kerrigan, "Offset-free receding horizon control of constrained linear systems," *AIChE J.*, vol. 51, no. 12, pp. 3134–3146, Aug. 2005.

[22] D. Limon, I. Alvarado, T. Alamo, and E. F. Camacho, "MPC for tracking piecewise constant references for constrained linear systems," *Automatica*, vol. 44, no. 9, pp. 2382–2387, Sep. 2008.

[23] U. Maeder, F. Borrelli, and M. Morari, "Linear offset-free model predictive control," *Automatica*, vol. 45, no. 10, pp. 2214–2222, Oct. 2009.

[24] D. Limon, I. Alvarado, T. Alamo, and E. F. Camacho, "Robust tube-based MPC for tracking of constrained linear systems with additive disturbances," *J. Process Control*, vol. 20, no. 3, pp. 248–260, Mar. 2010.

[25] N. Wada, "Model predictive tracking control for constrained linear systems using integrator resets," *IEEE Trans. Autom. Control*, vol. 60, no. 11, pp. 3113–3118, Nov. 2015.

[26] N. Wada, H. Tomosugi, and M. Saeki, "Model predictive tracking control for a linear system under time-varying input constraints," *Int. J. Robust Nonlinear Control*, vol. 23, no. 9, pp. 945–964, Mar. 2012.

[27] S. Chitraganti, R. Tóth, N. Meskin, and J. Mohammadpour, "Stochastic model predictive tracking of piecewise constant references for LPV systems," *IET Control Theory Appl.*, vol. 11, no. 12, pp. 1862–1872, Aug. 2017.

[28] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inqualities," *Automatica*, vol. 32, pp. 1361–1379, 1996.

[29] J.-W. Lee, W. Hyun Kwon, and J. Choi, "On stability of constrained receding horizon control with finite terminal weighting matrix," *Automatica*, vol. 34, no. 12, pp. 1607–1612, Dec. 1998.

[30] G. D. Zong and H. L. Ren, "Guaranteed cost finite-time control for semi-Markov jump systems with event-triggered scheme and quantization input," *Int. J. Robust Nonlinear Control*, vol. 29, no. 15, pp. 5251–5273, 2019, doi: 10.1002/RNC.4672.

[31] G. Zong, Y. Li, and H. Sun, "Composite anti-disturbance resilient control for Markovian jump nonlinear systems with general uncertain transition rate," *Sci. China Inf. Sci.*, vol. 62, no. 2, Jan. 2019, Art. no. 22205, doi: 10.1007/s11432-017-9448-8.

[32] D. Yang, G. D. Zong, and H. R. Karimi, "H_∞ refined anti-disturbance control of switched LPV systems with application to aero-engine," *IEEE Trans. Ind. Electron.*, vol. 67, no. 4, pp. 3180–3190, Apr. 2020.
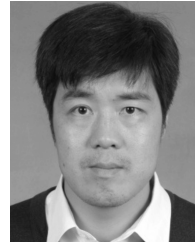
BAOCANG DING was born in Hebei, China. He received the master's degree from the China University of Petroleum, Beijing, in 2000, and the Ph.D. degree from Shanghai Jiaotong University, in 2003. From September 2003 to August 2007, he was an Associate Professor with the Hebei University of Technology, China. From September 2005 to September 2006, he was a Postdoctoral Research Fellow with the Department of Chemical and Materials Engineering, University of Alberta, Canada. From November 2006 to August 2007, he was a Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. From September 2007 to December 2008, he was a Professor with Chongqing University, China. From January 2009 to August 2019, he was a Professor with Xi'an Jiaotong University, China. He is currently a Professor with the Chongqing University of Posts and Telecommunications, China. His research interests include model predictive control, fuzzy control, robotic control, networked control, distributed control, and multienergy systems. He was a recipient of the 2009 Program for New Century Excellent Talents in University, China, and the 2016 Best Paper Award of *Asian Journal of Control*.

JUN WANG received the M.S. degree from the Chongqing University of Posts and Telecommunications, Chongqing, China, in 2017. He is currently pursuing the Ph.D. degree in control science and engineering with the School of Automation Science and Engineering, Xi'an Jiaotong University, Xi'an, China. His main research interest includes theory, algorithm, and application of model predictive control.

XIAOMING TANG received the B.S. degree from the College of Information and Electrical Engineering, Panzhihua University, Panzhihua, China, in 2008, and the Ph.D. degree in control theory and control engineering from the College of Automation, Chongqing University, Chongqing, China, in 2013.

From 2016 to 2017, he carried out his Postdoctoral research at The University of Texas at Arlington, Arlington, TX, USA. He is currently an Associate Professor with the College of Automation, Chongqing University of Posts and Telecommunications, Chongqing. His current research interests include model predictive control and networked control systems.

● ● ●