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Adaptive Neural Event-Triggered Control of MIMO Pure-Feedback Systems With Asymmetric Output Constraints and Unmodeled Dynamics

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ABSTRACT In this paper, the issue of adaptive neural event-triggered control (ETC) is studied for uncertain block-structure multi-input multi-output (MIMO) constrained non-affine nonlinear systems with unmodeled dynamics. A dynamic signal produced by the auxiliary system based on the property of unmodeled dynamics is employed to solve the dynamical disturbances. The unknown continuous function obtained at each step of recursion is estimated by using radial basis function neural networks (RBFNNs). Utilizing logarithmic function as an invertible mapping, the uncertain constrained MIMO non-affine system is changed into a novel unconstrained block-structure MIMO nonaffine system. Using improved dynamic surface control (DSC) strategy, adaptive event-triggered control scheme is developed for the transformed non-affine system based on relative threshold mechanism. According to the Lyapunov method, all the signals in the closed-loop system are shown to be semi-globally uniformly ultimately bounded (SGUUB). Output constraint requirements are not triggered, and Zeno behavior is avoided. A constrained pure-feedback system and a kind of 2-DOF flexible manipulator system are used to illustrate the theoretical findings.

INDEX TERMS Event-triggered control, dynamic surface control, block-structure nonlinear systems, unmodeled dynamics, output constraints.

I. INTRODUCTION

Since backstepping was proposed for a class of feedback linearizable systems in [1], it has widely been applied to construct adaptive controller for nonlinear systems as a popular tool in [2], [3]. Because it needs to differentiate for the designed virtual control at recursive each step, the controller design is quite complicated. This is its disadvantage in [1]–[3]. In order to remove this defect, dynamic surface control was proposed by introducing first-order filter in [4]. The computation complex in conventional backstepping is removed by using algebra operation to replace differential operation in DSC. Using mean value theorem and Nussbaum function, adaptive neural DSC method was developed for non-affine nonlinear systems in [6]. As we know that RBFNNs are a universal approximator. Based on the simple structure and infinite derivable properties, the RBFNNs has been widely used in adaptive backstepping control or DSC design. In [7], two robust adaptive DSC schemes were investigated by using RBFNNs for a class of pure-feeback systems with input nonlinearity and perturbed uncertainties. In addition, there widely exists unmodeled dynamics in modern industrial process, which usually degrades the system performance, sometimes, and leads to be instable for system. To deal with the impact of dynamical disturbances on the system, dynamic signal method in [8], [9] and Lyapunov function description in [10] as well as and normalization signal in [11] were usually employed to dispose of state and input unmodeled dynamics, respectively. However, the proposed design method was for the unmodeled dynamics in autonomous form in [8]. The dynamic signal method was developed for the unmodeled dynamics in nonautonomous form in [9].

Barrier Lyapunov function (BLF) in [12], [13], integral barrier Lyapunov function (iBLF) in [14] and nonlinear

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mapping (NM) in [15]–[18] were used to hand output constraints and full state constraints. However, single-input single-output (SISO) systems were discussed in [12]–[18]. In [19], fault-tolerant control was proposed by using BLF for uncertain parametric strict-feedback MIMO nonlinear systems with output restriction. Furthermore, based on BLF, decentralized adaptive finite time control was developed for uncertain pure-feedback system in [20]. Based on a novel BLF, robust adaptive control was investigated for uncertain MIMO nonlinear systems with state and input constraints in [21]. By introducing NM, adaptive neural control was proposed for MIMO nonlinear systems with time-varying output constraints in [22]. However, the dynamical disturbances were not studied in [19]–[22]. In [23], adaptive control scheme was developed for uncertain output constrained MIMO nonlinear systems in strict-feedback form based on both BLF and modified DSC. Furthermore, adaptive neural control was proposed for the MIMO pure-feedback systems with output constraints and dynamic uncertainties using both invertible nonlinear mapping and modified DSC in [24].

It is well known that the designed controller based on event-triggering mechanism can reduce energy consumption and occupation rate of transmission bandwidth. Event-triggered control (ETC) has become research hot topic in the past decade. By taking a given invariable difference value of the state norm as toggle condition, a simple event-triggered scheduler was proposed for a class of linear systems in [25]. In [26], integrating with model-based networked control theory, a novel ETC strategy with time-varying network delays was developed to a class of linear systems, and two fire-new error discriminants of states, fixed threshold strategy and relative threshold strategy, were designed as the toggle condition. In [27], [28], an event-sampled NN was invented to estimate the unknown terms in a class of strict-feedback systems. In [29]–[32], by structuring state-depended event-triggered condition, some adaptive control schemes were designed for a variety of uncertain systems. However, the above-mentioned achievements were dependent on the assumption that the closed loop system is input-to-state stable (ISS). To solve this problem, a new ETC controller design method was proposed for a class of affine systems with unknown parameters, and a new event-triggered condition called switching threshold strategy was proposed in [33]. Hereafter, some significant achievements based on [33] can be seen in [34]–[36]. However, the controlled plants in [34]–[36] were all SISO systems. Using RBFNNs to estimate the model had been done in many studies such as [37], [38].

In this paper, adaptive control is proposed based on event-triggering mechanism for uncertain constrained block-structure MIMO non-affine systems. To the best of our knowledge, the relative threshold strategy is first extended and applied to constrained MIMO uncertain pure-feedback systems. RBFNNs are used to appoximate the unknown continuous function vector, which is produced in the process of controller design. The unknown system functions have not been directly estimated by RBFNNs. The main contributions are listed as follows:

(1) Adaptive event-triggered DSC is proposed for output constrained block-structure pure-feedback nonlinear systems with dynamic uncertainties based on relative threshold strategy. By constructing first-order auxiliary system to produce a measurable signal, the dynamic uncertain terms are effectively dealt with. Furthermore, to fulfill the constraint requirements, using invertible nonlinear mapping (INM), the constrained block-structure non-affine nonlinear system is changed into an unconstrained one. Furthermore, the controller design is simplified based on the transformed system.
(2) The improved DSC approach is applied to the transformed unconstrained block-structure non-affine system, and the first-order auxiliary system is utilized to eliminate the repeated derivation of the intermediate variable in conventional backstepping.
(3) The updating laws with the only one tuning parameter for each approximated unknown function at recursive each step are proposed. Furthermore, a simple event-triggered control vector is developed using modified DSC and the property of hyperbolic tangent function in the final step.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following block-structure MIMO pure-feedback nonlinear systems

$$\begin{cases}\dot{\zeta} = Q(\zeta, \bar{x}_n, t),\\ \dot{x}_i = f_i(\bar{x}_i, x_{i+1}) + d_i(\zeta, \bar{x}_n, t), \quad 1 \le i \le n-1,\\ \dot{x}_n = f_n(\bar{x}_n, u) + d_n(\zeta, \bar{x}_n, t),\\ y = x_1,\end{cases} \tag{1}$$

where $x_i = [x_{i1}, \ldots, x_{im}]^T \in \mathbb{R}^m, i = 1, \ldots, n$ are the states, $f_i(\cdot) = [f_{i1}(\cdot), f_{i2}(\cdot), \ldots, f_{im}(\cdot)]^T \in \mathbb{R}^m$, $(i = 1, \ldots, n)$ are the unknown smooth nonlinear function vectors, $d_i(\cdot) = [d_{i1}(\cdot), d_{i2}(\cdot), \ldots, d_{im}(\cdot)]^T \in \mathbb{R}^m$ are the unknown smooth nonlinear dynamic disturbances, $\zeta \in \mathbb{R}^{n_0}$ is unmodeled dynamics, $Q(\zeta, \bar{x}_n, t) \in \mathbb{R}^{n_0}$ is an unknown smooth function vector satisfying the Lipschitz condition, $u = [u_1, \ldots, u_m]^T \in \mathbb{R}^m$ is the input, $\bar{x}_i = [x_1^T, \ldots, x_i^T]^T$, $y = [y_{11}, \ldots, y_{1m}]^T \in \mathbb{R}^m$ is the output.

In this paper, $\|A\| = \sqrt{tr(A^T A)}$ denotes the Frobenius norm of the matrix A, $\|\xi\| = \sqrt{\sum_{i=1}^n \xi_i^2}$ stands for the Euclidean norm of the vector $\xi = [\xi_1, \ldots, \xi_n]^T \in \mathbb{R}^n$, L_∞ denotes the set of all bounded functions.

The control objective is to design the adaptive event-triggered control u for system (1) such that the output $y(t) = [y_{11}(t), \ldots, y_{1m}(t)]^T$ can follow the desired tracking signal $y_d(t) = [y_{d1}(t), \ldots, y_{dm}(t)]^T$, meantime, it satisfies $-k_{j1} < y_{1j}(t) < k_{j2}$, $\forall t \ge 0$ with k_{j1} and k_{j2} being positive design constants, $j = 1, \ldots, m$, and all of signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB).

Definition 1 [9]: For system $\dot{\zeta} = Q(\zeta, \bar{x}_n, t)$, if there exist class K_∞ functions $\alpha_1(\cdot), \alpha_2(\cdot)$ and a Lyapunov function $U(\zeta)$ such that

$$\alpha_1(\|\zeta\|) \le U(\zeta) \le \alpha_2(\|\zeta\|), \tag{2}$$

and there exist two known constants $c > 0, d > 0$ and a known class K_∞ function $\gamma(\cdot)$ such that

$$\frac{\partial U(\zeta)}{\partial \zeta} Q(\zeta, \bar{x}_n, t) \le -cU(\zeta) + \gamma(\|x_1\|) + d, \tag{3}$$

then the system is called as exponentially input-state-practically stable (exp-ISpS).

Assumption 1: For block-structure system (1), $f_n(\cdot)$ is continuous differentiable, and $\frac{\partial f_n(\cdot)}{\partial u} \in L_\infty$ holds.

Assumption 2 [9]: For block-structure system (1), the disturbances $d_{ij}(\zeta, \bar{x}_n, t)$ satisfy the following inequalities

$$|d_{ij}(\zeta, \bar{x}_n, t)| \le \varphi_{ij1}(\|\bar{x}_{ij}\|) + \varphi_{ij2}(\|\zeta\|), \tag{4}$$

where $\varphi_{ij1}(\cdot)$ are nonnegative smooth functions, $\varphi_{ij2}(\cdot)$ are nonnegative monotone increasing smooth functions, $\bar{x}_{ij} = [x_1^T, \ldots, x_{i-1}^T, x_{i1}, \ldots, x_{ij}]^T, i = 1, \ldots, n, j = 1, \ldots, m$.

Assumption 3: The tracking signal $y_d(t)$, its first derivative $\dot{y}_d(t)$ and second derivative $\ddot{y}_d^T(t)$ are continuous and available, and $[y_d^T(t), \dot{y}_d^T(t), \ddot{y}_d^T(t)]^T \in \Omega_d = \Big\{[y_d^T(t), \dot{y}_d^T(t), \ddot{y}_d^T(t)]^T : y_d^T(t)y_d(t) + \dot{y}_d^T(t)\dot{y}_d(t) + \ddot{y}_d^T(t)\ddot{y}_d(t) \le B_0\Big\} \subset \mathbb{R}^{3m}$. Moreover, $|y_{dj}(t)| < B_{1j} < \min\{k_{j1}, k_{j2}\}$ hold, where B_0, B_{1j} are $m+1$ known positive constants, k_{j1}, k_{j2} are $2m$ positive design constants, satisfying $-k_{j1} < y_{1j}(t) < k_{j2}, \forall t \ge 0$.

Assumption 4 [9]: System $\dot{\zeta} = Q(\zeta, \bar{x}_n, t)$ is exponentially input-state-practically stable (exp-ISpS).

Lemma 1 [9]: If U is an exp-ISpS Lyapunov function for a system $\dot{\zeta} = Q(\zeta, \bar{x}_n, t)$, i.e., (2) and (3) hold, then $\forall \bar{c} \in (0, c)$, $\forall t_0 > 0$, $\forall \zeta_0 = \zeta(t_0)$, $\nu_0 > 0$, for any continuous class K_∞ function $\bar{\gamma}(\|x_1\|)$ such that $\bar{\gamma}(\|x_1\|) \ge \gamma(\|x_1\|)$, there exist a finite $T_0 = \max\{0, \ \ln[U(\zeta_0)/\nu_0]/(c - \bar{c})\}$, a nonnegative function $D(t_0, t)$, defined for all $t \ge t_0$ and a signal described by

$$\dot{\nu} = -\bar{c}\nu + \bar{\gamma}(\|x_1\|) + d, \quad \nu(t_0) = \nu_0 > 0 \tag{5}$$

such that $D(t_0, t) = 0$ for $t \ge t_0 + T_0$, and $U(\zeta) \le \nu(t) + D(t_0, t)$ with $D(t_0, t) = \max\Big\{0, \exp[-c(t - t_0)]U(\zeta_0) - \exp[-\bar{c}(t - t_0)]\nu_0\Big\}$.

Remark 1: $f_n(\bar{x}_n, u)$ can be written as

$$\begin{aligned} f_n(\bar{x}_n, u) &= [f_n(\bar{x}_n, u) - f_n(0, x_2, \ldots, x_n, u)] \\ &\quad + [f_n(0, x_2, \ldots, x_n, u) - f_n(0, 0, x_3, \ldots, x_n, u)] \\ &\quad + [f_n(0, 0, x_3, \ldots, x_n, u) - f_n(0, 0, 0, x_4, \ldots, x_n, u)] \\ &\quad + \ldots + [f_n(0, \ldots, 0, u) - f_n(0, \ldots, 0)] \\ &\quad + f_n(0, \ldots, 0). \end{aligned} \tag{6}$$

According to the discussion in [24], we have

$$f_n(\bar{x}_n, u) = G_{xn}(\bar{x}_n)\bar{x}_n + G_{un}(\bar{x}_n, u)u + f_n(0), \tag{7}$$

where $f_n(0) = f_n(0, \ldots, 0)$, $G_{xn}(\bar{x}_n) = [G_{n1}(\bar{x}_n), \ldots, G_{nn}(\bar{x}_n)]$, $\|G_{ni}\|$ and $\|G_{un}\|$ are bounded.

According to neural network universal theorem as discussed in [39], the unknown continuous function $\Sigma(Z) : \mathbb{R}^\ell \to \mathbb{R}$ over the compact set $\Omega_Z \subset \mathbb{R}^\ell$ can be denoted as follows:

$$\Sigma(Z) = \vartheta^{*T}\Xi(Z) + \varrho(Z), \tag{8}$$

where $Z \in \Omega_Z \subset \mathbb{R}^\ell$ is the neural network input vector, $\varrho(Z)$ is the approximation error, $\vartheta^* = [\vartheta_1^*, \vartheta_2^*, \ldots, \vartheta_\ell^*]^T \in \mathbb{R}^\ell$ is the ideal weight vector ($\ell > 0$), which has the form

$$\vartheta^* = \arg\min_{\vartheta \in \mathbb{R}^\ell}\left[\sup_{Z \in \Omega_Z} |\vartheta^T\Xi(Z) - \Sigma(Z)|\right], \tag{9}$$

$\Xi(Z) = [\Xi_1(Z), \Xi_2(Z), \ldots, \Xi_\ell(Z)]^T \in \mathbb{R}^\ell$ is the basis vector, the basis function $\Xi_i(Z)$ is selected as Gaussian function

$$\Xi_i(Z) = \exp\left[-\frac{\|Z - \mu_i\|^2}{\varsigma_i^2}\right], \quad i = 1, 2, \ldots, l, \tag{10}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the central of the receptive fields and ς_i is the width of the Gaussian function.

Remark 2: Assume $\Sigma_i(Z_i) = \Big[\Sigma_{i1}(Z_i), \Sigma_{i2}(Z_i), \ldots, \Sigma_{im}(Z_i)\Big]^T, i = 1, \ldots, n$, then $\Sigma_i(Z_i) = \Theta_i^{*T}\Xi_i(Z_i) + \varrho_i(Z_i)$, where $\Theta_i^* = \text{blockdiag}[\vartheta_{i1}^*, \ldots, \vartheta_{im}^*]$, $\vartheta_{ij}^* = \Big[\vartheta_{ij1}^*, \ldots, \vartheta_{ijl_{ij}}^*\Big]^T \in \mathbb{R}^{l_{ij}}$, $l_{ij} > 0, j = 1, \ldots, m$, $\Xi_i(Z_i) = [\Xi_{i1}^T(Z_i), \ldots, \Xi_{im}^T(Z_i)]^T$, $\Xi_{ij}(Z_i) = \Big[\Xi_{ij1}(Z_i), \ldots, \Xi_{ijl_{ij}}(Z_i)\Big]^T$, l_{ij} is the number of nodes in the ijth neural networks, $\varrho_i(Z_i) = \Big[\varrho_{i1}(Z_i), \varrho_{i2}(Z_i), \ldots, \varrho_{im}(Z_i)\Big]^T$ is the approximation error.

Inspired by [17], [18], to dispose of output restriction, define the invertible nonlinear mapping as follows:

$$\begin{cases} z_{1j} = \dfrac{1}{2}\ln\dfrac{(k_{j1} + x_{1j})}{(k_{j2} - x_{1j})} - \dfrac{1}{2}\ln\dfrac{k_{j1}}{k_{j2}}, & j = 1, \ldots, m, \\ z_i = x_i, & i = 2, \ldots, n, \end{cases} \tag{11}$$

where $k_{j1}, k_{j2} > 0$ are two known constraint constants subject to $-k_{j1} < x_{1j} < k_{j2}$.

Noting (11), its invertible mapping is

$$\begin{cases} x_{1j} = \dfrac{k_{j1} + k_{j2}}{2}\tanh\left(z_{1j} + \dfrac{1}{2}\ln\dfrac{k_{j1}}{k_{j2}}\right) - \dfrac{k_{j1} - k_{j2}}{2}, \\ j = 1, \ldots, m, \\ x_i = z_i, \quad i = 2, \ldots, n. \end{cases} \tag{12}$$

Differentiating x_{ij} for time t, it yields

$$\begin{cases} \dot{z}_{1j} = \varpi_{1j}(z_{1j})\dot{x}_{1j}, & j = 1, \ldots, m, \\ \dot{z}_i = \dot{x}_i, & i = 2, \ldots, n, \end{cases} \tag{13}$$

where

$$\varpi_{1j}(z_{1j}) = \frac{2}{(k_{j1} + k_{j2})\Big[1 - \tanh^2(z_{1j} + \frac{1}{2}\ln\frac{k_{j1}}{k_{j2}})\Big]}.$$

Let $\varpi(z_1) = \text{diag}[\varpi_{11}(z_{11}), \dots, \varpi_{1m}(z_{1m})]$, then system (1) can be rewritten as

$$\begin{cases} \dot{\zeta} = Q(\zeta, \bar{x}_n, t), \\ \dot{z}_1 = F_1(z_1, z_2) + D_1(\zeta, \bar{z}_n, t), \\ \dot{z}_i = F_i(\bar{z}_i, z_{i+1}) + D_i(\zeta, \bar{z}_n, t), \quad 2 \le i \le n-1, \\ \dot{z}_n = F_n(\bar{z}_n, u) + D_n(\zeta, \bar{z}_n, t), \\ y = z_1 \end{cases} \tag{14}$$

where $F_1(z_1, z_2) = \varpi(z_1)f(x_1, x_2)$, $F_i(\bar{z}_i, z_{i+1}) = f_i(\bar{x}_i, x_{i+1})$, $i = 2, \dots, n-1$, $F_n(\bar{z}_n, u) = f_n(\bar{x}_n, u)$, $D_1(\zeta, \bar{z}_n, t) = \varpi(z_1)d_1(\zeta, \bar{x}_n, t)$, $D_i(\zeta, \bar{z}_n, t) = d_i(\zeta, \bar{x}_n, t)$, $i = 2, \dots, n$, $z_i = [z_{i1}, \dots, z_{im}]^T$, $\bar{z}_i = [z_1^T, \dots, z_i^T]^T$, $i = 1, \dots, n$.

Remark 3: The uncertain terms $D_{ij}(\zeta, \bar{x}_n, t)$ satisfy $|D_{ij}(\zeta, \bar{x}_n, t)| \le \varphi_{ij1}(\|\bar{x}_{ij}\|) + \varphi_{ij2}(\|\zeta\|)$, $i = 1, \dots, n$, $j = 1, \dots, m$, so $|D_{1j}(\zeta, \bar{z}_n, t)| \le \varpi_{1j}(z_{1j})[\varphi_{1j1}(\|\bar{x}_{1j}\|) + \varphi_{1j2}(\|\zeta\|)]$, where $D_i(\cdot) = [D_{i1}(\cdot), \dots, D_{im}(\cdot)]^T$. Based on (2), it is easy to know $\|\zeta\| \le \alpha_1^{-1}(U(\zeta))$. Using Lemma 1, we have $\|\zeta\| \le \alpha_1^{-1}(\nu + D_0)$, $\forall t \ge 0$, it will be employed in the derivation later.

III. ADAPTIVE EVENT-TRIGGERED DYNAMIC SURFACE CONTROL

For convenience, we introduce the following notations $\bar{\chi}_i = [\chi_1^T, \dots \chi_i^T]^T$, $\bar{e}_i = [e_2^T, \dots e_i^T]^T$, $\bar{\hat{\beta}}_{im} = [\hat{\beta}_{11}, \dots, \hat{\beta}_{1m}, \dots, \hat{\beta}_{i1}, \dots, \hat{\beta}_{im}]^T$, $\beta_{ij}^* = \|\vartheta_{ij}^*\|^2$, $\beta_i^* = \text{diag}\left[\beta_{i1}^*, \dots, \beta_{im}^*\right]$, $\tilde{\beta}_{ij} = \hat{\beta}_{ij} - \beta_{ij}^*$, $\tilde{\beta}_i = \hat{\beta}_i - \beta_i^*$, $\hat{\beta}_i = \text{diag}\left[\hat{\beta}_{i1}, \dots, \hat{\beta}_{im}\right]$, $i = 1, \dots, n$, $j = 1, \dots, m$, where $\hat{\beta}_i$ and $\hat{\beta}_{ij}$ are the estimates of β_i^* and β_{ij}^* at time t.

The coordinate change and the error signals are defined as follows:

$$\begin{cases} \chi_1 = z_1 - \hat{y}_d, \\ \chi_2 = z_2 - \hbar_2, \\ \quad \vdots \\ \chi_n = z_i - \hbar_n, \end{cases} \tag{15}$$

$$\begin{cases} e_2 = \hbar_2 - \alpha_1, \\ \quad \vdots \\ e_n = \hbar_n - \alpha_{n-1}, \end{cases} \tag{16}$$

where $\hat{y}_{dj} = \frac{1}{2}\ln\frac{k_{j2}(k_{j1}+y_{dj})}{k_{j1}(k_{j2}-y_{dj})}$, $j = 1, \dots, m$, $\hat{y}_d = \left[\hat{y}_{d1}, \dots, \hat{y}_{dm}\right]^T$, $\alpha_1, \dots, \alpha_{n-1}, \hbar_2, \dots, \hbar_n$ will be given later.

Step 1: Let $\chi_1 = z_1 - \hbar_1$ with $\hbar_1 = \hat{y}_d$. Differentiating χ_1 for time t, we get

$$\begin{aligned} \dot{\chi}_1 &= F_1(z_1, z_2) + D_1(\zeta, \bar{z}_n, t) - \dot{\hbar}_1 \\ &= z_2 + F_1(z_1, z_2) - z_2 + D_1(\zeta, \bar{z}_n, t) - \dot{\hbar}_1. \end{aligned} \tag{17}$$

Choose V_1 as follows:

$$V_1 = \frac{1}{2}\chi_1^T\chi_1. \tag{18}$$

Using (17), the time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= \chi_1^T\left[z_2 + F_1(z_1, z_2) - z_2 + D_1(\zeta, \bar{z}_n, t) - \dot{\hbar}_1\right] \\ &= \chi_1^T\left[\chi_2 + e_2 + \alpha_1 + F_1(z_1, z_2) - z_2 + D_1(\zeta, \bar{z}_n, t) - \dot{\hbar}_1\right], \end{aligned} \tag{19}$$

where $z_2 = e_2 + \chi_2 + \alpha_1$.

Using the inequality $ab \le a^2 + \frac{b^2}{4}$, we obtain

$$\chi_1^T\chi_2 \le \chi_1^T\chi_1 + \frac{1}{4}\chi_2^T\chi_2, \tag{20}$$

$$\chi_1^Te_2 \le \chi_1^T\chi_1 + \frac{1}{4}e_2^Te_2. \tag{21}$$

From Assumption 2, we get

$$\begin{aligned} &\chi_1^TD_1(\zeta, \bar{z}_n, t) \\ &\le \|\chi_1\|\sum_{j=1}^m\left|D_{1j}(\zeta, z, t)\right| \\ &\le \|\chi_1\|\sum_{j=1}^m\varpi_{1j}(z_{1j})[\varphi_{1j1}\left(\left\|\bar{x}_{1j}\right\|\right) + \varphi_{1j2}(\|\zeta\|)] \\ &\le \chi_1^T\chi_1\sum_{j=1}^m\frac{\varpi_{1j}^2(z_{1j})[\varphi_{1j1}\left(\left\|\bar{x}_{1j}\right\|\right) + \varphi_{1j2}\left(\alpha_1^{-1}(\nu + D_0)\right)]^2}{2a_{1j}^2} \\ &\quad + \sum_{j=1}^m\frac{a_{1j}^2}{2}. \end{aligned} \tag{22}$$

Let

$$\Sigma_1(Z_1) = \chi_1\sum_{j=1}^m\frac{\varpi_{1j}^2(z_{1j})[\varphi_{1j1}(\|\bar{x}_{1j}\|) + \varphi_{1j2}(\alpha_1^{-1}(\nu + D_0))]^2}{2a_{1j}^2} + F_1(z_1, z_2) - z_2 - \dot{\hbar}_1. \tag{23}$$

Using RBFNNs to approximate the unknown continuous function $\Sigma_1(Z_1)$, we have

$$\Sigma_1(Z_1) = \Theta_1^{*T}\Xi_1(Z_1) + \varrho_1, \tag{24}$$

where $Z_1 = [z_1^T, z_2^T, \chi_1^T, \dot{\hbar}_1^T, \nu]^T \in \mathbb{R}^{4m+1}$, the ideal weight matrix is $\Theta_1^* = \text{blockdiag}[\vartheta_{11}^*, \dots, \vartheta_{1m}^*]$, the basis vector is $\Xi_1(Z_1) = [\Xi_{11}^T(Z_1), \dots, \Xi_{1m}^T(Z_1)]^T$, the approximation error is $\varrho_1 = [\varrho_{11}, \dots, \varrho_{1m}]^T$.

Design the virtual control α_1 and the updating law of the estimated diagonal matrix $\hat{\beta}_1$ as follows

$$\alpha_1 = -K_1\chi_1 - \frac{1}{2a_1^2}\hat{\beta}_1\Phi_1(Z_1), \tag{25}$$

$$\dot{\hat{\beta}}_1 = \text{diag}[r_{11}, \dots, r_{1m}] \times \left[\frac{1}{2a_1^2}\text{diag}[\chi_{11}, \dots, \chi_{1m}]\Psi_1(Z_1) - \sigma_1\hat{\beta}_1\right], \tag{26}$$

where $K_1 = K_1^T > 0$, $a_1 > 0$, $\sigma_1 > 0$, $r_{1j} > 0$, $\hat{\beta}_1 = \text{diag}[\hat{\beta}_{11}, \cdots, \hat{\beta}_{1m}]$, $\Phi_1(Z_1) = [\|\Xi_{11}(Z_1)\|^2\chi_{11}, \dots, \|\Xi_{1m}(Z_1)\|^2\chi_{1m}]^T$, $\Psi_1(Z_1) = \text{diag}[\|\Xi_{11}(Z_1)\|^2\chi_{11}, \dots, \|\Xi_{1m}(Z_1)\|^2\chi_{1m}]$, $\beta_{1k}^* = \|\vartheta_{1k}^*\|^2$,

$\hat{\beta}_{1k}$ are the estimates of unknown parameters $\beta_{1k}^*, k = 1, \ldots, m$.

Substituting (20)–(25) into (19) and using $\chi_1^T \varrho_1 \leq \chi_1^T \chi_1 + \frac{1}{4}\varrho_1^T \varrho_1$, we obtain

$$\begin{aligned}\dot{V}_1 \leq& -\chi_1^T K_1 \chi_1 + \chi_1^T \Theta_1^{*T} \Xi_1(Z_1) + \chi_1^T \varrho_1 \\ &- \frac{1}{2a_1^2}\chi_1^T \hat{\beta}_1 \Phi_1(Z_1) \\ &+ 2\chi_1^T \chi_1 + \frac{1}{4}\chi_2^T \chi_2 + \frac{1}{4} e_2^T e_2 + \sum_{j=1}^m \frac{a_{1j}^2}{2} \\ \leq& -\chi_1^T K_1 \chi_1 + \chi_1^T \Theta_1^{*T} \Xi_1(Z_1) \\ &- \frac{1}{2a_1^2}\chi_1^T \hat{\beta}_1 \Phi_1(Z_1) + 3\chi_1^T \chi_1 + \frac{1}{4}\chi_2^T \chi_2 \\ &+ \frac{1}{4} e_2^T e_2 + \frac{1}{4}\varrho_1^T \varrho_1 + \sum_{j=1}^m \frac{a_{1j}^2}{2}. \end{aligned} \tag{27}$$

There exist nonnegative continuous functions $\kappa_{1j}(\bar{\chi}_2, e_2, \bar{\hat{\beta}}_{1m}, \nu, y_d, \dot{y}_d), j = 1, \ldots, m$ such that $|\varrho_{1j}| \leq \kappa_{1j}$ hold. Let $\kappa_1 = [\kappa_{11}, \ldots, \kappa_{1m}]^T$, (27) can be written as

$$\begin{aligned}\dot{V}_1 \leq& -\chi_1^T (K_1 - 3I_{m\times m})\chi_1 - \frac{1}{2a_1^2}\chi_1^T \tilde{\beta}_1 \Phi_1(Z_1) + \frac{a_1^2}{2} \\ &+ \frac{1}{4}\chi_2^T \chi_2 + \frac{1}{4} e_2^T e_2 + \frac{1}{4}\kappa_1^T \kappa_1 + \sum_{j=1}^m \frac{a_{1j}^2}{2}. \end{aligned} \tag{28}$$

Define $\hbar_2$ as follows:

$$\tau_2 \dot{\hbar}_2 + \hbar_2 = \alpha_1, \hbar_2(0) = \alpha_1(0), \tag{29}$$

where $\tau_2 > 0$ is a positive design constant.

Noting (16), differentiating e_2 for time t, we have

$$\dot{e}_2 = -\frac{e_2}{\tau_2} - \dot{\alpha}_1. \tag{30}$$

Then

$$e_2^T \dot{e}_2 \leq -\frac{e_2^T e_2}{\tau_2} + \|e_2\| \eta_2(\bar{\chi}_3, \bar{e}_3, \bar{\hat{\beta}}_{1m}, \nu, y_d, \dot{y}_d, \ddot{y}_d), \tag{31}$$

where $\eta_2(\bar{\chi}_3, \bar{e}_3, \bar{\hat{\beta}}_{1m}, \nu, y_d, \dot{y}_d, \ddot{y}_d)$ is a nonnegative continuous function.

Using the inequality $ab \leq a^2 + \frac{b^2}{4}$ and (31), we get

$$e_2^T \dot{e}_2 \leq -\frac{\|e_2\|^2}{\tau_2} + \|e_2\|^2 + \frac{1}{4}\|\eta_2\|^2. \tag{32}$$

Step i $(2 \leq i \leq n-1)$: Let $\chi_i = z_i - \hbar_i$. Differentiating χ_i for time t, we obtain

$$\dot{\chi}_i = F_i(\bar{z}_i, z_{i+1}) + D_i(\zeta, \bar{z}_n, t) - \dot{\hbar}_i. \tag{33}$$

Define V_i as follows

$$V_i = \frac{1}{2}\chi_i^T \chi_i. \tag{34}$$

From (33), the time derivative of V_i is

$$\begin{aligned}\dot{V}_i =& \chi_i^T \dot{\chi}_i = \chi_i^T [F_i(\bar{z}_i, z_{i+1}) + D_i(\zeta, z, t) - \dot{\hbar}_i] \\ =& \chi_i^T [F_i(\bar{z}_i, z_{i+1}) + z_{i+1} - z_{i+1} + D_i(\zeta, \bar{z}_n, t) - \dot{\hbar}_i] \\ =& \chi_i^T [\chi_{i+1} + e_{i+1} + \alpha_i + F_i(\bar{z}_i, z_{i+1}) - z_{i+1} \\ &+ D_i(\zeta, \bar{z}_n, t) - \dot{\hbar}_i], \end{aligned} \tag{35}$$

where $z_{i+1} = \chi_{i+1} + e_{i+1} + \alpha_i$.

Using the inequality $ab \leq a^2 + \frac{b^2}{4}$, it yields

$$\chi_i^T \chi_{i+1} \leq \chi_i^T \chi_i + \frac{1}{4}\chi_{i+1}^T \chi_{i+1}, \tag{36}$$

$$\chi_i^T e_{i+1} \leq \chi_i^T \chi_i + \frac{1}{4} e_{i+1}^T e_{i+1}. \tag{37}$$

From Assumption 2, it yields

$$\begin{aligned}&\chi_i^T D_i(\zeta, \bar{z}_n, t) \\ &\leq \|\chi_i\| \sum_{j=1}^m \left| D_{ij}(\zeta, \bar{z}_n, t) \right| \\ &\leq \|\chi_i\| \sum_{j=1}^m [\varphi_{ij1}(\|\bar{x}_{ij}\|) + \varphi_{ij2}(\|\zeta\|)] \\ &\leq \chi_i^T \chi_i \sum_{j=1}^m \frac{[\varphi_{ij1}(\|\bar{x}_{ij}\|) + \varphi_{ij2}\left(\alpha_1^{-1}(\nu + D_0)\right)]^2}{2a_{ij}^2} \\ &\quad + \sum_{j=1}^m \frac{a_{ij}^2}{2}. \end{aligned} \tag{38}$$

Let

$$\begin{aligned}\Sigma_i(Z_i) =& \chi_i \sum_{j=1}^m \frac{[\varphi_{ij1}(\|\bar{x}_{ij}\|) + \varphi_{ij2}(\alpha_1^{-1}(\nu + D_0))]^2}{2a_{ij}^2} \\ &+ F_i(\bar{z}_i, z_{i+1}) - z_{i+1} - \dot{\hbar}_i. \end{aligned} \tag{39}$$

Using RBFNNs to appoximate the unknown function vector $\Sigma_i(Z_i)$, we have

$$\Sigma_i(Z_i) = \Theta_i^{*T} \Xi_i(Z_i) + \varrho_i, \tag{40}$$

where $Z_i = [z_1^T, \ldots, z_{i+1}^T, \chi_i^T, \dot{\hbar}_i^T, \nu]^T \in \mathbb{R}^{(i+3)m+1}$, the ideal weight matrix is $\Theta_i^* = \text{blockdiag}[\vartheta_{i1}^*, \ldots, \vartheta_{im}^*]$, the basis vector is $\Xi_i(Z_i) = [\Xi_{i1}^T(Z_i), \ldots, \Xi_{im}^T(Z_i)]^T$, the approximation error is $\varrho_i = [\varrho_{i1}, \ldots, \varrho_{im}]^T$.

Design the virtual control α_i and the updating law of the estimated diagonal matrix $\hat{\beta}_i$ as follows:

$$\alpha_i = -K_i \chi_i - \frac{1}{2a_i^2}\hat{\beta}_i \Phi_i(Z_i), \tag{41}$$

$$\begin{aligned}\dot{\hat{\beta}}_i =& \text{diag}[r_{i1}, \ldots, r_{im}] \\ &\times \left[\frac{1}{2a_i^2}\text{diag}[\chi_{i1}, \ldots, \chi_{im}]\Psi_i(Z_i) - \sigma_i \hat{\beta}_i\right], \end{aligned} \tag{42}$$

where $K_i = K_i^T > 0$, $a_i > 0$, $\sigma_i > 0$, $r_{ij} > 0$, $\hat{\beta}_i = \text{diag}[\hat{\beta}_{i1}, \cdots, \hat{\beta}_{im}]$, $\Phi_i(Z_i) = [\|\Xi_{i1}(Z_i)\|^2 \chi_{i1}, \ldots, \|\Xi_{im}(Z_i)\|^2 \chi_{im}]^T$, $\Psi_i(Z_i) = \text{diag}\left[\|\Xi_{i1}(Z_i)\|^2 \chi_{i1}, \ldots, \|\Xi_{im}(Z_i)\|^2 \chi_{im}\right]$, $\beta_{ik}^* = \|\vartheta_{ik}^*\|^2$, $\hat{\beta}_{ik}$ are the estimates of unknown parameters $\beta_{ik}^*, k = 1, \ldots, m$.

Substituting (36)–(41) into (35), we obtain

$$\begin{aligned}\dot{V}_i \le & -\chi_i^T K_i \chi_i + \chi_i^T \Theta_i^{*T} \Xi_i(Z_i) \\ & + \chi_i^T \varrho_i - \frac{1}{2a_i^2}\chi_i^T \hat{\beta}_i \Phi_i(Z_i) \\ & + 2\chi_i^T \chi_i + \frac{1}{4}\chi_{i+1}^T \chi_{i+1} + \frac{1}{4}e_{i+1}^T e_{i+1} + \sum_{j=1}^{m} \frac{a_{ij}^2}{2} \\ \le & -\chi_i^T (K_i - 3I_{m\times m})\chi_i - \frac{1}{2a_i^2}\chi_i^T \tilde{\beta}_i \Phi_i(Z_i) + \frac{a_i^2}{2} \\ & + \frac{1}{4}\chi_{i+1}^T \chi_{i+1} + \frac{1}{4}e_{i+1}^T e_{i+1} + \frac{1}{4}\varrho_i^T \varrho_i + \sum_{j=1}^{m} \frac{a_{ij}^2}{2}. \quad (43)\end{aligned}$$

There exist nonnegative continuous functions $\kappa_{ij}(\bar{\chi}_{i+1}, \bar{e}_{i+1}, \bar{\hat{\beta}}_{im}, \bar{\hbar}_i, \nu), j = 1, \ldots, m$ such that $|\varrho_{ij}| \le \kappa_{ij}$. Let $\kappa_i = [\kappa_{i1}, \ldots, \kappa_{im}]^T$, (43) can be written as

$$\begin{aligned}\dot{V}_i \le & -\chi_i^T (K_i - 3I_{m\times m})\chi_i - \frac{1}{2a_i^2}\chi_i^T \tilde{\beta}_i \Phi_i(Z_i) + \frac{a_i^2}{2} \\ & + \frac{1}{4}\chi_{i+1}^T \chi_{i+1} + \frac{1}{4}e_{i+1}^T e_{i+1} + \frac{1}{4}\kappa_i^T \kappa_i + \sum_{j=1}^{m} \frac{a_{ij}^2}{2}. \quad (44)\end{aligned}$$

Define $\hbar_{i+1}$ as follows

$$\tau_{i+1}\dot{\hbar}_{i+1} + \hbar_{i+1} = \alpha_i, \hbar_{i+1}(0) = \alpha_i(0), \quad (45)$$

where $\tau_{i+1} > 0$ is a positive design constant.

From (16), the time derivative of e_{i+1} is

$$\dot{e}_{i+1} = -\frac{e_{i+1}}{\tau_{i+1}} - \dot{\alpha}_i, \quad (46)$$

Then

$$e_{i+1}^T \dot{e}_{i+1} \le -\frac{e_{i+1}^T e_{i+1}}{\tau_{i+1}} + \|e_{i+1}\|\eta_{i+1}(\bar{\chi}_{i+2}, \bar{e}_{i+2}, \bar{\hat{\beta}}_{im}, \nu, y_d, \dot{y}_d, \ddot{y}_d), \quad (47)$$

where $\eta_{i+1}(\bar{\chi}_{i+2}, \bar{e}_{i+2}, \bar{\hat{\beta}}_{im}, \nu, y_d, \dot{y}_d, \ddot{y}_d)$ is a nonnegative continuous function.

From (47) and the inequality $ab \le a^2 + \frac{b^2}{4}$, we get

$$e_{i+1}^T \dot{e}_{i+1} \le -\frac{\|e_{i+1}\|^2}{\tau_{i+1}} + \|e_{i+1}\|^2 + \frac{1}{4}\|\eta_{i+1}\|^2. \quad (48)$$

Step n: Define $\chi_n = z_n - \hbar_n$. Differentiating χ_n for time t, we get

$$\dot{\chi}_n = F_n(\bar{z}_n, u) + D_n(\zeta, \bar{z}_n, t) - \dot{\hbar}_n \quad (49)$$

Define V_n as follows:

$$V_n = \frac{1}{2}\chi_n^T \chi_n \quad (50)$$

Differentiating V_n for time t and from to (56), we have

$$\dot{V}_n = \chi_n^T \dot{\chi}_n = \chi_n^T [F_n(\bar{z}_n, u) + D_n(\zeta, \bar{z}_n, t) - \dot{\hbar}_n]. \quad (51)$$

From (7), we obtain $F_n(\bar{z}_n, u) = f_n(\bar{x}_n, u) = f_n(0) + G_{xn}\bar{x}_n + G_{un}u$, it yields

$$\begin{aligned}\dot{V}_n &= \chi_n^T [f_n(0) + G_{xn}\bar{x}_n + G_{un}u + D_n(\zeta, \bar{z}_n, t) - \dot{\hbar}_n] \\ &= \chi_n^T [f_n(0) + G_{xn}\bar{x}_n + (G_{un} - I_{m\times m})u \\ &\quad + u + D_n(\zeta, \bar{z}_n, t) - \dot{\hbar}_n] \\ &= \chi_n^T [f_n(0) + u + D_n(\zeta, \bar{z}_n, t) - \dot{\hbar}_n] \\ &\quad + \chi_n^T G_{xn}\bar{x}_n + \chi_n^T (G_{un} - I_{m\times m})u. \quad (52)\end{aligned}$$

Using Assumption 2 and Young's inequality, we have

$$\begin{aligned}&\chi_n^T D_n(\zeta, \bar{z}_n, t) \\ &\le \|\chi_n\| \sum_{j=1}^{m} \left|D_{n,j}(\zeta, \bar{z}_n, t)\right| \\ &\le \|\chi_n\| \sum_{j=1}^{m} [\varphi_{nj1}(\|\bar{x}_{n,j}\|) + \varphi_{nj2}(\|\zeta\|)] \\ &\le \chi_n^T \chi_n \sum_{j=1}^{m} \frac{[\varphi_{nj1}(\|\bar{x}_{nj}\|) + \varphi_{nj2}(\alpha_1^{-1}(\nu + D_0))]^2}{2a_{nj}^2} \\ &\quad + \sum_{j=1}^{m} \frac{a_{nj}^2}{2}, \quad (53)\end{aligned}$$

and

$$\begin{aligned}\chi_n^T G_{xn}\bar{x}_n &\le \left|\chi_n^T G_{xn}\bar{x}_n\right| \le \left\|\chi_n^T G_{xn}\bar{x}_n\right\| \\ &\le \|\chi_n\|^2 \|\bar{x}_n\|^2 + \frac{1}{4}\|G_{xn}\|^2 \\ &\le \chi_n^T \chi_n \|\bar{x}_n\|^2 + \frac{1}{4}\|G_{xn}\|^2. \quad (54)\end{aligned}$$

Let

$$\begin{aligned}\Sigma_n(Z_n) = & \chi_n \sum_{j=1}^{m} \frac{[\varphi_{nj1}(\|\bar{x}_{nj}\|) + \varphi_{nj2}(\alpha_1^{-1}(\nu + D_0))]^2}{2a_{nj}^2} \\ & + f_n(0) + \chi_n \|\bar{x}_n\|^2 - \dot{\hbar}_n, \quad (55)\\ \Delta(u) = & (G_{un} - I_{m\times m})u. \quad (56)\end{aligned}$$

Using RBFNNs to estimate the unknown continuous function vector $\Sigma_n(Z_n)$, we have

$$\Sigma_n(Z_n) = \Theta_n^{*T} \Xi_n(Z_n) + \varrho_n, \quad (57)$$

where $Z_n = [z_1^T, \ldots, z_n^T, \chi_n^T, \dot{\hbar}_n^T, \nu]^T \in \mathbb{R}^{(n+2)m+1}$, the ideal weight matrix is $\Theta_n^* = \text{blockdiag}[\vartheta_{n1}^*, \ldots, \vartheta_{nm}^*]$, the basis vector is $\Xi_n(Z_n) = [\Xi_{n1}^T(Z_n), \ldots, \Xi_{nm}^T(Z_n)]^T$, the approximation error is $\varrho_n = [\varrho_{n1}, \ldots, \varrho_{nm}]^T$.

Design the updating law of the estimated diagonal matrix $\hat{\beta}_n$ as follows:

$$\dot{\hat{\beta}}_n = \text{diag}[r_{n1}, \ldots, r_{nm}]\Big[\frac{1}{2a_n^2} \times \text{diag}[\chi_{n1}, \ldots, \chi_{nm}]\Psi_n(Z_n) - \sigma_n\hat{\beta}_n\Big], \quad (58)$$

where $a_n > 0$, $\sigma_n > 0$, $r_{nj} > 0$, $j = 1, \ldots, m$, $\hat{\beta}_n = \text{diag}[\hat{\beta}_{n1}, \cdots, \hat{\beta}_{nm}]$, $\Phi_n(Z_n) = [\|\Xi_{n1}(Z_n)\|^2\chi_{n1}, \ldots, \|\Xi_{nm}(Z_n)\|^2\chi_{nm}]^T$, $\Psi_n(Z_n) = \text{diag}[\|\Xi_{n1}(Z_n)\|^2\chi_{n1}, \ldots, \|\Xi_{nm}(Z_n)\|^2\chi_{nm}]$, $\beta_{nk}^* = \|\vartheta_{nk}^*\|^2$, $\hat{\beta}_{nk}$ are the estimates of unknown parameters β_{nk}^*, $k = 1, \ldots, m$.

Inspired by [33], the event-triggered control is designed as follows:

$$\alpha_{nj} = -\Big[K_{nj}\chi_{nj} + \frac{1}{2a_n^2}\chi_{nj}\hat{\beta}_{nj}\|\Xi_{nj}(Z_n)\|^2\Big], \quad (59)$$

$$v_j(t) = -(1+\delta_j)\Big(\alpha_{nj}\tanh(\frac{\chi_{nj}\alpha_{nj}}{\varepsilon_j})+\bar{m}_{j1}\tanh(\frac{\chi_{nj}\bar{m}_{j1}}{\varepsilon_j})\Big), \quad (60)$$

$$u_j(t) = v_j(t_{jk}), \forall t \in [t_{jk}, t_{j,k+1}), \quad (61)$$

$$t_{j,k+1} = \inf\{t \in R|\ |v_j(t) - u_j(t)| \geq \delta_j|u_j(t)| + m_{j1}, t > t_{jk}\}, \quad (62)$$

where $K_{nj} > 0$, $t_{jk}, k \in Z^+$, $\varepsilon_j > 0$, $\delta_j \in [0, 1)$, $m_{j1} > 0$ and $\bar{m}_{j1} > \frac{m_{j1}}{1-\delta_j}$ are design constants, $v = [v_1, \ldots, v_m]^T$.
Let $q_{j2}(t) = \frac{v_j(t)-u_j(t)}{\delta_j|u_j(t)|+m_{j1}}$. From (62), we have $|q_{j2}(t)| \leq 1$ for $t \in [t_k, t_{k+1})$ and

$$v_j(t) = (1 + q_{j1}(t)\delta_j)u_j(t) + q_{j2}(t)m_{j1}, \forall t \in [t_{jk}, t_{j,k+1}), \quad (63)$$

where $q_{j1}(t) = q_{j2}(t)\text{sign}(u_j(t))$, and $|q_{j1}(t)| \leq 1$ for $t \in [t_k, t_{k+1})$. We denote $u_j(t)$ as follows:

$$u_j(t) = \frac{v_j(t)}{1 + q_{j1}(t)\delta_j} - \frac{q_{j2}(t)m_{j1}}{1 + q_{j1}(t)\delta_j}. \quad (64)$$

Using the inequality $0 \leq |x| - x\tanh(\frac{x}{\epsilon}) \leq 0.2785\epsilon$ with $\epsilon > 0$ in [40], we obtain

$$\begin{aligned}
\chi_n^T u(t) &= \sum_{j=1}^m \chi_{nj}u_j \\
&= \sum_{j=1}^m \Big[\frac{\chi_{nj}v_j(t)}{1 + q_{j1}(t)\delta_j} - \frac{\chi_{nj}q_{j2}(t)m_{j1}}{1 + q_{j1}(t)\delta_j}\Big] \\
&= \sum_{j=1}^m \Big[-\frac{1 + \delta_j}{1 + q_{j1}(t)\delta_j}\chi_{nj}\alpha_{nj}\tanh(\frac{\chi_{nj}\alpha_{nj}}{\varepsilon_j}) \\
&\quad - \frac{1 + \delta_j}{1+q_{j1}(t)\delta_j}\chi_{nj}\bar{m}_{j1}\tanh(\frac{\chi_{nj}\bar{m}_{j1}}{\varepsilon}) - \frac{\chi_{nj}q_{j2}(t)m_{j1}}{1 + q_{j1}(t)\delta_j}\Big] \\
&\leq \sum_{j=1}^m \Big[-\chi_{nj}\alpha_{nj}\tanh(\frac{\chi_{nj}\alpha_{nj}}{\varepsilon_j}) \\
&\quad - \chi_{nj}\bar{m}_{j1}\tanh(\frac{\chi_{nj}\bar{m}_{j1}}{\varepsilon_j}) + |\frac{\chi_{nj}m_{j1}}{1 - \delta_j}|\Big] \\
&\leq \sum_{j=1}^m \Big[-|\chi_{nj}\alpha_{nj}| - |\chi_{nj}\bar{m}_{j1}| + |\chi_{nj}\bar{m}_{j1}| + 0.557\varepsilon_j\Big] \\
&\leq \sum_{j=1}^m \Big[-K_{nj}\chi_{nj}^2 - \frac{1}{2a_n^2}\chi_{nj}^2\hat{\beta}_{nj}\|\Xi_{nj}(Z_n)\|^2 + 0.557\varepsilon_j\Big] \\
&= -\chi_n^T K_n\chi_n - \frac{1}{2a_n^2}\chi_n^T\hat{\lambda}_n\Phi(Z_n) + 0.557\sum_{j=1}^m \varepsilon_j,
\end{aligned} \quad (65)$$

where $K_n = \text{diag}[K_{n1}, \ldots, K_{nm}]$.

Substituting (53), (54), (58) and (65) into (52), we obtain

$$\begin{aligned}
\dot{V}_n &\leq -\chi_n^T K_n\chi_n + \chi_n^T\Theta_n^{*T}\Xi_n(Z_n) - \frac{1}{2a_n^2}\chi_n^T\hat{\beta}_n\Phi_n(Z_n) \\
&\quad + \chi_n^T\varrho_n + \frac{1}{4}\|G_{zn}\|^2 + \chi_n^T\Delta(u) \\
&\quad + \sum_{j=1}^m \frac{a_{ij}^2}{2} + 0.557\sum_{j=1}^m \varepsilon_j \\
&\leq -\chi_n^T K_n\chi_n - \frac{1}{2a_n^2}\chi_n^T\tilde{\beta}_n\Phi_n(Z_n) + 2\chi_n^T\chi_n \\
&\quad + \frac{1}{4}\varrho_n^T\varrho_n + \frac{1}{4}\|\Delta(u)\|^2 + \frac{1}{4}\|G_{zn}\|^2 \\
&\quad + \frac{a_n^2}{2} + \sum_{j=1}^m \frac{a_{nj}^2}{2} + 0.557\sum_{j=1}^m \varepsilon_j \\
&\leq -\chi_n^T(K_n - 2I_{m\times m})\chi_n - \frac{1}{2a_n^2}\chi_n^T\tilde{\beta}_n\Phi_n(Z_n) \\
&\quad + \frac{1}{4}\varrho_n^T\varrho_n + \frac{1}{4}||\Delta(u)||^2 + \frac{1}{4}\|G_{zn}\|^2 \\
&\quad + \frac{a_n^2}{2} + \sum_{j=1}^m \frac{a_{nj}^2}{2} + 0.557\sum_{j=1}^m \varepsilon_j.
\end{aligned} \quad (66)$$

There exist nonnegative continuous functions $\kappa_{nj}(\bar{\chi}_n, \bar{e}_n, \bar{\hat{\beta}}_{nm}, \hbar_i, \nu)$ such that $|\varrho_{nj}| \leq \kappa_{nj}$. Define $\kappa_n = [\kappa_{n1}, \ldots, \kappa_{nm}]^T$, it yields

$$\begin{aligned}
\dot{V}_n &\leq -\chi_n^T(K_n - 2I_{m\times m})\chi_n - \frac{1}{2a_n^2}\chi_n^T\tilde{\beta}_n\Phi_n(Z_n) \\
&\quad + \frac{1}{4}\kappa_n^T\kappa_n + \frac{1}{4}||\Delta(u)||^2 + \frac{1}{4}\|G_{zn}\|^2 \\
&\quad + \frac{a_n^2}{2} + \sum_{j=1}^m \frac{a_{nj}^2}{2} + 0.557\sum_{j=1}^m \varepsilon_j.
\end{aligned} \quad (67)$$

IV. MAIN RESULTS

Define V and the compact set A_n as follows:

$$\begin{aligned}
V &= \sum_{i=1}^n V_i + \frac{1}{2}\sum_{i=2}^n e_i^T e_i \\
&\quad + \frac{1}{2}\sum_{i=1}^n \text{tr}\Big(\tilde{\beta}_i^T\text{diag}[r_{i1}^{-1}, \ldots, r_{im}^{-1}]\tilde{\beta}_i\Big),
\end{aligned} \quad (68)$$

$$A_n = \Big\{[\bar{\chi}_n^T, \bar{e}_n^T, \bar{\hat{\beta}}_{nm}^T, \nu]^T:\ V \leq p\Big\} \subset \mathbb{R}^{p_n}, \quad (69)$$

where $p > 0$, $p_n = nm + m(2n - 1) + 1$.
On the compact set $A_n \times A_d \subset \mathbb{R}^{p_n+3m}$, $\|\eta_i\|$ and $\|\kappa_i\|$ have maximum values $N_i(p)$ and $M_i(p)$, respectively.

Theorem 1: If Assumptions 1–4 are true for system (1), and the event-triggered control (61), the virtual control (25) and (41), and the updating laws (26), (42), (58) are designed, then for any given constant $p > 0$, $V(0) \leq p$ and $-k_{j1} < y_{1j}(0) < k_{j2}$, there exist positive definite constant matrices K_i and constants τ_i, σ_i and r_{ij} subject to (70) such that all of the signals are bounded, and the output restrictions are not triggered, i.e., $-k_{j1} < y_{1j}(t) < k_{j2}, \forall t \geq 0$, and Zeno behavior can be avoided, and

$$\begin{cases} \min\limits_{1\leq i\leq n} \lambda_{\min}(K_i - \frac{13}{4}I_{m\times m}) \geq \frac{\Gamma}{2} \\ \min\limits_{2\leq i\leq n} \frac{1}{\tau_i} \geq \frac{5}{4} + \frac{\Gamma}{2} \\ \Gamma = \min\limits_{1\leq i\leq n} \{\sigma_i r_{i1}, \ldots, \sigma_i r_{im}\} \end{cases} \tag{70}$$

Proof: If $V \leq p$, then $\chi_1, \ldots \chi_n, e_2, \ldots e_n$ and $\hat{\beta}_i$ are bounded. Furthermore, α_i and $\hbar_{i+1} \in L_\infty$. Noting $y_d \in L_\infty$, it yields $\hat{y}_d \in L_\infty$. From (15) and $\chi_1 \in L_\infty, \hat{y}_d \in L_\infty$, we get $z_1 \in L_\infty$, this implies $y \in L_\infty$. According to (5) and $y \in L_\infty$, we have $\nu \in L_\infty$. Based on $\nu \in L_\infty$ and Assumption 4, Remark 3, we have $\zeta \in L_\infty$. According to Assumption 1 and using (59), (60) and (61), we get $v, u \in L_\infty$, $\|G_{zn}\|$ and $\|G_{un}\| \in L_\infty$, furthermore, it yields $\|\Delta(u)\| \in L_\infty$, i.e., $\|\Delta(u)\| \leq B_1(p)$, $\|G_{zn}\| \leq B_2(p)$ with B_1 and B_2 being two positive constants. From (25), we get $\alpha_1 \in L_\infty$, note that $z_2 = \chi_2 + e_2 + \alpha_1$ and $\chi_2, e_2, \alpha_1 \in L_\infty$, we have $z_2 \in L_\infty$, similarly, we get $\alpha_i, z_{i+1} \in L_\infty$. With the aid of (13), it yields $x_i \in L_\infty$. Thus, when $V \leq p$, we have $\|\eta_i\| \leq N_i$, $\|\kappa_i\| \leq M_i$. Substituting (26),(42), (58), (32) and (48) into the derivative of V, we obtain

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^{n} \dot{V}_i + \sum_{i=2}^{n} e_i^T \dot{e}_i + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{1}{r_{ij}} \tilde{\beta}_{ij} \dot{\hat{\beta}}_{ij} \\
&\leq -\sum_{i=1}^{n-1} \chi_i^T (K_i - 3I_{m\times m})\chi_i - \chi_n^T (K_n - 2I_{m\times m})\chi_n \\
&\quad - \sum_{i=1}^{n} \frac{1}{2a_i^2} \chi_i^T \tilde{\beta}_i \Phi_i(Z_i) - \sum_{i=2}^{n} (\frac{1}{\tau_i} - 1)\|e_i\|^2 \\
&\quad + \sum_{i=2}^{n} \frac{1}{4}\|e_i\|^2 + \sum_{i=2}^{n} \frac{1}{4}\|\chi_i\|^2 + \sum_{i=1}^{n} \frac{1}{4} M_i^2(p) \\
&\quad + \frac{1}{4}\|\Delta(u)\|^2 + \frac{1}{4}\|G_{zn}\|^2 + \sum_{i=1}^{n} \frac{a_i^2}{2} \\
&\quad + \sum_{i=1}^{n}\sum_{i=1}^{n} \frac{a_{ij}^2}{2} + \sum_{i=2}^{n} \frac{1}{4} N_i^2(p) + \frac{1}{4}\Sigma_n^2 + \frac{1}{4} W_n^2 \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{\beta}_{ij} \left[\frac{1}{2a_i^2} \|\Xi_{ij}(Z_i)\|^2 \chi_{ij}^2 - \sigma_i \hat{\beta}_{ij} \right] \\
&\quad + 0.557 \sum_{j=1}^{m} \varepsilon_j \\
&\leq -\sum_{i=1}^{n} \chi_i^T (K_i - \frac{13}{4} I_{m\times m})\chi_i - \sum_{i=1}^{n} \frac{1}{2a_i^2} \chi_i^T \tilde{\beta}_i \Phi_i(Z_i) \\
&\quad - \sum_{i=2}^{n} (\frac{1}{\tau_i} - \frac{5}{4})\|e_i\|^2 + \sum_{i=1}^{n} \frac{1}{4} M_i^2 + \sum_{i=1}^{n} \frac{a_i^2}{2} \\
&\quad + \sum_{i=1}^{n}\sum_{i=1}^{n} \frac{a_{ij}^2}{2} + \sum_{i=2}^{n} \frac{1}{4} N_i^2 + \frac{1}{4} B_1^2 + \frac{1}{4} B_2^2 \\
&\quad + \sum_{i=1}^{n} \frac{1}{2a_i^2} \sum_{j=1}^{m} \tilde{\beta}_{ij} \|\Xi_{ij}(Z_i)\|^2 \chi_{ij}^2 - \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\sigma_i}{2} \tilde{\beta}_{ij}^2 \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\sigma_i}{2} \beta_{ij}^{*2} + 0.557 \sum_{j=1}^{m} \varepsilon_j.
\end{aligned} \tag{71}$$

Let

$$\begin{aligned}
\Sigma_0 &= \sum_{i=1}^{n} \frac{1}{4} M_i^2(p) + \frac{1}{4} B_1^2(p) + \frac{1}{4} B_2^2(p) \\
&\quad + \sum_{i=2}^{n} \frac{1}{4} N_i^2(p) + \sum_{i=1}^{n} \frac{a_i^2}{2} + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{a_{ij}^2}{2} \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\sigma_i}{2} \beta_{ij}^{*2} + 0.557 \sum_{j=1}^{m} \varepsilon_j,
\end{aligned} \tag{72}$$

then

$$\begin{aligned}
\dot{V} \leq -\sum_{i=1}^{n} \chi_i^T (K_i - \frac{13}{4} I_{m\times m})\chi_i - \sum_{i=2}^{n} (\frac{1}{\tau_i} - \frac{5}{4})\|e_i\|^2 \\
- \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\sigma_i}{2} \tilde{\beta}_{ij}^2 + \Sigma_0.
\end{aligned} \tag{73}$$

Substituting (70) into (73), we obtain

$$\dot{V} \leq -\Gamma V + \Sigma_0. \tag{74}$$

If $V = p$, $\Gamma \geq \frac{\Sigma_0}{p}$, then $\dot{V} \leq 0$. It implies that with the aid of $V(0) \leq p$ and $\Gamma \geq \frac{\Sigma_0}{p}$, we have $V(t) \leq p, \forall t \geq 0$. From (74), we get

$$0 \leq V(t) \leq \frac{\Sigma_0}{\Gamma} + (V(0) - \frac{\Sigma_0}{\Gamma})e^{-\Gamma t}. \tag{75}$$

Therefore, we have all signals $\chi_i, e_i, \hat{\lambda}_i, z_i \in L_\infty$, then $\alpha_i, \hbar_i, u \in L_\infty$ are bounded. From (75), we obtain $\|\chi_1\| \leq \sqrt{\frac{2\Sigma_0}{\Gamma} + 2(V(0) - \frac{\Sigma_0}{\Gamma})e^{-\Gamma t}}$, therefore, for given p, $\sigma_1, \ldots, \sigma_n$, χ_1 as $t \to \infty$ can be made arbitrarily small by choosing large enough $r_{ij}, i = 1, \ldots, n, j = 1, \ldots, m$.

Define $E_j(t) = v_j(t) - u_j(t), \forall t \in [t_{jk}, t_{j,k+1})$. Therefore, the derivative of $|E_j(t)|$ is

$$\begin{aligned}
\frac{d|E_j(t)|}{dt} &= \frac{d}{dt}(E_j^2(t))^{\frac{1}{2}} = \text{sign}(E_j(t))\dot{E}_j(t) \\
&= \text{sign}(E_j(t))(\dot{v}_j(t) - \dot{u}_j(t)) \leq \left|\dot{v}_j(t)\right|.
\end{aligned} \tag{76}$$

with

$$\begin{aligned}
\dot{v}_j(t) &= \frac{\partial v_j}{\partial \alpha_{nj}} \dot{\alpha}_{nj} + \frac{\partial v_j}{\partial \chi_n} \dot{\chi}_n \\
&= -\frac{\partial v_j}{\partial \alpha_{nj}} \Big(K_{nj} \dot{\chi}_{nj} + \frac{1}{2a_n^2} \dot{\chi}_{nj} \hat{\beta}_{nj} \|\Xi_{nj}(Z_n)\|^2 \\
&\quad + \frac{1}{2a_n^2} \chi_{nj} \dot{\hat{\beta}}_{nj} \|\Xi_{nj}(Z_n)\|^2 \\
&\quad + \frac{1}{2a_n^2} \chi_{nj} \hat{\beta}_{nj} \frac{d\|\Xi_{nj}(Z_n)\|^2}{dt} \Big) \\
&\quad + \frac{\partial v_j}{\partial \chi_n^T} \Big(F_n(\bar{z}_n, u) + D_n(\zeta, \bar{z}_n, t) - \dot{\hbar}_n \Big).
\end{aligned} \tag{77}$$

For $V \le p$, all signals at the right side of (77) are bounded, so $\dot{v}_j(t)$ is bounded. Furthermore, there exists a positive constant C_j such that $|\dot{v}_j(t)| \le C_j$. In addition, since $|E_j(t_{jk})| = 0$ and $\lim_{t \to t^-_{j,k+1}} |E_j(t)| = \delta_j |u_j(t_{jk})| + m_{j1}$, for $t'_{j,k+1} \in (t_{jk}, t_{j,k+1})$, using mean value theorem and (76), we have

$$t'_{j,k+1} - t_{jk} = \frac{|E_j(t'_{j,k+1})| - |E_j(t_{jk})|}{\operatorname{sign}(E_j(t^*_j))\dot{E}_j(t^*_j)} \ge \frac{|E_j(t'_{j,k+1})| - |E_j(t_{jk})|}{|\dot{v}_j(t^*_j)|}, \tag{78}$$

where $t^*_j \in (t_{jk}, t'_{j,k+1})$.
Let $t'_{j,k+1}$ tend to $t^-_{j,k+1}$ above both sides of the inequality, we have

$$t_{j,k+1} - t_{jk} = \frac{|E_j(t^-_{j,k+1})| - |E_j(t_{jk})|}{\operatorname{sign}(e(t^*))\dot{e}(t^*)} \ge \frac{|E_j(t^-_{j,k+1})| - |E_j(t_{jk})|}{|\dot{v}_j(t^*_j)|} \ge \frac{\delta_j |u_j(t_{jk})| + m_{j1}}{C_j} \ge \frac{m_{j1}}{C_j} > 0. \tag{79}$$

Obviously, $(t_{j,k+1} - t_{jk})$ has a positive lower bound $\Delta t^*_j = \frac{m_{j1}}{C_j} > 0$, and Zeno behavior can be successfully avoided.

Remark 4: RBFNNs are used to appoximate the unknown continuous function vector, which is produced in the process of controller design, and the unknown system functions have not been directly estimated by them in this paper, wheras RBFNNs are used to estimate unknown system functions in [37], [38]; Unmodeled dynamic and asymmetric output constraints are considered in our paper, wheras they are not considered in [37], [38]; The considered systems are class of MIMO systems with block-structure, wheras the considered systems are a class of MIMO Systems with similar structure in [37], and they are a class of SISO systems without uncertainties in [38]. Therefore, with the help of event-triggered controller, our proposed method is more practical and the systems in this paper are more general.

Remark 5: Compared with the approach proposed in [24], a new nonlinear mapping based on hyperbolic tangent function is introduced to handle the asymmetric output constraints. By using this novel nonlinear mapping, the origin point between the original and transformed system can be mapped. It means that the controllability of the original system is guaranteed. Different from the time-driven control method proposed in [24], the new event-triggered approach can reduce energy consumption and occupation rate of transmission bandwidth without affecting the tracking accuracy and the system stability.

Remark 6: Σ_0 described in (72) is an unknown constant which depends on the size of the defined compact set A_n in (69). However, $\Gamma = \min_{1 \le i \le n} \{\sigma_i r_{i1}, \ldots, \sigma_i r_{im}\}$ does not depends on the size of the defined compact set A_n. In addition, Σ_0 does not include the constants $r_{i1}, \ldots, r_{im}, i = 1, \ldots, n$. Therefore, we can choose large enough constants $r_{i1}, \ldots, r_{im}, i = 1, \ldots, n$ for any given design constants $\sigma_{i1}, \ldots, \sigma_{im}, i = 1, \ldots, n$ and $p > 0$ such that $\Gamma \ge \frac{\Sigma_0}{p}$ and $\dot{V} \le 0$ hold.

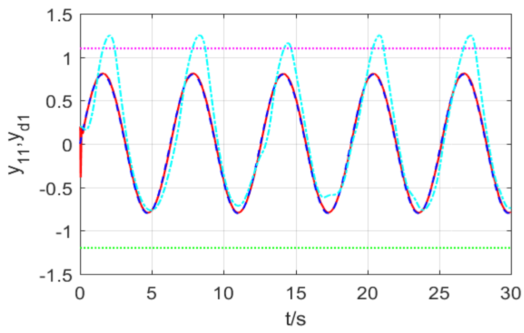

FIGURE 1. Case 1: output y_{11} (solid line) and tracking signal y_{d1} (dashed line), the upper bound k_{12} (dotted line) and the lower bound $-k_{11}$ (dotted line); Case 2: output y_{11} (dotted-dashed line).

V. NUMERICAL SIMULATION

To illustrate the theoretical findings of the proposed event-triggered control, a constrained pure-feedback system and a kind of 2-DOF flexible manipulator system are discussed.

Example 1: Consider the following constrained second-order uncertain MIMO system:

$$\begin{cases} \dot{\zeta} = Q(\zeta, y, t), \\ \dot{x}_1 = \begin{bmatrix} x_{11} \sin x_{11} + x_{21} + \frac{x_{22}^3}{7} \\ x_{11} + x_{12}x_{21} + (1 + x_{11}^2)x_{22} \end{bmatrix} + d_1, \\ \dot{x}_2 = \begin{bmatrix} x_{11}x_{12} + x_{21} + 0.2u_1^3 \\ x_{11}x_{12}x_{21} + (2 + \sin x_{22})u_2 \end{bmatrix} + d_2, \\ y = x_1, \end{cases} \tag{80}$$

where $Q(\zeta, y, t) = -\zeta + \|y\|^2 \sin t + 0.5$, $d_1 = [-0.5\zeta x_{21} \sin 10t, -0.5\zeta \sin x_{11} \cos t]^T$, $d_2 = [\sin(0.5x_{12}t) + x_{21}\zeta, x_{21}\zeta \cos(10x_{11}t) + x_{22}\zeta]^T$. The tracking signal is $y_d = \begin{bmatrix} 0.8 \sin t \\ 0.6 \sin t \end{bmatrix}$. The dynamic signal is taken as $\nu = -0.6\nu + 1.5\|x_1\|^4 + 1.5$. The output constraints are $k_{11} = 1.2$, $k_{12} = 1.3$, $k_{21} = 0.8$, $k_{22} = 0.9$.

Case 1: The design constants are chosen as $K_1 = \text{diag}[30, 40]$, $K_2 = \text{diag}[10, 30]$, $\tau_2 = 0.01$, $\sigma_1 = \sigma_2 = 25$, $r_{11} = r_{12} = r_{21} = r_{22} = 0.01$, $a_1 = a_2 = 25$, $\delta_1 = \delta_2 = 0.3$, $m_{11} = m_{21} = 0.2$. The initial conditions are selected as $x_1(0) = [0.2, 0.2]^T$, $x_2(0) = [0, 0]^T$, $\zeta(0) = 0.1$, $\hbar_2(0) = [0.1, 0.1]^T$, $\nu(0) = 0.1$, $\hat{\beta}_1(0) = \hat{\beta}_2(0) = diag[0.4, 0.4]$. The simulation results are shown in Figures 1–6.

Case 2: If output constraints and event-triggered controller are not considered, the design constants are chosen as $K_1 = \text{diag}[3.25, 3.25]$, $K_2 = \text{diag}[4, 4]$, $\sigma_1 = \sigma_2 = 0.5$, $\tau_2 = 0.001$, and the other conditions are the same, the simulation results without output constraints and event-triggered controller are shown in Figures 1 and 2.

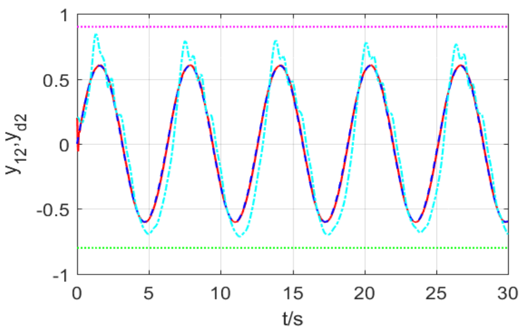

FIGURE 2. Case 1: output y_{12} (solid line) and tracking signal y_{d2} (dashed line), the upper bound k_{22} (dotted line) and the lower bound $-k_{21}$ (dotted line); Case 2: output y_{12} (dotted-dashed line).

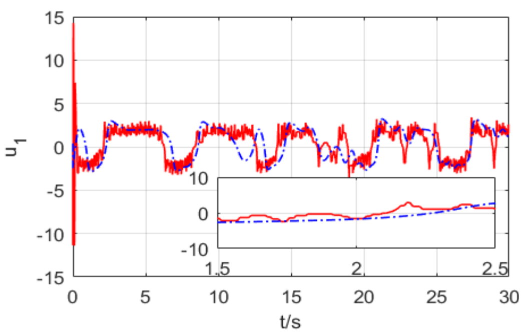

FIGURE 3. Control signals u_1 (solid line) for case 1 and (dotted-dashed line) for case 2.

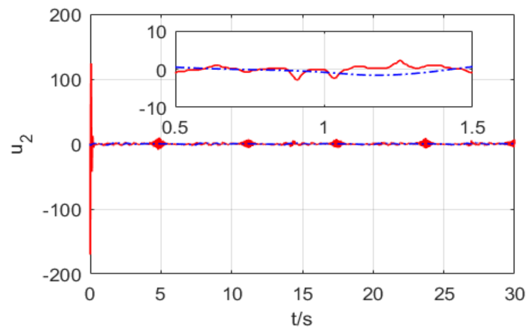

FIGURE 4. Control signals u_2 (solid line) for case 1 and (dotted-dashed line) for case 2.

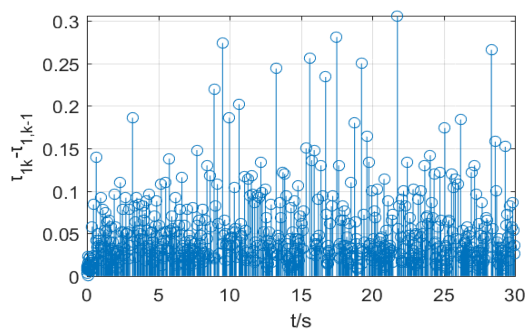

FIGURE 5. Time interval of trigger-event for u_1.

From Figures. 1 and 2, it is clearly to know that the output signal vector y can well track the desired trajectory and all the states are within the constraints. The tracking performance of case 1 is better than the tracking performance of case 2. In Figures. 5 and 6, we can find that the event-triggered numbers are 632 and 1411 in 30 seconds, respectively.

Remark 7: When the controller signal needs to be updated, the event-triggered condition $|E_j(t)| = v_j(t) - u_j(t) = \delta_j|u_j(t)| + m_{j1}$ is satisfied. Firstly, the traditional encoding method is used to transmit the initial controller value $u(t_0)$. In the following time point$t \in [t_{jk}, t_{j,k+1})$, $k = 0, 1, \ldots,$ the encoder can be designed as follows:

$$l_j = \begin{cases} 1, & E_j(t) \geq \delta_j|u_j(t)| + m_{j1} \\ & \quad = \delta_j|v_j(t_{kj})| + m_{j1} \\ 0, & E_j(t) \leq -\delta_j|u_j(t)| - m_{j1} \\ & \quad = -\delta_j|v_j(t_{kj})| - m_{j1} \end{cases} \tag{81}$$

The output of encoder l_j can be transmitted to the decoder with the help of network channel. The decoder only needs to store the event-triggered condition parameters and the last control signal value $u_j(t_k)$. The decoder is designed as follows:

$$u_j(t_{k+1}) = \begin{cases} u_j(t_{kj}) + \delta_j|u(t_{kj})| + m_{j1}, & l_j = 1 \\ u_j(t_{kj}) - \delta_j|u(t_{kj})| - m_{j1}, & l_j = 0 \end{cases} \tag{82}$$

By using this decoder, the controller signal $u_j(t)$ can be restored. When we update the controller signal, only "0" or "1" need to be transmitted through the network channel. Therefore, although the event-triggered number reaches 632 and 1411, that is, 5-12 times per second, the burden of signal transmission can be reduced.

Example 2: Consider 2-DOF flexible robotic system as follows:

$$\begin{cases} M(\omega)\ddot{\omega} + C(\omega, \dot{\omega})\dot{\omega} + G(\omega) + F\dot{\omega} \\ \quad + K_m(\omega - \omega_m) = 0 \\ J_m\ddot{\omega}_m + B_m\dot{\omega}_m + K_m(\omega_m - \omega) = u \end{cases} \tag{83}$$

The corresponding state equations of system (2) including unmodeled dynamics can be described as follows:

$$\begin{cases} \dot{\zeta} = Q(\zeta, y), \\ \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(x_1)\Big[C(x_1, x_2)x_2 + G(x_1) \\ \qquad + Fx_2 + K_m x_1\Big] + M^{-1}(x_1)x_3 + d_2, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = J_m^{-1}\left[-Bx_4 - K_m(x_3 - x_1)\right] + J_m^{-1}u + d_4, \\ y = x_1 \end{cases}$$

$$M(\omega) = \begin{pmatrix} p_1 + p_2 + 2p_3\cos(\omega_2) & p_2 + p_3\cos(\omega_2) \\ p_2 + p_3\cos(\omega_2) & p_2, \end{pmatrix}$$

$$C(\omega, \dot{\omega}) = \begin{pmatrix} -p_3\dot{\omega}_2\sin(\omega_2) & -p_3(\dot{\omega}_1 + \dot{\omega}_2)\sin(\omega_2) \\ p_3\dot{\omega}_1\sin(\omega_2) & 0 \end{pmatrix}$$

$$G(\omega) = \begin{pmatrix} p_4 g\cos(\omega_1) + p_5 g\cos(\omega_1 + \omega_2) \\ p_5 g\cos(\omega_1 + \omega_2) \end{pmatrix} \tag{84}$$

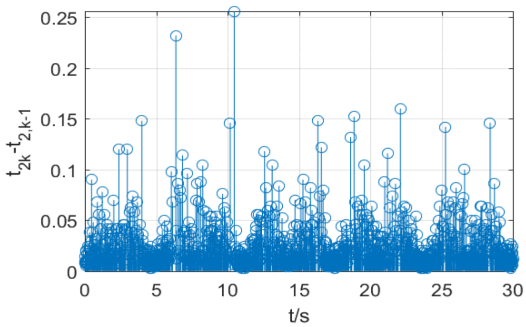

FIGURE 6. Time interval of trigger-event for u_2.

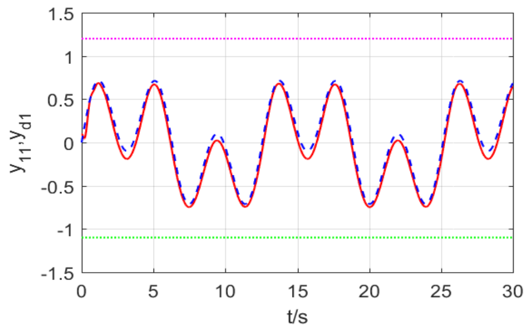

FIGURE 7. Output y_{11}(solid line) and tracking signal y_{d1} (dashed line), the upper bound k_{12} (dotted line) and the lower bound $-k_{11}$ (dotted line).

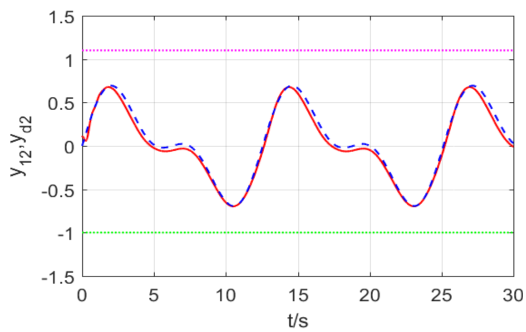

FIGURE 8. Output y_{12}(solid line) and tracking signal y_{d2}(dashed line), the upper bound k_{22} (dotted line) and the lower bound $-k_{21}$ (dotted line).

where $x_1 = \omega$, $x_2 = \dot{\omega}$, $\omega = [\omega_1, \omega_2]^T$, $Q(\zeta, y) = -\zeta + \|y\|^2 + 0.5$. $p_1 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1$, $p_2 = m_2 l_{c2}^2 + I_2$, $p_3 = m_2 l_1 l_{c2}$, $p_4 = m_1 l_{c2} + m_2 l_1$, $p_5 = m_2 l_{c2}$, m_i and l_i are the mass and length of the ith link, l_{ci} is the distance between the ith joint and the center of mass of the ith link, and I_i is the moment of inertia of the ith link, $d_2 = [0.1\zeta \sin 2t, 0.1\zeta \sin 2t]^T$, $d_2 = [0.5\sin(t) + \omega_1\zeta, 0.5\sin(t) + \omega_2\zeta]^T$ are the dynamic disturbances.

The system parameters are taken as $m_1 = 3kg$, $m_2 = 2kg$, $l_1 = 1m$, $l_2 = 1.2m$, $l_{c1} = 0.5m$, $l_{c2} = 0.6m$, $I_1 = 4 \times 10^{-3} kg \cdot m^2$, $I_2 = 3 \times 10^{-3} kg \cdot m^2$,

$$K_m = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}, \quad J_m = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.002 \end{bmatrix}$$

$$F = B = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

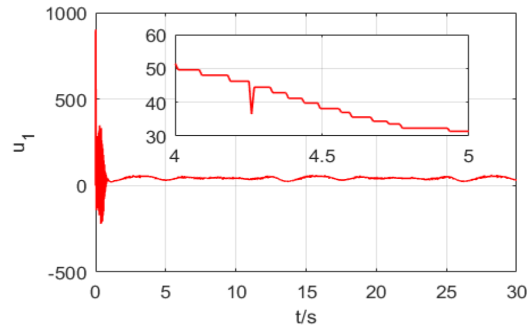

FIGURE 9. Control signal u_1.

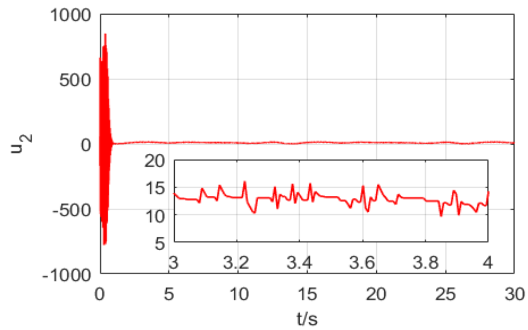

FIGURE 10. Control signal u_2.

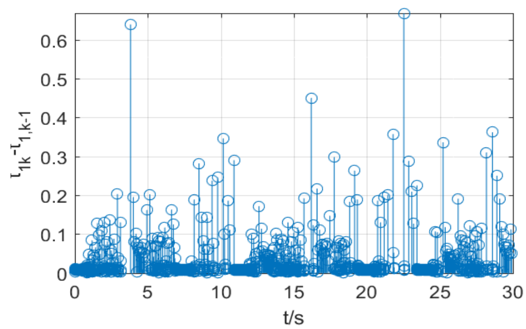

FIGURE 11. Time interval of trigger-event for u_1.

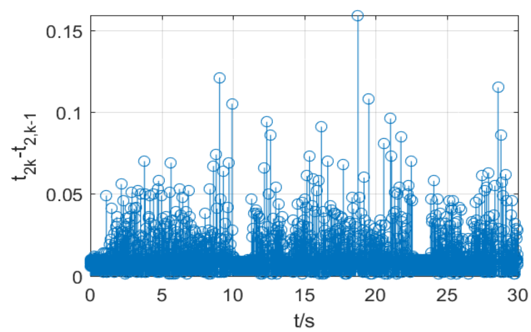

FIGURE 12. Time interval of trigger-event for u_2.

The controller parameters are selected as $K_1 = \text{diag}[30, 10]$, $K_2 = \text{diag}[4, 4]$, $K_3 = \text{diag}[5, 5]$, $K_4 = \text{diag}[3.5, 3.5]$, $\tau_2 = 0.001$, $\tau_3 = 0.003$, $\tau_4 = 0.001$, $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0.01$, $\hat{\beta}_1(0) = \hat{\beta}_2(0) = \hat{\beta}_3(0) = \hat{\beta}_4(0) = \text{diag}[0.2, 0.2]$, $\delta_1 = \delta_2 = 0.2$, $m_{11} = m_{21} = 0.3$, $k_{11} = 1.1$, $k_{12} = 1.2$, $k_{21} = 1.0$, $k_{22} = 1.1$; choose the tracking signal $y_d = [0.5(\sin(1.5t) + 0.8\sin(0.5t)), 0.5(\sin(0.5t) + 0.6\sin(t))]^T$, design unmodeled dynamics as $\dot{\zeta} = -\zeta + \|y\|^2 + 0.6$, construct the dynamic signal as $\nu = -0.5\nu + 1.6\|x_1\|^4 + 1.5$; select the initial values as $x_1(0) = [0.1, 0.1]^T$, $x_2(0) = x_3(0) = x_4(0) = [0.15, 0.15]^T$, $\hat{\beta}_2(0) = \hat{\beta}_4(0) = \text{diag}[0.1, 0.1]$, $\hbar_2(0) = \hbar_3(0) = \hbar_4(0) = [0.2, 0.2]^T$, $\nu(0) = 0.01$, $\zeta(0) = 0.01$. The plotted curves by matlab are shown in Figures 7-12. From Figures 7 and 8, we can see that the output restrictions can be abided by, Figures. 9 and 10 show that the designed event-triggered control signals are bounded. The event-triggered numbers are 1106 and 2267 for u_1 and u_2 in 30 seconds, respectively.

VI. CONCLUSION

Combining dynamic surface control technique with relative threshold strategy, adaptive neural event-triggered control has been developed for block-structure MIMO non-affine nonlinear systems including output restriction and dynamical uncertainties. The first-order auxiliary system designed based on property of unmodeled dynamics is employed to dispose of the dynamical uncertain terms. The output constraints can be carried out based on invertible nonlinear mapping. All the signals in the designed control system have been proved to be semi-global uniform ultimate bounded. A constrained pure-feedback system and a kind of 2-DOF flexible manipulator system are provided to verify the effectiveness of the designed adaptive event-triggered control algorithm.

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