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Dual Control for Stochastic Linear MIMO Systems With Parameter Uncertainty

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ABSTRACT This paper considers a dual control problem for stochastic linear MIMO (Multiple Input Multiple Output) systems with parameter uncertainty. A novel dual adaptive control law for MIMO systems is proposed. The design is based on the innovation of the dual control cost function, which was originally developed for conventional adaptive control of linear systems. However, the design process is modified and developed to cater to the stochastic MIMO case. This is a more challenging problem because the superposition of parameter uncertainty and the MIMO property makes the problem more intractable. As in all dual adaptive strategies, it leads to a control law that balances out the need for caution, due to parameter uncertainty, with the conflicting requirement of probing that acts to quickly reduce parameter uncertainty, which is the nature of dual control. The proposed control law has two parts. One reflects the goal of regulating the output, and the second reflects the ability to handle uncertain parameters. A learning factor is introduced to balance these two parts to obtain a control law that can be applied to the original system. In the simulation examples, the uncertain parameters can be estimated quickly and accurately for the unknown but constant case and the abrupt parameter case. Furthermore, it is shown that the novel dual control law is superior to other control strategies by comparing the performance of the cost function in a statistical sense.

INDEX TERMS Stochastic MIMO systems, parameter uncertainty, dual control, Kalman filter.

I. INTRODUCTION

In all control problems, there are certain degrees of uncertainty with respect to the process to be controlled. These uncertainties are probably the result of external noise (disturbance), structure uncertainty, parameter fluctuations and so on [1]. Simultaneously, the structure of the system or the parameters of the process may vary in an unknown way [2]. For example, during the working process of a gyroscope, due to the environment (temperature, humidity, etc.), creep degeneration of structural materials, the release of pretension stress, the vibration of the gyroscope itself, motor wear and other factors, the real-time parameters of the system are difficult to determine [3], [4]. There are also some systems where the parameters of the components are accurately known and the structural model of the system is fixed. However, due to modelling simplification, approximation, or environmental influence, there is no one-to-one correspondence between the equivalent parameters of the model and the practical physical parameters, which may lead to large variation or fluctuation in many different forms [5]. If these uncertainties cannot be effectively addressed, they may affect system performance and even have serious consequences. Therefore, it is necessary to study this uncertain control problem.

Considerable research has contributed to solving this uncertain control problem. Regarding this type of problem, most of the research has focused on uncertainty problems with unknown parameters in the system. For a stochastic system with unknown parameters, in the early 1960s, the former Soviet scholar Fel'dbaum noticed this type of problem and proposed a control design theory consisting of caution and probing, namely, dual control. Through caution, the control strategy drives the system output to the desired reference state. To obtain a good control effect or avoid generating unacceptable output responses, the control should be appropriate and not too large, and it must be cautious. By contrast,

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probing aims to continuously excite the system using control with a large amplitude so that the system generates richer information to improve the estimation quality effectively. Dual control aims to strike a balance between these two conflicting goals to yield an overall optimal response. The dual roles of optimal control, optimisation and estimation, in general situations, cannot be separated. This coupling between optimisation and estimation makes an analytical form of optimal control, in most situations, unattainable. For this reason, IEEE Control Systems Magazine listed this challenging topic with significant theoretical implications and practical value as one of the top 25 questions that had a significant impact on Control theory in the last century [2]. Although several such control laws have been proposed for standard adaptive control cases involving an externally predefined reference input [6]–[9], none of them addresses the dual adaptive control problem with the superposition of the MIMO case and two uncertainties.

Presently, many methods are dedicated to single-input single-output (SISO) systems with parameter uncertainty, while there are few current research results on MIMO systems with parameter uncertainty. For the SISO system with parameter uncertainty, many achievements have been made. Most of them focus on cases in which parameters are unknown but constant, such as dual adaptive control [15], innovation dual control [18], the variance minimisation algorithm [6], nominal dual control [19], [20], model predictive control (MPC) [9], and dual adaptive extremum control [10]. For the control problem of MIMO systems, in most cases, the common method is to decouple them into multiple SISO systems [11], [12]. Because modern industrial systems and aerospace systems are composed of many interconnected links, much important information is lost if they are forcibly decoupled. Therefore, this method only works for systems whose input and output are independent of each other, not for most systems. Other than this kind of approach, there are only a few methods for MIMO systems, such as dual MPC [13] and functional adaptive control [14]. In [13] and [14], the sigmoidal MLP and FRBF networks are used to estimate the system model because they are nonlinear systems. Therefore, the methods proposed work only for this kind of nonlinear MIMO system because of the diversity of nonlinear systems.

To sum up, in our problem, there exist two types of uncertainties, which make dynamic programming based on the optimality principle invalid. Previous efforts in dual control have thus mainly been devoting to developing certain suboptimal solution, by bypassing this essential feature of coupling in dual control. There are also some efforts to decompose the global optimisation problem into multiple single-stage optimisation problems and obtain the approximate optimal control. Because of these restrictions, we consider how to design a control law to make the resulting control tend to be optimal and can effectively deal with uncertainty. Besides, MIMO system has wide application in the development of modern science and technology. Motivated by the above discussion, the dual control problem for stochastic linear MIMO systems with parameter uncertainty is considered, which fills the theoretical gap in such problems. In this paper, we design a novel MIMO dual adaptive control law based on bicriterial dual control (BDC). One criterion minimises the performance of the cost function, and the other endows the control law with a learning ability, which leads to the proposed control law with two parts. One part reflects the goals of regulating the output, and the other reflects the ability of handling uncertain parameters. Meanwhile, a learning factor is introduced to balance these two parts to obtain a control law that can be applied to the original system. In contrast to the previous method, the learning factor of this paper can be automatically adjusted at any time so that the appropriate learning intensity can be applied to the practical system. Compared with the preset learning factor, this method prevents the system from appearing unacceptable after the parameters have been accurately estimated and the learning factor is still large.

The rest of the paper is organised as follows. In section 2, the control problem to be solved is described. In section 3, the estimation process of parameters with uncertainty is introduced in detail. A novel MIMO dual adaptive control law is designed for the control problem in section 4. In section 5, the effectiveness of the algorithm in this paper is verified by two numerical examples and compared statistically with the controls. The conclusion is presented in section 6.

Notation: Throughout this paper, we use $\mathbf{I}_n$ to represent the identity matrix of size n. $tr(\mathbf{A})$ indicates the trace of matrix $\mathbf{A}$, and $\mathbf{A}^T$ denotes the transposition of matrix $\mathbf{A}$. The expression $\xi \sim N(m, S)$ means that the variable ξ has a normal distribution with mean m and variance S, and the quadratic form $\boldsymbol{\xi}^T\mathbf{Q}\boldsymbol{\xi}$ will be denoted by $|\boldsymbol{\xi}|^2_{\mathbf{Q}}$.

II. PROBLEM STATEMENT

Consider a multiple-input multiple-output (MIMO) stochastic linear system with parameter uncertainty described by the following:

$$\begin{cases} y_1(k+1) \\ \quad = \sum_{i=1}^{m} a_{1i}(k)y_i(k) + \sum_{j=1}^{r} b_{1j}(k)u_j(k) + \varepsilon_1(k) \\ y_2(k+1) \\ \quad = \sum_{i=1}^{m} a_{2i}(k)y_i(k) + \sum_{j=1}^{r} b_{2j}(k)u_j(k) + \varepsilon_2(k) \\ \quad \vdots \\ y_m(k+1) \\ \quad = \sum_{i=1}^{m} a_{mi}(k)y_i(k) + \sum_{j=1}^{r} b_{mj}(k)u_j(k) + \varepsilon_m(k), \\ \quad k = 0, 1, \cdots, N-1, \end{cases} \tag{1}$$

where $y_i(k), i = 1, 2, \cdots, m$ are the control outputs and $u_j(k), j = 1, 2, \cdots, r$ are the control inputs. $a_{1i}, \cdots, a_{mi}, b_{1j}, \cdots, b_{mj}, i = 1, \cdots, m, j = 1, \cdots, r$ are the parameters describing the physical properties of the system, and they are uncertain time-variant parameters. $\varepsilon_i(k), i = 1, \cdots, m$ are external noise that are often Gaussian distributions with zero mean and variance $\sigma_i^2, i = 1, \cdots, m$, that is, they satisfy $\varepsilon_i \sim N(0, \sigma_i^2)$.

The above model representing the stochastic linear system is particularly common. Not only the MIMO case but also single-input single-output (SISO) systems, single-input multiple-output (SIMO) systems and multiple-input single-output (MISO) systems can be represented, although SIMO systems are very rare. For nonlinear systems, linearisation near the equilibrium point is considered in most cases to result in a linear system. Therefore, this system can also be used to describe a linearised nonlinear system.

For the sake of convenience in writing, in the rest of this paper, $a_{i1}, \cdots, a_{im}, b_{i1}, \cdots, b_{ir}, i = 1, \cdots, m$ are used to represent the time-variant parameters instead of $a_{i1}(k), \cdots, a_{im}(k), b_{i1}(k), \cdots, b_{ir}(k), i = 1, \cdots, m$.

By analysing the above system, it can be rewritten as the following state-space model:

$$\mathbf{y}(k+1) = \mathbf{A}\mathbf{y}(k) + \mathbf{B}\mathbf{u}(k) + \boldsymbol{\varepsilon}(k), \tag{2}$$

where $\mathbf{y}(k) = [y_1(k), y_2(k), \cdots, y_m(k)]^T$, $\mathbf{u}(k) = [u_1(k), u_2(k), \cdots, u_r(k)]^T$, $\boldsymbol{\varepsilon}(k) = [\varepsilon_1(k), \varepsilon_2(k), \cdots, \varepsilon_m(k)]^T$, $\boldsymbol{\varepsilon}(k) \sim N(0, \boldsymbol{\Sigma}_\varepsilon)$, $\boldsymbol{\Sigma}_\varepsilon = \text{diag}\,(\sigma_1^2, \sigma_2^2, \cdots, \sigma_m^2)$,

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ldots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & \cdots & b_{2r} \\ \vdots & \vdots & \ldots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mr} \end{bmatrix}.$$

Note that the model (2) is exactly identical to the original system. Therefore, the control algorithm designed for the model (2) is completely applicable to the original system (1).

Our control objective is design a control goal $\mathbf{u}(k)$ make the control output $\mathbf{y}(k)$ can track reference signal $\mathbf{r}(k)$. The following cost function is then introduced:

$$\mathbf{J} \triangleq E\left\{\sum_{k=0}^{N-1} |\mathbf{y}(k+1) - \mathbf{r}(k+1)|_{\mathbf{Q}}^2\right\} \tag{3}$$

where $\mathbf{r}(k+1), k = 0, 1, \cdots, N-1$ is the given reference output with appropriate dimension. $E\{\cdot\}$ denotes the mathematical expectation conditioned upon the information set z^k, and $\mathbf{Q}$ is a positive definite weight matrix. Note that z^k is the past information set before time k and will be defined in the next section.

From the system described in model (2), it can be seen that the system contains both the measurement noise that is unavoidable during the system operation and parameter uncertainty. Due to the existence of these two uncertainties, it is obviously impossible to obtain an optimal controller using direct dynamic programming. Therefore, we transformed the original global optimisation problem into multiple single-stage optimisation problems. The transformed cost function at time k can be described as follows:

$$\mathbf{J}_k = E\left\{|\mathbf{y}(k+1) - \mathbf{r}(k+1)|_{\mathbf{Q}}^2\right\} \tag{4}$$

Concerning the stochastic linear system model (2), the main aim of the work proposed in this paper is to design a dual control law

$$\mathbf{u}^*(k) = arg \min_{\mathbf{u}(k)} \mathbf{J}_k[\mathbf{u}(k), z^k] \tag{5}$$

such that the parameters can be estimated more accurately and the performance of the transformed cost function is minimised.

Before proceeding with the design of the noval dual control law, it is appropriate to first present the estimation process of the uncertain parameters.

III. PARAMETER ESTIMATION

Due to the existence of two uncertainties, it is not possible for dynamic programming based on the optimality principle to solve the problem of this paper. For control problems with only uncertain parameters, many researchers have proposed several suboptimal control approaches [16], [23], [24]. The main idea of these approaches is to better combine caution with probing and to achieve the best balance between the two. However, study results have shown that the one- or two-step control solution to the original problem obtained is very limited, and the controller designed has a disadvantage of approximation [22]. To better achieve control and parameter estimates for the original system, the key is to make full use of the information provided by the original system to eliminate the uncertainty as much as possible, as well as to minimise the performance of the cost function. Innovation is a key factor that cannot be ignored. The Kalman filter is clearly an effective tool for estimating uncertain parameters.

To make it easier to use the Kalman filter to deal with our problem effectively, the model (2) can be written as follows:

$$\mathbf{y}(k+1) = \boldsymbol{\Phi}(k)\boldsymbol{\Theta}(k) + \boldsymbol{\varepsilon}(k) \tag{6}$$

where, $\boldsymbol{\Phi}(k) = \text{diag}[\boldsymbol{\phi}(k), \boldsymbol{\phi}(k), \cdots, \boldsymbol{\phi}(k)]$, $\boldsymbol{\phi}(k) = [\mathbf{y}^T(k), \mathbf{u}^T(k)]$, $\boldsymbol{\Theta}(k) = [\boldsymbol{\theta}_1^T(k), \boldsymbol{\theta}_2^T(k), \cdots, \boldsymbol{\theta}_m^T(k)]^T$, $\boldsymbol{\theta}_i(k) = [a_{i1}, \cdots, a_{im}, b_{i1}, \cdots, b_{ir}]^T$, $i = 1, 2, \cdots, m$.

Here, $\boldsymbol{\Theta}(k)$ is the m(m + r)-dimensional vector consisting of all uncertain parameters. They are closely related to all links in the whole operation process of the system. Even if the physical parameters of each component in the practical system is accurately known, uncertainties are inevitable because of the difference in the operating environment or materials of components [25]; this motivates us to establish the following dynamic model:

$$\boldsymbol{\Theta}(k+1) = \boldsymbol{\Gamma}\boldsymbol{\Theta}(k) + \boldsymbol{\xi}(k), \tag{7}$$

where $\boldsymbol{\Gamma}$ is the coefficient matrix revealing the variation trend of uncertain parameters. $\boldsymbol{\xi}(k)$ is used to describe the process noise of parameter estimation and is assumed to be a Gaussian contribution with zero mean and variance $\mathbf{\Sigma}_\xi$, that is, to satisfy $\boldsymbol{\xi}(k) \sim N(0, \mathbf{\Sigma}_\xi)$.

As shown in (7), we treat this as a state equation, together with (6), to form a traditional state-space model. In the state equation (7), when the coefficient matrix $\mathbf{\Gamma}$ is the identity matrix, it is shown that the uncertain parameters are unknown constants only influenced by process noise, which is the ideal situation. However, this case is rare or even nonexistent. The case where $\mathbf{\Gamma}$ is not an identity matrix should therefore be highlighted. Here, the variation trend of parameters is revealed by the 2-norm. If $||\mathbf{\Gamma}||_2$ is greater than 1, the value of the parameters increases gradually with sampling instant k, e.g., in a power system, the temperature in the circuit increases with time, and the resistance value increases with the temperature increase. In contrast, if $||\mathbf{\Gamma}||_2$ is less than 1, the value of the parameters gradually decreases with sampling instant k, e.g., for a gyroscope, the core device of the inertial platform of a strategic missile, its performance gradually deteriorates because of the wear of the internal motor and the creep of the internal structure. When the parameter of the gyroscope is greater than the maintenance threshold, the coefficient matrix can be adjusted properly by correcting the gyroscope torque to reduce measurement error. However, because the wear of the internal motor is irreversible, the parameters after correction are still reduced. In our state-space model, the vector $\mathbf{\Theta}(k)$ represents nominal values of the system parameters, and the coefficient matrix $\mathbf{\Gamma}$ reflects the overall trend of parameter variation. Therefore, model (7) applies to all control systems with such characteristics.

For the model (7), uncertain parameters can be estimated using all available information. A standard definition of the information set [21] recorded up to and including time k is the set of all past inputs and measurements:

$$z^k \triangleq \{y_1(k), \cdots, y_m(k), u_1(k-1), \cdots, u_r(k-1), z^{k-1}\}.$$

Note that the initial information set $z^1 = \{y_1(1), \cdots, y_m(1), u_1(0), \cdots, u_r(0)\}$ is given in advance.

A recursive estimation algorithm needs to be used to achieve dual adaptive control in the presence of parameter uncertainty. Since the system is linear and subject to a Gaussian distribution, the Kalman filter is able to achieve optimal parameter estimation in a least-squares sense. Therefore, the Kalman filter algorithm can be used to recursively calculate an estimate $\hat{\mathbf{\Theta}}$ of the parameter vector $\mathbf{\Theta}$ at every sampling time k, as detailed in the equations of the following lemma.

Lemma 1: For a system consisting of (6) and (7) with the given initial state and the assumptions that the distribution of $\mathbf{\Theta}(k+1)$ conditioned on information set z^k is Gaussian distributed and the optimal mean-square predictive estimate of $\mathbf{\Theta}(k+1)$ conditional on z^k is given by its conditional expectation $E\{\mathbf{\Theta}(k+1)|z^k\}$, the system together with its conditional covariance, denoted as $\mathbf{P}(k+1)$, can be propagated through the following recursive relations:

$$\mathbf{P}(k+1|k) = \mathbf{\Gamma}\mathbf{P}(k)\mathbf{\Gamma}^T + \mathbf{\Sigma}_\varepsilon \tag{8a}$$

$$\mathbf{F}(k+1) = \mathbf{P}(k+1|k)\mathbf{\Phi}^T(k) \times [\mathbf{\Phi}(k)\mathbf{P}(k)\mathbf{\Phi}^T(k) + \mathbf{\Sigma}_\xi]^{-1} \tag{8b}$$

$$\hat{\mathbf{\Theta}}(k+1) = \mathbf{\Gamma}\hat{\mathbf{\Theta}}(k) + \mathbf{F}(k+1)\mathbf{e}(k+1) \tag{8c}$$

$$\mathbf{P}(k+1) = \mathbf{P}(k+1|k) - \mathbf{F}(k+1)\mathbf{\Phi}(k)\mathbf{P}(k+1|k) \tag{8d}$$

where $\mathbf{e}(k+1) \triangleq \mathbf{y}(k+1) - \mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k)$ is the innovation sequence. $\hat{\mathbf{\Theta}}(0)$ and $\mathbf{P}(0)$, reflecting the prior mean and covariance, are assumed known.

Proof: The proof follows directly by applying the standard Kalman filter. □

IV. DESIGN OF THE NOVEL DUAL CONTROL LAW

In this section, we propose a novel MIMO dual control strategy. The strategy not only can estimate the uncertain parameters in the system but also can motivate the control system to run towards the reference output so that the performance is minimized. To overcome the shortcoming of approximation after transformation and make the control law optimal, we superimposed the learning objective on the control objective. Therefore, in this dual control strategy, the control law with learning ability will be designed by bicriterial dual control (BDC). The basic idea of the bicriterial approach is based on minimisation in terms of $\mathbf{u}(k)$ of two criteria. These two criteria work together without conflict.

To minimise the performance of single objective optimisation problem after transformation, the following proposition is used.

Proposition: For the system consisting of (7) and (6), assuming that the external noise $\boldsymbol{\varepsilon}(k)$ and external noise $\boldsymbol{\xi}(k)$ are mutually independent and independent of the parameter vector $\mathbf{\Theta}$, the expectation of the innovation sequence at sampling time $k+1$ is as follows:

$$E\{|\mathbf{e}(k+1)|^2_{\mathbf{Q}}|z^k\} = tr(|\mathbf{\Phi}(k)|^2_{\mathbf{Q}}\mathbf{P}(k)) + tr(\mathbf{Q}\mathbf{\Sigma}_\varepsilon) \tag{9}$$

given $\mathbf{Q}$.

Proof: By the definition of innovation sequence and covariance matrix, we have the following:

$$\begin{aligned} &E\{|\mathbf{e}(k+1)|^2_{\mathbf{Q}}|z^k\} \\ &\quad = E\{|\mathbf{y}(k+1) - \mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k)|^2_{\mathbf{Q}}|z^k\} \\ &\quad = E\{|\mathbf{\Phi}(k)\mathbf{\Theta}(k) + \boldsymbol{\varepsilon}(k) - \mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k)|^2_{\mathbf{Q}}|z^k\} \\ &\quad = E\{|\mathbf{\Phi}(k)\tilde{\mathbf{\Theta}}(k) + \boldsymbol{\varepsilon}(k)|^2_{\mathbf{Q}}|z^k\} \\ &\quad = E\{|\mathbf{\Phi}(k)\tilde{\mathbf{\Theta}}(k)|^2_{\mathbf{Q}}|z^k\} + E\{|\boldsymbol{\varepsilon}(k)|^2_{\mathbf{Q}}\} \\ &\quad = tr(|\mathbf{\Phi}(k)|^2_{\mathbf{Q}}\mathbf{P}(k)) + tr(\mathbf{Q}\mathbf{\Sigma}_\varepsilon) \end{aligned}$$

which follows from expanding the quadratic form and collecting terms. □

The proposition allows the performance index (4) to be reformulated into the equivalent deterministic function in the following theorem.

Before describing the theorem, the following property of the matrix should be elaborated in advance.

Property of matrix trace:

$$tr(ABC) = tr(BCA) = tr(CAB). \quad (10)$$

Lemma 2: For the system consisting of (6) and (7), the performance index function J_k in (4) can be written over the available information set z_k as follows:

$$J_k = |\mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k) - \mathbf{r}(k+1)|^2_{\mathbf{Q}} + tr(|\mathbf{\Phi}(k)|^2_{\mathbf{Q}}\mathbf{P}(k)) + tr(\mathbf{Q}\mathbf{\Sigma}_\varepsilon). \quad (11)$$

Proof: By the proposition and the property of matrix trace, we have the following:

$$\begin{aligned} J_k &= E\{|\mathbf{y}(k+1) - \mathbf{r}(k+1)|^2_{\mathbf{Q}}|z^k\} \\ &= E\{|\mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k) + \mathbf{e}(k+1) - \mathbf{r}(k+1)|^2_{\mathbf{Q}}|z^k\} \\ &= E\{|\mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k) - \mathbf{r}(k+1)|^2_{\mathbf{Q}}|z^k\} \\ &\quad + E\{|\mathbf{e}(k+1)|^2_{\mathbf{Q}}\} \\ &= |\mathbf{\Phi}(k)\hat{\mathbf{\Theta}}(k) - \mathbf{r}(k+1)|^2_{\mathbf{Q}} \\ &\quad + tr(|\mathbf{\Phi}(k)|^2_{\mathbf{Q}}\mathbf{P}(k)) + tr(\mathbf{Q}\mathbf{\Sigma}_\varepsilon). \end{aligned}$$

Then, the reformulation of performance (11) is obtained. □

In the above formulation (11), the control action $\mathbf{u}(k)$ is included in $\mathbf{\Phi}(k)$. In the following partitioning of the observation vector $\boldsymbol{\phi}(k)$, the parameter vector conditional mean $\hat{\boldsymbol{\theta}}_i(k)$ and covariance matrix $\mathbf{P}(k)$ are introduced:

$$\boldsymbol{\theta}_i^T(k) = [\mathbf{a}_i^T, \mathbf{b}_i^T], i = 1, 2, \cdots, m \quad (12)$$

$$\mathbf{P}(k) = \begin{bmatrix} \mathbf{P}_1(k) & * & \cdots & * \\ * & \mathbf{P}_1(k) & \cdots & * \\ \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & \mathbf{P}_m(k) \end{bmatrix} \quad (13)$$

where $\mathbf{a}_i = [a_{i1}, a_{i2}, \cdots, a_{im}]^T$, $\mathbf{b}_i = [b_{i1}, b_{i2}, \cdots, b_{ir}]^T$, and $\mathbf{P}_1(t), \cdots, \mathbf{P}_m(t)$ are $(m+n)$-dimensional square matrices. $\mathbf{P}_i(t)$, $i = 1, \cdots, m$ can be further partitioned as follows:

$$\mathbf{P}_i(k) = \begin{bmatrix} \mathbf{P}_{ai}(k) & \mathbf{P}_{abi}(k) \\ \mathbf{P}_{abi}^T(k) & \mathbf{P}_{bi}(k) \end{bmatrix} \quad (14)$$

where $\mathbf{P}_{ai}(k)$ is the m-dimensional square matrix and $\mathbf{P}_{bi}(k)$ is the r-dimensional square matrix.

Corollary: For a system consisting of (6) and (7), the performance index function J_k in (4) explicitly including $\mathbf{u}(k)$ in (11) can be reformatted as follows:

$$\begin{aligned} J_k &= \sum_{i=1}^m q_i|\mathbf{u}(k)|^2_{\mathbf{P}_{bi}(k)} + \sum_{i=1}^m |\mathbf{u}^T(k)\hat{\mathbf{b}}_i|^2_{q_i} \\ &\quad + 2\sum_{i=1}^m q_i\mathbf{u}^T(k)\mathbf{P}^T_{abi}(k)\mathbf{y}(k) + \epsilon \\ &\quad + 2\sum_{i=1}^m (\mathbf{y}^T(k)\hat{\mathbf{a}}_i - r_i(k+1))q_i\mathbf{u}^T(k)\hat{\mathbf{b}}_i \end{aligned} \quad (15)$$

where ϵ represents the terms independent of $\mathbf{u}(k)$ and $\mathbf{y}(k)$, q_i represents the ith term on the principal diagonal line of $\mathbf{Q}$.

Proof: Considering the three terms of (11), the substitution of the partitioning (12), (13), (14) into (15) yields the following:

$$\begin{aligned} tr(|\mathbf{\Phi}(k)|^2_{\mathbf{Q}}\mathbf{P}(k)) &= \sum_{i=1}^m q_i tr(|\boldsymbol{\phi}(k)|^2_{\mathbf{P}_i(k)}) \\ &= \sum_{i=1}^m q_i\Big[|\mathbf{y}(k)|^2_{\mathbf{P}_{ai}(k)} \\ &\quad + 2\mathbf{u}^T(k)\mathbf{P}^T_{abi}(k)\mathbf{y}(k) + |\mathbf{u}(k)|^2_{\mathbf{P}_{bi}(k)}\Big] \\ |\mathbf{\Phi}(k)\hat{\Theta}(k) - \mathbf{r}(k+1)|^2_{\mathbf{Q}} &= \sum_{i=1}^m \Big[|\mathbf{y}^T(k)\hat{\mathbf{a}}_i - r_i(k+1)|^T_{q_i} \\ &\quad + 2(\mathbf{y}^T(k)\hat{\mathbf{a}}_i - r_i(k+1))q_i\mathbf{u}^T(k)\hat{\mathbf{b}}_i \\ &\quad + \hat{\mathbf{b}}_i^T\mathbf{u}(k)q_i\mathbf{u}^T(k)\hat{\mathbf{b}}_i\Big] \\ tr(\mathbf{Q}\Sigma_\varepsilon) &= \sum_{i=1}^m q_i\sigma^2 \end{aligned}$$

Then, the performance explicitly including $\mathbf{u}(k)$ is obtained. □

We can now formally express the result as follows.

Theorem 1: For a system consisting of (6) and (7), the optimal control $\mathbf{u}_c^*(k)$ based on the first criterion is represented as follows:

$$\mathbf{u}_c^*(k) = -\frac{\Xi(k) + \Upsilon(k)}{\Pi(k)}, \quad (16)$$

where

$$\begin{aligned} \Xi(k) &= \sum_{i=1}^m q_i\hat{\mathbf{b}}_i\Big[(\mathbf{y}^T(k)\hat{\mathbf{a}}_i - r_i(k+1)\Big], \\ \Upsilon(k) &= \sum_{i=1}^m q_i\mathbf{P}^T_{abi}(k)\mathbf{y}(k), \\ \Pi(k) &= \sum_{i=1}^m \Big[q_i\Big(\mathbf{P}_{bi}(k) + \mathbf{P}^T_{bi}(k)\Big) + 2\hat{\mathbf{b}}_i q_i\hat{\mathbf{b}}_i^T\Big]. \end{aligned}$$

Proof: The optimal control is determined by differentiating (15) with respect to $\mathbf{u}(k)$ and setting it equal to zero. The result of Theorem 1 can be obtained. □

It can be noted that $\mathbf{u}_c^*(k)$ is used to represent the control under the first criterion. Due to parameter uncertainty, the separation principle does not hold. It is forced to hold artificially in this paper. However, the design procedure of this controller does not deliberately take any measure to improve the accuracy of the parameter estimate. The second criterion is thus introduced to take into account the quality of the parameter estimate.

The second component of the designed controller should be evaluated for estimation quality and is shown as follows:

$$\Sigma_k = E\Big\{[\mathbf{y}(k) - \hat{\mathbf{y}}(k)][\mathbf{y}(k) - \hat{\mathbf{y}}(k)]^T\Big\} \quad (17)$$

Theorem 2: For a system consisting of (6) and (7), the learning control $\mathbf{u}_l^*(k)$ based on the second criterion is obtained as follows:

$$\mathbf{u}_l^*(k) = -\frac{\Upsilon_l(k)}{\Pi_l(k)} \tag{18}$$

where

$$\boldsymbol{\Upsilon}_l(k) = \sum_{i=1}^{m} \boldsymbol{P}_{abi}^T(k)\mathbf{y}(k)$$
$$\boldsymbol{\Pi}_l(k) = \sum_{i=m}^{m} \mathbf{P}_{bi}(k)$$

Proof: By the model (6) and the definition of the covariance matrix, the following relation can be obtained:

$$\begin{aligned} tr\Sigma_k &= trE\Big\{[\boldsymbol{\Phi}(k)\boldsymbol{\Theta}(k) + \boldsymbol{\varepsilon}(k) - \boldsymbol{\Phi}(k)\hat{\boldsymbol{\Theta}}(k)] \\ &\quad \times [\boldsymbol{\Phi}(k)\boldsymbol{\Theta}(k) + \boldsymbol{\varepsilon}(k) - \boldsymbol{\Phi}(k)\hat{\boldsymbol{\Theta}}(k)]^T\Big\} \\ &= tr(\boldsymbol{\Phi}(k)\mathbf{P}(k)\boldsymbol{\Phi}^T(k)) + tr(\boldsymbol{\Sigma}_\varepsilon) \end{aligned}$$

Differentiating (17) with respect to $\mathbf{u}(k)$ and setting it equal to zero, the learning control $\mathbf{u}_l^*(k)$ is obtained. □

For the control problem with performance (3), although the optimal control can be obtained based on the current model, the control effect is related to the model. The more accurate the model, the smaller the tracking error. Therefore, in order to improve the quality of model parameter estimation, this paper considers the learning objective (17) based on the control objective. There is no conflict between the two goals. The better the learning goal, the more accurate the control model. When the uncertain parameters gradually approach the truth values, the smaller the effect of the learning object item on the control target, and the control objectives play a major role. At this point, the corresponding control law should be very ideal. In order to reflect the learning ability of the controller for uncertain parameters, this paper superimposed the learning effect $\mathbf{u}_l^*(k)$ based on the second criteria on the control law $\mathbf{u}_c^*(k)$ based on the first criteria through the learning factor $\boldsymbol{\delta}$, and obtained $\mathbf{u}^*(k) = \mathbf{u}_c^*(k) + \boldsymbol{\delta}(k)\mathbf{u}_l^*(k)$. Obviously, the first term of $\mathbf{u}^*(k)$ reflects the control and optimisation functions of the control law, while the second term gives the learning function of the control law. $\boldsymbol{\delta}$ is a balance between the two functions.

The actual control based on the two criteria applied to the original system is thus expressed by the following theorem.

Theorem 3: For the system consisting of (6) and (7) with parameter uncertainty, the control law designed in this paper can be described as follows:

$$\mathbf{u}^*(k) = \mathbf{u}_c^*(k) + \boldsymbol{\delta}(k)\mathbf{u}_l^*(k) \tag{19}$$

where $\boldsymbol{\delta}(k)$ is the learning factor and stems from the reasoning that it is necessary to enrich the control with probing proportional to the parameter uncertainty. A common choice for $\boldsymbol{\delta}(k)$ is as follows:

$$\boldsymbol{\delta}(k) = \boldsymbol{\eta} tr\mathbf{P}(k+1) \tag{20}$$

where $\boldsymbol{\eta}$ is a fixed diagonal matrix with length r, which provides the amplitude of the probing signal.

Because the above formulation includes the term with learning control, the control law is endowed with a learning ability. At the same time, the designed controller deliberately takes measures to enrich the information about uncertain parameters to improve estimation quality. It is thus a dual adaptive control law.

The key to this paper is that the learning factor $\boldsymbol{\delta}(k)$ is constantly changing based on the covariance matrix throughout the control process. Of course, the value of the learning factor $\boldsymbol{\delta}(k)$ can be automatically adjusted to improve learning ability and does not dramatically affect the control effect. Even though $\boldsymbol{\eta}$ is set to be larger, as the estimated values approach the true values, the covariance matrix becomes smaller. Because the control law applied to the system makes full use of the latest information obtained in the control process as far as possible, the learning ability is also improved by the learning term, and the designed controller has a better control effect.

V. NUMERICAL EXPERIMENTS

The novel MIMO dual adaptive control algorithm in this paper can be summarised as follows:

Algorithm 1 Novel MIMO Dual Adaptive Control Algorithm

---

let $k = 0$ and let the initial information set z^0 be given
repeat
  Estimate the uncertain parameters $\hat{\boldsymbol{\Theta}}$ by Eq. (8c) based on the information set z^k at time k
  Calculate the control $\mathbf{u}_c^*(k)$ that minimises the performance by Eq. (16)
  Calculate the learning control $\mathbf{u}_l^*(k)$ by Eq. (18)
  Choose the appropriate $\boldsymbol{\eta}$ to determine the learning factor $\boldsymbol{\delta}(k)$
  Calculate the control $\mathbf{u}^*(k)$ applied to the original system by Eq. (19)
until $k = N - 1$

For parameter uncertainty, this paper separately considers two different kinds of uncertainties: 1. The system parameters are unknown but constant. 2. The parameters suddenly change at a certain moment. For these two cases, numerical experiments can be performed to illustrate the effectiveness of the algorithm. This section presents MATLAB simulation results for two stochastic system examples.

A. EXAMPLE 1

Considering a dynamic stochastic system with parameter uncertainty, the uncertainty under consideration is assumed to be unknown but constant. The original parameter vectors are as follows:

$$\boldsymbol{\theta}_1 = [0.2, 1.8, -0.8, 0.7]^T,$$
$$\boldsymbol{\theta}_2 = [-0.6, 0.5, 0.2, 1.5]^T.$$

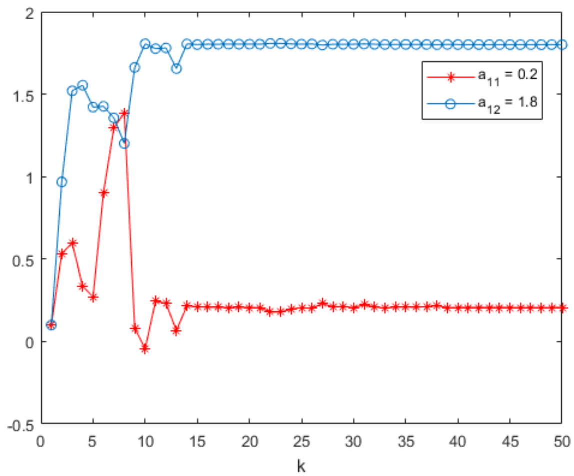

FIGURE 1. Estimation process of parameters a_{11} and a_{12}.

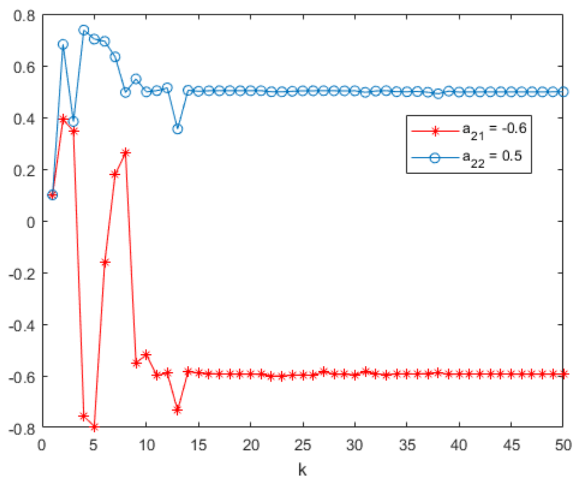

FIGURE 2. Estimation process of parameters a_{21} and a_{22}.

The initial values of the parameters are

$$\boldsymbol{\theta}_1(0) = \boldsymbol{\theta}_2(0) = [0.1, 0.1, 0.1, 0.1]^T,$$

The process noise variance and measurement noise variance of the parameters are: $\boldsymbol{\Sigma}_\xi = \mathbf{0}$, $\sigma_1^2 = \sigma_2^2 = 0.3$, $\boldsymbol{\Sigma}_\varepsilon = diag(\sigma_1^2, \sigma_2^2)$. The initial estimation covariance matrix is $\mathbf{P}(0) = \mathbf{I}_8$. The value of η is set to $\boldsymbol{\eta} = diag(0.6, 1.2)$.

The simulation results are as follows:

The estimation process of the parameters is shown in Fig. 1-4. As seen from the figure, at the beginning of the parameter estimate, the parameters change dramatically, however, as the sampling time increases, they tend to gradually become stable and converge to the true value from approximately $k = 15$. To better demonstrate the estimation ability of the proposed algorithm, we further illustrate the estimation accuracy by calculating the mean and variance of the estimated parameters, as shown in Tab. 1.

In Tab. 1, the left column of the term Mean represents the mean for the entire estimation process, while the right column represents the mean after time $k = 15$. This is because

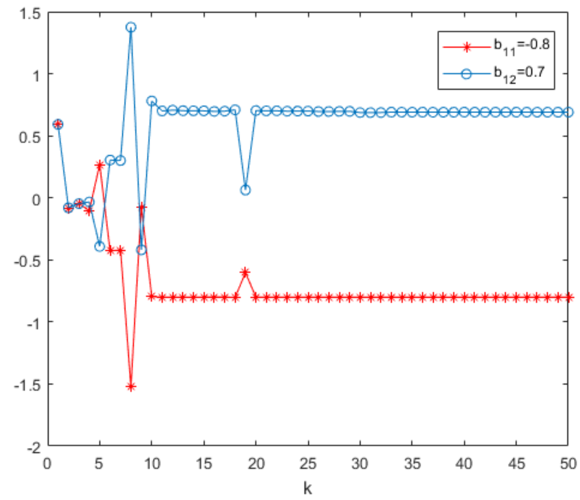

FIGURE 3. Estimation process of parameters b_{11} and b_{12}.

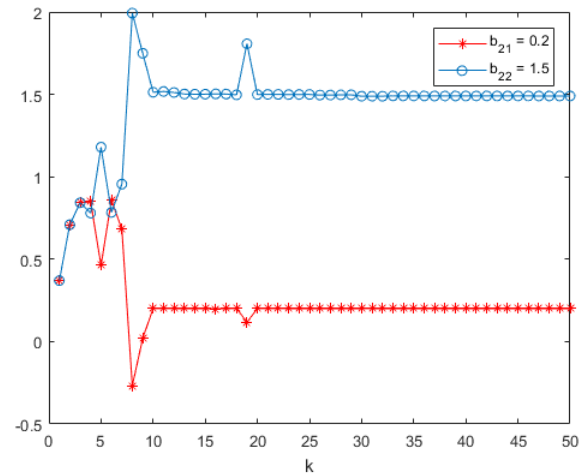

FIGURE 4. Estimation process of parameters b_{21} and b_{22}.

TABLE 1. Statistical result of the estimated parameters.

Parameter	Mean		Standard Deviation		Variance	
a_{11}	0.2809	0.2064	0.2560	0.0097	0.0655	0
a_{12}	0.7674	1.8019	0.1738	0.0022	0.0302	0
a_{21}	-0.5319	-0.5935	0.2557	0.0037	0.0654	0
a_{22}	0.5257	0.4996	0.0658	0.0026	0.0043	0
b_{11}	-0.7022	-0.7998	0.3316	0	0.1099	0
b_{12}	0.6026	0.6927	0.3113	0.0046	0.0969	0
b_{21}	0.2581	0.2001	0.2007	0	0.0403	0
b_{22}	1.4489	1.4929	0.2802	0.0040	0.0785	0

the parameters change dramatically at the beginning of the estimation, which may increase the variance even though the parameters are estimated accurately. The standard deviation and variance are the same. From the right column of the term Mean, it can be seen that the estimated values are very close to the true parameters. Additionally, the right columns of the standard deviation and variance are close to zero, which means that the effect of estimation is fairly good.

TABLE 2. Results of Monte Carlo under different control law.

Nominal Control	Pure Control	Novel Dual Control
86.735	92.359	51.946

In addition to accurately estimating parameters, the proposed control law is expected to minimise the performance of the cost function.

The statistical properties of the performance of the control law cannot be evaluated through single-trial analysis. Therefore, a Monte Carlo analysis was performed. The performance of the cost function at the end of each trial was quantified through the following measures:

$$\mathfrak{J} = \frac{1}{50}\sum_{k=0}^{49}\mathbf{y}^T(k+1)\mathbf{y}(k+1).$$

Since the system under consideration is a stochastic system, different control laws can only be applied to this system for the comparison of performance, which certainly includes the proposed control law of this paper. Nominal control and pure control are thus introduced. If the parameters of the model that have fluctuations are fixed at the nominal value, it is called nominal control. In this case, the control law is determined based on fixed parameters and its control progress does not need to be learned. If the diagonal matrix $\boldsymbol{\eta}$ of the algorithm in this paper is restricted to the zero matrix, that is, there are no learning terms, the control law is called pure control.

Imposing the above three control laws on the system, after 500 trials of Monte Carlo experiments are performed, the statistical results are shown in Tab. 2.

As shown in Tab. 2, there is little difference between the performance of nominal control and pure control. In other words, if there is stochastic parameter uncertainty in the system model, only Kalman filtering is used to estimate the parameters, and the control effect is not significant.

The algorithm in this paper is obviously better than the other two, which shows the effect of the proposed control law in this paper.

Except for the above case, the proposed control law is also effective for cases in which the parameters change abruptly. Therefore, numerical example 2 is carried out.

B. EXAMPLE 2

For the dynamic stochastic system (1), the abrupt parameters under consideration are as follows:

$$a_{11} = \begin{cases} 0.2, & t < 50 \\ 2.1, & t \geq 50 \end{cases} \quad a_{12} = \begin{cases} 1.8, & t < 50 \\ 3.5, & t \geq 50 \end{cases}$$

$$a_{21} = \begin{cases} -0.6, & t < 50 \\ 2.9, & t \geq 50 \end{cases} \quad a_{22} = \begin{cases} 0.5, & t < 50 \\ 2.5, & t \geq 50 \end{cases}$$

$$b_{11} = \begin{cases} -0.8, & t < 50 \\ 3, & t \geq 50 \end{cases} \quad b_{12} = \begin{cases} 0.7, & t < 50 \\ -2.5, & t \geq 50 \end{cases}$$

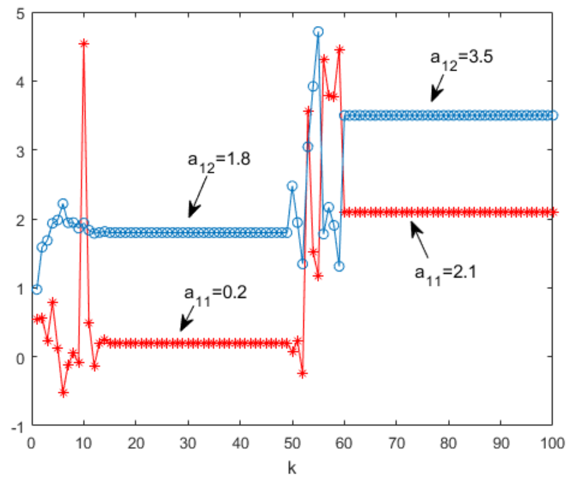

FIGURE 5. Estimation process of parameters a_{11} and a_{12}.

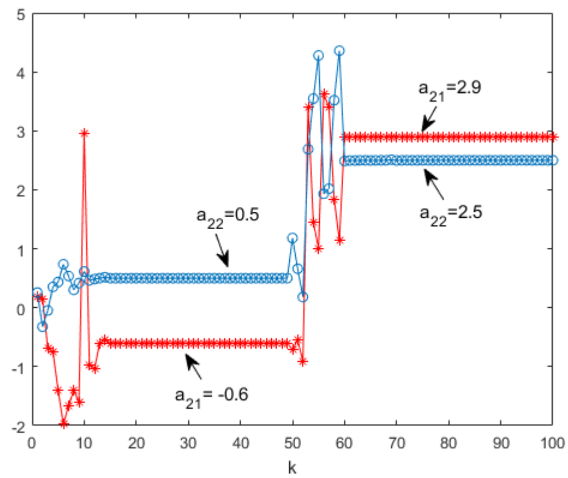

FIGURE 6. Estimation process of parameters a_{21} and a_{22}.

$$b_{21} = \begin{cases} 0.2, & t < 50 \\ 2.5, & t \geq 50 \end{cases} \quad b_{22} = \begin{cases} 1.5, & t < 50 \\ 4.5, & t \geq 50 \end{cases}$$

The process noise variance and measurement noise variance of the parameters are the same as in the first example. The initial estimation covariance matrix is $\mathbf{P}(0) = \mathbf{I}_8$. The fixed matrix $\boldsymbol{\eta} = diag(1, 1.2)$.

The simulation results are as follows:

In this case, the parameters change dramatically at the sampling time $k = 50$, as shown in Fig 5-8. For the sake of description, we take the interval $k < 50$ as stage 1 and $k \geq 50$ as stage 2. It can be seen from the figures that the estimation process of the parameters is very fast at stage 1, as in example 1, and remains stable. The most important point is that at stage 2, when the parameters change suddenly, the parameter can still converge quickly to the new variation value, even though the parameters are particularly volatile, even messy, at the beginning of stage 2.

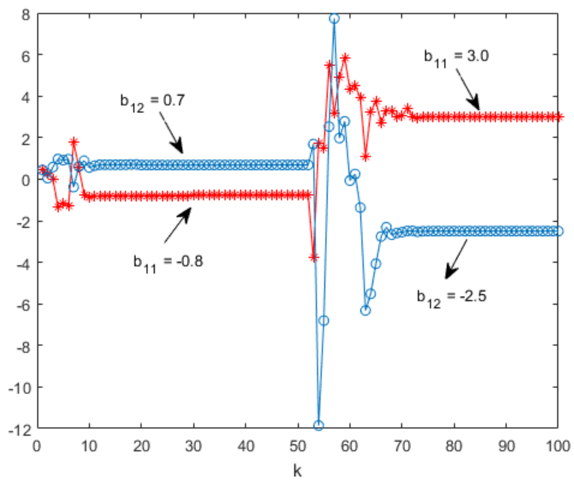

FIGURE 7. Estimation process of parameters b_{11} and b_{12}.

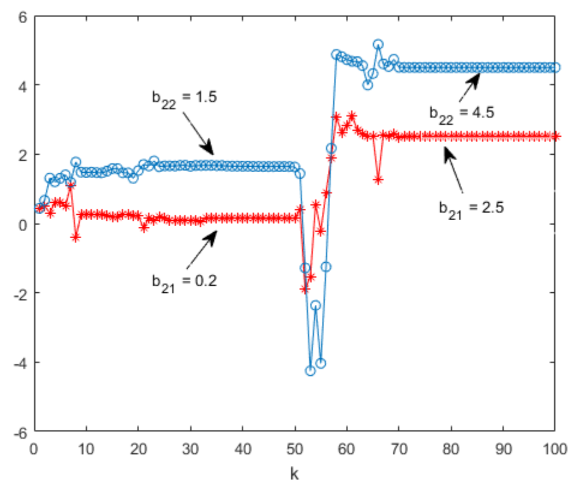

FIGURE 8. Estimation process of parameters b_{21} and b_{22}.

TABLE 3. Statistical result of the estimated parameters.

Parameter	Mean		Standard Deviation		Variance	
a_{11}	2.0909	2.1000	0.8508	0	0.6493	0
a_{12}	3.2329	3.5000	0.6312	0	0.3985	0
a_{21}	2.5498	2.9000	0.9573	0	0.9163	0
a_{22}	2.4395	2.5000	0.6297	0.0018	0.3965	0
b_{11}	2.7519	3.0137	1.5390	0.0795	2.3684	0.0063
b_{12}	-2.0169	-2.5002	2.6947	0.02	7.2615	0
b_{21}	2.0878	2.5000	1.05	0.0179	1.1025	0
b_{22}	3.6003	4.5001	2.2459	0.0426	5.0443	0.0018

Next, we analyse the statistical results at stage 2. Similar to stage 1, the mean and variance of the estimated parameters are shown in Tab. 3.

Similar to Tab. 1, the left column of the term Mean represents the mean for the entire estimation process at stage 2, while the right column represents the mean after time $k = 65$. From the right column of the term Mean, it can be seen that the estimated values are very close to the true parameters and some of them are already exactly equal to the true parameters.

TABLE 4. Results of Monte Carlo under different control laws.

Nominal Control	Pure Control	Dual Control
378.758	492.148	301.652

Meanwhile, the right columns of the standard deviation and variance are close to zero, which means that the effect of estimation is fairly good.

Next, the statistical properties of the performance of the control law can still be evaluated through Monte Carlo experiments. The results are shown in Tab. 4.

The result shown in Tab. 4 is the same as in Tab. 2, and there is still little difference between the performance of nominal control and pure control. The algorithm in this paper is obviously better than the other two, which also shows the effect of the control law proposed in this paper.

In this section, two numerical experiments are performed. By analysing the statistical properties of experimental results, the effectiveness of the proposed algorithm is verified.

VI. CONCLUSION

In this paper, a novel MIMO dual control law for a MIMO stochastic linear system with parameter uncertainty is proposed as an extension of a SISO stochastic linear system. The proposed control law has two parts. One can be used to drive output into the desired state, and the second improves the estimation ability of uncertain parameters. The key to this paper is that the learning factor can automatically change at any time and is closely related to the estimation covariance matrix, which balances the control target and parameter estimation. Even if the amplitude of the learning factor is set to be very large, the effect of the second term can be weakened due to the covariance matrix when the uncertain parameters converge to the true value. Furthermore, the statistical properties of estimation are analysed to verify the learning ability, and the comparison of system performance under three different control laws is made to illustrate the effectiveness of the proposed algorithm. The research results of the paper show that the learning property of the control algorithm is indispensable for the control problem with parameter uncertainty.

The practical system not only contains parameter uncertainty but also has a variety of other uncertainties that affect the performance of the system and even affect its normal operation. Therefore, the control problem for a MIMO system with mixed uncertainties will be considered in future work.

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