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# The Weighted Couple-Group Consensus for a Kind of Continuous Heterogeneous Multi-Agent Systems Based on Self-Adaptive and Cooperative-Competitive Mechanism

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**ABSTRACT** In this paper, the weighted couple-group consensus is investigated for a kind of continuous heterogeneous multi-agent systems in the cooperation-competition networks with time-delay. A novel weighted couple-group consensus protocol is proposed on basis of self-adaptive and cooperative-competitive mechanism. In this novel couple-group control protocol, self-adaptive regulation, weighted-distribution, cooperation-competition, position and velocity are considered. By applying the graph theory, complex frequency analysis method and linear algebra theory, some sufficient conditions have been given to guard the success of the couple-group consensus for this kind of heterogeneous multi-agent systems. Furthermore, the upper limit of the time delay may be computed if the weighted parameters are determined. The results show that the multi-agent system can converge to couple-group consensus state only if the upper bound input time-delay is satisfied the given time span. Some simulation examples show the validity of the obtained results.

**INDEX TERMS** Group consensus, heterogeneous, multi-agent systems, self-adaptive regulation, cooperative-competitive mechanism.

## I. INTRODUCTION

In the past decades, multi-agent systems (MASs) have become a hot topic for its potential application in formation control of multi-robot system, rendezvous in military, design of wireless sensor network, and so on. Among many topics of MASs, consensus is one of the most important problems and it has been intensively investigated in various disciplines [1]–[8]. In paper [1], consensus was discussed for three cases of networks of dynamic agents with fixed and switching topologies. In paper [2], discrete and continuous update schemes were proposed for consensus of multi-agent systems with dynamically changing interaction topologies. In literature [3], consensus was also investigated in multi-agent systems with non-uniform time delays and dynamically changing topologies. In paper [4], pinning consensus method for multi-agents networks was put forward. In [5], by employing stability theory and LMI (linear matrix inequality) technique, a sufficient condition of average consensus was obtained for the networks of continuous time agents with delayed information and jointly-connected topologies. In [6], consensus of multi-agent system with discrete-time and second-order dynamics was analyzed if the interaction graphs might not have spanning trees and the

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information was transmitted with time-varying delays. In [9], a novelty and interesting consensus method was used to solve the synchronization of networked systems based on the output information of neighbors.

Note that the states of all agents only converge to the same consensus value in literatures [1]–[9]. However, consensus values may be different for agents in different sub-networks with the changes of environment or tasks. Group consensus, as a special case of consensus, it was first discussed in paper [10]. Thereafter, many related results have been obtained for group consensus [10]–[37]. In [11], an algebraic condition was given for the couple-group consensus of a kind of multi-agent networks. In [12], a linear consensus protocol was proposed for a kind of continuous-time multi-agent systems under Markov switching topologies. In [13], a reduced-order system was introduced to analyze the group consensus problems in two cases. In [26], some sufficient conditions were given to solve the couple-group $L_2 - L_\infty$ consensus problem of nonlinear multi-agent systems.

It worth pointing out that most of aforementioned works is only related to homogeneous dynamic systems. In this situation, all agents of the whole complex systems only have the same dynamical behaviors. In fact, in many multi-agent systems, almost each agent has its own dynamics due to various external and interactional affects. Therefore, it is natural and necessary for us to analyze multi-agent systems with heterogeneous structure. Over the past ten years, some results on heterogeneous multi-agent systems have been achieved in [15], [17], [22]–[24], [28], [30]–[37]. In [31], a novel consensus protocol was proposed for high-ordered heterogeneous systems under the condition that the communication time-delays was uncertain and a necessary and sufficient condition was obtained to determine the existence of a high order consensus solution. In [37], the convergence properties of consensus algorithms were studied for agents in heterogeneous networks and the consensus could be realized even if the shared position and velocity information were different. At the same time, group consensus of heterogeneous systems has also received great interesting for many researchers. In [15],a novel group consensus protocol and a time-varying estimator of the uncertain parameters were put forward for the heterogeneous agents in the Euler–Lagrange system and the double-integrator system. In [22], some sufficient group consensus conditions were derived for a class of discrete heterogeneous systems which was composed of first-order and second-order agents with communication and input time delays. In [24], some sufficient algebraic criteria on couple-group consensus were derived and the upper bound of the input time delays could be computed for a heterogeneous multi-agent networks with communication and input time delays. In [23], group consensus control for heterogeneous multi-agent systems with fixed and switching topologies was discussed. In [32], by applying frequency-domain analysis method and matrix theory, some sufficient conditions were given to realize the group consensus for a heterogeneous multi-agent system with directed or undirected communication topologies. In [22], couple-group consensus of a kind of discrete-time heterogeneous systems was investigated. In [33], the weighted couple-group consensus was discussed for a kind of continuous-time heterogeneous multi-agent system with input and communication time delay and a novel weighted couple-group consensus protocol was designed based on cooperation and competition interaction. In [34], the distributed consensus tracking problem of heterogeneous multi-agent chaotic delayed nonlinear systems was investigated and a distributed adaptive control law was proposed. In [35], group synchronization problem was analyzed for interacting clusters of non-identical systems and a special sufficient group synchronization condition was given in terms of the structure and strength of the couplings. In [37], the consensus of heterogeneous networks with undirected topology was investigated. In [38], the definition of the weighted consensus was put forward and the weighted average consensus of multi-agent system was discussed. In [39], the robust consensus controller was designed for a class of MASs by using the dynamics weighted edge. Furthermore, in many ecosystems, both cooperation and competition co-exist, it is necessary for us to consider cooperation-competition interactions in heterogeneous multi-agent systems. In [40], couple-group consensus was discussed for a kind of discrete-time heterogeneous multi-agent systems with cooperative–competitive interactions and time delays. In [41], a novel weighted group consensus protocol was proposed for a class of discrete-time heterogeneous multi-agent systems which was composed of first-order and second-order agents with communication and input time delays.

Although consensus or group consensus have been deeply studied for heterogeneous multi-agent systems, and a lot of meaningful results have been obtained. As we know, heterogeneous systems have more complex structure than homogeneous multi-agent systems, the relevant crucial topics become more difficult to handle. To the best of our knowledge, most of the existing works [12]–[19], [21], [24], [32] are only based on the agents' cooperative or competitive relationship. Many significant results on group consensus of have been given for homogeneous multi-agent system in which all agents share a single value [22], [24]. There are few reports on the group consensus problem of heterogeneous multi-agent systems with communication and input time delays, especially for the case with the multiple and different time delays. Especially, the results of related works mainly rely on some specially assumptions [12]–[19], [21]–[24].

However, in many real multi-agent systems, the relationship among the agents can be either cooperative or competitive, or both cooperative and competitive [22], [33], [40], [41]. Furthermore, input time delays can arise due to various reasons, including data processing, decision-making, computational times and so on. It may seriously affect and even destroy the stable performance of the systems [34], [42].

In short, positions, velocity, cooperative-competitive interactions, communication and input time delays can all affect the performance of heterogeneous multi-agent systems.

Considering all of these factors, group consensus protocol should be designed according to environment, positions, velocity, self-adaptive regulation, cooperation-competition mechanism, or other constraint conditions.

In this paper, the weighted couple-group consensus of a kind of heterogeneous multi-agent systems is investigated. There are three main contributions in this paper. Firstly, based on [15], [22], [33], [35], [38], [40], [41], a novel weighted couple-group consensus control protocol is proposed for a kind of continuous heterogeneous multi-agent system. In this new protocol, positions, speed, self-adaptive regulation and competitive interaction are all considered, it extends the scope of the existing research [22], [33], [41] and it is more related to real situation. Secondly, some sufficient conditions have been obtained for the weighted couple-group consensus by using graph theory, complex frequency analysis method and linear algebra theory. The time delay upper limit of this system can be computed and the couple-group consensus can be realized only if the conditions of the weighted parameters are satisfied. The results are all effective no matter the topology of the multi-agent systems is undirected or directed graph. Thirdly, some simulation examples are given to illustrate the effectiveness of the conclusion in the topology of undirected and directed graph.

The rest of this article is organized as follows. The second and the third section list some preliminary knowledge and problem description. The fourth section presents the main results and the related proof. In firth section, some simulation examples are given to show the validity of the obtained results. Finally, the conclusion is completed for this paper.

## II. PRELIMINARIES AND LEMMAS

In this section, some basic related definitions and lemmas are introduced. In this paper, C is a complex set, $R$ is a real number set, $I_N$ is a unit matrix, $N$ is the dimension, $\mathrm{Re}(Z)$ is the real part, $|Z|$ is the module of complex number Z, $\lambda_i(A)$ is the *ith* eigenvalue of matrix $A$, and $\det(A)$ is the determinant of the matrix. A directed or undirected simple graph(without self-loop and parallel lines) with N nodes is denoted as $G = (V(G), E(G), A)$ where $V(G) = \{v_1, v_2, \cdots, v_N\}$ represents the vertex set of graph G, $E(G) \subseteq V(G)\times V(G) = \{< v_i, v_j > |v_i \in V(G) \text{ and } v_j \in V(G)\}$ (directed graph) or $E(G) \subseteq V(G)\&VE(G) = \{(v_i, v_j)|v_i \in V(G) \text{ and } v_j \in V(G)\}$ (undirected graph) represents the edge set of graph G,and $A = (a_{ij})_{N\times N} \in R_{N\times N}$ is the adjacency matrix. $e_{ij} \in E(G)$ if and only if $a_{ij} > 0$, in this case, the node $v_i$ (agent) can receive information from the node $v_j$ (agent). $N(v_i) = \{v_j \in V(G)|e_{ij} =< v_i, v_j >\in E(G) \text{ or } e_{ij} = (v_i, v_j) \in E(G)\}$ is the neighbors of node $v_i$, and $d_i = \deg(v_i)$ represents the degree of node $v_i$, so the degree matrix D of the graph G is diag $\{d_1, d_2, \cdots d_N\}$. Therefore, $L = D–A$ is defined as the Laplacian matrix of graph G. It is obvious that the adjacency matrix $A$ is a symmetric matrix if and only if the graph is an undirected graph. In the following section, some basic definitions and related lemmas are given as below.

*Definition 1:* Graph $G = (V(G), E(G), A)$ is a bipartite graph if and only if the vertex set V(G) and the edge set $E(G)$ satisfy the following two conditions:

(i)There are two non-empty subsets $V_1(G)$ and $V_2(G)$ of V(G) which have the following properties:

$$V(G) = V_1(G) \cup V_2(G),\ V_1(G) \cap V_2(G) = \Phi;$$

$\Phi$ is an empty subset.

(ii)For each edge $(v_i, v_j) \in E(G)$, $v_i \in V_1(G)$ and $v_j \in V_2(G)$.

*Definition 2.*Heterogeneous multi-agent systems (1) and (2) are said to achieve p-group consensus ($p \geq 2$, if $p = 2$, then p-group consensus is called couple-group consensus) asymptotically if for any initial position and velocity values, we have

(i)$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \pi_i = \pi_j$, $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| \neq 0, \pi_i \neq \pi_j$;

(ii)$\lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0, \pi_i = \pi_j$, $\lim_{t\to\infty} \|v_i(t) - v_j(t)\| \neq 0, \pi_i \neq \pi_j$.

where $i, j \in \sigma_1 \cup \sigma_2$, and $\pi_1, \pi_2, \ldots, \pi_p$ is a partition of the set $\sigma_1 \cup \sigma_2(\sigma_1 \cup \sigma_2 = \pi_1 \cup \pi_2 \cup \cdots \cup \pi_p$ and $\pi_i \cap \pi_j = \phi$ for $i \neq j$, $\phi$ is the null set).

*Lemma 1 [22]:* Suppose that $Z = [z_1, z_2, \ldots, z_n]$, and $z_i. \in R$, $L \in R^{n\times n}$ is a Laplacian matrix of G. Then, the following four conditions are equivalent:

(i)All of the eigenvalue of $L$ have positive real parts except a simple zero eigenvalue;

(ii)$Lz = 0$ implies that $z_1 = z_2 = \cdots = z_n$;

1) The consensus is reached asymptotically if the system $\dot{Z} = -LZ$ is stable at the origin;
2) The directed graph of G possesses a directed spanning tree.

*Lemma 2.* If $G =< V, E >$ is a connected bipartite graph, then zero is the unique simple eigenvalue of D-A, rank(D-A) $= n - 1$ and all the nonzero eigenvalues of G have positive real part, D and A are the degree and adjacency matrix of G respectively.

*Lemma 3.* The inequality $\sin x \leq x$ holds for all nonnegative number $x \geq 0$.

*Proof:* Let $F(x) = x - \sin x$, then the derivative of $F(x)$ is

$$F'(x) = 1 - \cos x \geq 0$$

Then $F(x)$ is a monotonous increasing function when $x \geq 0$, and $F(x) = x - \sin x \geq F(0) = 0$.

The proof is completed.

## III. PROBLEM STATEMENT

Based on the aforementioned discussion, couple-group consensus of a heterogeneous multi-agent system will be discussed in this section. In order not to lose generality, it is assumed that this system is composed of second-order and first-order dynamics. The first $n$ multi-agents are second-order, and the remaining $m$ multi-agents are first-order, their dynamic models are described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \quad i \in \sigma_1 \end{cases} \tag{1}$$

$$\dot{x}_i(t) = \dot{u}_i(t), \quad i \in \sigma_2 \tag{2}$$

where $\sigma_1 = \{1, 2, \cdots, n\}$, $\sigma_2 = \{n+1, n+2, \cdots, n+m\}$, and $\sigma = \sigma_1 \cup \sigma_2$.

$u_i(t) \in R^P.x_i(t) \in R^P, v_i(t) \in R^P$ represent the $i$ th control input of multiple agents, state and velocity respectively. Without loss of generality, we suppose $P = 1$, if $P > 1$, some similar conclusions can be achieved by kronecker product.

For this kind of continuous heterogeneous multi-agent systems, the neighbor of each agent may have first-order or second-order dynamics. $N_{i,f}$ and $N_{i,s}$ represent the first-order or second-order of the neighbor set of the ith agent, so $N_i = N_{i,f} \cup N_{i,s} = \{ \mathrm{v}_j | e_{ij} \in E(G)\}$. The adjacency matrix $A$ can be written as

$$A = \begin{pmatrix} A_{ss} & A_{sf} \\ A_{fs} & A_{ff} \end{pmatrix}$$

where $A_{ss}$ denotes the adjacency matrix of the second -order agents, $A_{ff} \in R^{m\times m}$ denotes the adjacency matrix of the first-order agents. $A_{sf}$ is the adjacency matrix of the second-order agents to the first-order agents and $A_{fs}$ is the adjacency matrix of the first-order agents to the second-order agents. Hence, the Laplace matrix $L$ of heterogeneous multi-agent systems can be represented as

$$L = D - A = \begin{pmatrix} L_{ss} + D_{sf} & -A_{sf} \\ -A_{sf} & L_{ff} + D_{fs} \end{pmatrix}$$

In the Laplace matrix $L, L_{ss} = D_{ss} - A_{ss}$ and $L_{ff} = D_{ff} - A_{ff}$ denote the corresponding Laplace matrices of the second-order and first-order agents respectively, $D_{sf} = diag\left\{ \sum_{j\in N_{i,f}} a_{ij}, i \in \sigma_1 \right\}$ and $D_{fs} = diag\left\{ \sum_{j\in N_{i,s}} a_{ij}, i \in \sigma_2 \right\}$.

## IV. MAIN RESULTS

In this section, we mainly seek for the condition of the couple-group consensus for systems (1) and (2). Based on [22], [33], [41],a novel couple-group consensus control protocol is designed as:

$$\begin{cases} \mathrm{u}_i(t) = \alpha_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[x_j(t-\tau_{ij}) - x_i(t-\tau)\right] \right. \\ \qquad \left. - \sum_{j\in N_{Di}} a_{ij} \left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right] \right] \\ \qquad + \beta_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[v_j(t-\tau_{ij}) - v_i(t-\tau)\right] \right. \\ \qquad \left. - \sum_{j\in N_{Di}} a_{ij} \left[v_j(t-\tau_{ij}) + v_i(t-\tau)\right] \right], i \in \sigma_1, \end{cases} \tag{3}$$

and

$$\mathrm{u}_i(t) = \gamma_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[x_j(t-\tau_{ij}) - x_i(t-\tau)\right] - \sum_{j\in N_{Di}} a_{ij} \left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right] \right] + w_i(t)$$

$$\dot{w}_i(t) = \kappa_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[x_j(t-\tau_{ij}) - x_i(t-\tau)\right] - \sum_{j\in N_{Di}} a_{ij} \left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right] \right], \quad i \in \sigma_2 \tag{4}$$

where $\tau_{ij}$ denotes the communication delay between agent $i$ and agent $j, \tau$ is the input delay of agent, $w_i(t)$ is the self-adaptive controller, $\alpha_i, \beta_i, k_i$ are the weighted control parameters.

*Definition 3:* If the heterogeneous multi-agent systems (1) and (2) can realize group consensus under the weighted control protocol (3) and (4), this kind of group consensus is called the weighted group consensus.

*Remark 1:* In fact, couple-group consensus protocols in literatures [22], [24], [33] and [41] are special cases of the consensus protocol (3) and (4). Hence, the control protocol (3) and (4) may be called the unified couple-group control consensus and related results are more applicable to real problems. On the other hand, both self-adaptive regulation and cooperative-competitive mechanism are considered, protocol (3) and (4) are also self-adaptive and cooperative-competitive.

The closed form of the heterogeneous systems (1) and (2) with the protocol (3) and (4) can be rewritten as (5) and (6).

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = \alpha_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[x_j(t-\tau_{ij}) - x_i(t-\tau)\right] \right. \\ \qquad \left. - \sum_{j\in N_{Di}} a_{ij} \left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right] \right] \\ \qquad + \beta_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[v_j(t-\tau_{ij}) - v_i(t-\tau)\right] \right. \\ \qquad \left. - \sum_{j\in N_{Di}} a_{ij} \left[v_j(t-\tau_{ij}) + v_i(t-\tau)\right] \right], \quad i \in \sigma_1. \end{cases} \tag{5}$$

and

$$\dot{x}_i(t) = \gamma_i \left[ \sum_{j\in N_{Si}} a_{ij} \left[x_j(t-\tau_{ij}) - x_i(t-\tau)\right] - \sum_{j\in N_{Di}} a_{ij} \left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right] \right] + w_i(t),$$

$$\dot{w}_i(t) = \kappa_i\left[\sum_{j\in N_{Si}} a_{ij}\left[x_j(t-\tau_{ij}) - x_i(t-\tau)\right] - \sum_{j\in N_{Di}} a_{ij}\left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right]\right], i\in\sigma_2 \quad (6)$$

From remark1, since multifactorial effects are considered in multi-agent systems (5) and (6), it is more difficult for us to obtain the sufficient condition to ensure the success of couple-group consensus. By using the Laplace transform, the graph theory, generalized Nyquist criterion and Gerschgorin disc theorem, a sufficient couple-group consensus condition for multi-agent network systems (5) and (6) is given and a upper limit of the time delay is obtained in Theorem 1.

*Theorem 1:* Suppose the topology of the heterogeneous multi-agent network systems (5) and (6) is an undirected bipartite graph, couple-group consensus of this system can be achieved asymptotically if the following conditions ($H_1$) and ($H_2$) hold:

$$(H_1)\alpha_i < \beta_i^2 d_i, \quad i\in\sigma_1;\ \kappa_i < \gamma_i^2\hat{d}_i,\ i\in\sigma_2;$$
$$(H_2)\tau \in \left\{0, \min\left\{\frac{1}{4\beta_i d_i(1+\alpha_i^2 d_i^2)}, \frac{1}{4\gamma_i\hat{d}_i(1+\kappa_i^2\hat{d}_i^2)}\right\}\right\}, \quad i\in\sigma_1\cup\sigma_2.$$

where $d_i = d(v_i)$ is the degree of $v_i$ in $\sigma_1$, $\hat{d}_i = d(v_i)$ is the degree of $v_i$ in $\sigma_2$, $d_i = \sum_{v_j\in N_i} a_{ij}, i\in\sigma_1$, $\hat{d}_i = \sum_{v_j\in N_i} a_{ij}, i\in\sigma_2$.

*Proof:* Do Laplace transforms to (5) and (6), we have:

$$\begin{cases} sx_i(s) = v_i(s) \\ sv_i(s) = \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j(s)e^{-\tau_{ij}s} - x_i(s)e^{-\tau s}\right] - \sum_{j\in N_{Di}} a_{ij}\left[x_j(s)e^{-\tau_{ij}s} + x_i(s)e^{-\tau s}\right]\right] \\ \quad +\beta_i\left[\sum_{j\in N_{si}} a_{ij}\left[v_j(s)e^{-\tau_{ij}s} - v_i(s)e^{-\tau s}\right] - \sum_{j\in N_{Di}} a_{ij}\left[v_j(s)e^{-\tau_{ij}s} + v_i(s)e^{-\tau s}\right]\right] \end{cases} i\in\sigma_1, \quad (7)$$

and

$$\begin{cases} sx_i(s) = \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j(s)e^{-\tau_{ij}s} - x_i(s)e^{-\tau s}\right] - \sum_{j\in N_{Di}} a_{ij}\left[x_j(s)e^{-\tau_{ij}s} + x_i(s)e^{-\tau s}\right]\right] + w_i(s) \\ sw_i(s) = \kappa_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j(s)e^{-\tau_{ij}s} - x_i(s)e^{-\tau s}\right] - \sum_{j\in N_{Di}} a_{ij}\left[x_j(s)e^{-\tau_{ij}s} + x_i(s)e^{-\tau s}\right]\right], \end{cases} i\in\sigma_2 \quad (8)$$

where $x_i(s)$, $v_i(s)$ and $w_i(s)$ represent the corresponding Laplace transforms of $x_i(t)$, $v_i(t)$ and $w_i(t)$.Next, we define

$$x_s(s) = [x_1(s), x_2(s), \cdots, x_n(s)]^T,$$

and

$$x_f(s) = [x_{n+1}(s), x_{n+2}(s), \cdots, x_{n+m}(s)]^T,$$

$$\tilde{L} = \left(\tilde{l}_{ij}\right)_{(n+m)\times(n+m)} = \begin{cases} \sum_{j\in N_i} a_{ij}e^{-\tau s}, & i=j \\ -e^{-\tau_{ij}s}a_{ij}, & i\neq j \end{cases}$$

After some manipulations, systems (7) and (8) can be rewritten as (9), shown at the bottom of this page, where $d_i = \sum_{v_j\in N_i} a_{ij}, i\in\sigma_1$, $\hat{d}_i = \sum_{v_j\in N_i} a_{ij}, i\in\sigma_2$, $d_i$ and $\hat{d}_i$ represent the degree of the agent $i$.

Let $x(s) = \left[x_s(s), x_f(s)\right]^T$, then systems (9) can be described as matrix form:

$$sx(s) = \Gamma(s)x(s) \quad (10)$$

and

$$\Gamma(s) = \begin{pmatrix} \frac{-s^2B_2 - B_2B_1^{-1}\left(\tilde{L}_{ss}+\tilde{D}_{sf}\right)}{e^{-\tau s}} & \frac{-B_2\left(B_1^{-1}+B_2^{-1}s\right)\tilde{A}_{sf}}{e^{-\tau s}} \\ \frac{-B_3\left(B_4^{-1}+B_3^{-1}s\right)\tilde{A}_{fs}}{e^{-\tau s}} & \frac{-s^2B_3 - B_3B_4^{-1}\left(\tilde{L}_{ff}+\tilde{D}_{fs}\right)}{e^{-\tau s}} \end{pmatrix}.$$

where,

$$B_1 = \begin{pmatrix} 1/\alpha_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\alpha_n, \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 1/\beta_1 d_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\beta_n d_n, \end{pmatrix}$$

$$\begin{cases} sx_s(s) = \frac{-s^2x_s(s) - \alpha_i(\tilde{L}_{ss}+\tilde{D}_{sf})x_s(s) + (\alpha_i+\beta_i s)(-\tilde{A}_{sf})x_f(s)}{\beta_i d_i e^{-\tau s}}, & i\in\sigma_1 \\ sx_f(s) = \frac{-s^2x_s(s) - \kappa_i(\tilde{L}_{ff}+\tilde{D}_{fs})x_s(s) + (\kappa_i+\gamma_i s)(-\tilde{A}_{fs})x_f(s)}{\gamma_i\hat{d}_i e^{-\tau s}}, & i\in\sigma_2 \end{cases} \quad (9)$$

$$\mathrm{B}_3 = \begin{pmatrix} 1/\gamma_{n+1}\hat{d}_{n+1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\gamma_{n+m}\hat{d}_{n+m}, \end{pmatrix}$$

$$\mathrm{B}_4 = \begin{pmatrix} 1/\kappa_{n+1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\kappa_{n+m} \end{pmatrix}.$$

Then the characteristic equation of the system (10) is

$$F(s) = \det(sI - \Gamma(s)) = 0 \tag{11}$$

When $s = 0$, $\det(sI - \Gamma(s)) = \left(-\mathrm{B}_2\mathrm{B}_1^{-1}\right)^n \left(-\mathrm{B}_3\mathrm{B}_4^{-1}\right)^m \det(\tilde{L})$.

Based on the Lemma 1, zero is the unique simple eigenvalue of $L$, then $s = 0$ is the unique simple root of equation (11).

When $s \neq 0$, $F(s) = \det(sI - \Gamma(s)) = s^{n+m}\det(I + G(s))$, and

$$G(s) = \frac{-\Gamma(s)}{s} = \begin{pmatrix} \frac{s^2\mathrm{B}_2 + \mathrm{B}_2\mathrm{B}_1^{-1}\left(\tilde{L}_{ss}+\tilde{D}_{sf}\right)}{se^{-\tau s}} & \frac{\mathrm{B}_2\left(\mathrm{B}_1^{-1}+\mathrm{B}_2^{-1}s\right)\tilde{A}_{sf}}{se^{-\tau s}} \\ \frac{\mathrm{B}_3\left(\mathrm{B}_4^{-1}+\mathrm{B}_3^{-1}s\right)\tilde{A}_{fs}}{se^{-\tau s}} & \frac{s^2\mathrm{B}_3 + \mathrm{B}_3\mathrm{B}_4^{-1}\left(\tilde{L}_{ff}+\tilde{D}_{fs}\right)}{se^{-\tau s}} \end{pmatrix}.$$

Let $s = j\omega(\forall\omega \in R)$, based on the generalized Nyquist criterion, if and only if $(-1, j0)$ is not located in the internal area of the Nyquist curve of $G(j\omega)$,then the roots of (11) are lied inside of the unit circle in the complex plane, and the couple-group consensus of multi-agent systems (5) and (6) is realized. Furthermore, by using Gerschgorin disc theorem, we can obtain

$$\lambda(G(j\omega)) \in \{G_i, i \in \sigma_1\} \cup \{G_i, i \in \sigma_2\} \tag{12}$$

When $i \in \sigma_1$,

$$G_i = \left\{ x : x \in \mathrm{C}, \left| x - \frac{-\omega^2 + \alpha_i d_i e^{-j\omega\tau}}{\beta_i d_i e^{-j\omega\tau} j\omega} \right| \le \sum_{j\in N_i} \left| \frac{-(\alpha_i + j\omega\beta_i)\, a_{ij} e^{-j\omega\tau_{ij}}}{\beta_i d_i e^{-j\omega\tau} j\omega} \right| \right\} \tag{13}$$

By the inequality (13), the center of the disk is

$$G_i(j\omega) = \frac{-\omega^2 + \alpha_i d_i e^{-j\omega\tau}}{\beta_i d_i e^{-j\omega\tau} j\omega} \tag{14}$$

Let $\omega_{i0}$ be the first cross of the curve $G_i(j\omega)$ on the real axis, based on (14), one can obtain the equation

$$\cos\omega_{i0}\tau = \frac{\alpha_i d_i}{\omega_{i0}^2} \tag{15}$$

Since $|\cos\omega\tau| = \left|\frac{\alpha_i d_i}{\omega^2}\right| < 1$, then we obtain

$$\omega > \sqrt{\alpha_i d_i} \tag{16}$$

Set $d_i = \sum_{v_j\in N_i} a_{ij}$, $i \in \sigma_1$, because the point $(-a, j0)$ $(a \geq 1)$ is not enclosed in $G_i$, then we have the following inequality

$$\left| -a - \frac{-\omega^2 + \alpha_i d_i e^{-j\omega\tau}}{\beta_i d_i e^{-j\omega\tau} j\omega} \right| > \sum_{j\in N_i} \left| \frac{-(\alpha_i + j\omega\beta_i)\, a_{ij} e^{-j\omega\tau_{ij}}}{\beta_i d_i e^{-j\omega\tau} j\omega} \right| \tag{17}$$

By the Euler formula and inequality (17), the following inequality is achieved

$$a^2 - 2\frac{\omega\sin\omega\tau}{\beta_i d_i}a + \frac{\omega^2}{\beta_i^2 d_i^2} - 2\frac{\alpha_i\omega\cos\omega\tau}{\beta_i} - 1 > 0 \tag{18}$$

Set

$$f(a) = a^2 - 2\frac{\omega\sin\omega\tau}{\beta_i d_i}a + \frac{\omega^2}{\beta_i^2 d_i^2} - 2\frac{\alpha_i\omega\cos\omega\tau}{\beta_i} - 1 \tag{19}$$

Then the conditions $a \geq 1$ and $f(a) > 0$ are satisfied if and only if $f(1) > 0$ and $f'(1) > 0$ are satisfied.

From $f(1) > 0$, we have

$$\omega > 2\beta_i d_i\sqrt{1 + \alpha_i^2 d_i^2} \tag{20}$$

From $f'(1) > 0$ and lemma3, we obtain

$$\tau < \frac{\beta_i d_i}{\omega^2} \tag{21}$$

Combining (20) and (21), we have

$$\tau < \frac{1}{4\beta_i d_i(1 + \alpha_i^2 d_i^2)} \tag{22}$$

On the other hand, $f'(1) = 2 \times 1 - 2\frac{\omega\sin\omega\tau}{\beta_i d_i} \times 1 > 0$, then we obtain

$$|\sin\omega\tau| < \frac{\beta_i d_i}{\omega} \tag{23}$$

If $\omega < \beta_i d_i$, then the inequality (23) holds obviously. Combing (16), the following inequality can be easily obtained

$$\sqrt{\alpha_i d_i} < \omega < \beta_i d_i \tag{24}$$

The inequality (24) means $\alpha_i < \beta_i^2 d_i$.

When $i \in \sigma_2$, let $\hat{d}_i = \sum_{v_j\in N_i} a_{ij}$, the corresponding inequality is obtained by Gerschgorin disc theorem

$$G_i = \left\{ x : x \in \mathrm{C}, \left| x - \frac{-\omega^2 + \kappa_i \hat{d}_i e^{-j\omega\tau}}{\gamma_i \hat{d}_i e^{-j\omega\tau} j\omega} \right| \le \sum_{j\in N_i} \left| \frac{-(\kappa_i + j\omega\beta_i)\, a_{ij} e^{-j\omega\tau_{ij}}}{\gamma_i \hat{d}_i e^{-j\omega\tau} j\omega} \right| \right\} \tag{25}$$

After similarly calculation, we obtain the following two inequalities

$$\kappa_i < \gamma_i^2 \hat{d}_i \quad \text{and } \tau < \frac{1}{4\gamma_i \hat{d}_i(1 + \kappa_i^2 \hat{d}_i^2)} \tag{26}$$

The proof of Theorem 1 is completed.

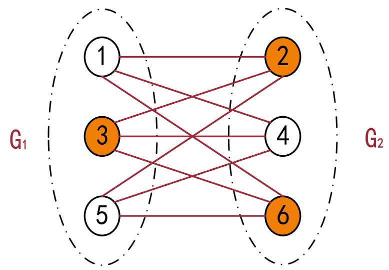



**FIGURE 1.** **The bipartite digraph topology of the heterogeneous multi-agent systems (5) and (6).**

*Remark 2:* In Theorem 1, a sufficient couple-group consensus condition of heterogeneous multi-agent systems (5) and (6) has been obtained. The conclusion shows that the couple-group consensus condition is not related to commu-nication time delay, it is only related to input time delay, and the upper bound of input time-delay can also be computed according to the given parameters.

*Remark 3:* In addition, the topology of the heterogeneous multi-agent network systems (5) and (6) is an undirected bipartite graph, the traditional in-degree balance condition needs not to be considered and the result is more practical to corresponding heterogeneous multi-agent systems.

By using the same technique, a similar result can be obtained for heterogeneous multi-agent systems with the topology of a directed bipartite graph.

*Corollary 1:* Suppose the topology of the heterogeneous multi-agent systems (5) and (6) is a directed bipartite graph, the systems' couple-group can be achieved asymptotically if the following conditions ($H_3$) and ($H_4$) hold:

$$(H_3)\alpha_i < \beta_i^2 d_i, \quad i \in \sigma_1, \kappa_i < \gamma_i^2 \hat{d}_i,\ i \in \sigma_2;$$

$$(H_4)\tau \in \left\{0, \min\left\{\frac{1}{4\beta_i d_i(1+\alpha_i^2 d_i^2)}, \frac{1}{4\gamma_i \hat{d}_i(1+\kappa_i^2 \hat{d}_i^2)}\right\}\right\}, \quad i \in \sigma_1 \cup \sigma_2.$$

where $d_i = \sum_{v_j \in N_i} a_{ij}, i \in \sigma_1$, $\hat{d}_i = \sum_{v_j \in N_i} a_{ij}, i \in \sigma_2$, $d_i$ and $\hat{d}_i$ represent the in-degree of the agent $i$.

The conclusion is obvious.

On basis of Theorem 1, the following corollary 2 gives a sufficient condition for the group consensus of multi-agent systems (5) and (6) in a topology of an undirected bipartite graph.

*Corollary 2:* Suppose that the topology of the heterogeneous multi-agent systems (5) and (6) is an undirected bipartite graph, the systems' couple-group consensus can be achieved asymptotically if the following conditions ($H_5$) and ($H_6$) hold:

$$(H_5)\alpha_i < \beta_i^2 d_i, i \in \sigma_1, \kappa_i < \gamma_i^2 \hat{d}_i, i \in \sigma_2;$$

$$(H_6)\tau \in \left[0, \min\left\{\sqrt{\frac{1}{2\alpha_i d_i}}, \sqrt{\frac{1}{2\kappa_i \hat{d}_i}}\right\}\right], \quad i \in \sigma_1 \cup \sigma_2.$$

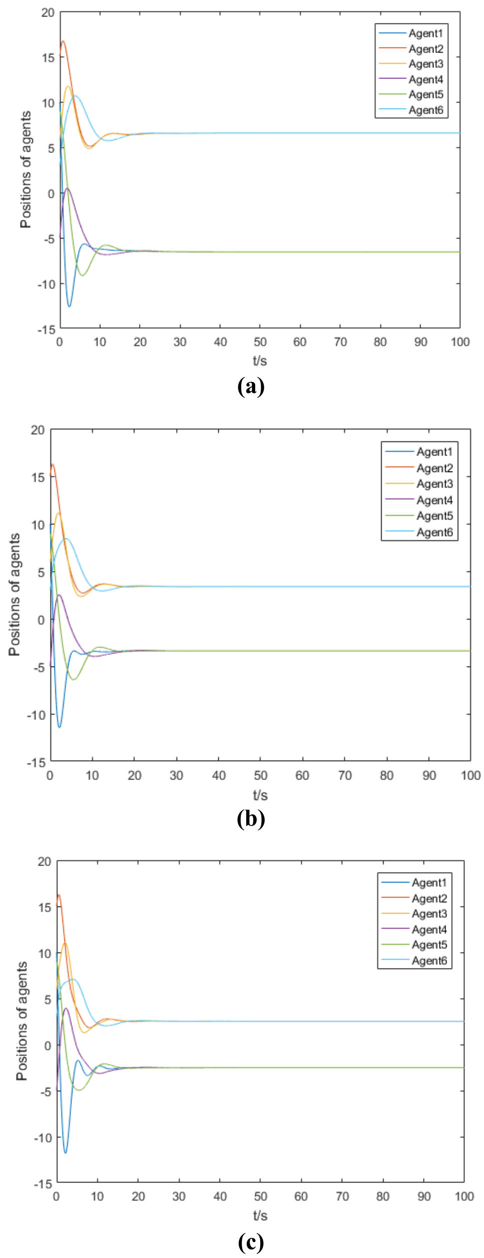



**FIGURE 2.** **The state trajectories of agents with bipartite topology Fig. 1 when the input time-delay is $\tau = 0.1$ under different communication time-delay (a) $\tau_{ij} = 0$ (b)$\tau_{ij} = 0.5$ (c)$\tau_{ij} = 0.9$.**

where $d_i = \sum_{v_j \in N_i} a_{ij}, i \in \sigma_1$, $\hat{d}_i = \sum_{v_j \in N_i} a_{ij}, i \in \sigma_2$, $d_i$ and $\hat{d}_i$ represent the degree of the agent $i$.

*Proof:* when $i \in \sigma_1$, according to Taylor formula, equation (15) can be written as

$$1 - \frac{\omega^2 \tau^2}{2} = \frac{\alpha_i d_i}{\omega^2} \tag{27}$$

Therefore,

$$\tau^2 = \frac{2\left(\omega^2 - \alpha_i d_i\right)}{\omega^4} \tag{28}$$

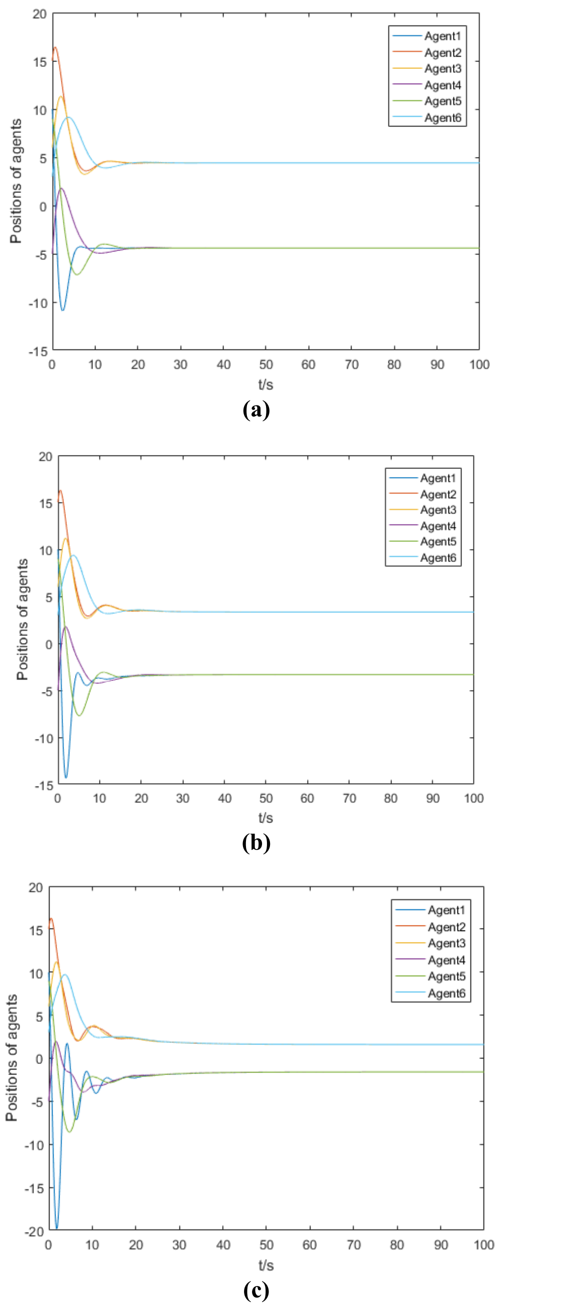



**FIGURE 3.** **The state trajectories of agents with bipartite topology in Fig.1 when the communication time-delay is $\tau = 0.3$ under different input time-delay (a) $\tau = 0$ (b)$\tau = 0.3$ (c)$\tau = 0.5$.**

and

$$\frac{d\tau^2}{d\omega} = \frac{-4(\omega^2 - 2\alpha_i d_i)}{\omega^5} \tag{29}$$

From equation (29),when $\omega^2 < 2\alpha_i d_i$, $\frac{d\tau^2}{d\omega} > 0$, the time-delay $\tau$ increases with the increase of $\omega$; when $\omega^2 > 2\alpha_i d_i$, $\frac{d\tau^2}{d\omega} < 0$, the delay $\tau$ decreases with the increase of $\omega$, so when $\omega^2 = 2\alpha_i d_i$, there is a delay upper bound $\tau_{\max} = \sqrt{\frac{1}{2\alpha_i d_i}}$.

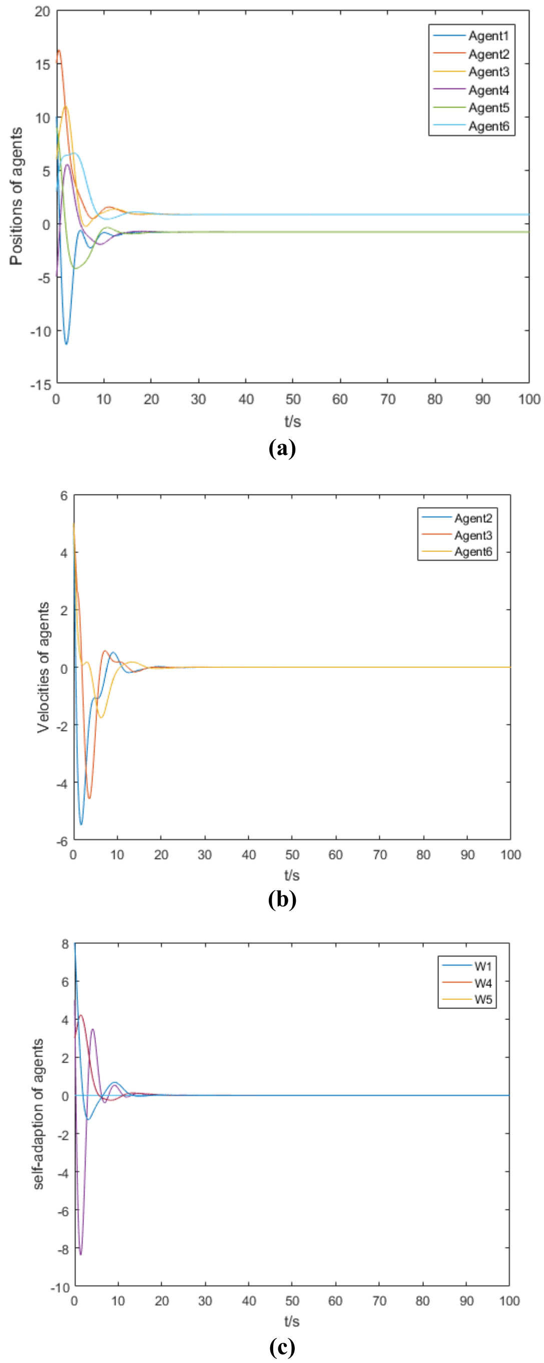



**FIGURE 4.** **In Fig.1, the state trajectories of the agent under undirected topology with different input time delays $\tau_1 = 0.1$, $\tau_2 = 0.37$, $\tau_3 = 0.25$, $\tau_4 = 0.68$, $\tau_5 = 0.38$, $\tau_6 = 0.53$ and same communication delay is $\tau_{ij} = 0.8$. (a) Positions, (b) Velocities, (c) self-adaptive regulations.**

Similarly, when $i \in \sigma_2$, we can also obtain $\tau_{\max} = \sqrt{\frac{1}{2\kappa_i \hat{d}_i}}$. The proof of corollary 2 is completed.

*Remark4:* In Corollary 1, the topology of the graph is different from the topology of the graph in Theorem 1; The topology of the graph in Corollary 2 is same as the topology of the graph in Theorem 1, but the obtained upper bound of time delay becomes bigger.

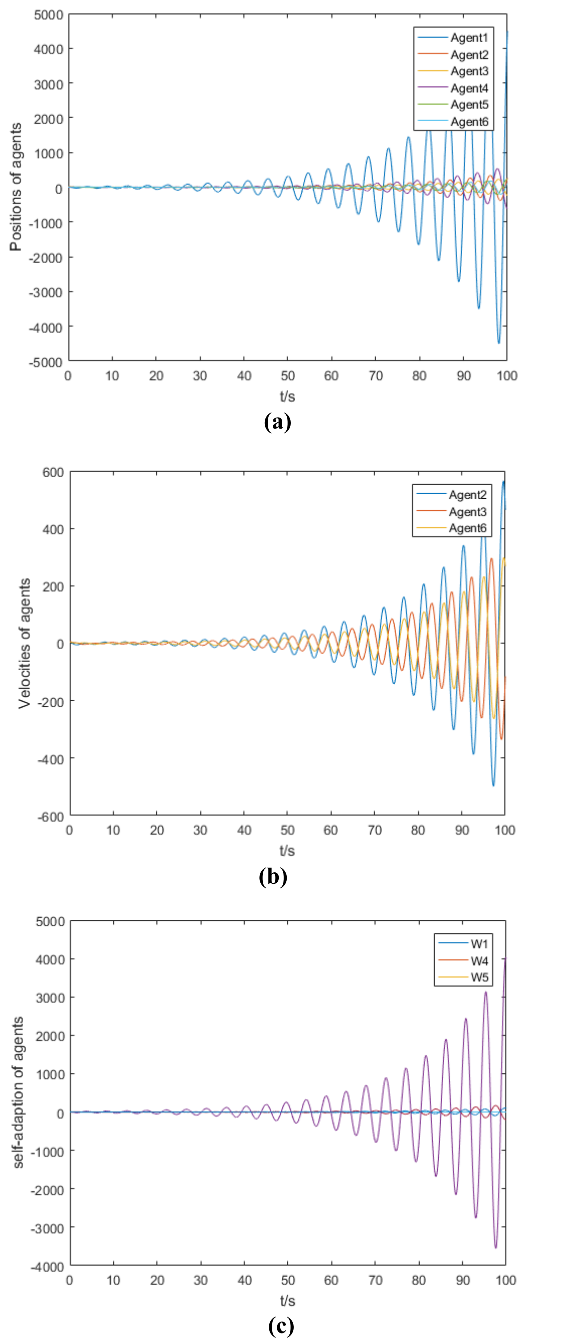



**FIGURE 5.** In Fig.1, the state trajectories of the agent under undirected topology with different input time delays $\tau_1 = 0.7$, $\tau_2 = 0.37$, $\tau_3 = 0.25$, $\tau_4 = 0.68$, $\tau_5 = 0.38$, $\tau_6 = 0.53$ and same communication delay is $\tau_{ij} = 0.8$. (a) Positions,(b )Velocities, (c) self-adaptive regulations.

*Remark5:* Obviously, when the weighted parameters are fixed, the upper limit of the time delay in corollary 2 is bigger than the upper limit of the time delay in Theorem 1. Hence, the interval of time delay in corollary 2 is wider than Theorem 1 under the same weighted parameters.

## V. SIMULATIONS EXAMPLES

In this section, some simulations are given to show the validity of the proposed results.

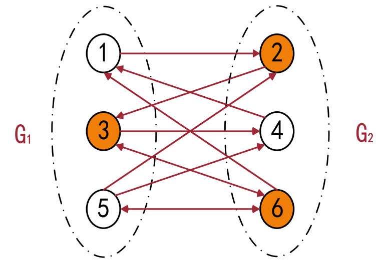



**FIGURE 6.** The directed bipartite graph topology of the heterogeneous multi-agent systems (5) and (6).

Fig.1 is a graph with bipartite topology of $G_1$ and $G_2$, and the agents belong to two subgroups. The agents 1,4,5 belong to a group, they are first-order agents. The agents 2,3,6 belong to another group, they are second-order agents. Hence, the agents 1,2,3,4,5,6, are composed of a heterogeneous multi-agents with bipartite topology.

*Example 1:* In Fig.1, the weight of each edge is $1(a_{ij} = 1, i, j \in \{1, 2, 3, 4, 5, 6\})$, then the degree of each vertex is $d_1 = 3, d_2 = 3, d_3 = 3, d_4 = 3, d_5 = 3, d_6 = 3, d_7 = 3$. The initial state values of the multi-agents systems (5) and (6) are $\{10, 15, 6, -5, 9, 3\}$.The control parameters are assumed as follows:

$$\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\} = \{3, 0.11, 0.18, 5, 2.3, 0.06\},$$

$$\{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\} = \{0.35, 1.2, 0.8, 0.12, 0.2, 1\}$$

$$\{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\} = \{1.5, 0.2, 0.25, 2, 0.9, 0.15\},$$

$$\{k_1, k_2, k_3, k_4, k_5, k_6\} = \{0.36, 2, 1.5, 0.04, 0.1, 1\}$$

Based on Theorem 1 and the control parameters, the spans of time delay span are calculated as

$$\tau_1 \in [0,0.109]\,,\ \tau_2 \in [0, 0.375]\,,\ \tau_3 \in [0,0.258]\,,$$

$$\tau_4 \in [0, 0.684]\,,\ \tau_5 \in [0, 0.382]\,,\ \tau_6 \in [0, 0.538]\,.$$

In Fig.2, the input time delay is $\tau = 0.1$(it satisfies the condition of Theorem 1), and communication delays are $\tau_{ij} = 0, \tau_{ij} = 0.5$ and $\tau_{ij} = 0.9$;In this case, the influence of the different communication delays which have made on group consensus of systems (5) and (6) is clear. In Fig.3, group consensus of systems (5) and (6) is realized when the input time delay is $\tau = 0, \tau = 0.03$ and $\tau = 0.07$ if the communication time-delay is $\tau_{ij} = 0.3$.

*Example 2:* The initial state values of the multi-agents systems (5) and (6) the control parameters are same as Example 1. In Fig.4, group consensus of systems (5) and (6) is realized when the input time delays are $\tau_1 = 0.1$, $\tau_2 = 0.37, \tau_3 = 0.25, \tau_4 = 0.68, \tau_5 = 0.38, \tau_6 = 0.53$ and the communication time-delay is $\tau_{ij} = 0.8$.

*Example 3:* The initial state values of the multi-agents systems (5) and (6) the control parameters are same as Example 1. In Fig.5, group consensus of systems (5) and (6) cannot be realized when the input time delays are $\tau_1 = 0.7, \tau_2 = 0.37, \tau_3 = 0.25, \tau_4 = 0.68, \tau_5 = 0.38, \tau_6 = 0.53$

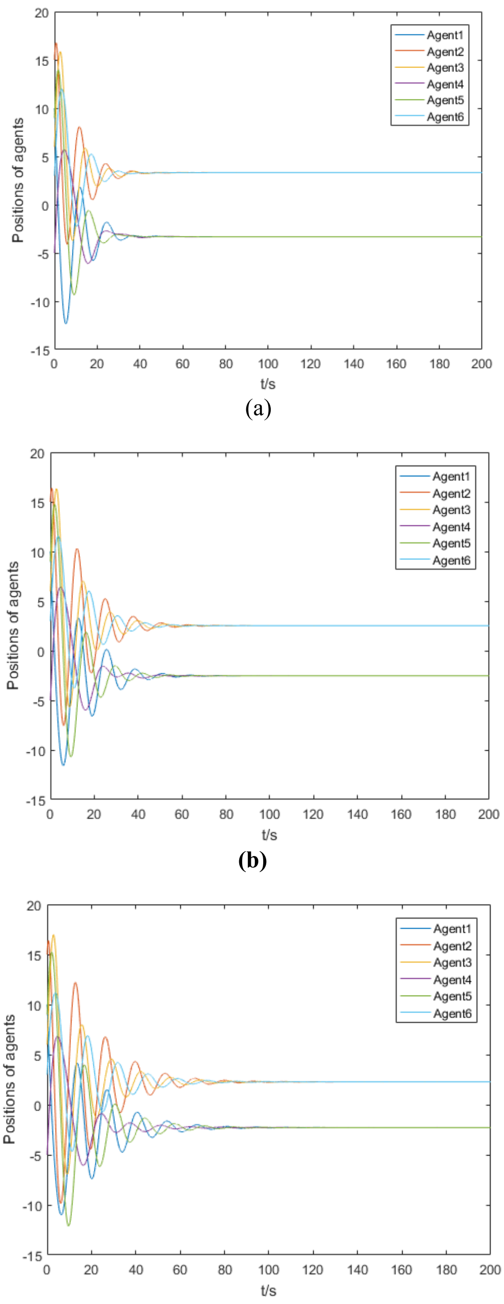



**FIGURE 7.** **The state trajectories of the agent under directed topology with same input time delays $\tau = 0.12$ and different communication delays: (a) $\tau_{ij} = 0$ (b)$\tau_{ij} = 0.5$ (c)$\tau_{ij} = 0.9$.**

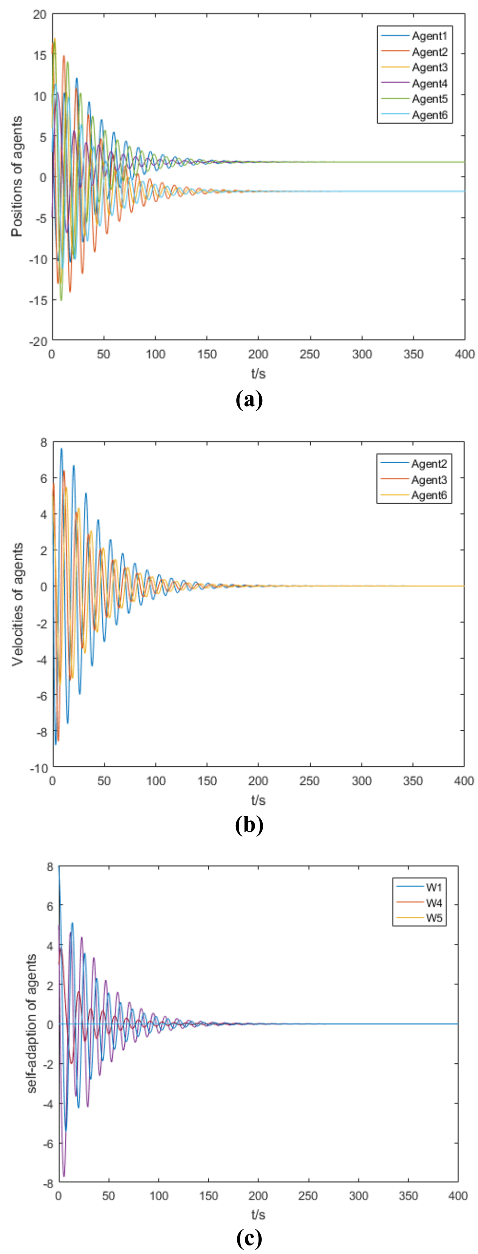



**FIGURE 8.** **The state trajectories of the agent under directed topology with different input time delays $\tau_1 = 0.23$, $\tau_2 = 0.58$, $\tau_3 = 0.44$, $\tau_4 = 1.03$, $\tau_5 = 1.23$,$\tau_6 = 0.82$ and same communication delay is $\tau_{ij} = 0.6$. (a) Positions, (b) Velocities, (c) self-adaptive regulations.**

and the communication time-delay is same as $\tau_{ij} = 0.8$. In this case, $\tau_1 = 0.7$ is not satisfied the condition of Theorem1.

*Example 4:* In Fig.6, the weight of each edge is $1(a_{ij} = 1, i, j \in \{1, 2, 3, 4, 5, 6\})$, then the in-degree of each vertex is $d_1 = 2, d_2 = 2, d_3 = 2, d_4 = 2, d_5 = 1, d_6 = 2$.The initial state values of the multi-agents systems (5) and (6) are $\{7, 3, 12, -6, 5, -15\}$.The control parameters are assumed as

$$\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\} = \{3, 0.11, 0.18, 5, 2.3, 0.06\},$$
$$\{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6\} = \{0.35, 1.2, 0.8, 0.12, 0.2, 1\},$$
$$\{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\} = \{1.5, 0.2, 0.25, 2, 0.9, 0.15\},$$
$$\{k_1, k_2, k_3, k_4, k_5, k_6\} = \{0.36, 2, 1.5, 0.04, 0.1, 1\}.$$

According to Corollary 1, the spans of time delay are calculated as $\tau_1 \in [0, 0.235]$, $\tau_2 \in [0, 0.596]$, $\tau_3 \in [0, 0.442]$, $\tau_4 \in [0,1.035]$, $\tau_5 \in [0, 1.237]$, $\tau_6 \in [0, 0.821]$. In Fig.7, the input delay is $\tau = 0.12$ (It satisfies the condition

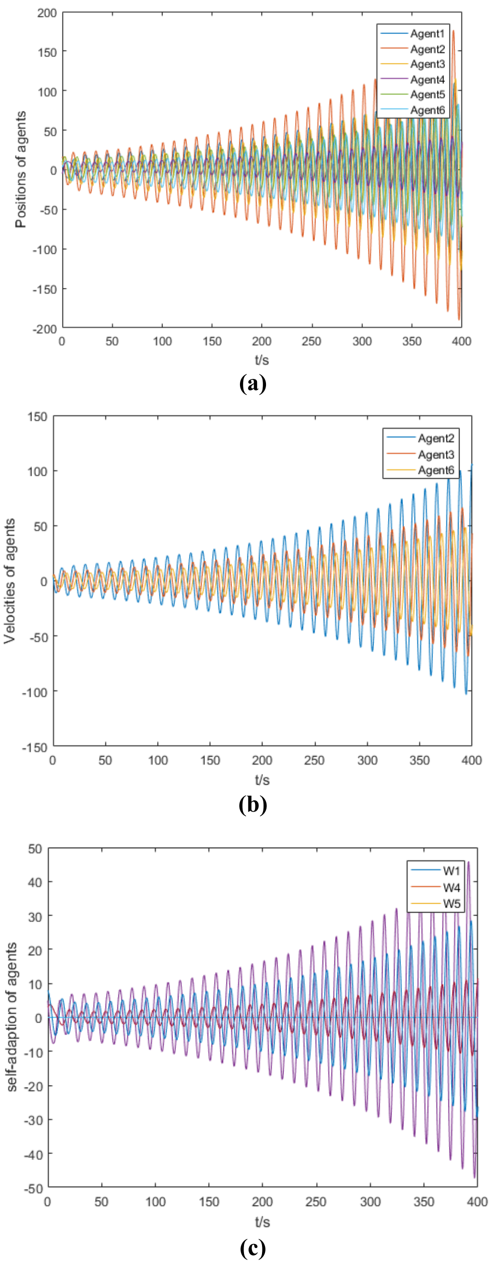



**FIGURE 9. The state trajectories of the agent directed topology with different input time delays $\tau_1 = 0.23$, $\tau_2 = 0.98$, $\tau_3 = 0.44$, $\tau_4 = 1.03$, $\tau_5 = 1.23$, $\tau_6 = 0.82$ and same communication delay is $\tau_{ij} = 0.6$. (a) Positions (b) Velocities (c) self-adaptive regulations.**

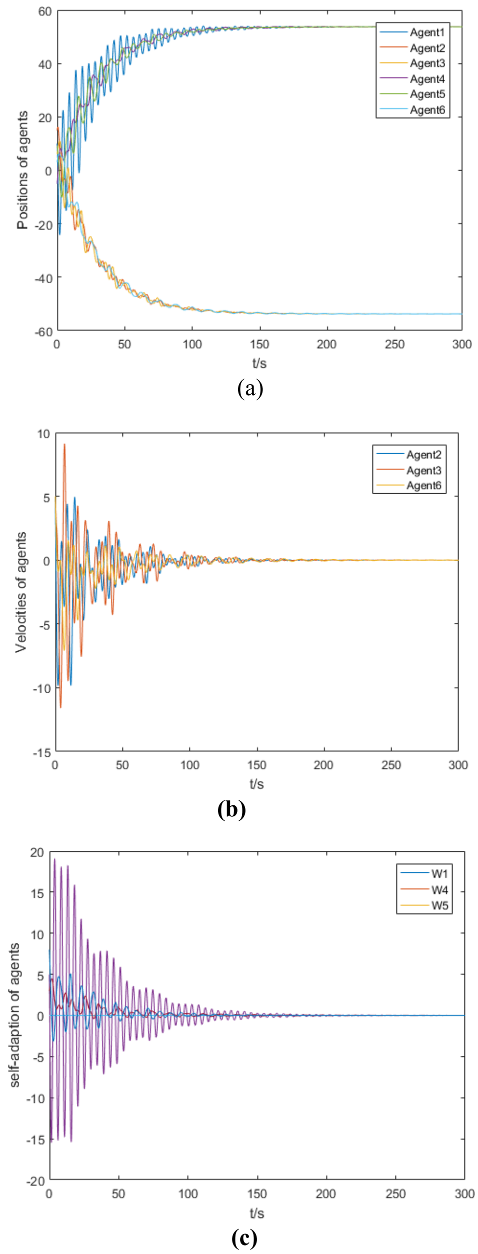


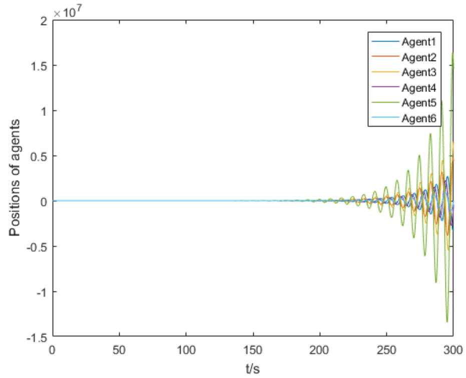

**FIGURE 10. The state trajectories of the agent undirected topology with different input time delays $\tau_1 = 0.6, \tau_2 = 1.2, \tau_3 = 0.9, \tau_4 = 2.0$, $\tau_5 = 1.2, \tau_6 = 1.6$ and same communication delay is $\tau_{ij} = 0.8$. (a) Positions, (b) Velocities, (c) self-adaptive regulations.**

of Corollary1 ) when the communication delays are $\tau_{ij} = 0$, $\tau_{ij} = 0.5$ and $\tau_{ij} = 0.9$ respectively. Fig.7 shows the weighted couple-group consensus of systems (5) and (6) is realized. Fig.8 also shows the weighted group consensus of system (5) and (6) is realized in a directed topology with different input time delays $\tau_1 = 0.23, \tau_2 = 0.58, \tau_3 = 0.44, \tau_4 = 1.03, \tau_5 = 1.23, \tau_6 = 0.82$ and the same communication delay $\tau_{ij} = 0.6$. In Fig.9, it is obvious that $\tau_2 = 0.98$ does not satisfy the condition of corollary1 and the group consensus of systems (5) and (6) cannot be realized.

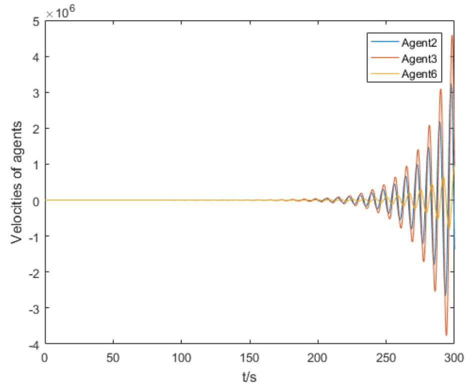
*Example 5:* The initial state values of the multi-agents systems (5) and (6) the control parameters are same as Example 1. Based on Corollary 2, the spans of time delay are calculated as

$$\tau_1 \in [0,0.680]\,,\ \tau_2 \in [0,1.230]\,,\ \tau_3 \in [0,0.961]\,,$$
$$\tau_4 \in [0,2.041]\,,\ \tau_5 \in [0,\ 1.290]\,,\ \tau_6 \in [0,1.666]\,.$$

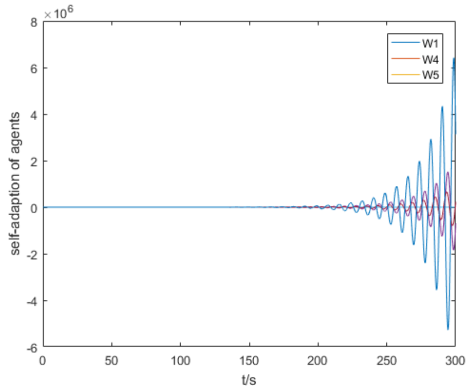
Fig.10 shows the weighted couple-group consensus of systems (5) and (6) is realized if $\tau_i (i = 1,2,3,4,5,6)$ are given as $\tau_1 = 0.6, \tau_2 = 1.2, \tau_3 = 0.9, \tau_4 = 2.0, \tau_5 = 1.2, \tau_6 = 1.6$.




(a)




(b)




(c)

**FIGURE 11. The state trajectories of the agent undirected topology with different input time delays $\tau_1 = 0.6, \tau_2 = 1.2, \tau_3 = 0.9, \tau_4 = 2.0, \tau_5 = 1.5, \tau_6 = 1.6$ and same communication delay is $\tau_{ij} = 0.8$. (a) Positions, (b) Velocities, (c) self-adaptive regulations.**

In Fig.11, $\tau_5 = 1.5 \notin [0, 1.290]$, the weighted group consensus of systems (5) and (6) cannot be realized.

## VI. CONCLUSION

In this paper, a novel weighted couple-group consensus protocol has been proposed for a kind of a continuous heterogeneous multi-agent systems based on self-adaptive regulation and cooperative-competitive mechanism. By using Laplace transform, algebra graph theory and linear algebra theory, some sufficient conditions have been achieved to ensure the success of the couple-group consensus of the heterogeneous multi-agent systems. Some simulation examples are given to show the validity of the results. In our future work, we will focus on the couple-group consensus of heterogeneous multi-agent systems in switching topology or stochastic environment.

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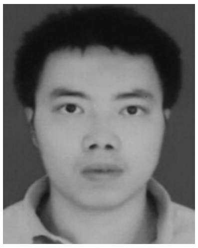

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