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A New Fuzzy Multiobjective Geometric Programming in Double Sampling in Presence of Non-Response

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ABSTRACT Non-linear programming problems can be solved efficiently by geometric programming in which non-linear expressions can be expressed by an exponential or power functions. In this paper, Double Sampling Stratification (DSS) strategy is used in multivariate stratified population with unidentified strata weights, for optimum sampling design (OSD) in the presence of non-response to estimate the unidentified population means. The double sampling problem in the presence of non-response is formulated as Fuzzy Multiobjective Convex Programming Problem (FMOCPP). Then, we convert FMOCPP into Fuzzy Single Objective Convex Problem (FSOCP) by using membership function. Dual solution is obtained using LINGO Software by solving FSOCP. The optimum allocations of sample sizes of respondents and non- respondents for both the phases are obtained by applying dual solutions and primal-dual relationship theorem. The given technique is illustrated by projecting a numerical problem.

INDEX TERMS Double sampling, dual solutions (DS), fuzzy programming (FP), geometric programming (GP), non-linear programming (NLP), non-response, non- respondents (NR), orthogonality condition (OC), respondents (R).

I. INTRODUCTION

The problem of non-response is frequently experiencing in conveyance of the sample surveys. The meaning of the non-response is that, we cannot get the requisite information for all selected units in the sample for one cause or the other. In case, the individual sampling unit is taken, then the chosen person may be reluctant to give the requisite information, or he/she may be out of station when the interviewer called. The incomplete sample data are obtained when non-response occurs, that affects the quality of estimates of the unidentified population parameters. The comprehensive details of the non-response problem in sample surveys have been given by [1]. The first classical non-response theory in mailed surveys was given by [2], in which mailing the questionnaires was done in the first attempt and in a second attempt, the individual interview to a sub-sample of the NR. The estimate for the population mean was constructed with these two attempts by them and the expression for the sampling variance of the estimate is obtained. They also worked out the optimum sampling fraction among the NR. In order to increase the response rate, [3] given the extensive study of the Hansen and Hurwitz's technique in which he send the waves of questionnaires to the non-respondent units. The comprehensive El-Badry's method and the application of Hansen and Hurwitz's technique with diverse models were discussed by [4]. The method proposed by [5] for the assortment of sub-samples of NR where the rate of sub-sampling was varying. The problem of optimum allocation in stratified sampling in presence of non-response for fixed cost as well as for fixed precision of the estimate was given by [6]. Many authors have discussed the problems of non-response and used different techniques and methods for the solution of the problem. Some of them are: [7]–[10]. The unidentified strata weights in stratified sampling are estimated by applying double sampling design. A huge simple random sample from

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the population with unstratification is strained and sampled units from each stratum are recorded to estimate the unidentified strata weights at first stage. A stratified random sample is then obtained comprising of simple random sub-samples out of the earlier selected units of the strata as described by [7]–[9], [12], [16]. In the case of non-response, the sub-samples may be separated into the classes of R and NR. A second sub-sample is then strained out of NR and an effort is completed to obtain the information. This method is called DSS. DSS estimators based on the sub-sampling of NR was derived by [11]. Many researchers have worked in the field of double sampling design, mathematical programming in double sampling design, [12]–[21] and many others.

The executive real life problems of sampling design, sociological, medical, hospital, household, economical, environmental, engineering and technological, finance, managerial problems, chemical problems and industrial areas are of conflicting objectives. It is significant to comprehend that multiple objectives are frequently non-commensurable and in divergence with each other. The system can neither be formulated nor be solved efficiently by conventional mathematics-based optimization techniques and also probability-based stochastic optimization approaches without distinguishable information. Nevertheless, for modeling and optimizing these types of systems, fuzzy set theory and fuzzy programming techniques afford a constructive and proficient method. Firstly the conception of fuzzy set theory was projected by [22]. Afterward, [23] have used the fuzzy set theory for decision-making problems. The research works in the field of fuzzy set theory and fuzzy programming have been done by many authors as: [24]–[28]. The researchers have worked in the area of fuzzy multiobjective programming such as: [29]–[36], [75]–[78], [79], [81]–[86] and many.

Geometric programming is an effective, efficient and powerful method for solving highly convex non-linear programming problems by converting it into linear forms with the help of strong duality theorem. The application of geometric programming in engineering design problems in early 1960s was done by Duffin and Zener. Further, [37] have solved extensive assortment of engineering design problems by creating basic theories of geometric programming. Geometric programming method was applied by many authors, some of them are: [38]–[45]. The researchers have worked in multiobjective geometric programming problem and applied in different fields as: [46]–[60] and many. The research work in fuzzy geometric programming and fuzzy goal programming has been done by many authors in different fields for obtaining the solution of the problems. Such as: [61]–[74] and many others. In this paper, we have used DSS strategy in multivariate stratified population with unidentified strata weights, for OSD is in the presence of non-response to estimate the unidentified population means. The double sampling problem in presence of non-response has been formulated of as FMOCPP. The FMOCPP is converted into FSOCP by using membership function of fuzzy programming. Dual solution is obtained by using LINGO [87] Software for solving FSOCP. The optimum allocations of SS of R and NR for both the phases have been obtained by applying DS and primal-dual relationship theorem as described by [44], [52], [66], [69], [70] and [80]. The given technique is illustrated by projecting a numerical problem.

II. FORMULATION OF THE DOUBLE SAMPLING FOR STRATIFICATION IN PRESENCE OF NON-RESPONSE

Let a multivariate survey be planned to estimate the number of people having different diseases like Thyroid, Hyper tension, Cataract, Glaucoma, Diabetes, Cancer, High B.P., HIV etc., in a district of population size N, divided into three strata on the basis of family income. The mailed questionnaires are used to get information. Again we let N_1, N_2 and N_3 be the unknown actual strata sizes. The non-response problem generally occurred in mailed questionnaire surveys. The surveyor is advised to use the proposed technique of this manuscript to solve the problems arising in the above situations. The population of size N is considered which consists of L non-overlapping strata of sizes $N_1, N_2, \cdots, N_L$, where $\sum_{h=1}^{L} N_{h,} = N$. If $N_1, N_2, \cdots, N_L$ are previously unknown then the strata weights $W_h = N_h/N; h = 1, 2, \cdots, L$ will remain unknown.

The double sampling technique is applied in these conditions to estimate the unidentified strata weights (W_h) by taking a huge preliminary sample of size n', where the population is treated as un-stratified. The SS $n'_h; h = 1, 2, \cdots, L$ are recorded coming in every stratum. The $w_h = n'_h/n'$ will be an unbiased estimate of W_h. Out of n'_h units, the sub-SS $n_h = \nu_h n'_h; h = 1, 2, \cdots, L; 0 < \nu_h \leq 1$ is drawn from every stratum for fixed ν_h using srswor. The DSS estimator of the population mean $\overline{Y}_j$ of the j^{th} characteristics out of p characteristics calculated on every chosen unit is as follows:

$$\bar{y}_{jds} = \sum_{h=1}^{L} w_h \bar{y}_{jh} \tag{1}$$

where $\bar{y}_{jh} = \frac{1}{n} \sum_{i=1}^{n_h} y_{jhi}$ is the sample mean of j^{th} characteristic, $j = 1, 2, \cdots, p$, based on $\mathrm{n_h}$ units for stratum h and ds stands for double sampling.

The sampling variance of $\bar{y}_{jds}$ is as follows:

$$V\left(\bar{y}_{jds}\right) = \left(\frac{1}{n'} - \frac{1}{N}\right) S_j^2 + \frac{1}{n'} \sum_{h=1}^{L} w_h \left(\frac{1}{\nu_h} - 1\right) S_{jh}^2 \tag{2}$$

where $S_j^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(y_{ji} - \overline{Y}_j\right)^2$ is the population variance of j^{th} the characteristics based on N units and $S_{jh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{jhi} - \overline{Y}_{jh}\right)^2$ is the population variance of j^{th} characteristic based on N_h units for stratum h. The total response will be considered by expressions 1 and 2.

Let the response to the first call will be n_{h1} units and n_{h2} units be the NR out of n_h units in the presence of non-response. The technique of Hansen and Hurwitz [2] is used to draw a sub-sample of NR of size $m_{h2} = k_h^* n_{h2}; 0 < k_h^* < 1$

out of n_{h2} units and interviewed with the improved technique, where known constant is k_h^*. In the second attempt, the unbiased estimator $\overline{y}_{jds}^*$ for $\overline{Y}_j$ based on sample means from the R and the NR group for j^{th} characteristic is as:

$$\overline{y}_{jds}^* = \sum_{h=1}^{L} w_h \overline{y}_{jh}^* \tag{3}$$

where $\overline{y}_{jh}^* = \frac{n_{h1}\overline{y}_{jh1} + n_{h2}\overline{y}_{jmh2}}{n_h}$.

The sample mean for R based on n_{h1} units is $\overline{y}_{jh1}$.

The sample mean for the NR based on m_{h2} units in second attempt is $\overline{y}_{jmh2}$.

The variance of $\overline{y}_{jds}^*$ is given as:

$$\begin{aligned}\overline{y}_{jds}^* &= \left(\frac{1}{n'} - \frac{1}{N}\right) S_j^2 + \frac{1}{n'} \sum_{h=1}^{L} w_h \left(\frac{1}{v_h} - 1\right) S_{jh}^2 \\ &\quad + \frac{1}{n'} \sum_{h=1}^{L} w_{h2} \left(\frac{1 - k_h^*}{k_h^* v_h}\right) S_{jh2}^2 \\ &= V_j; \quad j = 1, 2, \cdots, p \end{aligned} \tag{4}$$

where $w_{h2} = \frac{n_{h2}}{n_h}$ is the proportion of the NR and S_{jh2}^2 is the population variance of j^{th} characteristic, $j = 1, 2, \cdots, p$ of the NR in h^{th} stratum. The total cost of the survey is assumed to be linear that is given as:

$$C = c_0 n' + \sum_{h=1}^{L} c_{h1} n_h + \sum_{h=1}^{L} c_{h11} n_{h1} + \sum_{h=1}^{L} c_{h12} m_{h2} \tag{5}$$

where c_0 is per unit cost of receiving information from the preliminary sample, c_{h1} is per unit cost for the first attempt (phase-I), $c_{h11} = \sum_{j=1}^{p} c_{jh11}$ is per unit cost for processing the result of all the p characteristics on the n_{h1} selected units from R group in the h^{th} stratum at phase-I, $c_{h12} = \sum_{j=1}^{p} c_{jh12}$ is the per unit cost for measuring and processing the results of all the p characteristics on the m_{h2} units selected from the NR group in the h^{th} stratum at the second attempt (phase-II), the per unit costs of measuring the j^{th} characteristic of corresponding both the phases (phase-I and phase-II) are c_{jh11} and c_{jh12}. Because n_{h1} is unknown before the first is made, then the amount $w_{h1} n_h$ may be used as its estimated value. Thus, the total expected cost $\hat{C}$ of the survey is given as:

$$\hat{C} = c_0 n' + \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h + \sum c_{h12} m_{h2} \tag{6}$$

At phase-I, the optimum SS of R $n_h, h = 1, 2, \cdots, L$ are obtained by minimizing the variance $v_j, j = 1, 2, \cdots, L$ of (4) for the total expected cost given in (6) or by minimizing the cost for fixed precision given by [6]. At phase-II, the optimum SS of NR $m_{h2}, h = 1, 2, \cdots, L$ are obtained for a fixed cost of the survey.

The multiobjective formulation of the problem at phase-I may be given as:

$$\left.\begin{aligned} &Min\ V_j, j = 1, 2, \cdots, p \\ &Subject\ to \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h \\ &+ \sum c_{h12} m_{h2} \leq \left(\hat{C} - c_0 n'\right); \\ &and\ n_h \geq 0,\ h = 1, 2, \cdots, L \end{aligned}\right\} \tag{7}$$

The equivalent expression obtained by ignoring terms independent of n_h in minimizing V_j will be

$$\begin{aligned} &Z_j(n_1, n_2, \cdots, n_L) \\ &= \frac{1}{n'} \sum_{h=1}^{L} \left(\frac{w_h n_h' S_{jh}^2 + w_{h2}\left(\left(i - k_h^*\right)/k_h^*\right) n_h' S_{jh2}^2}{n_h} \right) \\ &= \sum_{h=1}^{L} \frac{a_{jh}}{n_h} = Z_j; \quad j = 1, 2, \cdots, p \end{aligned} \tag{8}$$

where $a_{jh} = \left(\frac{w_h n_h' S_{jh}^2 + w_{h2}\left(\left(i - k_h^*\right)/k_h^*\right) n_h' S_{jh2}^2}{n'} \right)$

The expression of the cost constraint may be given as:

$$\sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h \leq \hat{C}_0;$$

$$\text{where } \hat{C}_0 = \hat{C} - c_0 n' - \sum_{h=1}^{L} c_{h12} m_{h2}$$

By using the above transformations, the problem (7) can be rewritten as:

$$\left.\begin{aligned} &Min\ Z_j; j = 1, 2, \cdots, p \\ &Subject\ to \sum_{h=1}^{L} (c_{h1} + w_{h1} c_{h11}) n_h \\ &\leq \hat{C}_0\ and\ n_h \geq 0,\ h = 1, 2, \cdots, L \end{aligned}\right\} \tag{9}$$

At phase-II, the optimum SS of NR $m_{h2}, h = 1, 2, \cdots, L$ are obtained by minimizing the variance $v_j, j = 1, 2, \cdots, L$ of (4) for the total expected cost given in (6). The independent terms of m_{h2} ignored from RHS of (4), putting $k_h^* = m_{h2}/n_{h2} and v_h = n_h/n_h'$, in the equation (9) we get:

$$\left.\begin{aligned} &Min\ Z_j; j = 1, 2, \cdots, p \\ &Subject\ to \sum_{h=1}^{L} c_{h12} m_{h2} \leq \hat{C}_0' \\ &and\ m_{h2} \geq 0,\ h = 1, 2, \cdots, L \end{aligned}\right\} \tag{10}$$

where $Z_j; j = 1, 2, \cdots, p$ are the function of $m_{h2}; h = 1, 2, \cdots, L$ given by

$$\begin{aligned} &Z_j'(m_{12}, m_{22}, \cdots, m_{L2}) \\ &= \frac{1}{n'} \sum_{h=1}^{L} \frac{w_{h2} n_{h2} n_h' S_{jh2}^2}{m_{h2} n_h} = \sum_{h=1}^{L} \frac{b_{jh}}{m_{h2}}; \quad j = 1, 2, \cdots, p. \end{aligned}$$

$$b_{jh} = \frac{w_{h2}n_{h2}n'_hS^2_{jh2}}{n'n_h} and \hat{C}'_0 = \hat{C} - c_0n' - \sum_{h=}^{L}(c_{h1} + w_{h1}c_{h11})\,n_h. \quad (11)$$

III. GEOMETRIC PROGRAMMING APPROACH

The multiobjective nonlinear programming problem (MONLPP) at phase-I is given as follows:

$$\left.\begin{array}{l} Min\sum_{h=1}^{L}\frac{a_{jh}}{n_h},\ j=1,2,\cdots,p \\ Subject\ to \\ \sum_{h=1}^{L}(c_{h1}+w_{h1}c_{h11})\,n_h \leq \hat{C}_0 \\ and\ n_h \geq 0,\ h=1,2,\cdots,L \end{array}\right\} \quad (12)$$

The equation (12) can be written in the standard primal geometric programming problem as follows:

$$\left.\begin{array}{l} Max\ f_{0j}(n),j=1,2,\ldots,p\ (i) \\ Subject\ f_q(n) \leq 1,\ n_h \geq 0,\ h=1,2,\cdots,L\ (ii) \end{array}\right\} \quad (13)$$

where

$$f_q(n) = \sum_{i\in j[q]} t_i n_1^{p_{i1}} n_2^{p_{i2}} \cdots n_L^{p_{iL}}, \quad q=o,1,2,\cdots,k$$

or

$$f_q(n) = \sum_{i\in j[q]} t_i\left[\prod_{h=1}^{L} n_h^{p_{ih}}\right], \quad t_i>0,\ n_h>0, \quad q=o,1,2,\cdots,k,$$

p_{ih}: arbitrary real numbers,d_i :positive and $f_q(n)$: posinomials.

Let for simplicity

$$a_{jh} = \left(\frac{w_hn'_hS^2_{jh} + w_{h2}\left(\left(i-k^*_h\right)/k^*_h\right)n'_hS^2_{jh2}}{n'}\right)$$

$$\&t_i = a_{jh} = \frac{(c_{h1}+w_{h1}c_{h11})}{\hat{C}_0}$$

The multiobjective nonlinear programming problem (MONLPP) at phase-II is given by [16] as follows:

$$\left.\begin{array}{l} Min\sum_{h=1}^{L}\frac{b_{jh}}{m_{h2}},j=1,2,\cdots,p \\ Subject\ to\sum_{h=1}^{L}c_{h12}m_{h2} \leq \hat{C}'_0 \\ and\ m_{h2} \geq 0,\ h=1,2,\cdots,L \end{array}\right\} \quad (14)$$

In the same way, as described in (12), the equation (14) can be written in the standard primal GPP as follows:

$$\left.\begin{array}{l} Max\ f_{0j}(m),j=1,2,\ldots,p\ (i) \\ Subject\ f_q(m) \leq 1,\ m_{h2} \geq 0,\ h=1,2,\cdots,L\ (ii) \end{array}\right\} \quad (15)$$

where

$$f_q(m) = \sum_{i\in j[q]} t_i m_1^{p_{i1}} n_2^{p_{i2}} \cdots m_L^{p_{iL}}, \quad q=o,1,2,\cdots,k$$

or

$$f_q(m) = = \sum_{i\in j[q]} t_i\left[\prod_{h=1}^{L} m_h^{p_{ih}}\right], \quad t_i>0,\ m_h>0, \quad q=o,1,2,\cdots,k,$$

p_{ih}: arbitrary real numbers, t_i: positive and $f_q(m)$: posinomials.

Let for simplicity let

$$b_{jh} = \frac{w_{h2}n_{h2}n'_hS^2_{jh2}}{n'n_h}\&t_i = b_{jh} = \frac{c_{h12}}{\hat{C}'_0}$$

The R and non-respondent's dual form of the primal GPP is given in equation (16) as:

$$\left.\begin{array}{l} Maxv_{0j}(w) = \prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\frac{t_i}{w_i}\right)^{w_i}\right\} \\ \prod_{q=1}^{k}\left(\sum_{i\in j[q]} w_i\right)^{\sum_{i\in j[q]} w_i} \quad (i) \\ Subject\ \sum_{i\in[0]} w_i = 1 \quad (ii) \\ \sum_{q=0}^{k}\sum_{i\in j[q]} p_{ih}\,w_i = 0 \quad (iii) \\ w_i \geq 0,\ q=o,1,\cdots,k \\ and\ i=1,2,\ldots,m_k\ \ (iv) \end{array}\right\} j=1,\ldots,p. \quad (16)$$

The given two-steps can be used for solving dual GPP as given in equation (16):

Step I: The objective function constantly be in the given form, for obtaining the optimum value of the objective function

$$\begin{aligned} C_0(x^*) = &\left(\frac{1^{st}term'scoefficient}{w_{01}}\right)^{w_{01}} \\ &\times\left(\frac{2^{nd}term'scoefficient}{w_{02}}\right)^{w_{02}} \\ &\times\ldots\ldots\times\left(\frac{Lastterm'scoefficient}{w_k}\right)^{w_k} \\ &\times\left(1^{st}constraints's\sum w's\right)^{1^{st}constraints's\sum w's} \\ &\times\left(Lastconstraints's\sum w's\right)^{Lastconstraints's\sum w's} \end{aligned}$$

For our problem, the multiobjective function can be expressed as

$$\prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\frac{t_i}{w_i}\right)^{w_i}\right\}\prod_{q=1}^{k}\left(\sum_{i\in j[q]} w_i\right)^{\sum_{i\in j[q]} w_i}$$

Step II: The weighted equations for the GPP are given below.

In the objective function, the $\sum_{i\in j[o]} w_i = 1$ - Normality condition and for each primal variable n_h and m_{h2} having m terms.

$$\sum_{i=1}^{m_k}\{(w_i \text{foreachterm}) \times (\text{exponenton} n_h \text{ and } m_{h2} \text{inthatterm})\} = 0$$

- Orthogonality condition
and $w_i \geq 0$ -Positivity condition.

The step1 and step 2 can be used to solve the dual problem (16) and the unique solution w^*_{0i} to the dual constraints is obtained, which maximizes the objective function of the dual problem. Then, the primal-dual relationship theorem is applied to find the values of the SS of R (n^*_h) and NR (m^*_{h2}) of the original problem.

Primal-Dual Relationship Theorem: If the dual problem is maximized by the maximization point w^*_{0i} and the optimal SS of R and NR (n^*_h and m^*_{h2}) respectively are the minimization points for the Primal GPP that satisfies the system of equations:

$$f_{0j}(n, m_{h2}) = \begin{Bmatrix} w^*_{0i} v(w^*), & i \in J[0], \\ \dfrac{w_{ij}}{v_L(w^*_{0i})}, & i \in J[L], \end{Bmatrix} \quad (17)$$

where $v_L(w^*_{0i}) > 0$ for positive integers L.

IV. METHODOLOGY: FUZZY GEOMETRIC PROGRAMMING APPROACH

A. THE STEPS OF SOLUTION PROCEDURE FOR PHASE-I, IS AS FOLLOWS

Step-I: Select one of the objective functions amongst $f_{0j}(n), j = 1, 2, .., p$ and it can be solved as a FSOCP subject to the constraints 13(ii) with the application of geometric programming algorithm. Here the ideal solutions of the objective functions $f_{01}(n^{(1)}), f_{02}(n^{(2)}), \cdots, f_{0j}(n^{(j)}), \cdots, f_{0p}(n^{(p)})$ are $(n^{(1)}), (n^{(2)}), \cdots, (n^{(j)}), \cdots, (n^{(p)})$.

It is supposed that at least two of these ideal solutions amongst $f_{0j}(n), j = 1, 2, .., p$ are different and having different bound values. If the resulting values of all the optimal solutions $(n^{(1)}) = (n^{(2)}) = \cdots = (n^{(j)}) = \cdots = (n^{(p)})$ are identical, then stop and n^* is the optimal solution.

If they are not identical, then go to step-II.

Step-II: All the p objective functions $f_{0j}(n), j = 1, 2, .., p$ are evaluated having all the p ideal solutions $(n^{(1)}), (n^{(2)}), \cdots, (n^{(j)}), \cdots, (n^{(p)})$.

Step-III: The pay-off matrix can be constructed with all the p values of objective functions at p ideal solutions, in which the best solutions are diagonal solutions but off-diagonal solutions are the worst. We have the best value $L_j = Min\left\{f^*_{0j}(n^{(j)}), j = 1, 2, \ldots, p\right\}$ and worst value $U_j = Max\left\{f_{0j}(m^{(j)}), j = 1, 2, \cdots, p\right\}$ such that $L_j \leq f_{0j}(n) \leq U_j, j = 1, 2, .., p$.

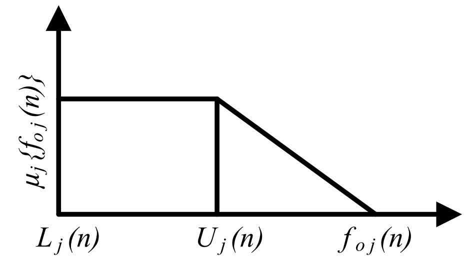

FIGURE 1. Membership function for minimization variances problem.

Step-IV: The membership function $\mu_j(f_{0j}(n))$ for the j^{th} objective function can be defined as:

$$\mu_j(f_{0j}(n)) = \begin{cases} 0, & \text{if } f_{0j}(n) \geq U_j \\ \dfrac{U_j(n) - f_{0j}(n)}{U_j(n) - L_j(n)}, & \text{if } L_j \leq f_{0j}(n) \geq U_j, \\ 1, & \text{if } f_{0j}(n) \leq L_j \end{cases} \quad j = 1, 2, \ldots, p \quad (18)$$

The membership function $\mu_j(f_{0j}(n))$ for phase –I, is shown in the following graph.

Where $L_j \neq U_j, j = 1, 2, .., p$. If $L_j = U_j$ then $\mu_j(f_{0j}(n)) = 1$ may be defined for any value of j.

The membership functions $\mu_j(f_{0j}(n)), j = 1, 2, \cdots, p$ can be maximized subject to the constraints 13(ii) and maxi-min operator is used to obtain the crisp model.

The universal aggregation function for R can be written as

$$\mu_{\tilde{D}}(n) = \mu_{\tilde{D}}\left\{\mu_1(f_{01}(n)), \mu_2(f_{02}(n)), \ldots, \mu_p(f_{0p}(n))\right\}$$

At phase-I, the FMOP for Respondents can be written as

$$\left.\begin{aligned} & Max\,\mu_{\tilde{D}}(n) \\ & Subject\ to \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})\,n_h \leq \hat{C}_0, \\ & n_h \geq 0\ and\ h = 1, 2, \cdots, L. \end{aligned}\right\} \quad (19)$$

The max-addition operator given by [75] is applied to the problem (19) for calculating the optimal values of R (n^*) for the given model as:

$$\left.\begin{aligned} & Max\,\mu_D(n^*) = \sum_{j=1}^{p}\mu_j(f_{0j}(n)) = \sum_{j=1}^{p}\frac{U_j - (f_{0j}(n))}{U_j - L_j} \\ & Subject\ to \sum_{h=1}^{L}(c_{h1} + w_{h1}c_{h11})\,n_h \leq \hat{C}_0; \\ & 0 \leq \mu_j(f_{0j}(n)) \leq 1, n_h \geq 0 \\ & and\ h = 1, 2, \cdots, L. \end{aligned}\right\} \quad (20)$$

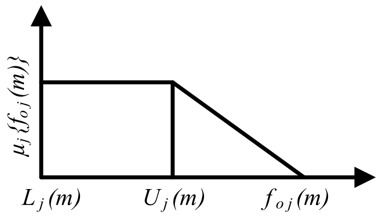

FIGURE 2. Membership function for minimization variances problem.

The problem (20) reduces to

$$\left.\begin{array}{l} Max\,\mu_D\,(n^*) \\ =\sum_{j=1}^{p}\left\{\dfrac{U_j}{U_j-L_j}-\dfrac{(f_{0j}\,(n))}{U_j-L_j}\right\} \\ Subject\ to \\ f_q\,(n)\le 1;\, n_h\ge 0 \\ and\ h=1,2,\cdots,L. \end{array}\right\},\quad j=1,2,\ldots,p. \tag{21}$$

where $f_q\,(n)=\sum_{h=1}^{L}\frac{(c_{h1}+w_{h1}c_{h11})}{\hat{C}_0}n_h$

The minimum values of $F_{oj}(n)=\left\{\frac{(f_{0j}(n))}{U_j-L_j}\right\}$ will maximize the problem (21).

The standard primal form for fuzzy multiobjective problem (FMOP) for phase-I, can be defined as:

$$\left.\begin{array}{l} Min\sum_{j=0}^{p}F_{oj}(n') \\ Subjectt\ of_q\left(n'\right)\le 1;\ and\ n'_h\ge 0,\, h=1,2,\cdots,L. \end{array}\right\} \tag{22}$$

B. THE STEPS OF SOLUTION PROCEDURE FOR PHASE-II, IS AS FOLLOWS

Step-I: Select one of the objective functions amongst $f_{0j}\,(m)\,,j=1,2,..,p$ and it can be solved as a FSOCPP subject to the constraints 15(ii) with the application of geometric programming algorithm. Here the ideal solutions of the objective function $f_{01}(m^{(1)}), f_{02}(m^{(2)}),\cdots,f_{0j}(m^{(j)}),\cdots,f_{0p}(m^{(p)})$ are $(m^{(1)}),(m^{(2)}),\cdots,(m^{(j)}),\cdots,(m^{(p)})$.

It is supposed that at least two of these ideal solutions amongst $f_{0j}\,(m)\,,j=1,2,..,p$ are different and having different bound values. If the resulting values of all the optimal solutions $(m^{(1)})=(m^{(2)})=\cdots=(m^{(j)})=\cdots=(m^{(p)})$ are identical, then stop and m^* is the optimal solution. If they are not identical, then go to step-II.

Step-II: All the p objective functions $f_{0j}\,(m)\,,j=1,2,..,p$ are evaluated having all the p ideal solutions $(m^{(1)}),(m^{(2)}),\cdots,(m^{(j)}),\cdots,(m^{(p)})$.

Step-III: The pay-off matrix can be constructed with all the p values of objective functions at p ideal solutions, in which the best solutions are diagonal solutions but off-diagonal solutions are the worst. We have the best value $L_j=Min\left\{f^*_{0j}(m^{(j)}),j=1,2,\ldots,p\right\}$ and worst value $U_j=Max\left\{f_{0j}(m^{(j)}),j=1,2,\cdots,p\right\}$ such that $L_j\le f_{0j}\,(m)\le U_j, j=1,2,..,p$.

Step-IV: The membership function $\mu_j\left(f_{0j}\,(m)\right)$ for the j^{th} objective function can be defined as

$$\mu_j\left(f_{0j}\,(m)\right)=\begin{cases} 0, & \\ \quad if\ f_{0j}\,(m)\ge U_j & \\ \dfrac{U_j\,(m)-f_{0j}\,(m)}{U_j\,(m)-L_j\,(m)}, & j=1,2,\ldots,p \\ \quad if\ L_j\le f_{0j}\,(m)\ge U_j, & \\ 1, & \\ \quad if\ f_{0j}\,(m)\le L_j & \end{cases} \tag{23}$$

The membership function $\mu_j\left(f_{0j}\,(m)\right)$ for phase –II, is shown in the following graph.

Where $L_j\ne U_j, j=1,2,..,p$. If $L_j=U_j$ then $\mu_j\left(f_{0j}\,(m)\right)=1$ may be defined for any value of j.

The membership functions $\mu_j\left(f_{0j}\,(m)\right),j=1,2,\cdots,p$ can be maximized subject to the constraints 15(ii) and then the crisp model can be obtained by maxi-min operator.

The universal aggregation function for NR can be written as

$$\mu_{\tilde{D}}\,(m)=\mu_{\tilde{D}}\left\{\mu_1\,(f_{01}\,(m))\,,\mu_2\,(f_{02}\,(m))\,,\ldots,\mu_p\left(f_{0p}\,(m)\right)\right\}$$

At phase-II, the FMOP for non-respondents can be written as

$$\left.\begin{array}{l} Max\,\mu_{\tilde{D}}\,(m) \\ Subject\ to\sum_{h=1}^{L}c_{h12}m_{h2}\le\hat{C}'_0 \\ and\ m_{h2}\ge 0,\ h=1,2,\cdots,L \end{array}\right\} \tag{24}$$

The max-addition operator, given by [75] is applied to the problem (24) for calculating the optimal values of NR (m^*) for the given model as:

$$\left.\begin{array}{l} Max\,\mu_D\left(m^*_{h2}\right)=\sum_{j=1}^{p}\mu_j\left(f_{0j}\,(m_{h2})\right) \\ =\sum_{j=1}^{p}\dfrac{U_j-\left(f_{0j}\,(m_{h2})\right)}{U_j-L_j} \\ Subject\ to\sum_{h=1}^{L}c_{h12}m_{h2}\le\hat{C}'_0; \\ 0\le\mu_j\left(f_{0j}\,(m_{h2})\right)\le 1, m_{h2}\ge 0\ and\ h=1,2,\cdots,L. \end{array}\right\} \tag{25}$$

The problem (25) reduces to

$$\left.\begin{array}{l} Max\,\mu_D\,(m^*) \\ =\sum_{j=1}^{p}\left\{\dfrac{U_j}{U_j-L_j}-\dfrac{(f_{0j}\,(m))}{U_j-L_j}\right\} \\ Subjectto \\ f_q\,(m)\le 1;\, m^*_{h2}\ge 0 \\ andh=1,2,\cdots,L. \end{array}\right\},\ j=1,2,\ldots,p. \tag{26}$$

where $f_q\,(m)=\sum_{h=1}^{L}\frac{c_{h12}}{\hat{C}'_0}m_{h2}$

TABLE 1. Data for four strata and two characteristics.

h	w_h	S^2_{1h}	S^2_{2h}	v_h	k^*_h	c_{h1}	c_{h11}	c_{h12}
1	0.32	4817.72	8121.15	0.4	0.5	1	2	3
2	0.21	6251.26	7613.52	0.5	0.6	1	3	4
3	0.27	3066.16	1456.4	0.6	0.7	1	4	5
4	0.20	6207.25	6977.72	0.65	0.75	1	5	6

The minimum values of $F_{oj}(m) = \left\{\frac{(f_{0j}(m))}{U_j - L_j}\right\}$ will maximize the problem (26).

The standard primal form for FMOP for phase-II, can be defined as

$$\left.\begin{array}{l} Min \sum_{j=0}^{p} F_{oj}(m') \\ Subject\ to\ f_q\left(m'\right) \leq 1; \\ and\ m^*_{h2} \geq 0, h = 1, 2, \cdots, L. \end{array}\right\} \quad (27)$$

where $f_q\left(n'\right) = \sum_{h=1}^{L} \frac{c_{h12}}{\hat{C}'_0} m'_{h2}$

The dual form of the standard primal GPP for R and NR can be given as:

$$\left.\begin{array}{l} Maxv(w) = \prod_{q=0}^{k} \prod_{i \in j[q]} \left\{\left(\frac{d_i}{w_i}\right)^{w_i}\right\} \\ \prod_{q=1}^{k} \left(\sum_{i \in j[q]} w_i\right)^{\sum_{i \in j[q]} w_i} \quad (i) \\ Subject \sum_{i \in [0]} w_i = 1 (ii) \\ \sum_{q=0}^{k} \sum_{i \in j[q]} p_{ih} w_i = 0 (iii) \\ w_i \geq 0, q = o, 1, \cdots, k \\ and\ i = 1, 2, \ldots, m_k (iv) \end{array}\right\} \quad (28)$$

The primal-dual relationship theorem (17) can be used for computing the optimal values of the SS of NR m^*_h

V. NUMERICAL EXAMPLE

We have considered the data for given numerical from [7] and [8] for the illustration of our proposed procedure. A population of size $N = 3850$ is divided into four strata. The existing information is shown by Table: 1. Table: 2 show the R and NR groups, in which $k = 1$ for the R group and $k = 2$ for the NR group. Now assume $C = 3{,}000$ units be the total sum existing for the survey. The values of $n'_h = w_h n'; h = 1, 2, \ldots, L$ are obtained as:

First of all, we solved the two sub-problems, for solving FMOCPP by using fuzzy programming

TABLE 2. Subdivided data as R and NR groups for four strata with two characteristics.

h	Groups	S^2_{1h}	S^2_{2h}	w_{hk}, K = 1,2
1	Respondent	2218.74	4318.28	$w_{11} = 0.70$
	Non-respondent	1908.37	2557.62	$w_{12} = 0.30$
2	Respondent	4056.75	5067.26	$w_{21} = 0.80$
	Non-respondent	3541.23	3984.85	$w_{22} = 0.20$
3	Respondent	2785.15	957.56	$w_{31} = 0.75$
	Non-respondent	1677.65	877.13	$w_{32} = 0.70$
4	Respondent	5015.17	3085.78	$w_{41} = 0.72$
	Non-respondent	2156.52	2756.62	$w_{42} = 0.28$

VI. ANALYSIS FOR THE RESPONDENTS AND NON-RESPONDENTS

A. FOR PHASE-I, THE OPTIMUM ALLOCATIONS OF RESPONDENTS CAN BE CALCULATED AS FOLLOWS

Sub problem1: By putting the values from given tables into the sub-problem 1, we get the required equation as:

$$\left.\begin{array}{l} Min\ f_{01} = \frac{676.53805}{n_1} + \frac{374.83501}{n_2} + \frac{272.05508}{n_3} \\ + \frac{288.54504}{n_4} \\ Subject\ to \\ 0.001263 n_1 + 0.001789 n_2 + 0.002105 n_3 \\ +0.002421 n_4 \leq 1; \\ n_h \geq 0\ and\ h = 1, 2, \ldots, 4 \end{array}\right\} \quad (29)$$

For the dual problem (30) of (29) is given as:

$$\left.\begin{array}{ll} Maxv(w^*_{0i}) = ((676.53805/w_{01})^{w_{01}}) & \\ \times ((374.83501/w_{02})^{w_{02}}) & \\ \times ((272.05508/w_{03})^{w_{03}}) & \\ \times ((288.54504/w_{04})^{w_{04}}) \times \left(\left(\frac{0.001263}{w_{11}}\right)^{w_{11}}\right) & \\ \times \left(\left(\frac{0.001789}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\frac{0.002105}{w_{13}}\right)^{w_{13}}\right) \times & \\ \left(\left(\frac{0.002421}{w_{14}}\right)^{w_{14}}\right) \times ((\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14}) & \\ \wedge(\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14})); & (i) \\ Subjectto & \\ w_{01} + w_{02} + w_{03} + w_{04} = 1 (\text{normality condition}) & (ii) \\ \left.\begin{array}{l} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{array}\right\} (\text{orthogonality condition}) & (iii) \\ w_{01}, w_{02}, w_{03}, w_{04}, w_{11}, w_{12}, w_{13}, w_{14} \geq 0\} & \\ (\text{positivity condition}) & (iv) \end{array}\right\}$$

(30)

The OC given in expression 30(iii) are calculated by using the following payoff matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{pmatrix} = \begin{Bmatrix} -w_{01}+w_{11}=0 \\ -w_{02}+w_{12}=0 \\ -w_{03}+w_{13}=0 \\ -w_{04}+w_{14}=0 \end{Bmatrix}$$

DS will be obtained from dual problem (30) as:

$w_{01} = 0.2771055$, $w_{02} = 0.2454836$, $w_{03} = 0.2268568$, $w_{04} = 0.2505541$ and $v(w^*) = 11.12770$.

The SS of R for phase-I, calculated by using equation (17) from sub-problem-1, are as follows:

$$f_{0j}(n) = w^*_{0i} v\left(w^*_{0i}\right)$$

The values of n for SS of R are evaluated by using above expressions as:

$$f_{01}(n) = w^*_{01} v\left(w^*_{01}\right)$$
$$\frac{676.53805}{n_1} = 0.277105 \times 11.12770 \Rightarrow n_1 \cong 219$$
$$f_{02}(n) = w^*_{02} v\left(w^*_{02}\right)$$
$$\frac{374.83501}{n_2} = 0.245483 \times 11.12770 \Rightarrow n_2 \cong 137$$
$$f_{03}(n) = w^*_{03} v\left(w^*_{03}\right)$$
$$\frac{272.05508}{n_3} = 0.226856 \times 11.12770 \Rightarrow n_3 \cong 108$$
$$f_{04}(n) = w^*_{04} v\left(w^*_{04}\right)$$
$$\frac{288.54504}{n_4} = 0.250554 \times 11.12770 \Rightarrow n_4 \cong 103$$

The values of objective function and SS of R are given as: 11.12770 and $n_1^* = 219$, $n_2^* = 137$, $n_3^* = 108$, $n_4^* = 103$.

Sub-problem 2: By putting the values from given tables into the sub-problem 2, we get the required equation as:

$$\left.\begin{array}{l} Min\, f_{02} = \dfrac{1077.13728}{n_1} + \dfrac{447.33203}{n_2} + \dfrac{131.54568}{n_3} \\ + \dfrac{330.56571}{n_4} \\ Subject\ to \\ 0.001263 n_1 + 0.001789 n_2 + 0.002105 n_3 \\ +0.02421 n_4 \leq 1; \\ n_h \geq 0\ and\ h = 1, 2, \ldots, 4 \end{array}\right\} \quad (31)$$

For the dual problem (32) of (31) is given as:

$$\left.\begin{array}{ll} Maxv(w^*_{0i}) = \left((1077.13728/w_{01})^{w_{01}}\right) & \\ \times\left((447.33203/w_{02})^{w_{02}}\right) \times \left((131.54568/w_{03})^{w_{03}}\right) & \\ \times\left((330.56571/w_{04})^{w_{04}}\right) \times \left(\left(\dfrac{0.001263}{w_{11}}\right)^{w_{11}}\right) & \\ \times\left(\left(\dfrac{0.001789}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\dfrac{0.002105}{w_{13}}\right)^{w_{13}}\right) \times & \\ \left(\left(\dfrac{0.002421}{w_{14}}\right)^{w_{14}}\right) \times \left((w_{11}+w_{12}+w_{13}+w_{14})\right. & \\ \left.\wedge(w_{11}+w_{12}+w_{13}+w_{14})\right); & (i) \\ Subjectto & \\ w_{01}+w_{02}+w_{03}+w_{04} = 1 (\text{normality condition}) & (ii) \\ \left.\begin{array}{l} -w_{01}+w_{11}=0 \\ -w_{02}+w_{12}=0 \\ -w_{03}+w_{13}=0 \\ -w_{04}+w_{14}=0 \end{array}\right\} (\text{orthogonality condition}) & (iii) \\ w_{01}, w_{02}, w_{03}, w_{04}, w_{11}, w_{12}, w_{13}, w_{14} \geq 0\} & \\ (\text{positivity condition}) & (iv) \end{array}\right\} \quad (32)$$

The OC given in expression 32(iii) are calculated by using the following payoff matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{pmatrix} = \begin{Bmatrix} -w_{01}+w_{11}=0 \\ -w_{02}+w_{12}=0 \\ -w_{03}+w_{13}=0 \\ -w_{04}+w_{14}=0 \end{Bmatrix}$$

DS will be obtained from dual problem (32) as:

$w_{01} = 0.3349945$, $w_{02} = 0.2569335$, $w_{03} = 0.1511349$, $w_{04} = 0.2569371$ and $v(w^*) = 12.12269$.

The values of objective function and SS of R are given as: 12.12269 and $n_1^* = 265$, $n_2^* = 144$, $n_3^* = 72$, $n_4^* = 106$.

For phase –I, the pay-off matrix is obtained as:

$$\begin{array}{c} \\ (n^{(1)}) \\ (n^{(2)}) \end{array} \begin{array}{cc} f_{01}(n) & f_{02}(n) \\ \multicolumn{2}{c}{\begin{bmatrix} 11.12770 & 12.61102 \\ 11.65666 & 12.12269 \end{bmatrix}} \end{array}$$

The limiting values $f_{01}(n)$ and $f_{02}(n)$ using pay-off matrix can be obtained as: $11.1277 \leq f_{01}(n) \leq 11.65666$ and $12.12269 \leq f_{02}(n) \leq 12.61102$.

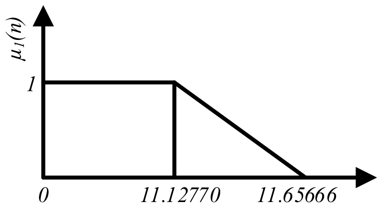

FIGURE 3. Membership function for minimization variances problem.

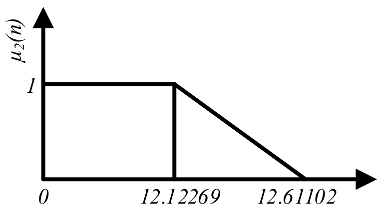

FIGURE 4. Membership function for minimization variances problem.

Let the fuzzy membership function $\mu_1(n)$ and $\mu_2(n)$ for $f_{01}(n)$ and $f_{02}(n)$ are defined respectively as:

$$\mu_1(n) = \begin{cases} 1, & \\ \quad if\ f_{01}(n) \le 11.12770 & \\ \dfrac{11.65666 - f_{01}(n)}{0.52896}, & \\ \quad if\ 11.12770 \le f_{01}(n) \le 11.65666 & \\ 0, & \\ \quad if\ f_{01}(n) \ge 11.65666 & \end{cases}$$

$$\mu_2(n) = \begin{cases} 1, & \\ \quad if\ f_{02}(n) \le 12.12269 & \\ \dfrac{12.61102 - f_{02}(n)}{0.48833}, & 12.61102 \\ qif\ 12.12269 \le f_{02}(n) \le . & \\ 0, & \\ \quad if\ f_{02}(n) \ge 12.61102 & \end{cases}$$

The MOGPP is reduced to the crisp problem, with the application of max-addition operator as:

$$\left.\begin{array}{l} Max\ (\mu_1(n) + \mu_2(n)) \\ i.e\ Max \left\{ 47.8633 - \left(\dfrac{f_{01}(n)}{0.52896} + \dfrac{f_{02}(n)}{0.48833} \right) \right\} \\ Subject\ to \\ 0.001263n_1 + 0.001789n_2 + 0.002105n_3 \\ +0.02421n_4 \le 1 \\ n_h \ge 0\ and\ n_h \text{are integers; } h = 1, 2, \ldots, 4. \end{array}\right\} \quad (33)$$

For maximizing the problem (33), $\left(\frac{f_{01}(n)}{0.52896} + \frac{f_{02}(n)}{0.48833}\right)$ is to be minimized as follows:

$$\left.\begin{array}{l} Min \left(\dfrac{f_{01}(n)}{0.52896} + \dfrac{f_{02}(n)}{0.48833} \right) \\ Subject\ to \\ 0.001263n_1 + 0.001789n_2 + 0.002105n_3 \\ +0.02421n_4 \le 1; \\ and\ n_h \ge 0,\ h = 1, 2, \ldots, 4 \end{array}\right\} \quad (34)$$

For minimizing above problem, the new problem is as:

$$\left.\begin{array}{l} Min \left\{ \left(\dfrac{3484.7569}{n_1} + \dfrac{1624.6721}{n_2} \right.\right. \\ \left.\left. + \dfrac{783.6994}{n_3} + \dfrac{1222.4269}{n_4} \right) \right\} \\ Subject\ to \\ 0.001263n_1 + 0.001789n_2 \\ +0.002105n_3 + 0.02421n_4 \le 1; \\ n_h \ge 0\ and\ h = 1, 2, \ldots, 4 \end{array}\right\} \quad (35)$$

Degree of Difficulty of the problem (35) is = (8-(4+1) =3.

$$\left.\begin{array}{l} Min = \dfrac{3484.7569}{n_1} + \dfrac{1624.6721}{n_2} + \dfrac{783.6994}{n_3} \\ + \dfrac{1222.4269}{n_4} \\ Subject\ to \\ 0.001263n_1 + 0.001789n_2 \\ +0.002105n_3 + 0.02421n_4 \le 1; \\ n_h \ge 0\ and\ h = 1, 2, \ldots, 4 \end{array}\right\} \quad (36)$$

For the dual problem (37) of (36) is given as:

$$\left.\begin{array}{ll} Maxv(w_{0i}^*) = \left((3484.7569/w_{01})^{w_{01}} \right) & \\ \times ((1624.6721/w_{02})^{w_{02}}) \times ((783.6994/w_{03})^{w_{03}}) & \\ \times ((1222.4269/w_{04})^{w_{04}}) \times \left(\left(\dfrac{0.001263}{w_{11}} \right)^{w_{11}} \right) & \\ \times \left(\left(\dfrac{0.001789}{w_{12}} \right)^{w_{12}} \right) \times \left(\left(\dfrac{0.002105}{w_{13}} \right)^{w_{13}} \right) \times & \\ \left(\left(\dfrac{0.002421}{w_{14}} \right)^{w_{14}} \right) \times ((\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14}) & \\ \wedge(\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14})); & (i) \\ Subjectto & \\ w_{01} + w_{02} + w_{03} + w_{04} = 1 \text{(normality condition)} & (ii) \\ \left.\begin{array}{l} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{array}\right\} \text{(orthogonality condition)} & (iii) \\ w_{01}, w_{02}, w_{03}, w_{04}, w_{11}, w_{12}, w_{13}, w_{14} \ge 0\} & \\ \text{(positivity condition)} & (iv) \end{array}\right\} \quad (37)$$

The OC given in expression 37(*iii*) are calculated by using the following payoff matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{Bmatrix}$$

DS will be obtained from dual problem (37) as: $w_{01} = 0.3081774$, $w_{02} = 0.2504384$, $w_{03} = 0.1886747$, $w_{04} = 0.2527096$ and $v(w^*) = 46.34195$. The optimal values of objective function and SS of R are given as: 46.34195 and $n_1^* = 244$, $n_2^* = 140$, $n_3^* = 90$, $n_4^* = 104$.

B. FOR PHASE II, THE OPTIMUM ALLOCATIONS OF NON-RESPONDENTS CAN BE CALCULATED AS FOLLOWS

Sub problem1: By putting the values from given tables into the sub-problem 1, we get the required equation (38) as:

$$\left.\begin{aligned} &Min f_{01} = \frac{54.81089}{m_{12}} + \frac{29.74633}{m_{22}} + \frac{28.93946}{m_{32}} + \frac{33.67489}{m_{42}} \\ &Subject\ to \\ &0.008571 n_1 + 0.0114 n_2 + 0.014295 n_3 + 0.01714 n_4 \le 1; \\ &m_h \ge 0\ and\ h = 1, 2, \ldots, 4 \end{aligned}\right\} \quad (38)$$

For the dual problem (39) of (38) is given as:

$$\left.\begin{aligned} &Maxv(w_{0i}^*) = \left((54.81089/w_{01})^{w_{01}}\right) \times \left((29.74633/w_{02})^{w_{02}}\right) \times \left((28.93946/w_{03})^{w_{03}}\right) \\ &\times \left((33.67489/w_{04})^{w_{04}}\right) \times \left(\left(\frac{0.008571}{w_{11}}\right)^{w_{11}}\right) \times \left(\left(\frac{0.0114}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\frac{0.01429}{w_{13}}\right)^{w_{13}}\right) \times \\ &\left(\left(\frac{0.01714}{w_{14}}\right)^{w_{14}}\right) \times ((\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14}) \wedge (\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14})); \quad (i) \\ &Subjectto \\ &w_{01} + w_{02} + w_{03} + w_{04} = 1 (\text{normality condition}) \quad (ii) \\ &\left.\begin{aligned} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{aligned}\right\} (\text{orthogonality condition}) \quad (iii) \\ &w_{01}, w_{02}, w_{03}, w_{04}, w_{11}, w_{12}, w_{13}, w_{14} \ge 0\} \\ &(\text{positivity condition}) \quad (iv) \end{aligned}\right\} \quad (39)$$

The OC given in expression 39(*iii*) are calculated by using the following payoff matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{Bmatrix}$$

DS will be obtained from dual problem (39) as:

$w_{01} = 0.2566551$, $w_{02} = 0.2180569$, $w_{03} = 0.2408031$, $w_{04} = 0.2844849$ and $v(w^*) = 7.131790$.

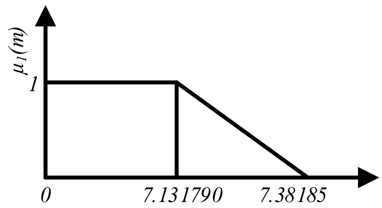

FIGURE 5. Membership function for minimization variances problem.

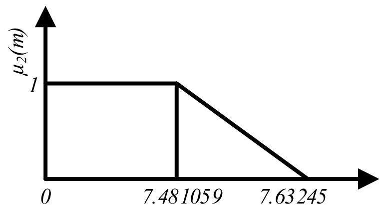

FIGURE 6. Membership function for minimization variances problem.

Similarly, the values of the objective function and SS of NR for phase-II, calculated by using equation (17) from sub-problem-1, are as follows: 7.131790 and $m_{1.12}^* = 30$, $m_{1.22}^* = 19$, $m_{1.32}^* = 17$, $m_{1.42}^* = 17$.

Sub problem 2: By putting the values from given tables into the sub-problem 2, we get the required equation as:

$$\left.\begin{aligned} &Min\, f_{02} = \frac{73.845820}{m_{12}} + \frac{33.747274}{m_{22}} + \frac{15.13049}{m_{32}} + \frac{43.04568}{m_{42}} \\ &Subject\ to \\ &0.001263 m_{12} + 0.001789 m_{22} + 0.002105 m_{32} + 0.02421 m_{42} \le 1; \\ &m_{h2} \ge 0\ and\ h = 1, 2, \ldots, 4 \end{aligned}\right\} \quad (40)$$

For the dual problem (41) of (40) is given as:

$$\left.\begin{aligned} &Maxv(w_{0i}^*) = \left((73.845820/w_{01})^{w_{01}}\right) \times \left((33.747274/w_{02})^{w_{02}}\right) \times \left((15.13049/w_{03})^{w_{03}}\right) \\ &\times \left((43.04568/w_{04})^{w_{04}}\right) \times \left(\left(\frac{0.008571}{w_{11}}\right)^{w_{11}}\right) \times \left(\left(\frac{0.0114}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\frac{0.01429}{w_{13}}\right)^{w_{13}}\right) \times \\ &\left(\left(\frac{0.01714}{w_{14}}\right)^{w_{14}}\right) \times ((\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14}) \wedge (\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14})); \quad (i) \\ &Subjectto \\ &w_{01} + w_{02} + w_{03} + w_{04} = 1 (\text{normality condition}) \quad (ii) \\ &\left.\begin{aligned} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{aligned}\right\} (\text{orthogonality condition}) \quad (iii) \\ &w_{01}, w_{02}, w_{03}, w_{04}, w_{11}, w_{12}, w_{13}, w_{14} \ge 0\} \\ &(\text{positivity condition}) \quad (iv) \end{aligned}\right\} \quad (41)$$

The OC given in expression 41(*iii*) are calculated by using the following payoff matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{Bmatrix}$$

DS will be obtained from dual problem (41) as: $w_{01} = 0.2901044$, $w_{02} = 0.2258481$, $w_{03} = 0.1700048$, $w_{04} = 0.3140427$ and $v(w^*) = 7.481059$.

The values of the objective function and SS of NR are given as: 7.481059 and $m^*_{2.12} = 34$, $m^*_{2.22} = 20$, $m^*_{2.32} = 12$, $m^*_{2.42} = 18$.

For phase –II, the pay-off matrix is obtained as:

$$\begin{array}{c} \\ (m^{(1)}) \\ (m^{(2)}) \end{array} \begin{array}{cc} f_{01}(m) & f_{02}(m) \\ \left[\begin{array}{c} 7.131790 \\ 7.38185 \end{array}\right. & \left.\begin{array}{c} 7.632457 \\ 7.481059 \end{array}\right] \end{array}$$

The limiting values $f_{01}(m)$ and $f_{02}(m)$ using pay-off matrix can be obtained as: $7.131790 \le f_{01}(m) \le 7.38185$ and $7.481059 \le f_{02}(m) \le 7.632457$.

Let the fuzzy membership function $\mu_1(m)$ and $\mu_2(m)$ for $f_{01}(m)$ and $f_{02}(m)$ are defined respectively as:

$$\mu_1(m) = \begin{cases} 1, & \text{if } f_{01}(m) \le 7.131790 \\ \dfrac{7.38185 - f_{01}(m)}{0.25006}, & \text{if } 7.131790 \le f_{01}(m) \le 7.38185 \\ 0, & \text{if } f_{01}(m) \ge 7.38185 \end{cases}$$

$$\mu_2(m) = \begin{cases} 1, & \text{if } f_{02}(m) \le 7.481059 \\ \dfrac{7.632457 - f_{02}(m)}{0.15140}, & \text{if } 7.481059 \le f_{02}(m) \le 7.632457. \\ 0, & \text{if } f_{02}(m) \ge 7.632457 \end{cases}$$

The final primal MOCPP for phase II can be obtained by using max-addition operator as described in phase I is as:

$$\left.\begin{array}{l} Min = \dfrac{704.5716}{m_{12}} + \dfrac{340.0430}{m_{22}} \\ + \dfrac{215.6658}{m_{32}} + \dfrac{418.9826}{m_{42}} \\ Subject\ to \\ 0.008571 n_1 + 0.0114 n_2 + 0.014295 n_3 \\ + 0.01714 n_4 \le 1; \\ m_h \ge 0\ and\ h = 1, 2, \ldots, 4 \end{array}\right\} \tag{42}$$

Dual of the final Primal problem for phase II:

$$\left.\begin{array}{l} Max\ v(w^*_{0i}) = \left((704.5716/w_{01})^{w_{01}}\right) \\ \times ((340.0430/w_{02})^{w_{02}}) \times ((215.6658/w_{03})^{w_{03}}) \\ \times ((418.9826/w_{04})^{w_{04}}) \times \left(\left(\dfrac{0.008571}{w_{11}}\right)^{w_{11}}\right) \\ \times \left(\left(\dfrac{0.0114}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\dfrac{0.01429}{w_{13}}\right)^{w_{13}}\right) \times \\ \left(\left(\dfrac{0.01714}{w_{14}}\right)^{w_{14}}\right) \times ((\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14}) \\ \wedge(\mathrm{w}_{11} + \mathrm{w}_{12} + \mathrm{w}_{13} + \mathrm{w}_{14})); \qquad (i) \\ Subject\ to \\ w_{01} + w_{02} + w_{03} + w_{04} = 1 \text{(normality condition)} \quad (ii) \\ \left.\begin{array}{l} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{array}\right\} \text{(orthogonality condition)} \quad (iii) \\ w_{01}, w_{02}, w_{03}, w_{04}, w_{11}, w_{12}, w_{13}, w_{14} \ge 0\} \\ \text{(positivity condition)} \qquad (iv) \end{array}\right\} \tag{43}$$

The OC given in expression 43(*iii*) are calculated by using the following payoff matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \end{Bmatrix}$$

DS will be obtained from dual problem (43) as: $w_{01} = 0.2773898$, $w_{02} = 0.2222447$, $w_{03} = 0.1981613$, $w_{04} = 0.30220427$ and $v(w^*) = 78.48303$.

The optimal values of objective function and SS of NR are obtained as follows: 78.48303 and $m^*_{2.12} = 32$, $m^*_{2.22} = 19$, $m^*_{2.32} = 14$, $m^*_{2.42} = 18$

VII. CONCLUSION

In this paper we have considered double sampling problem in the presence of non-response which is formulated as Fuzzy

Multiobjective Convex Programming Problem (FMOCPP). The formulated problem was very complicated and difficult to solve by general methods. The membership function was applied for converting the FMOCPP into Fuzzy Single Objective Convex Problem (FSOCP). We have used Fuzzy Geometric Programming Approach to solve FSOCP. There are many optimization softwares, now available for solving optimization problems, but we have used LINGO Software for solving FSOCP to get the dual solutions. We have used dual solutions and primal-dual relationship theorem to get optimum allocations of SS of R and NR. The given procedure is illustrated by a numerical problem. We have used the same problem as taken by [16], they have formulated their problem as a multiobjective programming problem and used goal programming to obtain the solution. We have also formulated the same problem and Fuzzy Geometric Programming Approach has been used to solve the same problem, the result is same as calculated by [16]. Thus, the projected method can be used for solving other complex problems.

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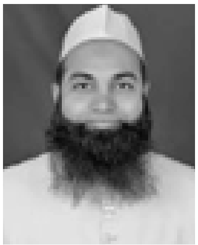

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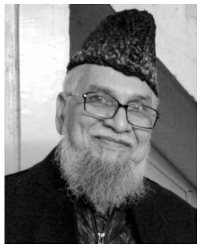

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