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# **Composition and Application of Extended Colored Logic Petri Nets to E-Commerce Systems**

# ZHEN WANG<sup>(D)</sup>, WENJING LUAN<sup>(D)</sup>, (Member, IEEE), YUYUE DU<sup>(D)</sup>, AND LIANG QI<sup>(D)</sup>, (Member, IEEE)

College of Computer Science and Engineering, Shandong University of Science and Technology, Qingdao 266590, China Corresponding authors: Wenjing Luan (wenjingmengjing@163.com) and Yuyue Du (yydu001@163.com)

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**ABSTRACT** Extended colored logic Petri nets (ECLPNs) are extensions of logic Petri nets (LPNs) and colored logic Petri nets (CLPNs). They are equivalent to LPNs and CLPNs, which can describe the batch processing and indeterminacy functions of resources in cooperative systems. The advantage of ECLPNs is that their net structures are much simpler than their equivalent CLPNs, and therefore, ECLPNs can be easily used to model and analyze cooperative systems. For systems containing several subsystems with the same function and structure, we can describe them by a single ECLPN. Then we propose a composition method of ECLPNs. We define the robustness of a system based on ECLPN which reflects the validity of the collaboration of subsystems. We define a strict conservativeness that guarantees data security. The robustness and strict conservativeness of composed ECLPNs are analyzed. An E-commerce example is presented to illustrate the modeling capacity and the advantage of ECLPNs.

**INDEX TERMS** Colored logic Petri net, extended colored logic Petri net, composition, e-commerce system, property analysis.

# I. INTRODUCTION

E-commerce systems are becoming more and more complicated, and their compatibility analysis is a co-NP-hardness problem [1]. A good way is to decompose the E-commerce system into several subsystems, analyze properties of each subsystem, and find the inheriting conditions of the properties when the subsystems are composed. Then the properties of a complete system are obtained. Thus, an E-commerce system can be divided into three classes of subsystems: customers, sellers and a third-party. The subsystems exchange massages and perform batch processes at the same time. However, batch processing brings the problem of data indeterminacy [2].

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Petri Nets (PNs) [3] are a class of graphs that consists of places, transitions and directed arcs between places and transitions. A siphon is a structure and closely related to properties like liveness and deadlock-free of PNs. Methods of siphon computation and interactive deadlock prevention policies have been proposed in [4]–[6]. PNs can be applied in process mining [7]–[9]. However, PNs with inhibitor arcs [10] cannot describe batch processing functions and passing value indeterminacy as clearly and concisely as logic Petri nets (LPNs) [11] do.

An LPN can describe passing value indeterminacy in cooperative systems. It is a high-level Petri net. Logic transitions in LPNs are attached with first-order predicate logic expressions. When logic expressions are attached to an input transition, they control whether the transition can fire or not. When they are attached to an output transition, they control which place tokens should flow into after the transition has fired. LPNs are applied to business process mining [12]–[14] and Web service discovery [15]. However, a logic output transition in LPNs cannot output tokens correctly according to just the attached logical output expression. LPNs cannot differentiate sources of tokens in the system, which may cause errors during the system running.

In colored Petri nets (CPNs) [16], tokens are differentiated by colors. Places of a CPN are combined with a color set, which represents the colors held in the place. Arcs have inscriptions (arc expressions), variables in an expression are assigned by a binding, and an arc expression indicates what type and how many colored tokens are needed when the corresponding transition fires. The state-of-the-art applications of CPNs are given in [17]–[20]. The extension of CPNs can help to solve the problems in daily-life systems [21]–[26]. In fact, CPNs can be simplified by logical expressions when describing passing value indeterminacy.

Like logical expressions can fold PNs with inhibitor arcs, logical expressions can fold CPNs as well. Colored logic Petri nets (CLPNs) proposed in [27] guarantee that output logic transitions can correctly output tokens by checking their colors and logical expressions. However, there are some defects in CLPNs. Many analysis methods and the firing rules of transitions in CLPNs are the same as those in LPNs. The scale of a CLPN model is nearly the same as that of an LPN model when a complicated system is modeled. Comparing with CLPN, the effectiveness of an ECLPN model can be illustrated.

ECLPNs [28] can describe multiple LPN models with the same structure by one model. Thus, the process can be further simplified by using ECLPNs. Customer subsystems share the same structure, so only one customer subsystem needs to be built, and all customers share the subsystem with distinct colored tokens. Therefore, ECLPNs can simplify net structures, and ensure the same modeling ability with CLPNs at the same time. Thus, we propose a composition method of ECLPNs to model E-commerce systems in this paper. Firstly, the basic design module is defined. The definition of the composition of ECLPNs is given. Then, the maintaining conditions of robustness and strict conservativeness of ECLPNs are analyzed.

The rest of the paper is organized as follows. Section II presents the basic concepts of LPNs, CPNs, CLPNs, and ECLPNs briefly. Section III gives the composition method of ECLPNs. Section IV describes an E-commerce system and makes a comparison between an ECLPN model and the corresponding LPN model. Section V concludes this paper and discusses the future work.

### **II. PRELIMINARIES**

In this section, concepts of PN [29]–[38], LPN [11] and CPN [16] are reviewed briefly, and the formal definition of ECLPN is presented. More information about these models can be referred to the corresponding references.

In the rest of this paper, we use  $\mathbb{B} \in \{false, true\}$  to denote the Boolean type,  $\mathbb{N}^+ = \{1, 2, ...\}$  to denote the set of all positive integers, and  $\mathbb{N}_n^+ = \{1, 2, ..., n\}$  to denote the first *n* elements in  $\mathbb{N}^+$ .

Definition 1 [3]: A Petri Net is described as a three-tuple PN = (P, T, F), where P is a finite set of places; T is a finite set of transitions; and F is a set of directed arcs.

Definition 2 [2]: Given a Petri net PN = (P, T, F). If  $x \in P \cup T$  is a node in PN, then  $\bullet x = \{y | (y, x) \in F\}$  and  $x^{\bullet} = \{y | (x, y) \in F\}$  are defined as its pre-set and post-set, respectively.

Definition 3 [11]: A Logic Petri Net is described as a sixtuple LPN = (P, T, F, I, O, M), where P is a finite set of places;  $T = T_D \cup T_I \cup T_O$  is a finite set of transitions; F is a set of directed arcs; I is a mapping function such that  $\forall t \in T_I, I(t)$  is a logical input expression denoted by  $f_I$ ; O is a mapping function such that  $\forall t \in T_O, O(t)$  is a logical output expression denoted by  $f_O$ ; and M is a marking function, which defines the number of tokens in each place.

Definition 4 [16]: A coloured Petri net is described as a nine-tuple CPN =  $(\sum, P, T, A, N, C, G, E, Init)$ , where  $\sum$  is a finite set of colors; P is a finite set of places; T is a finite set of transitions; A is a finite set of directed arcs; N is a node function; C is a color function; G is a guard function; E is an arc expression; and Init is an initialization function.

Definition 5 [27]: A coloured logic Petri net is described as an eight-tuple CLPN = (C, P, T, F, I, O, M, IC), where *C* is a finite set of consecutive prime numbers, where each prime represents a token color; *P* is a finite set of places, and each place can store a token; *T* is a finite set of transitions; *F* is a finite set of directed arcs; *I* is a mapping from a logic input transition to a logic input expression; *O* is a mapping from a logic output transition to a logic output expression; *M* is a marking function that maps *P* to the set  $\{0, 1\}$ ; and *IC* is a color marking function.

The formal definition of ECLPNs is presented as follows. *Definition 6 [28]:* An ECLPN is described as an eleven-tuple ECLPN =  $(\sum, P, T, A, C, G, E, Init, I, O, M)$ , where

- 1)  $\sum$  is a finite set of non-empty types, called a color set;
- *P* is a finite set of places, including one initial place *p*<sub>0</sub>, and one end place *p<sub>E</sub>*;
- 3)  $T = T_D \cup T_L$  is a finite set of transitions, where
  - a)  $T_D$  is the set of ordinary transitions.  $T_D$  is the same with the set of transitions in Colored Petri Nets; and
  - b)  $T_L$  is called a set of logic transitions, where  $\forall t \in T_L$ , tokens in input places for *t* to be enabled are restricted by a logic input expression  $f_I(t)$ , and tokens in output places after *t* fires are restricted by a logic output expression  $f_O(t)$ ;
- 4) *A* is a finite set of arcs such that  $P \cap A = T \cap A = \emptyset$ ;
- 5)  $C: P \to \sum$  is a color function, which specifies the set of token colors on a place.  $\forall p \in P, C(p) = \sum$  by default;

6) *G* is a guard function. It is defined from *T* into expressions such that:

$$\forall t \in T : Type(G(t)) = \mathbb{B} \land Type(Var(G(t))) \subseteq \sum,$$

where *Type* [16] means the type of an expression or a variable, *Var* [16] means the set of variables in an expression, and G(t) = true by default;

- 7) E is an arc expression function. It is defined from A into expressions with the binding b [16] such that
  - a) ∀(p, t) ∈ A, if t ∈ T<sub>L</sub>, then E(p, t)<b> is the set of all multisets of the token type. Otherwise, E(p, t)<b> is the same as those in CPNs; and
  - ∀(t, p) ∈ A, if t ∈ T<sub>L</sub>, then E(t, p)<b> is the set of all multisets of the token type. Otherwise, E(t, p)<b> is the same as those in CPNs;
- 8) *Init* is an initialization function. It is defined from *P* into closed expressions, such that:

$$\forall p \in P : Type(Init(p)) = C(p)_{MS};$$

- 9) *I* is a mapping from  $T_L$  to a logic input expression, such that  $\forall t \in T_L$ ,  $I(t) = f_I(t)$ ;
- 10) *O* is a mapping from  $T_L$  to a logic output expression, such that  $\forall t \in T_L$ ,  $O(t) = f_O(t)$ ; and
- 11)  $M: P \to \sum_{MS}$  is a marking function.  $\forall p \in P, M(p)$  denotes the multiset of colored tokens in *p*. At a time point in the net, the set of multisets  $\{M(p)|p \in P\}$  is called a marking, denoted by *M*. The initial marking is denoted by  $M_0$ , and the end marking is denoted by  $M_E$ .

Here is a detailed explanation about E(p, t) < b >.

For  $t \in T$  and  $p \in t$ , *b* is a binding, assigning colors to the tokens. The expression set on a directed arc (p, t)is E(p, t) < b >. Because of the passing value indeterminacy, there is more than one expression that can be accepted by the logic expression, which is attached to a logic transition. For instance, given a binding  $b = <v_1 = s$ ,  $v_2 = b_1$ ,  $v_3 = b_2 >$ , a logic expression is  $LE = \bullet T \bullet \lor b_1 \lor b_2$ , where  $\bullet T \bullet$  denotes the Boolean variable *true*. Thus any expression in the expression set  $E(p, t) < b > = \{\emptyset, b_1, b_2, b_1 + b_2\}$  can be accepted by logic expression *LE*.

To derive the firing rules of a transition in ECLPNs, some calculation methods of multisets are defined as follows.

Definition 7 [28]: MS is a multiset that has n types of element. The type set  $Elem = \{e_1, e_2, \ldots, e_n\}$  is formed by n types above. Function S maps a multiset to its type set. The coefficient of each type element composes the coefficient set  $H(Elem) = \{h(e)|e \in Elem\}$ , where h(e) is the coefficient of e in MS, which represents the number of e in MS.  $|MS| = \sum_{e \in E} h(e)$  denotes the total number of elements in  $MS. f_{int}(MS)$  maps a multiset to an integer. Let  $D = \{2, 3, 5, 7, 11, \ldots\}$  be the ascending prime number set, and  $D_n = \{d_1, d_2, \ldots, d_n\}$  be the set composed by the first n elements in D. Suppose that |Elem| = n, then  $f_{int}(MS) = \prod_{i=1}^{n} d_i^{h(ei)}$ .  $MS = \sum_{e \in E} h(e) \cdot e$  is called the sum form of MS, whereas  $f_{int}(MS) = \prod_{i=1}^{n} d_i^{h(ei)}$  is called the product form of MS.

For example,  $MS = 3 \ a + 4 \ b + c$ , then  $Elem = \{a, b, c\}$ ,  $S(3a + 4 \ b + c) = \{a, b, c\}$ .  $H(Elem) = \{3, 4, 1\}$ , |MS| = 3 + 4 + 1 = 8,  $f_{int}(MS) = 2^3 \times 3^4 \times 5 = 3240$ .  $(3a + 4 \ b + c)$  is the sum form of MS, whereas 3240 is the product form of MS.

Here we define the subset of a multiset.

Definition 8:  $MS_1$  and  $MS_2$  are two multisets.  $Elem_i$  is the type set of  $MS_i$ , and  $h_i(e)$  is the coefficient of e in  $MS_i$ ,  $i \in \{1, 2\}$ .  $MS_1$  is the subset of  $MS_2$  if

- 1)  $Elem_1 \subseteq Elem_2$ ; and
- 2)  $\forall e \in Elem_1: h_1(e) \leq h_2(e)$ , where if  $e \notin Elem_2$ , then  $h_2(e) = 0$ .

The firing rule of ECLPNs relies on colored tokens in pre-sets of a transition.

Definition 9 [28]: For  $ECLPN = \{\sum, P, T, A, C, G, E, Init, I, O\}, M$  is a marking, and b is a binding.

- 1) If  $t \in T_D$ , t is enabled if  $\forall p \in t: E(p, t) < b \ge M(p)$ ; and
- 2) If t ∈ T<sub>L</sub>, t is enabled if
  (a) ∀p ∈• t, ∃MS<sub>I</sub> ∈ E(p, t)<b>, such that MS<sub>I</sub> ⊆ M(p), where MS<sub>I</sub> is a multiset; and
  (b) ∪<sub>p∈•t</sub>S(MS<sub>I</sub>) makes f<sub>I</sub>(t) = true.

After t fires, M[t > M'], and it is necessary to meet the following conditions.

1) If  $t \in T_D$ ,

$$M'(p) = \begin{cases} M(p) - E(p, t) < b >, & p \in {}^{\bullet}t \\ M(p) + E(t, p) < b >, & p \in t^{\bullet} \\ M(p), & \text{else;} \end{cases}$$

- 2) If  $t \in T_L$ ,  $MS_{Imax}$  is the max in number of tokens in  $\{MS_I\}$ ,
  - a)  $\cup_{pa\in t \bullet} S(E(t, p_a)) < b >$  makes that  $f_O(t) = true$ ; b)  $\forall p_a \in t^{\bullet}, \exists MS_{out} \in E(t, p_a) < b >$  and  $\exists p_b \in {}^{\bullet} t$ such that  $MS_{out} \subseteq M(p_b)$ , where  $MS_{outMax}$  is the max in number of tokens in  $\{MS_{out}\}$ ; and  $M(p) - MS_{Imax}, p \in {}^{\bullet}t$ c)  $M'(p) = \begin{cases} M(p) + MS_{out}, p \in {}^{\bullet}t \\ M(p) + MS_{out}, p \in {}^{\bullet}t \end{cases}$

c) 
$$M'(p) = \begin{cases} M(p) + MS_{outMax}, & p \in t^{\bullet} \\ M(p), & \text{else.} \end{cases}$$

#### **III. THE COMPOSITION OF ECLPNS**

In this section, the input matrix and output matrix are defined. Basic CPN modules are given based on the matrix. The formal definition of composed ECLPNs is presented. Robustness and strict conservativeness of composed ECLPNs are analyzed. Finally, the procedure of ECLPN composition is given.

#### A. BASIC DESIGN MODULES

Definition 10: CPN = { $\sum$ , P, T, A, N, C, G, E, I},  $\forall p_j \in P$ ,  $t_i \in T$ , and binding b. Its input matrix is defined as  $IM^- = \{im_{ii}^-\}$ , where

$$im_{ij}^{-} = \begin{cases} f_{int}(E(p_j, t_i) < b >), & \text{if } (p_j, t_i) \in A \\ 1, & \text{else.} \end{cases}$$



FIGURE 1. A sequential CPN module.



FIGURE 2. A choice CPN module.



FIGURE 3. Graphical representation of elements: (a) a normal place, (b) a channel place, (c) a logic transition, (d) an ordinary transition, and (e) a directed arc.

The output matrix is defined as  $OM^+ = \{om_{ii}^+\}$ , where

$$om_{ij}^{+} = \begin{cases} f_{int}(E(t_i, p_j) < b >), & \text{if } (t_i, p_j) \in A \\ 1, & \text{else} \end{cases}$$

A system is composed of several subsystems, and the subsystems are composed of basic design modules. Two kinds of modules are introduced in this section: sequential CPN module (Fig. 1) and choice CPN module (Fig. 2). and the legend is presented in Fig. 3.

The sequential CPN module is defined below.

Definition 11: SCM = { $\sum$ , P, T, A, E,  $IM_S^-$ ,  $OM_S^+$ } is a sequential CPN module, and the default binding is b, where

- 1)  $\sum$  is a finite color set;
- 2)  $\overline{P} = \{p_1, p_2, \dots, p_n\}$  with  $n \in \mathbb{N}^+$ ;
- 3)  $T = \{t_1, t_2, \ldots, t_{n-1}\};$
- 4) A is a set of directed arcs;
- 5) *E* is an arc expression function;
- 6)  $IM_{S}^{-} = \begin{bmatrix} Int_{n-1} \\ 1_{n-1}^{T} \end{bmatrix}$ , where  $Int_{n-1}$  is an (n 1)-dimension square matrix, elements on the principal diagonal are  $f_{int}(E(p_{i}, t_{i}) < b>), i \in \mathbb{N}_{n-1}^{+}$ , and other elements are one.  $1_{n-1}$  is an (n 1)-dimension vector with each element being one; and
- 7)  $OM_{S}^{+} = \begin{bmatrix} 1_{n-1}^{T} \\ Int_{n-1}' \end{bmatrix}$ , where  $Int_{n-1}$  is an (n-1)-dimension square matrix, elements on the principal diagonal are  $f_{int}(E(t_{i}, p_{i+1}) < b >), i \in \mathbb{N}_{n-1}^{+}$ , and other elements are one.

The sequential CPN module is applied in describing a series of successive operations.

For the parallel PN module in [29],  $\exists P_{para} \subseteq P$ , if  $\forall p_i \in P_{para}$ ,  $p_j \in P_{para}$ :  $\bullet p_i = \bullet p_j$  and  $p_i^{\bullet} = p_j^{\bullet}$ , where  $i, j \in \mathbb{N}_n^+$  and  $i \neq j$ , then all places in  $P_{para}$  can be represented by

a place  $p_c$  in a sequential CPN module, and  $\forall p_i \in P_{para}$ ,  $i \in \mathbb{N}_n^+$ , is represented by a unique colored token in  $p_c$ .

For example, when n = 3, the input matrix  $IM_S^-$  and the output matrix  $OM_S^+$  are respectively given below.

$$IM_{S}^{-} = \begin{bmatrix} E_{1} & 1\\ 1 & E_{2}\\ 1 & 1 \end{bmatrix}, \quad OM_{S}^{+} = \begin{bmatrix} 1 & 1\\ E_{3} & 1\\ 1 & E_{4} \end{bmatrix}.$$

For  $IM_{S}^{-}$  and  $OM_{S}^{+}$ , the row label is  $[p_{1}, p_{2}, p_{3}]^{T}$ , the column label is  $[t_{1}, t_{2}]$ , and  $E_{1} = f_{int}(E(p_{1}, t_{1}) < b >)$ ,  $E_{2} = f_{int}(E(p_{2}, t_{2}) < b >)$ ,  $E_{3} = f_{int}(E(t_{1}, p_{2}) < b >)$ , and  $E_{4} = f_{int}(E(t_{2}, p_{3}) < b >)$ .

The choice CPN module is defined below.

Definition 12: CCM = { $\sum$ , P, T, A, E,  $IM_C^-$ ,  $OM_C^+$ } is a choice CPN module, and the default binding is b, where

- 1)  $\sum$  is a finite color set;
- 2)  $P = \{p_1, p_2, \dots, p_n\}$  with  $n \in \mathbb{N}^+$ ;
- 3)  $T = \{t_1, t_2, \ldots, t_{n-1}\};$
- 4) *A* is a set of directed arcs;
- 5) *E* is an arc expression function;

6) 
$$IM_C^- = \begin{bmatrix} 1_{n \times n} & Int_n'' \\ VInt_n^T & 1_n^T \\ 1_n^T & 1_n^T \end{bmatrix}$$
,

where  $1_{n \times n}$  is an  $n \times n$  matrix with each element being one.  $Int''_n$  is an *n*-dimension square matrix, elements on the principal diagonal are  $f_{int}(E(p_i, t_{i+n}) < b>)$ ,  $i \in \mathbb{N}_n^+$ , and other elements are one.  $VInt_n$  is an *n*-dimension with elements being  $f_{int}(E(p_{1+n}, t_i) < b>)$ ,  $i \in \mathbb{N}_n^+$ , and  $1_n$  is an *n*-dimension with elements being one; and

7) 
$$OM_C^+ = \begin{bmatrix} Int_n''' & 1_{n \times n} \\ 1_n^T & 1_n^T \\ 1_n^T & VInt_n'^T \end{bmatrix}$$
,

where  $Int'''_{n}$  is an *n*-dimension square matrix, elements on the principal diagonal are  $f_{int}(E(t_i, p_i) < b >)$ ,  $i \in \mathbb{N}_n^+$ , and other elements are one.  $VInt'_{n}$  is an *n*-dimension with elements being  $f_{int}(E(t_{i+n}, p_{n+2}) < b >)$ ,  $i \in \mathbb{N}_n^+$ .

A choice CPN module represents multiple competing choices for a successive operation.

For example, when n = 3, the input matrix  $IM_C^-$  and the output matrix  $OM_C^+$  are respectively given below.

In  $IM_C^-$  and  $OM_C^+$ , the row label is  $[p_1, p_2, p_3, p_4, p_5]^T$ , the column label is  $[t_1, t_2, t_3, t_4, t_5, t_6]$ , and  $E_1 = f_{int}(E(p_1, p_2, p_3))$ 



FIGURE 4. An ECLPN example.

 $\begin{array}{l} t_4 (>b>), E_2 = f_{int}(E(p_2,t_5) < b>), E_3 = f_{int}(E(p_3,t_6) < b>), \\ E_4 = f_{int}(E(p_4,t_1) < b>), E_5 = f_{int}(E(p_4,t_2) < b>), E_6 = \\ f_{int}(E(p_4,t_3) < b>), E_7 = f_{int}(E(t_1,p_1) < b>), E_8 = f_{int}(E(t_2,p_2) < b>), E_9 = f_{int}(E(t_3,p_3) < b>), E_{10} = f_{int}(E(t_4,p_5) < b>), \\ E_{11} = f_{int}(E(t_5,p_5) < b>), \text{ and } E_{12} = f_{int}(E(t_6,p_5) < b>). \end{array}$ 

# **B. THE COMPOSITION OF ECLPNS**

In this section, a complicated system S is modeled by the composition of ECLPNs. The subsystem ss of S is modeled by an ECLPN, and CPN modules form ss.

A complicated system is composed of multiple subsystems. There is no guarantee that different subsystems can run at exactly the same pace, and one subsystem process cannot be delayed by another slow subsystem. To solve this problem, we use an ECLPN to describe the composition in this work. A subsystem *S* in Fig. 4 is an example.

*S* is formed by one sequential CPN module *C* and two CPN modules  $S_1$  and  $S_2$ .  $S_1$  and  $S_2$  have different structures and are connected with *C* via places  $In_1$ ,  $In_2$ ,  $Out_1$ , and  $Out_2$ . If  $t_1$  fires by the CPN firing rules, it requires tokens in all its pre-sets. Therefore, it is necessary to attach a logic expression to  $t_1$ , for the sake of guaranteeing the independence of  $S_1$  and  $S_2$ . Similarly, if  $t_2$  fires by the CPN firing rule, it outputs tokens to all its post-sets, without checking whether *C* has imported a token from a certain subsystem or not. Thus, it is necessary to attach a logic expression to  $t_2$ , to ensure that  $t_2$  outputs tokens to correct places.

The binding for *S* is  $b = \{v_1 = c, v_2 = s_1, v_3 = s_2\}$ , where *c* denotes a token in *C*,  $s_1$  denotes a token in  $S_1$ , and  $s_2$  denotes a token in  $S_2$ . Logic expressions attached to both  $t_1$  and  $t_2$  are  $f_I(t_1) = f_O(t_2) = c \land (s_1 \lor s_2)$ . Expressions on directed arcs are set as follows:

 $E(p_1, t_1) < b > = c, E(In_1, t_1) < b > = s_1, E(In_2, t_1)$  $< b > = s_2,$ 

 $E(t_1, p_2) < b > = \{c, (c, s_1), (c, s_2), (c, s_1, s_2)\},\$ 

$$E(p_2, t_2) < b > = \{c, (c, s_1), (c, s_2), (c, s_1, s_2)\}$$

 $E(t_2, Out_1) < b > = s_1, E(t_2, p_3) < b > = c, E(t_2, Out_2) < b > = s_2.$ 

By the firing rule in Definition 9, the ECLPN model can describe the system soundly.

The composition of ECLPNs is defined below.

Definition 13: ECLPN<sub>l</sub> =  $(\sum_l, P_l, T_l, A_l, C_l, G_l, E_l, Init_l, I_l, O_l, M_l), l \in \mathbb{N}_n^+$ , are *n* ECLPNs in a complicated

system. ECLPN<sub>C</sub> =  $(\sum, P, T, A, C, G, E, Init, I, O, M)$  is the composition of ECLPN<sub>l</sub>,  $l \in \mathbb{N}_n^+$ , where

- 1)  $\sum = \bigcup_l \sum_l;$
- 2)  $P = (\bigcup_l P_l) \cup P_C$ , where  $P_C$  is the set of channel places. Different ECLPNs are connected with channel places;
- 3)  $T = \bigcup_l T_l;$
- 4) A = (∪<sub>l</sub>A<sub>l</sub>) ∪ A<sub>C</sub>, A<sub>C</sub> is a set of arcs. Arcs in A<sub>C</sub> are related to channel places;
- 5)  $C: P \to \sum$ ,

$$C(p) = \begin{cases} \Sigma_l, & p \in P_l \\ \Sigma_i \cup \Sigma_j, & p \in P_c, \end{cases}$$

where  $\sum_{i}$  and  $\sum_{j}$  are color sets of ECLPN<sub>i</sub> and ECLPN<sub>j</sub>;

- 6) *G* is a guard function, and  $G(t) = G_l(t), t \in T_l$ ;
- 7) *E* is an arc expression function, the binding  $b = \bigcup_l b_l$ ,

$$E(a) < b >= \begin{cases} E_l(a) < b_l >, & a \in A_l \\ E_c(a) < b_l >, & a \in A_c, \end{cases}$$

where  $E_c(a) < b >$  is the expression on arcs  $A \in A_c$ , which is set by the actual situation.  $\forall p_c \in P_c: E_c(\bullet p_c, p_c) < b > = E_c(p_c, p_c^{\bullet}) < b >$ , because channel places change no token;

8) Init is an initialization function, where

$$Init(p) = \begin{cases} Init_l, & p \in P_l \\ \emptyset, & p \in P_c \end{cases}$$

- 9) *I* is a mapping from  $T_L$  to a logic input expression, such that  $\forall t \in \bigcup_l T_{Ll}$ ,  $I(t) = I_l(t)$ ;
- 10) *O* is a mapping from  $T_L$  to a logic output expression, such that  $\forall t \in \bigcup_l T_{Ll}$ ,  $O(t) = f_O(t)$ ; and
- 11)  $M: P \to \sum_{MS}$  is the marking function.  $M_0$  denotes the initial marking, whereas  $M_E$  denotes the end marking.

#### C. THE PROPERTY ANALYSIS OF ECLPNC

In this section, we discuss the conditions for maintaining robustness and strict conservativeness of ECLPNs.

The robustness of an ECLPN shows that all the tokens in the initial place at the initial marking can flow to the end place at the end marking. It reveals that every subsystem can complete the task and there is no remaining token in the process. Let  $M_0$  denote the initial marking, and  $M_E$  denote the end marking. The definition of robustness for an ECLPN is presented below.

Definition 14: An ECLPN =  $\{\sum, P, T, A, C, G, E, Init, I, O\}$  is robust if

1) 
$$M_{0}(p) = \begin{cases} Init(p), & p = p_{0} \\ \emptyset, & p \in P \setminus \{p_{0}\}; \end{cases}$$
  
2) 
$$M_{E}(p) = \begin{cases} Init(p_{0}), & p = p_{E} \\ \emptyset, & p \in P \setminus \{p_{E}\}; \end{cases}$$
 and  
3) 
$$\forall t \in T, t \text{ is live.} \end{cases}$$

The composition of ECLPNs is an ECLPN. Conditions for the inheritance of robustness are discussed below.

If a component ECLPN is not robust, then, the composition is not robust. Provided that each system is robust before composition, the robustness of the composition depends on arc expressions which are connected to channel places. we have the following result.

*Theorem 1:* Let ECLPN<sub>C</sub> be the composition of ECLPN<sub>l</sub> =  $(\sum_l, P_l, T_l, A_l, C_l, G_l, E_l, Init_l, I_l, O_l, M_l), l \in \{1, 2\}$ , ECLPN<sub>C</sub> is robust if

- 1) ECLPN<sub>1</sub> and ECLPN<sub>2</sub> are robust; and
- 2) For the binding *b* of ECLPN<sub>C</sub>, there is a set of channel places  $P_{cI} \subseteq P_c$ ,  $P_{cI} = \{p_{cIj}|\bullet p_{cIj} \in \text{ECLPN}_1\}$ ,  $j \in \mathbb{N}^+_{|P_{cI}|}$  and  $a_{Ij} = (\bullet p_{cIj}, p_{cIj})$ . Similarly, there is a channel place set  $P_{cO} \subseteq P_c$ ,  $P_{cO} = \{p_{cOk} | p_{cOk}^{\bullet} \in \text{ECLPN}_1\}$ ,  $k \in \mathbb{N}^+_{|P_{cO}|}$  and  $a_{Ok} = (p_{cOk}, p_{cOk}^{\bullet})$ , and then,

then,  $\sum_{j=1}^{|P_{cl}|} E(a_{lj}) < b \ge \sum_{k=1}^{|P_{cO}|} E(a_{Ok}) < b >$ , where the subset of a multiset is defined in Definition 8.

*Proof:* Let  $Sum_1$  denote  $\sum_{j=1}^{|P_{cl}|} E(a_{lj}) < b >$  and  $Sum_2$  denote  $\sum_{k=1}^{|P_{cO}|} E(a_{Ok}) < b >$ . The type sets of  $Sum_1$  and  $Sum_2$  are respectively  $Elem_1$  and  $Elem_2$ . The coefficient sets of  $Sum_1$  and  $Sum_2$  are respectively  $H_1(Elem_1) = \{h_1(e)|e \in Elem_1\}$  and  $H_2(Elem_2) = \{h_2(e)|e \in Elem_2\}$ . The theorem is proved by 6 situations of relation between  $Sum_1$  and  $Sum_2$ , which can be divided into 3 classes of relation between  $Elem_1$  and  $Elem_2$  and whether the Condition a) holds or not. Condition a) is  $\forall e \in Elem_1$ :  $h_1(e) \leq h_2(e)$ , where if  $e \notin Elem_2$ , then  $h_2(e) = 0$ .

Relations between  $Sum_1$  and  $Sum_2$  are discussed by 6 situations as follows:

a): Condition a) fails and  $Elem_1 \supseteq Elem_2$ ;

b): Condition a) fails and  $Elem_1 \subseteq Elem_2$ ;

c): Condition a) fails and  $\exists e_1 \in Elem_1$ :  $e_1 \notin Elem_2$  and  $\exists e_2 \in Elem_2$ :  $e_2 \notin Elem_1$ ;

d): Condition a) holds and  $Elem_1 \supseteq Elem_2$ ;

e): Condition a) holds and  $Elem_1 \subseteq Elem_2$ ; and

f): Condition a) holds and  $\exists e_1 \in Elem_1$ :  $e_1 \notin Elem_2$  and  $\exists e_2 \in Elem_2$ :  $e_2 \notin Elem_1$ .

- 1) If Condition a) fails, then  $\exists e_1 \in Elem_1: h_1(e_1) > h_2(e_1)$ . Let  $r = h_1(e_1) \cdot h_2(e_1)$ , there is no transition in ECLPN<sub>2</sub> consumes r tokens colored as  $e_1$ , then  $r \cdot e_1 \in M_E(\bullet p_{cOk}), k = \mathbb{N}_{|P_{cO}|}^+, \bullet p_{cOk} \in P_2$ and  $\bullet p_{cOk} \neq p_{E2}, p_{E2}$  is the end place in ECLPN<sub>2</sub>. From  $\bullet p_{cOk} \neq \emptyset$ , ECLPN<sub>2</sub> is not robust. Therefore, situations a) - c) cannot ensure that ECLPN<sub>C</sub> is robust;
- 2) The situation d) does not exist, because  $Elem_1 \supseteq Elem_2$ which conflicts with Condition a). If  $\exists e_1 \in Elem_1: e_1 \notin Elem_2$ , then  $h_1(e_1) > h_2(e_1)$ ;
- 3) If  $Elem_1 \subseteq Elem_2$  and Condition a) holds, then  $Sum_1 \subseteq Sum_2$ . There is no redundant token in ECLPN<sub>2</sub>. When the system reaches the end marking, the robustness of ECLPN<sub>1</sub> and ECLPN<sub>2</sub>ensures that  $M_E(p_E) = Init(p_0)$ ,  $M_E(p) = \emptyset$ , and  $p \neq p_E$ . Therefore, ECLPN<sub>C</sub> is robust under situation e); and
- 4) If  $\exists e_1 \in Elem_1$ :  $e_1 \notin Elem_2$ ,  $\exists e_2 \in Elem_2$ :  $e_2 \notin Elem_1$ , and  $D_1 = \{d_1 | d_1 \in Elem_1 \setminus Elem_2\}$ , then the type set

of  $M_E(\bullet p_{cOk})$  is  $D_1$ ,  $k = \mathbb{N}^+_{|P_{cO}|}$ . For  $\bullet p_{cOk} \in P_2$ and  $\bullet p_{cOk} \neq p_{E2}$ ,  $p_{E2}$  is the end place in ECLPN<sub>2</sub>. From  $\bullet p_{cOk} \neq \emptyset$ , ECLPN<sub>2</sub> is not robust. Therefore, the situation f) cannot ensure ECLPN<sub>C</sub> be robust.

In 3), if  $\exists e_2 \in Elem_2$ :  $e_2 \notin Elem_1$ , there is no token colored as  $e_2$  that flows to ECLPN<sub>2</sub> from ECLPN<sub>1</sub>, a transition  $p_{cOk}$ ,  $k \in \mathbb{N}^+_{|P_{cO}|}$ , is still enabled. When it fires, it generates no token colored as  $e_2$  to  $p_{cOk}$  according to the firing rule of ECLPNs. Thus, ECLPN<sub>1</sub> is robust.

In an ECLPN, colored tokens represent data packages. Data are processed in different subsystems. The color indicates the source of a token. After the data processing, tokens return to the source according to their colors. The strict conservativeness of an ECLPN<sub>C</sub> reveals that types and quantities of tokens remain static from the initial marking to any reachable marking.

The strict conservativeness for an ECLPN is defined as follows.

Definition 15: An ECLPN = { $\sum$ , P, T, A, C, G, E, Init, I, O} is strictly conservative if

1)  $M_0$  is the initial marking, where  $M_0(p) = \begin{cases} Init(p), & p = p_0 \\ \vdots & \vdots \end{cases}$  and

$$[\emptyset, \quad p \in P \setminus \{p_0\}]$$

2)  $\forall M \in R(M_0): \Sigma_{j=1}^m M(p_j) = \Sigma_{j=1}^m M_0(p_j).$ 

Similar to the robustness, given that each system is strictly conservative, the inheritance of strict conservativeness also depends on arc expressions which are connected to channel places. Then we have the following theorem.

Theorem 2: Let ECLPN<sub>C</sub> be composed by ECLPN<sub>l</sub> =  $(\sum_l, P_l, T_l, A_l, C_l, G_l, E_l, Init_l, I_l, O_l, M_l), l \in \{1, 2\}.$ ECLPN<sub>C</sub> is strictly conservative if

- ECLPN<sub>1</sub> and ECLPN<sub>2</sub> are robust and strictly conservative; and
- 2) For the binding *b* of ECLPN<sub>C</sub>, there is a set of channel places  $P_{cI} \subseteq P_c$ ,  $P_{cI} = \{p_{clj}|^{\bullet}p_{clj} \in \text{ECLPN}_1\}$ ,  $j \in \mathbb{N}_{|PcI|}^+$  and  $a_{Ij} = (^{\bullet}p_{clj}, p_{clj})$ . Similarly, there is a channel place set  $P_{cO} \subseteq P_c$ ,  $P_{cO} = \{p_{cOk}|p_{cOk}^{\bullet} \in \text{ECLPN}_1\}$ ,  $k \in \mathbb{N}_{|PcO|}^+$  and  $a_{Ok} = (p_{cOk}, p_{cOk}^{\bullet})$ , and then,  $\Sigma_{j=1}^{|P_{cl}|} E(a_{Ij}) < b > \subseteq \Sigma_{k=1}^{|P_{cO}|} E(a_{Ok}) < b >$ ;

 $\sum_{j=1}^{k} E(a_{lj}) < b \ge \sum_{k=1}^{k} E(a_{Ok}) < b >;$ *Proof:* In ECLPN<sub>l</sub>,  $l \in \{1, 2\}, M_{l0}$  is the initial marking,

and  $\forall M_l \in R(M_{l0})$ .  $M_{lE}$  is the end marking,  $M_{l0}$  is the initial marking, place, and  $p_{lE}$  is the end place.  $Elem_l$  is the type set for  $M_{l0}(p_{l1})$ .  $H(Elem_l) = \{h(e)|e \in Elem_l\}$  is the coefficient set for  $Elem_l$ .

Given that ECLPN<sub>1</sub> is strictly conservative, we have  $\Sigma_{j=1}^{m_1} M_1(p_{1j}) = \Sigma_{j=1}^{m_1} M_{10}(p_{1j})$ . Given that ECLPN<sub>1</sub> is robust, we have  $\Sigma_{j=1}^{m_1} M_1(p_{1j}) = M_{10}(p_{11}) = M_{1E}(p_{1E})$ , and by the sum form, we have  $M_{10}(p_{11}) = M_{1E}(p_{1E}) = \sum_{e \in Elem1} h(e) \cdot e$ . Similarly, in ECLPN<sub>2</sub>, we have  $M_{20}(p_{21}) = M_{2E}(p_{2E}) =$ 

Similarly, in ECLPN<sub>2</sub>, we have  $M_{20}(p_{21}) = M_{2E}(p_{2E}) = \sum_{e \in Elem_2} h(e) \cdot e = \sum_{e \in Elem_2} h(e) \cdot e$ . Like Theorem 1, let  $Sum_1$  denote  $\sum_{j=1}^{|P_{cl}|} E(a_{lj}) < b >$  and

Like Theorem 1, let  $Sum_1$  denote  $\sum_{j=1}^{|I-c|} E(a_{Ij}) < b >$  and  $Sum_2$  denote  $\sum_{k=1}^{|P_{cO}|} E(a_{Ok}) < b >$ . The theorem is also proven

from 6 situations regarding the relation between  $Sum_1$  and  $Sum_2$ .

- When Condition a) fails, ∃e<sub>1</sub> ∈ Elem<sub>1</sub>, such that h<sub>1</sub>(e<sub>1</sub>)> h<sub>2</sub>(e<sub>1</sub>). Let r = h<sub>1</sub>(e<sub>1</sub>)-h<sub>2</sub>(e<sub>1</sub>). There is no transition in ECLPN<sub>2</sub> consuming the remaining r tokens colored as e<sub>1</sub>, and then r · e<sub>1</sub> ∈ M<sub>E</sub>(••p<sub>cOk</sub>), k = N<sup>+</sup><sub>|PcO|</sub>, where ••p<sub>cOk</sub> ∈ P<sub>2</sub> and ••p<sub>cOk</sub> ≠ p<sub>E2</sub>. Colored tokens in an ECLPN flow independently by the firing rule. Then in the marking M<sub>2E</sub>, the sum of tokens is ∑<sub>e∈Elem1</sub> h<sub>r</sub>(e) · e + ∑<sub>e∈Elem1</sub> h<sub>r</sub>(e) · e, where ∑<sub>e∈Elem1</sub> h<sub>r</sub>(e) · e is the sum of tokens in place ••p<sub>cOk</sub>. We have that ∑<sub>e∈Elem1</sub> h<sub>r</sub>(e) · e ⊇ r · e<sub>1</sub>. Therefore, ∑<sub>e∈Elem2</sub> h(e) · e + ∑<sub>e∈Elem1</sub> h<sub>r</sub>(e) · e ≥ r · e<sub>1</sub>. Therefore, situations a) c) cannot ensure that ECLPN<sub>C</sub> is strictly conservative;
- The situation d) does not exist, and the reason is the same with 2) in the proof for Theorem 1;
- 3) In situation e), if *Elem*<sup>1</sup> ⊆ *Elem*<sup>2</sup> and Condition a) holds, then *Sum*<sup>1</sup> ⊆ *Sum*<sup>2</sup>. Given that ECLPN<sup>1</sup> and ECLPN<sup>2</sup> are strictly conservative, we have the sum of tokens stay constant in ECLPN<sup>1</sup> and ECLPN<sup>2</sup>. ∀*p<sub>c</sub>* ∈ *P<sub>c</sub>*: *E<sub>c</sub>*(•*p<sub>c</sub>*, *p<sub>c</sub>*)*<b>* = *E<sub>c</sub>*(*p<sub>c</sub>*, *p<sup>c</sup>*)*<b>* (Definition 13). It shows that the sum of tokens does not change when tokens transferring between ECLPNs. Besides, *Sum*<sup>1</sup> ⊆ *Sum*<sup>2</sup> ensures that there is no redundant token in ECLPN<sup>2</sup>. Therefore, ECLPN<sub>C</sub> is strictly conservative; and
- 4) If  $\exists e_1 \in Elem_1$ :  $e_1 \notin Elem_2$ ,  $\exists e_2 \in Elem_2$ :  $e_2 \notin Elem_1$ , and  $D_1 = \{d_1 | d_1 \in Elem_1 \setminus Elem_2\}$ , then the type set of  $M_E(\bullet p_{cOk})$  is  $D_1$ ,  $k = \mathbb{N}_{|P_{cO}|}^+$ . There is no transition in ECLPN<sub>2</sub> consuming the remaining tokens in  $M_E(\bullet p_{cOk})$ ,  $k = \mathbb{N}_{|P_{cO}|}^+$ . In the marking  $M_{2E}$ , the sum of tokens is  $\sum_{e \in Elem_2} h(e) \cdot e + \sum_{e \in D1} h_r(e) \cdot e$ , where  $\sum_{e \in D1} h_r(e) \cdot e$  is the sum of tokens in place  $\bullet p_{cOk}$ , and  $h_r(e)$  is the coefficient of the element which is remaining in place  $\bullet p_{cOk}$ . Then we have  $\sum_{e \in D1} h_r(e) \cdot e$  $e \neq \emptyset$ , and therefore,  $\sum_{e \in Elem_2} h(e) \cdot e + \sum_{e \in D1} h_r(e) \cdot e$  $e \supseteq \sum_{e \in Elem_2} h(e) \cdot e$ . Thus, ECLPN<sub>2</sub> is not strictly conservative. As a result, situation f) cannot ensure that ECLPN<sub>C</sub> is strictly conservative.

### D. COMPOSITION PROCEDURE OF ECLPNs

Suppose that a complicated system is composed of multiple subsystems, and information transfer between subsystems. Then the composing procedure of an  $ECLPN_C$  model is presented below.

Step 1: Build a CPN subnet  $sn_1$  for a subsystem by sequential and choice CPN modules, and use colored token  $c_1$  to simulate the running process of the subsystem;

Step 2: For another subsystem, if it shares the same structure with  $sn_1$ , then it uses a colored token  $c_2$  in  $sn_1$  to simulate its running process. We then transform ordinary transitions



FIGURE 5. An ECLPN<sub>C</sub> example.

into logical transitions by attaching logical expressions. Arc expressions also make changes accordingly, and  $sn_1$  becomes an ECLPN. If the subsystem has a different structure from  $sn_1$ , then we construct another CPN subnet  $sn_i$ ,  $i = \{2, 3, ...\}$ , and use colored token  $c_i$  to simulate its running process. Repeat Step 2 until all subsystems are simulated in subnets;

Step 3: For a subnet  $sn_i$ ,  $i \in \mathbb{N}_n^+$ , if it receives tokens from  $sn_j$ ,  $j = \{1, 2, ..., i - 1, i + 1, ...\}$ , then we attach logic expressions to the transition in  $sn_j$ . The expression includes all types in  $sn_j$ . Arc expressions also make changes accordingly, and  $sn_j$  becomes an ECLPN; and

Step 4: Connect all  $sn_i$ ,  $i = \{1, 2, ...\}$ , by channel places and set arc expressions. The Composition of ECLPNs is complete.

*Remark 1:* In Step 1, building a CPN subnet begins with a simple, bounded, live, reversible CPN. It is elaborated by sequential and choice modules in a Top-down manner [30].

*Remark 2:* In Step 2, the purpose of attaching logic expressions is to keep  $c_1$  and  $c_2$  independent.

*Remark 3:* In Step 3, if there is a CPN subnet, it has only one type of token, and receives no token from any other subnets, and then, it can be regarded as an ECLPN, whose logic expression of all transitions is the Boolean constant *true*.

# **IV. ECLPN MODEL OF AN E-COMMERCE SYSTEM**

In this section, An ECLPN model  $ECLPN_{EC}$  is presented to describe an E-commerce system [29] and make a comparison with the LPN model  $LPN_{EC}$  in [29].

A CLPN [27] model is similar to LPN in scale. Therefore, we compare  $ECLPN_{EC}$  with  $LPN_{EC}$  instead of a CLPN model.

# A. INTRODUCTION OF AN E-COMMERCE SYSTEM

The E-commerce system is composed of 3 subsystems, i.e., customers, a merchant and a third-party. Customers accomplish the following operations: placing an order, making the payment or being refused, confirming the receipt of the commodity, and waiting for the confirmation of the receipt of payment from the merchant. The merchant sequentially receives an order, checks the inventory to decide whether to accept the order or not, checks the payment, delivers the product and confirms the receipt of payment. The third-party keeps the



**FIGURE 6.** Three subnets: (a) the seller subnet  $sn_c$ , (b) the merchant subnet  $sn_m$ , and (c) the third-party subnet  $sn_{tp}$ .

payment until the customer has confirmed the receipt of the commodity.

#### B. ECLPN<sub>EC</sub> MODEL

The composing procedure of  $ECLPN_{EC}$  according to Section III is given below.

Step 1: Construct the customer subnet  $sn_c$  by sequential and choice CPN modules. For all the customer subsystems that share the same structure, their operation processes are all described in  $sn_c$  by different colored tokens;

Step 2: Construct the merchant subnet  $sn_m$  and the thirdparty subnet  $sn_{tp}$  by sequential CPN modules.  $sn_c$ ,  $sn_m$  and  $sn_{tp}$  are shown in Fig. 6(a), (b), and (c), respectively;

In  $sn_c$ , the binding is  $b_1 = \langle con_1 = c, v_1 = c_1, v_2 = c_2, \ldots, v_n = c_n \rangle$ .  $\{v_1, v_2, \ldots, v_n\}$  denotes the set of tokens that are carrying data in customer subsystems.  $\{c_1, c_2, \ldots, c_n\}$  denotes the color of tokens, and colors implicate customers in  $sn_c$ .  $\{con_1\}$  denotes the control token, which is colored as  $\{c\}$ . The control token controls and records the operation process of other tokens.  $p_{in_j}$  is the initial place and  $p_{out_j}$  is the end place.

Let  $set_c$  be the set of  $MS_c$ , where  $MS_c$  makes the logic expression  $\bigcup_{i=1}^{n} c_i$  be *true*. There are  $2^n$  elements in  $set_c$ . In  $sn_c$ , all arc expressions are  $\{c\} \cup set_c$ .

In  $sn_m$ , the binding is  $b_2 = \langle con_2 = m \rangle$ , and  $\{con_2\}$  denotes the control token in  $sn_m$ , which is colored as  $\{m\}$ . All arc expressions are *m*.  $p_{in}$  is the initial place, and  $p_{out}$  is the end place.

In  $sn_{tp}$ , the binding is  $b_3 = \langle con_3 = tp \rangle$ , and  $\{con_3\}$  denotes the control token in  $sn_{tp}$ , which is colored as  $\{tp\}$ . All arc expressions are tp.  $p_{in_TP}$  is the initial place, and  $p_{out_TP}$  is the end place.



**FIGURE 7.** The composition of three ECLPNs model ECLPN<sub>EC</sub>. (a) the customer subnet  $sn_c$ , (b) the merchant subnet  $sn_m$ , and (c) the third-party subnet  $sn_{tp}$ .

Step 3: Connect the three subnets by channel places  $\{p_{c1}, p_{c2}, \ldots, p_{c9}\}$ , and the composition of ECLPNs is built as Fig. 7.

In  $sn_m$ , transition  $t_{decision}$  checks whether the product is in stock or not. If it is in stock, related tokens flow to  $p_{c2}$ , otherwise they flow to  $p_{c3}$ . Tokens flowing into  $sn_m$  via  $p_{c1}$ are the same with tokens flowing out of  $sn_m$  through  $p_{c2}$ and  $p_{c3}$ .

In Fig. 7, there are 23 places, 15 transitions, and 48 directed arcs.

The binding of  $ECLPN_{EC}$  is  $b_M = \langle con_1 = c, con_2 = m, con_3 = tp, v_1 = c_1, \ldots v_n = c_n \rangle$ ,  $con_1 - con_3$  are control tokens for  $sn_c$ ,  $sn_m$ , and  $sn_{tp}$ , respectively. { $v_1, v_2, \ldots, v_n$ } is the set of tokens carrying data in customer subsystems. They are produced in  $sn_c$ , flow into  $sn_m$  and  $sn_{tp}$ , and return to  $sn_c$  in the end.

Arc expressions in  $ECLPN_{EC}$  are updated as follows.

All tokens in the set  $\{v_1, v_2, ..., v_n\}$  transmit between subsystems. Therefore, all arc expressions which are related with channel places  $\{p_{c1}, p_{c2}, ..., p_{c9}\}$  are *set<sub>c</sub>*.

In  $sn_c$ , expressions on these arcs are  $\{c\}$ , and they are  $(t_{order\_c}, p_{order\_c})$ ,  $(p_{order\_c}, t_{payment\_c})$ ,  $(t_{payment\_c}, p_{payment\_c})$ ,  $(p_{product\_c}, p_{product\_c})$ ,  $(t_{product\_c}, p_{product\_c})$ ,  $(p_{product\_c}, t_{wait})$ , and  $(p_{order\_c}, t_{refused\_c})$ . Expressions on arcs  $(p_{in\_c}, t_{order\_c})$ ,  $(t_{wait}, p_{out\_c})$ ,  $(t_{refused\_c}, p_{out\_c})$ ,  $(p_{out\_c}, t_{c*})$ and  $(t_c*, p_{in\_c})$  remain  $\{c\} \cup set_c$ .

In  $sn_m$ , expressions on these arcs are  $\{m\} \cup set_c$ , and they are  $(t_{order}, p_{order})$ ,  $(p_{order}, t_{decision})$ ,  $(t_{payment}, p_{payment})$ , and  $(p_{payment}, t_{product})$ . Expressions on arcs  $(p_{in}, t_{order})$ ,  $(t_{decision}, p_{decision})$ ,  $(p_{decision}, t_{payment})$ ,  $(t_{product}, p_{product})$ ,  $(p_{product}, t_{get_payment})$ ,  $(t_{get_payment}, p_{out})$ ,  $(p_{out}, t^*)$ , and  $(t^*, p_{in})$ remain  $\{m\}$ .

In  $sn_{tp}$ , expressions on arcs  $(p_{in\_TP}, t_{payment\_step1})$ ,  $(t_{payment\_step1}, p_{payment\_step1})$ ,  $(p_{payment\_step1}, t_{payment\_step2})$ ,  $(t_{payment\_step2}, p_{out\_TP})$ ,  $(p_{out\_TP}, t_{TP*})$ , and  $(t_{TP*}, p_{in\_TP})$  remain  $\{tp\}$ .

Logic transitions and logical expressions are set below.

In  $sn_c$ , all transitions are logic transitions, and input and output expressions are  $f_I = f_O = \{c\} \cup set_c$ ;

$$M_0 \xrightarrow{t_{order\_c}} M_1 \xrightarrow{1} M_2 \xrightarrow{t_{payment\_c}} M_3 \xrightarrow{t_c^*} M_4$$

$$M_4 \xrightarrow{2} M_5 \xrightarrow{t_{product_c}} M_6 \xrightarrow{3} M_7 \xrightarrow{t_{wait}} M_8 \xrightarrow{t_c^*} M_0$$

FIGURE 8. The running process of subsystem snc.

$$M_{0} \xrightarrow{1} M_{1} \xrightarrow{t_{order}} M_{2} \xrightarrow{t_{decision}} M_{3} \xrightarrow{2} M_{4} \xrightarrow{t_{payment}} M_{5}$$

$$M_{5} \xrightarrow{t_{product}} M_{e} \xrightarrow{3} M_{7} \xrightarrow{t_{get\_payment}} M_{9} \xrightarrow{t^{*}} M_{0} \xrightarrow{4} M_{0}$$

FIGURE 9. The running process of subsystem snm.

In  $sn_m$ , all transitions are logic transitions except  $t^*$ , and input and output expressions are  $f_I = f_O = \{m\} \cup set_c$ .  $t^*$  is an ordinary transition; and

In  $sn_{tp}$ , all transitions are logic transitions except  $t_{TP}^*$ , and input and output expressions are  $f_I = f_O = \{tp\} \cup set_c$ .  $t_{TP}^*$  is an ordinary transition.

Suppose that there are three customers  $c_1$ ,  $c_2$  and  $c_3$  in the system.  $c_1$  and  $c_2$  can get the goods from the merchant, whereas  $c_3$  is refused, because the goods are sold out. Three processes are needed, representing three subsystems  $sn_c$ ,  $sn_m$  and  $sn_{tp}$ , respectively. Let  $set_c$  be the set of  $MS_c$ , where  $MS_c$  makes the logic expression  $c_1 \cup c_2 \cup c_3$  be *true* in this instance. While the system is running, different transitions fire concurrently when enabled.

The running process of subsystem  $sn_c$  is presented below in Fig. 8.

The operations 1)-3) in Fig. 8 are explained as follows.

- 1) Process  $sn_c$  initializes process  $sn_m$ , sends tokens in  $p_{c1}$  to  $sn_m$  and suspends process  $sn_c$ . Process  $sn_c$  awakes when tokens in  $p_{c2}$  or  $p_{c3}$  are received from  $sn_m$ ;
- 2) Process  $sn_c$  initializes process  $sn_{tp}$ , sends tokens in  $p_{c7}$  to  $sn_{tp}$  and suspends process  $sn_c$ . Process  $sn_c$  awakes when tokens in  $p_{c5}$  are received from  $sn_m$ ; and
- 3) Process  $sn_c$  sends tokens in  $p_{c8}$  to  $sn_{tp}$ , suspends process  $sn_c$  and wakes up process  $sn_{tp}$ . Process  $sn_c$  awakes when tokens in  $p_{c9}$  are received from  $sn_m$ .

With the column label being  $[p_{in_c}, p_{c1}, p_{order_c}, p_{c2}, p_{c7}, p_{payment_c}, p_{c5}, p_{c8}, p_{product_c}, p_{c9}, p_{c3}, p_{out_c}]$ , markings in Fig.8 are given below.

- $$\begin{split} M_0 &= (3c+c_1+c_2+c_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), \\ M_1 &= (0, c_1+c_2+c_3, 3c, 0, 0, 0, 0, 0, 0, 0, 0, 0), \\ M_2 &= (0, 0, 3c, c_1+c_2, 0, 0, 0, 0, 0, 0, c_3, 0), \\ M_3 &= (0, 0, 0, 0, c_1+c_2, 2c, 0, 0, 0, 0, 0, c_+c_3), \\ M_4 &= (c+c_3, 0, 0, 0, c_1+c_2, 2c, 0, 0, 0, 0, 0, 0), \end{split}$$
- $M_5 = (c + c_3, 0, 0, 0, 0, 2c, c_1 + c_2, 0, 0, 0, 0, 0),$
- $M_6 = (c + c_3, 0, 0, 0, 0, 0, 0, c_1 + c_2, 2c, 0, 0, 0),$
- $M_7 = (c + c_3, 0, 0, 0, 0, 0, 0, 0, 0, 2c, c_1 + c_2, 0, 0)$ , and
- $M_8 = (c + c_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2c + c_1 + c_2).$

The running process of subsystem  $sn_m$  is presented in Fig. 9.

$$M_0 \xrightarrow{1} M_1 \xrightarrow{t_{payment\_step1}} M_2 \xrightarrow{2} M_3 \xrightarrow{t_{payment\_step2}} M_4 \xrightarrow{t_{TP}^*} M_5 \xrightarrow{3} M_0$$

FIGURE 10. The running process of subsystem sn<sub>tp.</sub>

The operations 1)-4) in Fig. 9 are explained as follows.

- 1) Process  $sn_m$  receives tokens from  $sn_c$  in  $p_{c1}$ ;
- Process sn<sub>m</sub> wakes up process sn<sub>c</sub>, sends tokens in p<sub>c2</sub> and p<sub>c3</sub> to sn<sub>c</sub>, and suspends process sn<sub>m</sub>. Process sn<sub>m</sub> awakes when tokens in p<sub>c4</sub> are received from sn<sub>lp</sub>;
- 3) Process  $sn_m$  wakes up process  $sn_c$ , sends tokens in  $p_{c5}$  to  $sn_c$ , and suspends process  $sn_m$ . Process  $sn_m$  awakes when tokens in  $p_{c6}$  are received from  $sn_{tp}$ ; and
- Process sn<sub>m</sub> wakes up process sn<sub>c</sub>, and sends tokens in p<sub>c9</sub> to sn<sub>c</sub>.

With the column label being  $[p_{in}, p_{c1}, p_{order}, p_{c2}, p_{c3}, p_{decision}, p_{c4}, p_{payment}, p_{c5}, p_{product}, p_{c6}, p_{c9}, p_{out}]$ , markings in Fig. 9 are given below.

The operations 1)-3) in Fig. 10 are explained as follows.

- 1) Process  $sn_{tp}$  receives tokens from  $sn_c$  in  $p_{c7}$ ;
- 2) Process  $sn_{tp}$  wakes up process  $sn_m$ , sends tokens in  $p_{c4}$  to process  $sn_m$  and suspends process  $sn_{tp}$ . Process  $sn_{tp}$  awakes when tokens in  $p_{c8}$  are received from  $sn_c$ ; and
- 3) Process  $sn_{tp}$  wakes up process  $sn_m$ , and sends tokens in  $p_{c6}$  to process  $sn_m$ .

With the column label being  $[p_{in\_TP}, p_{c7}, p_{c4}, p_{payment\_step1}, p_{c8}, p_{c6}, p_{out\_TP}]$ , markings in Fig. 10 are given below.

 $M_0 = (2 tp, 0, 0, 0, 0, 0, 0), M_1 = (2 tp, c_1 + c_2, 0, 0, 0, 0, 0), M_2 = (0, 0, c_1 + c_2, 2 tp, 0, 0, 0), M_3 = (0, 0, 0, 2 tp, c_1 + c_2, 0, 0), M_4 = (0, 0, 0, 0, 0, c_1 + c_2, 2 tp), and M_5 = (2 tp, 0, 0, 0, 0, c_1 + c_2, 0).$ 

The property of  $ECLPN_{EC}$  is analyzed as follows.

Before the composition,  $sn_c$  is robust and strictly conservative. After the composition,  $sn_c$ 's output channel places are  $\{p_{c1}, p_{c7}, p_{c8}\}$ , input channels are  $\{p_{c2}, p_{c3}, p_{c5}, p_{c9}\}$ , and the following conclusions 1) - 3) hold in *ECLPN<sub>EC</sub>*.

- 1) Tokens flowing out from  $sn_c$  via  $p_{c1}$  are the same with tokens flowing in  $sn_c$  through  $p_{c2}$  and  $p_{c3}$ ;
- 2) Tokens flowing out from  $sn_c$  via  $p_{c7}$  are the same with tokens flowing in  $sn_c$  through  $p_{c5}$ ; and
- Tokens flowing out from sn<sub>c</sub> via p<sub>c8</sub> are the same with tokens flowing in sn<sub>c</sub> through p<sub>c9</sub>;

 TABLE 1. The comparison between ECLPN<sub>EC</sub> and LPN<sub>EC</sub>.

Item	$ECLPN_{EC}$	$LPN_{EC}$	Reduction rate
Number of	23	12 <i>n</i> +10	(12 <i>n</i> -
places			13)/(12 <i>n</i> +10)
Number of	15	5 <i>n</i> +10	(n-1)/(n+2)
transitions			
Number of	48	16 <i>n</i> +30	(8 <i>n</i> -9)/(8 <i>n</i> +15)
directed arcs			

**TABLE 2.** The comparison between  $ECLPN_{EC}$  and  $LPN_{EC}$  when n = 3.

Item	$ECLPN_{EC}$	$LPN_{EC}$	Reduction rate
Number of places	23	46	50%
Number of transitions	15	25	40%
Number of directed arcs	48	78	38.5%

For  $sn_m$ , it is robust and strictly conservative before the composition, and its input channel places are { $p_{c1}$ ,  $p_{c4}$ ,  $p_{c6}$ } and output channel places are { $p_{c2}$ ,  $p_{c3}$ ,  $p_{c5}$ ,  $p_{c9}$ } in the composed net, where conclusions 4) to 6) hold.

- Tokens flowing in sn<sub>m</sub> via p<sub>c1</sub> is the same with tokens flowing out from sn<sub>m</sub> through p<sub>c2</sub> and p<sub>c3</sub>;
- 5) Tokens flowing in  $sn_m$  via  $p_{c4}$  is the same with tokens flowing out from  $sn_m$  through  $p_{c5}$ ; and
- 6) Tokens flowing in  $sn_m$  via  $p_{c6}$  is the same with tokens flowing out from  $sn_m$  through  $p_{c9}$ ;

For  $sn_{tp}$ , it is robust and strictly conservative before the composition, and its input channel places are  $\{p_{c7}, p_{c5}\}$  and output channel places are  $\{p_{c4}, p_{c6}\}$  in the composed net, where conclusions 7) and 8) hold.

- 7) Tokens flowing in  $sn_{tp}$  via  $p_{c7}$  is the same with tokens flowing out from  $sn_{tp}$  through  $p_{c4}$ ; and
- 8) Tokens flowing in  $sn_{tp}$  via  $p_{c5}$  is the same with tokens flowing out from  $sn_{tp}$  through  $p_{c6}$ .

As a result,  $ECLPN_{EC}$  is robust and strictly conservative by 1) to 8), according to Theorems 1 and 2.

#### C. COMPARISON BETWEEN ECLPN<sub>EC</sub> AND LPN<sub>EC</sub>

Suppose that the complicated E-commerce system is composed of *n* customers,  $n \in \mathbb{N}^+$ , a seller and a third-party participant. The comparison between  $ECLPN_{EC}$  in this work and  $LPN_{EC}$  in [29] are presented in Table 1.

The reduction rate in Tables 1-2 is defined as:

Reduction rate = 
$$1 - \frac{\text{The item of } ECLPN_{EC}}{\text{The item of } LPN_{EC}}$$

From Table 1, we can see that the number of places, transitions and directed arcs are constant in  $ECLPN_{EC}$ , whereas  $LPN_{EC}$  needs to build a subnet for each subsystem. With the increase in the number of subsystems, the scale of  $LPN_{EC}$  continues to expand.

In the offseason, for instance, suppose that there are 3 customers participating the E-commerce system at the same time, and the comparison on the scale is shown in Table 2.

**TABLE 3.** The comparison between  $ECLPN_{EC}$  and  $LPN_{EC}$  when n = 15.

Item	$ECLPN_{EC}$	$LPN_{EC}$	Reduction rate
Number of places	23	190	87.9%
Number of	15	85	82.4%
Number of	48	270	82.2%
directed arcs			

In the peak season, suppose that there are 15 customers participating the E-commerce system at the same time, and the comparison on the scale is shown in Table 3.

According to Tables 2 and 3, with the increase of n, the reduction rate of  $ECLPN_{EC}$  is increasing, and thus  $ECLPN_{EC}$  is kept in a small scale.

If there are more sellers and third-party participants,  $ECLPN_{EC}$  can describe the process in those subsystems by using more colored tokens, without any change in the system structure and scale. The size of model can be further reduced.

#### **V. CONCLUSION**

In this paper, we give the method of the composition of ECLPNs and analyze its properties. The formal definition of the composition of ECLPNs is given. Conditions for the inheritance of robustness and strict conservativeness are discussed. We give the procedure of building a composition of ECLPNs from basic design modules. Besides, we apply ECLPN<sub>C</sub> in modeling and analyzing an E-commerce system, and make a comparison with an LPN model. The result shows that the ECLPN model has a simpler structure and lower complexity. Future work will study the maintenance of other properties, and apply ECLPNs in other fields, such as manufacturing systems [39], [40], knowledge-based systems [41], and transportation systems [42].

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**ZHEN WANG** received the B.S. degree in mathematics and applied mathematics from the Shandong University of Science and Technology, Qingdao, China, in 2014, where he is currently pursuing the master's degree in computer science. His current research interest is in Petri nets' theory and applications.



**WENJING LUAN** (Member, IEEE) received the B.S. and M.S. degrees from the Shandong University of Science and Technology, Qingdao, China, in 2009 and 2012, respectively, and the Ph.D. degree in computer software and theory from Tongji University, Shanghai, China, in 2018. From May 2017 to July 2017, she was a Visiting Student with the Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ, USA. She is currently a Lecturer of

computer science and technology with the Shandong University of Science and Technology. Her current research interests include location-based social networks, data mining, recommender systems, and intelligent transportation systems. She received the Best Student Paper Award (Finalist) at the 13th IEEE International Conference on Networking, Sensing and Control (ICNSC 2016).



**YUYUE DU** received the B.S. degree from Shandong University, Jinan, China, in 1982, the M.S. degree from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 1991, and the Ph.D. degree in computer application from Tongji University, Shanghai, China, in 2003. He is currently a Professor with the College of Computer Science and Engineering, Shandong University of Science and Technology, Qingdao, China. He has taken in over 20 projects supported by the National

Nature Science Foundation, the National Key Basic Research Developing Program, and other important and key projects at provincial levels. He has published over 200 articles in domestic and international academic publications, and they are embodied over 150 times by SCI and EI. His research interests are in formal engineering, process mining, Petri nets, real-time systems, web services, and workflows.



**LIANG QI** (Member, IEEE) received the B.S. degree in information and computing science and the M.S. degree in computer software and theory from the Shandong University of Science and Technology, Qingdao, China, in 2009 and 2012, respectively, and the Ph.D. degree in computer software and theory from Tongji University, Shanghai, China, in 2017. From 2015 to 2017, he was a Visiting Student with the Department of Electrical and Computer Engineering, New Jersey

Institute of Technology, Newark, NJ, USA. He is currently with the Shandong University of Science and Technology. He has published more than 50 articles in journals and conference proceedings. His research interests include Petri nets, machine learning, optimization, and intelligent transportation systems.