Generalized Single-Valued Neutrosophic Power Aggregation Operators Based on Archimedean Copula and Co-Copula and Their Application to Multi-Attribute Decision-Making

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ABSTRACT Single-valued neutrosophic set (SVN) can valid depict the incompleteness, nondeterminacy and inconsistency of evaluation opinion, and the Power average (PA) operator can take into account the correlation of multiple discussed data. Meanwhile, Archimedean copula and co-copula (ACC) can significant generate operational laws based upon diverse copulas. In this paper, we first redefine several novel operational laws of single-valued neutrosophic number (SVNN) based on ACC and discuss the associated properties of them. In view of these operational rules, we propound several novel power aggregation operators (AOs) to fuse SVN information, i.e., SVN copula power average (SVNCPA) operator, weighted SVNCPA (WSVNCPA) operator, order WSVNCPA operator, and SVN copula power geometric (SVNCPG) operator, weighted SVNCPG (WSVNCPG) operator, order WSVNCPG operator. At the same time, several significant characteristics and particular cases of these operators are examined in detail. Moreover, we extend these operators to their generalized form named generalized SVNCPA and SVNCPG operator. In addition, a methodology is designed based on these operators to cope with multi-attribute decision-making (MADM) problems with SVN information. Consequently, the effectiveness and utility of the designed approach is validated by a empirical example. A comparative and sensitivity analysis are carried out to elaborate the strength and preponderance of the propounded approach.

INDEX TERMS Multi-attribute decision-making, single-valued neutrosophic set, aggregation operator, archimedean copula and co-copula, power average.

I. INTRODUCTION

Multiple attribute decision making is an important constituent of modern decision science and management, which is a procevss of evaluating and selecting the optimal alternative by scientific and reasonable approaches based upon the selected multiple attributes and assessment information. Traditionally, managers expressing their preference information take in the

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form of a accurate numerical value. However, the precise numerical value is arduous for decision makers (DMs) to provide the incomplete and vague information in the complex system perfectly. In order to cope with this defect and describe the uncertainty and fuzziness of information more effectively, the fuzzy set (FS) [1] theory and its expansions form intuitionistic fuzzy set (IFS) [2], interval-valued intuitionistic fuzzy set (IVIFS) [3] are propounded and these fuzzy sets are wide-ranging utilized to handle decision making (DM) problems [4]–[12]. As the foundation of these

theories, numerous investigators have been favored on the operation laws of fuzzy numbers in the last two decades. Xu and Yager [13] pioneered several basic operational rules based on Algebraic t-norm and s-norm (ATS) and propounded some geometric operators of intuitionistic fuzzy number (IFN). Wang *et al.* [14] further developed the Einstein operatorts of IFN to aggregation intuitionistic fuzzy information. Xia *et al.* [15] defined the generalized operational laws of IFN on the basis of ATS and developed several generalized AOs. Lei and Xu [16] introduced the novel operational of IFN named derivative and differential operational laws. Besides, the neutral operational [17], exponential operational [18] and logarithmic operational [19] of IFN are put forward one after another. With the aid of these operations, a multitude of AOs have been brought forward to apply to the procedure of information aggregation. Luo *et al.* [20] propounded several exponential AOs based on ATS for settling MADM issues. Garg [21] developed some generalized AOs based on exponential and logarithmic operational laws under intuitionistic fuzzy circumstance. Garg [22] proposed a series of generalized intuitionistic fuzzy soft power AOs based upon ATS. Except for these, other researches also developed a sea of AOs based on different extension forms of IFS, which are summarized as [23]–[29]. However, these above approaches have a limitation that they disable to deal with uncertain and inconsistent evaluation information effectively in several special situations.

In order to overwhelm the aforementioned deficiency validly, Smarandache [30] originally pioneered a novel notion named neutrosophic set(NS). NS theory adds a independent indeterminacy membership degree on the basis of IFS, which is a novel generalization of FS and IFS. However, the NS theory is presented from the philosophical perspective in the early days, it is arduous to put it into particular issues because the membership functions of NS are bounded by in $]0^-, 1^+[$. For surmount it efficiently, Wang *et al.* [31] firstly developed the conception of single-valued neutrosophic and explored related operational laws and properties. Smarandache [32] defined the subtraction and division operational rules of SVNN. Thereafter, a lot of research achievements have been achieved with respect to the operations and AOs of SVN. Liu *et al.* [33] proposed a general operations of SVNNs based on ATT and several AOs. Lu *et al.* [34] developed the exponential operations of SVNN and corresponding AOs to settle MADM problems. Garg [35] defined the logarithmic operational laws for computing SVNN and propounded the logarithmic SVN weight averaging (L-SVNWA) operator and logarithmic SVN weight geometric (L-SVNWG) operator to integrate SVN information. However, the above-mention AOs assume that all attributes are independent of each other in the process of information aggregation. This will affects the final decision results. Based on this, several novel AOs are extended to SVN environment to overcome the above deficiencies. Ji *et al.* [36] defined the Frank operations of SVNN and propounded SVN Frank prioritized Bonferroni mean operator, which can take

into account the interrelationship between any two discussed parameters. Liu et al. [37] developed several power Muirhead mean operators under SVN context. Zhao et al. [38] introduced some SVN power Heronian mean AOs, which consider the importance and correlation between discussed arguments simultaneously. Peng et al. [39] presented power Shapley Choquet average operators for SVNNs to do with MADM problems. Liu et al. [40] defined the new operations based on Schweizer-Sklar norm and presented several SVN Schweizer-Sklar prioritized AOs. Based upon decision approaches, Sun et al. [41] presented the extended TODIM and extended ELECTRE III approaches to SVN environment on the basis of a novel distance measure to solve the problem of doctors assessment problems. Peng et al. [42] developed three MADM algorithms based on new similarity measure, TOPSIS and MABAC methods. Tian et al. [43] developed an extended QUALIFLEX approach on the basis of an improved SVN projection measure to resolve the problem of green supplier selection with weight information fully known. The more research results about SVN have been propounded in [44]–[52].

With the increasing complicacy of DM setting and the diversity of DMs' knowledge level and evaluation experience, the evaluation information provided by DMs may have subjectivity. For instance, a manager provides his assessment for safety and cost in a programme, he shall distribute a higher preference to safety than cost. For overwhelming this situation and improving the accuracy of decision results, Yager [53] propounded the power average operator which sticks out the support degree of data during information aggregating procedure. Besides, PA operator can acquire the weight from fused data, and cut down the the impact of too high and too low data. Since its appearance, the theory and application of it have received an increasingly number extensive attention in information fusion and data mining. Zhang [54] studied the generalized power geometric operator under intuitionistic fuzzy context to settle MAGDM issues. Liu *et al.* [55] combined PA operator and Maclaurin Symmetric mean (MSM) operator to developed sever power MSM AOs to fuse q-rung orthopair fuzzy information. For demonstrating the flexibility of the aggregation process, Zhou *et al.* [56] revealed the generalized power AOs to handle group DM problems. Ju *et al.* [57] extended it to q-ROF environment. Liu *et al.* [58] presented a novel MADM methodology based on generalized power operators under hesitant fuzzy linguistic context. Therefor, it is significant to study on AOs utilizing generalized PA operator to propound novel operators.

As a classical instance of traditional t-norm and s-norm, copulas and co-copulas [59] have received pervasive application in various flied. Among these utilizations, the operation of copulas is one of the essential core of research hot [60]–[66]. From these research achievements, the advantages of copulas are summarized in the following three categories: (i) it can generate operations under diverse fuzzy settings and the acquired operations based upon copulas are closed; (ii) it can effectively prevent the loss of information

in the process of information aggregation because it can seize the interrelationship among diverse attributes; (iii) it provides different copulas for managers to choose through their preference, and each copula has a parameter which makes the decision procedure more flexible. In allusion the above-mentioned merits, Tao *et al.* [67] firstly developed the copula operational on intuitionistic fuzzy environment and proposed a weighted aggregation operator to aggregate intuitionistic fuzzy information. Tao *et al.* [68] constructed a novel computation model based on the ACC, which provides a valid technique to fuse unbalanced linguistic information. Chen *et al.* [69] introduced several new AOs of linguistic neutrosophic on the basis of ACC. Based on these foregoing analysis, it is evident know that copulas and PA have their distinctive preponderance and acquire growing attention from scholars during the application in MADM issues but they are not generalized to SVN domain. Han *et al.* [70] propounded the conception of probabilistic unbalanced linguistic term set and several AOs bases upon Archimedean copula. In light of above discussion and motivations, the objective of this study are to combine copulas, PA operator and SVN to present several AOs and decision approach. Accordingly, the mainly intentions and contributions of this research are outline following:

1) To extend ACC to SVN and propound several novel general operational rules of SVNN;
2) To combine PA operator and the new operation to develop a series of SVN power AOs including SVN copula power average(SVNCPA) operator, weighted SVN copula power average (WSVNCPA) operator, order weighted SVN copula power average (OWSVNCPA) operator and corresponding geometric operators;
3) To extend the proposed AOs to their generalized version namely GWSVNCPA operator, GOWSVNCPA operator,GWSVNCPG operator and GOWSVNCPG operator;
4) To set up an MADM approach with the weight information unknown based on the propounded AOs;
5) To confirm the validity and strength of propounded method through empirical application and comparative study.

For achieving the objective, the remainder of this investigation is allocated as follows: Section 2 succinctly looks back several fundamental notions of SVNS and ACC. Section 3 develops several novel operations for SVNN based on ACC. Section 4 presents several aggregation operators based on the newly operations and discusses some properties and particular examples. Section 5 extends the propounded operators in Section 4 to their generalized form. Section 6 develops an approach based on those presented operators for tackling decision making issues. Section 7 exhibits an empirical example to demonstrate the validity and legitimacy of the presented strategy. The article ends with several conclusions in Section 8.

II. PRELIMINARIES
In this part, several fundamental concepts of SVN and ACC are succinctly reviewed as the preparation of the following research.

A. SVNN
Definition 1 [30]: Let $\mathcal{X}$ is a finite universe of discourse. A single-valued neutrosophic set $\mathfrak{A}$ in $\mathcal{X}$ is defined as below:

$$\mathfrak{A} = \{\langle x, (\mathcal{T}_{\mathfrak{A}}(x), \mathcal{I}_{\mathfrak{A}}(x), \mathcal{F}_{\mathfrak{A}}(x))\rangle \mid x \in \mathcal{X}\}, \qquad (1)$$

in which $\mathcal{T}_{\mathfrak{A}}(x) : \mathcal{X} \rightarrow [0, 1]$, $\mathcal{T}_{\mathfrak{A}}(x) : \mathcal{X} \rightarrow [0, 1]$ and $\mathcal{T}_{\mathfrak{A}}(x) : \mathcal{X} \rightarrow [0, 1]$ with the condition $0 \leq \mathcal{T}_{\mathfrak{A}}(x) + \mathcal{I}_{\mathfrak{A}}(x) + \mathcal{F}_{\mathfrak{A}}(x) \leq 3$, $\mathcal{T}_{\mathfrak{A}}(x), \mathcal{I}_{\mathfrak{A}}(x), \mathcal{F}_{\mathfrak{A}}(x)$ are called the truth-membership function, indeterminacy-membership function, falsity-membership function, respectively. For simplicity, $\alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F})$ is called a single-valued neutrosophic number and $\alpha^c = (\mathcal{F}, 1 - \mathcal{I}, \mathcal{T})$, Φ is a collection of all SVNNs.

B. POWER OPERATOR
Definition 2 [53]: For a family of real numbers $\mathfrak{a}(1, 2, \cdots, n)$, the power averaging operator is stated as below:

$$PA(\mathfrak{a}_1, \mathfrak{a}_2, \cdots, \mathfrak{a}_n) = \frac{\sum_{i=1}^{n}(1 + T(\mathfrak{a}_i))\mathfrak{a}_i}{\sum_{i=1}^{n}(1 + T(\mathfrak{a}_i))} \qquad (2)$$

where $T(\varepsilon_i) = \sum_{j=1, j\neq i}^{n} Sup(\varepsilon_i, \varepsilon_j)$, and $Sup(\varepsilon_i, \varepsilon_j)$ indicates the support degree of $\mathfrak{a}_i$ to $\mathfrak{a}_j$. The following properties are satisfied:

(1) $Sup(\mathfrak{a}_i, \mathfrak{a}_j) \in [0, 1]$;
(2) $Sup(\mathfrak{a}_i, \mathfrak{a}_j) = Sup(\mathfrak{a}_j, \mathfrak{a}_i)$;
(3) $Sup(\mathfrak{a}_i, \mathfrak{a}_j) \geq Sup(\mathfrak{a}_m, \mathfrak{a}_n)$ iff $\mid \mathfrak{a}_i - \mathfrak{a}_j \mid < \mid \mathfrak{a}_m - \mathfrak{a}_n \mid$.

C. ARCHIMEDEAN COPULA
Definition 3 [59]: A two-dimensional function $\mathfrak{C} : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a copula if it satisfies the following conditions:

$$(1) \mathfrak{C}(\mathfrak{u}, 1) = \mathfrak{C}(1, \mathfrak{u}) = \mathfrak{u};$$
$$(2) \mathfrak{C}(\mathfrak{u}, 0) = \mathfrak{C}(0, \mathfrak{u}) = 0;$$
$$(3) \mathfrak{C}(\mathfrak{u}_1, \mathfrak{v}_1) + \mathfrak{C}(\mathfrak{u}_2, \mathfrak{v}_2) \geq \mathfrak{C}(\mathfrak{u}_1, \mathfrak{v}_2) + \mathfrak{C}(\mathfrak{u}_2, \mathfrak{v}_1).$$

where $\mathfrak{u}, \mathfrak{u}_i, \mathfrak{v}_i \in [0, 1]$, $i = 1, 2$ with $\mathfrak{u}_1 \leq \mathfrak{u}_2$, $\mathfrak{v}_1 \leq \mathfrak{v}_2$.

Definition 4 [61]: A copula $\mathfrak{C} : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is said to be a Archimedean copula if there exist a continuous and strictly decreasing function ρ from $[0, 1]$ to $[0, \infty]$ with $\rho(1) = 0$, and function ϱ from $[0, \infty)$ to $[0, 1]$ with $\rho(1) = 0$ defined by

$$\varrho(m) = \begin{cases} \rho^{-1}(m), & m \in [0, \rho(0)]; \\ 0, & m \in [\rho(0), +\infty). \end{cases}$$

such that, for any $(\mathfrak{b}_1, \mathfrak{b}_2) \in [0, 1] \times [0, 1]$,

$$\mathfrak{C}(\mathfrak{b}_1, \mathfrak{b}_2) = \varrho(\rho(\mathfrak{b}_1) + \rho(\mathfrak{b}_2)). \qquad (3)$$

In light of the setting that if $\mathcal{C}$ is a strictly increasing on $[0, 1] \times [0, 1]$, then we can obtain $\rho(0) = +\infty$ and ϱ

TABLE 1. Archimedean copula.

Name	Generator $\xi(\mathfrak{t})$	Copula $\mathfrak{C}(a, b)$	Condition
Gumble	$(-\ln \mathfrak{t})^\sigma$	$e^{-((-\ln a)^\sigma + (-\ln b)^\sigma)^{\frac{1}{\sigma}}}$	$\sigma \geq 1$
Clayton	$\mathfrak{t}^{-\sigma} - 1$	$\left(a^{-\sigma} + b^{-\sigma} - 1\right)^{-\frac{1}{\sigma}}$	$\sigma \geq -1, \sigma \neq 0$
Frank	$\ln\left(\frac{e^{-\sigma \mathfrak{t}} - 1}{e^{-\sigma} - 1}\right)$	$-\frac{1}{\sigma} \ln\left(\frac{(e^{-\sigma a} - 1)(e^{-\sigma b} - 1)}{e^{-\sigma} - 1} + 1\right)$	$\sigma \neq 0$
Ali-Mikhail-Haq	$\ln\left(\frac{1 - \sigma(1 - \mathfrak{t})}{\mathfrak{t}}\right)$	$\frac{ab}{1 - \sigma(1-a)(1-b)}$	$\sigma \in [-1, 1)$
Joe	$-\ln\left(1 - (1-\mathfrak{t})^\sigma\right)$	$1 - ((1-a)^\sigma + (1-b)^\sigma - (1-a)^\sigma (1-b)^\sigma)^{\frac{1}{\sigma}}$	$\sigma \geq 1$

agrees with ρ^{-1} on $[0, +\infty]$. Genest *et al.* [63] originally developed the notion of Archimedean copula, then the Eq (3) is redefined as

$$\mathfrak{C}(\mathfrak{b}_1, \mathfrak{b}_2) = \rho^{-1}(\rho(\mathfrak{b}_1) + \rho(\mathfrak{b}_2)), \quad (4)$$

in which the function ρ is called as a strict generator and $\mathcal{C}$ is said to be a strict Archimedean copula, several common Archimedean copulas are displayed in Table 1.

What's more, Cherubini *et al.* [60] revealed the conception of co-copula.

Definition 5: Let $\mathfrak{C}: [0, 1] \times [0, 1] \rightarrow [0, 1]$ be a copula, then the co-copula is defined as

$$\mathfrak{C}^*(\mathfrak{b}_1, \mathfrak{b}_2) = 1 - \mathfrak{C}(1 - \mathfrak{b}_1, 1 - \mathfrak{b}_2). \quad (5)$$

III. OPERATIONS OF SVNN BASED ON COPULA AND CO-COPULA

In this section, we succinctly introduce several novel operations for SVNNs based on ACC and explore related properties and particular instances.

Inspired by the investigation about generalized intersection and generalized union of SVNNs completed by Smarandache [30]. We will redefine the definition of generalized intersection and generalized union of SVNNs based on the ACC.

Definition 6: Suppose $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2)$ be two SVNNs, then generalized intersection and generalized union operational are defined as below:

$$\alpha_1 \cap_{\mathfrak{C}, \mathfrak{C}^*} \alpha_2 = \left(\mathfrak{C}(\mathcal{T}_1, \mathcal{T}_2), \mathfrak{C}^*(\mathcal{I}_1, \mathcal{I}_2), \mathfrak{C}^*(\mathcal{F}_1, \mathcal{F}_2)\right), \quad (6)$$

$$\alpha_1 \cup_{\mathfrak{C}, \mathfrak{C}^*} \alpha_2 = \left(\mathfrak{C}^*(\mathcal{T}_1, \mathcal{T}_2), \mathfrak{C}(\mathcal{I}_1, \mathcal{I}_2), \mathfrak{C}(\mathcal{F}_1, \mathcal{F}_2)\right), \quad (7)$$

in which $\mathfrak{C}$ and $\mathfrak{C}^*$ denotes Archimedean copula and co-copula, respectively.

According to above-mentioned analysis, the operation laws of SVNNs on the basis of ACC can be depicted as follows:

$(1)\ \alpha_1 \oplus_{\mathfrak{C}} \alpha_2 = \left(\mathfrak{C}^*(\mathcal{T}_1, \mathcal{T}_2), \mathfrak{C}(\mathcal{I}_1, \mathcal{I}_2), \mathfrak{C}(\mathcal{F}_1, \mathcal{F}_2)\right) \quad (8)$

$$= \begin{pmatrix} 1 - \rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2)), \\ \rho^{-1}(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2)), \\ \rho^{-1}(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)) \end{pmatrix};$$

$(2)\ \alpha_1 \otimes_{\mathfrak{C}} \alpha_2 = \left(\mathfrak{C}(\mathcal{T}_1, \mathcal{T}_2), \mathfrak{C}^*(\mathcal{I}_1, \mathcal{I}_2), \mathfrak{C}^*(\mathcal{F}_1, \mathcal{F}_2)\right) \quad (9)$

$$= \begin{pmatrix} \rho^{-1}(\rho(\mathcal{T}_1) + \rho(\mathcal{T}_2)), \\ 1 - \rho^{-1}(\rho(1 - \mathcal{I}_1) + \rho(1 - \mathcal{I}_2)), \\ 1 - \rho^{-1}(\rho(1 - \mathcal{F}_1) + \rho(1 - \mathcal{F}_2)) \end{pmatrix};$$

$(3)\ \kappa\alpha = \left(1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T})), \rho^{-1}(\kappa\rho(\mathcal{I})), \rho^{-1}(\kappa\rho(\mathcal{F}))\right); \quad (10)$

$(4)\ \alpha^\kappa = \left(\rho^{-1}(\kappa\rho(\mathcal{T})), 1 - \rho^{-1}(\kappa\rho(1 - \mathcal{I})), \right.$

$$\left. 1 - \rho^{-1}(\kappa\rho(1 - \mathcal{F}))\right). \quad (11)$$

Theorem 1: For SVNNs $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2)$, then for $\kappa > 0$, $\alpha_1 \oplus_{\mathfrak{C}} \alpha_2$, $\alpha_1 \otimes_{\mathfrak{C}} \alpha_2$, $\kappa\alpha_1$ and α_1^κ are still SVNNs.

Proof: By utilizing fundament definition of ρ and SVNN, we have $0 \leq 1 - \mathcal{T} \leq 1$ and $0 = \rho(1) \leq \rho(1 - \mathcal{T}) \leq \rho(0) = +\infty$. Because ρ and ρ^{-1} are strictly decreasing function, then $0 = \rho^{-1}(+\infty) \leq \rho^{-1}(\kappa\rho(1 - \mathcal{T})) \leq \rho^{-1}(0) = 1$. Hence, $0 \leq 1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T})) \leq 1$. Analogously, we can prove $0 \leq \rho^{-1}(\kappa\rho(\mathcal{I})) \leq 1, 0 \leq \rho^{-1}(\kappa\rho(\mathcal{F})) \leq 1$. Accordingly, $0 \leq 1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T})) + \rho^{-1}(\kappa\rho(\mathcal{I})) + \rho^{-1}(\kappa\rho(\mathcal{F})) \leq 3$, i.e., $\kappa\alpha$ is a SVNN. Analogously, we can prove $\alpha_1 \oplus_{\mathfrak{C}} \alpha_2$, $\alpha_1 \otimes_{\mathfrak{C}} \alpha_2$ and α_1^κ are SVNNs.

In the next, we will discuss several relations of the presented operational laws.

Theorem 2: Let $\alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F})$, $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)$ $(i = 1, 2)$ are three SVNNs and $\kappa, \kappa_1, \kappa_2 \geq 0$, then

$(1)\ \alpha_1 \oplus_{\mathfrak{C}} \alpha_2 = \alpha_2 \oplus_{\mathfrak{C}} \alpha_1;$

$(2)\ \alpha_1 \otimes_{\mathfrak{C}} \alpha_2 = \alpha_2 \otimes_{\mathfrak{C}} \alpha_1;$

$(3)\ \kappa(\alpha_1 \oplus_{\mathfrak{C}} \alpha_2) = \kappa\alpha_1 \oplus_{\mathfrak{C}} \kappa\alpha_2;$

$(4)\ (\alpha_1 \otimes_{\mathfrak{C}} \alpha_2)^\kappa = \alpha_1^\kappa \otimes_{\mathfrak{C}} \alpha_2^\kappa;$

$(5)\ \kappa_1\alpha \oplus_{\mathfrak{C}} \kappa_2\alpha = (\kappa_1 + \kappa_2)\alpha;$

$(6)\ \alpha^{\kappa_1} \otimes_{\mathfrak{C}} \alpha^{\kappa_2} = \alpha^{\kappa_1 + \kappa_2}.$

For details of theorem proving, please refer to **Appendix A**.

Theorem 3: Let $\alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F})$, $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)$ $(i = 1, 2)$ are three SVNNs and $\kappa \geq 0$, then the following conclusions are valid:

$(1)\ (\alpha^c)^\kappa = (\kappa\alpha)^c;$

$(2)\ \kappa(\alpha^c) = (\alpha^\kappa)^c;$

$(3)\ \alpha_1^c \oplus_{\mathfrak{C}} \alpha_2^c = (\alpha_1 \otimes_{\mathfrak{C}} \alpha_2)^c;$

$(4)\ \alpha_1^c \otimes_{\mathfrak{C}} \alpha_2^c = (\alpha_1 \oplus_{\mathfrak{C}} \alpha_2)^c.$

where $\alpha^c = (\mathcal{F}, 1 - \mathcal{I}, \mathcal{T})$.

For details of theorem proving, please refer to **Appendix B**.

In what follows, we will explore several particular cases of operation laws for SVNNs with respect to the different form copulas.

Definition 7: Let $\alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F}), \alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)$ $(i = 1, 2)$ are three SVNNs and $\kappa \geq 0$, then the operation laws with respect to different generating function $\rho(t)$ are defined as follows.

For the details expression forms, please refer to **Appendix C**.

IV. THE SVN AGGREGATION OPERATORS BASED ON THE ACC

In this part, we shall develop several AOs based on ACC operations of SVNNs in Section 3.

A. SVNPA OPERATOR

Definition 8: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. The SVNPA operator is a mapping: $\Phi^n \to \Phi$ and

$$SVNPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = (\oplus_{\mathfrak{e}})_{i=1}^n \eta_i \alpha_i, \quad (12)$$

where $\eta_i = ((1 + \mathcal{R}_i)/(\sum_{i=1}^n (1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and $Sup(\alpha_i, \alpha_j)$ be symbol of the support for α_i from α_j.

Theorem 4: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by SVNCPA operator is still a SVNN and

$$
\begin{aligned}
&SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{e}})_{i=1}^n \eta_i \alpha_i \\
&= \left(1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(1 - \mathcal{T}_i)\right), \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{I}_i)\right), \right. \\
&\qquad \left. \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{F}_i)\right)\right).
\end{aligned}
\quad (13)
$$

For details of theorem proving, please refer to **Appendix D**.

In what follows, we shall investigate several anticipated theorems of SVNCPA operator.

Theorem 5 (Idempoyency): Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. If all $\alpha_i = \alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F})$, then

$$SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = \alpha.$$

Proof: Since $\alpha_i = \alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F})$ for all i, then we have

$$
\begin{aligned}
&SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{e}})_{i=1}^n \eta_i \alpha_i \\
&= \left(1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(1 - \mathcal{T}_i)\right), \right. \\
&\qquad \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{I}_i)\right), \\
&\qquad \left. \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{F}_i)\right) \right.
\end{aligned}
$$

$$
\begin{aligned}
&= \left(1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(1 - \mathcal{T})\right), \right. \\
&\qquad \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{I})\right), \\
&\qquad \left. \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{F})\right) \right) \\
&= \left(1 - \rho^{-1}(\rho(1 - \mathcal{T})), \rho^{-1}(\rho(\mathcal{I})), \rho^{-1}(\rho(\mathcal{F}))\right) \\
&= (\mathcal{T}, \mathcal{I}, \mathcal{F}) = \alpha.
\end{aligned}
$$

Theorem 6 (Monotonicity): Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)$ and $\beta_i = \left(\mathcal{T}_i', \mathcal{I}_i', \mathcal{F}_i'\right)(i = 1, 2, \cdots, n)$ are two collections of SVNNs. If $\mathcal{T}_i \leq \mathcal{T}_i', \mathcal{I}_i \geq \mathcal{I}_i', \mathcal{F}_i \geq \mathcal{F}_i'$ for all $(i = 1, 2, \cdots, n)$, then

$$SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \leq SVNCPA(\beta_1, \beta_2, \cdots, \beta_n)$$

Proof: Since

$$
\begin{aligned}
&SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{e}})_{i=1}^n \eta_i \alpha_i \\
&= \left(1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(1 - \mathcal{T}_i)\right), \right. \\
&\qquad \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{I}_i)\right), \\
&\qquad \left. \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{F}_i)\right) \right)
\end{aligned}
$$

and

$$
\begin{aligned}
&SVNCPA(\beta_1, \beta_2, \cdots, \beta_n) \\
&= (\oplus_{\mathfrak{e}})_{i=1}^n \eta_i \beta_i \\
&= \left(1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho\left(1 - \mathcal{T}_i'\right)\right), \right. \\
&\qquad \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho\left(\mathcal{I}_i'\right)\right), \\
&\qquad \left. \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho\left(\mathcal{F}_i'\right)\right) \right).
\end{aligned}
$$

In addition, $\mathcal{T}_i \leq \mathcal{T}_i', \mathcal{I}_i \geq \mathcal{I}_i', \mathcal{F}_i \geq \mathcal{F}_i'$ and $\rho(t)$ is strictly decreasing function, then we have

$$\rho(1 - \mathcal{T}_i) \leq \rho\left(1 - \mathcal{T}_i'\right), \rho(\mathcal{I}_i) \leq \rho\left(\mathcal{I}_i'\right) \text{ and } \rho(\mathcal{F}_i) \leq \rho\left(\mathcal{F}_i'\right).$$

Because $\rho^{-1}(t)$ is also strictly decreasing function, then we have

$$1 - \rho^{-1}(\rho(1 - \mathcal{T}_i)) \leq 1 - \rho^{-1}\left(\rho\left(1 - \mathcal{T}_i'\right)\right),$$

$$\rho^{-1}(\rho(\mathcal{I}_i)) \geq \rho^{-1}\left(\rho\left(\mathcal{I}_i'\right)\right)$$

$$\text{and } \rho^{-1}(\rho(\mathcal{F}_i)) \geq \rho^{-1}\left(\rho\left(\mathcal{F}_i'\right)\right).$$

Then $ACC - SVNWA(\alpha_1, \alpha_2, \cdots, \alpha_n) \leq ACC - SVNNWA(\beta_1, \beta_2, \cdots, \beta_n)$.

Theorem 7 (Boundedness): The SVNCPA operator is between the maximum and minimum operators, that is

$$min(\alpha, \alpha, \cdots, \alpha) \leq SVNNPA(\alpha_1, \alpha_2, \cdots, \alpha_n)$$
$$\leq min(\beta, \beta, \cdots, \beta).$$

Proof: According to Theorem 7, we can obtain

$$SVNCPA(\alpha, \alpha, \cdots, \alpha) = \alpha,$$
$$SVNCPA(\beta, \beta, \cdots, \beta) = \beta.$$

Then

$$\alpha \leq SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \leq \beta.$$

Accordingly,

$$min(\alpha, \alpha, \cdots, \alpha) \leq SVNNPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \leq min(\beta, \beta, \cdots, \beta).$$

Theorem 8 (Commutativity): Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)$ and $\beta_i = \left(\mathcal{T}_i', \mathcal{I}_i', \mathcal{F}_i'\right)(i = 1, 2, \cdots, n)$ are two collections of SVNNs. Then

$$SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = SVNCPA(\beta_1, \beta_2, \cdots, \beta_n).$$

Proof: Since $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)$ is any permutation of $\beta_i = \left(\mathcal{T}_i', \mathcal{I}_i', \mathcal{F}_i'\right)(i = 1, 2, \cdots, n)$, then we have

$$SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = SVNCPA(\beta_1, \beta_2, \cdots, \beta_n).$$

Theorem 9: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs and $\xi \geq 0$, then

$$SVNCPA(\xi\alpha_1, \xi\alpha_2, \cdots, \xi\alpha_n)$$
$$= \xi SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n).$$

Proof: Because
$$\xi\alpha_i = \left(1 - \rho^{-1}(\xi\rho(1 - \mathcal{T}_i)), \rho^{-1}(\xi\rho(\mathcal{I}_i)), \rho^{-1}(\xi\rho(\mathcal{F}_i))\right),$$
then

$$SVNCPA(\xi\alpha_1, \xi\alpha_2, \cdots, \xi\alpha_n)$$
$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho\left(\rho^{-1}(\xi\rho(1 - \mathcal{T}_i))\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho\left(\rho^{-1}(\xi\rho(\mathcal{I}_i))\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho\left(\rho^{-1}(\xi\rho(\mathcal{F}_i))\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\xi\rho(1 - \mathcal{T}_i)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\xi\rho(\mathcal{I}_i)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\xi\rho(\mathcal{F}_i)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\xi\sum_{i=1}^{n} \eta_i\rho(1 - \mathcal{T}_i)\right), \\ \rho^{-1}\left(\xi\sum_{i=1}^{n} \eta_i\rho(\mathcal{I}_i)\right), \\ \rho^{-1}\left(\xi\sum_{i=1}^{n} \eta_i\rho(\mathcal{F}_i)\right) \end{pmatrix}.$$

Besides,

$$\xi SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n)$$
$$= \begin{pmatrix} 1 - \rho^{-1}\left(\xi\rho\left(\rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho(1 - \mathcal{T}_i)\right)\right)\right), \\ \rho^{-1}\left(\xi\rho\left(\rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho(\mathcal{I}_i)\right)\right)\right), \\ \rho^{-1}\left(\xi\rho\left(\rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho(\mathcal{F}_i)\right)\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\xi\sum_{i=1}^{n} \eta_i\rho(1 - \mathcal{T}_i)\right), \\ \rho^{-1}\left(\xi\sum_{i=1}^{n} \eta_i\rho(\mathcal{I}_i)\right), \\ \rho^{-1}\left(\xi\sum_{i=1}^{n} \eta_i\rho(\mathcal{F}_i)\right) \end{pmatrix}.$$

Accordingly, SVNCPA $(\xi\alpha_1, \xi\alpha_2, \cdots, \xi\alpha_n) = \xi$ SVNCNWA $(\alpha_1, \alpha_2, \cdots, \alpha_n)$.

Theorem 10: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ with $w_i \geq 0$, $\sum_{i=1}^{n} = 1$. If $\gamma = (\mathcal{T}, \mathcal{I}, \mathcal{F})$ be a SVNN, then

$$SVNCPA(\alpha_1 \oplus_\mathfrak{c} \gamma, \alpha_2 \oplus_\mathfrak{c} \gamma, \cdots, \alpha_n \oplus_\mathfrak{c} \gamma)$$
$$= SVNNPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_\mathfrak{c} \gamma.$$

Proof: Since

$$\alpha_i \oplus_\mathfrak{c} \gamma = \begin{pmatrix} 1 - \rho^{-1}(\rho(1 - \mathcal{T}_i) + \rho(1 - \mathcal{T})), \\ \rho^{-1}(\rho(\mathcal{I}_i) + \rho(\mathcal{I})), \\ \rho^{-1}(\rho(\mathcal{F}_i) + \rho(\mathcal{F})) \end{pmatrix}.$$

Then

$$SVNCPA(\alpha_1 \oplus_\mathfrak{c} \gamma, \alpha_2 \oplus_\mathfrak{c} \gamma, \cdots, \alpha_n \oplus_\mathfrak{c} \gamma)$$
$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho\left(\rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}))\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho\left(\rho^{-1}(\rho(\mathcal{I}_1) + \rho(\mathcal{I}))\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n} \eta_i\rho\left(\rho^{-1}(\rho(\mathcal{F}_1) + \rho(\mathcal{F}))\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n}\eta_i(\rho(1-\mathcal{T}_1)+\rho(1-\mathcal{T}))\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i(\rho(\mathcal{I}_1)+\rho(\mathcal{I}))\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i(\rho(\mathcal{F}_1)+\rho(\mathcal{F}))\right) \end{pmatrix}.$$

In addition,

$SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_{\mathfrak{e}} \gamma$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho(1-\mathcal{T}_i)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho(\mathcal{I}_i)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho(\mathcal{F}_i)\right) \end{pmatrix} \oplus_c (\mathcal{T}, \mathcal{I}, \mathcal{F})$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\rho\left(\rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho(1-\mathcal{T}_i)\right)\right)+\rho(1-\mathcal{T})\right), \\ \rho^{-1}\left(\rho\left(\rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho(\mathcal{I}_i)\right)\right)+\rho(\mathcal{I})\right), \\ \rho^{-1}\left(\rho\left(\rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho(\mathcal{F}_i)\right)\right)+\rho(\mathcal{F})\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n}\eta_i(\rho(1-\mathcal{T}_1)+\rho(1-\mathcal{T}))\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i(\rho(\mathcal{I}_1)+\rho(\mathcal{I}))\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i(\rho(\mathcal{F}_1)+\rho(\mathcal{F}))\right) \end{pmatrix}.$$

Accordingly, $SVNCPA(\alpha_1 \oplus_{\mathfrak{e}} \gamma, \alpha_2 \oplus_{\mathfrak{e}} \gamma, \cdots, \alpha_n \oplus_{\mathfrak{e}} \gamma) = SVNCWA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_{\mathfrak{e}} \gamma$.

Theorem 11: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ with $w_i \geq 0$, $\sum_{i=1}^{n} = 1$. If $\xi \geq 0$ and $\gamma = (\mathcal{T}, \mathcal{I}, \mathcal{F})$ be a SVNN. Then,

$SVNCPA(\xi\alpha_1 \oplus_{\mathfrak{e}} \gamma, \xi\alpha_2 \oplus_{\mathfrak{e}} \gamma, \cdots, \xi\alpha_n \oplus_{\mathfrak{e}} \gamma)$
$$= \xi SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_c \gamma.$$

Proof: From the Theorem 12, we have

$SVNCPA(\xi\alpha_1 \oplus_{\mathfrak{e}} \gamma, \xi\alpha_2 \oplus_{\mathfrak{e}} \gamma, \cdots, \xi\alpha_n \oplus_{\mathfrak{e}} \gamma)$
$$= SVNCPA(\xi\alpha_1, \xi\alpha_2, \cdots, \xi\alpha_n) \oplus_{\mathfrak{e}} \gamma$$

Furthermore, according to Theorem 11, we have

$SVNCPA(\xi\alpha_1, \xi\alpha_2, \cdots, \xi\alpha_n)$
$$= \xi SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n).$$

Accordingly,

$SVNCPA(\xi\alpha_1 \oplus_{\mathfrak{e}} \gamma, \xi\alpha_2 \oplus_{\mathfrak{e}} \gamma, \cdots, \xi\alpha_n \oplus_{\mathfrak{e}} \gamma)$
$$= \xi SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_{\mathfrak{e}} \gamma.$$

which finishes the proof of Theorem 13.

Theorem 12: Let $\mathbb{A} = \{\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)|i = 1, 2, \cdots, n\}$ and $\mathbb{B} = \left\{\beta_i = \left(\mathcal{T}_i', \mathcal{I}_i', \mathcal{F}_i'\right)|i = 1, 2, \cdots, n\right\}$ are two collections of SVNNs, and $w = (w_1, w_2, \cdots, w_n)^T$ be the weight vector of $(\alpha_1, \alpha_2, \cdots, \alpha_n)$ with $w_i \geq 0$, $\sum_{i=1}^{n} = 1$. Then

$SVNCPA(\alpha_1 \oplus_{\mathfrak{e}} \beta_1, \alpha_1 \oplus_{\mathfrak{e}} \beta_1, \cdots, \alpha_n \oplus_{\mathfrak{e}} \beta_n)$
$$= SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_{\mathfrak{e}} SVNCWA(\beta_1, \beta_2, \cdots, \beta).$$

Proof: Since

$$\alpha_i \oplus_{\mathfrak{e}} \beta_i = \begin{pmatrix} 1 - \rho^{-1}\left(\rho(1-\mathcal{T}_i)+\rho\left(1-\mathcal{T}_i'\right)\right), \\ \rho^{-1}\left(\rho(\mathcal{I}_i)+\rho\left(\mathcal{I}_i'\right)\right), \\ \rho^{-1}\left(\rho(\mathcal{F}_i)+\rho\left(\mathcal{F}_i'\right)\right) \end{pmatrix}.$$

then we have

$SVNCPA(\alpha_1 \oplus_{\mathfrak{e}} \beta_1, \alpha_1 \oplus_{\mathfrak{e}} \beta_1, \cdots, \alpha_n \oplus_{\mathfrak{e}} \beta_n)$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho\left(\rho^{-1}\left(\rho(1-\mathcal{T}_i)+\rho\left(1-\mathcal{T}_i'\right)\right)\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho\left(\rho^{-1}\left(\rho(\mathcal{I}_i)+\rho\left(\mathcal{I}_i'\right)\right)\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\rho\left(\rho^{-1}\left(\rho(\mathcal{F}_i)+\rho\left(\mathcal{F}_i'\right)\right)\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\left(\rho(1-\mathcal{T}_i)+\rho\left(1-\mathcal{T}_i'\right)\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}\eta_i\left(\rho(\mathcal{I}_i)+\rho\left(\mathcal{I}_i'\right)\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^{n}w_i\left(\rho(\mathcal{F}_i)+\rho\left(\mathcal{F}_i'\right)\right)\right) \end{pmatrix}.$$

Besides, $SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_{\mathfrak{e}} SVNCPA(\beta_1, \beta_2, \cdots, \beta)$, as shown at the bottom of the next page. Hence, The Theorem 12 is proved.

In the following, we will discuss some particular SVN operators based on different generator $\rho(t)$.

Case 1: If $\rho(t) = (-\ln t)^{\sigma}$ with $\sigma \geq 1$, then we can obtain the Gumbel SVNPA (G-SVNCPA)operator,

$\mathcal{G} - SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n)$

$$= \begin{pmatrix} 1 - e^{-\left(\sum_{i=1}^{n}\eta_i(-\ln(1-\mathcal{T}_i)^{\sigma})\right)^{\frac{1}{\sigma}}}, \\ e^{-\left(\sum_{i=1}^{n}\eta_i(-\ln(\mathcal{I}_i)^{\sigma})\right)^{\frac{1}{\sigma}}}, \\ e^{-\left(\sum_{i=1}^{n}\eta_i(-\ln(\mathcal{F}_i)^{\sigma})\right)^{\frac{1}{\sigma}}} \end{pmatrix}. \qquad (14)$$

Case 2: If $\rho(t) = t^{-\sigma} - 1$ with $\sigma \geq 1$, then we can obtain the Clayton SVNPA (C-SVNPA) operator,

$$
\mathcal{C} - SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n)
$$

$$
= \left(\begin{array}{c} 1 - \left(\sum_{i=1}^{n} \eta_i \left((1 - \mathcal{T}_i)^{-\sigma} - 1 \right) + 1 \right)^{-\frac{1}{\sigma}}, \\ \left(\sum_{i=1}^{n} \eta_i \left(\mathcal{I}_i^{-\sigma} - 1 \right) + 1 \right)^{-\frac{1}{\sigma}}, \\ \left(\sum_{i=1}^{n} \eta_i \left(\mathcal{F}_i^{-\sigma} - 1 \right) + 1 \right)^{-\frac{1}{\sigma}} \end{array} \right). \quad (15)
$$

Case 3: If $\rho(t) = \ln\left(\frac{e^{-\sigma t} - 1}{e^{-\sigma} - 1} \right)$ with $\sigma \neq 0$, then we can obtain the Frank SVNCPA ($\mathcal{F}$-SVNCPA) operator,

$$
\mathcal{F} - SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n)
$$

$$
= \left(\begin{array}{c} 1 + \frac{1}{\sigma} \ln \left(1 + (e^{-\sigma} - 1) \prod_{i=1}^{n} \left(\frac{e^{-\sigma(1 - \mathcal{T}_i)} - 1}{e^{-\sigma} - 1} \right)^{\eta_i} \right), \\ -\frac{1}{\sigma} \ln \left(1 + (e^{-\sigma} - 1) \prod_{i=1}^{n} \left(\frac{e^{-\sigma \mathcal{I}_i} - 1}{e^{-\sigma} - 1} \right)^{\eta_i} \right), \\ -\frac{1}{\sigma} \ln \left(1 + (e^{-\sigma} - 1) \prod_{i=1}^{n} \left(\frac{e^{-\sigma \mathcal{F}_i} - 1}{e^{-\sigma} - 1} \right)^{\eta_i} \right) \end{array} \right).
$$
$$(16)$$

Case 4: If $\rho(t) = \ln\left(\frac{1 - \sigma(1 - t)}{t} \right)$ with $\sigma \in [-1, 1]$, then we can obtain the Ali-Mikhail-Haq SVNCPA (AMH-SVNCPA) operator,

$$
\mathcal{AHM} - SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n)
$$

$$
= \left(\begin{array}{c} \dfrac{\prod_{i=1}^{n}(1 - \sigma \mathcal{T}_i)^{\eta_i} - \prod_{i=1}^{n}(1 - \mathcal{T}_i)^{\eta_i}}{\prod_{i=1}^{n}(1 - \sigma \mathcal{T}_i)^{\eta_i} - \sigma \prod_{i=1}^{n}(1 - \mathcal{T}_i)^{\eta_i}}, \\[2em] \dfrac{(1 - \sigma) \prod_{i=1}^{n} \mathcal{I}_i^{\eta_i}}{\prod_{i=1}^{n}(1 - \sigma(1 - \mathcal{I}_i))^{\eta_i} - \sigma \prod_{i=1}^{n} \mathcal{I}^{\eta_i}}, \\[2em] \dfrac{(1 - \sigma) \prod_{i=1}^{n} \mathcal{F}_i^{\eta_i}}{\prod_{i=1}^{n}(1 - \sigma(1 - \mathcal{F}_i))^{\eta_i} - \sigma \prod_{i=1}^{n} \mathcal{F}^{\eta_i}} \end{array} \right). \quad (17)
$$

Case 5: If $\rho(t) = \ln\left(\frac{1 - \sigma(1-t)}{t} \right)$ with $\sigma \in [-1, 1]$, then we can obtain the Joe SVNPA (J-SVNCPA) operator,

$$
\mathcal{J} - SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n)
$$

$$
= \left(\begin{array}{c} 1 - \left(1 - \prod_{i=1}^{n}(1 - \mathcal{T}_i^{\sigma})^{\eta_i} \right)^{\frac{1}{\sigma}}, \\ \left(1 - \prod_{i=1}^{n}(1 - (1 - \mathcal{I}_i)^{\sigma})^{\eta_i} \right)^{\frac{1}{\sigma}}, \\ \left(1 - \prod_{i=1}^{n}(1 - (1 - \mathcal{F}_i)^{\sigma})^{\eta_i} \right)^{\frac{1}{\sigma}} \end{array} \right). \quad (18)
$$

$$
SVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \oplus_{\mathfrak{C}} SVNCPA(\beta_1, \beta_2, \cdots, \beta)
$$

$$
= \left(\begin{array}{c} 1 - \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho(1 - \mathcal{T}_i) \right), \\ \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho(\mathcal{I}_i) \right), \\ \rho^{-1} \left(\sum_{i=1}^{n} w_i \rho(\mathcal{F}_i) \right) \end{array} \right) \oplus_c \left(\begin{array}{c} 1 - \rho^{-1} \left(\sum_{i=1}^{n} w_i \rho\left(1 - \mathcal{T}_i'\right) \right), \\ \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho\left(\mathcal{I}_i'\right) \right), \\ \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho\left(\mathcal{F}_i'\right) \right) \end{array} \right)
$$

$$
= \left(\begin{array}{c} 1 - \rho^{-1} \left(\rho \left(\rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho(1 - \mathcal{T}_i) \right) \right) + \rho \left(\rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho\left(1 - \mathcal{T}_i'\right) \right) \right) \right), \\ \rho^{-1} \left(\rho \left(\rho^{-1} \left(\sum_{i=1}^{n} w_i \rho(\mathcal{I}_i) \right) \right) + \rho \left(\rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho\left(\mathcal{I}_i'\right) \right) \right) \right), \\ \rho^{-1} \left(\rho \left(\rho^{-1} \left(\sum_{i=1}^{n} \eta_i \rho(\mathcal{F}_i) \right) \right) + \rho \left(\rho^{-1} \left(\sum_{i=1}^{n} w_i \rho\left(\mathcal{F}_i'\right) \right) \right) \right) \end{array} \right)
$$

$$
= \left(\begin{array}{c} 1 - \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \left(\rho(1 - \mathcal{T}_i) + \rho\left(1 - \mathcal{T}_i'\right) \right) \right), \\ \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \left(\rho(\mathcal{I}_i) + \rho\left(\mathcal{I}_i'\right) \right) \right), \\ \rho^{-1} \left(\sum_{i=1}^{n} \eta_i \left(\rho(\mathcal{F}_i) + \rho\left(\mathcal{F}_i'\right) \right) \right) \end{array} \right).
$$

B. WEIGHTED AND OWSVNPA OPERATOR

Definition 9: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. WSVNCPA operator is a mapping: $\Phi^n \to \Phi$ and

$$WSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = (\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i, \qquad (19)$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^n \epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1$, $\sum_{i=1}^n \epsilon_i = 1$.

Theorem 13: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by WSVNPA operator is still a SVNN and

$$
\begin{aligned}
&WSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i \\
&= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho(1 - \mathcal{T}_i)\right), \\ \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho(\mathcal{I}_i)\right), \\ \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho(\mathcal{F}_i)\right) \end{pmatrix}.
\end{aligned} \qquad (20)
$$

The proof is similar to Theorem 4, so we omit it here.

Definition 10: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. OWSVNCPA operator is a mapping: $\Phi^n \to \Phi$ and

$$OWSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = (\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_{\varsigma(i)}, \qquad (21)$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^n \epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1$, $\sum_{i=1}^n \epsilon_i = 1$, and ς is permutation of $(1, 2, \cdots, n)$ such that $\alpha_{\varsigma(i-1)} \leq \alpha_{\varsigma(i)}$ for $i = 2, 3, \cdots, n$.

Theorem 14: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by OWSVNCPA operator is still a SVNNs and

$$
\begin{aligned}
&OWSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i \\
&= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho\left(1 - \mathcal{T}_{\varsigma(i)}\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho\left(\mathcal{I}_{\varsigma(i)}\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho\left(\mathcal{F}_{\varsigma(i)}\right)\right) \end{pmatrix}.
\end{aligned} \qquad (22)
$$

The proof is similar to Theorem 4, so we omit it here.

Obviously, the presented WSVNCPA and OWSVNCPA operators still possess the theorems proposed in Theorem 5-12.

C. SVNPG OPERATOR

Definition 11: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. SVNCPG operator is a mapping: $\Phi^n \to \Phi$ and

$$SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = ((\otimes_{\mathfrak{c}})_{i=1}^n)\alpha_i^{\eta_i}, \qquad (23)$$

where $\eta_i = ((1+\mathcal{R}_i)/(\sum_{i=1}^n(1+\mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and $Sup(\alpha_i, \alpha_j)$ be symbol of the support for α_i from α_j.

Theorem 15: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by SVNCPA operator is still a SVNN and

$$
\begin{aligned}
&SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= ((\otimes_{\mathfrak{c}})_{i=1}^n)\alpha_i^{\eta_i} \\
&= \begin{pmatrix} \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(\mathcal{T}_i)\right), \\ 1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(1 - \mathcal{I}_i)\right), \\ 1 - \rho^{-1}\left(\sum_{i=1}^n \eta_i \rho(1 - \mathcal{F}_i)\right) \end{pmatrix}.
\end{aligned} \qquad (24)
$$

The proof is similar to Theorem 4, we omit it here.

On the basis of Theorem 15, it can prove that the following theorems are contented by the presented SVNPG operator.

Theorem 16: Let $\alpha_i, \beta_i(i = 1, 2, \cdots, n)$ be two families of SVNNs. $\xi \geq 0$ and $\gamma = (\mathcal{T}, \mathcal{I}, \mathcal{F})$, then we have

(T1) If $\alpha_i = \alpha = (\mathcal{T}, \mathcal{I}, \mathcal{F})$, then $SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = \alpha$;

(T2) $SVNCPG(\xi\alpha_1, \xi\alpha_2, \cdots, \xi\alpha_n) = \xi SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n)$;

(T3) $SVNCPG(\alpha_1 \otimes_c \gamma, \alpha_2 \otimes_c \gamma, \cdots, \alpha_n \otimes_c \gamma) = SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \otimes_c \gamma$;

(T4) $SVNCPG(\xi\alpha_1 \otimes_c \gamma, \xi\alpha_2 \otimes_c \gamma, \cdots, \xi\alpha_n \otimes_c \gamma) = \xi SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \otimes_c \gamma$;

(T5) $SVNCPG(\alpha_1 \otimes_c \beta_1, \alpha_1 \otimes_c \beta_1, \cdots, \alpha_n \otimes_c \beta_n) = SVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \otimes_c SVNCPG(\beta_1, \beta_2, \cdots, \beta)$.

In the following, we will discuss several particular SVNPG operators based on different generator $\rho(t)$.

Case 6: If $\rho(t) = (-\ln t)^\sigma$ with $\sigma \geq 1$, then we can obtain the Gumbel SVNPA (G-SVNCPG)operator,

$$
\begin{aligned}
&G - SVNPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= \begin{pmatrix} e^{-\left(\sum_{i=1}^n \eta_i(-\ln(\mathcal{T}_i)^\sigma)\right)^{\frac{1}{\sigma}}}, \\ 1 - e^{-\left(\sum_{i=1}^n \eta_i(-\ln(1-\mathcal{I}_i)^\sigma)\right)^{\frac{1}{\sigma}}}, \\ 1 - e^{-\left(\sum_{i=1}^n \eta_i(-\ln(1-\mathcal{F}_i)^\sigma)\right)^{\frac{1}{\sigma}}} \end{pmatrix}.
\end{aligned} \qquad (25)
$$

Case 7: If $\rho(t) = t^{-\sigma} - 1$ with $\sigma \geq 1$, then we can obtain the Clayton SVNCPA (C-SVNCPG)operator,

$$
\begin{aligned}
&C - SVNPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= \begin{pmatrix} \left(\sum_{i=1}^{n} \eta_i(\mathcal{T}_i^{-\sigma} - 1) + 1\right)^{-\frac{1}{\sigma}}, \\ \left(1 - \left(\sum_{i=1}^{n} \eta_i((1 - \mathcal{I}_i)^{-\sigma} - 1) + 1\right)^{-\frac{1}{\sigma}}\right), \\ \left(1 - \left(\sum_{i=1}^{n} \eta_i((1 - \mathcal{F}_i)^{-\sigma} - 1) + 1\right)^{-\frac{1}{\sigma}}\right) \end{pmatrix}.
\end{aligned}
\tag{26}
$$

Case 8: If $\rho(t) = \ln\left(\frac{e^{-\sigma t} - 1}{e^{-\sigma} - 1}\right)$ with $\sigma \neq 0$, then we can obtain the Frank SVNCPA (F-SVNCPG)operator,

$$
\begin{aligned}
&F - SVNPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= \begin{pmatrix} -\frac{1}{\sigma} \ln\left(1 + (e^{-\sigma} - 1)\prod_{i=1}^{n}\left(\frac{e^{-\sigma \mathcal{T}_i} - 1}{e^{-\sigma} - 1}\right)^{\eta_i}\right) \\ 1 + \frac{1}{\sigma} \ln\left(1 + (e^{-\sigma} - 1)\prod_{i=1}^{n}\left(\frac{e^{-\sigma(1-\mathcal{I}_i)} - 1}{e^{-\sigma} - 1}\right)^{\eta_i}\right), \\ 1 + \frac{1}{\sigma} \ln\left(1 + (e^{-\sigma} - 1)\prod_{i=1}^{n}\left(\frac{e^{-\sigma(1-\mathcal{F}_i)} - 1}{e^{-\sigma} - 1}\right)^{\eta_i}\right) \end{pmatrix}.
\end{aligned}
\tag{27}
$$

Case 9: If $\rho(t) = \ln\left(\frac{1 - \sigma(1-t)}{t}\right)$ with $\sigma \in [-1, 1]$, then we can obtain the Ali-Mikhail-Haq SVNCPA (AMH-SVNCPG)operator,

$$
\begin{aligned}
&AHM - SVNPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= \begin{pmatrix} \frac{(1-\sigma)\prod_{i=1}^{n}\mathcal{T}_i^{\eta_i}}{\prod_{i=1}^{n}(1 - \sigma(1 - \mathcal{T}_i))^{\eta_i} - \sigma\prod_{i=1}^{n}\mathcal{T}^{\eta_i}} \\ \frac{\prod_{i=1}^{n}(1 - \sigma\mathcal{I}_i)^{\eta_i} - \prod_{i=1}^{n}(1 - \mathcal{I}_i)^{\eta_i}}{\prod_{i=1}^{n}(1 - \sigma\mathcal{I}_i)^{\eta_i} - \sigma\prod_{i=1}^{n}(1 - \mathcal{I}_i)^{\eta_i}}, \\ \frac{\prod_{i=1}^{n}(1 - \sigma\mathcal{F}_i)^{\eta_i} - \prod_{i=1}^{n}(1 - \mathcal{F}_i)^{\eta_i}}{\prod_{i=1}^{n}(1 - \sigma\mathcal{F}_i)^{\eta_i} - \sigma\prod_{i=1}^{n}(1 - \mathcal{F}_i)^{\eta_i}} \end{pmatrix}.
\end{aligned}
\tag{28}
$$

Case 10: If $\rho(t) = \ln\left(\frac{1 - \sigma(1-t)}{t}\right)$ with $\sigma \in [-1, 1]$, then we can obtain the Joe SVNCPA (J-SVNCPG)

operator,

$$
\begin{aligned}
&J - SVNPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= \begin{pmatrix} \left(1 - \prod_{i=1}^{n}(1 - (1 - \mathcal{T}_i)^{\sigma})^{\eta_i}\right)^{\frac{1}{\sigma}} \\ 1 - \left(1 - \prod_{i=1}^{n}(1 - \mathcal{I}_i^{\sigma})^{\eta_i}\right)^{\frac{1}{\sigma}}, \\ 1 - \left(1 - \prod_{i=1}^{n}(1 - \mathcal{F}_i^{\sigma})^{\eta_i}\right)^{\frac{1}{\sigma}} \end{pmatrix}.
\end{aligned}
\tag{29}
$$

D. WEIGHTED AND OWSVNCPG OPERATOR

Definition 12: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. WSVNPG operator is a mapping: $\Phi^n \to \Phi$ and

$$
WSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = (\oplus_{\mathfrak{c}})_{i=1}^{n}(\alpha_i)^{\vartheta_i}, \tag{30}
$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^{n}\epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1, \sum_{i=1}^{n}\epsilon_i = 1$.

Theorem 17: Let $\mathbb{A} = \{\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)|i = 1, 2, \cdots, n\}$ be a collection of SVNNs, then the fusion value produced by WSVNCPG operator is still a SVNN and

$$
\begin{aligned}
&WSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{c}})_{i=1}^{n}(\alpha_i)^{\vartheta_i} \\
&= \begin{pmatrix} \rho^{-1}\left(\sum_{i=1}^{n}\vartheta_i\rho(\mathcal{T}_i)\right), \\ 1 - \rho^{-1}\left(\sum_{i=1}^{n}\vartheta_i\rho(1 - \mathcal{I}_i)\right), \\ 1 - \rho^{-1}\left(\sum_{i=1}^{n}\vartheta_i\rho(1 - \mathcal{F}_i)\right) \end{pmatrix}.
\end{aligned}
\tag{31}
$$

Definition 13: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. OWSVNCPA operator is a mapping: $\Phi^n \to \Phi$ and

$$
OWSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = (\oplus_{\mathfrak{c}})_{i=1}^{n}\vartheta_i\alpha_{\varsigma(i)}, \tag{32}
$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^{n}\epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1, \sum_{i=1}^{n}\epsilon_i = 1$, and ς is permutation of $(1, 2, \cdots, n)$ such that $\alpha_{\varsigma(i-1)} \leq \alpha_{\varsigma(i)}$ for $i = 2, 3, \cdots, n$.

Theorem 18: Let $\mathbb{A} = \{\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)|i = 1, 2, \cdots, n\}$ be a collection of SVNNs, then the fusion value produced by

OWSVNCPG operator is still a SVNN and

$$
\begin{aligned}
&OWSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) \\
&= (\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i \\
&= \begin{pmatrix} \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho\left(\mathcal{T}_{\varsigma(i)}\right)\right), \\ 1 - \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho\left(1 - \mathcal{I}_{\varsigma(i)}\right)\right), \\ 1 - \rho^{-1}\left(\sum_{i=1}^n \vartheta_i \rho\left(1 - \mathcal{F}_{\varsigma(i)}\right)\right) \end{pmatrix}.
\end{aligned} \tag{33}
$$

Obviously, the presented WSVNCPA and OWSVNCPA operators still possess the theorems proposed in Theorem 16.

V. GENERALIZED WEIGHTED SVN PA OPERATORS

In this part, several generalized power operators including GWSVNPA, GOWSVNPA, GWSVNPG and GOWSVNPA operators are developed for aggregating the set of SVNNs Φ.

Definition 14: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. GWSVNPA operator is a mapping: $\Phi^n \to \Phi$ and

$$
GWSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = \left((\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i^q\right)^{1/q}, \tag{34}
$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^n \epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1, \sum_{i=1}^n \epsilon_i = 1$.

Remark 1: Particularly, if we assign $\chi = 1$, then the GWSVNCPA operator will degenerate into WSVNCPA operator.

Theorem 19: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by

WSVNCPA operator is still a SVNN and (35), as shown at the bottom of this page.

Proof: Since

$$
\alpha_i^q = \left(\rho^{-1}(q\rho(\mathcal{T})), 1 - \rho^{-1}(q\rho(1 - \mathcal{I})), 1 - \rho^{-1}(q\rho(1 - \mathcal{F}))\right),
$$

we can acquire

$$
\vartheta_i \alpha_i^q = \begin{pmatrix} 1 - \rho^{-1}\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(\mathcal{T}_i))\right)\right), \\ \rho^{-1}\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{I}_i))\right)\right), \\ \rho^{-1}\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{F}_i))\right)\right) \end{pmatrix}.
$$

and

$$
\begin{aligned}
&(\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i^q \\
&= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(\mathcal{T}_i))\right)\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{I}_i))\right)\right)\right), \\ \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{F}_i))\right)\right)\right) \end{pmatrix}.
\end{aligned}
$$

Accordingly, we have $\left((\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i^q\right)^{1/q}$, as shown at the bottom of this page.

Definition 15: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. GOWSVNCPA operator is a mapping: $\Phi^n \to \Phi$ and

$$
GOWSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = \left((\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_{\varsigma(i)}^q\right)^{1/q}, \tag{36}
$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^n \epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1, j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq$

$$
GWSVNPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = \begin{pmatrix} \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(\mathcal{T}_i))\right)\right)\right)\right)\right), \\ 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{I}_i))\right)\right)\right)\right)\right), \\ 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{F}_i))\right)\right)\right)\right)\right) \end{pmatrix}. \tag{35}
$$

$$
\left((\oplus_{\mathfrak{c}})_{i=1}^n \vartheta_i \alpha_i^q\right)^{1/q} = \begin{pmatrix} \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(\mathcal{T}_i))\right)\right)\right)\right)\right), \\ 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{I}_i))\right)\right)\right)\right)\right), \\ 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n\left(\vartheta_i \rho\left(1 - \rho^{-1}(q\rho(1 - \mathcal{F}_i))\right)\right)\right)\right)\right) \end{pmatrix}.
$$

$\epsilon_i \leq 1$, $\sum_{i=1}^n \epsilon_i = 1$, and ς is permutation of $(1, 2, \cdots, n)$ such that $\alpha_{\varsigma(i-1)} \leq \alpha_{\varsigma(i)}$ for $i = 2, 3, \cdots, n$.

Remark 2: Particularly, if we assign $\chi = 1$, then the GOWSVNCPA operator will degenerate into OWSVNCPA operator.

Theorem 20: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by WSVNCPA operator is still a SVNN and (37), as shown at the bottom of this page.

Definition 16: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. GWSVNCPG operator is a mapping: $\Phi^n \to \Phi$ and

$$GWSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = \frac{1}{q}\big((\oplus_{\mathfrak{C}})_{i=1}^n \vartheta_i \alpha_i^q\big), \quad (38)$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^n \epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1,j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1$, $\sum_{i=1}^n \epsilon_i = 1$.

Remark 3: Particularly, if we assign $\chi = 1$, then the GWSVNCPG operator will degenerate into WSVNCPG operator.

Theorem 21: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by WSVNPG operator is still a SVNN and (39), as shown at the bottom of this page.

Definition 17: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs. GWSVNCPG operator is a mapping: $\Phi^n \to \Phi$ and

$$GOWSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = \frac{1}{q}\big((\oplus_{\mathfrak{C}})_{i=1}^n \vartheta_i \alpha_{\varsigma(i)}^q\big), \quad (40)$$

where $\vartheta_i = (\epsilon_i(1 + \mathcal{R}_i)/(\sum_{i=1}^n \epsilon_i(1 + \mathcal{R}_i)))$, $\mathcal{R}_i = \sum_{j=1,j\neq i} Sup(\alpha_i, \alpha_j)$ and ϵ be the weight vector of α_i with $0 \leq \epsilon_i \leq 1$, $\sum_{i=1}^n \epsilon_i = 1$, and ς is permutation of $(1, 2, \cdots, n)$ such that $\alpha_{\varsigma(i-1)} \leq \alpha_{\varsigma(i)}$ for $i = 2, 3, \cdots, n$.

Remark 4: Particularly, if we assign $\chi = 1$, then the GOWSVNCPG operator will degenerate into OWSVNCPG operator.

Theorem 22: Let $\alpha_i = (\mathcal{T}_i, \mathcal{I}_i, \mathcal{F}_i)(i = 1, 2, \cdots, n)$ be a family of SVNNs, then the fusion value produced by GOWSVNCPG operator is still a SVNN and (41), as shown at the bottom of the next page.

Obviously, the presented GWSVNCPA, GOWSVNCPA, GWSVNCPG and GOWSVNCPG operators also have the same theorems which WSVNCPA and WSVNCPG operators have.

VI. THE APPLICATION TO DEVELOPED MADM APPROACH

Under this part, an approach is proposed to cope with MADM issues by utilizing the developed aggregation operator under sing-valued neutrosophic set context.

Assuming that $\Theta = \{\Theta_1, \Theta_2, \cdots, \Theta_m\}$ is a set of alternatives. $\mathcal{K} = \{\mathcal{K}_1, \mathcal{K}_2, \cdots, \mathcal{K}_n\}$ is a collection of attributes with the weight vector being $\tau = (\tau_1, \tau_2, \cdots, \tau_n)^T$ with $\tau_j \in [0, 1](j = 1, 2, \cdots, n)$ and $\sum_{j=1}^n \tau_j = 1$. A manager assesses those alternatives and provides assessment information under the attributes $\mathcal{K}_j$ by the form of SVNNs $g_{ij} = (\mathcal{T}_{ij}, \mathcal{I}_{ij}, \mathcal{F}_{ij})$, where $\mathcal{T}_{ij}, \mathcal{I}_{ij}, \mathcal{F}_{ij} \in [0, 1]$ and $0 \leq \mathcal{T}_{ij} + \mathcal{I}_{ij} + \mathcal{F}_{ij} \leq 3$. The assessment information of all alternatives $\Im$ are written as a decision-matrix $\mathcal{G}$, as shown at the bottom of the next page.

$$GOWSVNCPA(\alpha_1, \alpha_2, \cdots, \alpha_n) = \left((\oplus_{\mathfrak{C}})_{i=1}^n \vartheta_i \alpha_{\varsigma(i)}^q\right)^{1/q}$$
$$= \begin{pmatrix} \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n \left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(\mathcal{T}_{\varsigma(i)})\right)\right)\right)\right)\right)\right), \\ 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n \left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(1-\mathcal{I}_{\varsigma(i)})\right)\right)\right)\right)\right)\right), \\ 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n \left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(1-\mathcal{F}_{\varsigma(i)})\right)\right)\right)\right)\right)\right) \end{pmatrix}. \quad (37)$$

$$GWSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = \frac{1}{q}\left((\oplus_{\mathfrak{C}})_{i=1}^n \vartheta_i \alpha_i^q\right)$$
$$= \begin{pmatrix} 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n \left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(1 - \mathcal{I}_i)\right)\right)\right)\right)\right)\right), \\ \left(\rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n \left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(\mathcal{T}_i)\right)\right)\right)\right)\right)\right)\right), \\ \left(\rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^n \left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(\mathcal{T}_i)\right)\right)\right)\right)\right)\right)\right) \end{pmatrix}. \quad (39)$$

Aiming at the above-mentioned MADM issues, we design an approach on the basis of the presented aggregation operators and entropy weight method to cope with it in next section

A. THE SOLVING OF ATTRIBUTES WEIGHTS

Entropy notion was originally brought forward into information theory by Shannon, and it has been wide-ranging used in economic development, knowledge management and other domains. In information theory, entropy is a measure of uncertainty. The greater the uncertainty is, the greater the entropy is and the more information it possesses; the smaller the uncertainty is, the smaller the entropy is and the less information it possesses. As a valid tool to determine the weights of attributes, entropy measures are employed in varies decision problems under different evaluation contexts. Due to the influence of knowledge level, personal experience and decision setting, the attributes weights provided by decision makers are frequently subjective, which will make the finally decision resultant ambiguous. Accordingly, aiming at the decision problems with completely unknown attribute weight, the necessity of weight determination is bound to attract attention for us. In the following, an weight determination algorithm based on entropy measure under the SVN context.

Step i: Establish the score matrix $\mathcal{S} = \left(\mathcal{S}(g_{ij})\right)_{m \times n}$ of $\mathcal{Q} = \left(g_{ij}\right)_{m \times n}$

$$\mathcal{S} = \left(\mathcal{S}(g_{ij})\right)_{m \times n} = \begin{pmatrix} \mathcal{S}(g_{11}) & \mathcal{S}(g_{12}) & \cdots & \mathcal{S}(g_{1n}) \\ \mathcal{S}(g_{21}) & \mathcal{S}(g_{22}) & \cdots & \mathcal{S}(g_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ \mathcal{S}(g_{m1}) & \mathcal{S}(g_{m2}) & \cdots & \mathcal{S}(g_{mn}) \end{pmatrix}$$

Step ii: Normalizing the matrix $\mathcal{S}$ as:

$$\widetilde{\mathcal{S}} = \left(\widetilde{\mathcal{S}(g_{ij})}\right)_{m \times n} = \begin{pmatrix} \widetilde{\mathcal{S}(g_{11})} & \widetilde{\mathcal{S}(g_{12})} & \cdots & \widetilde{\mathcal{S}(g_{1n})} \\ \widetilde{\mathcal{S}(g_{21})} & \widetilde{\mathcal{S}(g_{22})} & \cdots & \widetilde{\mathcal{S}(g_{2n})} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{\mathcal{S}(g_{m1})} & \widetilde{\mathcal{S}(g_{m2})} & \cdots & \widetilde{\mathcal{S}(g_{mn})} \end{pmatrix}$$

where $\widetilde{\mathcal{S}(g_{ij})} = \frac{\mathcal{S}(g_{ij})}{\sum_{i=1}^{m} \mathcal{S}(g_{ij})}; (i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$.

Step iii: Obtain the attribute weights $\tau_j(j = 1, 2, \cdots, n)$ by

$$\tau_j = \frac{1 - E_j}{\sum_{j=1}^{n}\left(1 - E_j\right)} = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j}, \qquad (42)$$

where $E_j = -\frac{1}{\ln m} \sum_{i=1}^{m} \widetilde{\mathcal{S}(g_{ij})} \cdot \ln \widetilde{\mathcal{S}(g_{ij})}$.

B. SVN MADM APPROACH

Step 1: Normalize the decision matrix $\mathcal{G} = (g_{ij})_{m \times n}$ to $\tilde{\mathcal{G}} = (\tilde{g}_{ij})_{m \times n}$ by the following method.

$$\tilde{g}_{ij} = \left(\tilde{\mathcal{T}}_{ij}, \tilde{\mathcal{I}}_{ij}, \tilde{\mathcal{F}}_{ij}\right) \qquad (43)$$
$$= \begin{cases} \left(\mathcal{T}_{ij}, \mathcal{I}_{ij}, \mathcal{F}_{ij}\right), & \mathcal{K}_j \text{ is benefit type} \\ \left(\mathcal{F}_{ij}, 1 - \mathcal{I}_{ij}, \mathcal{T}_{ij}\right), & \mathcal{K}_j \text{ is cost type}. \end{cases}$$

Step 2: Determinate the attribute weights $\tau_j(j = 1, 2, \cdots, n)$ by the following formula;

$$\tau_j = \frac{1 - E_j}{\sum_{j=1}^{n}\left(1 - E_j\right)} = \frac{1 - E_j}{n - \sum_{j=1}^{n} E_j}, \qquad (44)$$

where $E_j = -\frac{1}{\ln m} \sum_{i=1}^{m} \widetilde{\mathcal{S}(g_{ij})} \cdot \ln \widetilde{\mathcal{S}(g_{ij})}$.

$$GOWSVNCPG(\alpha_1, \alpha_2, \cdots, \alpha_n) = \frac{1}{q}\left((\oplus_{\mathfrak{c}})_{i=1}^{n} \vartheta_i \alpha_i^q\right)$$
$$= \begin{pmatrix} 1 - \rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^{n}\left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(1 - \mathcal{T}_{\varsigma(i)})\right)\right)\right)\right)\right)\right), \\ \left(\rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^{n}\left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(\mathcal{I}_{\varsigma(i)})\right)\right)\right)\right)\right)\right), \\ \left(\rho^{-1}\left(\frac{1}{q}\rho\left(1 - \rho^{-1}\left(\sum_{i=1}^{n}\left(\vartheta_i\rho\left(1 - \rho^{-1}\left(q\rho(\mathcal{F}_{\varsigma(i)})\right)\right)\right)\right)\right)\right) \end{pmatrix}. \qquad (41)$$

$$\mathcal{G} = \left(g_{ij}\right)_{m \times n}$$
$$= \begin{pmatrix} (\mathcal{T}_{11}, \mathcal{I}_{11}, \mathcal{F}_{11}) & (\mathcal{T}_{12}, \mathcal{I}_{12}, \mathcal{F}_{12}) & \cdots & (\mathcal{T}_{1n}, \mathcal{I}_{1n}, \mathcal{F}_{1n}) \\ (\mathcal{T}_{21}, \mathcal{I}_{21}, \mathcal{F}_{21}) & (\mathcal{T}_{22}, \mathcal{I}_{22}, \mathcal{F}_{22}) & \cdots & (\mathcal{T}_{2n}, \mathcal{I}_{2n}, \mathcal{F}_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mathcal{T}_{m1}, \mathcal{I}_{m1}, \mathcal{F}_{m1}) & (\mathcal{T}_{m2}, \mathcal{I}_{m2}, \mathcal{F}_{m2}) & \cdots & (\mathcal{T}_{mn}, \mathcal{I}_{mn}, \mathcal{F}_{mn}) \end{pmatrix}$$

Step 3: Compute the supports

$$Sup\left(\tilde{g}_{ij}, \tilde{g}_{i\ell}\right) = 1 - \hat{d}\left(\tilde{g}_{ij}, \tilde{g}_{i\ell}\right) \qquad (45)$$

$$i = 1, 2, \cdots, m; j, \quad \ell = 1, 2, \cdots, n; \ j \neq \ell.$$

where $\hat{d} = \frac{1}{3}\left(\mid \mathcal{T}_{ij} - \mathcal{T}_{i\ell} \mid + \mid \mathcal{I}_{ij} - \mathcal{I}_{i\ell} \mid + \mid \mathcal{F}_{ij} - \mathcal{F}_{i\ell} \mid\right)$ is the normalized Hamming distance.

Step 4: Compute the total support $\tilde{g}_{ij}$

$$T\left(\tilde{g}_{ij}\right) = \sum_{i=1, j\neq\ell}^{n} Sup\left(\tilde{g}_{ij}, \tilde{g}_{i\ell}\right). \qquad (46)$$

Step 5: Acquire the weight ϖ_{ij} corresponding assessment value $\tilde{g}_{ij}$

$$\varpi_{ij} = \frac{\tau_j\left(1 + T\left(\tilde{g}_{ij}\right)\right)}{\sum\limits_{j=1}^{n}\left(\tau_j\left(1 + T\left(\tilde{g}_{ij}\right)\right)\right)}. \qquad (47)$$

where $\varpi_j \in [0, 1](j = 1, 2, \cdots, n)$ and $\sum_{i=j}^{n} \varpi_j = 1$.

Step 6: Fuse the overall comprehensive assessment value g_i for alternative $\mathcal{H}_i$ by ACC-SVNPWA operator and ACC-SVNPWG operator

$$\tilde{g}_i = WSVNCPA(\tilde{g}_{i1}, \tilde{g}_{i2}, \cdots, \tilde{g}_{in}) \qquad (48)$$

Or

$$\tilde{g}_i = WSVNCPG(\tilde{g}_{i1}, \tilde{g}_{i2}, \cdots, \tilde{g}_{in}) \qquad (49)$$

Step 7: Calculate the cosine similarity measure of the comprehensive assessment value.

Step 8: Acquire the order relation of alternatives and select optimal one(s).

VII. EMPIRICAL EXAMPLE

A. EXAMPLE APPLICATION

In order to solve the problem of poor Internet connection in remote areas, the effective implementation of goods and services tax (GST) operation plan should be effectively improved [35]. The government plans to work with state-owned enterprises to provide private mobile service providers. To this end, the Indian government has published a global bidding announcement in newspapers to select suppliers for these projects, and has considered five required attributes, namely, technical expertise ($\mathcal{A}_1$), service quality ($\mathcal{A}_2$), bandwidth ($\mathcal{A}_3$), Internet speed ($\mathcal{A}_4$) and customer service ($\mathcal{A}_5$). These importance attributes are considered completely unknown. The five suppliers (alternatives) preparing

for bidding are $\Theta_i(i = 1, 2, \cdots, 5)$. The assessment information provided by managers in the form of SVNN are listed in the bottom of this page.

Then, the objective of government is to choose the optimal supplier to provide the best Internet services for its citizens. The optimal selection process by utilizing the presented approach is summarized below.

Step 1: The normalization is omitted because all attributes are considered as the benefit type.

Step 2: Computing the attributes weight by the Eq (44): $\tau_1 = 0.2292$, $\tau_2 = 0.2499$, $\tau_3 = 0.1348$, $\tau_4 = 0.1958$, $\tau_5 = 0.1903$.

Step 3: Compute the supports $Sup\left(\tilde{g}_{ij}, \tilde{g}_{i\ell}\right)$

$$Sup\left(\tilde{g}_{1j}, \tilde{g}_{1\ell}\right) = \begin{pmatrix} 0.0000 & 0.9833 & 0.8000 & 0.9000 & 0.9333 \\ 0.9833 & 0.0000 & 0.8000 & 0.9000 & 0.8667 \\ 0.8000 & 0.8000 & 0.0000 & 0.9000 & 0.8667 \\ 0.9000 & 0.9000 & 0.9000 & 0.0000 & 0.9667 \\ 0.9333 & 0.8667 & 0.8667 & 0.9667 & 0.0000 \end{pmatrix},$$

$$Sup\left(\tilde{g}_{2j}, \tilde{g}_{2\ell}\right) = \begin{pmatrix} 0.0000 & 0.9667 & 0.8667 & 0.8333 & 0.9667 \\ 0.9667 & 0.0000 & 0.9000 & 0.8667 & 0.9333 \\ 0.8667 & 0.9000 & 0.0000 & 0.9667 & 0.9000 \\ 0.8333 & 0.8667 & 0.9667 & 0.0000 & 0.8667 \\ 0.9667 & 0.9333 & 0.9000 & 0.8667 & 0.0000 \end{pmatrix},$$

$$Sup\left(\tilde{g}_{3j}, \tilde{g}_{3\ell}\right) = \begin{pmatrix} 0.0000 & 0.9333 & 0.8333 & 0.9000 & 0.9000 \\ 0.9333 & 0.0000 & 0.9000 & 0.9000 & 0.9000 \\ 0.8333 & 0.9000 & 0.0000 & 0.9333 & 0.8667 \\ 0.9000 & 0.9000 & 0.9333 & 0.0000 & 0.8667 \\ 0.9000 & 0.9000 & 0.8667 & 0.8667 & 0.0000 \end{pmatrix},$$

$$Sup\left(\tilde{g}_{4j}, \tilde{g}_{4\ell}\right) = \begin{pmatrix} 0.0000 & 0.9333 & 0.8333 & 0.9000 & 0.9000 \\ 0.9833 & 0.0000 & 0.8000 & 0.9000 & 0.8667 \\ 0.9333 & 0.8000 & 0.0000 & 0.9000 & 0.8667 \\ 0.9000 & 0.9000 & 0.9000 & 0.0000 & 0.9667 \\ 0.9000 & 0.8667 & 0.8667 & 0.9667 & 0.0000 \end{pmatrix},$$

$$Sup\left(\tilde{g}_{5j}, \tilde{g}_{5\ell}\right) = \begin{pmatrix} 0.0000 & 0.9667 & 0.8333 & 0.9000 & 0.7667 \\ 0.9667 & 0.0000 & 0.8000 & 0.9333 & 0.7333 \\ 0.8333 & 0.8000 & 0.0000 & 0.8667 & 0.9333 \\ 0.9000 & 0.9333 & 0.8667 & 0.0000 & 0.8000 \\ 0.7667 & 0.7333 & 0.9333 & 0.8000 & 0.0000 \end{pmatrix}.$$

$$\mathcal{Q} = \left(g_{ij}\right)_{5\times5} = \begin{pmatrix} (0.5, 0.3, 0.4) & (0.5, 0.2, 0.3) & (0.2, 0.2, 0.6) & (0.3, 0.2, 0.4) & (0.3, 0.3, 0.4) \\ (0.7, 0.1, 0.3) & (0.7, 0.2, 0.3) & (0.6, 0.3, 0.2) & (0.6, 0.4, 0.2) & (0.7, 0.1, 0.2) \\ (0.5, 0.3, 0.4) & (0.6, 0.2, 0.4) & (0.6, 0.1, 0.2) & (0.5, 0.1, 0.3) & (0.6, 0.4, 0.3) \\ (0.7, 0.3, 0.4) & (0.7, 0.2, 0.2) & (0.4, 0.5, 0.2) & (0.5, 0.2, 0.2) & (0.4, 0.5, 0.4) \\ (0.4, 0.1, 0.3) & (0.5, 0.1, 0.2) & (0.4, 0.1, 0.5) & (0.4, 0.3, 0.6) & (0.3, 0.2, 0.4) \end{pmatrix}.$$

TABLE 2. The fusion result by WSVNPA operator.

	Gumbel($\sigma = 1$)	Clayton($\sigma = 1$)	Frank($\sigma = 1$)	Ali-Mikhail-Haq($\sigma = -1$)	Joe($\sigma = 1$)
Θ_1	(0.4011, 0.2325, 0.3920)	(0.4118, 0.1193, 0.1514)	(0.4048, 0.2320, 0.3900)	(0.2837, 0.2331, 0.3943)	(0.4011, 0.2325, 0.3920)
Θ_2	(0.6677, 0.1899, 0.2474)	(0.6707, 0.0992, 0.1231)	(0.6682, 0.1870, 0.2467)	(0.4001, 0.1931, 0.2481)	(0.6677, 0.1899, 0.2474)
Θ_3	(0.5565, 0.1871, 0.3288)	(0.5592, 0.0991, 0.1401)	(0.5572, 0.1847, 0.3273)	(0.3572, 0.1898, 0.3306)	(0.5565, 0.1871, 0.3288)
Θ_4	(0.5987, 0.2811, 0.2164)	(0.6170, 0.1282, 0.1147)	(0.6029, 0.2779, 0.2157)	(0.3724, 0.2849, 0.2172)	(0.5987, 0.2811, 0.2164)
Θ_5	(0.4179, 0.1364, 0.3474)	(0.4210, 0.0832, 0.1402)	(0.4189, 0.1353, 0.3420)	(0.2941, 0.1377, 0.3540)	(0.4179, 0.1364, 0.3474)

TABLE 3. The fusion result by WSVNPG operator.

	Gumbel($\sigma = 1$)	Clayton($\sigma = 1$)	Frank($\sigma = 1$)	Ali-Mikhail-Haq($\sigma = -1$)	Joe($\sigma = 1$)
Θ_1	(0.3965, 0.2387, 0.4093)	(0.1454, 0.2403, 0.4186)	(0.2899, 0.2394, 0.4119)	(0.3741, 0.1812, 0.2888)	(0.3695, 0.2387, 0.4093)
Θ_2	(0.6626, 0.2284, 0.2541)	(0.1814, 0.2377, 0.2557)	(0.5371, 0.2323, 0.2548)	(0.6635, 0.1538, 0.2020)	(0.6626, 0.2284, 0.2541)
Θ_3	(0.5515, 0.2196, 0.3416)	(0.1718, 0.2269, 0.3452)	(0.4712, 0.2227, 0.3430)	(0.5524, 0.1549, 0.2536)	(0.5515, 0.2196, 0.3416)
Θ_4	(0.5626, 0.3155, 0.2257)	(0.1415, 0.3289, 0.2292)	(0.4150, 0.3202, 0.2271)	(0.5689, 0.2136, 0.1830)	(0.5626, 0.3155, 0.2257)
Θ_5	(0.4102, 0.1577, 0.3972)	(0.1547, 0.1622, 0.4178)	(0.4361, 0.1597, 0.4037)	(0.4114, 0.1202, 0.2799)	(0.4102, 0.1577, 0.3972)

TABLE 4. The cosine similarity measure of alternatives based on diverse copulas.

Copulas	Operators	The Cosine similarity				
		Θ_1	Θ_2	Θ_3	Θ_4	Θ_5
Gumbel		0.6607	0.9061	0.8270	0.8603	0.7458
Clayton		0.9057	0.9733	0.9560	0.9633	0.9325
Frank	SVNPA	0.6656	0.9074	0.8291	0.8637	0.7515
Ali-Mikhail-Haq		0.5266	0.7863	0.6838	0.7027	0.6121
Joe		0.6607	0.9061	0.8270	0.8603	0.7458
Gumbel		0.6149	0.8888	0.8052	0.8233	0.6925
Clayton		0.2885	0.4610	0.3840	0.3933	0.3262
Frank	SVNPG	0.5185	0.8416	0.7552	0.7565	0.6122
Ali-Mikhail-Haq		0.7391	0.9340	0.8807	0.8964	0.8036
Joe		0.6149	0.8888	0.8052	0.8233	0.6925

Step 4: Compute the total support $T(\tilde{g}_{ij})$

$$T(\tilde{g}_{ij}) = \begin{pmatrix} 3.5667 & 3.5000 & 3.3667 & 3.6667 & 3.6333 \\ 3.6333 & 3.6667 & 3.6333 & 3.5333 & 3.6667 \\ 3.5667 & 3.6333 & 3.5333 & 3.6000 & 3.5333 \\ 3.4667 & 3.4333 & 3.4333 & 3.5000 & 3.2333 \\ 3.6000 & 3.4000 & 3.6000 & 3.3667 & 3.5000 \end{pmatrix}.$$

Step 5: Deduce the weight ϖ corresponding assessment value $\tilde{g}_{ij}$,

$$\varpi = (\varpi_{ij})_{5\times5}$$
$$= \begin{pmatrix} 0.2504 & 0.2690 & 0.1408 & 0.2186 & 0.1212 \\ 0.2498 & 0.2744 & 0.1469 & 0.2088 & 0.1200 \\ 0.2485 & 0.2749 & 0.1451 & 0.2138 & 0.1176 \\ 0.2513 & 0.2720 & 0.1467 & 0.2163 & 0.1136 \\ 0.2559 & 0.2668 & 0.1505 & 0.2075 & 0.1194 \end{pmatrix}.$$

Step 6: Fuse the overall comprehensive assessment value by WSVNPA and WSVNPG operator, the fused result are shown in Table 2 and Table 3.

Step 7: Computing the cosine similarity measure of the aggregation values, which are listed in Table 4.

Step 8: The order relation of alternatives can be concluded in the Table 5.

Furthermore, if we utilize other propounded operators including SVNPA, SVNPG, OWSVNPA and OWSVNPG on the basis of different copulas to cope with the example, the corresponding aggregation results deduced by different operators and cosine similarity of every alternatives $\Theta_i(i = 1, 2, 3, 4, 5)$ are summarized in Table 6. From it, we can derive that although the ranking of alternatives are a little diverse, the optimal programme is Θ_2.

B. SENSITIVITY ANALYSIS

Under this subsection, we will explore the effect of parameters σ and q on decision results.

In order to analyze the influence of parameter q on decision results, we select the Gumbel type copula to conduct an analysis with respect to different values of q. On the basis of these values, the propounded operators such as GWSVNPA, GOWSVNPA, GWSVNPG and GOWSVNPG operator are applied to fuse the each evaluation information of alternative and their corresponding cosine similarity degree with the order relation are listed in Table 7. From it, we can derive the following conclusions: (1) the cosine similarity degrees of alternatives based on diverse values of q are different; (2) although the final decision resultants of alternatives are slightly diverse, the optimal programme are the same; (3) the

TABLE 5. The order relation of alternatives based upon different copulas.

Copulas	Order relation of alternatives	
	WSVNPA operator	WSVNPG operator
Gumbel	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
Clayton	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
Frank	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
Ali-Mikhail-Haq	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
Joe	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$

TABLE 6. The decision results based on the propounded operators.

Copulas	Operators	The Cosine similarity					Sorting
		Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	
Gumbel ($\sigma = 1$)	SVNPA	0.4867	0.8312	0.7308	0.7084	0.5746	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	SVNPG	0.7016	0.9303	0.8792	0.8534	0.7747	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	WSVNPA	0.6607	0.9057	0.6656	0.5266	0.6607	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	WSVNPG	0.6149	0.8888	0.8052	0.8233	0.6925	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	OWSVNPA	0.6581	0.9184	0.8575	0.8476	0.7490	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	OWSVNPG	0.6152	0.9035	0.8325	0.8055	0.7025	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
Clayton ($\sigma = 1$)	SVNPA	0.8536	0.9653	0.9438	0.9417	0.8938	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	SVNPG	0.3180	0.5332	0.4491	0.4095	0.3616	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	WSVNPA	0.9057	0.9733	0.9560	0.9633	0.9325	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	WSVNPG	0.2885	0.4610	0.3840	0.3933	0.3262	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	OWSVNPA	0.9047	0.9757	0.9716	0.9616	0.9311	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	OWSVNPG	0.2884	0.4907	0.4124	0.3753	0.3415	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
Frank ($\sigma = 1$)	SVNPA	0.5058	0.8457	0.7488	0.7308	0.5952	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	SVNPG	0.6022	0.8955	0.8397	0.7623	0.6952	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	WSVNPA	0.6656	0.9074	0.8291	0.8637	0.7515	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	WSVNPG	0.5185	0.8416	0.7552	0.7565	0.6122	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	OWSVNPA	0.6628	0.9194	0.8595	0.8516	0.7539	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	OWSVNPG	0.5444	0.8752	0.7726	0.7511	0.6533	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
Ali-Mikhail-Haq ($\sigma = 1$)	SVNPA	0.3673	0.6554	0.5548	0.5268	0.4369	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	SVNPG	0.8157	0.9580	0.9278	0.9158	0.8680	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	WSVNPA	0.5266	0.7863	0.6838	0.7027	0.6121	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	WSVNPG	0.7391	0.9340	0.8807	0.8964	0.8036	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	OWSVNPA	0.5235	0.8096	0.7261	0.7018	0.6194	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	OWSVNPG	0.7385	0.9417	0.8969	0.8868	0.8072	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
Joe ($\sigma = 1$)	SVNPA	0.4867	0.8312	0.7308	0.7084	0.5746	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	SVNPG	0.7016	0.9303	0.8792	0.8534	0.7747	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	WSVNPA	0.6607	0.9057	0.6656	0.5266	0.6607	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	WSVNPG	0.6149	0.8888	0.8052	0.8233	0.6925	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	OWSVNPA	0.6581	0.9184	0.8575	0.8476	0.7490	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	OWSVNPG	0.6152	0.9035	0.8325	0.8055	0.7025	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$

parameter q can be regarded as a risk preference attitude of managers, i.e., the parameter q has a better control capability for the comprehensive evaluation value of alternatives.

To analyze the effect of diverse parameter value of σ, we select the Gumbel-WSVNPA operator to perform the parameter analysis. The values of parameter σ change from 1 to 10 incrementing by 1 and the the corresponding parameter analysis resultants are summarized in Table 8 and Figure 1. From those, although the ranking orders of programmes with respect to different value of σ are slightly different, the best programme are the same. Furthermore, when $1 \leq \theta \leq 6$, the ranking relation of alternatives is $\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$, when $\sigma > 6$, the ranking relation of alternatives is $\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_1 \succ \Theta_5$.

In summary, we can derive that the sorting orders of alternatives are relatively stable with respect to different values of σ. Accordingly, the propounded approach can validly avoid interferences and acquire the best alternative.

C. COMPARISON ANALYSIS
In what follows, we perform a comparison exploration between the propounded MADM technique with the existing approaches to testify the effectiveness and to reveal the significant merits of the propounded approach. we compare the propounded approach with diverse approaches developed in [33]–[35], [44], [45]. The derived ranking order results are depicted in Table 9. From it, we can obtain that the decision results acquired from the existing methods are basically the

TABLE 7. The decision results of generalize aggregation operators based on diverse parameter values.

q	Operators	Gumbel type($\sigma = 1$)					Sorting
		Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	
q=0.5	GWSVNPA	0.8833	0.9512	0.9264	0.9358	0.9179	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GWSVNPG	0.3111	0.5385	0.4530	0.4666	0.3611	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GOWSVNPA	0.8815	0.9579	0.9402	0.9298	0.9219	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GOWSVNPG	0.3116	0.5517	0.4677	0.4568	0.3579	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
q=1	GWSVNPA	0.6607	0.9057	0.6656	0.5266	0.6607	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GWSVNPG	0.6149	0.8888	0.8052	0.8233	0.6925	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	OWSVNPA	0.6581	0.9184	0.8575	0.8476	0.7490	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
	OWSVNPG	0.6152	0.9035	0.8325	0.8055	0.7025	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
q=2	GWSVNPA	0.1939	0.6390	0.4136	0.5063	0.2476	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GWSVNPG	0.4833	0.7260	0.6388	0.6407	0.5151	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GOWSVNPA	0.1928	0.6705	0.4632	0.4839	0.2441	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GOWSVNPG	0.4877	0.7197	0.6357	0.6328	0.4904	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
q=5	GWSVNPA	0.0079	0.0965	0.0320	0.0621	0.0089	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GWSVNPG	0.3428	0.6513	0.5102	0.5443	0.4060	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GOWSVNPA	0.0078	0.1054	0.0379	0.0590	0.0085	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
	GOWSVNPG	0.3465	0.6540	0.5509	0.4998	0.3916	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$

TABLE 8. The decision results of G-WSVNPA operator based on different values of σ.

| σ	The Cosine similarity					Sorting
	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	
$\sigma = 1$	0.6607	0.9057	0.6656	0.5266	0.6607	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
$\sigma = 2$	0.8035	0.9380	0.8968	0.9179	0.8430	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
$\sigma = 3$	0.8139	0.9284	0.8904	0.9133	0.8357	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
$\sigma = 4$	0.8087	0.9169	0.8785	0.9046	0.8205	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
$\sigma = 5$	0.8011	0.9066	0.8673	0.8964	0.8067	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
$\sigma = 6$	0.7938	0.8979	0.8577	0.8893	0.7953	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
$\sigma = 7$	0.7872	0.8057	0.8497	0.8831	0.7861	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_1 \succ \Theta_5$
$\sigma = 8$	0.7813	0.8843	0.8429	0.8778	0.7787	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_1 \succ \Theta_5$
$\sigma = 9$	0.7761	0.8789	0.8371	0.8732	0.7725	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_1 \succ \Theta_5$
$\sigma = 10$	0.7714	0.8743	0.8321	0.8692	0.7674	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_1 \succ \Theta_5$
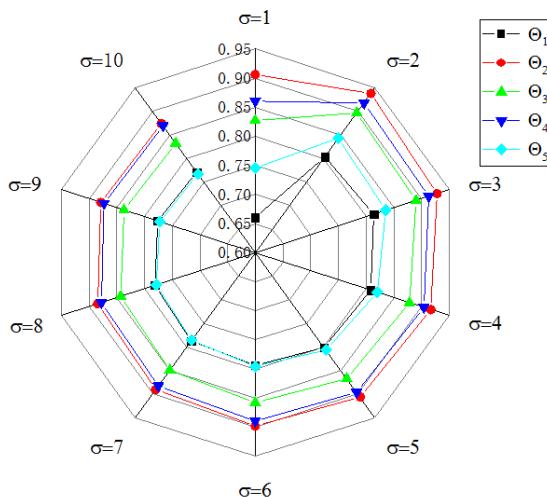
FIGURE 1. The resultants of diverse σ values.

Next, we further conduct a detailed analysis between the propounded method with the existing approach proposed by [44] and [33].

✓ Compared with the method based on the SVNHWA and SVNHWG operator propounded by Liu [33], the final decision results are the same as the propounded method. Their common advantage is that they all have a universal parameter that makes the process of information fusion more flexible. However, the method proposed by [33] can not clear up the awkward arguments on the process of sorting alternatives. On the contrary, the propounded approach has the ability to eliminate the influence of singular data. Consequently, the developed method is more universal and rational in view of these aggregation operators.

✓ Compared with the approach on the basis of the SVNWA operator propounded by Ye [44], the SVNWA operator can aggregate vague information and the computational process is not relatively complex. However, it has the following defects (i) the SVNWA operator based on Algebraic T-norm and T-conorm lacks of flexibility and robustness; (ii) the SVNWA operator does not take into consideration the impact of extreme information provided by manager and the correlation of fusion information. Nevertheless, the

same as those of the presented approach. Accordingly, the effectiveness of the propounded method can be elaborated. Besides, we perform several characteristic analysis between the presented approach with the prevailing methods, the analysis are displayed in Table 10.

TABLE 9. Comparative with previous operators under SVN environment.

Operators	The Cosine similarity					Sorting
	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	
NWA [44]	0.6210	0.8983	0.8248	0.8035	0.6874	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
SVNWA [45]	0.6374	0.9149	0.8483	0.8304	0.7303	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
SVNWG [45]	0.5858	0.8969	0.8224	0.7819	0.6793	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_5 \succ \Theta_1$
SVNHWA [33]	0.6162	0.9132	0.8401	0.8268	0.7050	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
SVNHWG [33]	0.6055	0.9162	0.8459	0.8199	0.8123	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
SVNWEA [34]	0.0048	0.1325	0.0711	0.0298	0.0226	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_5 \succ \Theta_1$
L-SVNWA [35]	0.5628	0.9122	0.7981	0.8459	0.4837	$\Theta_2 \succ \Theta_4 \succ \Theta_3 \succ \Theta_1 \succ \Theta_5$
L-SVNWG [35]	0.5775	0.9418	0.8613	0.8111	0.6649	$\Theta_2 \succ \Theta_3 \succ \Theta_4 \succ \Theta_1 \succ \Theta_5$

TABLE 10. Characteristic comparison with existing operators under SVN environment.

	Approaches	Information relevance	Flexibility	The support degree of attributes
Existing Approaches	NWA [44]	×	×	×
	SVNWA [45]	×	×	×
	SVNWG [45]	×	×	×
	SVNHWA [33]($\gamma = 4$)	×	√	×
	SVNHWG [33]($\gamma = 4$)	×	√	×
	SVNWEA [34]	×	×	×
	L-SVNWA [35]	×	√	×
	L-SVNWA [35]	×	√	×
Propounded methods	G-SVNPWA	√	√	√
	C-SVNPWA	√	√	√
	F-SVNPWA	√	√	√
	AHM-SVNPWA	√	√	√
	J-SVNPWA	√	√	√

novel method propounded in this paper can not only take into account the interrelationship among the input data, but also eliminate the effect of extreme information and the presented method; on the other hand, the presented method give a general and flexible aggregation function because of the different copulas and flexible parameters. Particularly, when the parameter $\sigma = 1$ in the type Gumbel-copula, it reduces to Algebraic T-norm and T-conorm. Accordingly, the propounded method on the basis of those presented operators is more universal and rational to cope with decision problems.

All in all, based on the comparatives and analysing, the propounded method in this study has the following superiorities (**i**) it supplies a generalized aggregation function based on copula and gives more selections for managers to chose appropriate function to fuse information; (**ii**) it can take into consideration the correlation among multiple aggregated arguments and has a flexibility for aggregation process by the general parameter; (**iii**) it can highlight the importance of the aggregated information and vanish the influence of embarrassed data by taking diverse weight obtain by the support degree.

VIII. CONCLUSION
In this paper, we first set up several novel operational laws of SVN numbers on the basis of copulas, and explore corresponding properties and several representative cases based on different copulas. Secondly, we propound a series of power aggregation operators namely SVNCPA, WSVNCPA,

OWSVNCPA and their geometric operators. Meanwhile, we extend those operators to their generalized version. We also explore several attractive properties and some particular cases of those operators. Besides, we put forward an novel algorithm to deal with MADM problem with SVN information on the basis of the propounded operators. Finally, the efficacy of the propounded algorithm is confirmed by a actual application and the advantages are shown by comparing with other previous approaches. The main superiorities of the presented approach are (**i**) it can portray inconsistent and incomplete information in several practical issues through single-valued neutrosophic numbers; (**ii**) it can remove the influence of awkward information according to power averaging operator; (**iii**) it can make the decision process more flexible and make decision makers have more freedoms to choice appropriate integration operator. In the future, we will generalize the Archimedean copula and co-copula to other fuzzy environment such as Hesitant fuzzy linguistic term set [71], 2-tuple linguistic neutrosophic [72], [73] and so forth, and research the application of emergency management strategy selection and venture investment decision in single-valued neutrosophic environment.

APPENDIXES
APPENDIX A
THE PROOF OF THEOREM 2
Proof: (1) and (2) are evident, the proofs are omitted here, we prove the others:

(3) According to Eq (8 and Eq (10),

$$\kappa(\alpha_1 \oplus_{\mathfrak{C}} \alpha_2)$$
$$= \kappa\Big(1 - \rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2)),$$
$$\rho^{-1}(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2)), \rho^{-1}(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2))\Big)$$
$$= \Big(1 - \rho^{-1}\Big(\kappa\rho\Big(1 - \big(1 - \rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2))\big)\Big)\Big),$$
$$\rho^{-1}\Big(\kappa\rho\Big(\rho^{-1}(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2))\Big)\Big),$$
$$\rho^{-1}\Big(\kappa\rho\Big(\rho^{-1}(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2))\Big)\Big)\Big)$$
$$= \Big(1 - \rho^{-1}\Big(\kappa\rho\Big(\rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2))\Big)\Big),$$
$$\rho^{-1}(\kappa(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2))),$$
$$\rho^{-1}(\kappa(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)))\Big)$$
$$= \Big(1 - \rho^{-1}(\kappa(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2))),$$
$$\rho^{-1}(\kappa(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2))),$$
$$\rho^{-1}(\kappa(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)))\Big).$$

Moreover,

$$\kappa\alpha_1 \oplus_{\mathfrak{C}} \kappa\alpha_2$$
$$= \Big(1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T}_1)), \rho^{-1}(\kappa\rho(\mathcal{I}_1)), \rho^{-1}(\kappa\rho(\mathcal{F}_1))\Big)$$
$$\oplus_{\mathcal{C}} \Big(1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T}_2)), \rho^{-1}(\kappa\rho(\mathcal{I}_2)), \rho^{-1}(\kappa\rho(\mathcal{F}_2))\Big)$$
$$= \Big(1 - \rho^{-1}\Big(\rho\Big(1 - \big(1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T}_1))\big)\Big)$$
$$+ \rho\Big(1 - \big(1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T}_2))\big)\Big)\Big),$$
$$\rho^{-1}\Big(\rho\Big(\rho^{-1}(\kappa\rho(\mathcal{I}_1))\Big)$$
$$+ \rho\Big(\rho^{-1}(\kappa\rho(\mathcal{I}_2))\Big)\Big),$$
$$\rho^{-1}\Big(\rho\Big(\rho^{-1}(\kappa\rho(\mathcal{F}_1))\Big) + \rho\Big(\rho^{-1}(\kappa\rho(\mathcal{F}_2))\Big)\Big)\Big)$$
$$= \Big(1 - \rho^{-1}\Big(\rho\Big(\rho^{-1}(\kappa\rho(1 - \mathcal{T}_1))\Big)$$
$$+ \rho\Big(\rho^{-1}(\kappa\rho(1 - \mathcal{T}_2))\Big)\Big), \rho^{-1}(\kappa(\rho(\mathcal{I}_1)$$
$$+ \rho(\mathcal{I}_2))), \rho^{-1}(\kappa(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)))\Big)$$
$$= \Big(1 - \rho^{-1}(\kappa(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2))),$$
$$\rho^{-1}(\kappa(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2))),$$
$$\rho^{-1}(\kappa(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)))\Big).$$

Accordingly, $\kappa(\alpha_1 \oplus_{\mathfrak{C}} \alpha_2) = \kappa\alpha_1 \oplus_{\mathfrak{C}} \kappa\alpha_2$ holds.
(5)

$$\kappa_1\alpha \oplus_{\mathfrak{C}} \kappa_2\alpha$$
$$= \Big(1 - \rho^{-1}(\kappa_1\rho(1 - \mathcal{T})), \rho^{-1}(\kappa_1\rho(\mathcal{I})), \rho^{-1}(\kappa_1\rho(\mathcal{F}))\Big)$$
$$\oplus_{\mathcal{C}}\Big(1 - \rho^{-1}(\kappa_2\rho(1 - \mathcal{T})), \rho^{-1}(\kappa_2\rho(\mathcal{I})), \rho^{-1}(\kappa_2\rho(\mathcal{F}))\Big)$$
$$= \Big(1 - \rho^{-1}\Big(\rho\Big(1 - \big(1 - \rho^{-1}(\kappa_1\rho(1 - \mathcal{T}))\big)\Big)$$
$$+ \rho\Big(1 - \big(1 - \rho^{-1}(\kappa_2\rho(1 - \mathcal{T}))\big)\Big)\Big),$$

$$\rho^{-1}\Big(\rho\Big(\rho^{-1}(\kappa_1\rho(\mathcal{I}))\Big)$$
$$+ \rho\Big(\rho^{-1}(\kappa_2\rho(\mathcal{I}))\Big)\Big),$$
$$\rho^{-1}\Big(\rho\Big(\rho^{-1}(\kappa_1\rho(\mathcal{F}))\Big) + \rho\Big(\rho^{-1}(\kappa_2\rho(\mathcal{F}))\Big)\Big)\Big)$$
$$= \Big(1 - \rho^{-1}\Big(\rho\Big(\rho^{-1}(\kappa_1\rho(1 - \mathcal{T}))\Big)$$
$$+ \rho\Big(\rho^{-1}(\kappa_2\rho(1 - \mathcal{T}))\Big)\Big), \rho^{-1}(\kappa_1\rho(\mathcal{I}) + \kappa_2\rho(\mathcal{I})),$$
$$\rho^{-1}(\kappa_1\rho(\mathcal{F}) + \kappa_2\rho(\mathcal{F}))\Big)$$
$$= \Big(1 - \rho^{-1}((\kappa_1 + \kappa_2)\rho(1 - \mathcal{T})),$$
$$\rho^{-1}((\kappa_1 + \kappa_2)\rho(\mathcal{I})), \rho^{-1}((\kappa_1 + \kappa_2)\rho(\mathcal{F}))\Big).$$

Furthermore

$$(\kappa_1 + \kappa_2)\alpha$$
$$= \Big(1 - \rho^{-1}((\kappa_1 + \kappa_2)\rho(1 - \mathcal{T})), \rho^{-1}((\kappa_1 + \kappa_2)\rho(\mathcal{I})),$$
$$\rho^{-1}((\kappa_1 + \kappa_2)\rho(\mathcal{F}))\Big).$$

Hence, $\kappa_1\alpha \oplus_{\mathfrak{C}} \kappa_2\alpha = (\kappa_1 + \kappa_2)\alpha$ is kept.

Similarly, (4) and (6) can be proved by the same method, which finishes the proof of Theorem 2.

APPENDIX B
THE PROOF OF THEOREM 3
Proof:

(1) $(\alpha^c)^\kappa$
$$= (\mathcal{F}, 1 - \mathcal{I}, \mathcal{T})^\kappa$$
$$= \Big(\rho^{-1}(\kappa\rho(\mathcal{F})), 1 - \rho^{-1}(\kappa\rho(\mathcal{I})), 1 - \rho^{-1}(\kappa\rho(1 - \mathcal{T}))\Big)$$
$$= (\kappa\alpha)^c.$$

(2) $\kappa(\alpha^c)$
$$= \kappa(\mathcal{F}, 1 - \mathcal{I}, \mathcal{T})$$
$$= \Big(1 - \rho^{-1}(\kappa\rho(1 - \mathcal{F})), \rho^{-1}(\kappa\rho(1 - \mathcal{I})), \rho^{-1}(\kappa\rho(\mathcal{T}))\Big)$$
$$= (\alpha^\kappa)^c.$$

(3) $\alpha_1^c \oplus_{\mathfrak{C}} \alpha_2^c$
$$= (\mathcal{F}_1, 1 - \mathcal{I}_1, \mathcal{T}_1) \oplus_{\mathcal{C}} (\mathcal{F}_2, 1 - \mathcal{I}_2, \mathcal{T}_2)$$
$$= \Big(1 - \rho^{-1}(\rho(1 - \mathcal{F}_1) + \rho(1 - \mathcal{F}_2)), \rho^{-1}(\rho(1 - \mathcal{I}_1)$$
$$+ \rho(1 - \mathcal{I}_2)), \rho^{-1}(\rho(\mathcal{T}_1) + \rho(\mathcal{T}_2))\Big)$$
$$= \Big(\rho^{-1}(\rho(\mathcal{T}_1) + \rho(\mathcal{T}_2)), \rho^{-1}(\rho(1 - \mathcal{I}_1) + \rho(1 - \mathcal{I}_2)),$$
$$1 - \rho^{-1}(\rho(1 - \mathcal{F}_1) + \rho(1 - \mathcal{F}_2))\Big)^c$$
$$= (\alpha_1 \otimes_{\mathfrak{C}} \alpha_2)^c.$$

(4) $\alpha_1^c \otimes_{\mathfrak{C}} \alpha_2^c$
$$= (\mathcal{F}_1, 1 - \mathcal{I}_1, \mathcal{T}_1) \otimes_{\mathcal{C}} (\mathcal{F}_2, 1 - \mathcal{I}_2, \mathcal{T}_2)$$
$$= \Big(\rho^{-1}(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)), \rho^{-1}(\rho(\mathcal{I}_1) + \rho(\mathcal{I}_2)),$$
$$1 - \rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2))\Big)$$
$$= \Big(1 - \rho^{-1}(\rho(1 - \mathcal{T}_1) + \rho(1 - \mathcal{T}_2)), \rho^{-1}(\rho(\mathcal{I}_1)$$

$$+ \rho(\mathcal{I}_2)), \rho^{-1}(\rho(\mathcal{F}_1) + \rho(\mathcal{F}_2)) \Big)^c$$
$$= (\alpha_1 \oplus_{\mathfrak{C}} \alpha_2)^c.$$

APPENDIX C
Several operational laws based upon diverse copulas.
1. If $\rho(t) = (-\ln t)^\sigma$, $\sigma \geq 1$, then we have

(1) $\alpha_1 \oplus_{\mathfrak{C}} \alpha_2 = \Big(1 - e^{-\left((-\ln(1-\mathcal{T}_1))^\sigma + (-\ln(1-\mathcal{T}_2))^\sigma\right)^{\frac{1}{\sigma}}},$

$$e^{-\left((-\ln(\mathcal{I}_1))^\sigma + (-\ln(\mathcal{I}_2))^\sigma\right)^{\frac{1}{\sigma}}},$$
$$e^{-\left((-\ln(\mathcal{F}_1))^\sigma + (-\ln(\mathcal{F}_2))^\sigma\right)^{\frac{1}{\sigma}}} \Big);$$

(2) $\alpha_1 \otimes_{\mathfrak{C}} \alpha_2 = \Big(e^{-\left((-\ln(\mathcal{T}_1))^\sigma + (-\ln(\mathcal{T}_2))^\sigma\right)^{\frac{1}{\sigma}}},$

$$1 - e^{-\left((-\ln(1-\mathcal{I}_1))^\sigma + (-\ln(1-\mathcal{I}_2))^\sigma\right)^{\frac{1}{\sigma}}},$$
$$1 - e^{-\left((-\ln(1-\mathcal{F}_1))^\sigma + (-\ln(1-\mathcal{F}_2))^\sigma\right)^{\frac{1}{\sigma}}} \Big);$$

(3) $\kappa\alpha = \Big(1 - e^{-\left(\kappa(-ln(1-\mathcal{T}))^\sigma\right)^{\frac{1}{\sigma}}}, e^{-\left(\kappa(-ln(\mathcal{I}))^\sigma\right)^{\frac{1}{\sigma}}},$

$$e^{-\left(\kappa(-ln(\mathcal{I}))^\sigma\right)^{\frac{1}{\sigma}}} \Big);$$

$\alpha^\kappa = \Big(e^{-\left(\kappa(-ln(\mathcal{T}))^\sigma\right)^{\frac{1}{\sigma}}},$

$$1 - e^{-\left(\kappa(-ln(1-\mathcal{I}))^\sigma\right)^{\frac{1}{\sigma}}}, 1 - e^{-\left(\kappa(-ln(1-\mathcal{F}))^\sigma\right)^{\frac{1}{\sigma}}} \Big).$$

2. If $\rho(t) = t^{-\sigma} - 1$ with $\sigma \geq -1$, $\sigma \neq 0$, then we have

(1) $\alpha_1 \oplus_{\mathfrak{C}} \alpha_2$
$$= \Big(1 - \left((1 - \mathcal{T}_1)^{-\sigma} + (1 - \mathcal{T}_2)^{-\sigma} - 1\right)^{-\frac{1}{\sigma}},$$
$$\left((\mathcal{I}_1)^{-\sigma} + (\mathcal{I}_2)^{-\sigma} - 1\right)^{-\frac{1}{\sigma}},$$
$$\left((\mathcal{F}_1)^{-\sigma} + (\mathcal{F}_2)^{-\sigma} - 1\right)^{-\frac{1}{\sigma}} \Big);$$

(2) $\alpha_1 \otimes_{\mathfrak{C}} \alpha_2 = \Big(\left((\mathcal{T}_1)^{-\sigma} + (\mathcal{T}_2)^{-\sigma} - 1\right)^{-\frac{1}{\sigma}},$

$$1 - \left((1 - \mathcal{I}_1)^{-\sigma} + (1 - \mathcal{I}_2)^{-\sigma} - 1\right)^{-\frac{1}{\sigma}},$$
$$1 - \left((1 - \mathcal{F}_1)^{-\sigma} + (1 - \mathcal{F}_2)^{-\sigma} - 1\right)^{-\frac{1}{\sigma}} \Big);$$

(3) $\kappa\alpha = \Big(1 - \left(\kappa\left((1 - \mathcal{T})^{-\sigma} - 1\right) + 1\right)^{-\frac{1}{\sigma}},$

$$\left(\kappa\left(\mathcal{I}^{-\sigma} - 1\right) + 1\right)^{-\frac{1}{\sigma}},$$
$$\left(\kappa\left(\mathcal{F}^{-\sigma} - 1\right) + 1\right)^{-\frac{1}{\sigma}} \Big);$$

(4) $\alpha^\kappa = \Big(\left(\kappa\left(\mathcal{T}^{-\sigma} - 1\right) + 1\right)^{-\frac{1}{\sigma}},$

$$1 - \left(\kappa\left((1 - \mathcal{I})^{-\sigma} - 1\right) + 1\right)^{-\frac{1}{\sigma}},$$
$$1 - \left(\kappa\left((1 - \mathcal{F})^{-\sigma} - 1\right) + 1\right)^{-\frac{1}{\sigma}} \Big).$$

3. If $\rho(t) = \ln\left(\frac{e^{-\sigma t} - 1}{e^{-\sigma} - 1}\right)$ with $\sigma \neq 0$, then we have

(1) $\alpha_1 \oplus_{\mathfrak{C}} \alpha_2$
$$= \Big(1 + \frac{1}{\sigma} \ln\left(\frac{\left(e^{-\sigma(1-\mathcal{T}_1)} - 1\right)\left(e^{-\sigma(1-\mathcal{T}_2)} - 1\right)}{e^{-\sigma} - 1} + 1 \right),$$
$$-\frac{1}{\sigma} \ln\left(\frac{\left(e^{-\sigma\mathcal{I}_1} - 1\right)\left(e^{-\sigma\mathcal{I}_2} - 1\right)}{e^{-\sigma} - 1} + 1 \right),$$
$$-\frac{1}{\sigma} \ln\left(\frac{\left(e^{-\sigma\mathcal{F}_1} - 1\right)\left(e^{-\sigma\mathcal{F}_2} - 1\right)}{e^{-\sigma} - 1} + 1 \right) \Big);$$

(2) $\alpha_1 \otimes_{\mathfrak{C}} \alpha_2$
$$= \Big(-\frac{1}{\sigma} \ln\left(\frac{\left(e^{-\sigma\mathcal{T}_1} - 1\right)\left(e^{-\sigma\mathcal{T}_2} - 1\right)}{e^{-\sigma} - 1} + 1 \right),$$
$$1 + \frac{1}{\sigma} \ln\left(\frac{\left(e^{-\sigma(1-\mathcal{I}_1)} - 1\right)\left(e^{-\sigma(1-\mathcal{I}_2)} - 1\right)}{e^{-\sigma} - 1} + 1 \right),$$
$$1 + \frac{1}{\sigma} \ln\left(\frac{\left(e^{-\sigma(1-\mathcal{F}_1)} - 1\right)\left(e^{-\sigma(1-\mathcal{F}_2)} - 1\right)}{e^{-\sigma} - 1} + 1 \right) \Big);$$

(3) $\kappa\alpha$
$$= \Big(1 - \frac{1}{\sigma} \ln\left(1 + \left(e^{-\sigma} - 1\right)^{1-\kappa}\left(e^{-\sigma(1-\mathcal{T})} - 1\right)^\kappa\right),$$
$$\frac{1}{\sigma} \ln\left(1 + \left(e^{-\sigma} - 1\right)^{1-\kappa}\left(e^{-\sigma\mathcal{I}} - 1\right)^\kappa\right),$$
$$\frac{1}{\sigma} \ln\left(1 + \left(e^{-\sigma} - 1\right)^{1-\kappa}\left(e^{-\sigma\mathcal{F}} - 1\right)^\kappa\right) \Big);$$

(4) α^κ
$$= \Big(\frac{1}{\sigma} \ln\left(1 + \left(e^{-\sigma} - 1\right)^{1-\kappa}\left(e^{-\sigma\mathcal{T}} - 1\right)^\kappa\right),$$
$$1 - \frac{1}{\sigma} \ln\left(1 + \left(e^{-\sigma} - 1\right)^{1-\kappa}\left(e^{-\sigma(1-\mathcal{I})} - 1\right)^\kappa\right),$$
$$1 - \frac{1}{\sigma} \ln\left(1 + \left(e^{-\sigma} - 1\right)^{1-\kappa}\left(e^{-\sigma(1-\mathcal{F})} - 1\right)^\kappa\right) \Big).$$

4. If $\rho(t) = \ln\left(\frac{1 - \sigma(1-t)}{t}\right)$ with $\sigma \in [-1, 1]$, then we have

(1) $\alpha_1 \oplus_{\mathfrak{C}} \alpha_2$
$$= \Big(1 - \frac{(1 - \mathcal{T}_1)(1 - \mathcal{T}_2)}{1 - \sigma\mathcal{T}_1\mathcal{T}_2}, \frac{\mathcal{I}_1\mathcal{I}_2}{1 - \sigma(1 - \mathcal{I}_1)(1 - \mathcal{I}_2)},$$
$$\frac{\mathcal{F}_1\mathcal{F}_2}{1 - \sigma(1 - \mathcal{F}\mathcal{I}_1)(1 - \mathcal{F}_2)} \Big);$$

(2) $\alpha_1 \otimes_{\mathfrak{C}} \alpha_2$
$$= \Big(\frac{\mathcal{I}_1\mathcal{I}_2}{1 - \sigma(1 - \mathcal{I}_1)(1 - \mathcal{I}_2)},$$
$$1 - \frac{(1 - \mathcal{I}_1)(1 - \mathcal{I}_2)}{1 - \sigma\mathcal{I}_1\mathcal{I}_2},$$
$$1 - \frac{(1 - \mathcal{F}_1)(1 - \mathcal{F}_2)}{1 - \sigma\mathcal{F}_1\mathcal{F}_2} \Big);$$

(3) $\kappa\alpha = \Big(\frac{(1 - \sigma\mathcal{T})^\kappa - (1 - \mathcal{T})^\kappa}{(1 - \sigma\mathcal{T})^\kappa - \sigma(1 - \mathcal{T})^\kappa},$
$$\frac{(1 - \sigma)\mathcal{I}^\kappa}{(1 - \sigma(1 - \mathcal{I}))^\kappa - \sigma\mathcal{I}^\kappa},$$

$$\left. \frac{(1-\sigma)\mathcal{F}^{\kappa}}{(1-\sigma(1-\mathcal{F}))^{\kappa} - \sigma\mathcal{F}^{\kappa}} \right);$$

$$(4)\ \alpha^{\kappa} = \left(\frac{(1-\sigma)\mathcal{T}^{\kappa}}{(1-\sigma(1-\mathcal{T}))^{\kappa} - \sigma\mathcal{T}^{\kappa}}, \right.$$
$$\frac{(1-\sigma\mathcal{I})^{\kappa} - (1-\mathcal{I})^{\kappa}}{(1-\sigma\mathcal{I})^{\kappa} - \sigma(1-\mathcal{I})^{\kappa}},$$

$$\left. \frac{(1-\sigma\mathcal{F})^{\kappa} - (1-\mathcal{F})^{\kappa}}{(1-\sigma\mathcal{F})^{\kappa} - \sigma(1-\mathcal{F})^{\kappa}} \right).$$

5. If $\rho(\mathfrak{t}) = -\ln(1 - (1-\mathfrak{t})^{\sigma})$, $\sigma \geq -1$, then we have

(1) $\alpha_1 \oplus_{\mathfrak{c}} \alpha_2$

$$= \left(\left(\mathcal{T}_1^{\sigma} + \mathcal{T}_2^{\sigma} - \mathcal{T}_1^{\sigma}\mathcal{T}_2^{\sigma} \right)^{\frac{1}{\sigma}}, \right.$$

$$SVNCWA(\alpha_1, \alpha_2) = \eta_1\alpha_1(\oplus_{\mathfrak{c}})\eta_2\alpha_2$$

$$= \begin{pmatrix} 1 - \rho^{-1}(\eta_1\rho(1-\mathcal{T}_1)), \\ \rho^{-1}(\eta_1\rho(\mathcal{I}_1)), \\ \rho^{-1}(\eta_1\rho(\mathcal{F}_1)) \end{pmatrix} \oplus_c \begin{pmatrix} 1 - \rho^{-1}(\eta_2\rho(1-\mathcal{T}_2)), \\ \rho^{-1}(\eta_2\rho(\mathcal{I}_2)), \\ \rho^{-1}(\eta_2\rho(\mathcal{F}_2)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\rho\left(\rho^{-1}(\eta_1\rho(1-\mathcal{T}_1)) \right) + \rho\left(\rho^{-1}(\eta_2\rho(1-\mathcal{T}_2)) \right) \right), \\ \rho^{-1}\left(\rho\left(\rho^{-1}(\eta_1\rho(\mathcal{I}_1)) \right) + \rho\left(\rho^{-1}(\eta_2\rho(\mathcal{I}_2)) \right) \right), \\ \rho^{-1}\left(\rho\left(\rho^{-1}(\eta_1\rho(\mathcal{F}_1)) \right) + \rho\left(\rho^{-1}(\eta_2\rho(\mathcal{F}_2)) \right) \right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}(\eta_1\rho(1-\mathcal{T}_1) + \eta_2\rho(1-\mathcal{T}_2)), \\ \rho^{-1}(\eta_1\rho(\mathcal{I}_1) + \eta_2\rho(\mathcal{I}_2)), \\ \rho^{-1}(w_1\rho(\mathcal{F}_1) + \eta_2\rho(\mathcal{F}_2)) \end{pmatrix}$$

$$= \left(1 - \rho^{-1}\left(\sum_{i=1}^{2} \eta_i\rho(1-\mathcal{T}_i) \right), \rho^{-1}\left(\sum_{i=1}^{2} \eta_i\rho(\mathcal{I}_i) \right), \rho^{-1}\left(\sum_{i=1}^{2} \eta_i\rho(\mathcal{F}_i) \right) \right).$$

$$SVNCWA(\alpha_1, \alpha_2, \cdots, \alpha_{k+1})$$

$$= SVNCWA(\alpha_1, \alpha_2, \cdots, \alpha_k) \oplus_{\mathfrak{c}} (\eta_{k+1}\alpha_{k+1})$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(1-\mathcal{T}_k) \right), \\ \rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(\mathcal{I}_k) \right), \\ \rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(\mathcal{F}_k) \right) \end{pmatrix} \oplus_{\mathfrak{c}} \begin{pmatrix} 1 - \rho^{-1}(\eta_{k+1}\rho(1-\mathcal{T}_{k+1})), \\ \rho^{-1}(\eta_{k+1}\rho(\mathcal{I}_{k+1})), \\ \rho^{-1}(\eta_{k+1}\rho(\mathcal{F}_{k+1})) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\rho\left(\rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(1-\mathcal{T}_k) \right) \right) + \rho\left(\rho^{-1}(\eta_{k+1}\rho(1-\mathcal{T}_{k+1})) \right) \right), \\ \rho^{-1}\left(\rho\left(\rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(\mathcal{I}_k) \right) \right) + \rho\left(\rho^{-1}(\eta_{k+1}\rho(\mathcal{I}_{k+1})) \right) \right), \\ \rho^{-1}\left(\rho\left(\rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(\mathcal{F}_k) \right) \right) + \rho\left(\rho^{-1}(\eta_{k+1}\rho(\mathcal{F}_{k+1})) \right) \right) \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(1-\mathcal{T}_k) + \eta_{k+1}\rho(1-\mathcal{T}_{k+1}) \right), \\ \rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(\mathcal{I}_k) + \eta_{k+1}\rho(\mathcal{I}_{k+1}) \right), \\ \rho^{-1}\left(\sum_{i=1}^{k} \eta_k\rho(\mathcal{F}_k) + \eta_{k+1}\rho(\mathcal{F}_{k+1}) \right) \end{pmatrix} = \begin{pmatrix} 1 - \rho^{-1}\left(\sum_{i=1}^{k+1} \eta_{k+1}\rho(1-\mathcal{T}_{k+1}) \right), \\ \rho^{-1}\left(\sum_{i=1}^{k+1} \eta_{k+1}\rho(\mathcal{I}_{k+1}) \right), \\ \rho^{-1}\left(\sum_{i=1}^{k+1} \eta_{k+1}\rho(\mathcal{F}_{k+1}) \right) \end{pmatrix}.$$

$$1 - \left((1 - \mathcal{I}_1)^\sigma + (1 - \mathcal{I}_2)^\sigma - (1 - \mathcal{I}_1)^\sigma (1 - \mathcal{I}_2)^\sigma\right)^{\frac{1}{\sigma}},$$
$$1 - \left((1 - \mathcal{F}_1)^\sigma + (1 - \mathcal{F}_2)^\sigma - (1 - \mathcal{F}_1)^\sigma (1 - \mathcal{F}_2)^\sigma\right)^{\frac{1}{\sigma}}\right);$$

(2) $\alpha_1 \otimes_\mathfrak{c} \alpha_2$

$$= \left(1 - \left((1 - \mathcal{T}_1)^\sigma + (1 - \mathcal{T}_2)^\sigma - (1 - \mathcal{T}_1)^\sigma (1 - \mathcal{T}_2)^\sigma\right)^{\frac{1}{\sigma}},\right.$$
$$\left(\mathcal{I}_1^\sigma + \mathcal{I}_2^\sigma - \mathcal{I}_1^\sigma \mathcal{I}_2^\sigma\right)^{\frac{1}{\sigma}}, \left(\mathcal{F}_1^\sigma + \mathcal{F}_2^\sigma - \mathcal{F}_1^\sigma \mathcal{F}_2^\sigma\right)^{\frac{1}{\sigma}},\right);$$

(3) $\kappa\alpha = \left(\left(1 - (1 - \mathcal{T}^\sigma)^\kappa\right)^{\frac{1}{\sigma}}, 1 - \left(1 - (1 - (1 - \mathcal{I})^\sigma)^\kappa\right)^{\frac{1}{\sigma}},\right.$
$$\left. 1 - \left(1 - (1 - (1 - \mathcal{F})^\sigma)^\kappa\right)^{\frac{1}{\sigma}}\right);$$

(4) $\alpha^\kappa = \left(1 - \left(1 - (1 - (1 - \mathcal{T})^\sigma)^\kappa\right)^{\frac{1}{\sigma}}, \left(1 - (1 - \mathcal{I}^\sigma)^\kappa\right)^{\frac{1}{\sigma}},\right.$
$$\left.\left(1 - (1 - \mathcal{F}^\sigma)^\kappa\right)^{\frac{1}{\sigma}}\right).$$

APPENDIX D
THE PROOF OF THEOREM 4

Proof: For Theorem 4, the first conclusion is obvious, and then we prove Eq (13) by mathematical induction on n.

For $n = 2$, we have $SVNCWA(\alpha_1, \alpha_2)$, as shown at the bottom of the previous page.

Suppose that Eq (13) is kept for $n = k$, then we have

$$SVNCWA(\alpha_1, \alpha_2, \cdots, \alpha_k)$$
$$= \left(1 - \rho^{-1}\left(\sum_{i=1}^{k} \eta_k \rho(1 - \mathcal{T}_k)\right), \rho^{-1}\left(\sum_{i=1}^{k} \eta_k \rho(\mathcal{I}_k)\right),\right.$$
$$\left. \rho^{-1}\left(\sum_{i=1}^{k} \eta_k \rho(\mathcal{F}_k)\right)\right).$$

Then, when $n = k + 1$, we have $SVNCWA(\alpha_1, \alpha_2, \cdots, \alpha_{k+1})$, as shown at the bottom of the previous page, i.e., Eq (13) is valid for $n = k + 1$. Accordingly, Eq (13) is kept for all n.

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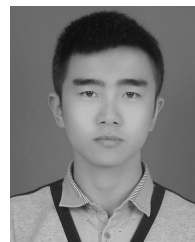
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