IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received December 29, 2019, accepted January 20, 2020, date of publication February 12, 2020, date of current version March 12, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2973494

High-Density Parking for Automated Vehicles: A Complete Evaluation of Coordination Mechanisms

JOSÉ AZEVEDO, PEDRO M. D'OREY, (Member, IEEE), AND MICHEL FERREIRA
Instituto de Telecomunicações, 4200-465 Porto, Portugal
Department of Computer Science, Faculty of Sciences, University of Porto, 4169-007 Porto, Portugal

Corresponding author: Pedro M. d'Orey (pedro.dorey@dcc.fc.up.pt)

This work was supported in part by the Project MobiWise under grant POCI-01-0145-FEDER-016426, funded by the FEDER (European Regional Development Fund) through the Operational Competitiveness and Internationalization Programme (COMPETE 2020) and by national funds through Fundação para a Ciência e Tecnologia, the project S2MovingCity under grant CMUP-ERI/TIC/0010/2014, and grants UID/EEA/50008/2019 and UIDB/EEA/50008/2020 funded by FCT/MCTES through national funds and when applicable co-funded EU funds.

ABSTRACT Three main changes are currently taking place in the context of the automotive industry: electrification of vehicle propulsion, automated driving and a shift towards mobility as a service. While the first two represent opportunities for the industry growth, the later questions the private ownership of a car. Keeping the concept of a privately owned car will involve reducing the economical and environmental cost of such ownership. In this paper, we address the reduction of the parking footprint of cars, leveraging on electrification and low-level driving automation to more than double the density of cars parked in a given area, compared to conventional parking lots. We perform a complete evaluation of different strategies of vehicle coordination based on large-scale datasets of parking sessions in distinct scenarios and under varying demand patterns. Our results on the key metrics, namely area per vehicle, travel distance while parked, and removal time - clearly highlight the relevance and efficiency of this novel approach to parking. We also empirically derive guidelines for designing high-density parking systems (e.g. parking lot layout or capacity) and show the involved trade-offs.

INDEX TERMS Autonomous vehicles, high-density parking, parking, path planning, vehicular networks.

I. INTRODUCTION

Private transport is the dominant form of transportation in most of the world, relying increasingly on automobiles to move people from one place to another [1]. Car-based transport has proved unsustainable [2], consuming excessive energy, affecting the health of populations, and being greatly responsible for the phenomena of urban sprawl, which consumes natural habitat and agricultural lands. Most measures to mitigate the negative effects of a car-based society are centered on the use of fiscal policies to influence vehicle purchase decisions with a low CO_2 emissions figure often resulting in reduced taxation [3]. However, centering such policies on the potential emissions of each car captures only the environmental impact caused during the *mobile* existence of the vehicles, which typically represents only 5% of their overall lifespan. The impact of a *parked* car is non-existent in terms of pollutant emissions or noise, but clearly exists in terms of land use, where it is a major consumer, leaving less land available for other purposes. Note that in some urban areas the parcel of land solely devoted to parking can exceed 20% (e.g. Atlanta, USA [4]), becoming the single biggest land use in terms of public infrastructure.

The straightforward measure that would capture the impact of a parked car in terms of land use is obviously the area occupied by each vehicle. However, fiscal and pricing policies built around the area of each car are very rare. As a result, and in contrast to what we observe in terms of CO_2 emissions of cars during the last decades, the area of the top selling automobiles in the world has been increasing steadily. The top selling car in Europe in 2017 and in 2008 was the Volkswagen Golf, which grew almost 2% in area over that period. A similar or even higher increase in area occurred with all of the most representative cars sold worldwide over

The associate editor coordinating the review of this manuscript and approving it for publication was Razi Iqbal.

the last years (e.g. the area of the Volkswagen Polo grew by more than 8% in the last decade).

A simple reason that explains why area-related measures have been absent from fiscal and pricing policies for cars is the fact that the area occupied by a parked car is determined rather by the area of the delimited parking spot. As the areas of these parking spots are essentially identical, the areas occupied by a small versus a large car parked in such parking spots are exactly the same, becoming irrelevant for the purpose of policy making. The fact is that in terms of land use, the ownership of a large car or a small car is not differentiated, given the current paradigm of delimited spaces, either for curbside or off-street parking.

To consider the area of each car in fiscal and pricing policies that can influence the buying decisions of car owners, we need a new paradigm of urban car parking where parking spaces are unbound and the actual measures of each car are what defines the occupied area while parked. In addition, and to significantly reduce the land use devoted to parking, the new paradigm shall achieve a much higher density in terms of the number of cars parked in a given area, compared to current parking lots. By achieving a 2 or 3 times higher density, this new approach to parking has the potential to change the landscape of our cities in a very significant way.

Few works in the literature focused on improving space efficiency being the vast majority of the research devoted to sensing and disseminating parking availability (e.g. [5]) or automatic parking of vehicles (e.g. [6]). In a seminal paper [7], we have presented the concept of high-density parking based on the collaborative mobility of parked cars that autonomously move to accommodate entering cars, or to create exit paths to blocked vehicles. This slow and autonomous mobility of parked cars is highly compatible with the current technology of new cars, both in terms of sensing and control, as well as in terms of energy efficiency, given the market shift to hybrid and fully electric propulsion.

The optimization of this collaborative mobility is a highly interesting problem, for which we have done preliminary studies [8], [9]. The main challenge is related to the determination of movement plans for (cohorts of) vehicles whose motions might be interdependent due to the parallel task execution and the high-density parking configuration. In this paper we make the first complete study of the high-density parking concept, evaluating large parking lots in different settings, using different strategies and spatial configurations.

To summarize our main contributions are as follows:

- we propose a planning and control framework to efficiently move (a cohort of) vehicle(s) in a high-density parking configuration considering the interdependent motions of vehicles;
- we present several algorithms for selecting the original destination for each entering vehicle and the associated reallocation strategies for in-park vehicles;
- we perform a large-scale evaluation of the proposed framework making use of empirical data from several parking lots considering different parking technologies, layouts, capacities and demand patterns.

The remainder of the paper is organized as follows. We review the relevant state-of-the-art in Section II. The proposed planning and control framework is detailed in Section III, while section IV presents the four strategies for reallocating vehicles in a high-density parking configuration. In Section V, we characterize the three empirical datasets collected in San Francisco, USA. The proposed methods are evaluated making use of empirical data in Section VI. Concluding remarks and future work directions are given in Section VII.

II. RELATED WORK

A. (AUTOMATIC) PARKING ASSISTANCE SYSTEMS

In recent years, much of the literature in the area of parking has focused on smart parking solutions, where information collection based on networked sensors and real-time dissemination to drivers are the main topics. Numerous works in the area of parking assistance systems propose the use of sensing, computation and communication technologies to solve the problem of finding available on- and off-street parking spaces. Sensing technology [e.g. WiFi [10], LIDAR [11], GPS [12] or sensors [13]] can be used to detect available parking spaces. Parking availability is then shared through parking information systems to assist on parking space selection, guidance to parking space [14], [15] and space reservation. Klappenecker *et al.* [16] proposed the dissemination of parking information (e.g. occupancy and arrival rate) through vehicular networks so that vehicles can predict the availability of free parking spaces upon arrival. A complete survey on smart parking is presented in [5].

In addition to these research efforts in the context of smart city sensing and information dissemination related to parking, vehicle manufacturers have also presented several systems that automate parking operations to simplify the process, with a more local and maneuver-oriented perspective. In this context, self-parking systems (e.g. [6]) automatically perform parallel and perpendicular maneuvers by controlling the vehicle actuators depending on sensing information. Specifically, several authors proposed specific path/motion planning methods (e.g. [17]) for automated parking given the more compact nature of parking lots (i.e. tight environments). Automated Valet Parking (e.g. [18], [19]) combines self-parking with autonomous driving, allowing the passenger to leave at its destination rather than at the parking lot. A survey on automatic parking is available in [6].

B. HIGH-DENSITY PARKING

In the above mentioned works the architecture of parking lots and parking spaces are unchanged, with no implications in terms of the space occupied by parked cars. To reduce the space occupied by parked cars, automated robotic parking systems (e.g. Parkmatic[1]) - that resort to electric elevators

[1]http://www.parkmatic.com

and rotating/sliding platforms to automatically move vehicles in a high-density parking configuration - have been proposed decades ago. However, due to their complexity, these systems present high capital and operational costs, which results in high costs for the end user. Additionally, access times to retrieve vehicles can be high due to the limited availability of moving platforms.

To reduce space requirements for parking, while simultaneously maintaining low capital and operational costs, we proposed a novel concept for autonomous parking enabled by drive-by-wireless (DbWl) and vehicular networks [7]. A high-density parking configuration - where inter-vehicle distance is kept to a minimum - improves considerably space utilization. Instead of using mechanical moving platforms, our system relies on collaborative vehicle mobility that replaces the concept of a static parking session by a slow motion idle state that is particularly compatible with electric vehicles. The collaborative in-park vehicle mobility is governed by a controller that determines planning strategies and controls remotely vehicles through wireless communications in combination with drive-by-wire. Note that currently an increasing number of high-end cars offer the ability to slowly move and maneuver the car from a smartphone (e.g. [20]). A video[2] with a small-scale prototype done with radio-controlled miniatures provides a clear and visual explanation of the concept. In [21], the authors also study parking space optimization for automated valet parking making an in-depth theoretical analysis of the parking lot properties under various aspects, including the worst-case extraction time, total traveled distance, and the number of movements per car. A later work [22] also studied high-density parking but made several unrealistic assumptions (e.g. fixed demand).

The design of a high-density parking lot configuration allows improving the space requirements but demands the implementation of **collision-free path plans** for multiple vehicles to allow the entry or exit of selected vehicles from the parking area while minimizing a cost function (e.g., total travel distance). This task relates to the Multi-agent Path Planning Problem. Multi-agent path planning has been shown to be a PSPACE-hard problem [23]. Wang and Botea [24] proposed a multi-agent path planning algorithm for grid maps that runs in low-polynomial time. Wang *et al.* [25] proposed a Scalable Multi-Agent Path Planning Algorithm with Tractability and Completeness Guarantees for the class of problems termed Slidable. Wilde *et al.* [26] presented the Push and Rotate algorithm to move agents to specific positions through evasive movements of other agents. Similar tasks are found in other application domains, namely goods transfer [27], warehouse storage [28], transshipment [29], container stacking [30], among others. Ma *et al.* [31] discussed the challenges arising from the application of multi-agent path finding to real-world scenarios and present several research directions for the generalization of this problem.

[2]https://youtu.be/eD9KUGIzTfA

In the classical bin-packing problem a set objects of different sizes must be packed into a minimum number of identical bins. This problem has been extensively studied during the last decades due to the its computational complexity and the number of possible application domains (e.g., [32]). Compacting vehicles of variable size in a high-density parking configuration relates to the family of two-dimensional packing problems (e.g. [33]). More specifically, the herein considered problem is to allocate a set of rectangular items (i.e., vehicles) to the minimum number of bins (of a rectangular high-density parking area) fulfilling certain constraints, such as fixed orientation (i.e., vehicles should be parked bumper-to-bumper), vehicle constraints (e.g., minimum door-to-door distance or minimum turning radius) and possibly fixed vehicle ordering.

III. PLANNING AND CONTROL FRAMEWORK

A. SYSTEM FUNCTIONING

In a setting of automated parking lots, users will leave their cars at the entrance of these infrastructures, where for safety reasons only empty vehicles will enter. Upon arrival, the vehicle and the Parking Lot Controller (PLC) exchange information (e.g. user identification, estimated exit time, parking map) via vehicular networks. The PLC is responsible for controlling the access to the parking area and for coordinating the vehicle mobility inside the parking lot. Depending on the automation level of the vehicle, the PLC will 1) remotely drive the vehicle by sending control messages (e.g. accelerate, brake, steer) to the onboard unit (OBU) that forwards them to the vehicle actuators using Drive-by-Wire (DbW) technology, or 2) assign new parking position to fully autonomous vehicles. The vehicles will report the execution of the maneuvers to the PLC. Due to the high-density parking configuration, the parking lot controller often needs to move vehicles to allow the entry or exit of vehicles, which will be coordinated by the in-park systems. The system functioning relies on several assumptions:

- vehicles can be controlled remotely with high precision (e.g. already available in commercial vehicle for remote vehicle parking in tight spaces [20]);
- accurate positioning and environment sensing information is available;
- low-latency, reliability and secure Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communications (e.g. [34]);
- selected system information (e.g. vehicle exit times) is available or can be estimated with high accuracy.

B. SETTING

The automated parking lot depicted in Fig. 1 is composed by:

- **parking area** (*P*) for long-term storage of vehicles; *P* can be viewed as $m \times n$ matrix with each parking space being identified by p_{ij}.
- **buffer area(s)** (B_i) for transferring vehicles between (different) stacks, short-term storage of vehicles (e.g.

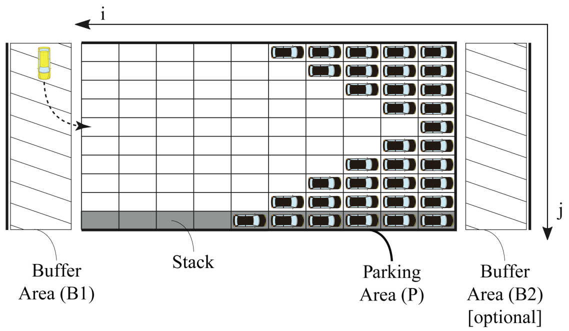


FIGURE 1. High-density parking lot with *n* interacting stacks (10 stacks in the example) each with capacity to hold up to *m* vehicles (10 places per stack in the example) and two buffers areas (B1, B2) at each end of the parking area (P). Buffer area B2 is optional.

for blocking vehicles) and circulation area for for entering/exiting vehicle;
- (optional) **entry/exit pathways**.

Vehicles are parked in rectangular cells in a grid structure P. The bump-to-bump and door-to-door distance between vehicles in P is kept to a minimum to achieve the high-density configuration. The grid P can be viewed as being composed by n parallel interacting *stacks* (i.e. columns). In this study, we consider that the high-density parking makes use of a single buffer area $B1$.

The parking lot demand is affected by a wide number of factors, namely time of the day, day of the week (e.g. weekend vs. weekday), special events (e.g. soccer match), location (e.g. business districts vs. residential area), among many others. The demand can be formalized by a sequence of independent vehicle entry or exit events; each event could contain the following information: 1) entry time, 2) estimated exit time, 3) vehicle characteristics, etc. The variable user demand will greatly impact the performance of the automated parking lot system.

C. PROBLEM DEFINITION

This system has as basic operation to criteriously assign vehicles V to parking spaces. In the parking lot vehicles can travel individually or in a cohort (i.e. platoon) reducing the in-park mobility. Due to the high-density configuration, the entry or exit of vehicles from the parking lots usually requires moving additional vehicles as frequently no obstacle-free path exists between two given cells of P. For instance, vehicles in the parking lot might be required to move to allow the entry of vehicle(s) to reach more favorable parking positions (e.g. entering vehicle has a later departure time). The proposed system should be able to optimize the in-park mobility, including (1) safely and efficiently navigate all vehicles from their start to target positions, while avoiding movable obstacles, and (2) select the vehicle destination (i.e. final parking position) taking into consideration a number of criteria (e.g. total travel distance).

The problem of automatic plan and control of vehicles in a high-density parking lot has unique features that differentiates it from other similar problems (e.g., container stacking problem). First, in our setting several vehicles can be moved simultaneously in a platoon between different stacks of the parking lot (as opposed to the container stacking problem where only one container can be moved at once). Second, obstacles (i.e., vehicles) are self-movable, which creates opportunities for further optimization. The selection of the vehicle destination is tightly coupled with path planning in high-density parking lots. Additionally, exchange of positioning information between vehicles via V2X networks allows improving the system observability and predicting possible conflicts. Also, the regularity and predictability of certain system parameters (e.g. vehicle exit times), which are used by people in their daily routines, can further enrich the optimization strategies. On the other side, there are strict requirements on the algorithmic execution times since (i) the frequent storage and retrieval of vehicles creates re-configurations of the system layout in a high-density parking area that can contain hundreds of vehicles and (ii) the system should guarantee short vehicle retrieval times to provide a good Quality of Experience (QoE) to the end user.

IV. PLANNING AND CONTROL STRATEGIES

In the following, we present four planning and control strategies for parking vehicles in a high-density configuration that make use of the framework presented in the previous section. More specifically, we describe heuristic-based procedures for selecting the *Vehicle Destination*, which are performed when a new vehicle enters the system (*storage*) or whenever vehicles within the system need to be relocated (*relocation*) to allow the entry or exit of other vehicles. The criteria for selecting the vehicle destination should ensure low in-park mobility and fast access to vehicles on exit (*removal*) and could include a set of geometric (e.g. number of vehicles in each column), temporal (e.g. vehicle exit times), distance (e.g. in-park travel distance) parameters, among others. Note that these planning strategies might have access or able to estimate with precision future vehicle entry and/or exit times.

Mobility in the automated parking lot is triggered by the entry or exit of vehicles. The interaction between stacks is materialized by the transfer of vehicles between columns depending on a number of conditions (e.g. turning radius) and the selected vehicle control strategy. The transfer of vehicles between stacks can assume three forms:

- **1-1 strategy:** vehicles are moved in convoy to a single destination stack;
- **1-N strategy:** vehicles are moved from their current position to potentially n different stacks (i.e., new vehicle destinations are computed individually);
- **1-back strategy:** vehicles are redirected to the stack that they left to allow the entry or exit of other vehicle.

We consider the following two planning and control strategies without future system knowledge:

Smallest Length Stack (SLS): The criteria for selecting the target vehicle position is the current stack occupancy. Stack occupancy is defined as the number of parking spaces that are currently occupied in a given stack. A vehicle arriving to the parking lot is directed to the stack with smallest stack occupancy; if there exist several stacks with the minimum occupancy, the vehicle is placed in the leftmost stack. The exit of a vehicle from the parking lot might cause the relocation of other vehicles if these are blocking the current path to a parking exit. The SLS procedure makes use of the *1-N* strategy. In this pattern, vehicles being relocated are assigned a new parking position as if they were now entering the parking lot (i.e., the vehicle destination procedure is applied to each relocating vehicle). Note that vehicles can potentially be re-directed to *N* different stacks. After all blocking vehicles have been moved to new stack(s), the exiting vehicle proceeds to the closest exit.

Smallest Length Stack Platoon (SLSP): The criteria for selecting the target vehicle position is the current stack occupancy. A vehicle arriving to the parking lot is directed to the stack with smallest occupancy; if there exist several stacks with the minimum occupancy, vehicles are placed in the leftmost stack (similarly to SLS). If a vehicle leaving a stack *i* is blocked by other cars, all blocking vehicles move in platoon to the buffer area to allow the 'exiting' vehicle to proceed to the closest exit. Afterwards, the blocking vehicles return in platoon to the initial stack *i*. Note that all blocking vehicles will be redirected to solely one stack. The SLSP procedure makes use of the *1-back* strategy.

We consider the following two planning and control strategies with future system knowledge:

Conditional Order on Arrival (COA): In this scheme the main objective is to have vehicles in stacks ordered by exit time (i.e., vehicles with smallest parking duration are parked closer to the buffer area). In the following, we assume that the vehicle exit times are known in forehand. Consider that $t^{out}(v)$ and $t^{out}(v^{st}_{first})$ are the exit time of a vehicle v entering the parking lot and the exit time of the vehicle in stack st closest to the buffer area, respectively. In order to reduce the probability of moving vehicles to allow the exit of a given car, the vehicle destination strategy parks together vehicles with similar exit times. Thus, the selection of the vehicle destination is done based on the heuristic δ_{st}: minimum positive time interval between $t^{out}(v)$ and $t^{out}(v^{st}_{last})$. The procedure iterates over the different rows, selects the vehicle closest to buffer area and selects the stack with minimum positive δ_{st}. We solely select the vehicle closest to the buffer area to reduce the in-park mobility due to the introduction of a new vehicle into the system. Since often the number of available parallel stacks will be small (i.e. all stack would produce $\delta_{st} < 0$), the vehicle ordering might be lost implying additional maneuvers during vehicle exit and the vehicle is parked in the stack with smallest occupancy; if there exist several stacks with the minimum occupancy, the vehicle is placed in the leftmost stack (similarly to the SLS procedure).

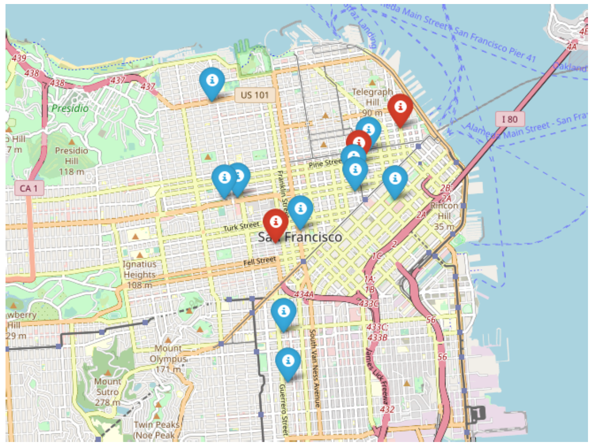


FIGURE 2. Geographic location of parking garages operated by the San Francisco Municipal Transportation Agency (SFMTA). The garages used in the study are represented by red icons, while the remaining parking lots are depicted in blue.

In general, the exit of a vehicle from the parking lot will not require maneuvering of other vehicles as vehicles in the stack are ordered by exit time, reducing the in-park mobility. However, the COA strategy does not guarantee that vehicles are *always* ordered by exit time in the stacks. Thus, whenever an exiting vehicle is blocked by other vehicles, it will require that the path to a parking lot exit has to be freed. This strategy makes use of the *1-N* vehicle relocation pattern. In this pattern, each vehicle being relocated is assigned a new parking position in the stack with smallest δ_{st}.

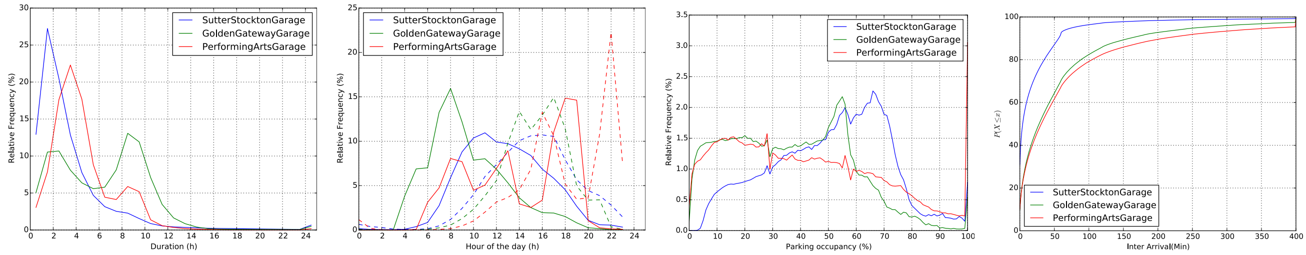
Order on Arrival (OoA): The criteria for selecting the target vehicle position is the time interval between vehicle exit times δ_{st}. Similarly, a vehicle arriving to the parking lot is directed to the stack with the minimal (positive) exit time difference δ_{st}. The system determines the vehicles that will leave the parking lot after the entering vehicle ($\delta_{st} > 0$) and selects the stack spot where the difference between exit times is minimal. If $\delta_{st} < 0$, the vehicle v_i is placed on the lowermost row of the stack s_j where the difference between exit times is minimized. This strategy ensures that vehicles are ordered in stacks by exit time from top to bottom (i.e. vehicles with later exit times placed at the topmost row). A vehicle leaving the parking lot will proceed directly to the closest exit without requiring the execution of maneuvers by other vehicles in all situations.

V. DATASETS CHARACTERIZATION

Detailed information on off-street parking lots for the city of San Francisco, USA, was retrieved from the SFPark website (http://sfpark.org/). The dataset comprises around 11.6 million parking records for the year 2012 and contains the following information for each parking record: 1) facility name, 2) entry time, 3) exit time, 4) payment amount and 5) payment type (e.g. cash). The dataset has information on fifteen distinct parking lots. To emulate the parking functioning, we split the data into different datasets per parking lot and separate vehicle entry and exit events into different rows

TABLE 1. Dataset characterization.

Metric	PAG	GGG	SSG
Capacity (#)	598	1095	1865
Effective Capacity (#)	418	766	1305
Maximum Occupancy (#)	814	1363	1666
Parking Events ('000)	389	631	2324
Fare ($) [peak fare: period]	2-4 [$4:12-15h]	7 [$7:00-18h]	2-5 [$5:12-15h]
Location	37.778047,-122.422796	37.781389,-122.415703	37.790351,-122.407126
Neighborhoods	Downtown/Civic Center Western Addition	Financial District	Chinatown Financial District Downtown/Civic Center



(a) Parking Duration (b) Entry (—)/ Exit (- -) Events (c) Parking Occupancy (d) Vehicle Inter-Arrival Time

FIGURE 3. Analysis of San Francisco parking datasets.

ordered by event time to have the sequential events order for the simulation.

In this study, we assess the performance of the high-density parking system in three parking lots with distinct parking patterns, namely Performing Arts Garage (PAG), Golden Gateway Garage (GGG) and Sutter Stockton Garage (SSG). We chose these datasets because they have distinct parking locations, distinct entry distributions and are located in different areas of the city (see Fig. 2). Table 1 summarizes the characteristics of each dataset. The SF garages have very distinct number of vehicles spaces, being the average occupancy clearly below these figures due to San Francisco Municipal Transportation Agency (SFMTA) target of maximum parking occupancy of 70% of parking capacity [35]. Valet parking enables garages to hold more vehicles than its capacity as stated in [35]. The number of parking events is clearly different for the three garages having SSG six times more demand than PAG despite its capacity being solely around 3 times larger. The fare system for each parking lot is clearly different due to the different demand and local characteristics having the GGG the highest fare throughout the day (until 06:00 pm).

In the following, the surroundings of each of the parking lots are characterized in more detail. Table 1 presents the geographical location and the main neighborhoods in the vicinity of each garage. The Performing Arts Garage is situated in a predominantly artistic area, full of theaters and nightlife attractions. The Golden Gateway Garage is situated in the financial district of the city. *Banks*, *Financial Center* and *Gryphon Investors* are some of the main points of interest in this area. The Sutter Stockton Garage is located in a more general area, with several restaurants, hotels and churches, being nearby of the *Old St Mary's Cathedral*, *Pantheon* and *St Mary's Square*.

Dataset Analysis: Fig. 3 presents a comparative analysis of the three San Francisco parking datasets, which have clearly distinct temporal usage patterns:

- Due to the parking location and the associated activities, the parking duration (Fig. 3a) differs widely between parking lots: 1) the GGG park exhibits a bimodal distribution with the main mode at 8-hour that can be associated with work-related activities and 2) the other parks typical have parking duration below 4 hours (modes below 2-hour and 4-hour for the SSG and PAG, respectively).
- Fig. 3b depicts the frequency of vehicle entries and exits from the parking lot as a function of the hour of the day. The GGG and PAG parking lots have more concentrated entry/exit times (e.g. vehicle entry at 8h for GGG while 19/20h for PAG), while users arrive throughout the day to SSG. Specifically, the GGG park has a pronounced entry peak at 8h, while the PAG park has 3 modes (8h, 13h, 19/20h) demonstrating the different usage pattern. Similar results are obtained for the vehicle exits: the PAG has exits modes at 16h and 22h, while the GGG has exits modes at 14h and 17h.
- Fig. 3c presents a histogram of the parking lot occupancy, i.e. the ratio of number of vehicles inside the parking lot to the parking lot capacity. The result show that 1) the SSG park has consistently higher levels of parking occupancy (peak around 70%) and 2) that the parking occupancy varies widely (throughout the day) resulting in different demand patterns for the high-density parking system.
- Fig. 3d depicts the inter-arrival time, i.e. time between consecutive parking entry events. The results show that this metric is very small for all parking lots. The vehicle

TABLE 2. Characterization of conventional and high-density parking lots used in the simulation study for assessing the system performance.

Metric	Conv.	PAG Conf. 1	PAG Conf. 2	GGG	SSG
Capacity (#)	286	297		640	1000
Width (m)	175	54	22	80	100
Height (m)	28	55	135	80	100
Shape	n/a ()	11x27	27x11	16x40	20x50
Aspect Ratio (in meters)	n/a (6.25)	2.45 (1.02)	2.45 (6.14)	2.5 (1)	2.5 (1)
Area (m^2)	5950	3240	3080	6800	10500
Buffer areas (area in m^2)	n/a (-)	1 (270)	(110)	(400)	(500)
Area/car (m^2)	20.8	10.9	10.4	10.62	10.5

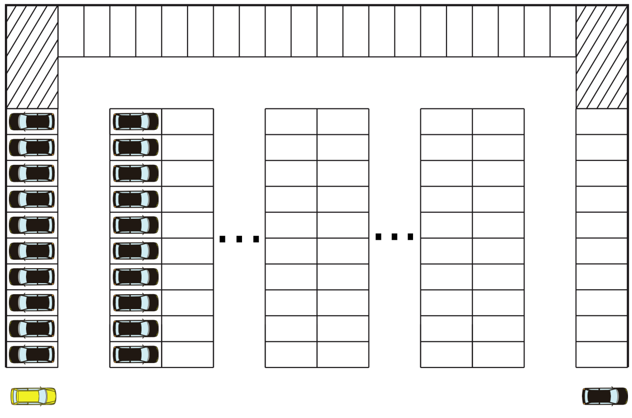


FIGURE 4. Conventional parking lot layout. The conventional parking lot can hold up to 286 vehicles. For visual clarity reasons, we do not depict all parking spaces.

arrival time is below 100 minutes in at least 80% and 95% of the cases for the PAG/GGG and the SSG, respectively, which is also clearly related with the larger garage capacities and higher demand patterns.

VI. RESULTS AND DISCUSSION

A. SYSTEM SIMULATION

To assess the different planning and control strategies, we resorted to a discrete event simulator that makes use of the empirical data previously described. The simulator keeps track of the (current) parking lot state (e.g., assignment of vehicles to parking spaces, pending action list) and determines the vehicle trajectories. Vehicles perform kinematically-valid motions (e.g., we consider maximum velocities, accelerations and turning radius for vehicles moving inside the parking lots). The actions performed by the vehicles are dependent on the different planning and control strategies. We also compare the performance of the high-density parking system with a conventional parking lot design.

Parameter Setting: The settings for the main system simulation parameters are given in Table 2. As show in Section V the garages have different characteristics, namely in terms of demand and capacity. In order to further increase the average parking occupancy (and consequently the system demand), we have further reduced the effective parking capacities (Table 1) for this simulation study as shown in Table 2.

The generic layout of our high-density parking lot is depicted in Fig. 1. The high-density parking lots considered in the study have different number of n of parallel stacks and each stack can hold up to m vehicles (shape: m x n). Each parking space has an area of 10 m^2 (width: 2 m and height: 5 m). We defined conservative limits for maximum speed (10 km/h) and maximum acceleration (0.4 m/s^2). Vehicles enter or leave the parking lot through the closest exit.

B. METRICS

In this simulation study, we consider four metrics for the evaluation of the different planning and control strategies, which are defined as follows for mobility inside the garage:

- **Parking space per vehicle** (m^2): quotient between the total parking area to the parking capacity. This metric allow determining how space efficiency varies with the system implementation.
- **Maneuvers (#)**: total number of maneuvers performed per vehicle between each vehicle start until the last full stop for vehicle entry, vehicle exit and inter-stack mobility. For conventional parking lots this metric is fixed to two (1 entry + 1 exit). This metric allows assessing the number of additional maneuvers and vehicle start-ups required by automated parking lots.
- **Travel distance (m)**: total distance traveled including entry/exit and inter-stack mobility, if applicable. This metric allows understanding the economic and environmental implications of the system implementation.
- **Removal time (s)**: time elapsed between pickup request and vehicle arrival to the parking lot entrance. This metric allows understanding the Quality of Service (QoS) that the end-user can experience.

Note that these metrics are aggregated on a per-vehicle basis and that lower values represent better performance. For the last three metrics, we estimate the corresponding empirical Cumulative Distribution Function (eCDF) and present the percentiles 5, 50, 85 and 99 % of the eCDF - that represent critical values of the distribution - whenever comparing the different parking types, capacities and planning & control strategies. We highlight in dark blue the best performing algorithm for each tuple (*metric, percentile, position*), and in light blue the best performing algorithm for the other *position* value in the tables given in Section VI-D.

Parameter Exploration: In this study, we aim at better understanding the performance of the planning & control algorithms in the following conditions:

- *different parking technologies*, i.e. comparing the high-density parking concept with the functioning of conventional parking lots. For fairness reasons, we consider the optimal operation of the conventional parking lot, where vehicles park at the closest available parking spot and leave the garage through the closest exit.

TABLE 3. Conventional parking lot.

	Travel distance (m)				Maneuvers (#)				Removal Time (s)			
	5%	50%	85%	99%	5%	50%	85%	99%	5%	50%	85%	99%
Conventional	100	490	840	1060	2	2	2	2	15	25	30	35

TABLE 4. Performing Arts Garage (PAG) with 297 spaces [config. 1] (11 × 27 shape).

		Travel distance (m)				Maneuvers (#)				Removal Time (s)			
		5%	50%	85%	99%	5%	50%	85%	99%	5%	50%	85%	99%
SLS	*Top*	105	225	435	780	2	2	5	10	17	23	26	28
	Bottom	45	145	260	440	2	9	17	31	11	17	21	27
SLSP	*Top*	105	245	505	790	2	2	6	11	17	24	29	35
	Bottom	45	185	465	730	2	7	11	15	12	21	27	35
COA	*Top*	55	120	195	600	2	2	3	9	15	22	26	33
	Bottom	45	145	255	425	2	9	17	30	11	18	26	35
OoA	*Top*	85	235	495	945	2	8	12	27	10	15	18	26
	Bottom	45	185	390	650	2	11	19	31	9	14	17	22

- *different vehicle starting positions*, i.e. we consider two different settings: *Bottom* setting where vehicles initially park on the row closest to the buffer area $B1$ and a *Top* setting where vehicles initially park on the furthest row from the buffer area.
- *distinct layouts*, i.e. how the parking layout (shape) affects the system performance. For instance, we will compare the performance of a deep layout (e.g. PAG shape 27 × 11) with a stretched layout (PAG shape 11 × 27) with the same capacity;
- *distinct parking capacities*, i.e. how the system performance varies for increasing parking sizes with the same aspect ratio (i.e. ratio between the max(columns, rows) to min(columns, rows)).

C. RESULTS (STATIC PARAMETERS)

Herein, we compare the space efficiency of the high-density parking system against a conventional parking lot. For instance, the Performing Arts Garage (PAG) occupies a total area of 3240 m^2 considering that the buffer $B1$ has an area of 270 m^2. Note that the size of the buffer area is proportional to the parking lot width. On the other hand, the considered conventional parking lot (CPL) with a similar capacity has an area of approximately 5950 m^2. Then, the area occupied per vehicle is then 10.9 m^2 and 20.8 m^2 for the PAG and the CPL, respectively. Other high-density parking lots present similar area/car values [range: 10.5-10.9 m^2] as shown in Table 2.

The implementation of the high-density parking lot allows decreasing by approximately 50% the requirements for parking space, i.e., in the same parking area it is possible to park twice as many vehicles. The space optimization is mainly due to the compact structure of the automated parking lot and the absence of circulation lanes. Depending on the conventional parking lot layout, the benefits of high-density parking lots could even be considerably higher as 22 m^2 per parking space is considered good static efficiency of a car park with 100 parking spaces at 90° [36]. Note also that adding a second buffer area would still allow to hold *almost* twice the number of vehicles in a given parking area but would improve the flexibility of the system operation and decrease the number of conflicting trajectories in the buffer area.

D. RESULTS (DYNAMIC PARAMETERS)

1) COMPARING DIFFERENT PARKING TECHNOLOGIES

First, we compare the performance of a conventional parking lot (Table 3) against a high-density parking system (Table 4) with similar capacity. For space constraints, herein, we solely focus the analysis on the PAG parking lot but the drawn conclusions hold also for the larger parking lots (GGG and SSG). The results show that a properly designed high-density parking system (i.e. making use of a suitable planning & control strategy, layout and starting position as evaluated later on) can reduce considerably the metrics travel distance and removal time. The compact structure of the high-density parking (e.g. with nearly half the parking area) allows reducing the: 1) travel distance/time (by 2 to 3 times for the CoA/Bottom strategy,[3] which is specially evident for 50% and 85% percentiles) despite the in-park mobility to allow the entry/exit of vehicles and 2) removal time as vehicles are parked closer to the exits, which improves the quality of the service provided to the end customer.

The reduced travel distance can have important implications in terms of energy consumption (e.g. electricity or fuel) and pollutant emissions for vehicles equipped with combustion engines. In our high-density parking lots, the travel distance is not the only parameter that has to be accounted for in terms of energy consumption, since the number of start/stop events of each vehicle also has implications in terms of energy conversion. However, note that - in a paradigm of electric propulsion - the conversion of travel distance to energy consumption is much more linear than with combustion engines.

[3]Even the worst performing algorithm, *OoA/Top*, gives substantially better results in terms of distance compared to the conventional parking lot (-30%).

TABLE 5. Performing Arts Garage (PAG) with 297 spaces [config. 2] (27 × 11 shape).

		Metric											
		Travel distance (m)				Maneuvers (#)				Removal Time (s)			
		5%	50%	85%	99%	5%	50%	85%	99%	5%	50%	85%	99%
SLS	*Top*	240	900	2075	3700	2	5	12	21	21	29	32	33
	Bottom	65	450	1010	1330	4	33	79	104	9	17	24	32
SLSP	*Top*	240	1030	2510	3940	2	5	13	25	23	32	41	54
	Bottom	60	750	2280	3730	3	16	26	33	13	29	40	55
COA	*Top*	180	560	1910	3770	2	4	12	22	19	31	41	54
	Bottom	70	450	1010	1720	4	34	78	100	11	21	35	54
OoA	*Top*	210	990	2450	4360	3	20	30	61	9	13	24	32
	Bottom	60	520	1360	2570	4	26	44	70	1	12	14	24

TABLE 6. Golden Gate Garage (GGG) with 640 spaces (16 × 40 shape).

		Metric											
		Travel distance (m)				Maneuvers (#)				Removal Time (s)			
		5%	50%	85%	99%	5%	50%	85%	99%	5%	50%	85%	99%
SLS	*Top*	150	430	960	1630	2	5	10	15	19	27	31	34
	Bottom	90	310	580	930	6	41	69	100	12	20	27	32
SLSP	*Top*	130	480	1240	1930	2	4	11	15	19	30	38	45
	Bottom	80	460	950	1430	2	14	32	43	15	27	36	44
COA	*Top*	60	170	370	1240	2	2	5	13	15	26	32	41
	Bottom	90	310	580	920	5	30	80	100	12	23	35	45
OoA	*Top*	120	470	1040	1950	2	23	41	85	11	16	20	30
	Bottom	70	370	730	1340	4	32	56	95	9	16	19	25

TABLE 7. Sutter Stockton Garage (SSG) with 1000 spaces (20 × 50 shape).

		Metric											
		Travel distance (m)				Maneuvers (#)				Removal Time (s)			
		5%	50%	85%	99%	5%	50%	85%	99%	5%	50%	85%	99%
SLS	*Top*	210	650	1420	3170	2	7	14	20	19	29	34	37
	Bottom	140	490	1120	2480	14	64	148	249	14	24	30	36
SLSP	*Top*	210	740	1760	4320	2	7	14	20	21	34	43	53
	Bottom	140	830	1700	2770	2	19	38	54	20	34	43	52
COA	*Top*	110	310	940	2360	2	2	13	19	16	30	41	52
	Bottom	140	480	1110	2480	10	55	166	250	15	29	42	53
OoA	*Top*	230	1230	2490	4210	7	49	90	182	11	19	22	34
	Bottom	120	680	1510	2860	7	46	90	197	11	17	21	32

Obviously, the number of maneuvers is larger for the automated parking system as vehicles need to move to allow the entry or exit of other vehicles that might be blocked. However, we observe that for the majority of the vehicles (85%) the number of maneuvers is only slightly higher than for conventional parking lots (e.g. 3 maneuvers for CoA/Bottom for PAG - config. 1) which demonstrates the feasibility of the system. The results also show that a small subset of vehicles ($\leq$ 1%) could have large number of maneuvers per parking session, which could be mitigated through the implementation of additional mechanisms to ensure that all vehicles execute lesser maneuvers (e.g. by penalizing moving vehicles that already have above average number of movements).

After demonstrating the advantages of the high-density parking system, we now present in more detail how different parameters (i.e. starting position within the stack, layout and parking capacity) impact on the system performance in terms of the previously defined metrics. The following analysis will resort to Tables 4, 5, 6 and 7.

2) COMPARING DIFFERENT VEHICLE STARTING POSITIONS

The selection of the initial row within a stack to park a given vehicle impacts differently the performance of the four planning & control strategies. In this study, we solely focus on two extreme positions (i.e. *Bottom* or *Top*) as the other intermediate starting positions would not provide new insights as the results could be scaled based on these two core positions. The following analysis is based on the evaluation of the results of all automated parking lots, namely for the PAG, GGG and SSG parking lots.

In general, parking vehicles closer to the buffer area decreases the travel distance and removal time but, as expected, increases the number of per-vehicle maneuvers as the probability of moving these vehicles due to the entry or exit of other vehicles increases considerably. This is specially evident for layouts with low number of stacks with high capacity (i.e. *deep layouts* as for example PAG config. 2) where the number of options to park vehicles is smaller and consequently the probability of assigning vehicles a given stack is larger. The COA strategy does not follow entirely this

TABLE 8. Ratio of the GGG/SSG results for a given algorithm, start position, metric and percentile to the corresponding PAG result. This metric allows better understanding the impact on the system performance of increasing the parking capacity maintaining the same aspect ratio.

		Metric											
		Travel distance (m)				Maneuvers (#)				Removal Time (s)			
		5%	50%	85%	99%	5%	50%	85%	99%	5%	50%	85%	99%
SLS	*T*	1.4/2.0	1.9/2.9	2.2/3.3	2.1/4.1	1.0/1.0	2.5/3.5	2.0/2.8	1.5/2.0	1.1/1.1	1.2/1.2	1.2/1.3	1.2/1.3
	B	2.0/3.1	2.1/3.4	2.2/4.3	2.1/5.6	3.0/7.0	4.6/7.1	4.1/8.7	3.2/8.0	1.1/1.3	1.1/1.4	1.3/1.4	1.2/1.3
SLSP	*T*	1.2/2.0	1.9/3.0	2.5/3.5	2.4/5.5	1.0/1.0	2.0/3.5	1.8/2.3	1.4/1.8	1.1/1.2	1.2/1.4	1.3/1.5	1.3/1.5
	B	1.8/3.1	2.5/4.5	2.0/3.7	2.0/3.8	1.0/2.0	2.0/2.7	2.9/3.5	2.9/3.6	1.2/1.7	1.3/1.6	1.3/1.6	1.2/1.5
COA	*T*	1.1/2.0	1.4/2.6	1.9/4.8	2.1/3.9	1.0/1.0	2.0/2.0	1.7/4.3	1.4/2.1	1.0/1.1	1.1/1.3	1.2/1.6	1.2/1.6
	B	2.0/3.1	2.1/3.3	2.2/4.3	2.1/5.8	2.5/5.0	3.3/6.1	4.7/9.8	3.3/8.3	1.1/1.4	1.3/1.6	1.3/1.6	1.3/1.5
OoA	*T*	1.4/2.7	2.0/5.2	2.1/5.0	2.1/4.5	1.0/3.5	2.9/6.1	3.4/7.5	3.15/6.7	1.1/1.1	1.1/1.3	1.1/1.2	1.1/1.3
	B	1.6/2.7	2.0/3.7	1.9/3.9	2.1/4.4	4.0/7.0	3.0/4.2	3.0/4.7	3.1/6.3	1.0/1.2	1.1/1.2	1.1/1.2	1.1/1.4

pattern of larger travel distances for the *Top* setup for the two parking lots with larger capacities (i.e. GGG and SSG). Thus, the selection of this parameter has an import effect on the system performance and - in general - it constitutes a trade-off between total travel distance and per-vehicle maneuvers, which should be taken carefully taken into consideration in conjunction with other factors by the car park operator during the design phase.

3) COMPARING DISTINCT LAYOUTS

We now study the impact of park layout in the performance of the automated parking system resorting to the PAG dataset. Specifically, we compare a *wide layout*, i.e PAG dataset config. 1 with 11 rows and 27 columns (Table 4) with a *deep layout*, i.e. PAG dataset config. 2 with 27 rows and 11 columns (Table 5) for the different planning & control strategies. Note that the following conclusions could have been also obtained for the GGG and SSG dataset but are not presented here for brevity.

Analyzing the results, we can observe that the *COA* strategy performs best in both parking layouts for achieving low travel distance, low number of maneuvers and reasonable removal time. Now comparing a given algorithm in both park layouts we can conclude that deep layouts: 1) impact differently the distinct parking algorithms and starting positions (e.g. as expected, the number of per-vehicle maneuvers increases considerably more for the *Bottom* setting for park with lower number of stacks when comparing with the *Top* setting) and 2) lead to an increase in all dynamic metrics, namely travel distance, number of maneuvers and removal time. This results demonstrate that the parking layout is another critical parameter for the performance of automated parking lots. The results show that *wide layouts* should be favored as these allow optimizing the placement of vehicles within the parking lot as a wider range of non-conflicting options is available. Certainly, this consideration has to combined with the needs of other disciplines (e.g. Architecture) that contribute to the design, construction or renovation of buildings or other public spaces. Due to this reason in the following evaluation we solely consider *wide layouts* for the GGG and SSG datasets.

4) COMPARING DISTINCT PARKING CAPACITIES

We now assess how the system performance is impaired by the parking capacity taking the PAG as reference, while maintaining fixed all other parameters (i.e. *Wide layout*, and *Top* or *Bottom* starting position):

- **PAG**: 297 spaces
- **GGG**: 640 spaces (2.15 times larger than PAG)
- **SSG**: 1000 spaces (3.37 time larger than PAG)

Note that for fairness reasons all parks have a similar aspect ratio, i.e. the ratio between the rows and columns in terms of quantity (or distance). For instance, the PAG config. 1, GGG and SSG parks have a aspect ratio of 2.45 (1.02), 2.5 (1) and 2.5 (1), respectively. Resorting to the information provided in Tables 4, 6 and 7, we can conclude that the *COA* is the strategy with overall best balanced performance (i.e. low travel distance and maneuvers with a reasonable removal time) for all datasets, having excellent results on the *Top* position.

Table 8 presents the ratio between the GGG/SSG results for a given algorithm, start position, metric and percentile to the corresponding PAG result. The results show that this increase is different for the i) four planning & control algorithms (e.g., the *SLS/Bottom* strategy suffers the highest overall impact in both cases), ii) starting positions (the *Bottom* position having larger increases in all metrics when comparing with the *Top* position due to the more varying and larger demand), iii) different percentiles (as expected, the larger percentiles are more affected by the increase in the park size) and iv) different metrics.

Regarding the different metrics, as expected, all metrics increase as the parking lots become larger: the travel distance, maneuvers and removal time increase between $[1.1, 2.5]$ (average: 1.95), $[1.0, 4.7]$ (average: 2.51) and $[1.0, 1.3]$ (average: 1.17) times for the GGG dataset, respectively. On the other hand, the travel distance, maneuvers and removal time increase between $[2.0, 5.8]$ (average: 3.74), $[1.0, 9.8]$ (average: 4.53) and $[1.1, 1.7]$ (average: 1.37) times for the SSG dataset, respectively. We can conclude that the metric: 1) *maneuvers* increases more quickly for larger parks for all considered planning & control strategies, 2) *travel distance* increases almost linearly with the parking size increase and 3) *removal time* is less influenced by the increase of the parking size (up to 60% and 70% for the GGG and SSG datasets, respectively) being the values still within acceptable bounds for the end costumer.

E. DISCUSSION

The present evaluation allows better understanding the performance limits of a low-complexity high-density parking system that can be implemented with technology available in today's vehicles. Furthermore, we have demonstrated that this new parking paradigm can substantially improve how cars are parked nowadays, reducing the parking area to nearly half and decreasing significantly the in-park mobility (in terms of distance), while still reducing the vehicle removal times. The increased in-park start & stops and low velocity travel within the parking area are very compatible with electric propulsion that is rapidly increasing worldwide.

This study can also support the design of future automated parking lots. First, the results have shown that *COA* strategy presents the best overall performance. Herein, we also identified the main parameters that impair the system performance and that exist complex interactions between them and with the planning & control strategies. For instance, the results have demonstrated that *wide layouts* should be favored in favor of *deep layouts* for improved performance, and that the *Top* position decreases considerably the number of maneuvers while having low in-park travel distance in selected planning strategies. The evaluation has also demonstrated that the system is scalable. However, to further improve the performance, buildings could be designed with a number of smaller *interacting* parking areas.

The selection of the most appropriate planning strategy and its configuration will depend also on the car park operator's objectives. Furthermore, the design of new high-density parking lots should also consider interaction with other disciplines (e.g. architecture for the integration of these structures into buildings) and city planners (e.g. to allocate capacity closer to the user demand) to further improve the system performance. Vehicle automation in parking can also lead to decreasing the *cruising for parking* phenomena.

VII. CONCLUSION

We presented an exhaustive evaluation of different planning strategies for automated, high-density parking lots, based on a novel concept of cooperative parking that leverages the current low-level driving automation of vehicles and their electrification in terms of propulsion. The system allows halving the space requirements for parking vehicles while simultaneously providing fast access times to vehicles entering and exiting the parking lot. Through a comprehensive simulation study, we have shown that the cooperative in-park mobility in terms of number of maneuvers and travel distance is low, and that the parking lot layout significantly affects these two metrics. Our results demonstrate consistency over different patterns of parking lots utilization, from work-related to event-driven, highlighting the importance of planning strategies specifically suited for each pattern of parking sessions in different parking lots.

Clearly, the main result of our novel concept of cooperative and high-density parking, which is implementable with the current existing technology in modern vehicles, is the reduction to half of the space necessary to park vehicles. This reduction and the magnitude of freed space that results from it, holds a substantial potential to reshape the landscape of our cities. The controlled environment of such car-only spaces also configures a much simpler scenario in terms of the legal barriers that have been presenting challenges for high-level autonomous driving.

As future work, we intend to work on the accurate prediction of entry and exit times for parking vehicles, as well as understand and quantify how randomness on these times can impact the overall system performance. Another research line we are pursuing is the design of multi-level parking buildings that are specifically conceived to improve the efficiency of this concept of cooperative and automated parking, including the assessment of novel layout patterns (e.g. concentric circles, 3D spirals). We also intend to develop an automatic procedure to design high-density parking facilities based on the parking operators' and real-world requirements together with predicted demand.

REFERENCES

[1] Eurostat. *Statistics Explained: Passenger Transport Statistics*. Accessed: Dec. 2019. [Online]. Available: https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Passenger_transport_statistics#Modal_split_of_inland_passengers

[2] B. van Wee, *The Unsustainability of Car Use*. Dordrecht, The Netherlands: Springer, 2014, pp. 69–83.

[3] R. Gerlagh, I. van den Bijgaart, H. Nijland, and T. Michielsen, "Fiscal policy and CO_2 emissions of new passenger cars in the EU," *Environ. Resource Econ.*, vol. 69, no. 1, pp. 103–134, Jan. 2018.

[4] D. Edwards. (2012). *Cars Kill Cities*, Progressive Transit Blog. Accessed: Dec. 2019. [Online]. Available: https://progressivetransit.wordpress.com/2012/01/25/cars-kill-cities/

[5] T. Lin, H. Rivano, and F. Le Mouel, "A survey of smart parking solutions," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 12, pp. 3229–3253, Dec. 2017.

[6] W. Wang, Y. Song, J. Zhang, and H. Deng, "Automatic parking of vehicles: A review of literatures," *Int. J. Automot. Technol.*, vol. 15, no. 6, pp. 967–978, Oct. 2014.

[7] M. Ferreira, L. Damas, H. Conceicao, P. M. d'Orey, R. Fernandes, P. Steenkiste, and P. Gomes, "Self-automated parking lots for autonomous vehicles based on vehicular ad hoc networking," in *Proc. IEEE Intell. Vehicles Symp. Proc.*, Jun. 2014, pp. 472–479.

[8] P. M. d'Orey, J. Azevedo, and M. Ferreira, "Automated planning and control for high-density parking lots," in *Proc. 27th Int. Conf. Automated Planning Scheduling (ICAPS)*, 2017, pp. 367–372.

[9] P. M. d'Orey, J. Azevedo, and M. Ferreira, "Exploring the solution space of self-automated parking lots: An empirical evaluation of vehicle control strategies," in *Proc. IEEE 19th Int. Conf. Intell. Transp. Syst. (ITSC)*, Nov. 2016, pp. 1134–1140.

[10] S. Nawaz, C. Efstratiou, and C. Mascolo, "Parksense: A smartphone based sensing system for on-street parking," in *Proc. 19th Annu. Int. Conf. Mobile Comput. Netw.*, 2013, pp. 75–86.

[11] D. A. Thornton, K. Redmill, and B. Coifman, "Automated parking surveys from a LIDAR equipped vehicle," *Transp. Res. C, Emerg. Technol.*, vol. 39, pp. 23–35, Feb. 2014.

[12] B. Xu, O. Wolfson, J. Yang, L. Stenneth, P. S. Yu, and P. C. Nelson, "Real-time street parking availability estimation," in *Proc. IEEE 14th Int. Conf. Mobile Data Manage.*, Jun. 2013, pp. 16–25.

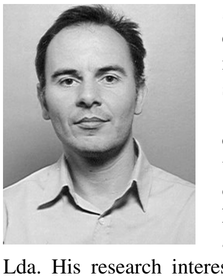
[13] J. K. Suhr and H. G. Jung, "Sensor fusion-based vacant parking slot detection and tracking," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 21–36, Feb. 2014.
[14] R. Lu, X. Lin, H. Zhu, and X. Shen, "SPARK: A new VANET-based smart parking scheme for large parking lots," in *Proc. IEEE 28th Conf. Comput. Commun. (INFOCOM)*, Apr. 2009, pp. 1413–1421.
[15] T. Rajabioun, B. Foster, and P. Ioannou, "Intelligent parking assist," in *Proc. 21st Medit. Conf. Control Autom.*, Jun. 2013, pp. 1156–1161.
[16] A. Klappenecker, H. Lee, and J. L. Welch, "Finding available parking spaces made easy," *Ad Hoc Netw.*, vol. 12, pp. 243–249, Jan. 2014.
[17] H. Banzhaf, L. Palmieri, D. Nienhuser, T. Schamm, S. Knoop, and J. M. Zollner, "Hybrid curvature steer: A novel extend function for sampling-based nonholonomic motion planning in tight environments," in *Proc. IEEE 20th Int. Conf. Intell. Transp. Syst. (ITSC)*, Oct. 2017, pp. 1–8.
[18] D. C. Conner, H. Kress-Gazit, H. Choset, A. A. Rizzi, and G. J. Pappas, "Valet parking without a valet," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Oct. 2007, pp. 572–577.
[19] K. Min, J. Choi, H. Kim, and H. Myung, "Design and implementation of path generation algorithm for controlling autonomous driving and parking," in *Proc. 12th Int. Conf. Control, Automat. Syst. (ICCAS)*, Oct. 2012, pp. 956–959.
[20] E. Uhlemann, "Active safety vehicles evolving toward automated driving [Connected Vehicles]," *IEEE Veh. Technol. Mag.*, vol. 10, no. 4, pp. 20–23, Dec. 2015.
[21] J. Timpner, S. Friedrichs, J. van Balen, and L. Wolf, "K-stacks: high-density valet parking for automated vehicles," in *Proc. IEEE Intell. Vehicles Symp. (IV)*, Jun. 2015, pp. 895–900.
[22] M. Nourinejad, S. Bahrami, and M. J. Roorda, "Designing parking facilities for autonomous vehicles," *Transp. Res. B, Methodol.*, vol. 109, pp. 110–127, Mar. 2018.
[23] K.-H. C. Wang and A. Botea, "Fast and memory-efficient multi-agent pathfinding," in *Proc. Int. Conf. Automated Planning Scheduling (ICAPS)*, 2008, pp. 380–387.
[24] K.-H. C. Wang and A. Botea, "Tractable multi-agent path planning on grid maps," in *Proc. 21st Int. Joint Conf. Artif. Intell. (IJCAI)*, San Francisco, CA, USA, 2009, pp. 1870–1875.
[25] K.-H. C. Wang and A. Botea, "MAPP: A scalable multi-agent path planning algorithm with tractability and completeness guarantees," *J. Artif. Intell. Res.*, vol. 42, pp. 55–90, Sep. 2011.
[26] B. de Wilde, A. W. ter Mors, and C. Witteveen, "Push and rotate: Cooperative multi-agent path planning," in *Proc. Int. Conf. Auto. Agents Multi-Agent Syst. (AAMAS)*, 2013, pp. 87–94.
[27] H. Ma, C. Tovey, G. Sharon, T. S. Kumar, and S. Koenig, "Multi-agent path finding with payload transfers and the package-exchange robot-routing problem," in *Proc. AAAI Conf. Artif. Intell.*, 2016, pp. 1–7.
[28] C. Huetter, "More shuttles, less cost: Energy efficient planning for scalable high-density warehouse environments," in *Proc. 26th Int. Conf. Automated Planning Scheduling (ICAPS)*, 2016, pp. 403–411.
[29] J. Yu and S. M. LaValle, *Multi-Agent Path Planning and Network Flow*. Berlin, Germany: Springer, 2013, pp. 157–173.
[30] M. A. Salido, O. Sapena, M. Rodriguez, and F. Barber, "A planning tool for minimizing reshuffles in container terminals," in *Proc. 21st IEEE Int. Conf. Tools with Artif. Intell.*, Nov. 2009, pp. 567–571.
[31] H. Ma, S. Koenig, N. Ayanian, L. Cohen, W. Hoenig, T. S. Kumar, T. Uras, H. Xu, C. Tovey, and G. Sharon, "Overview: Generalizations of multi-agent path finding to real-world scenarios," in *Proc. Int. Joint Conf. Artif. Intell. (IJCAI)*, 2016, pp. 1–4.
[32] E. G. Coffman, Jr., and J. G. S. D. Csirik Galambos Martello Vigo, *Bin Packing Approximation Algorithms: Survey and Classification*. New York, NY, USA: Springer, 2013, pp. 455–531.
[33] A. Lodi, S. Martello, and M. Monaci, "Two-dimensional packing problems: A survey," *Eur. J. Oper. Res.*, vol. 141, no. 2, pp. 241–252, Sep. 2002.
[34] E. G. Ström, P. Popovski, and J. Sachs, "5G ultra-reliable vehicular communication," *CoRR*, vol. abs/1510.01288, 2015. [Online]. Available: http://arxiv.org/abs/1510.01288
[35] "SFpark: Pilot project evaluation," San Francisco Municipal Transp. Agency, San Francisco, CA, USA, Tech. Rep., Jun. 2014.
[36] J. Hill, G. Rhodes, S. Vollar, and C. Whapples, *Car Park Designers' Handbook*. London, U.K.: Thomas Telford, 2005.

JOSÉ AZEVEDO received the master's degree in network and electronic systems engineering from the Faculty of Sciences, University of Porto, Portugal, in 2015, where he is currently pursuing the Ph.D. degree in computer science.

He is currently a Researcher with the Instituto de Telecomunicações, Porto. His current research interests include automated vehicles, vehicular networks, and data mining applied in the transportation domain.

PEDRO M. D'OREY (Member, IEEE) received the *Licenciatura* degree in electrical and computer engineering from the University of Porto, Portugal, in 2004, the M.Sc. degree in telecommunications from the Queen Mary University of London, London, U.K., in 2008, and the Ph.D. degree in telecommunications from the University of Porto, in 2014.

He is currently a Research Scientist with the Instituto de Telecomunicações, Porto, and an Invited Assistant Professor with the University of Porto. Previously, he was a Research Scientist at NEC Laboratories Europe, Germany, and an R&D Engineer in the area of mobile communications. His current research interests include automated vehicles, vehicular networks, and data mining applied in the transportation domain.



MICHEL FERREIRA graduated in computer science from the University of Porto, Porto, Portugal, in 1994, and received the Ph.D. degree in computer science from the University of Porto, in 2002.

He is currently an Assistant Professor of computer science at the Faculty of Sciences, University of Porto, and a Researcher of the Porto Laboratory of Instituto de Telecomunicações, where he leads the Geo-Networks Group. He is also a co-founder of the spin-off company Geolink, Lda. His research interests include vehicular networks, spatio-deductive databases, and computer simulation. He has led several research projects in the areas of logic-based spatial databases, vehicular sensing, and inter-vehicle communication.