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Impulsive Synchronization of Fractional-Order Chaotic Systems With Actuator Saturation and Control Gain Error

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ABSTRACT This paper investigates the impulsive synchronization scheme of fractional-order chaotic systems with actuator saturation and control gain error. Based on the theory of fractional order system and impulsive differential system, discontinuous Lyapunov stability and matrix inequality approach, some new sufficient conditions are derived to guarantee the impulsive synchronization of a general class of fractional order chaotic systems. It is worth mentioning that the actuator saturation and control gain error are discussed simultaneously, which is more rigorous and practical in real systems. Finally, some simulation results verify the correctness and effectiveness of the theoretical results.

INDEX TERMS Fractional-order chaotic systems, impulsive synchronization, control gain error, actuator saturation.

I. INTRODUCTION

In the past few decades, the synchronization scheme for a myriad of chaotic systems has been wildly applied to many occasions, such as neural networks [1], mechanical systems [2] and data transmission privacy [3]. In [4], the chaos synchronization case was discussed for the first time, and a variety of significant and representative control protocols were designed subsequently to achieve the synchronization objective of chaotic dynamical systems. The representative synchronization protocols consist of linear and nonlinear feedback control [5], [6], sliding mode control [7], [8], event-triggered control [9], [10], nonlinear observer approach [11], [12], fuzzy approach [13], [14], adaptive control [15], [16], etc. It should be noted that the impulsive control approach, one discontinuous control protocol, has special advantages over the above continuous ones. For the synchronization control process with impulsive approach, the response (slave) system obtains the drive (master) system's state information

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only at the discrete instants which described by an impulsive sequence. Therefore, the state transmission burdens between the response (slave) and drive (master) systems will be relieved in large extent. It is obviously concluded that the impulsive approach can obtain higher robustness and lower control cost in practical applications than the continuous control methods [17]–[20].

Fractional order calculus and fractional order system are the old mathematic research fields for more than 300 years, and they are rarely used to the actual physical system owing to the poor application background. However, it has been confirmed that the fractional order calculus can describe a large number of systems more accurately than the integer one, such as viscoelastic systems [21], electromagnetic wave systems [22], macroeconomic systems [23] and so on. In recent years, the synchronization case for the fractional order chaotic systems obtained wide and considerable attention. Several typical fractional-order chaotic systems are analyzed and proved such as Lorenz systems [24], hyperchaotic Lü systems [25], Liu systems [26] and Bloch equations [27]. Numerous studies focused on the synchronization case of many kinds of

fractional-order nonlinear chaotic systems owing to its wild applications in genetic characteristics [28], and it was a challenging research task owing to the high sensitivity to initial parameters. In addition, various kinds of control methods has been explored to accomplish the synchronization goal for the fractional order systems. In [29], the robust synchronization case of the fractional order unified systems was studied by the linear control approach. The active synchronization case between two identical (or nonidentical) fractional order chaotic systems was discussed in [30]. In [31], the complete synchronization case of the commensurate fractional order systems with sliding mode control approach was studied. The adaptive synchronization of fractional order chaotic systems with uncertain system parameters via fuzzy sliding mode control approach was explored in [32]. The lag projective synchronization of delayed fractional-order systems via comparison system theory of linear fractional equation was discussed in [33]. In [34], the synchronization case of different fractional order chaotic systems with time-varying orders and parameters was investigated. Note that the above papers [29]–[34] are concern with the dynamical control systems in ideal system models. In fact, in many real-world systems, the control with restricted conditions and disturbances occurs periodically or aperiodically, which is not negligible in real systems and deserves further in-depth study.

Actuator saturation, which is also called control input saturation, is a serious defect in control process. The reason for this defect is that the actuators cannot provide persistent efforts in the practical engineering applications. It often destroy the control performance and effectiveness or even the stability to a great extent if the effect of the actuator saturation is ignored [35]. Owing to the importance and significance of the saturation, there are many results about the actuator saturation in recent years [11], [36]–[40]. For instance, in [36], the synchronization of nonlinear master and slave systems with input saturation and input delay (delay-range dependent) was investigated. In [37], the design of adaptive feedback controllers for chaos synchronization with unknown parameters and input saturation constraints was studied. In [38], the adaptive synchronization for the unknown chaotic systems with external unknown disturbances and input saturation was investigated, and the prescribed performance can be assured. In [11], the chaos synchronization with model uncertainties, non-symmetric input saturation and external disturbances was discussed. In [39], the synchronization case of delayed complex networks with actuator saturations via intermittent controller was studied. Note that the above results focus on the actuator saturation constraint problems via continuous control approaches. Based on the remarkable control advantage of the impulsive control approach (fast response speed, low energy consumption and simple implementation), it is significant and important to explore the impulsive synchronization of nonlinear systems with actuator saturation [40]. In addition, so far the impulsive synchronization of fractional order systems with actuator saturation has not investigated yet, which deserves further investigate intensively.

On the other hand, another nonnegligible disturbance element is the control gain error. Specifically, the disturbance occurring at the impulsive control instant has great adverse influence on the synchronization performance. In [41], the fuzzy adaptive control scheme of nonlinear fractional-order chaotic systems with unknown control gain sign was studied. In [42], the impulsive stabilization of nonlinear system with bounded gain error was explored. The impulsive consensus of multi-agent systems with bounded and unknown control gain error was further investigated in [43], [44]. Considering the analytical complexity of fractional order systems, the extension of control gain error case to fractional order systems is more significant and challenging.

Motivated by the above discussions, this paper mainly investigates the impulsive synchronization case of nonlinear fractional-order chaotic systems with actuator saturation and control gain error, which goes deep into investigation firstly in this paper. By conducting the synchronization error vector, the master-slave synchronization case of the fractional order chaotic systems is transformed into the asymptotic stability case of the synchronization error system. Combined with the fractional order derivative, impulsive differential system and some matrix inequality techniques, the impulsive controller is designed and some novel sufficient synchronization conditions are derived. It should be noted that the actuator saturation and control gain error are discussed simultaneously, which is very rigorous and practical in real systems. Moreover, the impact of the key controller design parameters, saturation level and control gain error intensity on the synchronization performance is analyzed intensively, which provides helpful guideline for obtaining better control performance.

The rest of this paper is organized as follows. In Section 2, some preliminaries are given, which introduces Caputo fractional-order derivative and a useful inequality lemma. Section 3 studies the synchronization case with actuator saturation and control gain error. The simulation results are presented in Section 4. Finally, the conclusion is drawn in Section 5.

Notation: In this paper, \otimes and I_n denote the Kronecker product and the n dimensional identity matrix. $\mathbb{R}^{n \times m}$ denotes the set of all $n \times m$ real matrices, specially, \mathbb{R} and \mathbb{R}^n denote the real number and n -dimensional Euclidean space. $\mathbb{N} = \{1, 2, \dots\}$.

II. PRELIMINARIES

Definition 1 [45]: The Caputo fractional order derivative for function $f(t) \in ([t_0, +\infty), \mathbb{R})$ is defined as

$${}_{t_0}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, \quad (1)$$

where $n \in \mathbb{N}$, $\alpha > 0$ with $n-1 < \alpha < n$, and $\Gamma(\cdot)$ is the Gamma function $\Gamma(p) = \int_0^{+\infty} t^{p-1} e^{-t} dt$.

In the following, let ${}_{t_0}D_t^\alpha x(t)$ as $D^\alpha x(t)$ for convenience.

Lemma 1 [45]: If the continuous function $V(t) \in ([t_0, +\infty), \mathbb{R})$ satisfies

$$D^\alpha V(t) \leq \zeta V(t), \quad (2)$$

where $0 < \alpha < 1$ and $\zeta \in \mathbb{R}$, then

$$V(t) \leq V(t_0)E_\alpha(\zeta(t - t_0)^\alpha), \quad (3)$$

where $E_\alpha(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(\alpha k + 1)}$ is the Mittag-Leffler function.

Property 1: For any constants $v \in \mathbb{R}$ and $\omega \in \mathbb{R}$, it satisfies

$$D^\alpha(vp(t) + \omega q(t)) = vD^\alpha p(t) + \omega D^\alpha q(t). \quad (4)$$

Lemma 2 [46]: For a continuous and derivable function $x(t) \in \mathbb{R}, t \geq t_0$, it has

$$\frac{1}{2}D^\alpha x^2(t) \leq x(t)D^\alpha x(t). \quad (5)$$

III. MAIN RESULTS

The master nonlinear system to be considered is given as

$$D^\alpha x(t) = Ax(t) + f(x(t)), \quad (6)$$

where $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}, f(x(t)) = [f_1(x_1(t)), \dots, f_n(x_n(t))]^T \in \mathbb{R}^n$ is the nonlinear dynamical function, and satisfies the Lipschitz condition $\|f(y) - f(x)\| \leq \sigma \|y - x\|, \sigma > 0, x, y \in \mathbb{R}$.

From the master-slave synchronization topic, the slave system can be described as

$$D^\alpha y(t) = Ay(t) + f(y(t)) + u(t), \quad (7)$$

where $u(t) = [u_1(t), \dots, u_n(t)]^T \in \mathbb{R}^n$ is the designed controller, $y(t) = [y_1(t), \dots, y_n(t)]^T \in \mathbb{R}^n$. Then, from (6) and (7), one can get the following synchronization error system,

$$D^\alpha e(t) = Ae(t) + f(y(t)) - f(x(t)) + u(t), \quad (8)$$

where $e(t) = y(t) - x(t)$ denotes the error vector.

Remark 1: Many nonlinear dynamical systems (including many typical chaotic systems, such as Chen system, Chua's system, Lorenz system and so on) meet the description (6).

The controller $u(t)$ is designed as

$$u(t) = \text{sat}(b_k e(t_k))\delta(t - t_k), \quad k \in \mathbb{N}, \quad (9)$$

where $\{t_k\}$ denotes the impulsive instants with $0 \leq t_0 < t_1 < \dots < t_k < \dots, \delta(t)$ is the Dirac delta function and satisfies $\delta(t) = 0$ for $t \neq 0$, the saturation function $\text{sat}(b_k e(t_k)) \doteq (\text{sat}(b_k e_1(t_k)), \dots, \text{sat}(b_k e_n(t_k)))^T$ with $\text{sat}(s) = \text{sign}(s) \min\{\Delta, |s|\}$, where $s \in \mathbb{R}, \Delta > 0$ is the saturation level, $b_k \in \mathbb{R}$ is the impulsive control gain.

Assumption 1: The controller is disturbed with the control parametric uncertainty Δb_k , which satisfies

$$\Delta b_k = \kappa \varphi(t_k) b_k, \quad (10)$$

where $\kappa > 0$ is a known constant, $|\varphi(t_k)| < 1$. Thus, the real controller is

$$\begin{aligned} u(t) &= \text{sat}((b_k + \Delta b_k)e(t_k))\delta(t - t_k) \\ &= \text{sat}((1 + \kappa \varphi(t_k))b_k e(t_k))\delta(t - t_k). \end{aligned} \quad (11)$$

Define a time-varying parameter $h_i(t_k)$ as

$$h_i(t_k) = \begin{cases} \frac{\Delta}{|(1 + \kappa \varphi(t_k))b_k e_i(t_k)|} & |(1 + \kappa \varphi(t_k))b_k e_i(t_k)| > \Delta \\ 1 & |(1 + \kappa \varphi(t_k))b_k e_i(t_k)| \leq \Delta \end{cases} \quad (12)$$

Obviously, it has $h_i(t) \in (0, 1]$ and the saturation input can be expressed as

$$\text{sat}((1 + \kappa \varphi(t_k))b_k e_i(t_k)) = (1 + \kappa \varphi(t_k))b_k h_i(t_k) e_i(t_k). \quad (13)$$

Then one can get

$$\begin{aligned} &\text{sat}((1 + \kappa \varphi(t_k))b_k e(t_k)) \\ &= (\text{sat}((1 + \kappa \varphi(t_k))b_k e_1(t_k)), \text{sat}((1 + \kappa \varphi(t_k))b_k e_2(t_k)), \\ &\quad \dots, \text{sat}((1 + \kappa \varphi(t_k))b_k e_n(t_k)))^T \\ &= ((1 + \kappa \varphi(t_k))b_k h_1(t_k) e_1(t_k), (1 + \kappa \varphi(t_k))b_k h_2(t_k) e_2(t_k), \\ &\quad \dots, (1 + \kappa \varphi(t_k))b_k h_n(t_k) e_n(t_k))^T \\ &= (1 + \kappa \varphi(t_k))b_k H(t_k) e(t_k), \end{aligned} \quad (14)$$

where $H(t_k) = \text{diag}\{h_1(t_k), h_2(t_k), \dots, h_n(t_k)\}$.

From (10)~(14), the error system (8) can be rewritten as

$$\begin{cases} D^\alpha e(t) = Ae(t) + f(y(t)) - f(x(t)), & t \neq t_k, \\ \Delta e(t_k) = e(t_k^+) - e(t_k) = (1 + \kappa \varphi(t_k))b_k H(t_k) e(t_k). \end{cases} \quad (15)$$

Theorem 1: The asymptotical synchronization between systems (6) and (7) is realized, if the following conditions are satisfied

$$2(A + \sigma I_n) \leq \lambda I_n, \quad (16)$$

$$\Lambda_k^T \Lambda_k \leq \eta_k I_n, \quad (17)$$

$$E_\alpha(\lambda \tau_k^\alpha) \eta_k < \theta, \quad (18)$$

where $\Lambda_k = (1 + \kappa \varphi(t_k))b_k H(t_k) + I_n, 0 < \theta < 1, 0 < \eta_k < 1, \lambda > 0$ are constants, $\tau_k = t_{k+1} - t_k$ is the impulsive interval.

Proof: Choose the Lyapunov functions as

$$V(t) = e^T e. \quad (19)$$

It is easy to verify that $V(t)$ is nonnegative for $[t_0, \infty)$.

When $t \neq t_k$, it has the following fractional order derivative

$$\begin{aligned} D^\alpha V(t, e) &\leq 2e^T D^\alpha e \\ &= 2e^T (Ae + f(y) - f(x)) \\ &\leq 2e^T (A + \sigma I_n)e. \end{aligned} \quad (20)$$

From (16), there exists

$$D^\alpha V(t, e) \leq \lambda e^T e = \lambda V(t, e). \quad (21)$$

From Lemma 1, let each t_{k-1} as the initial time, then

$$V(t_k) \leq V(t_{k-1}^+)E_\alpha(\lambda(t_k - t_{k-1})^\alpha). \quad (22)$$

When $t = t_k$, it follows from (15) that

$$e(t_k^+) = ((1 + \kappa\varphi(t_k))b_k H(t_k) + I_n)e(t_k), \quad (23)$$

then it yields from (17) that

$$\begin{aligned} V(t_k^+) &= e^T(t_k^+)e(t_k^+) \\ &= e^T(t_k)((1 + \kappa\varphi(t_k))b_k H(t_k) + I_n)^T \\ &\quad \times ((1 + \kappa\varphi(t_k))b_k H(t_k) + I_n)e(t_k) \\ &\leq \eta_k V(t_k). \end{aligned} \quad (24)$$

Therefore

$$\begin{aligned} V(t_k^+) &\leq \eta_k V(t_k, e) \\ &\leq \eta_k V(t_{k-1}^+)E_\alpha(\lambda\tau_k^\alpha). \end{aligned} \quad (25)$$

Indeed, when $k = 1$, we get

$$V(t_1^+) \leq \eta_1 V(t_1) \leq \eta_1 V(t_0)E_\alpha(\lambda\tau_1^\alpha). \quad (26)$$

Thus, (18) holds for $k = 1$, we have

$$V(t_1^+) \leq \eta_1 V(t_0)E_\alpha(\lambda\tau_1^\alpha) \leq \theta V(t_0). \quad (27)$$

Similarly, for $k = 2$, we have

$$V(t_2^+) \leq \eta_2 V(t_1)E_\alpha(\lambda\tau_2^\alpha) \leq \theta^2 V(t_0). \quad (28)$$

For $t = t_k$, one can obtain the following recursive result

$$\begin{aligned} V(t_k^+) &\leq V(t_{k-1}^+)\eta_k E_\alpha(\lambda(t_k - t_{k-1})^\alpha) \\ &\leq \eta_1 E_\alpha(\lambda\tau_1^\alpha)V(t_0) \cdots \eta_{k-1} E_\alpha(\lambda\tau_{k-1}^\alpha)\eta_k E_\alpha(\lambda\tau_k^\alpha) \\ &\leq \theta^k V(t_0). \end{aligned} \quad (29)$$

Since $0 < \theta < 1$ is a constant, $\theta^k \rightarrow 0$ as $k \rightarrow \infty$. It is obvious that $V(t_0)$ is bounded, so $\|V(t)\| \rightarrow 0$ as $k \rightarrow \infty$. Since $V(t) = e^T(t)e(t)$, the synchronization of fractional order chaotic systems is realized. This completes the proof.

Remark 2: If the parameters b_k and κ satisfy $((1 + \kappa\varphi(t_k))b_k \in (-2, -1) \cup (-1, 0)$, it follows from (17) and $h_i(t_k) \in (0, 1]$ that $\eta_k \in (0, 1)$. One can choose suitable control gain b_k and impulsive interval τ_k to achieve the master-slave synchronization goal.

Corollary 1: The asymptotical synchronization without the control gain error between systems (6) and (7) is realized, if the following conditions satisfied

$$2(A + \sigma I_n) \leq \lambda I_n, \quad (30)$$

$$((b_k H(t_k) + I_n)^T (b_k H(t_k) + I_n)) \leq \eta_k I_n, \quad (31)$$

$$E_\alpha(\lambda\tau_k^\alpha) \cdot \eta_k < \theta, \quad (32)$$

where θ, λ, η_k and τ_k have the same meanings with Theorem 1.

Conducting $\kappa = 0$ into the dynamic error system (15), and the detailed analysis process is omitted for brevity (the similar proof is similar to that of Theorem 1).

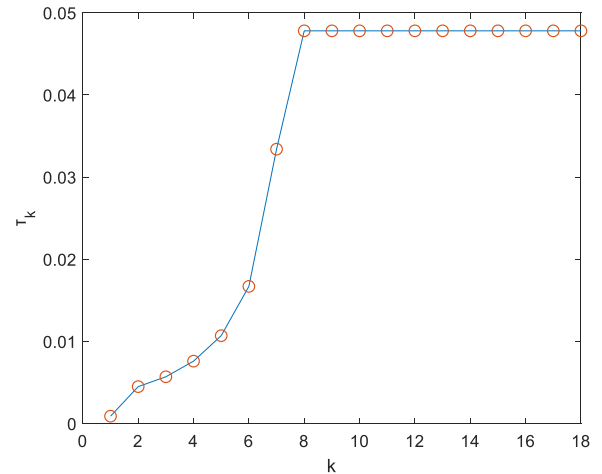


FIGURE 1. The impulsive interval $\tau_k = t_{k+1} - t_k$ vs k (Example A).

IV. NUMERICAL EXAMPLES

Subsequently, the fractional order chaotic Chua's system is considered to verify the results, and the model to be considered is

$$\begin{cases} D^\alpha x_1 = a(x_2 - \xi(x_1)), \\ D^\alpha x_2 = x_1 - x_2 + x_3, \\ D^\alpha x_3 = -bx_2, \end{cases} \quad (33)$$

where $\xi(x_1) = \omega x_1 + 0.5(\zeta - \omega)(|x_1 + 1| - |x_1 - 1|)$, ζ and ω are two given constants.

Let $\alpha = 0.97, a = 10, b = 14.7, \zeta = -0.144, \omega = 0.256, H(0) = \text{diag}[0.5, 0.4, 0.1]$. Correspondingly, the system can be rewritten as

$$\begin{aligned} A &= \begin{bmatrix} -\omega a & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \\ f(x) &= \begin{bmatrix} -0.5a(\zeta - \omega)(|x_1 + 1| - |x_1 - 1|) \\ 0 \\ 0 \end{bmatrix}, \end{aligned}$$

where $\sigma = |av| = 1.44$.

A. SYNCHRONIZATION WITH ACTUATOR SATURATION/WITHOUT CONTROL GAIN ERROR

Let $\Delta = 0.3, b_k = -0.678$. Due to different η_k at different t_k , the $\tau_k = t_{k+1} - t_k$ is different obviously. By simple calculating, the corresponding curves of τ_k is shown in Fig. 1. It can be observed that the impulsive interval reaches a constant value finally. Since the error is decreasing over time, there is no saturation appearance any more when error is small enough ($|b_k e_i(t_k)| \leq \Delta$) and $H(t_k)$ becomes a unit matrix. Because η_k is related to τ_k , since η_k is a constant, from (18) we can get that the impulsive interval is a constant value. Therefore, it can conclude that there is no saturation appearance when the impulsive interval reaches a constant value. The synchronization errors are reflected

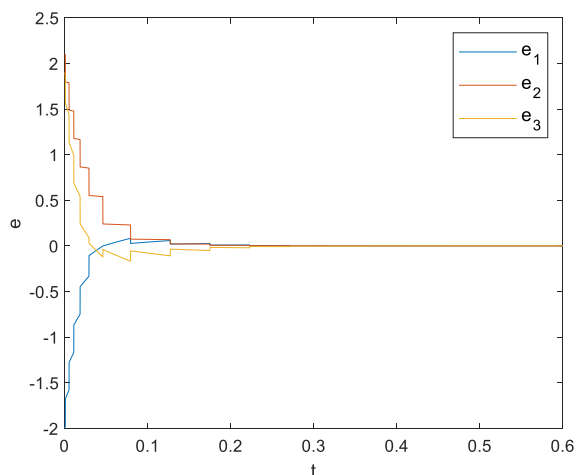


FIGURE 2. Error states of the master and slaver systems (Example A).

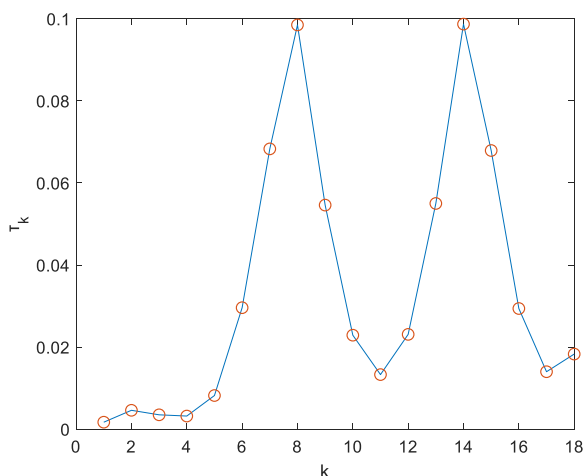


FIGURE 3. The impulsive interval $\tau_k = t_{k+1} - t_k$ vs k (Example B).

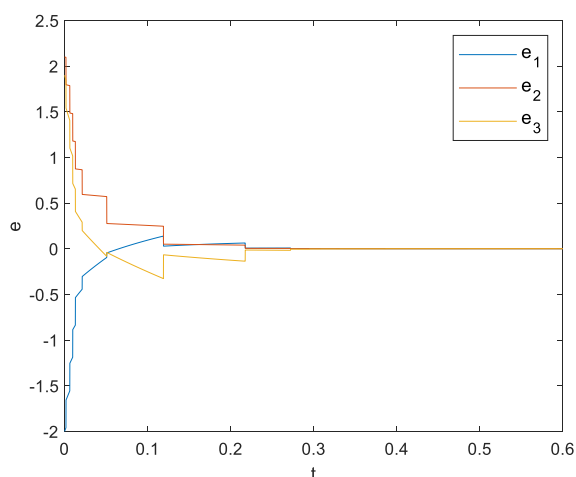


FIGURE 4. Error states of the master and slaver systems (Example B).

in Fig. 2, which shows that the error states of the master and slaver systems cover to zero, then the system can realize synchronization.

B. SYNCHRONIZATION WITH ACTUATOR SATURATION AND CONTROL GAIN ERROR

When control gain error exists, let $\Delta = 0.3, b_k = -0.6, \kappa = 0.5, \varphi(t_k) = \sin(k)$, thus $\Delta b_k = 0.5 * \sin(k)b_k$. Due to different η_k at different t_k , the $\tau_k = t_{k+1} - t_k$ is different obviously. By simple calculating, the corresponding curves of τ_k is shown in Fig. 3. Distinguished from Fig. 1, the impulsive interval cannot reach a constant value finally because of the influence of control gain error. Even if the synchronization error is small enough and there isn't saturation appearance, η_k is changing on the effect of Δb_k . Since η_k is related to τ_k , the impulsive interval can't reach a constant value finally. The synchronization errors are reflected in Fig. 4, which shows that the error states of the master and slaver systems converge to zero, then the system can realize synchronization.

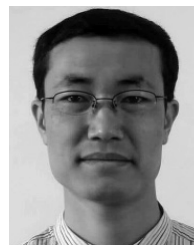
V. CONCLUSION

The synchronization problem of fractional-order chaotic systems by impulsive approach subject to actuator saturation and control gain error is mainly discussed. The controller is provided based on the Caputo fractional-order derivative method. Then, based on the Lyapunov stability and impulsive differential system theory, the error system with impulsive control is analyzed and the control guideline for impulsive control parameters is studied. The relations of the impulsive controller, actuator saturation level and control gain error function are considered, and the method is proved to be effective under the derived inequality conditions. Considering the real engineering applications, the above results are meaningful in the synchronization problems.

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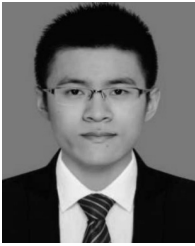
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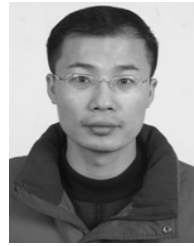
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