An Improved Method to Manage Conflict Data Using Elementary Belief Assignment Function in the Evidence Theory

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ABSTRACT Dempster-Shafer evidence theory plays an important role in many applications such as multi-sensor data fusion and pattern recognition. However, if there are conflicts among evidences, the results of data fusion using Dempster combination rule may lead to counter-intuitive results. In this paper, we propose a new method named elementary belief assignment function for conflict data fusion. The proposed method aims at getting a more rational data fusion result by preprocessing the mass function before implementing data fusion with Dempster's combination rule. The elementary belief assignment function takes into consideration not only the number of focal elements in the current body of evidence but also the proposition in the power set space. By assigning the mass value of potential conflict focal element to other related propositions in the power set space, we can reduce the conflict level among different bodies of evidences effectively. We verify the rationality and efficiency of the proposed method according to several experiment examples.

INDEX TERMS Dempster-Shafer evidence theory, conflict management, conflict data fusion, basic belief assignment, elementary belief assignment function.

I. INTRODUCTION

Information fusion technology has been applied to a variety of scenarios, such as military situation assessment, artificial intelligence, cloud computing and so on. Information fusion needs to deal with a large number of uncertain information with the characteristics of multi-source heterogeneity, imprecision, unreliability and imperfection. Theoretical tools for processing uncertain information include probability theory [1], fuzzy set theory [2], rough set theory [3], Dempster-Shafer evidence theory (D-S evidence theory) [4]–[6], information entropy theory [7], [8] etc.

As a typical information fusion theory, D-S evidence theory is an extension of probability theory, that the basic probability assignment (BPA) function is a kind of imprecise probability [9]. It has been applied to solve problems in diverse areas. In [10]–[12], new methods are proposed in the framework of D-S theory for revealing the data structure in

clustering. In [13], [14], D-S theory is adopted for improving the reliability and efficiency in decision-making process. D-S theory is also effective in problems related to decision-making [15], [16], classification [17]–[19], sensor data fusion [20], and so on [21]–[23].

D-S evidence theory can deal with uncertainty information. However, Zadeh points out the potential problems in the evidence theory: if there is a conflict among the bodies of evidence, the fusion result may be counter-intuitive [24], [25]. Consequently, how to manage the conflict evidence is an important issue for uncertain information processing in the frame of D-S evidence theory [26]–[28]. The mathematical basis, validity and rationality of data fusion in evidence theory have become a focus of current research [29], [30]. The negation of BPA is a new mathematical tool for uncertain or conflict information processing in evidence theory [30]. D-numbers theory breaks through the mutual exclusion of elements in the frame of discernment [31], [32]. Dependent evidence fusion [33] is also a promising method for conflict evidence processing. DSmT theory redistributes

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the conflict information in the frame of discernment through logical operation to avoid conflicts in subsequent data fusion [34].

To address the conflict data fusion problem in evidence theory, current researches focus on modifying the Dempster's combination rule or modify the original mass function [35]–[37]. For the first class, some researchers argue that Dempster's combination rule leads to the lost of conflict information. So, the new combination rules need to focus on how to distribute the conflict. In particular, we should consider the conflict to be distributed to which subset and in what proportion. The method of modifying combination rules can solve the conflict problem in some degree. But the modification of classical combination rule may leads to losing some good properties in the combination rule, e.g. additivity and associativity in Dempster combination rule. For the second class, the method of modifying origin mass function retains the characteristics of the classic Dempster combination rule. Some methods proposed are (1) using a weighted average of the modified source data evidence [38], (2) the method of conflict evidence correction based on uncertainty measurement of evidence [39], (3) evidence assessment method for measuring the degree of conflict [40], [41]. There is no definite method for evidence modification. Recently, a new method named base belief function is proposed [42]. Base belief function modifies the origin mass function which is a new perspective for conflict data processing. However, base belief function assigns the base belief function among the power set space, which assigns too much mass value to the non-focal element propositions. In this paper, we improved the base belief function method by proposing a new method named the elementary belief assignment function. Compared with the previous method, the proposed method can reduce the amount of computation in some cases.

The rest of this paper is organized as follows. In Section II, the basic concepts in D-S evidence theory are briefly introduced. The elementary belief assignment function is proposed in Section III. Section IV presents some experiment results with the proposed method. Finally, conclusions and the possible following work are provided in Section V.

II. PRELIMINARIES
In this section, we will introduce some preliminaries in detail.

A. DEMPSTER-SHAFER EVIDENCE THEORY
Dempster-Shafer evidence theory (D-S evidence theory) is an imprecise reasoning theory first proposed by Dempster in 1967 [5] and further developed by Shafer in 1976 [4].

Definition 1: Frame of Discernment (FOD)

Frame of Discernment Ω is defined as a non-empty set $\Omega = \{\theta_1, \theta_2, \ldots, \theta_i \ldots, \theta_n\}$, it has N mutually exclusive elements. Its power set contains 2^N elements, which is represented as follows:

$$2^\Omega = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_N\}, \{\theta_1, \theta_2, \ldots, \theta_i\}, \ldots, \Omega\}. \quad (1)$$

Definition 2: Mass function

In FOD, the basic evidence function is defined to represent uncertain information. Mass function, also known as basic probability assignment (BPA) or basic belief assignment (BBA), $m(A)$ is a mapping of powerset 2^Ω on interval [0,1]. This mapping satisfies the following relation:

$$m(\emptyset) = 0, \quad \sum_{A \in \Omega} m(A) = 1. \quad (2)$$

If $m(A) > 0$, then A is called a focal element.

Definition 3: Body of evidence

Body of evidence is an evidence unit for uncertain information modeling based on FOD. A body of evidence is a tuple for proposition sets and their mass functions, defined as follows:

$$(X, m) = \{\langle A, m(A) \rangle : A \in 2^\Omega, m(A) > 0\}. \quad (3)$$

where, X is a subset on the powerset 2^Ω.

Definition 4: Dempster combination rule

In the framework of evidence theory, two separate sets of mass functions m_1, m_2 can be used for data fusion using the following rule (($m = m_1 \oplus m_2$)):

$$m(A) = \begin{cases} \dfrac{1}{1-k} \displaystyle\sum_{B \cap C = A} m_1(B)m_2(C), & A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \quad (4)$$

where

$$k = \sum_{B \cap C = \emptyset} m_1(B)m_2(C).$$

k is a normalized factor.

B. OPEN ISSUES IN THE CLASSICAL COMBINATION RULE
The following example clarifies a shortcoming in the classical Dempster's combination rule.

Example 1: Assuming that the FOD is $\Omega = \{a, b, c\}$, The BPA values for the two groups are as follows (adopted from Zadeh [24], [25]):

$$m_1(b) = 0.01, \quad m_1(c) = 0.99;$$
$$m_2(a) = 0.99, \quad m_2(b) = 0.01.$$

With Dempster combination rule, the fusion result is $m(b) = 1, m(a) = m(c) = m(a, b) = m(a, c) = m(b, c) = m(a, b, c) = 0$. Obviously, this result is counterintuitive, because the first evidence source strongly supports the proposition $\{c\}$ and the second evidence source strongly supports proposition $\{a\}$, but the fusion result assigns a belief of 0 for $\{a\}$ and $\{c\}$. Simultaneously, both sources of evidence have a very low belief for proposition $\{b\}$ in comparison with $\{a\}$ and $\{c\}$, but the fusion result assigns a value of 100% belief on $\{b\}$.

The example shows a shortcoming in classical Dempster combination rule. If the mass function of a subset is 0, then in the final fusion result, the corresponding proposition is still assigned a mass value of 0. However, in some cases, even if the mass function of some proposition is 0, the corresponding proposition should also be taken into consideration for addressing potential conflict data.

III. THE ELEMENTARY BELIEF ASSIGNMENT FUNCTION

In this section, we construct a new function named the elementary belief assignment function for evidence modification to get a reasonable fusion result regarding the open issue in the classical combination rule. We give some numerical examples to analyze the fusion results obtained by the proposed methods in comparison with the existed method.

Assuming that A is a set of N elements, we know that A has 2^N subsets. We discuss this issue in the closed world assumption, thus, $m(\emptyset) = 0$. Based on the above assumptions, we define the elementary belief assignment function $n(R_i)$ as follows:

$$n(R_i) = \frac{1}{\lambda}, \qquad (5)$$

where R_i represents a subset in FOD Ω. λ represents the number of focal elements contained in each body of evidence. For the same focal element mentioned in multiple body of evidence, the calculation is only once. For example, we have the following two bodies of evidence:

$$m_1(a) = 0.1, \quad m_1(b) = 0.9.$$
$$m_2(a) = 0.1, \quad m_2(a, c) = 0.2, \quad m_2(a, b, c) = 0.7.$$

Consequently, the number of focal elements contained in all the body of evidences is 4: $\{a\}$, $\{b\}$, $\{a, c\}$, $\{a, b, c\}$. It should be noted that $\{a\}$ is mentioned in both body of evidence, and it is only calculated once.

Once we have the elementary belief assignment function, the next step is to modify the data. The method is to take the arithmetic mean of the focal elements in each body of evidence. The mean value of the modified data is defined as follows:

$$M(R_i) = \frac{m(R_i) + n(R_i)}{1 + \frac{2^\Omega - 1}{\lambda}}. \qquad (6)$$

An illustration to the complete calculation process is shown in Figure 1. The elementary belief assignment function is used to modify the basic belief assignment as well as weaken its conflict. The proposed method considers both the size of the FOD and the number of focal elements in the body of evidence. For the initial belief distribution that contains approximate definite belief assignment such as 0.99 or a small belief assignment such as 0.01, the proposed method can weaken the potential conflict for a better result.

IV. EXPERIMENT RESULTS

To illustrate the rationality of the elementary belief assignment function, we give the following examples with different conflict cases.

Example 2: Assuming that the FOD is $\Omega = \{a, b, c\}$, the BPA values for the body of evidence are as follows:

$$m_1(a) = 0.99, \quad m_1(a, b) = 0.01;$$
$$m_2(b) = 0.01, \quad m_2(c) = 0.99.$$

With the proposed method, the calculation process is as follows.
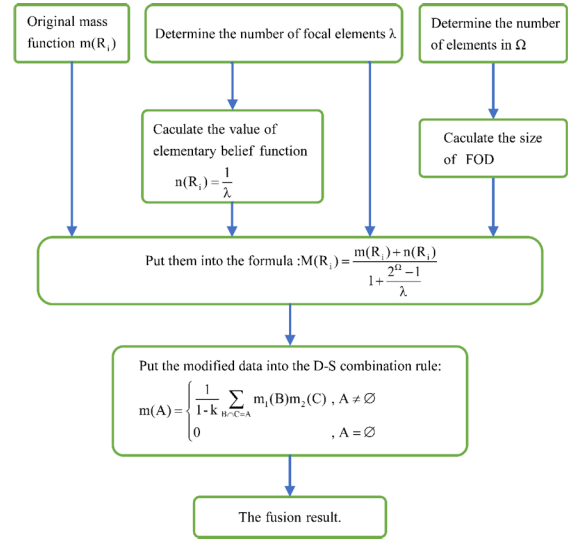

FIGURE 1. The flowchart of data processing with the elementary belief assignment function.

Step 1 : Determine the number of focal elements.

In this case, there are four focal elements denoted as $\{a\}$, $\{b\}$, $\{c\}$ and $\{a, b\}$. So, $\lambda = 4$.

Step 2 : Calculate the value of elementary belief assignment function based on Eq. (5).

$$n(R_i) = \frac{1}{\lambda} = \frac{1}{4}.$$

Step 3 : Calculate the size of FOD :

There are 3 elements in FOD, so the size of FOD is:

$$2^3 - 1 = 7.$$

Step 4 : Calculate the modified BPA values.

$$M_1(a) = \frac{m_1(a) + n(R_i)}{1 + \frac{2^\Omega - 1}{\lambda}} = \frac{0.99 + \frac{1}{4}}{1 + \frac{7}{4}} = 0.45.$$

$$M_1(a, b) = \frac{m_1(a, b) + n(R_i)}{1 + \frac{2^\Omega - 1}{\lambda}} = \frac{0.01 + \frac{1}{4}}{1 + \frac{7}{4}} = 0.09.$$

$$M_1(b) = M_1(c) = M_1(a, c) = M_1(b, c) = M_1(a, b, c)$$

$$= \frac{m_1(R_i) + n(R_i)}{1 + \frac{2^\Omega - 1}{\lambda}} = \frac{\frac{1}{4}}{1 + \frac{7}{4}} = 0.09.$$

$$M_2(b) = \frac{m_2(b) + n(R_i)}{1 + \frac{2^\Omega - 1}{\lambda}} = \frac{0.01 + \frac{1}{4}}{1 + \frac{7}{4}} = 0.09.$$

$$M_2(c) = \frac{m_2(c) + n(R_i)}{1 + \frac{2^\Omega - 1}{\lambda}} = \frac{0.99 + \frac{1}{4}}{1 + \frac{7}{4}} = 0.45.$$

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TABLE 1. Results of two combination methods of Example 3.

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(a, b, c)$
Classical Dempster's rule	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Proposed method	0.36	0.13	0.36	0.04	0.04	0.04	0.01
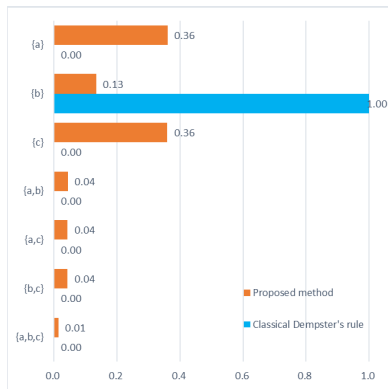
(a) Comparison of fusion results
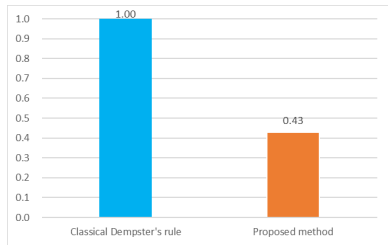
(b) Comparison of normalization factor k

FIGURE 2. Comparison of fusion results of Example 3.

$$M_2(a) = M_2(a, b) = M_2(a, c) = M_2(b, c) = M_2(a, b, c)$$

$$= \frac{m_2(c) + n(R_i)}{1 + \dfrac{2^\Omega - 1}{\lambda}} = \frac{0.99 + \dfrac{1}{4}}{1 + \dfrac{7}{4}} = 0.09.$$

Step 5 : Data fusion based on Dempster combination rule. The fusion result is shown as follows:

$$m(a) = 0.36, \quad m(b) = 0.13, \ m(c) = 0.36,$$
$$m(a, b) = 0.04, \quad m(a, c) = 0.04, \ m(b, c) = 0.04,$$
$$m(a, b, c) = 0.01.$$

The fusion result and the normalization factor k after data modification with the proposed method are compared with the result without using data modification, as shown in Table 1 and Figure 2.

In this case, the results of the classical Dempster's combination rule are anomalous. Both evidential bodies assign a mass value of almost 0 to $\{b\}$, but the fusion result indicates that the belief on $\{b\}$ is 100%. The value of the normalization factor k can be reduced to 0.43 from by using the proposed method. Accordingly, the fusion results indicated that $\{a\}$ and $\{c\}$ were assigned the same and highest levels

of BPA with a mass value of 0.36, which is conform to our intuition.

Example 3: Assuming that the FOD is $\Omega = \{a, b, c\}$, the BPA values for the body of evidence are as follows:

$$m_1(a) = 0.30, \quad m_1(b) = 0.30,$$
$$m_1(c) = 0.30, \quad m_1(a, b, c) = 0.10;$$
$$m_2(a, b) = 0.30, \quad m_2(b, c) = 0.30,$$
$$m_2(a, c) = 0.30, \quad m_2(a, b, c) = 0.10.$$

We calculate the fusion results by Dempster's combination rule and the normalization factor k with the modified data using the proposed method. The result is also compared with the result without data modification, as shown in Table 2 and Figure 3.
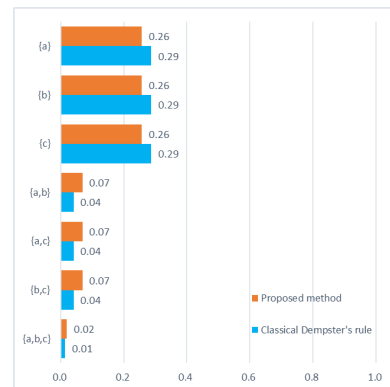
(a) Comparison of fusion results
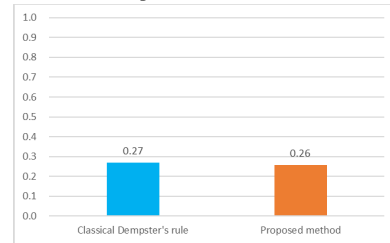
(b) Comparison of normalization factor k

FIGURE 3. Comparison of fusion results of Example 4.

It can be seen from Figure 2, the fusion results and the conflict coefficient obtained by the two methods are relatively close. This is decided by the characteristics of the original mass function values. Each subset in the identification framework is a focal element, and their mass function values are close to each other with low conflict. No evidence source strongly supports a specific subset. It also shows that, in the condition of low conflict, using the proposed method can achieve a rational fusion result.

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TABLE 2. Results of two combination methods of Example 4.

	$m(a)$	$m(b)$	$m(c)$	$m(a,b)$	$m(a,c)$	$m(b,c)$	$m(a,b,c)$
Classical Dempster's rule	0.29	0.29	0.29	0.04	0.04	0.04	0.01
Proposed method	0.26	0.26	0.26	0.07	0.07	0.07	0.02

Example 4: Assuming that the FOD is $\Omega = \{a, b\}$. The BPA values for the body of evidence are as follows:
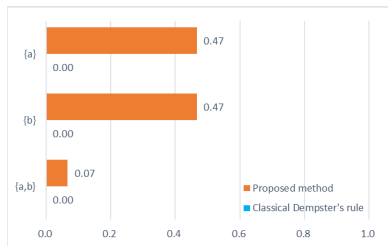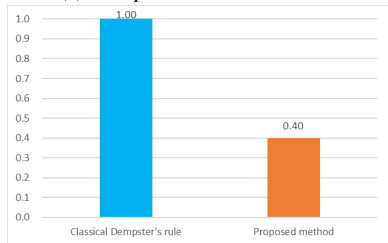
$$m_1(a) = 1;$$
$$m_2(b) = 1.$$

The fusion result and normalization factor k with and without the proposed method are shown in Table 3 and Figure 4.

TABLE 3. Results of two combination methods of Example 5.

	$m(a)$	$m(b)$	$m(a,b)$
Classical Dempster's rule	0.00	0.00	0.00
Proposed method	0.47	0.47	0.07

(a) Comparison of fusion results

(b) Comparison of normalization factor k

FIGURE 4. Comparison of fusion results of Example 5.

The method without data modification gives the result that $\{a\} = \{b\} = \{a, b\} = 0$, which is abnormal. In the original BPA distribution, the two sources of evidence strongly support $\{a\}$ and $\{b\}$. Thus, the belief support for $\{a\}$ and $\{b\}$ should be approximately equal in the fusion result. The result with the proposed method is $\{a\} = \{b\} = 0.47$ which is in line with expectations. In addition, the proposed method assigns a belief of 0.07 on $\{a, b\}$, which shows the superiority of D-S evidence theory in modeling uncertainty in a power set space rather than the set of basic events.

Example 5: Assuming that the FOD is $\Omega = \{a, b, c\}$. The BPA values for the body of evidence are as follows:

$$m_1(a) = 0.90, \quad m_1(b) = 0.00, \quad m_1(c) = 0.00,$$
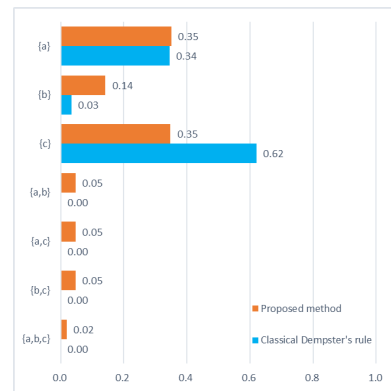$$m_1(a, b) = 0.00, \quad m_1(a, c) = 0.00,$$

$$m_1(b, c) = 0.00, \quad m_1(a, b, c) = 0.1;$$
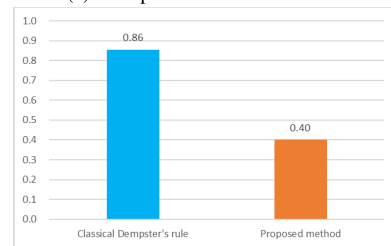$$m_2(a) = 0.05, \quad m_2(b) = 0.05, \quad m_2(c) = 0.90,$$
$$m_2(a, b) = 0.00, \quad m_2(a, c) = 0.00,$$
$$m_2(b, c) = 0.00, \quad m_2(a, b, c) = 0.00.$$

We compare the fusion results by using the two methods and the normalization factor, as shown in Table 4 and Figure 5.

(a) Comparison of fusion results

(b) Comparison of normalization factor k

FIGURE 5. Comparison of fusion results of Example 6.

In this case, the two bodies of evidence for $\{a\}$ and $\{c\}$ are assigned a belief of 0.9 respectively. However, in the fusion result without using the proposed method, the belief for $\{c\}$ is nearly two times than that for $\{a\}$. The value of k is 0.86, which can be seen as a high conflict in this case, the fusion result without data modification is not rational. The proposed method reduces the normalization factor k to 0.4. The fusion result with the proposed method is $\{a\} = \{c\} = 0.35$ which satisfies our intuitive.

Example 6: Assuming that the FOD is $\Omega = \{a, b\}$, the BPA values for the body of evidence are as follows:

$$m_1(a) = 1;$$
$$m_2(a, b) = 1.$$

The fusion result by using the classical Dempster's combination rule directly and the proposed method are shown

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TABLE 4. Results of two combination methods of Example 6.

	$m(a)$	$m(b)$	$m(c)$	$m(a,b)$	$m(a,c)$	$m(b,c)$	$m(a,b,c)$
Classical Dempster's rule	0.34	0.03	0.62	0.00	0.00	0.00	0.00
Proposed method	0.35	0.14	0.35	0.05	0.05	0.05	0.02

TABLE 5. Results of two combination methods of Example 7.

	$m(a)$	$m(b)$	$m(a,b)$
Classical Dempster's rule	1.00	0.00	0.00
Proposed method	0.62	0.24	0.14

(a) Comparison of fusion results

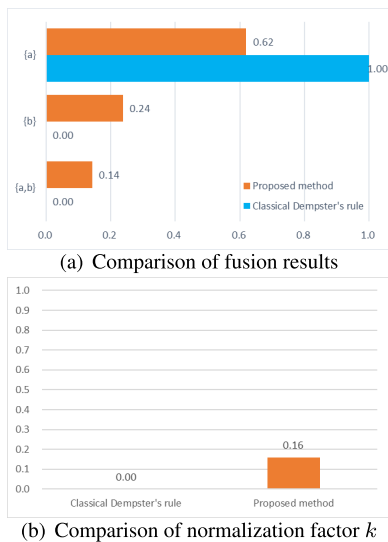(b) Comparison of normalization factor k

FIGURE 6. Comparison of fusion results of Example 7.

in Table 5 and Figure 6, as well as a comparison on the normalization factor k.

In the two bodies of evidence, the first source assigns 100% mass value on $\{a\}$ while the second source assigns 100% mass value on $\{a, b\}$. Therefore, from the intuition, the fusion results should assign a belief on $\{a\}$ and $\{a, b\}$ respectively. But the fusion results using Dempster's combination rule directly are $\{a\} = 1$, $\{b\} = \{a, b\} = 0$. It's clearly a breach of the intuition. In contrast, the proposed method assigns a certain mass value to $\{a\}$, $\{b\}$ and $\{a, b\}$ respectively and the belief on $\{a\}$ is the largest which is consist with the initial belief assignment in the sources of evidence.

V. CONCLUSION

If there is high conflict among evidence sources when using Dempster's combination rule for data fusion, unreasonable fusion results may occur. Solutions to this problem can be divided into two categories: modifying the original BPA and modifying the combination rules. In this paper, we improve the fusion result by proposing the elementary belief assignment function to modify the initial BPA functions.

The proposed method takes into account not only the focal element in the current body of evidence but also the proposition in the power set space, which can assign the belief to other related propositions in the power set space and subsequently reduce the conflict level effectively. In addition, if the conflict is small, the proposed method is compatible with the fusion result of using Dempster's combination rule directly. The proposed method does not modify the Dempster combination rule, thus, the superiority in the properties of the classical combination rule is satisfied, e.g. additivity and associativity in Dempster combination rule.

The proposed method still has some shortcomings. It can only be applied to the closed world assumption. In the open world assumption, the variety of uncertainties will increase, e.g. the type and number of unknown elements, the size of incomplete FOD [43], [44]. In this case, the elementary belief assignment function should be improved in a new method, which can be the following research direction of this work.

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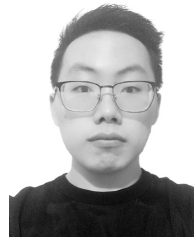
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