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A Novel Identification Method for a Class of Closed-Loop Systems Based on Basis Pursuit De-Noising

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ABSTRACT This paper presents a novel method to identify a class of closed-loop systems, in which both the forward channel and the feedback channel have unknown time-delays. Taking into account the time-delays, an overparameterized identification model with a sparse parameter vector is established. Based on the basis pursuit de-noising criterion, the sparse parameter vector is estimated by solving a quadratic programming. The time-delays and the parameters are estimated according to the structure of the parameter estimation vector and the model equivalence principle, respectively. The proposed method is applicable even in the case of a few number of sampled data. The effectiveness of the proposed algorithm is verified by the numerical simulation results.

INDEX TERMS Closed-loop system, basis pursuit de-noising, system identification, time-delay estimation.

I. INTRODUCTION

The identification of the closed-loop systems is an active research in system identification due to its stability, security and safety. There has been continuing interest in methods of identifying the closed-loop systems [1]–[4]. For example, a modified least-squares algorithm was presented to identify the closed-loop systems in the presence of noises [5]. A hierarchical multi-innovation stochastic gradient algorithm was developed for the identification of a class of closed-loop systems [6]. The algorithms used in [5], [6] are suitable for the closed-loop systems without time-delays. However, time-delays are common in closed-loop engineering systems due to the measurement and transmission. For example, in a networked controlled system, due to the network transmission, the time-delays not only include in the control plant, but also in the feedback channel [7], [8]. Thus, it is necessary to address the identification problems of the closed-loop systems with unknown time-delays both in the forward and feedback channels.

For the closed-loop systems with unknown time-delays in the forward and feedback channels, the relationship between

the reference input and the output can be modeled as a high dimensional regression vector form, where the parameter vector is sparse. The identification goal is to estimate the sparse parameter vector, and then to extract the feedback channel parameters and time-delays from the estimation vector. Traditional identification methods need a large amount of sampled data to identify a high dimensional model, which may not be available in many cases, such as the linear time-variant system identification [9]. We aim to develop identification methods to simultaneously estimate the time-delays and parameters of the closed-loop systems with a few number of sampled data.

Since the parameter vector of the parameterized model is sparse, we need to perform a sparse system identification first. The compressive sensing recovery techniques discussed in [10], [11] have been successfully applied to the sparse system identification [9], [12]–[14]. Among these techniques, the greedy algorithms and the convex optimization algorithms are most widely used. Greedy algorithms such as the orthogonal matching pursuit (OMP) algorithm and its variations have been developed to identify linear and nonlinear sparse systems for the merits of fast speed and easy implementation. For example, a block orthogonal matching pursuit (BOMP) algorithm was proposed for the identification

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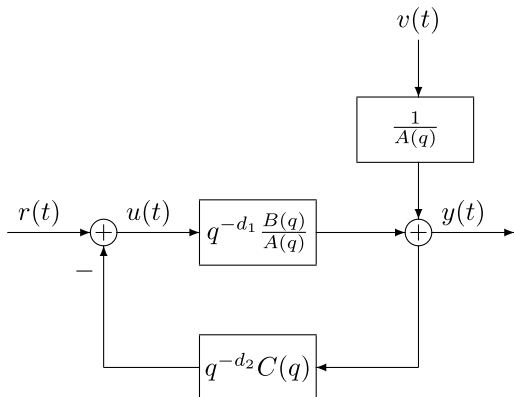


FIGURE 1. A closed-loop system.

of the sparse ARX models [9]. A threshold orthogonal matching pursuit algorithm was presented for multiple input finite impulse response models and a class of Hammerstein models [13]–[15]. An auxiliary model based orthogonal matching pursuit iterative algorithm was proposed for the joint parameter and time-delay estimation of multiple input output-error models [16]. Convex optimization algorithms such as the basis pursuit, the basis pursuit de-noising (BPDN) can recovery sparse signals with the advantages of high stability and strong applicability [17]. In the literature of system identification, the BPDN criterion was combined with the auxiliary model idea to jointly estimate the parameters and time-delays of multivariable output-error systems [18]. In the current paper, the BPDN criterion is used to identify the parameters and time-delays of a class of closed-loop systems simultaneously due to its robustness.

The structure of this paper is as follows. In Section II, the introduction and identification model of the closed-loop system are given. The BPDN based algorithm is described in Section III. A simulation example is given to verify the feasibility and effectiveness of the proposed algorithm in Section IV. Finally, Section V contains some conclusions.

II. PROBLEM DESCRIPTION

Consider a closed-loop system in Figure 1, where $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the input and output of the system, $r(t) \in \mathbb{R}$ is the reference input, $v(t) \in \mathbb{R}$ is a white noise with zero mean and variance σ^2 , d_1 and d_2 are unknown time-delays, $A(q)$, $B(q)$ and $C(q)$ are the polynomials in the unit backward shift operator q^{-1} ,

$$A(q) = 1 + \sum_{i=1}^{n_a} a_i q^{-i},$$

$$B(q) = \sum_{i=1}^{n_b} b_i q^{-i},$$

$$C(q) = 1 + \sum_{i=1}^{n_c} c_i q^{-i}.$$

The orders n_a , n_b and n_c are known, while the parameters a_i , b_i and c_i are unknown. Without loss of generality, we suppose

that $d_1 + d_2 > n_a$, $y(t) = 0$, $u(t) = 0$ and $v(t) = 0$ for $t \leq 0$. The identification goal is to estimate the coefficients in $A(q)$, $B(q)$ and $C(q)$, and the unknown time-delays d_1 and d_2 from the input and output data $\{r(t), y(t), t = 1, 2, \dots\}$.

From Figure 1, the relationship between the reference input $r(t)$ and the output $y(t)$ can be expressed by an ARX model,

$$(A(q) + q^{-(d_1+d_2)}B(q)C(q))y(t) = q^{-d_1}B(q)r(t) + v(t). \quad (1)$$

Define

$$\beta(q) := B(q)C(q) = \beta_1 q^{-1} + \beta_2 q^{-2} + \dots + \beta_{n_\beta} q^{-n_\beta},$$

$$n_\beta := n_b + n_c,$$

$$\alpha(q) := A(q) + q^{-(d_1+d_2)}B(q)C(q)$$

$$= 1 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_{n_a} q^{-n_a}$$

$$+ q^{-(d_1+d_2)}(\beta_1 q^{-1} + \beta_2 q^{-2} + \dots + \beta_{n_\beta} q^{-n_\beta}). \quad (2)$$

Then Equation (1) can be rewritten as

$$\alpha(q)y(t) = q^{-d_1}B(q)r(t) + v(t). \quad (3)$$

In order to ensure the identifiability of the closed-loop system, the following assumptions are generally made [19]:

- The noise $v(t)$ is stationary.
- The reference input $r(t)$ is a stationary random process, and independent of the noise $v(t)$.
- The polynomials $\alpha(q)$ and $B(q)$ are coprime.

Taking into account the unknown time-delays, an overparameterization method is applied to form an identification model by setting a maximum output regression length l_1 and a maximum input regression length l_2 , satisfying $l_1 > d_1 + d_2 + n_\beta$ and $l_2 > d_1 + n_b$ [15]. Define the information vector $\varphi(t)$ and the parameter vector θ as

$$\varphi(t) := [-y(t-1), \dots, -y(t-l_1), r(t-1), \dots, r(t-l_2)]^T \in \mathbb{R}^n, \quad n = l_1 + l_2,$$

$$\theta := [a_1, \dots, a_{n_a}, \underbrace{0, \dots, 0}_{d_1+d_2-n_a}, \beta_1, \dots, \beta_{n_\beta},$$

$$\underbrace{0, \dots, 0}_{l_1-d_2-n_\beta}, \underbrace{b_1, \dots, b_{n_b}}_{l_2-n_b-d_1}, \underbrace{0, \dots, 0}_{l_2-n_b-d_1}]^T \in \mathbb{R}^n. \quad (4)$$

Then Equation (3) can be modeled by a simplified vector form

$$y(t) = \varphi^T(t)\theta + v(t). \quad (5)$$

It is worth to note that the parameter vector θ is sparse since most of the entries are zero and the sparsity level is $K := n_a + n_\beta + n_b$. The first task of the identification is to estimate the coefficients in $\alpha(q)$ and $B(q)$, i.e., the nonzero parameters in θ . Generally, under the case of m observations, the parameter vector θ can be estimated by the least squares (LS) algorithm, i.e.,

$$\hat{\theta}_{LS} = (\Phi^T \Phi)^{-1} \Phi^T Y, \quad (6)$$

where

$$Y := [y(1), y(2), \dots, y(m)]^T \in \mathbb{R}^m,$$

$$\Phi := [\varphi(1), \varphi(2), \dots, \varphi(m)]^T \in \mathbb{R}^{m \times n},$$

$$V := [v(1), v(2), \dots, v(m)]^T \in \mathbb{R}^m.$$

However, it may require a lot of identification cost and take a long time to get enough data since the dimension of the parameter vector θ is high, and the computational burden is heavy because of the matrix inversion in (6). Moreover, using the LS algorithm, the sparse solution as well as the time-delays cannot be obtained [12]. In order to effectively identify the sparse systems and to save the identification cost, we aim to find identification methods to get the parameter vector θ and the time-delays from a few number of measurements.

III. IDENTIFICATION ALGORITHM

The sparse identification of θ can be expressed as

$$\hat{\theta} = \arg \min \|\theta\|_0, \text{ s.t. } \|Y - \Phi\theta\| \leq \epsilon, \quad (7)$$

where $\hat{\theta}$ is the estimate of θ , $\|\theta\|_0$ represents the number of non-zero parameters in θ , $\|\theta\|$ denotes the l_2 norm of θ , and $\epsilon > 0$ is the error tolerance. However, the noncontinuous sparse optimization problem in (7) is difficult to solve in practice. An alternative is the following relaxed l_1 norm [20],

$$\hat{\theta} = \arg \min \|\theta\|_1, \text{ s.t. } \|Y - \Phi\theta\| \leq \epsilon, \quad (8)$$

where $\|\theta\|_1$ represents the l_1 norm of θ . Many greedy algorithms have been proposed to solve this problem [9], [12], [15]. For comparison, we list the steps of the threshold orthogonal matching pursuit (Th-OMP) algorithm in Figure 2.

Several conditions on the information matrix have been established to guarantee the recovery of the sparse vector. It has been proved that the sparse recovery can be guaranteed as long as the information matrix satisfies the restricted isometry property (RIP) [21]. However, it is difficult to verify the RIP in system identification because the columns of the information matrix are correlated with each other. A weaker condition based on the mutual coherence was discussed in [9]. In order to guarantee the recovery, the mutual coherence of the information matrix should be as low as possible. In the ARX framework, the mutual coherence is bounded and can be reduced by pre-filtering the measurements. However, how to choose the filter is still an issue. An exact recovery condition (ERC) has been developed in [22]. The consistent properties of identifying the sparse ARX models was discussed in [12] by using the ERC.

In this paper, we focus on the BPDN criterion to solve this optimization problem due to its robustness. First we normalize the information matrix Φ by dividing the elements in each column by the l_2 norm of that column and let Φ_o be the normalized information matrix, i.e.,

$$\Phi_o := \Phi\Phi_l^{-1},$$

$$\Phi_l := \begin{bmatrix} \Phi(1) & 0 & 0 & \dots & 0 \\ 0 & \Phi(2) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & \Phi(n) \end{bmatrix} \in \mathbb{R}^{n \times n}, \quad (9)$$

Input: the output vector Y ,
the information matrix Φ .

Output: the parameter estimation vector $\hat{\theta}_p$.

Initialization: initialize $p = 0$, and set
the initial estimation vector $\hat{\theta}_0 = \mathbf{0}$,
the initial residual $r_0 = Y$,
the initial index set $\Lambda_0 = \emptyset$.

Main Iteration:
increment p by 1 and perform the following steps:
for $p = 1 : K$

1. Compute the index λ_p by

$$\lambda_p = \arg \max_{j=1,2,\dots,n} \left\langle r_{p-1}, \frac{\phi_j}{\|\phi_j\|} \right\rangle.$$
2. Update the index set Λ_p by $\Lambda_p := \Lambda_{p-1} \cup \lambda_p$.
3. Form the sub-information matrix Φ_{Λ_p} according to the index set Λ_p .
4. Compute the p -th parameter estimation vector

$$\hat{\theta}_p = \arg \min_{\theta_p} \|Y - \Phi_{\Lambda_p} \theta_p\|^2$$

$$= [\Phi_{\Lambda_p}^T \Phi_{\Lambda_p}]^{-1} \Phi_{\Lambda_p}^T Y.$$
5. Denote the j -th of $\hat{\theta}_p$ by $\hat{\theta}_j$.
If $|\hat{\theta}_j| < \epsilon$, let $\hat{\theta}_j = 0$.
6. Compute the p -th residual $r_p = Y - \Phi_{\Lambda_p} \hat{\theta}_p$.

end

FIGURE 2. The threshold orthogonal matching pursuit algorithm.

where $\Phi(j) := \sqrt{\sum_{i=1}^m \Phi_{ij}^2}$. From (9), we have $\Phi = \Phi_o \Phi_l$. Thus, the system in Equation (5) can be modeled by the matrix form

$$Y = \Phi_o \theta_o + V,$$

where

$$\theta_o := \Phi_l \theta. \quad (10)$$

Note that the positions of the nonzero parameters in the parameter vectors θ_o is the same as that in the parameter vector θ . Thus, the sparse optimization problem in Equation (8) equals to

$$\hat{\theta}_o = \arg \min \|\theta_o\|_1, \text{ s.t. } \|Y - \Phi_o \theta_o\| \leq \epsilon. \quad (11)$$

The constrained optimization problem in Equation (11) can be cast as a convex unconstrained optimization problem, which consists of a quadratic l_2 error and an l_1 regularization,

$$\min_{\theta_o} \frac{1}{2} \|Y - \Phi_o \theta_o\|^2 + \lambda \|\theta_o\|_1, \quad (12)$$

where λ is a non-negative parameter and can be set as $\lambda = \sigma\sqrt{2\log(n)}$ [17]. Two non-negative vectors \mathbf{u}_o and \mathbf{v}_o are introduced to convert the problem in (12) into a quadratic program [18], [23], and let $u_{oj} := (\theta_{oj})_+$, $v_{oj} := (-\theta_{oj})_+$ and $(\theta_o)_+ := \max\{0, \theta_o\}$ for all $j = 1, 2, \dots, n$, where θ_{oj} , u_{oj} and v_{oj} represent the j -th element of the vectors $\boldsymbol{\theta}_o$, \mathbf{u}_o and \mathbf{v}_o , respectively. Then the parameter vector $\boldsymbol{\theta}_o$ can be expressed as

$$\boldsymbol{\theta}_o := \mathbf{u}_o - \mathbf{v}_o.$$

Let $\mathbf{1}_n := [1, \dots, 1]^T \in \mathbb{R}^n$, $\mathbf{1}_{2n} := [1, \dots, 1]^T \in \mathbb{R}^{2n}$ and

$$\mathbf{z}_o := [\mathbf{u}_o^T, \mathbf{v}_o^T]^T \in \mathbb{R}^{2n}, \quad (13)$$

then the l_1 norm of $\boldsymbol{\theta}_o$ can be expressed as

$$\|\boldsymbol{\theta}_o\|_1 = \mathbf{1}_n^T \mathbf{u}_o + \mathbf{1}_n^T \mathbf{v}_o = \mathbf{1}_{2n}^T \mathbf{z}_o. \quad (14)$$

It is obvious that all elements in \mathbf{z}_o are nonnegative. Using the above expression, the quadratic error can be written as

$$\begin{aligned} & \|Y - \Phi_o \boldsymbol{\theta}_o\|^2 \\ &= \|Y - [\Phi_o, -\Phi_o] \mathbf{z}_o\|^2 \\ &= Y^T(Y - [\Phi_o, -\Phi_o] \mathbf{z}_o) - \mathbf{z}_o^T [\Phi_o, -\Phi_o]^T (Y - [\Phi_o, -\Phi_o] \mathbf{z}_o) \\ &= Y^T Y - Y^T [\Phi_o, -\Phi_o] \mathbf{z}_o - \mathbf{z}_o^T [\Phi_o, -\Phi_o]^T Y \\ & \quad + \mathbf{z}_o^T [\Phi_o, -\Phi_o]^T [\Phi_o, -\Phi_o] \mathbf{z}_o. \end{aligned} \quad (15)$$

Since $Y^T [\Phi_o, -\Phi_o] \mathbf{z}_o$ is a scalar, we can derive that

$$\begin{aligned} Y^T [\Phi_o, -\Phi_o] \mathbf{z}_o &= (Y^T [\Phi_o, -\Phi_o] \mathbf{z}_o)^T \\ &= \mathbf{z}_o^T [\Phi_o, -\Phi_o]^T Y \\ &= \begin{bmatrix} \Phi_o^T Y \\ -\Phi_o^T Y \end{bmatrix}^T \mathbf{z}_o. \end{aligned}$$

Let $\boldsymbol{\gamma} := \Phi_o^T Y \in \mathbb{R}^n$ and $\mathbf{B} := \begin{bmatrix} \Phi_o^T \Phi_o & -\Phi_o^T \Phi_o \\ -\Phi_o^T \Phi_o & \Phi_o^T \Phi_o \end{bmatrix} \in \mathbb{R}^{(2n) \times (2n)}$. It further follows that

$$\begin{aligned} & \|Y - \Phi_o \boldsymbol{\theta}_o\|^2 \\ &= Y^T Y - 2 \begin{bmatrix} \boldsymbol{\gamma} \\ -\boldsymbol{\gamma} \end{bmatrix}^T \mathbf{z}_o + \mathbf{z}_o^T [\Phi_o, -\Phi_o]^T [\Phi_o, -\Phi_o] \mathbf{z}_o \\ &= Y^T Y - 2 \begin{bmatrix} \boldsymbol{\gamma} \\ -\boldsymbol{\gamma} \end{bmatrix}^T \mathbf{z}_o + \mathbf{z}_o^T \mathbf{B} \mathbf{z}_o, \end{aligned} \quad (16)$$

and

$$\begin{aligned} & \min_{\boldsymbol{\theta}_o} \frac{1}{2} \|Y - \Phi_o \boldsymbol{\theta}_o\|^2 + \lambda \|\boldsymbol{\theta}_o\|_1 \\ &= \min_{\mathbf{z}_o} \frac{1}{2} Y^T Y - \begin{bmatrix} \boldsymbol{\gamma} \\ -\boldsymbol{\gamma} \end{bmatrix}^T \mathbf{z}_o + \frac{1}{2} \mathbf{z}_o^T \mathbf{B} \mathbf{z}_o + \lambda \mathbf{1}_{2n}^T \mathbf{z}_o \\ &= \min_{\mathbf{z}_o} \frac{1}{2} Y^T Y + C^T \mathbf{z}_o + \frac{1}{2} \mathbf{z}_o^T \mathbf{B} \mathbf{z}_o, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathbf{C} &:= \begin{bmatrix} -\boldsymbol{\gamma} \\ \boldsymbol{\gamma} \end{bmatrix} + \lambda \mathbf{1}_{2n} \\ &= \begin{bmatrix} -\boldsymbol{\gamma} \\ \boldsymbol{\gamma} \end{bmatrix} + \sigma\sqrt{2\log(n)} \mathbf{1}_{2n} \in \mathbb{R}^{2n}. \end{aligned} \quad (18)$$

Since $\frac{1}{2} Y^T Y$ is a constant, the problem (17) equals to a standard quadratic program form

$$\min_{\mathbf{z}_o} C^T \mathbf{z}_o + \frac{1}{2} \mathbf{z}_o^T \mathbf{B} \mathbf{z}_o, \quad \text{s.t. } \mathbf{z}_o \geq 0. \quad (19)$$

Generally, the above inequality constrained quadratic program can be solved by the active set method [24]. For simplicity, the quadratic program problem can be solved by invoking the function of the standard scientific software toolbox. For example, the MATLAB provides a function named ‘quadprog’ to obtain the optimum solution $\hat{\mathbf{z}}_o$. Then the parameter estimation vector $\hat{\boldsymbol{\theta}}_o$ can be obtained by

$$\hat{\boldsymbol{\theta}}_o = \hat{\mathbf{z}}_o(1:n) - \hat{\mathbf{z}}_o(n+1:2n). \quad (20)$$

Since the system is corrupted by noise, the parameter estimation error may be large. In order to reduce the estimation error, the parameter estimation vector $\hat{\boldsymbol{\theta}}_o$ is filtered by setting a small threshold $\epsilon > 0$. The filtered parameter estimation vector can be written as $\hat{\boldsymbol{\theta}}_\epsilon$. Thus, the parameter estimation vector $\hat{\boldsymbol{\theta}}$ can be obtained from (10) as

$$\hat{\boldsymbol{\theta}} = \Phi_l^{-1} \hat{\boldsymbol{\theta}}_\epsilon. \quad (21)$$

The coefficients $a_i (i = 1, 2, \dots, n_a)$, $b_j (j = 1, 2, \dots, n_b)$ and $\beta_r (r = 1, 2, \dots, n_\beta)$ can be directly obtained from (21) while parameters $c_k (k = 1, 2, \dots, n_c)$ cannot. The following step is to estimate the feedback channel parameters. According to the model equivalence principle [25], and from (2), we have

$$\hat{\mathbf{S}} = \hat{\boldsymbol{\eta}} \hat{\boldsymbol{\theta}}_c,$$

where

$$\hat{\boldsymbol{\eta}} := \begin{bmatrix} \hat{b}_1 & 0 & \dots & 0 \\ \hat{b}_2 & \hat{b}_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{b}_{n_c} & \hat{b}_{n_c-1} & \dots & \hat{b}_1 \\ \hat{b}_{n_c+1} & \hat{b}_{n_c} & \dots & \hat{b}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{b}_{n_b-1} & \hat{b}_{n_b-2} & \dots & \hat{b}_{n_b-n_c} \\ \hat{b}_{n_b} & \hat{b}_{n_b-1} & \dots & \hat{b}_{n_b-n_c+1} \\ 0 & \hat{b}_{n_b} & \dots & \hat{b}_{n_b-n_c+2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{b}_{n_b} \end{bmatrix},$$

$$\begin{aligned} \hat{\mathbf{S}} &:= [\hat{\beta}_2 - \hat{b}_2, \hat{\beta}_3 - \hat{b}_3, \dots, \hat{\beta}_{n_c+1} - \hat{b}_{n_c+1}, \hat{\beta}_{n_c+2} - \\ & \quad \hat{b}_{n_c+2}, \dots, \hat{\beta}_{n_b} - \hat{b}_{n_b}, \hat{\beta}_{n_b+1}, \hat{\beta}_{n_b+2}, \dots, \hat{\beta}_{n_\beta}]^T, \\ \hat{\boldsymbol{\theta}}_c &:= [\hat{c}_1, \hat{c}_2, \dots, \hat{c}_{n_c}]^T. \end{aligned}$$

Using the LS algorithm, the parameters c_i can be estimated by

$$\hat{\boldsymbol{\theta}}_c = (\hat{\boldsymbol{\eta}}^T \hat{\boldsymbol{\eta}})^{-1} \hat{\boldsymbol{\eta}}^T \hat{\mathbf{S}}. \quad (22)$$

Once the sparse parameter vector is recovered, the unknown time-delays can be estimated based on the positions and

Input: the output vector Y ,
the information matrix Φ .

Output: the parameter estimation vectors $\hat{\theta}$ and $\hat{\theta}_c$,
the time-delays \hat{d}_1 and \hat{d}_2 .

Steps:

1. Normalize the information matrix Φ by

$$\Phi_o = \Phi\Phi_l^{-1}.$$
2. Construct vectors and matrix

$$\gamma = \Phi_o^T Y, B = \begin{bmatrix} \Phi_o^T \Phi_o & -\Phi_o^T \Phi_o \\ -\Phi_o^T \Phi_o & \Phi_o^T \Phi_o \end{bmatrix},$$

$$C = \begin{bmatrix} -\gamma \\ \gamma \end{bmatrix} + \sigma \sqrt{2 \log(n)} \mathbf{1}_{2n}.$$
3. Solve the quadratic program and invoke the function of the standard scientific software for the optimum solution \hat{z}_o .
4. Compute the parameter estimation vector $\hat{\theta}_o$ by

$$\hat{\theta}_o = \hat{z}_o(1:n) - \hat{z}_o(n+1:2n).$$
5. Set a small threshold to get the filtered parameter estimation vector $\hat{\theta}_\epsilon$.
6. Compute $\hat{\theta}$ and $\hat{\theta}_c$ by

$$\hat{\theta} = \Phi_l^{-1} \hat{\theta}_\epsilon, \hat{\theta}_c = (\hat{\eta}^T \hat{\eta})^{-1} \hat{\eta}^T \hat{S}.$$
7. Estimate the time-delays \hat{d}_1 and \hat{d}_2 by

$$\hat{d}_1 = l_2 - n_b - z_3, \hat{d}_2 = l_1 - n_\beta - z_2.$$

FIGURE 3. The Th-BPDN based identification algorithm.

the number of zeros of each zero-block in the parameter estimation vector. Obviously, there are three zero-blocks in $\hat{\theta}$ from (4). Let the number of zeros in each zero-block be $z_p (p = 1, 2, 3)$. Then the time-delays can be estimated as follows,

$$\begin{aligned} \hat{d}_1 &= l_2 - n_b - z_3, \\ \hat{d}_2 &= l_1 - n_\beta - z_2. \end{aligned} \tag{23}$$

It is noteworthy that the noise vector V can be both the white noise and the colored noise. Equations (7)-(23) form the threshold basis pursuit de-noising (Th-BPDN) based identification algorithm for a class of closed-loop systems. The steps of the Th-BPDN based identification algorithm are shown in Figure 3.

IV. SIMULATION EXAMPLE

Example 1: Consider a closed-loop system in Figure 1 with

$$\begin{aligned} A(q) &= 1 - 0.20q^{-1} + 0.30q^{-2}, \\ B(q) &= 3.50q^{-1} + 2.90q^{-2}, \\ C(q) &= 1 + 0.30q^{-1}, \\ d_1 &= 45, \quad d_2 = 50. \end{aligned} \tag{24}$$

TABLE 1. The parameter estimation errors δ versus different noise variances σ^2 .

σ^2	0.40 ²	0.50 ²	0.60 ²	0.70 ²	0.80 ²
Th-BPDN	0.0417	0.0524	0.0632	0.0741	0.0850
Th-OMP	0.1850	0.1881	0.1914	0.1947	0.3004

TABLE 2. The parameter estimation errors δ with different sampled data lengths m .

m	100	150	300	500	800
Th-BPDN	0.0584	0.0311	0.0280	0.0248	0.0103
LS	3.0013	1.6995	0.3448	0.2383	0.1810

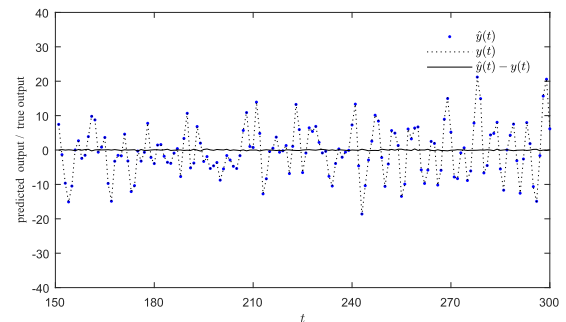


FIGURE 4. Predicted output $\hat{y}(t)$, true output $y(t)$, and errors.

Then

$$\beta(q) = B(q)C(q) = 3.50q^{-1} + 3.95q^{-2} + 0.87q^{-3}.$$

Let the input and output maximum regression lengths be $l_1 = 80$ and $l_2 = 120$. Then the parameter vector to be recovered is

$$\theta = [-0.2, 0.3, \mathbf{0}_{93}, 3.5, 3.95, 0.87, \mathbf{0}_{27}, 3.5, 2.9, \mathbf{0}_{73}]^T \in \mathbb{R}^{200},$$

where $\mathbf{0}_j$ denotes a j dimensional vector. Obviously, there are seven nonzero parameters in θ , i.e., $K = 7$.

In simulation, we take the reference input $\{r(t)\}$ as a persistent excitation signal sequence with zero mean and unit variance and $\{v(t)\}$ as a white noise sequence with zero mean and different variances σ^2 .

Let $\epsilon = 0.03$. For a sampled data length $m = 150$, the Th-BPDN algorithm and the Th-OMP algorithm are used to estimate the system, respectively. To test the robustness of the two algorithms [26], the parameter estimation errors $\delta := \|\theta - \hat{\theta}\| / \|\theta\|$ with different noise levels of the two algorithms are shown in Table 1. It is observed from Table 1 that the Th-BPDN algorithm is more robust than the Th-OMP algorithm.

When the noise variance is $\sigma^2 = 0.30^2$, applying the Th-BPDN algorithm and the LS algorithm to identify the close-loop system, respectively, Table 2 shows the parameter estimation errors versus different sampled data lengths. It can be seen from Table 2 that the estimation accuracy of the Th-BPDN algorithm is much higher than the LS algorithm.

For a sampling data length $m = 300$ and the noise variance $\sigma^2 = 0.10^2$, using the proposed Th-BPDN algorithm to estimate the sparse vector with the first 150 data, and using the remaining 150 data to validate the model, the parameter estimation vector is

$$\hat{\theta} = [-0.1960, 0.2913, \mathbf{0}_{93}, 3.4841, 3.9644, 0.8180, \mathbf{0}_{27}, 3.4622, 2.8900, \mathbf{0}_{73}]^T \in \mathbb{R}^{200}, \quad (25)$$

and the root mean square error of the output is

$$\delta_s := \sqrt{\frac{1}{m_e} \sum_{t=151}^{300} [\hat{y}(t) - y(t)]^2} = 0.1258,$$

which is close to standard deviation of the noise $\sigma = 0.10$. Therefore, the estimation model can well capture the dynamic performance of the true system. Figure 4 shows the true outputs, the predicted outputs of the estimation model and their errors. As can be seen, the predicted outputs $\hat{y}(t)$ are very close to the true outputs $y(t)$, and their errors are close to zero. From (25), we have

$$\begin{aligned} \hat{a}_1 &= -0.1960, & \hat{a}_2 &= 0.2913, \\ \beta_1 &= 3.4841, & \beta_2 &= 3.9644, & \beta_3 &= 0.8180, \\ \hat{b}_1 &= 3.4622, & \hat{b}_2 &= 2.8900. \end{aligned}$$

Using the model equivalence principle, we have

$$\hat{c}_1 = 0.2991.$$

From (25), it can be seen that the numbers of zeros in each zero-block are $z_1 = 93$, $z_2 = 27$ and $z_3 = 73$. Then, according to (23), the estimated time-delays can be computed by

$$\begin{aligned} \hat{d}_1 &= l_2 - n_b - z_3 = 45, \\ \hat{d}_2 &= l_1 - n_b - z_2 = 50. \end{aligned}$$

It is obvious that the estimated time-delays are identical to the true time-delays.

Example 2: Consider a closed-loop system with the same parameters as Example 1 but a colored noise.

Taking $m = 150$, $\epsilon = 0.03$, applying the Th-BPDN algorithm to identify the system, the parameter estimation vector is

$$\hat{\theta} = [-0.1991, 0.2921, \mathbf{0}_{93}, 3.4751, 3.9282, 0.8131, \mathbf{0}_{27}, 3.4712, 2.8679, \mathbf{0}_{73}]^T \in \mathbb{R}^{200}. \quad (26)$$

From (26), it can be seen that the numbers of zeros in each zero-block are $z_1 = 93$, $z_2 = 27$ and $z_3 = 73$. Then, the estimated time-delays are $\hat{d}_1 = 45$, $\hat{d}_2 = 50$, which are the same as Example 1.

The parameter estimation errors with different noise levels of the Th-BPDN algorithm and the Th-OMP algorithm are shown in Table 3. It can be seen from Table 3 that the Th-BPDN algorithm is more robust than the Th-OMP algorithm.

From Examples 1 and 2, we can see that the proposed method can effectively estimate the parameters and the

TABLE 3. The parameter estimation errors δ versus different noise variances μ^2 .

μ^2	0.40 ²	0.50 ²	0.60 ²	0.70 ²	0.80 ²
Th-BPDN	0.0447	0.0557	0.0667	0.0776	0.0901
Th-OMP	0.1956	0.1998	0.2663	0.2668	0.2673

time-delays for the closed-loop systems with white Gaussian noise and non-Gaussian noise. Moreover, the simulation results show that the proposed Th-BPDN algorithm is robust to white Gaussian noise and non-Gaussian noise.

V. CONCLUSION

This paper presents a novel identification method based on the Th-BPDN algorithm and the model equivalence principle for a class of closed-loop systems with unknown time-delays. The system parameters and the time-delays can be jointly estimated with a few number of data by the proposed method. For the sparse system identification in the first step, the proposed Th-BPDN algorithm is more robust than the Th-OMP algorithm. The simulation results verify that the proposed method is effective.

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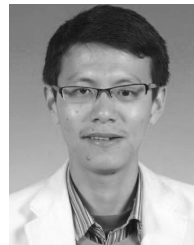
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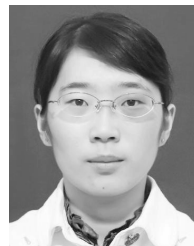
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