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# Distributed Fusion Filter for Nonlinear Multi-Sensor Systems With Correlated Noises

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**ABSTRACT** This paper is concerned with distributed fusion (DF) estimation problem for nonlinear multi-sensor systems with correlated noises. Based on a recursive linear minimum variance estimation (RLMVE) framework, a novel filter is developed. It is proved that the RLMVE-based filter and the existing de-correlated filter have the functional equivalence. Then, for multi-sensor cases, cross-covariance matrices between any two local filters are derived. Based on the RLMVE-based filter and cross-covariance matrices, a DF filter weighted by matrices is proposed in the sense of linear minimum variance. Finally, based on the existing de-correlated filter, the algorithm of cross-covariance for de-correlated systems and the DF algorithm weighted by matrices, a de-correlated DF filtering algorithm is proposed. An example verifies the effectiveness of the proposed RLMVE-based DF filter.

**INDEX TERMS** Multi-sensor, nonlinear system, distributed fusion filter, cross-covariance matrix, linear minimum variance estimation.

## I. INTRODUCTION

INFORMATION fusion is one of the important technologies in intelligent detection and estimation fields, and has been widely applied in target tracking, navigation, monitoring, fault-tolerant control, Big Data, and so on [1]–[6]. The fusion estimation is one of the significant research topics in information fusion technologies. In general, fusion estimation processing can be performed in a centralized or distributed manner [2], [5]–[7]. In a centralized fusion (CF) framework [8], [9], measurements of all sensors are sent to fusion center for estimation and output. The advantage of CF is that its estimation accuracy is globally optimal when all sensors are faultless. However, in large sensor networks, the tremendous computing workload in the fusion center will embarrass the real-time performance of systems. Even in some situations, CF is often impossible to be realized, due to limitations in communication bandwidth, sensor power, and available communication hardware.

In a distributed fusion (DF) framework, local estimates are sent to a fusion center for processing according to a

certain performance criterion. Estimation accuracy of DF is usually locally optimal and globally suboptimal under different performance criteria. So, its estimation accuracy is usually lower than CF's. But DF has better robustness and flexibility because of the parallel computing architecture. DF is usually preferable in wireless sensor networks (WSN), as it allows each sensor node to process locally measurements and it is much more efficient in communication compared to CF [2], [10]. Currently, the federated Kalman filters, fusion algorithms weighted by matrices, diagonal matrices, and scalars [5], [6], [11], covariance intersection (CI) fusion algorithm [12]–[18], etc. have been used for the DF framework.

In general, almost all systems are nonlinear. Therefore, information fusion estimation for nonlinear systems has attracted more and more attention, and has always been one of the hot issues in information fusion fields [19]–[25]. Due to the complexity and uncertainty of nonlinear systems, the information fusion estimation for nonlinear systems has not been well solved. Linearizing nonlinear parts and using linear methods are often used for fusion estimation of nonlinear systems [9]. But the Taylor series expansion used for linearization often makes

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deviations or even divergence when system states change drastically.

In practical applications, due to the discretization of continuous-time systems and the influence of the internal and external environments, the system noise and measurement noise are often correlated. Therefore, estimation problems with correlated noises have received much attention. The estimators proposed in [26]–[29] and [30]–[37] could be applied respectively to linear and nonlinear systems with correlated noises. Based on the RLMVE framework, there are two fundamental methods for solving the estimation problem with correlated noises. One is the de-correlated method, which can convert correlated noises into uncorrelated noises by substituting the measurement equation into the state equation [28], [34], [35]. The other method is a Gaussian approximation recursive filter framework [32], [33], [36]. When the process noise and the measurement noise are correlated, it is very difficult to directly solve the state one-step predictor based on the RLMVE framework. The literature [36] avoids solving one-step predictor by using a two-step predictor. The literature [32] proposes a method for solving the state one-step predictor by noise estimator. However, these filtering algorithms are only for single sensor systems. So, it is necessary to study the information fusion filtering problem for nonlinear multi-sensor systems with correlated noises.

In this paper, a RLMVE-based filter and two DF filters are proposed for nonlinear multi-sensor systems with correlated noises. The main original results of this article are listed below:

1) A novel filtering algorithm for nonlinear systems with correlated noises is proposed based on the RLMVE framework. It is proved that the proposed RLMVE-based filter and the existing de-correlated filter have the functional equivalence.

2) Based on the RLMVE framework, an algorithm for calculating cross-covariance matrices is proposed. Based on the cross-covariance matrices, the RLMVE-based filter and the DF algorithm weighted by matrices [6], a RLMVE-based DF filter is proposed. It can effectively fuse local estimates, and has high accuracy and good flexibility.

3) Based on the existing de-correlated filter, the algorithm of cross-covariance for de-correlated systems and the DF algorithm weighted by matrices, a de-correlated DF filtering algorithm is proposed.

The rest of the paper is organized as follows. A filter based on RLMVE framework is proposed for nonlinear systems with correlated noises in Section II. A RLMVE-based DF filter is proposed in Section III. A de-correlated DF filter is proposed in Section IV. The simulation analysis is given in Section V. The conclusions are summarized in Section VI.

*Notations:*  $\mathfrak{R}^n$  denotes the  $n$ -dimensional Euclidean space.  $\mathbf{I}_{n \times n}$  is the  $n$ -dimensional identity matrix.  $\mathbb{E}$  denotes the mathematical expectation. Superscripts T and  $-1$  denote the transpose and inverse, respectively.  $\delta_{tk}$  is the Kronecker delta function, i.e.,  $\delta_{tt} = 1$  and  $\delta_{tk} = 0 (t \neq k)$ .

$\hat{\theta}_{k|k-\gamma}^{(j)} = \mathbb{E} \left\{ \theta_k | z_{0 \sim k-\gamma}^{(j)} \right\}$ , ( $\gamma = 0, \dots, k$ ;  $\theta = \mathbf{x}, \mathbf{z}, \mathbf{w}$ ) is the estimator of  $\theta$  based on the measurements  $z_{0 \sim k-\gamma}^{(j)} = \{z_0^{(j)}, \dots, z_{k-\gamma}^{(j)}\}$ .  $\tilde{\theta}_{k|k-\gamma}^{(j)} = \theta_k - \hat{\theta}_{k|k-\gamma}^{(j)}$  is the estimation error.  $(\mathbf{g}(\xi) | z_{0 \sim k-\gamma}^{(j)})$  is a function with independent variable  $\xi$  which is a random variable conditioned by  $z_{0 \sim k-\gamma}^{(j)}$ .  $\mathbf{P}_{\theta\phi, k|k-\gamma}^{(ij)} = \mathbb{E} \left\{ \left( \tilde{\theta}_{k|k-\gamma}^{(i)} | z_{0 \sim k-\gamma}^{(i)} \right) \left( \tilde{\phi}_{k|k-\gamma}^{(j)} | z_{0 \sim k-\gamma}^{(j)} \right)^T \right\}$ , ( $\phi = \mathbf{x}, \mathbf{z}, \mathbf{w}$ ) is the estimation error cross-covariance matrix and  $\mathbf{P}_{xx, k|k-\gamma}^{(ij)}$  will be abbreviated to  $\mathbf{P}_{k|k-\gamma}^{(ij)}$ .

## II. RLMVE-BASED FILTER

Consider a nonlinear multi-sensor dynamic system with correlated noises:

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k) + \mathbf{w}_k \quad (1)$$

$$\mathbf{z}_k^{(j)} = \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{v}_k^{(j)}, \quad j = 1, 2, \dots, L \quad (2)$$

where  $\mathbf{f}_k(\cdot) \in \mathfrak{R}^n$  and  $\mathbf{h}_k^{(j)}(\cdot) \in \mathfrak{R}^{m_j}$  are the known nonlinear functions,  $\mathbf{x}_k \in \mathfrak{R}^n$  is the state vector at time  $k$ ,  $\mathbf{z}_k^{(j)} \in \mathfrak{R}^{m_j}$  is the measurement vector of the  $j$ th sensor at time  $k$ ,  $\mathbf{w}_k \in \mathfrak{R}^n$  is the process noise, and  $\mathbf{v}_k^{(j)} \in \mathfrak{R}^{m_j}$  is the measurement noise of the  $j$ th sensor.  $\mathbf{w}_k$  and  $\mathbf{v}_k^{(j)}$  are correlated Gaussian noises with zero-mean and satisfy:

$$\mathbb{E} \left\{ \begin{bmatrix} \mathbf{w}_t \\ \mathbf{v}_t^{(i)} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k^T & (\mathbf{v}_k^{(j)})^T \end{bmatrix} \right\} = \begin{bmatrix} \mathbf{Q}_w & \mathbf{S}^{(j)} \\ (\mathbf{S}^{(i)})^T & \mathbf{R}^{(ij)} \end{bmatrix} \delta_{tk} \quad (3)$$

For the  $j$ th subsystem, based on measurements  $z_{0 \sim k}^{(j)}$ , the RLMVE framework can be presented by the following Lemma.

*Lemma 1* [38]: For system in Equations (1) and (2), based on measurements  $z_{0 \sim k}^{(j)}$ , the estimator  $\hat{\mathbf{x}}_{k|k}^{(j)}$  of the state  $\mathbf{x}_k$  in the sense of linear minimum variance has the RLMVE framework:

$$\hat{\mathbf{x}}_{k|k}^{(j)} = \hat{\mathbf{x}}_{k|k-1}^{(j)} + \mathbf{K}_k^{(j)} \mathbf{z}_{k|k-1}^{(j)}, \quad j = 1, 2, \dots, L \quad (4)$$

where the filtering gain is computed by:

$$\mathbf{K}_k^{(j)} = \mathbf{P}_{xz, k|k-1}^{(j)} \left( \mathbf{P}_{zz, k|k-1}^{(j)} \right)^{-1} \quad (5)$$

and the filtering error variance is computed by

$$\mathbf{P}_{k|k}^{(j)} = \mathbf{P}_{k|k-1}^{(j)} - \mathbf{K}_k^{(j)} \mathbf{P}_{zz, k|k-1}^{(j)} \left( \mathbf{K}_k^{(j)} \right)^T \quad (6)$$

*Remark 1:* The estimator  $\hat{\mathbf{x}}_{k|k}^{(j)}$  of the state  $\mathbf{x}_k$  is a function of  $z_{0 \sim k}^{(j)}$ , with the performance of minimizing  $\mathbf{J} = \mathbb{E} \left[ (\tilde{\mathbf{x}}_{k|k}^{(j)})^T \tilde{\mathbf{x}}_{k|k}^{(j)} \right]$ . When the probability distributions of the state  $\mathbf{x}_k$  and the measurements  $\mathbf{z}_k^{(j)}$  are Gaussian, the linear minimum variance estimate is optimal estimation. For linear systems with Gaussian noises, the well-known Kalman filter just has the RLMVE framework [38]. For nonlinear systems with Gaussian noises, the Unscented Kalman Filter (UKF) [39], [40] and the Cubature Kalman

Filter (CKF) [41]–[43] also have such a recursive form. In this paper, a filtering framework is proposed based on RLMVE framework which can deal with the filtering problem for nonlinear systems with correlated Gaussian noises.

Since the process noise  $\mathbf{w}_k$  is correlated to the measurement noise  $\mathbf{v}_k^{(j)}$ ,  $\mathbf{w}_k$  has become one of system states, and estimators will be biased while the correlation is ignored. Many references have treated estimation problems with correlated noises via an augmentation method, which increases computational cost. For system in Equations (1) and (2), as a special state,  $\mathbf{w}_k$  is zero-mean and additive. Based on the two characteristics, a novel RLMVE framework is proposed as the following Theorem.

*Theorem 1:* For system in Equations (1) and (2), the local filter  $\hat{\mathbf{x}}_{k|k}^{(j)}$  has the RLMVE framework as Equations (4)–(6), where

$$\mathbf{P}_{xz,k|k-1}^{(j)} = \mathbb{E} \left\{ \mathbf{x}_k \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \right)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} - \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \quad (7)$$

$$\mathbf{P}_{zz,k|k-1}^{(j)} = \mathbb{E} \left\{ \mathbf{h}_k^{(j)}(\mathbf{x}_k) \left( \cdot \right)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} - \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right) \left( \cdot \right)^T + \mathbf{R}^{(j)} \quad (8)$$

where  $(\cdot)$  means the same as the previous one.

The prediction error covariance matrix is computed by:

$$\mathbf{P}_{k|k-1}^{(j)} = \mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \left( \cdot \right)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} - \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \cdot \right)^T + \mathbf{P}_{ww,k-1|k-1}^{(j)} \quad (9)$$

The state predictor is computed by:

$$\hat{\mathbf{x}}_{k|k-1}^{(j)} = \tilde{\mathbf{x}}_{k|k-1}^{(j)} + \hat{\mathbf{w}}_{k-1|k-1}^{(j)} \quad (10)$$

where

$$\tilde{\mathbf{x}}_{k|k-1}^{(j)} = \mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \quad (11)$$

The local process noise estimator  $\hat{\mathbf{w}}_{k-1|k-1}^{(j)}$  has the RLMVE framework:

$$\hat{\mathbf{w}}_{k-1|k-1}^{(j)} = \mathbf{K}_{w,k-1}^{(j)} \tilde{\mathbf{z}}_{k-1|k-2}^{(j)} \quad (12)$$

$$\mathbf{K}_{w,k-1}^{(j)} = \mathbf{S}_{k-1}^{(j)} \left( \mathbf{P}_{zz,k-1|k-2}^{(j)} \right)^{-1} \quad (13)$$

$$\mathbf{P}_{ww,k-1|k-1}^{(j)} = \mathbf{Q}_w - \mathbf{K}_{w,k-1}^{(j)} \mathbf{P}_{zz,k-1|k-2}^{(j)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \quad (14)$$

*Proof:* See Appendix A.

Based on the RLMVE framework, a filtering framework for nonlinear systems with correlated noises is proposed by Theorem 1. It is just a framework and the conditional mean needs to be calculated in Equations (7), (8), (9) and (11). The conditional mean can be written as the integrals:

$$\mathbb{E} \left\{ \mathbf{x}_k \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \right)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} = \int_{\mathbb{R}^n} \mathbf{x}_k \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \right)^T \mathcal{N} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)}, \mathbf{P}_{k|k-1}^{(j)} \right) d\mathbf{x}_k \quad (15)$$

$$\mathbb{E} \left\{ \mathbf{h}_k^{(j)}(\mathbf{x}_k) \left( \cdot \right)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} = \int_{\mathbb{R}^n} \mathbf{h}_k^{(j)}(\mathbf{x}_k) \left( \cdot \right)^T \mathcal{N} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)}, \mathbf{P}_{k|k-1}^{(j)} \right) d\mathbf{x}_k \quad (16)$$

$$\mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \left( \cdot \right)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} = \int_{\mathbb{R}^n} \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \left( \cdot \right)^T \mathcal{N} \left( \hat{\mathbf{x}}_{k-1|k-1}^{(j)}, \mathbf{P}_{k-1|k-1}^{(j)} \right) d\mathbf{x}_{k-1} \quad (17)$$

$$\mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} = \int_{\mathbb{R}^n} \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \mathcal{N} \left( \hat{\mathbf{x}}_{k-1|k-1}^{(j)}, \mathbf{P}_{k-1|k-1}^{(j)} \right) d\mathbf{x}_{k-1} \quad (18)$$

For linear systems, integrals can be achieved by recursion. But for nonlinear systems, integrals are not easy to be calculated. The Monte Carlo method [44] is used to deal with the integrals for non-Gaussian systems. The spherical-radial rule [41]–[43], UT transformation [39], [40], etc. are often used for Gaussian systems. Here, we use spherical-radial cubature rule to realize Theorem 1.

The proposed RLMVE-based filter can deal with estimation problems for systems with single sensor. Moreover, the filter is also available for multi-sensor systems by CF framework.

### III. RLMVE-BASED DF FILTER

The proposed RLMVE-based filter is available for systems with single sensor or multi-sensor CF framework. However, CF for multi-sensor systems has some disadvantage as mentioned above. Next, a RLMVE-based DF framework weighted by matrices in the sense of linear unbiased minimum variance (LUMV) will be proposed.

*Lemma 2 [6]:* Assume  $\hat{\mathbf{x}}^{(j)}$  ( $j = 1, 2, \dots, L$ ) is the local unbiased estimator based on the measurements of the  $j$ th sensor, and its error covariance and cross-covariance matrices are  $\mathbf{P}^{(i)}$  and  $\mathbf{P}^{(ij)}$  ( $i \neq j$ ). Then, the optimal DF estimator  $\hat{\mathbf{x}}^{(M)}$  weighted by matrices in the sense of LUMV is:

$$\hat{\mathbf{x}}^{(M)} = \sum_{j=1}^L \mathbf{A}^{(j)} \hat{\mathbf{x}}^{(j)} \quad (39)$$

where the weighting matrices  $\mathbf{A}^{(j)}$ ,  $j = 1, 2, \dots, L$  are given by:

$$\left( \mathbf{A}^{(1)} \quad \mathbf{A}^{(2)} \quad \dots \quad \mathbf{A}^{(L)} \right) \triangleq \left( \mathbf{e}^T \mathbf{P}^{-1} \mathbf{e} \right)^{-1} \mathbf{e}^T \mathbf{P}^{-1} \quad (40)$$

where

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}^{(1)} & \dots & \mathbf{P}^{(1L)} \\ \vdots & \ddots & \vdots \\ \mathbf{P}^{(L1)} & \dots & \mathbf{P}^{(L)} \end{bmatrix}_{nL \times nL} \quad (41)$$

$\mathbf{e} = [\mathbf{I}_n \dots \mathbf{I}_n]_{nL \times n}^T$ ,  $\mathbf{P}^{(ij)} = \mathbb{E} \left\{ \left( \mathbf{x} - \hat{\mathbf{x}}^{(i)} \right) \left( \mathbf{x} - \hat{\mathbf{x}}^{(j)} \right)^T \right\}$  and  $\mathbf{P}^{(ij)} = \mathbf{P}^{(ji)}$  when  $i = j$ . The covariance of the DF filter weighted by matrices is given as:

$$\mathbf{P}^{(M)} = \left( \mathbf{e}^T \mathbf{P}^{-1} \mathbf{e} \right)^{-1} \quad (42)$$

and  $\mathbf{P}^{(M)} \leq \mathbf{P}^{(j)}$ ,  $j = 1, 2, \dots, L$ .

**Algorithm 1** The RLMVE-Based Filtering Algorithm for Nonlinear Systems With Correlated Noises

Initialization ( $k = 0$ ):  $\hat{\mathbf{x}}_{0|-1}^{(j)} = E\{\mathbf{x}_0\}$ ,  $\hat{\mathbf{z}}_{-1|-2}^{(j)} = 0$ ,  $\mathbf{P}_{-1|-1}^{(j)} = \mathbf{I}$ ,  $\mathbf{P}_{zz,-1|-2}^{(j)} = \mathbf{I}$ ;

Generating filtering particles:

$$\mathbf{P}_{k-1|k-1}^{(j)} = \mathbf{S}_{k-1|k-1}^{(j)} (\cdot)^T \quad (19)$$

$$\mathbf{x}_{\mu, k-1|k-1}^{(j)} = \mathbf{S}_{k-1|k-1}^{(j)} \boldsymbol{\xi}_{\mu}^{(j)} + \hat{\mathbf{x}}_{k-1|k-1}^{(j)}, \quad \mu = 1, \dots, 2n \quad (20)$$

$$\boldsymbol{\xi}_{\mu}^{(j)} = \sqrt{n} [1]_{\mu}^{(j)} \quad (21)$$

where  $[1]_{\mu}^{(j)}$  is defined by:

$$[1]_{\mu}^{(j)} = \begin{bmatrix} \mathbf{I}_{n \times n} & -\mathbf{I}_{n \times n} \end{bmatrix} \quad (22)$$

$$\mathbf{X}_{\mu, k|k-1}^{(j)} = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \Big|_{\mathbf{x}_{k-1} = \mathbf{x}_{\mu, k-1|k-1}^{(j)}} \quad (23)$$

White noise filter  $\hat{\mathbf{w}}_{k-1|k-1}^{(j)}$ :

$$\mathbf{K}_{w,k-1}^{(j)} = \mathbf{S}_{k-1}^{(j)} \left( \mathbf{P}_{zz,k-1|k-2}^{(j)} \right)^{-1} \quad (24)$$

$$\hat{\mathbf{w}}_{k-1|k-1}^{(j)} = \mathbf{K}_{w,k-1}^{(j)} \left( \mathbf{z}_{k-1}^{(j)} - \hat{\mathbf{z}}_{k-1|k-2}^{(j)} \right) \quad (25)$$

$$\mathbf{P}_{ww,k-1|k-1}^{(j)} = \mathbf{Q}_w - \mathbf{K}_{w,k-1}^{(j)} \mathbf{P}_{zz,k-1|k-2}^{(j)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \quad (26)$$

State predictor  $\hat{\mathbf{x}}_{k|k-1}^{(j)}$ :

$$\hat{\mathbf{x}}_{k|k-1}^{(j)} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{X}_{\mu, k|k-1}^{(j)} \quad (27)$$

$$\hat{\mathbf{x}}_{k|k-1}^{(j)} = \hat{\mathbf{x}}_{k|k-1}^{(j)} + \hat{\mathbf{w}}_{k-1|k-1}^{(j)} \quad (28)$$

$$\mathbf{P}_{k|k-1}^{(j)} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{X}_{\mu, k|k-1}^{(j)} (\cdot)^T - \hat{\mathbf{x}}_{k|k-1}^{(j)} (\cdot)^T + \mathbf{P}_{ww,k-1|k-1}^{(j)} \quad (29)$$

Generating prediction particles:

$$\mathbf{P}_{k|k-1}^{(j)} = \mathbf{S}_{k|k-1}^{(j)} (\cdot)^T \quad (30)$$

$$\mathbf{x}_{\mu, k|k-1}^{(j)} = \mathbf{S}_{k|k-1}^{(j)} \boldsymbol{\xi}_{\mu}^{(j)} + \hat{\mathbf{x}}_{k|k-1}^{(j)}, \quad \mu = 1, \dots, 2n \quad (31)$$

$$\mathbf{Z}_{\mu, k|k-1}^{(j)} = \mathbf{h}_k^{(j)}(\mathbf{x}_k) \Big|_{\mathbf{x}_k = \mathbf{x}_{\mu, k|k-1}^{(j)}} \quad (32)$$

Measurement predictor  $\hat{\mathbf{z}}_{k|k-1}^{(j)}$ :

$$\hat{\mathbf{z}}_{k|k-1}^{(j)} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{Z}_{\mu, k|k-1}^{(j)} \quad (33)$$

State local filter  $\hat{\mathbf{x}}_{k|k}^{(j)}$ :

$$\mathbf{P}_{xz,k|k-1}^{(j)} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{x}_{\mu, k|k-1}^{(j)} \left( \mathbf{Z}_{\mu, k|k-1}^{(j)} \right)^T - \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \quad (34)$$

**Algorithm 1 (Continued.)** The RLMVE-Based Filtering Algorithm for Nonlinear Systems With Correlated Noises

$$\mathbf{P}_{zz,k|k-1}^{(j)} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{Z}_{\mu, k|k-1}^{(j)} (\cdot)^T - \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right) (\cdot)^T + \mathbf{R}^{(j)} \quad (35)$$

$$\mathbf{K}_k^{(j)} = \mathbf{P}_{xz,k|k-1}^{(j)} \left( \mathbf{P}_{zz,k|k-1}^{(j)} \right)^{-1} \quad (36)$$

$$\hat{\mathbf{x}}_{k|k}^{(j)} = \hat{\mathbf{x}}_{k|k-1}^{(j)} + \mathbf{K}_k^{(j)} \mathbf{z}_{k|k-1}^{(j)} \quad (37)$$

$$\mathbf{P}_{k|k}^{(j)} = \mathbf{P}_{k|k-1}^{(j)} - \mathbf{K}_k^{(j)} \mathbf{P}_{zz,k|k-1}^{(j)} \left( \mathbf{K}_k^{(j)} \right)^T \quad (38)$$

*Remark 2:* The DF estimator in Lemma 1 only needs local estimates, estimation error covariance matrices  $\mathbf{P}^{(j)}$  and cross-covariance matrices  $\mathbf{P}^{(ij)}$ , whatever the systems are linear or nonlinear.

Usually, most estimators can provide the corresponding estimation error covariance matrices. But for multi-sensor systems, cross-covariance matrices between local estimators are hard to obtain or even impossible, particularly for nonlinear systems [12]–[18], [45], [46]. Unfortunately, cross-covariance matrices are essential in DF estimator weighted by matrices in Lemma 2. If ignoring cross-covariance matrices, it will bring accuracy loss to the estimator.

Next, taking filtering problem as an example, filtering error cross-covariance matrices  $\mathbf{P}_{k|k}^{(ij)}$  between any two local filters for multi-sensor systems will be given.

*Theorem 2:* Based on the RLMVE framework, the filtering error cross-covariance  $\mathbf{P}_{k|k}^{(ij)}$  ( $i \neq j$ ;  $i, j = 1, \dots, L$ ) can be calculated as:

$$\mathbf{P}_{k|k}^{(ij)} = \mathbf{P}_{k|k-1}^{(ij)} - \mathbf{P}_{xz,k|k-1}^{(ij)} \left( \mathbf{K}_k^{(j)} \right)^T - \mathbf{K}_k^{(i)} \mathbf{P}_{zx,k|k-1}^{(ij)} + \mathbf{K}_k^{(i)} \mathbf{P}_{zz,k|k-1}^{(ij)} \left( \mathbf{K}_k^{(j)} \right)^T \quad (43)$$

*Proof:* See Appendix B.

In Equation (43), the filtering gain  $\mathbf{K}_k^{(j)}$  can be obtained via local filters, but the prediction error cross-covariance  $\mathbf{P}_{k|k-1}^{(ij)}$ ,  $\mathbf{P}_{zz,k|k-1}^{(ij)}$ ,  $\mathbf{P}_{xz,k|k-1}^{(ij)}$  and  $\mathbf{P}_{zx,k|k-1}^{(ij)}$  cannot be got. We consider that the state  $\mathbf{x}_k$  is a random variable with the conditional probability density function (PDF)  $P\{\mathbf{x}_k | \mathbf{z}_{0 \sim k}^{(j)}\} = \mathcal{N}(\hat{\mathbf{x}}_{k|k}^{(j)}, \mathbf{P}_{k|k}^{(j)})$ , when  $\mathbf{w}_k$  and  $\mathbf{v}_k^{(j)}$  are Gaussian. Then, the joint conditional PDF of two Gaussian random variables  $(\mathbf{x}_k | \mathbf{z}_{0 \sim k}^{(j)})$  and  $(\mathbf{x}_k | \mathbf{z}_{0 \sim k}^{(i)})$  is also Gaussian and with the conditional PDF  $\mathcal{N} \left( \begin{bmatrix} \hat{\mathbf{x}}_{k|k}^{(i)} \\ \hat{\mathbf{x}}_{k|k}^{(j)} \end{bmatrix}, \begin{bmatrix} \mathbf{P}_{k|k}^{(i)} & \mathbf{P}_{k|k}^{(ij)} \\ \mathbf{P}_{k|k}^{(ij)} & \mathbf{P}_{k|k}^{(j)} \end{bmatrix} \right)$ . Based on them, we will give these prediction error cross-covariance matrices, respectively.

*Theorem 3:* Based on RLMVE framework, the prediction error cross-covariance  $\mathbf{P}_{k|k-1}^{(ij)}$  in (43) can be calculated as:

$$\begin{aligned} \mathbf{P}_{k|k-1}^{(ij)} &= \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(i)} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} \right)^T + \mathbf{P}_{ww,k-1|k-1}^{(ij)} \end{aligned} \quad (44)$$

where  $\mathbf{P}_{ww,k-1|k-1}^{(ij)}$  is computed by:

$$\begin{aligned} \mathbf{P}_{ww,k-1|k-1}^{(ij)} &= \mathbf{Q}_w - \mathbf{S}_{k-1}^{(j)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \\ &\quad - \mathbf{K}_{w,k-1}^{(i)} \left( \mathbf{S}_{k-1}^{(i)} \right)^T + \mathbf{K}_{w,k-1}^{(i)} \mathbf{P}_{zz,k-1|k-2}^{(ij)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \end{aligned} \quad (45)$$

*Proof:* See Appendix C.

*Theorem 4:* Based on RLMVE framework, the measurement prediction error covariance  $\mathbf{P}_{zz,k|k-1}^{(ij)}$  in (43) can be calculated as:

$$\begin{aligned} \mathbf{P}_{zz,k|k-1}^{(ij)} &= \mathbb{E} \left\{ \left( \mathbf{h}_k^{(i)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &\quad - \left( \hat{\mathbf{z}}_{k|k-1}^{(i)} \right) \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T + \mathbf{R}^{(ij)} \end{aligned} \quad (46)$$

*Proof:* See Appendix D.

*Theorem 5:* Based on RLMVE framework, the state and measurement prediction error cross-covariance  $\mathbf{P}_{xz,k|k-1}^{(ij)}$  in (43) can be calculated as:

$$\begin{aligned} \mathbf{P}_{xz,k|k-1}^{(ij)} &= \mathbb{E} \left\{ \left( \mathbf{x}_k \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}^{(j)}(\mathbf{x}_k, k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(i)} \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \end{aligned} \quad (47)$$

Similar to  $\mathbf{P}_{xz,k|k-1}^{(ij)}$ ,  $\mathbf{P}_{zx,k|k-1}^{(ij)}$  can be calculated as:

$$\begin{aligned} \mathbf{P}_{zx,k|k-1}^{(ij)} &= \mathbb{E} \left\{ \left( \mathbf{h}^{(i)}(\mathbf{x}_k, k) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{x}_k \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &\quad - \hat{\mathbf{z}}_{k|k-1}^{(i)} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} \right)^T \end{aligned} \quad (48)$$

*Proof:* See Appendix E.

The conditional mean in Equations(44), (46) and (47) can be written as the integrals:

$$\begin{aligned} \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ = \int_{\mathbb{R}^{2n}} \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \\ \times \mathcal{N} \left( \begin{bmatrix} \hat{\mathbf{x}}_{k-1|k-1}^{(i)} \\ \hat{\mathbf{x}}_{k-1|k-1}^{(j)} \end{bmatrix}, \begin{bmatrix} \mathbf{P}_{k-1|k-1}^{(i)} & \mathbf{P}_{k-1|k-1}^{(ij)} \\ \mathbf{P}_{k-1|k-1}^{(ji)} & \mathbf{P}_{k-1|k-1}^{(j)} \end{bmatrix} \right) \\ \times d\mathbf{X}_{k-1|k-1} \end{aligned} \quad (49)$$

$$\text{where } \mathbf{X}_{k-1|k-1} = \begin{bmatrix} \mathbf{x}_{k-1} & \mathbf{z}_{0 \sim k-1}^{(i)} \\ \mathbf{x}_{k-1} & \mathbf{z}_{0 \sim k-1}^{(j)} \end{bmatrix}.$$

$$\begin{aligned} \mathbb{E} \left\{ \left( \mathbf{h}_k^{(i)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ = \int_{\mathbb{R}^{2n}} \left( \mathbf{h}_k^{(i)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \\ \times \mathcal{N} \left( \begin{bmatrix} \hat{\mathbf{x}}_{k|k-1}^{(i)} \\ \hat{\mathbf{x}}_{k|k-1}^{(j)} \end{bmatrix}, \begin{bmatrix} \mathbf{P}_{k|k-1}^{(i)} & \mathbf{P}_{k|k-1}^{(ij)} \\ \mathbf{P}_{k|k-1}^{(ji)} & \mathbf{P}_{k|k-1}^{(j)} \end{bmatrix} \right) d\mathbf{X}_{k|k-1} \end{aligned} \quad (50)$$

$$\text{where } \mathbf{X}_{k|k-1} = \begin{bmatrix} \mathbf{x}_k & \mathbf{z}_{0 \sim k-1}^{(i)} \\ \mathbf{x}_k & \mathbf{z}_{0 \sim k-1}^{(j)} \end{bmatrix}.$$

$$\begin{aligned} \mathbb{E} \left\{ \left( \mathbf{x}_k \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}^{(j)}(\mathbf{x}_k, k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ = \int_{\mathbb{R}^{2n}} \left( \mathbf{x}_k \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}^{(j)}(\mathbf{x}_k, k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \\ \times \mathcal{N} \left( \begin{bmatrix} \hat{\mathbf{x}}_{k|k-1}^{(i)} \\ \hat{\mathbf{x}}_{k|k-1}^{(j)} \end{bmatrix}, \begin{bmatrix} \mathbf{P}_{k|k-1}^{(i)} & \mathbf{P}_{k|k-1}^{(ij)} \\ \mathbf{P}_{k|k-1}^{(ji)} & \mathbf{P}_{k|k-1}^{(j)} \end{bmatrix} \right) d\mathbf{X}_{k|k-1} \end{aligned} \quad (51)$$

Up to now, a computing formula of the filtering error cross-covariance  $\mathbf{P}_{k|k}^{(ij)}$  ( $i \neq j$ ;  $i, j = 1, \dots, L$ ) has been proposed based on RLMVE framework. It is just a framework that can be implemented by unscented transformation, spherical-radial cubature rule or Monte Carlo method. Here we only give an implementation method of spherical-radial cubature rule.

Combining Algorithm 1 with Algorithm 2, a DF filter can be obtained for nonlinear multi-sensor systems with correlated noises by applying Lemma 2.

A distributed fusion filtering algorithm has been presented in Algorithm 1 to Algorithm 3, which can fuse estimates from local subsystems. Because DF framework allows each sensor node to process local measurements and only transmit local estimates, it is much more efficient in communication compared to CF which must transmit all measurements [2]. Moreover, DF has better robustness and flexibility, because of the parallel computing architecture.

#### IV. DE-CORRELATED DF FILTER

In this section, a existing de-correlation filter for nonlinear systems with correlated noises is introduced [28], [34], [35]. The de-correlated method can make the noises uncorrelated, and effectively deal with estimation problems with correlated noises by reconstructing the state equation.

*Lemma 3* [28], [34], [35]: For system in Equations (1) and (2), the de-correlated system can be written as:

$$\mathbf{x}_{k+1} = \mathbf{f}_k^{(j),dc}(\mathbf{x}_k) + \mathbf{w}_k^{(j),dc}, \quad j = 1, \dots, L \quad (66)$$

$$\mathbf{z}_k = \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{v}_k^{(j)} \quad (67)$$

where

$$\mathbf{f}_k^{(j),dc}(\mathbf{x}_k) = \mathbf{f}_k(\mathbf{x}_k) - \mathbf{M}^{(j)} \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{M}^{(j)} \mathbf{z}_k^{(j)} \quad (68)$$

**Algorithm 2** RLMVE-Based Algorithm of the Filtering Error Cross-Covariance  $\mathbf{P}_{k|k}^{(ij)}$ 

Initialization ( $k = 0$ ):  $\mathbf{P}_{0|0}^{(ij)} = \mathbf{I}$ ,  $\mathbf{P}_{1|0}^{(ij)} = \mathbf{I}$ ;

Generating joint filtering particles:

$$\begin{bmatrix} \mathbf{P}_{k-1|k-1}^{(i)} & \mathbf{P}_{k-1|k-1}^{(ij)} \\ \mathbf{P}_{k-1|k-1}^{(ji)} & \mathbf{P}_{k-1|k-1}^{(j)} \end{bmatrix} = \mathbf{S}_{k-1|k-1} (\cdot)^T \quad (52)$$

$$\mathbf{x}_{\mu, k-1|k-1} = \mathbf{S}_{k-1|k-1} \boldsymbol{\xi}_{\mu} + \begin{bmatrix} \hat{\mathbf{x}}_{k-1|k-1}^{(i)} \\ \hat{\mathbf{x}}_{k-1|k-1}^{(j)} \end{bmatrix}, \quad \mu = 1, \dots, 4n \quad (53)$$

$$\boldsymbol{\xi}_{\mu} = \sqrt{2n} [1]_{\mu} \quad (54)$$

where  $[1]_{\mu}$  is defined by:

$$[1]_{\mu} = [\mathbf{I}_{2n \times 2n} \quad -\mathbf{I}_{2n \times 2n}] \quad (55)$$

$$\mathbf{X}_{\mu, k|k-1}^{(1)} = \mathbf{f}(\mathbf{x}_{k-1}, k-1) \Big|_{\mathbf{x}_{k-1} = (\mathbf{x}_{\mu, k-1|k-1})_{1 \sim 2n}}$$

$$\mathbf{X}_{\mu, k|k-1}^{(2)} = \mathbf{f}(\mathbf{x}_{k-1}, k-1) \Big|_{\mathbf{x}_{k-1} = (\mathbf{x}_{\mu, k-1|k-1})_{2n+1 \sim 4n}} \quad (56)$$

where  $(\mathbf{x}_{\mu, k-1|k-1})_{1 \sim 2n}$  means the first  $2n$  elements of  $\mathbf{x}_{\mu, k-1|k-1}$  and  $(\mathbf{x}_{\mu, k-1|k-1})_{2n+1 \sim 4n}$  means the last  $n$  elements of  $\mathbf{x}_{\mu, k-1|k-1}$ ;

Prediction error cross-covariance  $\mathbf{P}_{k|k-1}^{(ij)}$ :

$$\mathbf{P}_{k|k-1}^{(ij)} = \frac{1}{4n} \sum_{\mu=1}^{4n} \mathbf{X}_{\mu, k|k-1}^{(1)} (\mathbf{X}_{\mu, k|k-1}^{(2)})^T - \hat{\mathbf{x}}_{k|k-1}^{(i)} (\hat{\mathbf{x}}_{k|k-1}^{(j)})^T + \mathbf{P}_{ww, k-1|k-1}^{(ij)} \quad (57)$$

Generating joint prediction particles:

$$\begin{bmatrix} \mathbf{P}_{k|k-1}^{(i)} & \mathbf{P}_{k|k-1}^{(ij)} \\ \mathbf{P}_{k|k-1}^{(ji)} & \mathbf{P}_{k|k-1}^{(j)} \end{bmatrix} = \mathbf{S}_{k|k-1} \mathbf{S}_{k|k-1}^T \quad (58)$$

$$\mathbf{x}_{\mu, k|k-1} = \mathbf{S}_{k|k-1} \boldsymbol{\xi}_{\mu} + \begin{bmatrix} \hat{\mathbf{x}}_{k|k-1}^{(i)} \\ \hat{\mathbf{x}}_{k|k-1}^{(j)} \end{bmatrix}, \quad \mu = 1, \dots, 4n \quad (59)$$

$$\mathbf{Z}_{\mu, k|k-1}^{(1)} = \mathbf{h}(\mathbf{x}_k, k-1) \Big|_{\mathbf{x}_{k-1} = (\mathbf{x}_{\mu, k|k-1})_{1 \sim 2n}}$$

$$\mathbf{Z}_{\mu, k|k-1}^{(2)} = \mathbf{h}(\mathbf{x}_k, k-1) \Big|_{\mathbf{x}_{k-1} = (\mathbf{x}_{\mu, k|k-1})_{2n+1 \sim 4n}} \quad (60)$$

$$\mathbf{M}^{(j)} = \mathbf{S}^{(j)} (\mathbf{R}^{(j)})^{-1} \quad (69)$$

$$\mathbf{w}_k^{(j), dc} = \mathbf{w}_k - \mathbf{M}^{(j)} \mathbf{v}_k^{(j)} \quad (70)$$

and the statistical property of the process noise  $\mathbf{w}^{(j), dc}(k)$  is given as:

$$\mathbb{E} \left\{ \mathbf{w}^{(j), dc}(k) \right\} = \mathbf{0} \quad (71)$$

$$\begin{aligned} \mathbf{Q}_k^{(j), dc} &= \mathbb{E} \left\{ \mathbf{w}^{(j), dc}(k) (\cdot)^T \right\} \\ &= \mathbf{Q}_w - \mathbf{S}^{(j)} (\mathbf{R}^{(j)})^{-1} (\mathbf{S}^{(j)})^T \end{aligned} \quad (72)$$

**Algorithm 2 (Continued.)** RLMVE-Based Algorithm of the Filtering Error Cross-Covariance  $\mathbf{P}_{k|k}^{(ij)}$ 

The calculation of the filtering error cross-covariance  $\mathbf{P}_{k|k}^{(ij)}$ :

$$\mathbf{P}_{zz, k|k-1}^{(ij)} = \frac{1}{4n} \sum_{\mu=1}^{4n} \mathbf{Z}_{\mu, k|k-1}^{(1)} (\mathbf{Z}_{\mu, k|k-1}^{(2)})^T - (\hat{\mathbf{z}}_{k|k-1}^{(i)}) (\hat{\mathbf{z}}_{k|k-1}^{(j)})^T + \mathbf{R}^{(ij)} \quad (61)$$

$$\mathbf{P}_{xz, k|k-1}^{(ij)} = \frac{1}{4n} \sum_{\mu=1}^{4n} (\mathbf{x}_{\mu, k|k-1})_{1 \sim n} (\mathbf{Z}_{\mu, k|k-1}^{(2)})^T - \hat{\mathbf{x}}_{k|k-1}^{(i)} (\hat{\mathbf{z}}_{k|k-1}^{(j)})^T \quad (62)$$

$$\begin{aligned} \mathbf{P}_{k|k}^{(ij)} &= \mathbf{P}_{k|k-1}^{(ij)} - \mathbf{P}_{xz, k|k-1}^{(ij)} (\mathbf{K}_k^{(j)})^T \\ &\quad - \mathbf{K}_k^{(i)} \mathbf{P}_{zx, k|k-1}^{(ij)} + \mathbf{K}_k^{(i)} \mathbf{P}_{zz, k|k-1}^{(ij)} (\mathbf{K}_k^{(j)})^T \end{aligned} \quad (63)$$

**Algorithm 3** RLMVE-Based DF Filtering Algorithm for Nonlinear Multi-Sensor Systems With Correlated Noises

Initialization ( $k = 0$ );

Local filter  $\hat{\mathbf{x}}_{k|k}^{(j)}$ ,  $j = 1, 2, \dots, L$  (Algorithm 1);

The calculation of the filtering error cross-covariance matrices  $\mathbf{P}_{k|k}^{(ij)}$  (Algorithm 2);

Fusion and output:

$$(\mathbf{A}^{(1)} \quad \mathbf{A}^{(2)} \quad \dots \quad \mathbf{A}^{(L)}) \triangleq (\mathbf{e}^T \mathbf{P}_{k|k}^{-1} \mathbf{e})^{-1} \mathbf{e}^T \mathbf{P}_{k|k}^{-1} \quad (64)$$

$$\hat{\mathbf{x}}_{k|k}^{(M)} = \sum_{j=1}^L \mathbf{A}^{(j)} \hat{\mathbf{x}}_{k|k}^{(j)} \quad (65)$$

*Lemma 4* [28], [34]–[35]: For de-correlated system in Equations (66) and (67), the de-correlated filter based on RLMVE framework can be written as:

$$\hat{\mathbf{x}}_{k|k}^{(j), dc} = \hat{\mathbf{x}}_{k|k-1}^{(j), dc} + \mathbf{K}_k^{(j), dc} \mathbf{z}_{k|k-1}^{(j), dc} \quad (73)$$

where

$$\hat{\mathbf{x}}_{k|k-1}^{(j), dc} = \mathbb{E} \left\{ \mathbf{f}_k^{(j), dc}(\mathbf{x}_k) \Big| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \quad (74)$$

$$\mathbf{K}_k^{(j), dc} = \mathbf{P}_{xz, k|k-1}^{(j), dc} (\mathbf{P}_{zz, k|k-1}^{(j), dc})^{-1} \quad (75)$$

$$\begin{aligned} \mathbf{P}_{xz, k|k-1}^{(j), dc} &= \mathbb{E} \left\{ \mathbf{x}_k (\mathbf{h}_k^{(j)}(\mathbf{x}_k))^T \Big| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(j), dc} (\hat{\mathbf{z}}_{k|k-1}^{(j), dc})^T \end{aligned} \quad (76)$$

$$\begin{aligned} \mathbf{P}_{zz, k|k-1}^{(j), dc} &= \mathbb{E} \left\{ \mathbf{h}_k^{(j)}(\mathbf{x}_k) (\cdot)^T \Big| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad - (\hat{\mathbf{z}}_{k|k-1}^{(j), dc}) (\cdot)^T + \mathbf{R}^{(j)} \end{aligned} \quad (77)$$

$$\hat{\mathbf{z}}_{k|k-1}^{(j), dc} = \mathbb{E} \left\{ \mathbf{z}_k^{(j)} \Big| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \quad (78)$$

The filtering error covariance matrix and the prediction error covariance matrix are computed by:

$$\mathbf{P}_{k|k}^{(j),dc} = \mathbf{P}_{k|k-1}^{(j),dc} - \mathbf{K}_k^{(j),dc} \mathbf{P}_{zz,k|k-1}^{(j),dc} \left( \mathbf{K}_k^{(j),dc} \right)^T \quad (79)$$

$$\mathbf{P}_{k|k-1}^{(j),dc} = \mathbb{E} \left\{ \mathbf{f}_{k-1}^{(j),dc}(\mathbf{x}_{k-1}) (\cdot)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right. \\ \left. - \hat{\mathbf{x}}_{k|k-1}^{(j),dc} (\cdot)^T + \mathbf{Q}^{(j),dc} \right\} \quad (80)$$

*Theorem 6:* The de-correlated filter and the proposed RLMVE-based filter in Section II have the functional equivalence.

*Proof:* See Appendix F.

*Remark 3:* The de-correlated filter and the proposed RLMVE-based filter have the same computational complexity. But the de-correlated filter needs to substitute the measurement equation into the state equation. That is, the de-correlated filter is suitable when the measurement models are not too complicated.

The de-correlated filter is available for systems with single sensor or CF framework. Next, a de-correlated DF filter is proposed.

*Theorem 7:* For de-correlated system in Equations (66) and (67), the cross-covariance  $\mathbf{P}_{k|k}^{(ij),dc}$  based on RLMVE framework can be calculated as:

$$\mathbf{P}_{k|k}^{(ij),dc} \\ = \mathbf{P}_{k|k-1}^{(ij),dc} - \mathbf{P}_{xz,k|k-1}^{(ij),dc} \left( \mathbf{K}_k^{(j),dc} \right)^T \\ - \mathbf{K}_k^{(i),dc} \mathbf{P}_{zx,k|k-1}^{(ij),dc} + \mathbf{K}_k^{(i),dc} \mathbf{P}_{zz,k|k-1}^{(ij),dc} \left( \mathbf{K}_k^{(j),dc} \right)^T \quad (81)$$

$$\mathbf{P}_{k|k-1}^{(ij),dc} \\ = \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}^{(i),dc}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{f}_{k-1}^{(j),dc}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ - \hat{\mathbf{x}}_{k|k-1}^{(i),dc} \left( \hat{\mathbf{x}}_{k|k-1}^{(j),dc} \right)^T + \mathbf{Q}_w - \mathbf{M}^{(i)} \left( \mathbf{S}^{(i)} \right)^T \\ - \mathbf{S}^{(j)} \left( \mathbf{M}^{(j)} \right)^T + \mathbf{M}^{(i)} \left( \mathbf{R}^{(ij)} \right)^{-1} \left( \mathbf{M}^{(j)} \right)^T \quad (82)$$

$$\mathbf{P}_{zz,k|k-1}^{(ij),dc} \\ = \mathbb{E} \left\{ \left( \mathbf{h}_k^{(i)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ - \hat{\mathbf{z}}_{k|k-1}^{(i),dc} \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right)^T + \mathbf{R}^{(ij)} \quad (83)$$

$$\mathbf{P}_{xz,k|k-1}^{(ij),dc} \\ = \mathbb{E} \left\{ \left( \mathbf{x}_k \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{h}^{(j)}(\mathbf{x}_k, k) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ - \hat{\mathbf{x}}_{k|k-1}^{(i),dc} \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right)^T \quad (84)$$

*Proof:* Similar to Theorem 2-5, Theorem 7 can be proved.

Using Lemma 4, Theorem 7 and spherical-radial cubature rule, a de-correlated DF filter algorithm for nonlinear multi-sensor systems with correlated noises is given below.

The particles  $\mathbf{X}_{\mu, k-1|k-1}^{(j),dc}$ ,  $\mathbf{Z}_{\mu, k-1|k-1}^{(j),dc}$ ,  $\mathbf{X}_{\mu, k-1|k-1}^{(1),dc}$ ,  $\mathbf{X}_{\mu, k-1|k-1}^{(2),dc}$ ,  $\mathbf{Z}_{\mu, k|k-1}^{(1),dc}$ ,  $\mathbf{Z}_{\mu, k|k-1}^{(2),dc}$  and  $\mathbf{X}_{\mu, k|k-1}^{dc}$

#### Algorithm 4 De-Correlated DF Filtering Algorithm

Initialization ( $k = 0$ ):  $\hat{\mathbf{x}}_{0|-1}^{(j)} = \mathbb{E} \{ \mathbf{x}_0 \}$ ,  $\hat{\mathbf{z}}_{-1|-2}^{(j)} = 0$ ,  $\mathbf{P}_{-1|-1}^{(j)} = \mathbf{I}$ ,  $\mathbf{P}_{zz,-1|-2}^{(j)} = \mathbf{I}$  ( $j = 1, 2, \dots, L$ );

Local filter:

State predictor  $\hat{\mathbf{x}}_{k|k-1}^{(j),dc}$ :

$$\hat{\mathbf{x}}_{k|k-1}^{(j),dc} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{X}_{\mu, k-1|k-1}^{(j),dc} \quad (85)$$

$$\mathbf{P}_{k|k-1}^{(j),dc} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{X}_{\mu, k-1|k-1}^{(j),dc} (\cdot)^T - \hat{\mathbf{x}}_{k|k-1}^{(j),dc} (\cdot)^T + \mathbf{Q}^{(j),dc} \quad (86)$$

Measurement predictor  $\hat{\mathbf{z}}_{k|k-1}^{(j),dc}$ :

$$\hat{\mathbf{z}}_{k|k-1}^{(j),dc} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{Z}_{\mu, k-1|k-1}^{(j),dc} \quad (87)$$

State filter  $\hat{\mathbf{x}}_{k|k}^{(j),dc}$ :

$$\mathbf{P}_{xz,k|k-1}^{(j),dc} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{X}_{\mu, k-1|k-1}^{(j),dc} \left( \mathbf{Z}_{\mu, k-1|k-1}^{(j),dc} \right)^T \\ - \hat{\mathbf{x}}_{k|k-1}^{(j),dc} \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right)^T \quad (88)$$

$$\mathbf{P}_{zz,k|k-1}^{(j),dc} = \frac{1}{2n} \sum_{\mu=1}^{2n} \mathbf{Z}_{\mu, k-1|k-1}^{(j),dc} (\cdot)^T - \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right) (\cdot)^T + \mathbf{R}^{(j)} \quad (89)$$

$$\mathbf{K}_k^{(j),dc} = \mathbf{P}_{xz,k|k-1}^{(j),dc} \left( \mathbf{P}_{zz,k|k-1}^{(j),dc} \right)^{-1} \quad (90)$$

$$\hat{\mathbf{x}}_{k|k}^{(j),dc} = \hat{\mathbf{x}}_{k|k-1}^{(j),dc} + \mathbf{K}_k^{(j),dc} \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right) \quad (91)$$

$$\mathbf{P}_{k|k}^{(j),dc} = \mathbf{P}_{k|k-1}^{(j),dc} - \mathbf{K}_k^{(j),dc} \mathbf{P}_{zz,k|k-1}^{(j),dc} \left( \mathbf{K}_k^{(j),dc} \right)^T \quad (92)$$

involved in Algorithm 4 can be handled by the method similar to Algorithm 1 and Algorithm 2.

## V. SIMULATION

Consider a target tracking process in a horizontal plane:

$$\mathbf{x}_{k+1} = \Phi \mathbf{x}_k + \Gamma \mathbf{w}_k \quad (99)$$

where  $\mathbf{x}_k = (x_k \dot{x}_k y_k \dot{y}_k)^T$  is the state vector,  $\Phi =$

$$\begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \Gamma = \begin{bmatrix} 0.5T^2 & 0 \\ T & 0 \\ 0 & 0.5T^2 \\ 0 & T \end{bmatrix}.$$

Assume that there are four sensors locating at four points  $\mathbf{z}^j(x^j, y^j)$ ,  $j = 1, \dots, 4$ . The measurement equations of the four sensors can be written as:

$$\mathbf{z}_k^{(j)} = \begin{bmatrix} \sqrt{(x_k - x^j)^2 + (y_k - y^j)^2} \\ \arctan((y_k - y^j)/(x_k - x^j)) \end{bmatrix} + \mathbf{v}_k^{(j)} \quad (100)$$

**Algorithm 4** (Continued.) De-Correlated DF Filtering Algorithm

The calculation of cross-covariance matrices  $\mathbf{P}_{k|k}^{(ij),dc}$ :

$$\begin{aligned} \mathbf{P}_{k|k-1}^{(ij),dc} &= \frac{1}{4n} \sum_{\mu=1}^{4n} \mathbf{X}_{\mu, k-1|k-1}^{(1),dc} \left( \mathbf{X}_{\mu, k-1|k-1}^{(2),dc} \right)^T \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(i),dc} \left( \hat{\mathbf{x}}_{k|k-1}^{(j),dc} \right)^T + \mathbf{Q}_w - \mathbf{M}^{(i)} \left( \mathbf{S}^{(i)} \right)^T \\ &\quad - \mathbf{S}^{(j)} \left( \mathbf{M}^{(j)} \right)^T + \mathbf{M}^{(i)} \left( \mathbf{R}^{(ij)} \right)^{-1} \left( \mathbf{M}^{(j)} \right)^T \end{aligned} \quad (93)$$

$$\begin{aligned} \mathbf{P}_{z_z, k|k-1}^{(ij),dc} &= \frac{1}{4n} \sum_{\mu=1}^{4n} \mathbf{Z}_{\mu, k|k-1}^{(1),dc} \left( \mathbf{Z}_{\mu, k|k-1}^{(2),dc} \right)^T \\ &\quad - \left( \hat{\mathbf{z}}_{k|k-1}^{(i),dc} \right) \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right)^T + \mathbf{R}^{(ij)} \end{aligned} \quad (94)$$

$$\begin{aligned} \mathbf{P}_{z_x, k|k-1}^{(ij),dc} &= \frac{1}{4n} \sum_{\mu=1}^{4n} \left( \mathbf{x}_{\mu, k|k-1}^{dc} \right)_{1 \sim n} \left( \mathbf{Z}_{\mu, k|k-1}^{(2),dc} \right)^T \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(i),dc} \left( \hat{\mathbf{z}}_{k|k-1}^{(j),dc} \right)^T \end{aligned} \quad (95)$$

$$\begin{aligned} \mathbf{P}_{k|k}^{(ij),dc} &= \mathbf{P}_{k|k-1}^{(ij),dc} - \mathbf{P}_{z_x, k|k-1}^{(ij),dc} \left( \mathbf{K}_k^{(j),dc} \right)^T \\ &\quad - \mathbf{K}_k^{(i),dc} \mathbf{P}_{z_x, k|k-1}^{(ij),dc} + \mathbf{K}_k^{(i),dc} \mathbf{P}_{z_z, k|k-1}^{(ij),dc} \left( \mathbf{K}_k^{(j),dc} \right)^T \end{aligned} \quad (96)$$

Fusion and output:

$$\left( \mathbf{A}^{(1)} \quad \mathbf{A}^{(2)} \quad \dots \quad \mathbf{A}^{(L)} \right) \triangleq \left( \mathbf{e}^T \mathbf{P}_{k|k}^{-1} \mathbf{e} \right)^{-1} \mathbf{e}^T \mathbf{P}_{k|k}^{-1} \quad (97)$$

$$\hat{\mathbf{x}}_{k|k}^{(M),dc} = \sum_{j=1}^L \mathbf{A}^{(j)} \hat{\mathbf{x}}_{k|k}^{(j)} \quad (98)$$

The process noise  $\mathbf{w}_k = \alpha \xi_k + \mathbf{w}_k^e$  and measurement noise  $\mathbf{v}_k^{(j)} = \beta^{(j)} \xi_k + \mathbf{v}_k^{e(j)}$ , where  $\xi_k$  is the background noise and  $\xi_k \sim \mathcal{N}(0, \sigma_\xi^2)$ ,  $\mathbf{w}_k^e \sim \mathcal{N}(0, \sigma_{we}^2)$ ,  $\mathbf{v}_k^{e(j)} \sim \mathcal{N}(0, \sigma_{vej}^2)$ , then we have

$$\mathbf{S}_k^{(j)} = \alpha \sigma_\xi^2 \left( \beta^{(j)} \right)^T, \quad \mathbf{R}^{(ij)} = \beta^{(i)} \sigma_\xi^2 \left( \beta^{(j)} \right)^T \quad (101)$$

In simulation, we set  $\sigma_\xi^2 = 0.08^2 \mathbf{W}$ ,  $\sigma_{we}^2 = \begin{bmatrix} 0.2^2 \text{m}^2 \text{s}^{-2} & 0 \\ 0 & 0.2^2 \text{m}^2 \text{s}^{-2} \end{bmatrix}$ ,  $\sigma_{ve1}^2 = \begin{bmatrix} 1.5^2 \text{m}^2 & 0 \\ 0 & 0.09^2 \text{rad}^2 \end{bmatrix}$ ,  $\sigma_{ve2}^2 = \begin{bmatrix} 2 \text{m}^2 & 0 \\ 0 & 0.07^2 \text{rad}^2 \end{bmatrix}$ ,  $\sigma_{ve3}^2 = \begin{bmatrix} 1.8^2 \text{m}^2 & 0 \\ 0 & 0.08^2 \text{rad}^2 \end{bmatrix}$ ,  $\sigma_{ve4}^2 = \begin{bmatrix} 1.4^2 \text{m}^2 & 0 \\ 0 & 0.1^2 \text{rad}^2 \end{bmatrix}$ ,  $\alpha = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ ,  $\beta^{(1)} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}$ ,  $\beta^{(2)} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}$ ,  $\beta^{(3)} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$ , and  $\beta^{(4)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ . The sampling period is  $T = 0.2\text{s}$ , the sensors' positions are located at  $p_1(100\text{m}, 100\text{m})$ ,  $p_2(-100\text{m}, 100\text{m})$ ,

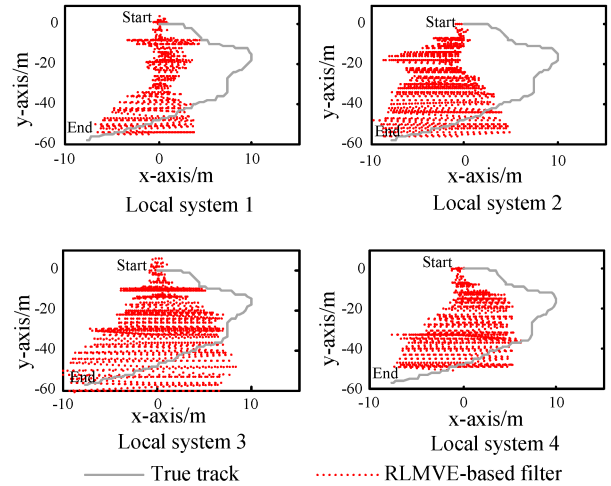


FIGURE 1. True track and local estimation curves of RLMVE-based filter.

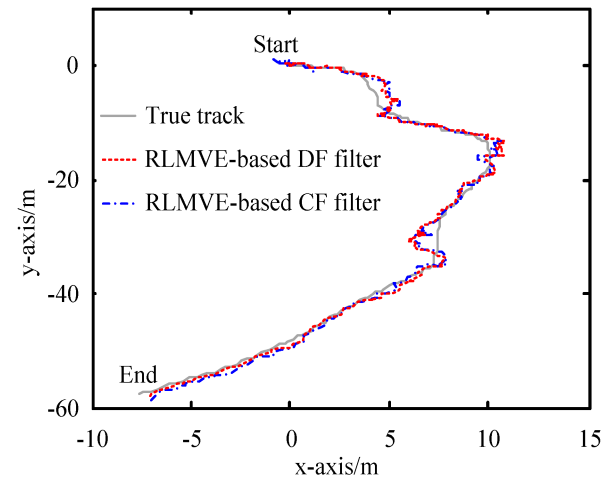


FIGURE 2. True track and estimation curves of RLMVE-based DF filter and CF filter.

$p_3(-100\text{m}, -100\text{m})$ ,  $p_4(100\text{m}, -100\text{m})$ , and the initial state is  $\mathbf{x}_0 = [0\text{m} \ 0\text{m} \ 0\text{m} \ 0\text{m}]^T$ .

The performance index of estimation is accumulative mean square error (AMSE) of position at time  $k$  [41], [9]:

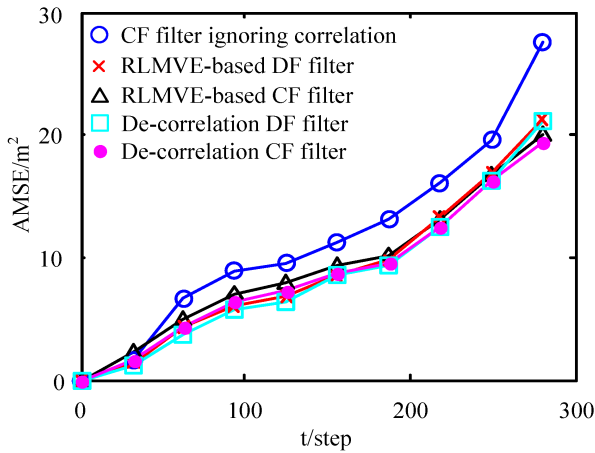
$$\text{AMSE} = \sum_{t=0}^k \frac{1}{N} \sum_{i=1}^N \left( (x_t^i - \hat{x}_{t|t}^i)^2 + (y_t^i - \hat{y}_{t|t}^i)^2 \right) \quad (102)$$

where  $(x_t^i, y_t^i)$  and  $(\hat{x}_{t|t}^i, \hat{y}_{t|t}^i)$  are the true and estimated positions of the  $i$ th Monte Carlo experiment at time  $t$ .

Local estimation curves of the RLMVE-based filter are shown in Figure 1. It is evident that the tracking effects are very unsatisfactory, and it means that the filter with single sensor is not available for such systems. So, we introduce the CF framework. Estimation curve of RLMVE-based CF filter is shown in Figure 2. It illustrates that the proposed RLMVE-based filter is effective.

In consideration of the limitations of CF, DF framework is introduced. Estimation curve of the RLMVE-based DF filter is also shown in Figure 2. It is obviously that RLMVE-based





**FIGURE 3.** AMSEs of distances between actual position and their estimates of CF filter ignoring correlation, RLMVE-based DF filter, RLMVE-based CF filter, De-correlated DF filter and De-correlated CF filter.

DF filter can work well, which illustrates that the proposed RLMVE-based DF filter is effective.

In order to analyze the effectiveness and superiority of the proposed filtering algorithms, the classical CKF is introduced, which ignores the correlation, i.e.,  $S_k^{(j)} = 0$  ( $j = 1, \dots, 4$ ). The AMSEs of distances between actual position and their estimates of CF filter ignoring correlation, RLMVE-based DF filter, RLMVE-based CF filter, de-correlated DF filter and de-correlated CF filter are shown in Figure 3 with 50 Monte Carlo experiments. From Figure 3, it is evident that the accuracy of CF filter ignoring correlation is lower than that of other filters. It illustrates that the performance of the RLMVE-based filter in Algorithm 1 is better than that of the classical filter ignoring correlation.

The AMSEs of the RLMVE-based DF filter and de-correlated DF filter is slightly lower than the ones of RLMVE-based CF filter and de-correlated CF filter, and it verifies the effectiveness of the proposed DF filters.

The AMSEs of the RLMVE-based CF filter and de-correlated CF filter are approximate, which confirms the correctness of Theorem 6. The slight difference in value is due to the calculation of the approximate integrals.

## VI. CONCLUSION

In this paper, a RLMVE-based filter and two DF filters are proposed for nonlinear multi-sensor systems with correlated noises. Firstly, a novel RLMVE-based filter for nonlinear systems with correlated noise is proposed. It is proved that the RLMVE-based filter and the existing de-correlated filter have the functional equivalence. Then, an algorithm for calculating the cross-covariance is proposed. Based on the RLMVE-based filter, the algorithm of cross-covariance and the DF algorithm weighted by matrices, a RLMVE-based DF filter is proposed. Finally, based on the existing de-correlated filter, the algorithm of cross-covariance for de-correlated systems and the DF algorithm weighted by matrices, a de-correlated DF filtering algorithm is proposed. Both the DF filtering algorithms can handle the filtering problem for

nonlinear multi-sensor systems with correlated noises. Due to the different ways of dealing with correlated noises, the de-correlated DF filter is suitable when the measurement models are not too complicated.

## APPENDICES

### APPENDIX A

From

$$\begin{aligned} \hat{\mathbf{x}}_{k|k-1}^{(j)} &= E \left\{ \mathbf{x}_k \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= E \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \right) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= E \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} + E \left\{ \mathbf{w}_{k-1} \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \end{aligned} \quad (103)$$

Equation (10) can be obtained. The prediction error covariance matrix  $\mathbf{P}_{k|k-1}^{(j)}$  is derived as follows:

$$\begin{aligned} \mathbf{P}_{k|k-1}^{(j)} &= E \left\{ \hat{\mathbf{x}}_{k|k-1}^{(j)} (\hat{\mathbf{x}}_{k|k-1}^{(j)})^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= E \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} - \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} + \hat{\mathbf{w}}_{k-1|k-1}^{(j)} \right) \right) \right. \\ &\quad \left. (\cdot)^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= E \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \mathbf{f}_{k-1}^T(\mathbf{x}_{k-1}) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad + E \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \tilde{\mathbf{w}}_{k-1}^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad + E \left\{ \tilde{\mathbf{w}}_{k-1} \mathbf{f}_{k-1}^T(\mathbf{x}_{k-1}) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad - E \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} \right)^T \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(j)} E \left\{ \mathbf{f}_{k-1}^T(\mathbf{x}_{k-1}) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad + \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} \right)^T + \mathbf{P}_{ww,k-1|k-1}^{(j)} \end{aligned} \quad (104)$$

and  $\hat{\mathbf{x}}_{k|k-1}^{(j)} = E \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\}$ . Because  $\mathbf{f}_{k-1}(\mathbf{x}_{k-1})$  and  $\tilde{\mathbf{w}}_{k-1}$  are mutually independent, and  $\hat{\mathbf{w}}_{k-1|k-1}^{(j)}$  is unbiased estimation, i.e.,  $E \left\{ \tilde{\mathbf{w}}_{k-1}^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} = 0$ ,  $E \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \tilde{\mathbf{w}}_{k-1}^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} = 0$ , Equation (104) can be written as Equation (9).

The prediction error cross-covariance matrix  $\mathbf{P}_{xz,k|k-1}^{(j)}$  is derived as:

$$\begin{aligned} \mathbf{P}_{xz,k|k-1}^{(j)} &= E \left\{ \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= E \left\{ \left( \mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}^{(j)} \right) \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{v}_k^{(j)} - \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= E \left\{ \mathbf{x}_k \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \right)^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} - \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(j)} E \left\{ \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) \right)^T \mid \mathbf{z}_{0 \sim k-1}^{(j)} \right\} + \hat{\mathbf{x}}_{k|k-1}^{(j)} \left( \hat{\mathbf{z}}_{k|k-1}^{(j)} \right)^T \end{aligned} \quad (105)$$

Because  $\mathbf{v}_k^{(j)}$  is Gaussian noise and independent of  $\mathbf{w}_{k-1}$ , (7) can be obtained.

The measurement prediction error covariance matrix  $\mathbf{P}_{zz,k|k-1}^{(j)}$  is derived as:

$$\begin{aligned} \mathbf{P}_{zz,k|k-1}^{(j)} &= \mathbb{E} \left\{ \tilde{\mathbf{z}}_{k|k-1}^{(j)} (\cdot)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{v}_k^{(j)} - \hat{\mathbf{z}}_{k|k-1}^{(j)} \right) (\cdot)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \end{aligned} \quad (106)$$

Similar to  $\mathbf{P}_{xz,k|k-1}^{(j)}$ , Equation (106) can be written as Equation (8).

Based on RLMVE framework, the estimation  $\hat{\mathbf{w}}_{k-1|k-1}^{(j)}$  of  $\mathbf{w}_{k-1}$  can be calculated as:

$$\hat{\mathbf{w}}_{k-1|k-1}^{(j)} = \hat{\mathbf{w}}_{k-1|k-2}^{(j)} + \mathbf{K}_{w,k-1}^{(j)} \tilde{\mathbf{z}}_{k-1|k-2}^{(j)} \quad (107)$$

Because  $\mathbf{w}_{k-1}$  is Gaussian noise and independent of  $\mathbf{v}_{k-2}$ ,  $\hat{\mathbf{w}}_{k-1|k-2}^{(j)} = \mathbb{E} \left\{ \mathbf{w}_{k-1} \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} = 0$ , Equation (12) can be obtained. From RLMVE framework, the filter gain  $\mathbf{K}_{w,k-1}^{(j)}$  is derived as:

$$\begin{aligned} \mathbf{K}_{w,k-1}^{(j)} &= \mathbf{P}_{wz,k-1|k-2}^{(j)} \left( \mathbf{P}_{zz,k-1|k-2}^{(j)} \right)^{-1} \\ &= \mathbb{E} \left\{ \tilde{\mathbf{w}}_{k-1|k-2}^{(j)} \left( \tilde{\mathbf{z}}_{k-1|k-2}^{(j)} \right)^T \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} \left( \mathbf{P}_{zz,k-1|k-2}^{(j)} \right)^{-1} \\ &= \mathbb{E} \left\{ \mathbf{w}_{k-1} \left( \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) + \mathbf{v}_{k-1}^{(j)} - \hat{\mathbf{z}}_{k-1|k-2}^{(j)} \right)^T \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} \\ &\quad \left( \mathbf{P}_{zz,k-1|k-2}^{(j)} \right)^{-1} \end{aligned} \quad (108)$$

Because  $\mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1})$  and  $\mathbf{w}_{k-1}$  are mutually independent,  $\mathbb{E} \left\{ \mathbf{w}_{k-1} \left( \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) \right)^T \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} = 0$  and

$$\begin{aligned} \mathbf{P}_{wz,k-1|k-2}^{(j)} &= \mathbb{E} \left\{ \mathbf{w}_{k-1} \left( \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) + \mathbf{v}_{k-1}^{(j)} - \hat{\mathbf{z}}_{k-1|k-2}^{(j)} \right)^T \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} \\ &= \mathbf{S}_{k-1}^{(j)} \end{aligned} \quad (109)$$

Equation (13) can be obtained.

The filtering error covariance matrix  $\mathbf{P}_{ww,k-1|k-1}^{(j)}$  is derived as:

$$\begin{aligned} \mathbf{P}_{ww,k-1|k-1}^{(j)} &= \mathbb{E} \left\{ \tilde{\mathbf{w}}_{k-1|k-1}^{(j)} (\cdot)^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{w}_{k-1} - \mathbf{K}_{w,k-1}^{(j)} \tilde{\mathbf{z}}_{k-1|k-2}^{(j)} \right) (\cdot)^T \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} \\ &= \mathbb{E} \left\{ \mathbf{w}_{k-1} (\cdot)^T \middle| \mathbf{z}_{0 \sim k-2}^{(j)} \right\} - \mathbf{K}_{w,k-1}^{(j)} \mathbf{P}_{zz,k-1|k-2}^{(j)} \\ &\quad \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \\ &= \mathbf{Q}_w - \mathbf{K}_{w,k-1}^{(j)} \mathbf{P}_{zz,k-1|k-2}^{(j)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \end{aligned} \quad (110)$$

The proof is completed.

## APPENDIX B

From the definition of the filtering error cross-covariance,  $\mathbf{P}_{k|k}^{(ij)}$  ( $i \neq j$ ;  $i, j = 1, \dots, L$ ) can be written as:

$$\mathbf{P}_{k|k}^{(ij)} = \mathbb{E} \left\{ \left( \tilde{\mathbf{x}}_{k|k}^{(i)} \middle| \mathbf{z}_{0 \sim k}^{(i)} \right) \left( \tilde{\mathbf{x}}_{k|k}^{(j)} \middle| \mathbf{z}_{0 \sim k}^{(j)} \right)^T \right\} \quad (111)$$

From Equation (4), Equation (111) can be rewritten as:

$$\begin{aligned} \mathbf{P}_{k|k}^{(ij)} &= \mathbb{E} \left\{ \left( \mathbf{x}_k - \left( \hat{\mathbf{x}}_{k|k-1}^{(i)} + \mathbf{K}_k^{(i)} \tilde{\mathbf{z}}_{k|k-1}^{(i)} \right) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \right. \\ &\quad \left. \left( \mathbf{x}_k - \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} + \mathbf{K}_k^{(j)} \tilde{\mathbf{z}}_{k|k-1}^{(j)} \right) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \left( \tilde{\mathbf{x}}_{k|k-1}^{(i)} \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \tilde{\mathbf{x}}_{k|k-1}^{(j)} \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &\quad - \mathbb{E} \left\{ \left( \tilde{\mathbf{x}}_{k|k-1}^{(i)} \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \tilde{\mathbf{z}}_{k|k-1}^{(j)} \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \left( \mathbf{K}_k^{(j)} \right)^T \\ &\quad - \mathbf{K}_k^{(i)} \mathbb{E} \left\{ \left( \tilde{\mathbf{z}}_{k|k-1}^{(i)} \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \tilde{\mathbf{x}}_{k|k-1}^{(j)} \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &\quad + \mathbf{K}_k^{(i)} \mathbb{E} \left\{ \left( \tilde{\mathbf{z}}_{k|k-1}^{(i)} \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \tilde{\mathbf{z}}_{k|k-1}^{(j)} \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \left( \mathbf{K}_k^{(j)} \right)^T \end{aligned} \quad (112)$$

From the definitions of  $\mathbf{P}_{k|k-1}^{(ij)}$ ,  $\mathbf{P}_{xz,k|k-1}^{(ij)}$ ,  $\mathbf{P}_{zx,k|k-1}^{(ij)}$  and  $\mathbf{P}_{zz,k|k-1}^{(ij)}$ , (43) can be obtained. The proof is completed.

## APPENDIX C

The prediction error cross-covariance  $\mathbf{P}_{k|k-1}^{(ij)}$  in Equation (43) can be calculated as:

$$\begin{aligned} \mathbf{P}_{k|k-1}^{(ij)} &= \mathbb{E} \left\{ \left( \tilde{\mathbf{x}}_{k|k-1}^{(i)} \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \tilde{\mathbf{x}}_{k|k-1}^{(j)} \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} - \hat{\mathbf{x}}_{k|k-1}^{(i)} \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \right. \\ &\quad \left. \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} - \hat{\mathbf{x}}_{k|k-1}^{(j)} \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \end{aligned} \quad (113)$$

Because  $\mathbf{w}_{k-1}$  is assumed to be zero-mean and independent of the past states, from Equation (10) we have:

$$\begin{aligned} \mathbf{P}_{k|k-1}^{(ij)} &= \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} - \left( \hat{\mathbf{x}}_{k|k-1}^{(i)} + \hat{\mathbf{w}}_{k-1|k-1}^{(i)} \right) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \right. \\ &\quad \left. \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} - \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} + \hat{\mathbf{w}}_{k-1|k-1}^{(j)} \right) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{f}_{k-1}^T(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right) \right\} \\ &\quad + \mathbb{E} \left\{ \left( \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \tilde{\mathbf{w}}_{k-1}^T \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right) \right\} \\ &\quad + \mathbb{E} \left\{ \left( \tilde{\mathbf{w}}_{k-1}^T \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right) \left( \mathbf{f}_{k-1}^T(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right) \right\} \\ &\quad - \mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(i)} \right\} \left( \hat{\mathbf{x}}_{k|k-1}^{(j)} \right)^T \\ &\quad - \hat{\mathbf{x}}_{k|k-1}^{(i)} \mathbb{E} \left\{ \mathbf{f}_{k-1}^T(\mathbf{x}_{k-1}) \middle| \mathbf{z}_{0 \sim k-1}^{(j)} \right\} \\ &\quad + \hat{\mathbf{x}}_{k|k-1}^{(i)} \left( \hat{\mathbf{x}}_{k|k-1}^{(i)} \right)^T + \mathbf{P}_{ww,k-1|k-1}^{(ij)} \end{aligned} \quad (114)$$

Because  $f_{k-1}(\mathbf{x}_{k-1})$  is independent of  $\tilde{\mathbf{w}}_{k-1}$  and  $\hat{\mathbf{w}}_{k|k}^{(j)}$  is an unbiased estimate, Equation (114) can be rewritten as Equation (44), where the  $\hat{\mathbf{x}}_{k|k-1}^{(i)}$  is as Equation (11).

$$\begin{aligned} & \mathbf{P}_{ww,k-1|k-1}^{(ij)} \text{ in Equation (44) is computed as:} \\ & \mathbf{P}_{ww,k-1|k-1}^{(ij)} \\ &= \mathbb{E} \left\{ \left( \tilde{\mathbf{w}}_{k-1|k-1}^{(i)} \Big| \mathbf{z}_{0\sim k-1}^{(i)} \right) \left( \tilde{\mathbf{w}}_{k-1|k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{w}_{k-1} - \mathbf{K}_{w,k-1}^{(i)} \tilde{\mathbf{z}}_{k-1|k-2}^{(i)} \Big| \mathbf{z}_{0\sim k-2}^{(i)} \right) \right. \\ & \quad \left. \left( \mathbf{w}_{k-1} - \mathbf{K}_{w,k-1}^{(j)} \tilde{\mathbf{z}}_{k-1|k-2}^{(j)} \Big| \mathbf{z}_{0\sim k-2}^{(j)} \right)^T \right\} \\ &= \mathbf{Q}_w - \mathbf{P}_{wz,k-1|k-2}^{(ij)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T - \mathbf{K}_{w,k-1}^{(i)} \mathbf{P}_{zw,k-1|k-2}^{(ij)} \\ & \quad + \mathbf{K}_{w,k-1}^{(i)} \mathbf{P}_{zz,k-1|k-2}^{(ij)} \left( \mathbf{K}_{w,k-1}^{(j)} \right)^T \quad (115) \end{aligned}$$

$\mathbf{P}_{wz,k-1|k-2}^{(ij)}$  in Equation (115) is given by:

$$\begin{aligned} & \mathbf{P}_{wz,k-1|k-2}^{(ij)} \\ &= \mathbb{E} \left\{ \left( \tilde{\mathbf{w}}_{k-1|k-2}^{(i)} \Big| \mathbf{z}_{0\sim k-2}^{(i)} \right) \left( \tilde{\mathbf{z}}_{k-1|k-2}^{(j)} \Big| \mathbf{z}_{0\sim k-2}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \mathbf{w}_{k-1} \left( \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) + \mathbf{v}_{k-1}^{(j)} - \hat{\mathbf{z}}_{k-1|k-2}^{(j)} \Big| \mathbf{z}_{0\sim k-2}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \mathbf{w}_{k-1} \left( \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) \Big| \mathbf{z}_{0\sim k-2}^{(j)} \right)^T \right\} \\ & \quad + \mathbb{E} \left\{ \mathbf{w}_{k-1} \left( \mathbf{v}_{k-1}^{(j)} \right)^T \right\} - \mathbb{E} \left\{ \mathbf{w}_{k-1} \right\} \left( \hat{\mathbf{z}}_{k-1|k-2}^{(j)} \right)^T \\ &= \mathbf{S}_{k-1}^{(j)} \quad (116) \end{aligned}$$

and similar to Equation (116),  $\mathbf{P}_{zw,k-1|k-2}^{(ij)} = \left( \mathbf{S}_{k-1}^{(i)} \right)^T$ .

From Equation (116),  $\mathbf{P}_{ww,k-1|k-1}^{(ij)}$  can be rewritten as Equation (45). The proof is completed.

#### APPENDIX D

From Equation (2), the measurement prediction error covariance  $\mathbf{P}_{zz,k|k-1}^{(ij)}$  in Equation (43) can be calculated as:

$$\begin{aligned} & \mathbf{P}_{zz,k|k-1}^{(ij)} = \mathbb{E} \left\{ \left( \tilde{\mathbf{z}}_{k|k-1}^{(i)} \Big| \mathbf{z}_{0\sim k-1}^{(i)} \right) \left( \tilde{\mathbf{z}}_{k|k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{h}_k^{(i)}(\mathbf{x}_k) + \mathbf{v}_k^{(i)} - \hat{\mathbf{z}}_{k|k-1}^{(i)} \Big| \mathbf{z}_{0\sim k-1}^{(i)} \right) \right. \\ & \quad \left. \times \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{v}_k^{(j)} - \hat{\mathbf{z}}_{k|k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right)^T \right\} \quad (117) \end{aligned}$$

Because  $\mathbf{v}_k^{(j)}$  is zero-mean and independent of  $\mathbf{w}_{k-1}$ , Equation (117) can be written as Equation (46). The proof is completed.

#### APPENDIX E

The cross-covariance  $\mathbf{P}_{xz,k|k-1}^{(ij)}$  in Equation (43) can be calculated as:

$$\begin{aligned} & \mathbf{P}_{xz,k|k-1}^{(ij)} = \mathbb{E} \left\{ \left( \tilde{\mathbf{x}}_{k|k-1}^{(i)} \Big| \mathbf{z}_{0\sim k-1}^{(i)} \right) \left( \tilde{\mathbf{z}}_{k|k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right)^T \right\} \\ &= \mathbb{E} \left\{ \left( \mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}^{(i)} \Big| \mathbf{z}_{0\sim k-1}^{(i)} \right) \right. \\ & \quad \left. \times \left( \mathbf{h}_k^{(j)}(\mathbf{x}_k) + \mathbf{v}_k^{(j)} - \hat{\mathbf{z}}_{k|k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right)^T \right\} \quad (118) \end{aligned}$$

Because  $\mathbf{v}_k^{(j)}$  and  $\mathbf{w}_k$  are zero-mean and  $\mathbf{w}_k$  is independent of the past states, Equation (47) can be obtained. The proof is completed.

#### APPENDIX F

Structurally, the de-correlated filter and proposed filter in section II differ only in the predictor  $\hat{\mathbf{x}}_{k|k-1}^{(j)}$ . If the predictors have the same structure, Theorem 6 can be proved. From Equations (68) and (69), the predictor  $\hat{\mathbf{x}}_{k|k-1}^{(j),dc}$  can be rewritten as:

$$\begin{aligned} & \hat{\mathbf{x}}_{k|k-1}^{(j),dc} = \mathbb{E} \left\{ \mathbf{f}_{k-1}^{(j),dc}(\mathbf{x}_k) \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\} \\ &= \mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{S}^{(j)} \left( \mathbf{R}^{(j)} \right)^{-1} \right. \\ & \quad \left. \times \left( \mathbf{y}_{k-1}^{(j)} - \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) \right) \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\} \\ &= \mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\} \\ & \quad + \mathbf{S}^{(j)} \left( \mathbf{R}^{(j)} \right)^{-1} \mathbb{E} \left\{ \mathbf{v}_{k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\} \quad (119) \end{aligned}$$

From RLMVE framework, the filter  $\hat{\mathbf{v}}_{k-1|k-1}^{(j),dc} = \mathbb{E} \left\{ \mathbf{v}_{k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\}$  of the measurement noise  $\mathbf{v}_{k-1}^{(j)}$  can be written as:

$$\begin{aligned} & \hat{\mathbf{v}}_{k-1|k-1}^{(j),dc} = \mathbb{E} \left\{ \mathbf{v}_{k-1}^{(j)} \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\} \\ &= \hat{\mathbf{v}}_{k-1|k-2}^{(j),dc} + \mathbf{P}_{vz,k-1|k-2}^{(j),dc} \left( \mathbf{P}_{zz,k-1|k-2}^{(j),dc} \right)^{-1} \tilde{\mathbf{z}}_{k-1|k-2}^{(j),dc} \quad (120) \end{aligned}$$

where  $\hat{\mathbf{v}}_{k-1|k-2}^{(j),dc} = 0$  and  $\mathbf{P}_{vz,k-1|k-2}^{(j),dc}$  is computed by:

$$\begin{aligned} & \mathbf{P}_{vz,k-1|k-2}^{(j),dc} \\ &= \mathbb{E} \left\{ \tilde{\mathbf{v}}_{k-1|k-2}^{(j),dc} \left( \tilde{\mathbf{z}}_{k-1|k-2}^{(j),dc} \right)^T \Big| \mathbf{z}_{0\sim k-2}^{(j)} \right\} \\ &= \mathbb{E} \left\{ \mathbf{v}_{k-1}^{(j)} \left( \mathbf{h}_{k-1}^{(j)}(\mathbf{x}_{k-1}) + \mathbf{v}_{k-1}^{(j)} - \hat{\mathbf{z}}_{k-1|k-2}^{(j),dc} \right)^T \Big| \mathbf{z}_{0\sim k-2}^{(j)} \right\} \\ &= \mathbf{R}^{(j)} \quad (121) \end{aligned}$$

So,  $\hat{\mathbf{v}}_{k-1|k-1}^{(j),dc}$  in Equation (120) is obtained as:

$$\hat{\mathbf{v}}_{k-1|k-1}^{(j),dc} = \mathbf{R}^{(j)} \left( \mathbf{P}_{zz,k-1|k-2}^{(j),dc} \right)^{-1} \tilde{\mathbf{z}}_{k-1|k-2}^{(j),dc} \quad (122)$$

From Equation (122), Equation (119) can be rewritten as:

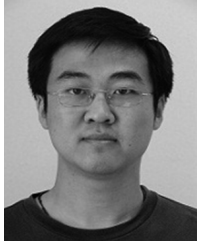
$$\begin{aligned} & \hat{\mathbf{x}}_{k|k-1}^{(j),dc} = \mathbb{E} \left\{ \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) \Big| \mathbf{z}_{0\sim k-1}^{(j)} \right\} \\ & \quad + \mathbf{S}^{(j)} \left( \mathbf{P}_{zz,k-1|k-2}^{(j),dc} \right)^{-1} \tilde{\mathbf{z}}_{k-1|k-2}^{(j),dc} \quad (123) \end{aligned}$$

Compared with Equations (10), (11), (12) and (13),  $\hat{\mathbf{x}}_{k|k-1}^{(j),dc}$  has the same form as  $\hat{\mathbf{x}}_{k|k-1}^{(j)}$  in Algorithm 1. The proof is completed.

## REFERENCES

- [1] E. Waltz and J. Llinas, *Multisensor Data Fusion*, Boston, MA, USA: Artech House, 1990, pp. 25–69.
- [2] S. Din, A. Ahmad, A. Paul, M. M. Ullah Rathore, and G. Jeon, “A cluster-based data fusion technique to analyze big data in wireless multi-sensor system,” *IEEE Access*, vol. 5, pp. 5069–5083, 2017, doi: [10.1109/ACCESS.2017.2679207](https://doi.org/10.1109/ACCESS.2017.2679207).
- [3] Y. Bar-Shalom and X. R. Li, *Estimation and Tracking: Principle, Techniques and Software*. Boston, MA, USA: Artech House, 1993, pp. 15–89.
- [4] J. Xiong, L. Shu, Q. Wang, W. Xu, and C. Zhu, “A scheme on indoor tracking of ship dynamic positioning based on distributed multi-sensor data fusion,” *IEEE Access*, vol. 5, pp. 379–392, 2017, doi: [10.1109/ACCESS.2016.2607232](https://doi.org/10.1109/ACCESS.2016.2607232).
- [5] S. Sun, H. Lin, J. Ma, and X. Li, “Multi-sensor distributed fusion estimation with applications in networked systems: A review paper,” *Inf. Fusion*, vol. 38, pp. 122–134, Nov. 2017, doi: [10.1016/j.inffus.2017.03.006](https://doi.org/10.1016/j.inffus.2017.03.006).
- [6] S.-L. Sun and Z.-L. Deng, “Multi-sensor optimal information fusion Kalman filter,” *Automatica*, vol. 40, no. 6, pp. 1017–1023, Jun. 2004, doi: [10.1016/j.automatica.2004.01.014](https://doi.org/10.1016/j.automatica.2004.01.014).
- [7] A. Simoes and J. Xavier, “FADE: Fast and asymptotically efficient distributed estimator for dynamic networks,” *IEEE Trans. Signal Process.*, vol. 67, no. 8, pp. 2080–2092, Apr. 2019, doi: [10.1109/TSP.2019.2901355](https://doi.org/10.1109/TSP.2019.2901355).
- [8] E. Song, Y. Zhu, J. Zhou, and Z. You, “Optimal Kalman filtering fusion with cross-correlated sensor noises,” *Automatica*, vol. 43, no. 8, pp. 1450–1456, Aug. 2007, doi: [10.1016/j.automatica.2007.01.010](https://doi.org/10.1016/j.automatica.2007.01.010).
- [9] G. Hao, S.-L. Sun, and Y. Li, “Nonlinear weighted measurement fusion unscented Kalman filter with asymptotic optimality,” *Inf. Sci.*, vol. 299, pp. 85–98, Apr. 2015, doi: [10.1016/j.ins.2014.12.013](https://doi.org/10.1016/j.ins.2014.12.013).
- [10] L. Zhang, D. Niu, E. Song, J. Zhou, Q. Shi, and Y. Zhu, “Joint optimization of dimension assignment and compression in distributed estimation fusion,” *IEEE Trans. Signal Process.*, vol. 67, no. 9, pp. 2453–2468, May 2019, doi: [10.1109/TSP.2019.2904935](https://doi.org/10.1109/TSP.2019.2904935).
- [11] G.-Q. Duan and S.-L. Sun, “Self-tuning distributed fusion filter for multi-sensor systems subject to unknown model parameters and missing measurement rates,” *IEEE Access*, vol. 6, pp. 61519–61528, 2018, doi: [10.1109/ACCESS.2018.2874444](https://doi.org/10.1109/ACCESS.2018.2874444).
- [12] Z. Deng, P. Zhang, W. Qi, J. Liu, and Y. Gao, “Sequential covariance intersection fusion Kalman filter,” *Inf. Sci.*, vol. 189, pp. 293–309, Apr. 2012, doi: [10.1016/j.ins.2011.11.038](https://doi.org/10.1016/j.ins.2011.11.038).
- [13] J. K. Uhlmann, “Covariance consistency methods for fault-tolerant distributed data fusion,” *Inf. Fusion*, vol. 4, no. 3, pp. 201–215, Sep. 2003, doi: [10.1016/S1566-2535\(03\)00036-8](https://doi.org/10.1016/S1566-2535(03)00036-8).
- [14] S. J. Julier and J. K. Uhlmann, “A non-divergent estimation algorithm in the presence of unknown correlations,” in *Proc. Amer. Control Conf.*, Albuquerque, NM, USA, 1997, pp. 2369–2373.
- [15] S. J. Julier and J. K. Uhlmann, “Using covariance intersection for SLAM,” *Robot. Auto. Syst.*, vol. 55, no. 1, pp. 3–20, Jan. 2007, doi: [10.1016/j.robot.2006.06.011](https://doi.org/10.1016/j.robot.2006.06.011).
- [16] S. J. Julier, J. K. Uhlmann, “General decentralized data fusion with covariance intersection,” in *Handbook of Multisensor Data Fusion: Theory and Practice*, 2nd ed. M. E. Liggins, D. L. Hall, J. Llinas Eds. Boca Raton, FL, USA: CRC Press, 2009, pp. 319–342.
- [17] D. Franken and A. Hupper, “Improved fast covariance intersection for distributed data fusion,” in *Proc. 7th Int. Conf. Inf. Fusion*, Philadelphia, PA, USA, 2005, pp. 154–160.
- [18] W. Niehsen, “Information fusion based on fast covariance intersection filtering,” in *Proc. 5th Int. Conf. Inf. Fusion*, Annapolis, MD, USA, 2002, pp. 901–904.
- [19] O. Straka, J. Dunfk, and M. Šimandl, “Truncation nonlinear filters for state estimation with nonlinear inequality constraints,” *Automatica*, vol. 48, no. 2, pp. 273–286, Feb. 2012, doi: [10.1016/j.automatica.2011.11.002](https://doi.org/10.1016/j.automatica.2011.11.002).
- [20] I. Urteaga and P. M. Djurić, “Sequential estimation of hidden ARMA processes by particle filtering—Part II,” *IEEE Trans. Signal Process.*, vol. 65, no. 2, pp. 494–504, Jan. 2017, doi: [10.1109/TSP.2016.2598324](https://doi.org/10.1109/TSP.2016.2598324).
- [21] Q. Ge, T. Shao, Q. Yang, X. Shen, and C. Wen, “Multisensor nonlinear fusion methods based on adaptive ensemble fifth-degree iterated cubature information filter for biomechanics,” *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 46, no. 7, pp. 912–925, Jul. 2016, doi: [10.1109/TSMC.2016.2523911](https://doi.org/10.1109/TSMC.2016.2523911).
- [22] B. Jia, M. Xin, K. Pham, E. Blasch, and G. Chen, “Multiple sensor estimation using a high-degree cubature information filter,” *Proc. SPIE Sensors Syst. Space Appl.*, vol. 8739, May 2013, Art. no. 87390, doi: [10.1117/12.2015546](https://doi.org/10.1117/12.2015546).
- [23] J. Huang, G. Li, and H. Yuan, “Multi-sensor information fusion cubature Kalman particle filter,” in *Proc. 9th Int. Symp. Comput. Intell. Design (ISCID)*, Dec. 2016, pp. 410–413.
- [24] Q. Ge, T. Shao, S. Chen, and C. Wen, “Carrier tracking estimation analysis by using the extended strong tracking filtering,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 2, pp. 1415–1424, Feb. 2017, doi: [10.1109/TIE.2016.2610403](https://doi.org/10.1109/TIE.2016.2610403).
- [25] R. Carelli, R. Kelly, O. H. Nasisi, C. Soria, and V. Mut, “Control based on perspective lines of a non-holonomic mobile robot with camera-on-board,” *Int. J. Control*, vol. 79, no. 4, pp. 362–371, Aug. 2006, doi: [10.1080/00207170500537180](https://doi.org/10.1080/00207170500537180).
- [26] S. Zhang, Y. Zhao, F. Wu, and J. Zhao, “Robust recursive filtering for uncertain systems with finite-step correlated noises, stochastic nonlinearities and autocorrelated missing measurements,” *Aerosp. Sci. Technol.*, vol. 39, pp. 272–280, Dec. 2014, doi: [10.1016/j.ast.2014.09.013](https://doi.org/10.1016/j.ast.2014.09.013).
- [27] T. Tian, S. Sun, and N. Li, “Multi-sensor information fusion estimators for stochastic uncertain systems with correlated noises,” *Inf. Fusion*, vol. 27, pp. 126–137, Jan. 2016, doi: [10.1016/j.inffus.2015.06.001](https://doi.org/10.1016/j.inffus.2015.06.001).
- [28] X. Wang, S.-L. Sun, K.-H. Ding, and J.-Y. Xue, “Weighted measurement fusion white noise deconvolution filter with correlated noise for multisensor stochastic systems,” *Math. Problems Eng.*, vol. 2012, pp. 857–868, 2012, doi: [10.1155/2012/257619](https://doi.org/10.1155/2012/257619).
- [29] A. Fu, Y. Zhu, and E. Song, “The optimal Kalman type state estimator with multi-step correlated process and measurement noises,” in *Proc. Int. Conf. Embedded Softw. Syst.*, Nice, France, 2008, pp. 215–220.
- [30] P. Florchinger, “Finite-dimensional filter for a class of nonlinear systems with correlated noises,” *Stochastic Anal. Appl.*, vol. 34, no. 5, pp. 882–892, Jul. 2016, doi: [10.1080/07362994.2016.1182871](https://doi.org/10.1080/07362994.2016.1182871).
- [31] F. Li and W. J. Zeng, “Information fusion for multi-sensor system with finite-time correlated process noises,” *Lect. Notes Electr. Eng.*, vol. 241, no. 1, pp. 719–729, 2014, doi: [10.1007/978-3-642-40078-0-62](https://doi.org/10.1007/978-3-642-40078-0-62).
- [32] Y. L. Huang, Y. Zhang, X. Wang, and L. Zhao, “Gaussian filter for nonlinear systems with correlated noises at the same epoch,” *Automatica*, vol. 60, pp. 122–126, Oct. 2015, doi: [10.1016/j.automatica.2015.06.035](https://doi.org/10.1016/j.automatica.2015.06.035).
- [33] Y. Huang, Y. Zhang, N. Li, and Z. Shi, “Design of Gaussian approximate filter and smoother for nonlinear systems with correlated noises at one epoch apart,” *Circuits, Syst., Signal Process.*, vol. 35, no. 11, pp. 3981–4008, Feb. 2016, doi: [10.1007/s00034-016-0256-0](https://doi.org/10.1007/s00034-016-0256-0).
- [34] H. M. Qian, L. GE, W. Huang, and X. Liu, “Design of CKF with correlative noises based on Bayesian estimation,” *Syst. Eng. Electron.*, vol. 34, no. 11, pp. 2215–2218, Nov. 2012.
- [35] X. Wang, Y. Liang, Q. Pan, C. Zhao, and F. Yang, “Design and implementation of Gaussian filter for nonlinear system with randomly delayed measurements and correlated noises,” *Appl. Math. Comput.*, vol. 232, pp. 1011–1024, Apr. 2014, doi: [10.1016/j.amc.2013.12.168](https://doi.org/10.1016/j.amc.2013.12.168).
- [36] X. Wang, Y. Liang, Q. Pan, and F. Yang, “A Gaussian approximation recursive filter for nonlinear systems with correlated noises,” *Automatica*, vol. 48, no. 9, pp. 2290–2297, 2012, doi: [10.1016/j.automatica.2012.06.035](https://doi.org/10.1016/j.automatica.2012.06.035).
- [37] J. Xu, G. M. Dimirovski, Y. Jing, and C. Shen, “UKF design and stability for nonlinear stochastic systems with correlated noises,” in *Proc. 46th IEEE Conf. Decis. Control*, New Orleans, LA, USA, 2007, pp. 6226–6231.
- [38] R. E. Kalman, “A new approach to linear filtering and prediction problems,” *J. Basic Eng.*, vol. 82, no. 1, pp. 35–45, Mar. 1960.
- [39] S. J. Julier, J. K. Uhlmann, and H. F. Durrant-Whyte, “A new approach for filtering nonlinear systems,” in *Proc. Amer. Control Conf.*, Seattle, WA, USA, vol. 1995, pp. 1628–1632.
- [40] S. J. Julier and J. K. Uhlmann, “Unscented filtering and nonlinear estimation,” *Proc. IEEE*, vol. 92, no. 3, pp. 401–422, Mar. 2004, doi: [10.1109/JPROC.2003.823141](https://doi.org/10.1109/JPROC.2003.823141).
- [41] I. Arasaratnam and S. Haykin, “Cubature Kalman filters,” *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1254–1269, Jun. 2009, doi: [10.1109/TAC.2009.2019800](https://doi.org/10.1109/TAC.2009.2019800).
- [42] I. Arasaratnam and S. Haykin, “Cubature Kalman smoothers,” *Automatica*, vol. 47, no. 10, pp. 2245–2250, Oct. 2011, doi: [10.1016/j.automatica.2011.08.005](https://doi.org/10.1016/j.automatica.2011.08.005).
- [43] I. Arasaratnam, S. Haykin, and T. R. Hurd, “Cubature Kalman filtering for continuous-discrete systems: Theory and simulations,” *IEEE Trans. Signal Process.*, vol. 58, no. 10, pp. 4977–4993, Oct. 2010, doi: [10.1109/TSP.2010.2056923](https://doi.org/10.1109/TSP.2010.2056923).
- [44] T. B. Schön, A. Svensson, L. Murray, and F. Lindsten, “Probabilistic learning of nonlinear dynamical systems using sequential Monte Carlo,” *Mech. Syst. Signal Process.*, vol. 104, pp. 866–883, May 2018, doi: [10.1016/j.ymsp.2017.10.033](https://doi.org/10.1016/j.ymsp.2017.10.033).

- [45] Y. Wang and X. Rong Li, "Distributed estimation fusion with unavailable cross-correlation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 259–278, Jan. 2012, doi: [10.1109/TAES.2012.6129634](https://doi.org/10.1109/TAES.2012.6129634).
- [46] J. L. Cong, Y. Y. Li, G. Q. Qi, and A. D. Sheng, "An order insensitive sequential fast covariance intersection fusion algorithm," *Inf. Sci.*, vols. 367–368, pp. 28–40, Nov. 2016, doi: [10.1016/j.ins.2016.06.001](https://doi.org/10.1016/j.ins.2016.06.001).



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