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Decentralized Adaptive Tracking Control of a Class of Nonlinear Discrete-Time Coupled Multi-Agent Systems With Unknown Dynamics

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ABSTRACT This paper studies the trajectory tracking control of a class of multi-agent systems, in which, each agent is expressed by a nonlinear discrete-time unknown dynamics and can interact with its neighbors via the history outputs of its neighbors. In order to tackle each unknown dynamics, based on its neighbors' and its own history I/O data and neural network, an approximate model is established by the direct data-driven method. Using the neighbors' history information and the reference trajectory, the decentralized adaptive indirect data-driven control is designed; then, the feedback gain matrix online is designed and adjusted by measured output data and previous estimates. For each agent, this is an adaptive control process of prediction, estimation, and adjustment, which needs to solve some nonlinear optimization problems online, can surmount the negative effects of the modeling errors caused by neural networks, and is the key to making each agent output asymptotically track the given reference trajectory. The convergence analysis shows that the applied method is effective and feasible.

INDEX TERMS Multi-agent system, tracking control, indirect data-driven method, adaptive control, nonlinear discrete-time system.

I. INTRODUCTION

In recent years, the tracking control for multi-agent systems (MASs) has attracted significant research attentions in the control community [1]–[4], due to its wide application background in engineering and scientific fields, such as spacecraft formation flying [5], cooperative control in robotic systems [6], unmanned systems [7], target tracking in sensor networks [8], and so on. Thus, a great of progress has been made on the tracking control of MASs [9]–[12], to name a few.

However, due to various kinds of uncertainties in the real world, for the multi-agent system (MAS), each agent is often difficult and almost impossible to get precise model although it is producing, measuring, transmitting, and storing huge amount of valuable data. As well known, for systems with uncertainties, system identification is basic and crucial. To identify parameter, many estimation algorithms have been emerged such as the least-squares algorithm, the projection algorithm, the backstepping approach and so on [13]–[15]. For identifying model structure uncertainty, fuzzy logic algorithm, neural networks (NNs) and wavelet are applied in [16]–[19]. Thus, facing to uncertainties, in the tracking control community, the adaptive control of the MASs with model parameters and model structure uncertainties have been investigated in [20]–[22].

In fact, most results on adaptive control are still concentrating on dealing with various uncertainties in single-agent systems [23]–[27]. Compared with single-agent systems, the effects of uncertainties of MASs on the overall performances are closely related to the pattern of information interaction. Hence, the decentralized adaptive control of MASs with uncertainties has been paid much attention to by the systems and control community. Although due to the interactions among agents and complexity of performance indices, studying the decentralized adaptive tracking control of MASs with uncertainties brings intrinsic difficulties and challenges, there is still a lot of literature dedicated to discussing the the

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decentralized adaptive tracking control of MASs [28]–[31]. For example, the work in [28] investigated the distributed adaptive consensus tracking control without such requirements for nonlinear high-order multi-agent systems subjected to mismatched unknown parameters and uncertain external disturbances. The distributed consensus tracking of unknown nonlinear chaotic delayed fractional-order multi-agent systems with external disturbances was studied in [29]. The cooperative preview tracking problem of discrete-time linear multi-agent systems under the fixed directed acyclic communication topology was analysed in [30]. And in [31], the adaptive dynamic programming algorithm of iteration in policy evaluation and policy improvement was developed to solve the optimal tracking control problem of discrete-time multi-agent systems.

Most of the above literature assume that each agent is either linear or continuous dynamics. Due to the approaches in the linear and continuous systems may not work or even fail in designing discrete-time nonlinear adaptive systems, which is caused by the inherent limitations of the feedback mechanism in the discrete-time nonlinear adaptive systems control. This phenomenon brings great difficulties, which yield relatively limited research work, in discrete-time nonlinear adaptive systems. Little results have been obtained to study the decentralized adaptive tracking control of discrete-time and nonlinear multi-agent systems. The paper [32] investigated the decentralized adaptive tracking control for a class of discrete-time nonlinear hidden leader follower multi-agent systems. Among all the agents, there exists a hidden leader that knows the desired reference trajectory, while the followers do not know the desired reference signal or are not aware of which agent is a leader. And then the work in [33] discussed the decentralized adaptive tracking control for a class of coupled hidden leader-follower multi-agent systems with unknown internal parameters and unknown high-frequency gains.

Given the above discussion of [32] and [33], in the two papers, the decentralized adaptive tracking control of discrete-time nonlinear hidden leader follower multi-agent systems was studied. Furthermore reading them, the common natures are that each dynamics contains the unknown parameter and the projection algorithm is taken to identify parameters. Inspired by them, combined with [27], the decentralized adaptive tracking control of discrete-time nonlinear in model structure uncertainty is tackled in this paper, and based on the its neighbors' and its own history I/O data and neural network (NN), an approximate model is established by the direct data-driven method.

The data-driven control of the MASs has been investigated in [34]–[36]. For instance, in [34], data-driven consensus control for networked agents was discussed an iterative learning control approach to achieve accurate coordination performances of the output data sequences for multiple plants. And the optimal consensus control problem for discrete-time multi-agent systems with completely unknown dynamics by utilizing a data-driven reinforcement learning method was investigated in [35]. The work in [36] studied the output consensus problem for a class of nonlinear networked multi-agent systems with switching topology and time-varying delays and the distributed data-driven consensus protocols were proposed to synchronise the outputs of the agents.

Based on the above analysis, in this paper, the decentralized tracking control of discrete-time coupled MASs with unknown nonlinear structure dynamics is addressed. And each agent can only obtain its own and its neighborhood history information, and link may propagate over the whole network along with the information exchange among agents, in addition, the uncertainties of each dynamics and the interactions among agents and complexity of performance indices for MAS, thus the difficulty in this paper arises from the fully decentralized protocol design with the unknown discrete-time dynamics for the tracking control considering coupling among agents. The main contributions of this paper are listed as follows. (1) Under the mild conditions, based on Lagrange's mean value theorem, for each agent, the given reference trajectory, there exists a unique control input such that the dynamics is satisfied. This is an linearization technique which is applicable to nonlinear discrete-time systems. (2) The indirect data-driven method is adopted to predict and estimate some relative model structure matrices, and then, design and adjust the feedback gains such that outputs of each agent tracks the given reference trajectory. In this process, the approximate models are established to estimate the unknown functions using the recursive NN. (3) And then it is shown that the system output asympotically converges to the reference trajectory, especially, the given same reference signal to all agents, the whole system eventually achieves synchronization in the presence of strong couplings.

The rest of this paper is organized as follows. The problem formulation and basic assumptions in Section II. Section III designs indirect data-driven output decentralized trajectory tracking control law. Section IV analyses effectiveness and feasibility of the algorithm convergence. Finally, some concluding remarks are given in Section V by highlighting certain unsolved problems. And here we give the nomenclature in the following Table 1.

TABLE 1. Nomenclature.

f_i	unknown structure function of agent i
$\hat{f}_i$	approximate structure function of agent i
$y_i(k)$	output of agent i at the time k
u_i	control input of agent i
R	real number
R^n	n-dimensional Euclidean space
I_n	n-dimensional identity matrix
D_u	bounded convex set with $u = 0$
D_y	bounded convex set with $y = 0$
$\frac{\partial f}{\partial x}$	partial derivative of the function f with respect to the variable x
$\hat{g}_i$	high-frequency estimate of agent i
$[\cdot]^{-1}$	inverse of matrix
$\|\cdot\|$	vector or matrix Euclidian norm
$\otimes$	Kronecker product
$[\cdot]^T$	vector or matrix transpose

II. PROBLEM FORMULATION AND BASIC ASSUMPTIONS

A. ALGEBRAIC GRAPH THEORY

Under an MAS study, each agent may be coupled to other agents through its neighbors' available information. If each agent is taken as a node, let the communicated topology be represented by a directed graph from algebraic graph theory. A directed graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ with a set of N agents $\mathcal{V} = \{1, 2, \cdots, N\}$, and $\varepsilon = \mathcal{V} \times \mathcal{V}$ is a set of M ordered edges of the form (i, j), representing that agent j has access to the information of agent j and calling agent j is a neighbor of agent i. The set of all neighbors of agent i is expressed by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \varepsilon\}$. Matrix $\mathcal{A}(a_{ij} = 0, 1) \in R^{N\times N}$ is an adjacency matrix, whose entries

$$\begin{cases} a_{ij} = 1, & \text{if } j \text{ is } i's \text{ neighbor} \\ a_{ij} = 0, & \text{otherwise}. \end{cases}$$

The indegree matrix is defined as a diagonal matrix $D = diag[d_1, d_2, \cdots, d_N]$ with $d_i = \sum_{j=1}^{N} a_{ij}, i = 1, \cdots, N$. Obviously, d_i is the number of agent i's neighbors.

Definition 1 [33]: The whole MAS is globally reachable if each node/agent is globally reachable.

Definition 2 [32]: An adjacency matrix $\mathcal{A}(a_{ij} = 0, 1)$ is a strongly connected matrix if there exists a path that follows the direction of the edges of the directed graph such that any two agents i and j are connected.

B. SYSTEM REPRESENTATION AND ASSUMPTIONS

Let us consider the multi-agent system consisting of N agents. The dynamic of agent i is described by

$$y_i(k+1) = f_i(y_i(k), \phi_i(k), u_i(k)), \quad k \geq 0, \ i = 1, 2, \cdots, N, \tag{1}$$

where

$$u_i(k) = [u_{i1}(k), u_{i2}(k), \cdots, u_{im}(k)]^T \in D_u \subseteq R^m$$

and

$$y_i(k) = [y_{i1}(k), y_{i2}(k), \cdots, y_{in}(k)]^T \in D_y \subseteq R^n (m \leq n)$$

are the input and the output of agent i at the time k, respectively. The two bounded convex sets D_u and D_y contain $u = 0$ and $y = 0$, respectively. The neighbor measured outputs $\phi_i(k) = [y_i^{(1)T}(k), y_i^{(2)T}(k), \cdots, y_i^{(m_i)T}(k)]^T \in R^{n\times m_i}$, in which m_i is the number of neighbors of agent i and $y_i^{(j)}(k)(1 \leq j \leq m_i)$ denotes measured output of the j^{th} neighbor. The explicit mathematical function $f_i(y_i(k), \phi_i(k), u_i(k))$ is unknown. $f_i(\cdot) = [f_{i1}(\cdot), f_{i2}(\cdot), \cdots, f_{in}(\cdot)]^T \in R^n$ and $f_i(0, 0, 0) = 0$. Suppose that the values of $y_i(k)$ and $u_i(k)$ can be measured and recorded, and $y_i(0)$ and $\phi_i(0)$ are set.

Assumption A1: The function $f_i(y_i(k), \phi_i(k), u_i(k))$ has continuous partial derivatives with respect to $y_i(k)$, $\phi_i(k)$ and $u_i(k)$, respectively.

Assumption A2: $Rank[\frac{\partial f_i(y_i(k),\phi_i(k),u_i(k))}{\partial u_i(k)}] = m$ for $u_i(k) \in D_u$.

Assumption A3: The directed graph of the MAS is strongly connected.

Remark 1: Assumption **A3** indicates that any agent of MAS has either directly or indirectly access to the information of other agents.

For any $y_i(k), \bar{y}_i(k), \phi_i(k), \bar{\phi}_i(k), u_i(k), \bar{u}_i(k)$, in which, $(y_i(k), \phi_i(k), u_i(k))$ and $(\bar{y}_i(k), \bar{\phi}_i(k), \bar{u}_i(k))$ satisfy (1), respectively. On the basis of Assumption **A1** and Lagrange's mean value theorem we obtain that

$$\begin{aligned} y_i(k+1) - \bar{y}_i(k+1) &= f_i(y_i(k), \phi_i(k), u_i(k)) - f_i(\bar{y}_i(k), \\ &\qquad \bar{\phi}_i(k), \bar{u}_i(k)) \\ &= \bar{F}_i(k, \alpha_i(k))[x_i(k) - \bar{x}_i(k)]. \end{aligned} \tag{2}$$

In (2), we denote

$$\begin{cases} x_i(k) = [y_i^T(k), \phi_i^T(k), u_i^T(k)]^T \\ \bar{x}_i(k) = [\bar{y}_i^T(k), \bar{\phi}_i^T(k), \bar{u}_i^T(k)]^T \\ \bar{F}_i(k, \alpha_i(k)) = [\bar{F}_{i,y_i}(k, \alpha_i(k)), \bar{F}_{i,\phi_i}(k, \alpha_i(k)), \\ \bar{F}_{i,u_i}(k, \alpha_i(k))], \end{cases} \tag{3}$$

where

$$\begin{cases} \bar{F}_{i,y_i}(k, \alpha_i(k)) = [\frac{\partial f_{i1}^T}{\partial y_i} z_{i1}(k), \cdots, \frac{\partial f_{in}^T}{\partial y_i} z_{in}(k)]^T \\ \frac{\partial f_{ij}}{\partial y_i}(z_{ij}(k)) = [\frac{\partial f_{ij}(z_{ij}(k))}{\partial y_{i1}}, \cdots, \frac{\partial f_{ij}(z_{ij}(k))}{\partial y_{in}}] \\ \bar{F}_{i,\phi_i}(k, \alpha_i(k)) = [\frac{\partial f_{i1}^T}{\partial \phi_i} z_{i1}(k), \cdots, \frac{\partial f_{in}^T}{\partial \phi_i} z_{in}(k)]^T \\ \frac{\partial f_{ij}}{\partial \phi_i}(z_{ij}(k)) = [\frac{\partial f_{ij}(z_{ij}(k))}{\partial \phi_{i1}}, \cdots, \frac{\partial f_{ij}(z_{ij}(k))}{\partial \phi_{im_i}}] \\ \bar{F}_{i,u_i}(k, \alpha_i(k)) = [\frac{\partial f_{i1}^T}{\partial u_i} z_{i1}(k), \cdots, \frac{\partial f_{in}^T}{\partial u_i} z_{in}(k)]^T \\ \frac{\partial f_{ij}}{\partial u_i}(z_{ij}(k)) = [\frac{\partial f_{ij}(z_{ij}(k))}{\partial u_{i1}}, \cdots, \frac{\partial f_{ij}(z_{ij}(k))}{\partial u_{im}}] \\ z_{ij}(k) \triangleq \bar{x}_i(k) + \alpha_{ij}(k)[x_i(k) - \bar{x}_i(k)] \\ 0 < \alpha_{ij}(k) < 1, \quad 1 \leq j \leq n, k \geq 0 \\ \alpha_i(k) = [\alpha_{i1}(k), \alpha_{i2}(k), \cdots, \alpha_{in}(k)]. \end{cases} \tag{4}$$

It is easy to see that, the derivatives $\bar{F}_{i,y_i}(k, \alpha_i(k)) \in R^{n\times n}$, $\bar{F}_{i,\phi_i}(k, \alpha_i(k)) \in R^{n\times m_i}$, $\bar{F}_{i,u_i}(k, \alpha_i(k)) \in R^{n\times m}$ and $\bar{F}_i(k, \alpha_i(k)) \in R^{n\times(n+m_i+m)}$. The coefficient $\alpha_i(k)$ is defined by Lagrange's mean value theorem, which varies with the time k. By (2) and (3), one has

$$\begin{aligned} y_i(k+1) - \bar{y}_i(k+1) &= \bar{F}_{i,y_i}(k, \alpha_i(k))[y_i(k) - \bar{y}_i(k)] \\ &\quad + \bar{F}_{i,\phi_i}(k, \alpha_i(k))[\phi_i(k) - \bar{\phi}_i(k)] \\ &\quad + \bar{F}_{i,u_i}(k, \alpha_i(k))[u_i(k) - \bar{u}_i(k)]. \end{aligned} \tag{5}$$

Theorem 1: If system (1) satisfies Assumptions **A1** and **A2**, then for given $y_i(k) \in D_y, k = 1, 2, \cdots, i = 1, 2, \cdots, N$, there exists a unique control input $u_i(k) \in D_u$ such that $y_i(k)$, $\phi_i(k)$ and $u_i(k)$ satisfy (1).

Proof 1: First, the existence of $u_i(k)$ is to be proven. According to Assumption **A1**, (1), (5) and $f_i(0, 0, 0) = 0$, it is not difficult to get that

$$\begin{aligned} &y_i(k+1)-0\\ &\quad = f_i(y_i(k),\phi_i(k),u_i(k))-f_i(0,0,0)\\ &\quad = \bar{F}_{i,y_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}y_i(k)\\ &\qquad +\bar{F}_{i,\phi_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}\phi_i(k)\\ &\qquad +\bar{F}_{i,u_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}u_i(k). \end{aligned} \tag{6}$$

By the basis of Assumption **A2**, it is easy to see that matrix

$$\bar{F}_{i,u_i}(k,\alpha_i(k))|_{\bar{y}_i(k)}=0,\quad \bar{\phi}_i(k)=0,\bar{u}_i(k)=0$$

is invertible. Thus, the solution of (6) is obtained, that is

$$\begin{aligned} u_i(k) = &[\bar{F}^T_{i,u_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}\\ &\times \bar{F}_{i,u_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}]^{-1}\\ &\times \bar{F}^T_{i,u_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}\\ &\times [y_i(k+1)-\bar{F}_{i,y_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}\\ &\quad y_i(k)-\bar{F}_{i,\phi_i}(k,\alpha_i(k))|_{\bar{y}_i(k)=0,\bar{\phi}_i(k)=0,\bar{u}_i(k)=0}\phi_i(k)]. \end{aligned} \tag{7}$$

Remark 2: (7) shows that for given $y_i(k) \in D_y, k = 1, 2, \cdots, i = 1, 2, \cdots, N$, there exists a control input $u_i(k) \in D_u$ such that $y_i(k)$, $\phi_i(k)$ and $u_i(k)$ satisfy (1).

Next, the uniqueness of the $u_i(k)$ is to be proven using the proof by contradiction. For given $y_i(k) \in D_y, k = 1, 2, \cdots, i = 1, 2, \cdots, N$, assume that there exist two different control inputs $u_{i,a}(k) \in D_u$ and $u_{i,b}(k) \in D_u$, that is, for given $y_i(k+1)$, there are $u_{i,a}(k), u_{i,b}(k)(u_{i,a}(k) \neq u_{i,b}(k))$ such that $y_i(k+1) = f_i(y_i(k), \phi_i(k), u_i(k))$. In other words, the vectors $(y_i(k), \phi_i(k), u_{i,a}(k))$ and $(y_i(k), \phi_i(k), u_{i,b}(k))$ satisfy (1). In (5), using $y_i(k), \phi_i(k), u_{i,a}(k), u_{i,b}(k)$ in place of $\bar{y}_i(k), \bar{\phi}_i(k), u_i(k), \bar{u}_i(k)$, respectively, we have

$$\tilde{F}^T_{i,u_i}(k,\alpha_i(k))\tilde{F}_{i,u_i}(k,\alpha_i(k))[u_i(k)-\bar{u}_i(k)] = 0, \tag{8}$$

where $\tilde{F}_{i,u_i}(k,\alpha_i(k))$ is $\bar{F}_{i,u_i}(k,\alpha_i(k))$ with $\bar{y}_i(k) = y_i(k)$, $\bar{\phi}_i(k) = \phi_i(k), u_i(k) = u_{i,a}(k)$, and $\bar{u}_i(k) = u_{i,b}(k)$. That is to say that (8) can be written as

$$\tilde{F}^T_{i,u_{i,a}}(k,\alpha_i(k))\tilde{F}_{i,u_{i,a}}(k,\alpha_i(k))[u_{i,a}(k)-u_{i,b}(k)] = 0.$$

According to Assumption **A2**, it is easy to know that $\tilde{F}^T_{i,u_i}(k,\alpha_i(k))\tilde{F}_{i,u_i}(k,\alpha_i(k))$ is invertible. Thus, one has

$$u_{i,a}(k)-u_{i,b}(k) = 0,$$

that is

$$u_{i,a}(k) = u_{i,b}(k), \tag{9}$$

which indicates that this result is contrary to the assumption that two control inputs $u_{i,a}(k) \neq u_{i,b}(k)$. Based on the proof by contradiction, we can get that for given $y_i(k) \in D_y$, $k = 1, 2, \cdots, i = 1, 2, \cdots, N$, if there exists a control input $u_i(k) \in D_u$ such that $y_i(k)$, $\phi_i(k)$ and $u_i(k)$ satisfy (1), then this control input is unique.

Let the signal $y_i^*(k) \in D_y$ is the reference trajectory of system (1). For the whole system, each agent has its own reference trajectory. Then, each vector $\phi_i(k)$ has the corresponding vector $\phi_i^*(k)$. Assume that each $\|y_i^*(k)\| < \infty$. Obviously, $\|\phi_i^*(k)\| < \infty$. Then there exists a control input sequence $\{u_i^*(k)\}$ such that $\|u_i^*(k)\| < \infty$ and

$$y_i^*(k+1) = f_i(y_i^*(k), \phi_i^*(k), u_i^*(k)). \tag{10}$$

Remark 3: From Theorem 1, it is easy to see that for given $y_i(k), i = 1, 2, \cdots, N$, there exists a unique $u_i(k)$. In fact, Assumption *A*2 is the sufficient condition for the uniqueness. If this condition cannot be satisfied, a proper control input cannot be obtained to make the output of each agent to track the reference trajectory.

Remark 4: From (10) and Theorem 1, it is easy to see that for the reference trajectory $y_i^*(k), i = 1, 2, \cdots, N$, there exists a unique control input $u_i^*(k)$ such that $y_i^*(k+1) = f_i(y_i^*(k), \phi_i^*(k), u_i^*(k))$.

By (1), (5) and (10), it yields that

$$\begin{aligned} &y_i(k+1)-y_i^*(k+1)\\ &\quad = f_i(y_i(k),\phi_i(k),u_i(k))-f_i(y_i^*(k),\\ &\qquad \phi_i^*(k),u_i^*(k))\\ &\quad = F_{i,y_i}(k,\alpha_i(k))[y_i(k)-y_i^*(k)]\\ &\qquad +F_{i,\phi_i}(k,\alpha_i(k))[\phi_i(k)-\phi_i^*(k)]\\ &\qquad +F_{i,u_i}(k,\alpha_i(k))[u_i(k)-u_i^*(k)], \end{aligned} \tag{11}$$

where $F_{i,y_i}(k,\alpha_i(k)), F_{i,\phi_i}(k,\alpha_i(k))$ and $F_{i,u_i}(k,\alpha_i(k))$ are equal to $\bar{F}_{i,y_i}(k,\alpha_i(k))$, $\bar{F}_{i,\phi_i}(k,\alpha_i(k))$ and $\bar{F}_{i,u_i}(k,\alpha_i(k))$ in (4), respectively, with $\bar{y}_i(k) = y_i^*(k), \bar{\phi}_i(k) = \phi_i^*(k)$ and $\bar{u}_i(k) = u_i^*(k)$.

However, according to the function $f_i(y_i(k), \phi_i(k), u_i(k))$ is unknown, we cannot obtain the control input sequence $\{u_i^*(k)\}$. We will discuss how to solve this problem in the next section.

III. INDIRECT DATA-DRIVEN OUTPUT DECENTRALIZED TRAJECTORY TRACKING CONTROL

In this section, our task is to control each output $y_i(k)$ to track the given reference trajectory $y_i^*(k)$ at any time k, the control law of agent i is designed as follows:

$$\begin{aligned} u_i(k) = u_i^*(k) + I_{m\times n}[h_i(k)-h_i^*(k)] + C_i(k)\\ \times [y_i(k)-y_i^*(k)],\quad k \geq 0, \end{aligned} \tag{12}$$

where $C_i(k) \in R^{m\times n}$ is the feedback gain, and

$$h_i(k) = \frac{1}{d_i}\sum_{l\in\mathcal{N}_i} y_l(k), \tag{13}$$

$$h_i^*(k) = \frac{1}{d_i}\sum_{l\in\mathcal{N}_i} y_l^*(k), \tag{14}$$

where d_i and N_i are the number and set of the neighbors of agent i at the time k, respectively.

Denote

$$Y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_N(k) \end{bmatrix}, \quad Y^*(k) = \begin{bmatrix} y_1^*(k) \\ y_2^*(k) \\ \vdots \\ y_N^*(k) \end{bmatrix} \tag{15}$$

and

$$U(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_N(k) \end{bmatrix}, \quad U^*(k) = \begin{bmatrix} u_1^*(k) \\ u_2^*(k) \\ \vdots \\ u_N^*(k) \end{bmatrix}. \tag{16}$$

Thus, from (12) and Assumption **A3**, one has

$$U(k) = U^*(k) + [\Lambda\mathcal{A} \otimes I_{m\times n} + C(k)][Y(k) - Y^*(k)], \tag{17}$$

where

$$\Lambda = \begin{bmatrix} \frac{1}{d_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{d_N} \end{bmatrix}, \tag{18}$$

$$\mathcal{A} = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1N} \\ a_{21} & 0 & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & 0 \end{bmatrix} \tag{19}$$

and

$$C(k) = \begin{bmatrix} C_1(k) & 0 & \cdots & 0 \\ 0 & C_2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_N(k) \end{bmatrix} = diag\begin{bmatrix} C_1(k) & C_2(k)\cdots & C_N(k) \end{bmatrix}, \tag{20}$$

in which, d_i is the number of neighbors, a_{ij} is the element defined the adjacency matrix, and $C_i(k)$ the feedback gain. According to (11), we have the following equation

$$\begin{aligned} Y(k+1) - Y^*(k+1) &= F_Y(k, \alpha(k))[Y(k) - Y^*(k)] \\ &\quad + F'_Y(k, \alpha(k))[Y(k) - Y^*(k)] \\ &\quad + F_U(k, \alpha(k))[U(k) - U^*(k)]. \end{aligned} \tag{21}$$

In (21), we denote

$$\begin{aligned} F_Y(k, \alpha(k)) &= \begin{bmatrix} F_{1,y_1}(k, \alpha_1(k)) & 0 & \cdots & 0 \\ 0 & F_{2,y_2}(k, \alpha_2(k)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_{N,y_N}(k, \alpha_N(k)) \end{bmatrix} \\ &= diag\begin{bmatrix} F_{1,y_1}(k, \alpha_1(k)) & F_{2,y_2}(k, \alpha_2(k)) & \cdots & F_{N,y_N}(k, \alpha_N(k)) \end{bmatrix} \end{aligned} \tag{22}$$

$$F'_Y(k, \alpha(k)) = \begin{bmatrix} F_{1,Y}(k, \alpha_1(k)) \\ F_{2,Y}(k, \alpha_2(k)) \\ \vdots \\ F_{N,Y}(k, \alpha_N(k)) \end{bmatrix}, \tag{23}$$

in which, $F_{i,Y}(k, \alpha_i(k))$ is dragged in and denoted by

$$F_{i,Y}(k, \alpha_i(k)) = \frac{\partial f_i}{\partial Y^T} z_i(k), \tag{24}$$

here, $F_{i,Y}(k, \alpha_i(k))$ is equal to $\bar{F}_{i,Y}(k, \alpha_i(k))$ with $\bar{y}_i(k) = y_i^*(k)$, $\bar{u}_i(k) = u_i^*(k)$, if $y_l \in \mathcal{N}_i$, taking $\bar{y}_l(k) = y_l^*(k)$; otherwise, taking $\bar{y}_l(k) = 0$. And

$$\begin{aligned} F_U(k, \alpha(k)) &= \begin{bmatrix} F_{1,u_1}(k, \alpha_1(k)) & 0 & \cdots & 0 \\ 0 & F_{2,u_2}(k, \alpha_2(k)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_{N,u_N}(k, \alpha_N(k)) \end{bmatrix} \\ &= diag\begin{bmatrix} F_{1,u_1}(k, \alpha_1(k)) & F_{2,u_2}(k, \alpha_2(k)) & \cdots & F_{N,u_N}(k, \alpha_N(k)) \end{bmatrix}. \end{aligned} \tag{25}$$

By (17) and (21), one has

$$\begin{aligned} Y(k+1) - Y^*(k+1) = \{&F_Y(k, \alpha(k)) + F'_Y(k, \alpha(k)) \\ &+ F_U(k, \alpha(k))[\Lambda\mathcal{A} \otimes I_{m\times n} \\ &+ C(k)]\}[Y_i(k) - Y_i^*(k)]. \end{aligned} \tag{26}$$

Denote

$$e_i(k) = y_i(k) - y_i^*(k), \tag{27}$$

$$\begin{aligned} G(k) &= F_Y(k, \alpha(k)) + F'_Y(k, \alpha(k)) + F_U(k, \alpha(k)) \\ &\quad \times [\Lambda\mathcal{A} \otimes I_{m\times n} + C(k)] \end{aligned} \tag{28}$$

and

$$E(k) = \begin{bmatrix} e_1(k) & e_2(k) & \cdots & e_N(k) \end{bmatrix}^T. \tag{29}$$

Thus, putting (29) into (26), one has

$$E(k+1) = G(k)E(k). \tag{30}$$

How to control the output of each agent to track the corresponding reference trajectory is transformed into how to design and adjust $C(k)$ to such that $\|G(k)\| < 1$. Like this, each $\|e_i(k)\|$ will keep on decreasing until it is less than the maximum tolerable tracking error ϵ_i. The design of $C_i(k)$ is the important to deal with the control problem, and is necessary to be end before the time k. Otherwise, the system cannot be controlled on time.

For each agent, its neighbors are fixed. Thus, $\Lambda\mathcal{A}$ is known in advance. According to (28), if we wish for $C(k)$, be prepared for $G(k)$, $F_Y(k, \alpha(k))$, $F'_Y(k, \alpha(k))$ and $F_U(k, \alpha(k))$.

However, they cannot be analytically calculated owning to the model structure uncertainties in the multi-agent system (1). To solve this problem, a data-driven method is adopted to predict and estimate these matrices. In this paper, we assume that there is no measurement noise (if any measurement noise, it has been filtered) contained in the measured output data.

A. ESTABLISH APPROXIMATE MODELS AND ESTIMATES

Let $\bar{y}_i(0) = 0, i = 1, 2, \cdots, N$, obviously know $\bar{\phi}_i(0)$, select appropriate sets of inputs $\bar{u}_i(k) \in D_u, i = 1, 2, \cdots, N, k \geq 0$ and record their values. Let the whole system run under the input control $U(k)$ or $u_i(k), i = 1, 2, \cdots, N, k \geq 0$ for quite a time. The outputs $Y(k)$ or $y_i(k), k \geq 1$ are measured and recorded. Then we estimate the unknown functions $f_i, i = 1, 2, \cdots, N$ using a recursive NN based on recording input-output data. For the system (1), an approximate model is established. The approximate model is expressed by

$$\bar{y}_i(k+1)=\hat{f}_i(\bar{y}_i(k), \bar{\phi}_i(k), \bar{u}_i(k)), \quad (k<0),\ i=1, 2, \cdots, N, \tag{31}$$

where $\hat{f}_i(\cdot)$ is an approximate model of $f_i(\cdot)$, which $\hat{f}_i(\cdot)$ also satisfies Assumptions **A1**, **A2** and $\hat{f}_i(0, 0, 0) = 0$.

By the $\hat{f}_i(\cdot)$ and the given reference trajectory $y_i^*(k)$, obviously, $\phi_i^*(k)$ is given. The nonlinear optimization problem is solved.

$$J(\hat{u}_i^*(k))= \min_{u_i(k)\in D_u} \left\| y_i^*(k+1)-\hat{f}_i(y_i^*(k), \phi_i^*(k), u_i(k))\right\|. \tag{32}$$

Then, we can get $\hat{u}_i^*(k)$, which is the estimate of $u_i^*(k)$. So far, many approaches for solving the nonlinear optimization problems have been studied [8], [9]. The details will not be dealt with in this paper due to the limited space.

It is clear that $\hat{f}_i(\cdot) \neq f_i(\cdot)$ and $\hat{u}_i^*(k) \neq u_i^*(k)$. Thus, we have to design and adjust $C_i(k)$ for every $k \geq 0$, even though $\hat{u}_i^*(k)$ is close to $u_i^*(k)$.

B. SET AND ESTIMATE THE INITIAL VALUES

After establishing the approximate model (31), we start to control system (1). For each agent, in the initial stage, the actual measured and recorded I/O data are not sufficient to design $C_i(0)$ and $C_i(1)$; thus, we have to artificially set the values of them. Besides, presetting the initial values of $C_i(k)$ is also necessary to control the system when the proper controller has not been obtained yet. On this account, the first two feedback gain matrices are designed and recorded as follows

$$C_i(0) = C_i(1) = \begin{bmatrix} c_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & c_2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_m & 0 & \cdots & 0 \end{bmatrix},$$

where $c_j(0 < |c_j| < 1, 1 \leq j \leq m)$ satisfy

$$Rank[C_i(1)] = Rank[C_i(0)] = m.$$

Then, for agent i, $y_i(0) \in D_y$ and the maximum tolerable tracking error $0 < \epsilon_i < \infty$ are set. In (12), substituting $\hat{u}_i^*(0)$ for $u_i^*(0)$, we have

$$u_i(0)=\hat{u}_i^*(0)+I_{m\times n}[h_i(0) - h_i^*(0)]+C_i(0)[y_i(0)-y_i^*(0)], \tag{33}$$

where $\hat{u}_i^*(0)$ is obtained from (32).

Because that the $y_i^*(k)$ is given, and from (12), it is clear that the value of $y_i(k)$ is necessary to design $u_i(k)$ for each subsystem. But, except the preset $y_i(0)$, after the time $k \geq 1$, every $y_i(k)$ can only be measured and recorded. To deal with this contradiction, we will make a slight modification in (12) using the prediction of $y_i(k)$ in place of $y_i(k)$. According to the approximate model (31), we can get the $\hat{y}_i(k)$ as follows

$$\hat{y}_i(k) = \hat{f}_i(y_i(k-1), \phi_i(k-1), u_i(k-1)), \quad (k \geq 1), \quad i = 1, 2, \cdots, N, \tag{34}$$

where the values of $y_i(k-1)$, $\phi_i(k-1)$ and $u_i(k-1)$ are actually measured and recorded before the time k. That is, for the whole system, the prediction of each subsystem output at the time k is obtained. Thus, the real control input for subsystem i at the time $k = 1$, $u_i(1)$ is designed as follows

$$u_i(1) = \hat{u}_i^*(1) + I_{m\times n}[\hat{h}_i(1)-h_i^*(1)]+C_i(1)[\hat{y}_i(1)-y_i^*(1)], \tag{35}$$

where

$$\hat{h}_i(1) = \frac{1}{d_i}\sum_{l\in\mathcal{N}_i} \hat{y}_l(1). \tag{36}$$

Obviously, $\hat{u}_i^*(1)$ and $\hat{y}_i(1)$ are obtained from (32) and (34), respectively. Then, the real value of each subsystem output $y_i(1)$ can be measured and recorded.

To achieve control objectives for the whole system and each subsystem, the estimate values of $F_{i,y_i}(k, \alpha_i(k))$, $F_{i,\phi_i}(k, \alpha_i(k))$ and $F_{i,u_i}(k, \alpha_i(k))$ at the time $k = 0, 1$ are written as follows:

$$\begin{cases} \hat{F}_{i,y_i}(k, \hat{\alpha}_i(k)) = [\frac{\partial \hat{f}_{i1}^T}{\partial y_i}\hat{z}_{i1}(k), \cdots, \frac{\partial \hat{f}_{in}^T}{\partial y_i}\hat{z}_{in}(k)]^T \\ \frac{\partial \hat{f}_{ij}}{\partial y_i}(\hat{z}_{ij}(k)) = [\frac{\partial \hat{f}_{ij}(\hat{z}_{ij}(k))}{\partial y_{i1}}, \cdots, \frac{\partial \hat{f}_{ij}(\hat{z}_{ij}(k))}{\partial y_{in}}] \\ \hat{F}_{i,\phi_i}(k, \hat{\alpha}_i(k)) = [\frac{\partial \hat{f}_{i1}^T}{\partial \phi_i}\hat{z}_{i1}(k), \cdots, \frac{\partial \hat{f}_{in}^T}{\partial \phi_i}\hat{z}_{in}(k)]^T \\ \frac{\partial \hat{f}_{ij}}{\partial \phi_i}(\hat{z}_{ij}(k)) = [\frac{\partial \hat{f}_{ij}(\hat{z}_{ij}(k))}{\partial \phi_{i1}}, \cdots, \frac{\partial \hat{f}_{ij}(\hat{z}_{ij}(k))}{\partial \phi_{im_i}}] \\ \hat{F}_{i,u_i}(k, \hat{\alpha}_i(k)) = [\frac{\partial \hat{f}_{i1}^T}{\partial u_i}\hat{z}_{i1}(k), \cdots, \frac{\partial \hat{f}_{in}^T}{\partial u_i}\hat{z}_{in}(k)]^T \\ \frac{\partial \hat{f}_{ij}}{\partial u_i}(\hat{z}_{ij}(k)) = [\frac{\partial \hat{f}_{ij}(\hat{z}_{ij}(k))}{\partial u_{i1}}, \cdots, \frac{\partial \hat{f}_{ij}(\hat{z}_{ij}(k))}{\partial u_{im}}] \\ \hat{z}_{ij}(k) \triangleq \hat{x}_i^*(k) + \hat{\alpha}_{ij}(k)[x_i(k) - \hat{x}_i^*(k)] \\ x_i(k) = [y_i^T(k), \phi_i^T(k), u_i^T(k)]^T \\ \hat{x}_i^*(k) = [y_i^{*T}(k), \phi_i^{*T}(k), \hat{u}_i^{*T}(k)]^T \\ 0 < \hat{\alpha}_{ij}(k) < 1, \quad 1 \leq j \leq n, k \geq 0 \\ \hat{\alpha}_i(k) = [\hat{\alpha}_{i1}(k), \hat{\alpha}_{i2}(k), \cdots, \hat{\alpha}_{in}(k)], \end{cases} \tag{37}$$

where

$$\hat{f}_i = \hat{f}_i(y_i(k), \phi_i(k), u_i(k)) \tag{38}$$

and $\hat{\alpha}_i(k)$ is the estimate of $\alpha_i(k)$. And according to (26) and Assumption **A3**, $\hat{\alpha}(k)$, $k = 0, 1$ are obtained by the following equations

$$\begin{cases} J(\hat{\alpha}(0)) = \min\limits_{\alpha(0)} \Big\| Y(1) - Y^*(1) - [\hat{F}_Y(0, \alpha(0)) + \\ \hat{F}'_Y(0, \alpha(0)) + \hat{F}_U(0, \alpha(0))(\Lambda \mathcal{A} \otimes I_{m\times n} \\ +C(0))]E(0)\| \\ 0 < \alpha_i(0) < 1, \quad 1 \le i \le N \\ \hat{\alpha}(0) = [\hat{\alpha}_1(0), \hat{\alpha}_2(0), \cdots, \hat{\alpha}_N(0)] \end{cases} \tag{39}$$

and

$$\begin{cases} J(\hat{\alpha}(1)) = \min\limits_{\alpha(1)} \Big\| \hat{Y}(2) - Y^*(2) - [\hat{F}_Y(1, \alpha(1)) + \\ \hat{F}'_Y(1, \alpha(1)) + \hat{F}_U(1, \alpha(1))(\Lambda \mathcal{A} \otimes I_{m\times n} + \\ C(1))]E(1)\| \\ 0 < \alpha_i(1) < 1, 1 \le i \le N \\ \hat{\alpha}(1) = [\hat{\alpha}_1(1), \hat{\alpha}_2(1), \cdots, \hat{\alpha}_N(1)], \end{cases} \tag{40}$$

where the predicition $\hat{Y}(2)$ can be obtained from (34); and at the time $k = 0, 1$, we denote

$$\hat{F}_Y(k, \alpha(k)) = diag\left[\hat{F}_{1,y_1}(k, \alpha_1(k)) \quad \hat{F}_{2,y_2}(k, \alpha_2(k)) \cdots \hat{F}_{N,y_N}(k, \alpha_N(k))\right],$$

$$\hat{F}'_Y(k, \alpha(k)) = \begin{bmatrix} \hat{F}_{1,Y}(k, \alpha_1(k)) \\ \hat{F}_{2,Y}(k, \alpha_2(k)) \\ \vdots \\ \hat{F}_{N,Y}(k, \alpha_N(k)) \end{bmatrix}$$

and

$$\hat{F}_U(k, \alpha(k)) = diag\left[\hat{F}_{1,u_1}(k, \alpha_1(k))\ \hat{F}_{2,u_2}(k, \alpha_2(k)) \cdots \hat{F}_{N,u_N}(k, \alpha_N(k))\right].$$

From (28) and (37), $G(0)$ and $G(1)$ are to be estimated by

$$\hat{G}(k) = \hat{F}_Y(k, \hat{\alpha}(k)) + \hat{F}'_Y(k, \hat{\alpha}(k)) + \hat{F}_U(k, \hat{\alpha}(k))B(k), \quad k = 0, 1, \tag{41}$$

where

$$\hat{F}_Y(k, \hat{\alpha}(k)) = diag\left[\hat{F}_{1,y_1}(k, \hat{\alpha}_1(k)) \quad \hat{F}_{2,y_2}(k, \hat{\alpha}_2(k)) , \cdots \hat{F}_{N,y_N}(k, \hat{\alpha}_N(k))\right],$$

$$\hat{F}'_Y(k, \hat{\alpha}(k)) = \begin{bmatrix} \hat{F}_{1,Y}(k, \hat{\alpha}_1(k)) \\ \hat{F}_{2,Y}(k, \hat{\alpha}_2(k)) \\ \vdots \\ \hat{F}_{N,Y}(k, \hat{\alpha}_N(k)) \end{bmatrix},$$

$$\hat{F}_U(k, \hat{\alpha}(k)) = diag\left[\hat{F}_{1,u_1}(k, \hat{\alpha}_1(k)) \quad \hat{F}_{2,u_2}(k, \hat{\alpha}_2(k)) \cdots \quad \hat{F}_{N,u_N}(k, \hat{\alpha}_N(k))\right]$$

and

$$B(k) = \Lambda \mathcal{A} \otimes I_{m\times n} + C(k).$$

C. DESIGN AND ADJUST THE FEEDBACK MATRIX GAINS

Based on the above theoretical analyses, it is easy to see that $\hat{F}_Y(k, \hat{\alpha}(k))$, $\hat{F}'_Y(k, \hat{\alpha}(k))$, $\hat{F}_U(k, \hat{\alpha}(k))$ and $\hat{G}(k)$ are all obtained after the time k. However, $C(k)$ need be designed before the time k. This contradiction is solved by predicting $\hat{F}_Y(k, \hat{\alpha}(k))$, $\hat{F}'_Y(k, \hat{\alpha}(k))$, $\hat{F}_U(k, \hat{\alpha}(k))$ and presetting the desired value of $G(k)$ before the time k. We preset the desired value of $G(k)$ in the form of the following equation

$$\check{G}(k) = \frac{2\hat{G}(k-1) - \hat{G}(k-2)}{(1 + \beta + \|E(k-1)\|)\left\|2\hat{G}(k-1) - \hat{G}(k-2)\right\|}, \quad k \ge 2, \tag{42}$$

where $0 < \beta < +\infty$ can be adjusted to improve the convergence rate. From (41), it is clear to get $\hat{G}(0)$ and $\hat{G}(1)$. The real value of $Y(1)$ is measured and recorded. And the reference signal $Y^*(1)$ is given in advance. Thus, $\check{G}(2)$ is preset. Obviously, the presetting desired value $\check{G}(k)$ satisfies the expectation $\left\|\check{G}(k)\right\| < 1$.

The values $\hat{U}^*(k)$ and $\hat{Y}(k)$ are calculated by (32) and (34), respectively. Let $B(k-1)E(k-1) + [B(k-1)E(k-1) - B(k-2)E(k-2)]$ be the prediction of $B(k)[Y(k) - Y^*(k)]$ or $B(k)E(k)$, in which, the real values of $Y(k-2)$, $Y(k-1)$ are measured and recorded at the time $k-2, k-1$, respectively; The reference signals $Y^*(k-2)$ and $Y^*(k-1)$ are given in advance. From (17) and Assumption **A3**, the prediction value of $U(k)$ can be written as

$$\hat{U}(k) = \hat{U}^*(k) + 2B(k-1)E(k-1) - B(k-2)E(k-2). \tag{43}$$

Then the predictions of $\hat{F}_Y(k, \hat{\alpha}(k))$, $\hat{F}'_Y(k, \hat{\alpha}(k))$, $\hat{F}_U(k, \hat{\alpha}(k))$ can be expressed as

$$\begin{cases} \check{F}_{i,y_i}(k) = [\frac{\partial \hat{f}_{i1}^T}{\partial y_i}\check{z}_{i1}(k), \cdots, \frac{\partial \hat{f}_{in}^T}{\partial y_i}\check{z}_{in}(k)]^T \\ \frac{\partial \hat{f}_{ij}}{\partial y_i}(\check{z}_{ij}(k)) = [\frac{\partial \hat{f}_{ij}(\check{z}_{ij}(k))}{\partial y_{i1}}, \cdots, \frac{\partial \hat{f}_{ij}(\check{z}_{ij}(k))}{\partial y_{in}}] \\ \check{F}_{i,Y}(k) = \frac{\partial \hat{f}_i^T}{\partial Y}\check{z}_i(k) \\ \check{F}_{i,\phi_i}(k) = [\frac{\partial \hat{f}_{i1}^T}{\partial \phi_i}\check{z}_{i1}(k), \cdots, \frac{\partial \hat{f}_{in}^T}{\partial \phi_i}\check{z}_{in}(k)]^T \\ \frac{\partial \hat{f}_{ij}}{\partial \phi_i}(\check{z}_{ij}(k)) = [\frac{\partial \hat{f}_{ij}(\check{z}_{ij}(k))}{\partial \phi_{i1}}, \cdots, \frac{\partial \hat{f}_{ij}(\check{z}_{ij}(k))}{\partial \phi_{im_i}}] \\ \check{F}_{i,u_i}(k) = [\frac{\partial \hat{f}_{i1}^T}{\partial u_i}\check{z}_{i1}(k), \cdots, \frac{\partial \hat{f}_{in}^T}{\partial u_i}\check{z}_{in}(k)]^T \\ \frac{\partial \hat{f}_{ij}}{\partial u_i}(\check{z}_{ij}(k)) = [\frac{\partial \hat{f}_{ij}(\check{z}_{ij}(k))}{\partial u_{i1}}, \cdots, \frac{\partial \hat{f}_{ij}(\check{z}_{ij}(k))}{\partial u_{im}}] \\ \check{z}_{ij}(k) \triangleq \hat{x}_i^*(k) + \frac{1}{2}[x_i(k) - \hat{x}_i^*(k)] \\ x_i(k) = [y_i^T(k), \phi_i^T(k), u_i^T(k)]^T \\ \hat{x}_i^*(k) = [y_i^{*T}(k), \phi_i^{*T}(k), \hat{u}_i^{*T}(k)]^T \\ 1 \le j \le n, k \ge 0, \end{cases} \tag{44}$$

where

$$\hat{f}_i = \hat{f}_i(y_i(k), \phi_i(k), u_i(k)).$$

Under these predictions and the following assumption, the feedback matrix gain $B(k)$ for $k \geq 2$.

Assumption A4: $\forall k \geq 2, Rank[\check{F}_{i,u_i}(k)] = m$, where $\check{F}_{i,u_i}(k)$ is defined in (44).

This assumption is reasonable due to the learning capability of NNs for nonlinear time-invariant systems. $\check{F}_{i,y_i}(k)$, $\check{F}_{i,u_i}(k)$ should also satisfy Assumptions **A1** and **A2**, respectively. And $[\check{F}^T_{i,u_i}(k)\check{F}_{i,u_i}(k)]^{-1}$ exists. Then we can design the gain $B(k)$ under (44) and Assumptions **A3** and **A4**. Let

$$\check{G}(k) = \check{F}_Y(k) + \check{F}'_Y(k) + \check{F}_U(k)B(k), k \geq 2, \quad (45)$$

where

$$\check{F}'_Y(k) = \begin{bmatrix} \check{F}_{1,Y}(k) \\ \check{F}_{2,Y}(k) \\ \vdots \\ \check{F}_{N,Y}(k) \end{bmatrix}. \quad (46)$$

From (47), one has

$$B(k)=[\check{F}^T_U(k)\check{F}_U(k)]^{-1}\check{F}^T_U(k)[\check{G}(k)-\check{F}_Y(k)-\check{F}'_Y(k)]. \quad (47)$$

The real control input at the time is designed as

$$U(k) = \hat{U}^*(k) + B(k)[\hat{Y}(k) - Y^*(k)], \quad k \geq 2, \quad (48)$$

where the values $\hat{U}^*(k)$ and $\hat{Y}(k)$ are calculated by (32) and (34), respectively. Using $B(k-1)$ in place of $B(k)$, the real value of $Y(k)$ can be measured and recorded. By (37), $\hat{F}_{i,y_i}(k, \alpha_i(k))$, $\hat{F}_{i,\phi_i}(k, \alpha_i(k))$ and $\hat{F}_{i,u_i}(k, \alpha_i(k))$ are obtained under the real values of $U(k)$ and $Y(K)$. Thus, we can obtain the estimate $\hat{\alpha}(k) = [\hat{\alpha}_1(k), \hat{\alpha}_2(k), \cdots, \hat{\alpha}_N(k)]$ from

$$\begin{cases} J(\hat{\alpha}(k)) = \min \|Y(k+1) - Y^*(k+1) - \hat{G}(k)E(k)\| \\ 0 < \alpha_i(k) < 1, \quad 1 \leq i \leq N, \end{cases} \quad (49)$$

where the prediction $\hat{Y}(k+1)$ is obtained from (34). The same to (41), we can get that

$$\hat{G}(k) = \hat{F}_Y(k, \hat{\alpha}(k)) + \hat{F}'_Y(k, \hat{\alpha}(k)) + \hat{F}_U(k, \hat{\alpha}(k))B(k), \quad k \geq 2, \quad (50)$$

where

$$\hat{F}_Y(k, \hat{\alpha}(k)) = diag\left[\hat{F}_{1,y_1}(k, \hat{\alpha}_1(k)) \quad \hat{F}_{2,y_2}(k, \hat{\alpha}_2(k)) \quad \cdots \quad \hat{F}_{N,y_N}(k, \hat{\alpha}_N(k))\right],$$

$$\hat{F}'_Y(k, \hat{\alpha}(k)) = \begin{bmatrix} \hat{F}_{1,Y}(k, \hat{\alpha}_1(k)) \\ \hat{F}_{2,Y}(k, \hat{\alpha}_2(k)) \\ \vdots \\ \hat{F}_{N,Y}(k, \hat{\alpha}_N(k)) \end{bmatrix}$$

and

$$\hat{F}_U(k, \hat{\alpha}(k)) = diag\left[\hat{F}_{1,u_1}(k, \hat{\alpha}_1(k)) \quad \hat{F}_{2,u_2}(k, \hat{\alpha}_2(k)) \quad \cdots \quad \hat{F}_{N,u_N}(k, \hat{\alpha}_N(k))\right].$$

Store the values $B(k)$ and $\hat{G}(k)$ for presetting $\check{G}(k+1)$ and designing and $B(k+1)$ during the next time interval $(k, (k+1))$.

D. CHECK THE TRAJECTORY TRACKING ERROR

For any the time $k \geq 2$, we need check each $\|e_i(k)\|$. If $\|e_i(k)\| < \epsilon_i$, where ϵ_i is the maximum tolerable tracking error for agent i, then, from this moment, feedback gain matrix is to be continued to use $B(k)$; otherwise, the feedback gain matrix is obtained from Step 3, until $\|e_i(k)\| < \epsilon_i$.

IV. ALGORITHM CONVERGENCE ANALYSIS

In the section, the convergence condition of the indirect data-driven output trajectory tracking control (IDDOTTC) algorithm is studied.

Theorem 2: For the multi-agent system, under Assumptions **A1-A4**, if for any $k \geq 2$

$$\sup_{D_y \times D_u} \left[\left\|G(k) - \hat{G}(k)\right\|\right] = \delta_c \leq \infty,$$

$$\sup_{D_y \times D_u} \left[\left\|\hat{G}(k) - \check{G}(k)\right\|\right] = \delta_p \leq \infty,$$

$$\max\left\{\delta_c, \delta_p\right\} < \frac{\beta}{2(1+\beta)}, \quad (51)$$

where $0 < \beta < \infty$ is emerged in (42). Then the system outputs $Y(k)$ converge to the reference trajectory $Y^*(k)$. In other words, for each subsystem, the output $y_i(k)$ tracks the reference trajectory $y^*_i(k)$.

Proof 2: For the multi-agent system, when (51) is satisfied, thus for any $k \geq 2$, one has

$$\begin{aligned} \|G(k)\| &= \left\|G(k) - \hat{G}(k) + \hat{G}(k) - \check{G} + \check{G}\right\| \\ &\leq \left\|G(k) - \hat{G}(k)\right\| + \left\|\hat{G}(k) - \check{G}\right\| + \left\|\check{G}\right\| \\ &\leq \delta_c + \delta_p + \frac{1}{1 + \beta + \|E(k-1)\|} \\ &< \frac{\beta}{2(1+\beta)} + \frac{\beta}{2(1+\beta)} + \frac{1}{1+\beta} \\ &= 1. \end{aligned} \quad (52)$$

Thus, $\forall k \geq 2$, $\|E(k+1)\| \leq \|G(k)\| \|E(k)\| < \|E(k)\|$. It is easy to know that the system output $Y(k)$ asymptotically converges to the reference trajectory $Y^*(k)$ after the time $k = 2$.

Remark 5: In (51), δ_c reflects the nonlinear approximation capability of NN and δ_p reflects the approximation precision. A sufficient condition for the convergence of the indirect data-driven output trajectory tracking control algorithm by Theorem 2, which is to guarantee that $\|G(k)\| < 1$.

This theorem also shows that the NN has to perform well in establishing approximate model due to the strong learning capability of NNs. It is noted that with the NN alone, the modeling errors are inevitable and hence it cannot make $y_i(k) \rightarrow y_i^*(k)$ as $k \rightarrow \infty$. The IDDOTTC algorithm not only uses the NN to establish the approximate model, but also compensates the modeling errors online. output trajectory tracking In this sense, this algorithm integrates the merits of both the data-driven decoupling control (DDDC) algorithm and indirect data-driven control (IDDC) algorithm.

V. CONCLUSION

In this study, the trajectory tracking control of a class of nonlinear discrete-time multi-agent systems with unknown dynamics. Based on Lagrange's mean value theorem, for each agent, the given reference trajectory, there exists a unique control input such that the dynamics is satisfied. Meanwhile, based on the its neighbors' and its own history I/O data and neural network, the approximate models are established to estimate the unknown functions, the indirect data-driven method is adopted to predict and estimate some relative model structure matrices, and then, design and adjust the feedback gains such that outputs of each agent tracks the given reference trajectory. In the future work, we will be to investigate the trajectory tracking control of a class of nonlinear discrete-time multi-agent systems with stochastic noise.

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