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Adaptive Finite-Time Neural Control for a Class of Stochastic Nonlinear Systems With Known Hysteresis

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ABSTRACT This manuscript considers the finite time adaptive neural tracking control problem for a class of the nonstrict-feedback stochastic nonlinear systems with Bouc-Wen hysteresis input. During the design process, a Bouc-Wen model is first adopted to obtain the input hysteresis phenomenon. By fusion with the backstepping technique and the neural network approximation capability, the unknown nonlinearities are coped with, a constructive finite time adaptive neural network control strategy is proposed. Furthermore, the finite-time mean square stability of stochastic nonlinear systems is proved, and at last the effectiveness of the proposed control strategy is validated by the simulations.

INDEX TERMS Bouc-Wen hysteresis, stochastic system, adaptive neural control, finite-time mean square stability.

I. INTRODUCTION

In the wide range of practical control application, the hysteresis are always founded. Hysteresis characteristics are generally nondifferentiable nonlinearities and usually unknown. Due to its nonlinear characteristics, the hysteresis in the system always give rise to undesirable inaccuracies, reduce the tracking control performance of the system and even cause the instability of the system [1]. In order to handle the control problems of stochastic nonlinear systems, a series of adaptive control schemes are proposed in the literature [3]–[12] by constructing a quadratic Lyapunov function, combined with the backstepping technique presented in [2]. Many results have been obtained in [13]–[23] for the nonlinear systems with hysteresis. The authors in previous references [13], [24]–[29] have studied the adaptive control of single-input and single-output (SISO) nonlinear systems with unknown backlash-like hysteresis. In [13], [15]–[23], considering the backlash-like hysteresis phenomenon as a bounded interference, the backlash-like hysteresis model was dealt with, and some adaptive robust control techniques were developed.In [23], the novel hysteresis inverse was acquired and efficiently eliminated the hysteresis effects. The system performance was improved by taking the hysteresis into consideration in controller design. The literatures mentioned above have proposed some control schemes for the stochastic nonlinear systems with hysteresis input. As a note, the results obtained in the above schemes can only ensure system performances when the time converges to be infinite.

However, in a large of practical controlled systems, the infinite-time stability is usually not feasible. We hope that the systems can obtain the stable performance in finite-time, which can rapidly achieve system transient performance. Thus, the investigation on control design for nonlinear systems is a valuable problem. The literatures [30], [31] have established the finite-time Lyapunov stability theorem, based on the Lyapunov theorem, finite-time control strategies for nonlinear systems were proposed in [34]–[48]. However, the control results obtained in [30]–[46] are only suitable for deterministic system. In practical system applications, the existence of random disturbance usually leads the systems instable, which cause the above control schemes to be unavailable. Therefore, it needs to take a further consideration for the finite time stability problem of stochastic systems. The limited time control can offer more advantages, for instance, the better disturbance resistance ability

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and strong robustness [47], [48]. A rigorous restricted time stability analysis for stochastic systems was first made by Yin *et al.* for the first time and the random finite time stability theorem was established in [49], the theory was gradually applied to construct the finite-time control strategy for nonlinear systems in [50]. The point is that the nonlinearities terms in [49], [50] must satisfy some growth condition assumptions. If random disturbances are unknown, the proposed finite-time stability standard is ineffective. Therefore, in [51], which has proposed an novel finite time stability theorem for nonlinear systems, it removes the linear growth condition. Nevertheless, the hysteresis problem is not considered in the control schemes mentioned above.

Inspired by the above related studies, this article will develop a finite-time stabilization issue of a class of stochastic systems with hysteresis input. We assume that the nonlinearities and the stochastic interference terms are unknown in this article. By employing the approximation ability of RBF neural network systems, a new finite-time neural control scheme is proposed. Combining with the new finite-time stability criterion proposed in [51], the effectiveness of the proposed control strategy is proved. To sum up, the main innovation points of this article can be concluded as follows.

(1) Different from the existing studies of finite-time adaptive control strategies, the randomness and hysteresis nonlinearity are taken into account in this article. The state of the systems is unknown, and all the unknown functions contain the whole state variables, i.e., a non-strict form.

(2) Compared with the studies of stochastic systems with hysteresis, these control schemes are only able to guarantee the infinite time stability of random nonlinear system in theory. The control problem based on Lyapunov finite-time stability theory is considered in this paper, which can guarantee the stability of the system in a limited time. Therefore, the proposed control strategy is more meaningful in the realistic controlled system.

II. PRELIMINARY KNOWLEDGE AND PROBLEM DESCRIPTION

A. PRELIMINARIES

In order to facilitate the later stability analysis, the necessary basic lemmas and definitions are given in this part. Now consider the following stochastic nonlinear system:

$$dx = f(x,t)dt + g^T(x,t)dw, \tag{1}$$

where $x \in R^n$ represents the state variable, $f : R^{n+m} \to R^n$, and $g : R^{n+m} \to R^{n\times r}$ are Borel measurable and continuous in x which satisfied $f(0,t) = g(0,t) = 0$ for any $t \geq 0$; w stands for an r-dimension Brownian motion defined on a complete probability space $(\Theta, F, \{F_t\}_{t\geq 0}, P)$ with Θ denoting a sample space, F is a $\sigma-$field, $\{F_t\}_{t\geq 0}$ denoting a filtration, P representing a probability measure.

Definition 1: Corresponded with system(1), and arbitrary functions $V(x,t) \in C^2$, we give a definition of the differential operator of V as bellow:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\{g^T \frac{\partial^2 V}{\partial x^2} g\}, \tag{2}$$

where Tr is a matrix trace.

Definition 2: If for $\forall x(t_0) = x_0$, existing the constant $\varepsilon > 0$, the retention time $\Lambda(\varepsilon, x_0) < \infty$ is to let $E(|x(t)|^2) < \varepsilon$, for $\forall t > t_0 + \Lambda$. The stochastic nonlinear system (1) is called to be practical finite-time stable in mean square.

Lemma 1 (see [51]): Consider the system $\zeta = f(\zeta, \tau)$, for smooth positive definite function $V(\zeta) \in C^1$, suppose the scalars $k > 0$, $0 < \alpha < 1$, $c > 0$, the following relation can be obtained:

$$LV(\zeta) \leq -kV^\alpha(\zeta) + c, t \geq 0. \tag{3}$$

We define $LV(\zeta) = f(\zeta, \tau)$ is SGPFS.

Lemma 2 (see [52]): For $x_k \in R$, $k = 1, \ldots, p$, $\tau \in (0,1)$, we can get the following relation:

$$\Big(\sum_{k=1}^{p} |x_k|\Big)^\tau \leq \sum_{k=1}^{p} |x_k|^\tau \leq p^{1-\tau}\Big(\sum_{k=1}^{p} |x_k|\Big)^\tau. \tag{4}$$

Lemma 3 (see [53]): For the dynamic system:

$$\dot{\hat{\eta}}(t) = -\kappa\hat{\eta}(t) + \gamma\rho(t) \tag{5}$$

where $\kappa > 0$, $\rho(t) \geq 0$, and $\gamma > 0$. If the $\hat{\eta}(t)$ is satisfied the condition $\hat{\eta}(t_0) \geq 0$, $\forall t \geq t_0$, we can obtain $\hat{\eta}(t) \geq 0$.

Lemma 4 (see [54]): For $\forall p \in R$, $q \in R$ and positive scalars γ, η, κ, the following relation holds:

$$|p|^\gamma |q|^\kappa \leq \frac{\gamma}{\gamma+\kappa}\eta|p|^{\gamma+\kappa} + \frac{\kappa}{\gamma+\kappa}\eta^{\frac{-\gamma}{\kappa}}|q|^{\gamma+\kappa}. \tag{6}$$

Lemma 5 (see [14]): For $\kappa \in [0,t]$ and the smooth function $h(t)$, if $h(t)$ satisfies the relation as follows:

$$\int_\kappa^t h(\vartheta)d\vartheta \leq 0, \tag{7}$$

then we can obtain $h(t) \leq 0$, $\forall\, t \in [0, +\infty)$.

Lemma 6 (see [14]): For the system $\dot{\chi} = f(\chi, u)$ and $s \in [0,t]$, if the function $\varpi(\chi(t)) \in C^2$, three constants $a, b \in (0, +\infty)$, and $\beta \in (0,1)$,$\kappa_\infty-$ functions η_1 and η_2, which satisfied the following inequality

$$\begin{cases} \eta_1(\|\chi\|) \leq \zeta(\chi(t)) \leq \eta_2(\|\chi\|) \\ \varpi(\chi(t)) - \varpi(\chi(s)) \leq -a\int_s^t \varpi^\beta(\chi(\vartheta))d\vartheta + b(t-s), \end{cases} \tag{8}$$

Then, there are two constants $\tau > 0$ and $\Lambda > 0$ which satisfy $\|\chi(t)\| < \tau$, $\forall t \geq \Lambda$.

B. PROBLEM FORMULATION

Consider the following stochastic nonlinear system:

$$\begin{cases} dx_i = (x_{i+1} + f_i(x))dt + \varphi_i^T(x)dw, \quad 1 \leq i \leq n-1 \\ dx_n = (u + f_n(x))dt + \varphi_n^T(x)dw, \\ y = x_1. \end{cases} \tag{9}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ represent the state vector; $y \in R$ is a system output; $f_i(.) : R^n \to R$ and $\varphi_i(.)$:

$R^n \rightarrow R^r (i = 1, 2, \ldots, n)$ represent unknown smooth nonlinear function; $u \in R$ denotes the control signal with known direction hysteresis nonlinearity. u will be constructed specified in the later.

C. HYSTERESIS CHARACTERISTIC

The Bouc-Wen hysteresis model is adapted to capture the output hysteresis phenomenon in this manscript which can be formulated as follows

$$H(v) = \mu_1 v + \mu_2 \varsigma \tag{10}$$

where μ_1 and μ_2 are known hysteresis parameters with the same sign and μ_1 satisfy the equalities:

$$0 < \delta_{min} \leq \mu_1. \tag{11}$$

where μ_1 governs the direction of hysteresis model. When $\mu_1 > 0$, the direction of the hysteresis is positive, if $\mu_1 < 0$, the direction of hysteresis is negative. The known hysteresis direction implies the sign of μ_1 is known in this paper. $v(t) \in R$ is the input of the hysteresis, ς is the auxiliary variable which satisfied the following differential equalition:

$$\frac{d\varsigma}{dt} = \frac{dv}{dt} - \beta\left|\frac{dv}{dt}\right||\varsigma|^{n-1}\varsigma - \chi\frac{dv}{dt}|\varsigma|^n \equiv \dot{v}f(\varsigma, \dot{v}) \tag{12}$$

where ς satisfies the equalities:

$$|\varsigma| \leq \sqrt[n]{1/(\beta + \chi)} \tag{13}$$

D. RBF NEURAL NETWORKS

In this section, the unknown continuous function like $f_i(\cdot) : R^n \rightarrow R$ need to be approximated by radial basis function neural networks as the following form:

$$f(Z) = W^T S(Z), \tag{14}$$

where $\forall Z \in \Theta_Z \subset R^q$, and denotes an input vector, $W = [w_1, \cdots, w_N]^T \in R^N$ with $N > 1$ represent the weight vector, $S(Z) = [S_1(Z), S_2(Z), \ldots, S_N(Z)]^T$ denotes a basis function vector with $S_i(Z)$ being the Gaussian function as follows:

$$S_i(Z) = \exp[-\frac{(Z - \phi_i)^T (Z - \phi_i)}{\eta^2}], \quad i = 1, 2, \ldots, N, \tag{15}$$

where $\phi_i = [\phi_{i1}, \phi_{i2}, \ldots, \phi_{iq}]^T$ is the center of the receptive field and η is the width of the basis function. For given accuracy $\epsilon > 0$ with sufficiently large node number N, the RBF NN can approximate any continuous function $f(Z)$ over a compact set $\Theta_z \subset R^q$ such that

$$f(Z) = W^{*T} S(Z) + \delta(Z), \tag{16}$$

where $\forall Z \in \Theta_Z \subset R^q$, W^* denotes an ideal weight vector which defined as:

$$W^* = \arg \min_{W \in R^N} \{ \sup_{Z \in \Omega_Z} |f(Z) - W^T S(Z)|\},$$

and $\delta(Z)$ is an approximation error satisfying $| \delta(Z) | \leq \varepsilon$.

Lemma 7: We denote $\hat{z}_p = [z_1, z_2, \ldots, z_p]^T$, and $S(\hat{z}_p) = [S_1(\hat{z}_p), S_2(\hat{z}_p), \ldots, S_q(\hat{z}_p)]$ are the basis function vector of a RBF NN. Then for any positive integer $k \leq p$, we can obtain that:

$$||S(\hat{z}_p)||^2 \leq ||S(\hat{z}_k)||^2 \tag{17}$$

Remark 1: This lemma illustrates one of the characteristic of RBF NN. We can apply it to system (9) to complete the adaptive neural backstepping design.

This manuscript contributes to design an adaptive neural network controller, for system(9) to make all signals in the closed loop system are practical finite-time stable in mean square, the output of controller will follow the designed reference signal y_d in limited time.

III. MAIN RESULT

This part aims to construct a controller for system(9) by backstepping. The neural network $W_i(Z_i)$ are utilized to approximate unknown functions $\bar{f}$. Firstly, let $\theta_i = \|W_i\|^2, i = 1, 2, \cdots, n$, $\hat{\theta}_i$ is the estimate of θ_i and $\tilde{\theta} = \theta_i - \hat{\theta}_i$ as the estimation error, then make the following transformation:

$$\begin{cases} z_1 = x_1 - y_d, \\ z_i = x_i - \alpha_{i-1}. \quad 2 \leq i \leq n. \end{cases} \tag{18}$$

where α_{i-1} being an designed virtual control function constructed in the step i.

Step 1. For stochastic system, we can get the following equation.

$$dz_1 = (f_1(x) + x_2 - \dot{y}_d)dt + \xi_1^T(x)dw. \tag{19}$$

Think about the following candidate Lyapunov function:

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{2q_1}\tilde{\theta}_1^2, \tag{20}$$

where $q_1 > 0$ denotes a design parameter, and $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$. From (2), (18), (19), differentiating V_1 yields

$$LV_1 = z_1^3(z_2 + \alpha_1 + f_1(x) - \dot{y}_d) + \frac{3}{2}z_1^2\xi_1^T\xi_1 - \frac{1}{q_1}\tilde{\theta}_1\dot{\hat{\theta}}_1. \tag{21}$$

Applying Young's inequality, it is easily yielded that

$$\frac{3}{2}z_1^2\xi_1^T\xi_1 \leq \frac{3}{4l_1^2}z_1^4\|\xi_1\|^4 + \frac{3}{4}l_1^2, \tag{22}$$

$$z_1^3 z_2 \leq \frac{3z_1^4}{4} + \frac{z_2^4}{4}, \tag{23}$$

where $l_1 > 0$ represents the design parameter.

Combining (22)–(23), (21) can be rewritten as

$$\begin{aligned} LV_1 \leq z_1^3 f_1 + z_1^3\alpha_1 - z_1^3\dot{y}_d + \frac{3z_1^4}{4} + \frac{z_2^4}{4} \\ + \frac{3}{4l_1^2}z_1^4\|\xi_1\|^4 + \frac{3}{4}l_1^2 - \frac{1}{q_1}\tilde{\theta}_1\dot{\hat{\theta}}_1. \end{aligned} \tag{24}$$

where let $\bar{f}_1 = f_1 + \frac{3z_1}{4} + \frac{3}{4l_1^2}z_1\|\xi_1\|^4 - \dot{y}_d$, then (24) can be rewritten as

$$LV_1 \leq z_1^3\bar{f}_1 + z_1^3\alpha_1 + \frac{z_2^4}{4} + \frac{3}{4}l_1^2 - \frac{1}{q_1}\tilde{\theta}_1\dot{\hat{\theta}}_1. \tag{25}$$

Hence, the unknown functions f_1, ξ_1 are contained in $\bar{f}_1$, according to (17), which illuminates the approximation ability of neural network $W_1^{*T}S_1$, so the neural network $W_1^{*T}S_1$ will be used to model it. For any given constant $\varepsilon_i > 0$, one has

$$\bar{f}_1 = W_1^{*T}S_1(Z_1) + \delta_1(Z_1), |\delta_1(Z_1)| \le \varepsilon_1, \tag{26}$$

where the definitions of W_1^{*T} and $S_1(Z_1)$ can be found in (16), $Z_1 = [x^T, y_d^T, \dot{y}_d^T]$, combining the Young's inequality, one has:

$$z_1^3\bar{f}_1 \le \frac{1}{2a_1^2}z_1^6\theta_1 S_1^T S_1 + \frac{1}{2}a_1^2 + \frac{3z_1^4}{4} + \frac{1}{4}\varepsilon_1^4. \tag{27}$$

where $\theta_1 = ||W_1^*||^2$, it follows immediately from substituting (27) into (25) that

$$LV_1 \le \frac{3z_1^4}{4} + \frac{1}{2a_1^2}z_1^6\theta_1 S_1^T S_1 + z_1^3\alpha_1 + \frac{z_2^4}{4} + \frac{3}{4}l_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{1}{2}a_1^2 - \frac{1}{q_1}\tilde{\theta}_1\dot{\hat{\theta}}_1. \tag{28}$$

Choose the virtual control signal and adaptive law as follows:

$$\alpha_1 = -k_1 z_1^{4\nu-3} - \frac{3z_1}{4} - \frac{{z_1}^3}{2a_1^2}\hat{\theta}_1 S_1^T S_1, \tag{29}$$

$$\dot{\hat{\theta}}_1 = \frac{q_1}{2a_1^2}z_1^6 S_1^T S_1 - \sigma_1\hat{\theta}_1. \tag{30}$$

where k_1, r_1 and σ_1 are positive design parameters and $\hat{\theta}_1(0) \ge 0$.

Remark 2: It is not difficult to see from the adaptive law (30) that it satisfies the conditions of Lemma 3. Consequently, suppose that the initial condition $\hat{\theta}_1(t_0) \ge 0$, the $\hat{\theta}_1(t) \ge 0$ $\forall t \ge t_0$. Actually, in practical situations, it is invariably rational to select $\hat{\theta}_1(t_0) \ge 0$, due to the $\hat{\theta}_1$ is an estimation of θ_1. This nature will be used in subsequent design procedure.

Taking (29) and (30) into (28), we have

$$LV_1 \le -k_1 z_1^{4\nu} + \frac{z_2^4}{4} + \frac{3}{4}l_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{1}{2}a_1^2 + \frac{\sigma_1}{q_1}\tilde{\theta}_1\hat{\theta}_1. \tag{31}$$

For the term $\frac{\sigma_1}{q_1}\tilde{\theta}_1\hat{\theta}_1$, applys Young's inequality, the following holds.

$$\frac{\sigma_1}{q_1}\tilde{\theta}_1\hat{\theta}_1 \le -\frac{\sigma_1}{2q_1}\tilde{\theta}_1^2 + \frac{\sigma_1}{2q_1}\hat{\theta}_1^2 \tag{32}$$

So, (31) can be rewritten as follows:

$$LV_1 \le -k_1 z_1^{4\nu} + \frac{z_2^4}{4} + \frac{3}{4}l_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{1}{2}a_1^2 - \frac{\sigma_1}{2q_1}\tilde{\theta}_1^2 + \frac{\sigma_1}{2q_1}\hat{\theta}_1^2. \tag{33}$$

Let $\iota = \frac{3}{4}l_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{1}{2}a_1^2 + \frac{\sigma_1}{2q_1}\hat{\theta}_1^2$, then

$$LV_1 \le -k_1 z_1^{4\nu} + \frac{z_2^4}{4} - \frac{\sigma_1}{2q_1}\tilde{\theta}_1^2 + \iota. \tag{34}$$

step i:

$$dz_i = (f_i(x) + x_{i+1} - L\alpha_{i-1})dt + (\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j)^T dw \tag{35}$$

where

$$L\alpha_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}[f_j + x_{j+1}] + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p x_q}\xi_p^T\xi_q + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^j}y_d^{j+1}. \tag{36}$$

for $\frac{\partial\alpha_0}{\partial x_k} = 0$, $\frac{\partial\alpha_0}{\partial\theta_k} = 0$.

Now select the following Lyapunov function candidate as:

$$V_i = V_{i-1} + \frac{z_i^4}{4} + \frac{\tilde{\theta}_i^2}{2q_i} \tag{37}$$

where q_i represents the design parameter, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ represents the parameter error. Similarly, follow the step 1, yields:

$$LV_i = LV_{i-1} + z_i^3(z_{i+1} + \alpha_i + f_i(x) - L\alpha_{i-1}) - \frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i + \frac{3}{2}z_i^2(\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j)^T(\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j). \tag{38}$$

For the purpose of simplifying the writing of the latter part, it can be rewritten as: $\frac{3}{2}z_i^2(\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j)^T(\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j) = \frac{3}{2}z_i^2\left\|\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j\right\|^2$, by adapting the Young's inequality, the following inequalities obtained easily:

$$z_i^3 z_{i+1} \le \frac{3z_i^4}{4} + \frac{z_{i+1}^4}{4}, \tag{39}$$

$$\frac{3}{2}z_i^2\left\|\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j\right\|^2 \le \frac{3}{4l_i^2}z_i^4\left\|\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j\right\|^4 + \frac{3}{4}l_i^2. \tag{40}$$

Taking (39) and (40) into (38), we can get the following result:

$$LV_i \le -\sum_{j=1}^{i-1}(k_j z_j^{4\nu} + \frac{\sigma_i}{2q_j}\tilde{\theta}_j^2) + \sum_{j=1}^{i-1}\iota_j + \frac{1}{4}z_{i+1}^4 + z_i^3\alpha_i + z_i^3\bar{f}_i - \frac{1}{q_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \tag{41}$$

where

$$\bar{f}_i = f_i - L\alpha_{i-1} + \frac{3}{4l_i^2}z_i\left\|\xi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\xi_j\right\|^4 + \frac{3}{4}z_i$$

with k_i being the positive design parameters. Apparently, f_i is an unknown function of z_i, and the neural network $W_i^{*T}S_i(Z_i)$ is utilized to approximate it. Since $\bar{f}_i$ can expressed as follows with $Z_i = [x^T, \bar{\hat{\theta}}_i, \bar{y_d}^T]^T$, $\bar{y}_d = [y_d, y_d', \ldots, y_d^{(i)}]^T$ and $\bar{\hat{\theta}}_i = [\tilde{\theta}_1, \ldots, \tilde{\theta}_i]^T$.

$$\bar{f}_i = W_i^{*T}S_i(Z_i) + \delta_i(Z_i), \quad |\delta_i(Z_i)| \le \varepsilon_i, \tag{42}$$

with δ_i being the approximation error which satisfied $|\delta_i| \le \varepsilon_i$, $\varepsilon_i \ge 0$. Using the same procedure as (27), we have

$$z_i^3\bar{f}_i \le \frac{1}{2a_i^2}z_i^6\theta_i S_i^T S_i + \frac{1}{2}a_i^2 + \frac{3z_i^4}{4} + \frac{1}{4}\varepsilon_i^4, \tag{43}$$

where $a_i > 0$, then, similar to (29) and (30), yields:

$$\alpha_i = -k_i z_i^{4\nu-3} - \frac{3}{4}z_i - \frac{1}{2a_i^2}\hat{\theta}_i z_i^3 S_i^T S_i \tag{44}$$

$$\dot{\hat{\theta}}_i = \frac{q_i}{2a_i^2}z_i^6 S_i^T S_i - \sigma_i\hat{\theta}_i, \tag{45}$$

where $q_i \ge 0$ and $\hat{\theta}_i(0) \ge 0$. Similar to (32), we can easily get that

$$\frac{\sigma_i}{q_i}\tilde{\theta}_i\hat{\theta}_i \le -\frac{\sigma_i}{2q_i}\tilde{\theta}_i^2 + \frac{\sigma_i}{2q_i}\hat{\theta}_1^2 \tag{46}$$

so substituting (43)-(46) into (41), we can obtain that

$$LV_i \le -\sum_{j=1}^{i-1}(k_j z_j^{4\nu} + \frac{\sigma_j}{2q_j}\tilde{\theta}_j^{\,2}) + \sum_{j=1}^{i-1}\iota_j + \frac{z_{i+1}^4}{4}, \tag{47}$$

where $\iota_j = \frac{3}{4}l_j^2 + \frac{1}{4}\varepsilon_j^4 + \frac{1}{2}a_j^2 + \frac{\sigma_j}{2q_j}\hat{\theta}_j^2, j = 1, 2 \ldots, i$.

Step n: In this procedure, the real control signal u is constructed. From $z_i = x_i - \alpha_{i-1}, (i = 2, \ldots, n)$, we have the following equation.

$$dz_n = (f_n(x)+H(v)-L\alpha_{n-1})dt+\left(\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j\right)^T dw \tag{48}$$

with

$$L\alpha_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}[f_j + x_{j+1}] + \frac{1}{2}\sum_{p,q=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_p\partial x_q}\xi_p^T\xi_q + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j + \sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j)}}y_d^{(j+1)}. \tag{49}$$

So we define Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{z_n^4}{4} + \frac{\tilde{\theta}_n^2}{2q_n} \tag{50}$$

then we can get the result easily:

$$LV_n = LV_{n-1} + z_n^3(H(v) + f_n(x) - L\alpha_{n-1}) - \frac{1}{q_n}\tilde{\theta}_n\dot{\hat{\theta}}_n + \frac{3}{2}z_n^2(\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j)^T(\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j), \tag{51}$$

and similar to (40), we have:

$$\frac{3}{2}z_n^2\left|\left|\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j\right|\right|^2 \le \frac{3}{4l_n^2}z_n^4\left|\left|\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j\right|\right|^4 + \frac{3}{4}l_n^2. \tag{52}$$

Hence, the LV_n can be rewritten as:

$$LV_n = LV_{n-1} + z_n^3(\mu_1 v + \mu_2\varsigma + f_n(x) - L\alpha_{n-1}) + \frac{3}{4l_n^2}z_n^4\left|\left|\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j\right|\right|^4 + \frac{3}{4}l_n^2. \tag{53}$$

Then, the following equality yields:

$$z_n^3\mu_2\varsigma \le \frac{3}{4}z_n^4 + \frac{1}{4}(\mu_2\varsigma)^4, \tag{54}$$

From the inequalities (52), (53) and (54), the following form yields:

$$LV_n \le -\sum_{j=1}^{n-1}(k_j z_j^{4\nu} + \frac{\sigma_j}{2q_j}\tilde{\theta}_j^2) + \sum_{j=1}^{i-1}\iota_j + z_n^3\bar{f}_n - \frac{3}{4}z_n^4 + z_n^3\mu_1 v + \frac{3}{4}l_n^2 + \frac{1}{4}(\mu_2\varsigma)^4 - \frac{\tilde{\theta}_n\dot{\hat{\theta}}_n}{q_n}. \tag{55}$$

where $\bar{f}_n$ denotes as follows:

$$\bar{f}_n = f_n - L\alpha_{n-1} + \frac{7}{4}z_n + \frac{3z_n^4}{4l_n^2}\left|\left|\xi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\xi_j\right|\right|^4. \tag{56}$$

For the constant ε_n, the neural networks is used again to approximate $\bar{f}_n$. So that

$$\bar{f}_n = W_n^{*T}S_n(Z_n) + \delta_n(Z_n), \quad |\delta_n(Z_n)| \le \varepsilon_n. \tag{57}$$

Furthermore, applying the Young's inequality, we have

$$z_n^3\bar{f}_n \le \frac{1}{2a_n^2}z_i^6\theta_n S_n^T S_n + \frac{1}{2}a_n^2 + \frac{3z_n^4}{4} + \frac{1}{4}\varepsilon_n^4. \tag{58}$$

Now, we choose the ideal control signal v and adaptation laws as

$$v = -k_n z_n^{4\nu-3} - \frac{z_n^3\hat{\theta}_n S_n^T S_n}{2a_n^2\delta_{min}}, \tag{59}$$

$$\dot{\hat{\theta}}_n = \frac{q_n}{2a_n^2}z_n^6 S_n^T S_n - \sigma_n\hat{\theta}_n, \quad \hat{\theta}_n(0) \ge 0 \tag{60}$$

with $k_n > 0$, $a_n > 0$ and $q_n > 0$ represent design parameters. Applying to Lemma 3, the following inequality holds.

$$z_n^3\mu_1 v \le -\delta_{min}k_n z_n^{4\nu} - \frac{z_n^6\hat{\theta}_n S_n^T S_n}{2a_n^2}. \tag{61}$$

So combined the (58)-(61), we can obtain that

$$LV_n \le -\sum_{j=1}^{n}(k_j z_j^{4\nu} + \frac{\sigma_j}{2q_j}\tilde{\theta}_j^{\,2}) + \sum_{j=1}^{n-1}\iota_j - \delta_{min}k_n z_n^{4\nu} + \frac{3l_n^2}{4} + \frac{a_n^2}{2} + \frac{\varepsilon_n^4}{4} + \frac{(\mu_2\varsigma)^4}{4} + \frac{\sigma_n}{q_n}\tilde{\theta}_n\hat{\theta}_n. \tag{62}$$

Based on the following inequality

$$\frac{\sigma_n}{q_n}\tilde{\theta}_n\hat{\theta}_n \le -\frac{\sigma_n}{2q_n}\tilde{\theta}_n^2 + \frac{\sigma_n}{2q_n}\hat{\theta}_1^2, \tag{63}$$

the following result yields:

$$LV_n \le -\sum_{j=1}^{n}(m_j z_j^{4\nu} + \frac{\sigma_j}{2q_j}\tilde{\theta}_j^{\,2}) + \sum_{j=1}^{n}\iota_j + \frac{1}{4}(\mu_2\varsigma)^4 \tag{64}$$

where $m_j = k_j (1 \leq j \leq n-1), m_n = \delta_{min} k_n$, and $\iota_j = \frac{\sigma_j}{2q_j}\theta_j^2 + \frac{a_j^2}{2} + \frac{3l_j^2}{4} + \frac{\varepsilon_j^4}{4} (1 \leq 1 \leq n)$. Until now, we can conclude the main result below.

Theorem 1: For nonlinear stochastic system (9), through the above recursive procedure, by applying backstepping technique and the packaged unknown function $\bar{f}_n$ which is approximated by neural network approximation, the controller (59) together with the adaptive law (60) what we have designed can guarantee all the signal of closed-system are practical finite-time stable in mean square.

Proof: Consider a Lyapunov function as $V = V_n$. Let $m = min\{m_j, q_j, j = 1, 2, \ldots, n\}$, then (64) can be expressed as follows

$$LV_n \leq -m \sum_{j=1}^{n} (z_j^{4\nu} + \frac{\tilde{\theta}_j^2}{2q_j}) + \sum_{j=1}^{n} \iota_j + \frac{1}{4}(\mu_2 \varsigma)^4 \tag{65}$$

If $z = \sum_{j=1}^{n} \frac{\tilde{\theta}_j^2}{2q_j}$, $\chi = 1$, and $\kappa = \sigma, \eta = 1-\sigma, \mu = \frac{1}{\sigma}$ are applied to Lemma 4, we get

$$(\sum_{j=1}^{n} \frac{\tilde{\theta}_j^2}{2q_j})^{\sigma} \leq (1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}} + \sum_{j=1}^{n} \frac{\tilde{\theta}_j^2}{2q_j} \tag{66}$$

Combining Lemma 2, (65) and (66), we then have

$$LV(Z_n(t)) \leq -4^{\sigma} m (\sum_{j=1}^{n} \frac{z_j^4}{4})^{\sigma} - m(\sum_{j=1}^{n} \frac{\tilde{\theta}_j^2}{2\lambda_j})^{\sigma} + c. \tag{67}$$

where $c = m(1-\sigma)\sigma^{\frac{\sigma}{1-\sigma}} + \sum_{j=1}^{n} c_j + \frac{1}{4}(\mu_2\varsigma)^4$. Then, by applying Lemma 1, one has

$$LV(Z_n(t)) \leq -mV^{\sigma}(Z_n(t)) + c. \tag{68}$$

According to Itô formula, we have

$$\begin{aligned} EV(Z_n(t)) &= EV(Z_n(s)) + E\int_s^t LV(Z_n(\vartheta))d\vartheta \\ &= EV(Z_n(s)) + \int_s^t E[LV(Z_n(\vartheta))]d\vartheta. \end{aligned} \tag{69}$$

where $s \in [0, t]$, by using the Jessen's inquality and (68), one obtains

$$\begin{aligned} E[LV(Z_n(s))] &\leq -mE[V^{\sigma}(Z_n(s)] + c \\ &\leq -m[EV(Z_n(s))]^{\sigma} + c. \end{aligned} \tag{70}$$

Substituting (70) into (69), yields

$$EV(Z_n(t)) \leq EV(Z_n(s)) + \int_s^t \{-m[EV(Z_n(\vartheta))]^{\sigma} + c\} d\vartheta. \tag{71}$$

Therefore

$$EV(Z_n(t)) - EV(Z_n(s)) \leq -m\int_s^t [EV(Z_n(\vartheta))]^{\sigma} d\vartheta + c(t-s). \tag{72}$$

Applying $\omega(t) = EV(Z_n(t))$, from Lemma 6, it is concluded that the setting time $\Psi = \frac{1}{(1-\sigma)\beta m}\Big[(EV(Z_n(0)))^{1-\sigma} - \left(\frac{c}{(1-\beta)m}\right)^{(1-\sigma)/\sigma}\Big]$, which makes $EV(Z_n(t)) \leq \rho$ for $\forall t \geq \Psi$, where $\rho = 4(\frac{c}{(1-\beta)m})^{1/4\sigma}$. Using the expression of $V(Z_n(t))$, one has

$$E\Big(\sum_{j=1}^{n} z_j^4\Big) \leq 4E[V(Z_n(t))] \leq 4\rho, \quad t \geq \Psi. \tag{73}$$

From the property of mathematical expectation, we have

$$\Big[E(z_j^2)\Big]^2 \leq E\Big(z_j^4\Big) \leq E\Big(\sum_{j=1}^{n} z_j^4\Big) \leq 4\rho, \quad t \geq \Psi. \tag{74}$$

Thus

$$E(z_j^2) \leq 2\sqrt{\rho}, \quad t \geq \Psi. \tag{75}$$

Therefore, $E(|y - y_d|^2) \leq 2\sqrt{\rho}, \quad t \geq \Psi$. So, we can obtain

$$E(\theta_j^2) \leq 2\lambda_{\max}\rho, \quad t \geq \Psi, \tag{76}$$

where $\lambda_{\max} = \max\{\lambda_j, \ 1 \leq j \leq n\}$. (75), (76) mean that the system is practical finite-time stable in mean square.

Remark 3: Consider the stochastic nonliear system (9) with Bouc-Wen hysteresis. From the aforementioned proof, under the stability condition, a settling time Ψ has been found and the finite time stability has been proved. The actual controller (59) with the adaptive laws (30), (45) and (60) can guarantee that all the signal in the closed-loop system remain bounded in probability, and the tracking error converges to a small neighborhood of zero. Therefore, we will give some specific examples to verify the effectiveness of the scheme.

IV. SIMULATION STUDY

In this section, a simulations example is given to validate the feasibility of the proposed control programme. There is a non-strict feedback nonlinear system model with finite-time and hysteresis in example 1.

Example 1: The stochastic non-strict feedback nonlinear system with hysteresis mechanism is given as:

$$\begin{cases} dx_1 = (0.5x_1 sinx_2 + x_2)dt + (0.5x_1^2 sinx_1)dw, \\ dx_2 = (x_1^2 cosx_2 + u)dt + (x_1x_2)dw, \\ y = x_1, \end{cases}$$

where x_1, x_2 represent the state variables, the $y(t)$ is system output. The u denotes the system input and hysteresis output which can be shown as following Bouc-Wen hysteresis:

$$\begin{aligned} u &= H(v) = \mu_1 v + \mu_2 \varsigma \\ \frac{d\varsigma}{dt} &= \frac{dv}{dt} - \beta|\frac{dv}{dt}||\varsigma|^{n-1}\varsigma - \chi\frac{dv}{dt}|\varsigma|^n \end{aligned} \tag{77}$$

where $\varsigma(t_0) = 0$, and the hysteresis parameters are $\mu_1 = 3$, $\mu_2 = 5, \beta = 1.5, n = 2$ and $\chi = 0.5$. The adaptive neural controller we designed should keep all signals remain bounded in probability, the system output y follows the reference trajectory chosen as $y_d = 1.5\sin(0.5t)$. Now, to guarantee all signals are stable in a mean square for a

TABLE 1. Design parameters.

Parameters	Values
$x_1(0), x_2(0)$	0.8,-0.1
$\hat{\theta}_1(0), \hat{\theta}_2(0)$	0.5,0.6
k_1, k_2	10,8
a_1, a_2	0.6,1.2
q_1, q_2	16,28
λ_1, λ_2	1,2
ν	$\frac{99}{100}$
δ_{min}	0.5
η	2
v	$[-1.5, -1, -0.5, 0, 0.5, 1, 1.5]^T$

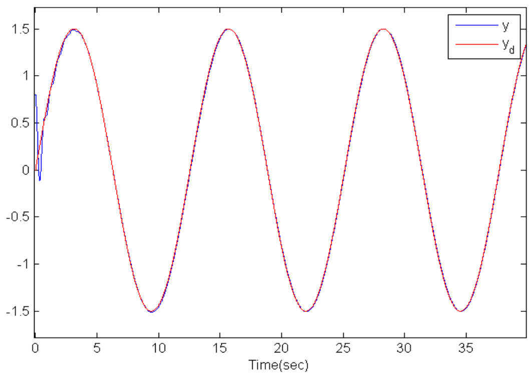

FIGURE 1. System output y and ideal signal y_d.

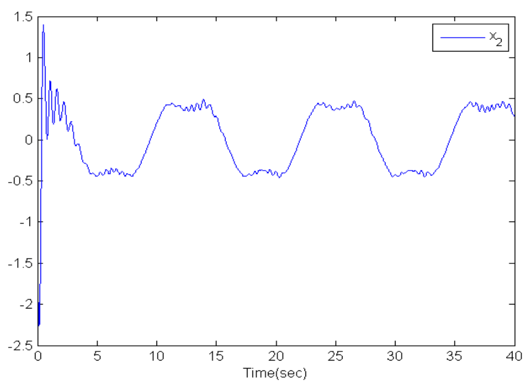

FIGURE 2. System state variable x_2.

limited time, we consider establishing the following actual control signal, virtual controller and the adaption laws:

$$\alpha_1 = -k_1 z_1^{4\nu-3} - \frac{3z_1}{4} - \frac{{z_1}^3}{2a_1^2}\hat{\theta}_1 \xi_1^T \xi_1,$$

$$v = -k_2 z_2^{4\nu-3} - \frac{3z_2}{4} - \frac{z_2^3}{2a_2^2 \delta_{min}}\hat{\theta}_2 \xi_2^T \xi_2,$$

$$\dot{\hat{\theta}}_i = \frac{q_i}{2a_i^2} z_i^6 \xi_i^T \xi_i - \sigma_i \hat{\theta}_i, \quad i = 1, 2.$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$. In simulation, the initial conditions are chosen in Table 1.

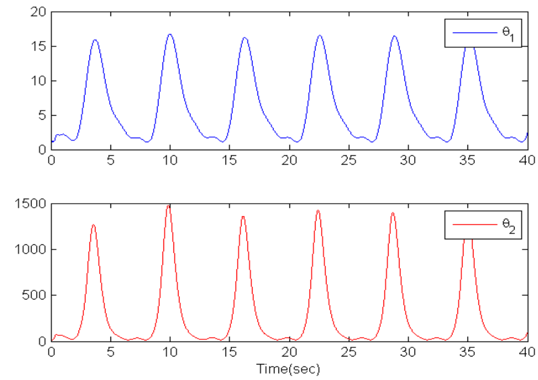

FIGURE 3. Adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.

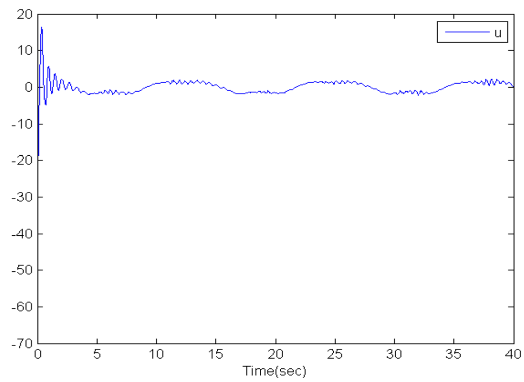

FIGURE 4. The actual controller u.

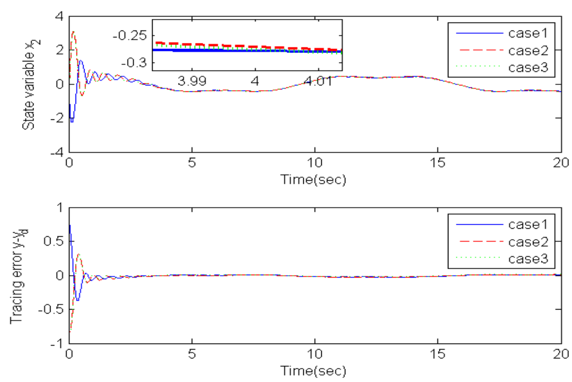

FIGURE 5. The tracing error y_d and state x_2 under three disturbance.

The following pictures 1-4 show the main results, respectively. Fig.1 provides the tracking performance of the systems. Fig.2 demonstrates the system state x_2. Fig.3 illustrates the adaptive laws θ_1 and θ_2, and Fig.4 shows the actual controller u.

Through the above simulation analysis, the results demonstrate that the control signals are stable for a limited time.

In order to show the effectiveness of the proposed scheme under different uncertain stochastic disturbance conditions, for random variable with normal distribution $dw \sim N(0, dt)$, we make simulation under three random disturbance

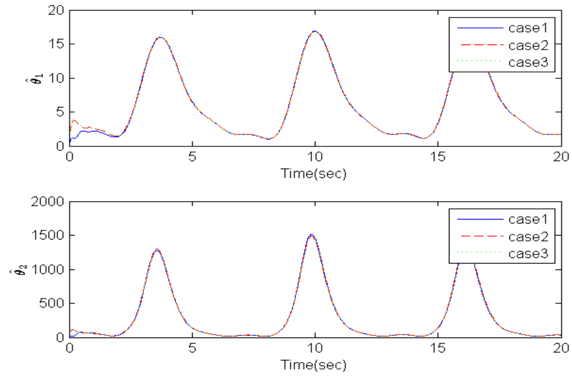

FIGURE 6. The adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ under three disturbance.

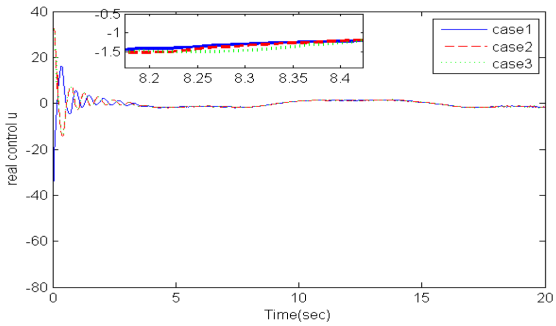

FIGURE 7. The actual controller u under three disturbance.

TABLE 2. Design parameters.

Parameters	Values
$x_1(0), x_2(0), x_2(0)$	7.5,7.5,7.5
$\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0)$	0.5,0.6,0.6
k_1, k_2, k_3	10,8,8
a_1, a_2, a_3	0.6,1.2,0.5
q_1, q_2, q_3	16,28,26
ν	$\frac{99}{100}$
δ_{min}	0.5
η	2
v	$[-1.5, -1, -0.5, 0, 0.5, 1, 1.5]^T$

situations, where w indicates an independent standard Brownian motion. In the simulation, the same design parameters and the same initial conditions as the ones in Example1 are used. The simulation results are displayed by Figs.5-7. From Figs.5-7, it can be seen that the desired control performance can be guaranteed even if the system is affected by random disturbances.

Example 2: Consider a third-order stochastic nonlinear system with hysteresis:

$$\begin{cases} dx_1 = (0.5x_1 sinx_2 + x_2)dt + (x_1^2 sinx_1)dw, \\ dx_2 = (x_2^2 cosx_2 + x_3)dt + (cos(x_1)x_2)dw, \\ dx_3 = (x_3^2 + u)dt + (0.5x_1)dw, \\ y = x_1, \end{cases}$$

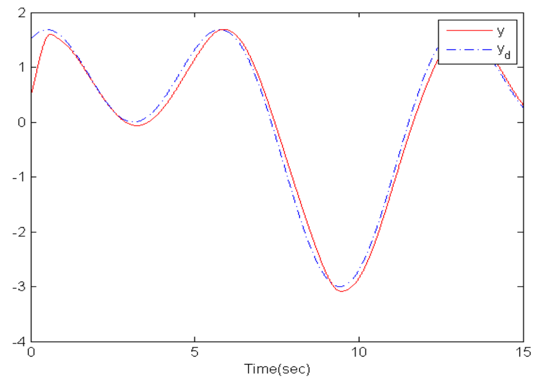

FIGURE 8. y and y_d.

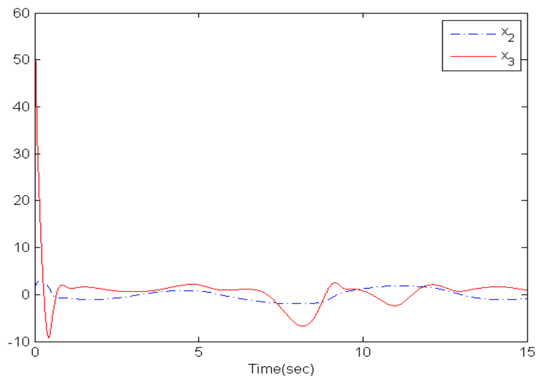

FIGURE 9. The State variables x_2 and x_3.

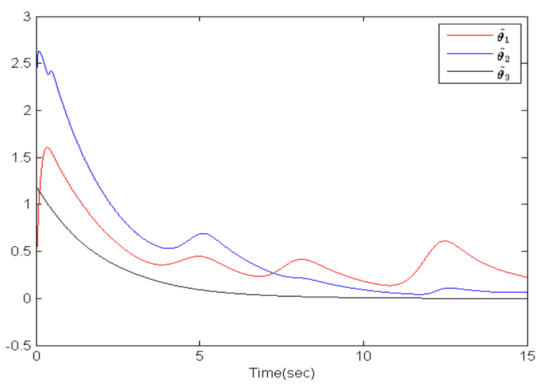

FIGURE 10. The adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$.

where x_1, x_2, x_3 represent the state variables, the $y(t)$ is system output. u and the hysteresis parameters are the same as the one in example 1. The reference signal is $y_d = 1.5(sin(0.5t) + cos(t))$ and the initial conditions are chosen in Table 2.

The simulation results are as Figs. 8-11. According to the four figures show, the system output locates in a small neighborhood of the reference signal in finite time and the other signals are also bounded under the hysteresis input by the proposed control scheme.

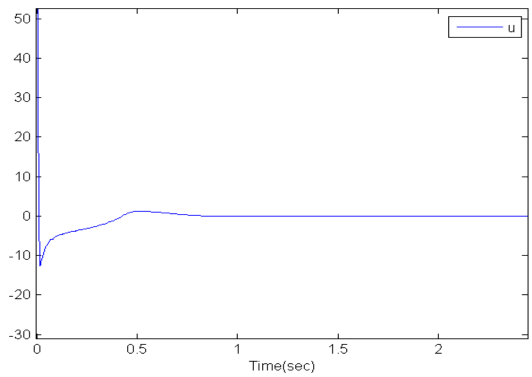

FIGURE 11. The actual controller u.

V. CONCLUSION

In this paper, on the basis of the approximation capacity of neural network systems and the important lemma of stochastic systems finite-time stability, an novel adaptive control scheme is proposed for a class of stochastic systems with Bouc-wen hysteresis input. Furthermore, the finite-time stability of the adaptive system can be guaranteed by stability analysis, and the simulation examples further proves that the nonlinear performance will be achieved in finite time. This control scheme will be extended to a larger class of suitable systems with hysteresis.

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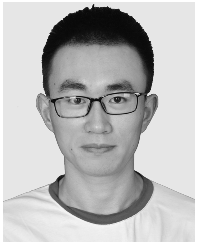

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