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Novel Dual-Purpose Algorithm for Principal and Minor Component Analysis

XU ZHONGYING, GAO YINGBIN, AND KONG XIANGYU
Xi'an Research Institute of High Technology, Xi'an 710025, China

Corresponding author: Gao Yingbin (545436624@qq.com)

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ABSTRACT A dual-purpose algorithm is capable of estimating the principal component and minor component from input signals by simply switching the sign of some terms in the same learning rule. Compared with single-purpose algorithms, a dual-purpose algorithm has many advantages. In this paper, a novel dual-purpose algorithm is proposed based on the study of some existing algorithms. The dynamic behavior of this dual-purpose algorithm is investigated by the deterministic discrete time method. Some constraint conditions, which provide a way to choose the initial weight vector and learning factor, are also derived to guarantee its convergence. Numerical simulation results not only demonstrate the fast convergence of the proposed algorithm but also demonstrate the correctness of the convergence conditions.

INDEX TERMS Dual-purpose algorithm, principal component analysis, minor component analysis, dynamic behavior analysis.

I. INTRODUCTION

In the field of signal processing, principal component analysis (PCA) is a technique that can be performed to estimate the eigenvector that corresponds to the maximum eigenvalue of the signal autocorrelation matrix. Minor component analysis (MCA) can be used to extract the eigenvector that corresponds to the minimum eigenvalue of the signal autocorrelation matrix [1]. PCA and MCA have been applied in many areas of signal processing [2]–[4]. Currently, neural network-based PCA and MCA algorithms are research hotspots, and many outstanding algorithms have been proposed [5]–[8]. However, these algorithms can perform only PCA or only MCA. Is it possible to use the same algorithm to simultaneously implement PCA and MCA by changing only the plus-minus sign of some terms in one learning rule? Some investigators have studied this problem and designated algorithms that can achieve these functions as dual-purpose algorithms [9].

Relative to the PCA or MCA algorithm with a single function, the study of a dual-purpose algorithm is significant in three aspects [10]: (1) the dual-purpose algorithm can be applied to both PCA fields and MCA fields, with a wider range of applications; (2) because the dual-purpose algorithm uses different signs on only some terms in its rules of operation when implementing PCA or MCA, the hardware cost for algorithms can be reduced; and (3) the study of the dual-purpose algorithm can interpret the intrinsic link between PCA and MCA algorithms and thus is of theoretical importance. Minimal attention has been paid to dual purpose algorithms, and few researchers have examined the convergence speed of existing algorithms [11]–[15]. Therefore, developing a fast dual purpose algorithm is the aim of this paper.

Dynamic property analysis, which can describe the movement trajectory of the weight vector of a neural network during the whole iteration procedure and can help us understand why algorithms can converge to the desired component after iterations, has become an important aspect of studying neural network algorithms. In recent years, the deterministic discrete time (DDT) analysis method has become the mainstream method for analyzing the dynamic properties of algorithms [13], [16], [17]. When the DDT method is employed, the dynamic property of a weight vector can be obtained by projecting the weight vector onto all of the eigenvectors of the autocorrelation matrix and analyzing the changing laws of these projections. Compared with other methods, the DDT analysis method can retain the discrete nature and the dynamic properties of the algorithm [18] and thus has been favored by many investigators and extensively

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applied in many PCA or MCA algorithms [18]–[20]. Compared with PCA or MCA algorithms, the structure of dual-purpose algorithms is more complicated, and the difficulty in analysis increases correspondingly. Therefore, it is necessary to investigate how to use the DDT method to examine the dynamic properties of dual-purpose algorithms.

In this study, based on some existing algorithms, we propose a fast dual-purpose algorithm and analyze its dynamic properties using the DDT method. This paper is organized as follows: in the second section, we introduce the symbol usage rule and describe important symbols; in the third section, we propose the dual-purpose algorithm; in the fourth section, we perform a dynamic property analysis of the dual-purpose algorithm; in the fifth section, we present the experimental simulation results; and in the sixth section, we summarize our findings.

II. NOTATIONS AND ACRONYMS

In this paper, unless otherwise noted, symbol naming adheres to the following rules: matrices are represented by capital bold-italic letters (e.g., $\boldsymbol{R}$), vectors are represented by lowercase bold-italic letters (e.g., $\boldsymbol{v}$), and scalars are represented by lowercase italic letters (e.g., k). The notations of some important symbols are as follows:

$\boldsymbol{R}$: autocorrelation matrix of signal
$\boldsymbol{x}$: input signal vector
$\boldsymbol{w}$: weight vector of neural network
$\boldsymbol{v}$: eigenvector of autocorrelation matrix
η : learning factor

III. NOVEL DUAL-PURPOSE ALGORITHM

Assume that the input signal sequence $\{\boldsymbol{x}(k)|\,\boldsymbol{x}(k) \in \mathbb{R}^{n\times 1}\}$ is a stationary random input signal of dimension n. We then consider the neural network model with the following form:

$$y(k) = \boldsymbol{w}^T(k)\boldsymbol{x}(k), \quad (k = 0, 1, 2\cdots) \tag{1}$$

where $y(k)$ is the output and $\boldsymbol{w}(k)$ is the weight vector of the neural network. The core step in developing the neural network algorithm is to construct an appropriate weight vector update rule, which enables the weight vector to converge to the principal component (PC) or minor component (MC) of the signal after several iterations.

Among neural network-based algorithms, the Oja algorithm is famous; its updating rule is given by

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \eta\left[y(k)\boldsymbol{x}(k) - y^2(k)\boldsymbol{w}(k)\right] \tag{2}$$

where η is the learning factor of the algorithm, which satisfies $0 < \eta < 1$. However, this algorithm can be utilized only for PCA. By adding a penalty term to (2) and changing the sign of the learning factor, we propose a novel dual-purpose algorithm as follows:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) \pm \eta\left[y(k)\boldsymbol{x}(k) - y^2(k)\boldsymbol{w}(k) + \boldsymbol{w}(k)\left(\boldsymbol{w}^T(k)\boldsymbol{w}(k)\right)^2 - \boldsymbol{w}(k)\right] \tag{3}$$

When "+" is used, Eq. (3) is a PCA algorithm; conversely, when " − " is used, it is an MCA algorithm. By applying the conditional expectation factor to Eq. (3) and substituting the obtained conditional expectation value in the next iteration, we can obtain the DDT system of Eq. (3) as

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) \pm \eta\left[\boldsymbol{R}\boldsymbol{w}(k) - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)\boldsymbol{w}(k) + \boldsymbol{w}(k)\left(\boldsymbol{w}^T(k)\boldsymbol{w}(k)\right)^2 - \boldsymbol{w}(k)\right] \tag{4}$$

where $\boldsymbol{R} = E[\boldsymbol{x}(k)\boldsymbol{x}^T(k)]$ is the autocorrelation matrix. Assuming $\lambda_i(i = 1, 2, \cdots, n)$ and $\boldsymbol{v}_i$ are the eigenvalues of the autocorrelation matrix $\boldsymbol{R}$ and its corresponding eigenvectors, respectively, then $\boldsymbol{v}_i(i = 1, 2, \cdots, n)$ constitutes a set of orthogonal bases of the space of $\mathbb{R}^{n\times n}$. For convenience, we sort the eigenvalues in descending order; i.e.,

$$\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0 \tag{5}$$

Similarly, the eigenvectors are also sorted. According to matrix theory, any vector in the space of $\mathbb{R}^{n\times n}$ can be expressed as a linear combination of bases. Therefore, $\boldsymbol{w}(k)$ and $\boldsymbol{R}\boldsymbol{w}(k)$ can be expressed as

$$\boldsymbol{w}(k) = \sum_{i=1}^{n} z_i(k)\boldsymbol{v}_i, \quad \boldsymbol{R}\boldsymbol{w}(k) = \sum_{i=1}^{n} \lambda_i z_i(k)\boldsymbol{v}_i \tag{6}$$

where $z_i(i = 1, 2, \cdots, n)$ represent some constants, the projection lengths of $\boldsymbol{w}(k)$ on $\boldsymbol{v}_i(i = 1, 2, \cdots, n)$. Based on Eqs. (4) and (6), if $k \geq 0$, we have

$$z_i(k+1) = \left\{1 \pm \eta\left[\lambda_i - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1\right]\right\} z_i(k) \tag{7}$$

Based on the nature of the Rayleigh quotient [21], if $\boldsymbol{w}(k) \neq \boldsymbol{0}$, then

$$0 < \lambda_n\boldsymbol{w}^T(k)\boldsymbol{w}(k) < \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) < \lambda_1\boldsymbol{w}^T(k)\boldsymbol{w}(k) \tag{8}$$

This equation will be extensively applied in the following proofs.

IV. DYNAMIC PROPERTIES ANALYSIS

Since Algorithm (4) must address the valuation of the learning factor for the plus-minus sign, we must discuss the dynamic properties separately for each sign.

A. DYNAMIC PROPERTIES OF PCA ALGORITHM

When (3) takes the "+" sign, it is a PCA algorithm.

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \eta\left[\boldsymbol{R}\boldsymbol{w}(k) - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)\boldsymbol{w}(k) + \boldsymbol{w}(k)\left(\boldsymbol{w}^T(k)\boldsymbol{w}(k)\right)^2 - \boldsymbol{w}(k)\right] \tag{9}$$

Eq. (7) can be expressed as follows:

$$z_i(k+1) = \left\{1 + \eta\left[\lambda_i - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1\right]\right\} z_i(k) \tag{10}$$

The dynamic property analysis of Algorithm (9) is performed based on the following theorems.

Theorem 1: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_1 \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then if $k \geq 0$, $\|\boldsymbol{w}(k+1)\| < 1 + \eta\lambda_1$ is always true.

Proof: Please refer to Appendix A.

Lemma 1: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_1 \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$; then, if $k \geq 0$, $\beta > 0$ is always true, in which

$$\beta = 1 + \eta[\lambda_i - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1] \quad (11)$$

Proof: Please refer to Appendix B.

Premultiplying Eq. (6) simultaneously with $\boldsymbol{v}_i^T$, we have $z_i(k) = \boldsymbol{v}_i^T\boldsymbol{w}(k)(i = 1, 2, \cdots, n)$; i.e., $z_i(k)$ can be regarded as the projection length of $\boldsymbol{w}^T(k)$ on $\boldsymbol{v}_i$ at the k th iteration. Given $\beta > 0$, z_i will not change its sign in the iteration process. Because $\boldsymbol{w}^T(0)\boldsymbol{v}_1 \neq 0$, if $k \geq 0$, $z_1(k) \neq 0$ is always true. Thus, Eq. (6) can be decomposed into

$$\boldsymbol{w}(k) = \sum_{i=1}^{n} z_i(k)\boldsymbol{v}_i = z_1(k)\boldsymbol{v}_1 + \sum_{i=2}^{n} z_i(k)\boldsymbol{v}_i \quad (12)$$

The dynamic properties of $\boldsymbol{w}(k)$ are clearly determined by $z_1(k)(i = 1, 2, \cdots, n)$. Therefore, we focus on analyzing the dynamic properties of $z_1(k)(i = 1, 2, \cdots, n)$ in iterations.

Lemma 2: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_1 \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, and if $k \to \infty$, then $\lim_{k\to\infty} z_i(k) = 0$, $(i = 2, 3, \cdots, n)$.

Proof: Please see Appendix C.

Lemma 3: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_1 \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then if $k \to \infty$, $\lim_{k\to\infty} z_1(k) = \pm 1$.

Proof: Please refer to Appendix D.

Theorem 2: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_1 \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, when $k \to \infty$, $\lim_{k\to\infty} \boldsymbol{w}(k) = \pm\boldsymbol{v}_1$.

Proof: According to Lemma 2, we have

$$\lim_{k\to\infty} z_i(k) = 0, \quad (i = 2, 3, \cdots, n) \quad (13)$$

According to Lemma 3, we have

$$\lim_{k\to\infty} z_1(k) = \pm 1 \quad (14)$$

According to Eq. (6), we have

$$\lim_{k\to\infty} \boldsymbol{w}(k) = \lim_{k\to\infty} z_1(k)\boldsymbol{v}_1 + \lim_{k\to\infty} \sum_{i=2}^{n} z_i(k)\boldsymbol{v}_i = \pm\boldsymbol{v}_1 \quad (15)$$

We have accomplished the dynamic property analysis of the algorithm for the case in which the "+" sign is chosen for Eq. (3). Next, we will analyze the case in which the "−" sign is chosen. Since the proof process is similar, we will simplify the proof process and present only the conclusion.

B. DYNAMIC PROPERTIES OF THE MCA ALGORITHM

When the "-" sign is utilized, (3) is an MCA algorithm.

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) - \eta\Big[\boldsymbol{R}\boldsymbol{w}(k) - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)\boldsymbol{w}(k) + \boldsymbol{w}(k)\left(\boldsymbol{w}^T(k)\boldsymbol{w}(k)\right)^2 - \boldsymbol{w}(k)\Big] \quad (16)$$

Accordingly, Eq. (7) can be expressed as follows:

$$z_i(k+1) = \left\{1 - \eta\left[\lambda_i - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1\right]\right\} z_i(k) \quad (17)$$

The dynamic property analysis of Algorithm (16) will be conducted through two theorems and three lemmas.

Theorem 3: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_n \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then if $k \geq 0$, $\|\boldsymbol{w}(k+1)\| < 1 + \eta + \eta\lambda_1$ is always true.

Proof: Please refer to Appendix E.

Lemma 4: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_n \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then if $k \geq 0$, $\beta' > 0$ is always true, where

$$\beta' = 1 - \eta\left[\lambda_i - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1\right] \quad (18)$$

Proof: Please see Appendix F.

Eq. (6) can be broken down into another form:

$$\boldsymbol{w}(k) = \sum_{i=1}^{n} z_i(k)\boldsymbol{v}_i = \sum_{i=1}^{n-1} z_i(k)\boldsymbol{v}_i + z_n(k)\boldsymbol{v}_n \quad (19)$$

Lemma 5: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_n \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then when $k \to \infty$, $\lim_{k\to\infty} z_i(k) = 0$, $(i = 1, 2, 3, \cdots, n-1)$.

Proof: Please see Appendix G.

Lemma 6: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_n \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then when $k \to \infty$, $\lim_{k\to\infty} z_n(k) = \pm 1$.

Proof: Please see Appendix H.

Theorem 4: Assuming $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, if the initialization condition satisfies $\boldsymbol{w}^T(0)\boldsymbol{v}_n \neq 0$ and $\|\boldsymbol{w}(0)\| \leq 1$, then when $k \to \infty$, $\lim_{k\to\infty} \boldsymbol{w}(k) = \pm\boldsymbol{v}_n$.

Proof: By substituting the conclusions of Lemmas 5 and 6, we have

$$\lim_{k\to\infty} \boldsymbol{w}(k) = \lim_{k\to\infty} \sum_{i=1}^{n-1} z_i(k)\boldsymbol{v}_i + \lim_{k\to\infty} z_n(k)\boldsymbol{v}_n = \pm\boldsymbol{v}_n \quad (20)$$

Therefore, we complete the dynamic property analysis of the MCA algorithm.

Remark: The DDT system of the proposed algorithm has a computation complexity of $n^2 + 4n$ flops per update, which is the same as $n^2 + 4n$ of the Chen algorithm's DDT system [14], and is cheaper than $n^2 + 8n$ of the Hasan algorithm's DDT system in [15] and $n^2 + 5n$ the projection approximation subspace tracking with deflation (PASTd) algorithm's DDT system in [16]. In addition, the operations involved in (4) are simple matrix addition and multiplication, which are easy for the systolic array implementation.

V. SIMULATION EXPERIMENT

In this section, we verify the performance of the proposed algorithm through two experiments. In the first, we verify the algorithm's ability to extract PC and MC and compare the results with those of some existing algorithms. In the second experiment, we examine the dynamic properties of the algorithm in the iterative process. Assume that the autocorrelation matrix of the input signal is a symmetric positive definite matrix [22]:

$$\boldsymbol{R}_1 = \begin{bmatrix} 7.8465 & -1.0608 & -0.2722 & -0.3463 & 0.5346 \\ -1.0608 & 3.2566 & 0.5156 & 1.2748 & -0.9590 \\ -0.2722 & 0.5156 & 4.4051 & 1.0389 & -0.3293 \\ -0.3463 & 1.2748 & 1.0389 & 6.1511 & -1.4831 \\ 0.5346 & -0.9590 & -0.3293 & -1.4831 & 3.7119 \end{bmatrix} \tag{21}$$

The maximum eigenvalue and minimum eigenvalue of the matrix can be calculated, using MATLAB, as $\lambda_1 = 8.8423$ and $\lambda_5 = 2.4187$, respectively, with the following corresponding eigenvectors:

$$\boldsymbol{v}_1 = [-0.7259, 0.3195, 0.2177, 0.4890, -0.2907]^T \tag{22}$$

$$\boldsymbol{v}_5 = [0.1093, 0.8532, -0.0940, -0.0573, 0.4979]^T \tag{23}$$

According to the signal processing theory [23], $\boldsymbol{v}_1$ constitutes the PC of the input signal, and $\boldsymbol{v}_5$ constitutes the MC of the input signal.

A. ALGORITHM PERFORMANCE COMPARISON EXPERIMENT

To verify the performance and advantages of the proposed algorithm, we compare the proposed algorithm with some existing algorithms, namely, the PASTd dual-purpose algorithm [13], modified novel information criterion (MIC) PCA algorithm [24] and Feng MCA algorithm [25]. The direction cosine (DC) values of the weight vector and the PC or MC in the iterative process are calculated using the following equation [26]:

$$\mathrm{DC}(k) = \left|\boldsymbol{w}^T(k)\boldsymbol{v}_i\right| / \|\boldsymbol{w}(k)\| \cdot \|\boldsymbol{v}_i\| \tag{24}$$

where $i = 1, 5$. Clearly, when and only when the weight vector converges to be in line with the direction of the PC or MC, the DC curve converges to 1.

Fig. 1 shows that after several iterations, the DC curve of the proposed algorithm converges to unit 1; i.e., the proposed algorithm can extract the PC of the signal; similarly, Fig. 2 also shows that the proposed algorithm can extract the MC of the signal. The comparison of the proposed algorithm with the existing algorithms indicates that the proposed algorithm outperforms these existing algorithms in terms of convergence speed.

B. HIGH-DIMENSIONAL VECTOR EXPERIMENT

This experiment is designed to examine the ability of the proposed algorithm to address the high-dimensional matrix.

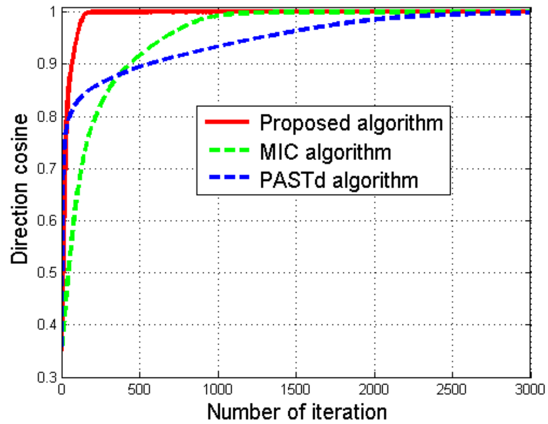

FIGURE 1. Direction cosine curve of PC.

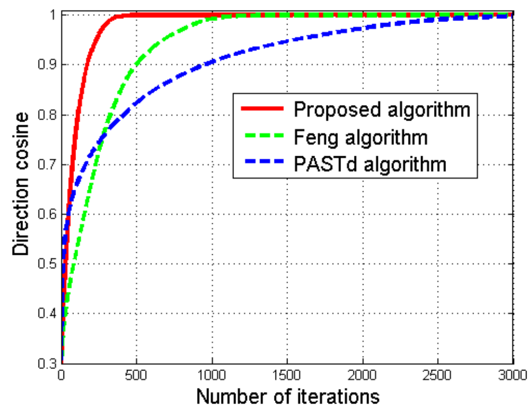

FIGURE 2. Direction cosine curve of MC.

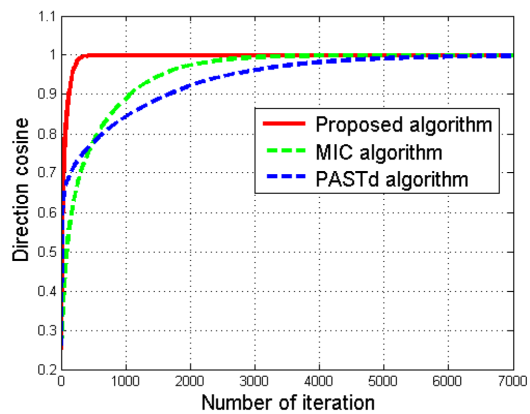

FIGURE 3. DC curve of PCA with high-dimensional matrix.

Consider the 30 × 30 positive symmetric matrix, which is randomly generated; its largest eigenvalue and smallest eigenvalue are given by $\lambda'_1 = 9.8327$ and $\lambda'_{30} = 1.0795$, respectively. The proposed algorithm is also compared with the MIC algorithm, Feng algorithm and PASTd algorithm.

Fig. 3 and Fig. 4 provide the DC curves of these algorithms for PCA and MCA, respectively. We note that the

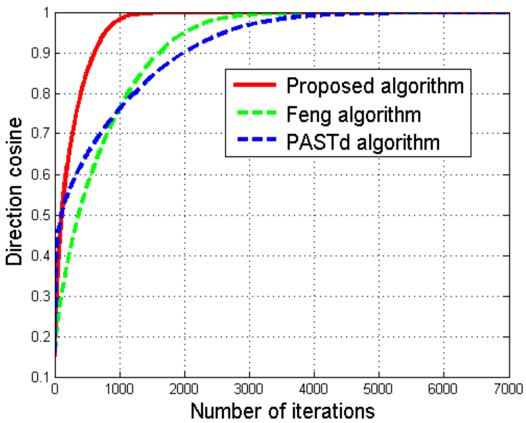

FIGURE 4. DC curve of MCA with high-dimensional matrix.

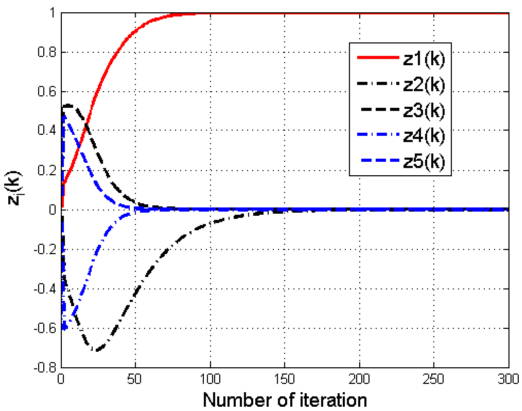

FIGURE 5. Weight vector dynamic behavior of PCA.

proposed algorithm exhibits excellent performance with the high-dimensional matrix. For high-dimensional data, however, the proposed algorithm achieves a faster convergence speed than the other three PCA or MCA algorithms.

C. DYNAMIC PROPERTIES OF THE ALGORITHM

This experiment was mainly designed to examine the dynamic behavior of the proposed algorithm in the iterative process, in which the matrix was employed as the subject, with the following initialization condition: the learning factor was set to $\eta = 0.02$. The following conditions were satisfied: $\eta \leq 0.2$ and $\eta\lambda_1 \leq 0.2$, The initialization weight vector was randomized with a normalized modulus value of 0.1. In the experiment, the weight vector $\boldsymbol{w}(k)$ was projected onto the feature vector $\boldsymbol{v}_i (i = 1, 2, \cdots, 5)$, i.e.,

$$z_i(k) = \boldsymbol{w}^T(k)\boldsymbol{v}_i, \quad (i = 1, 2, \cdots, 5) \tag{25}$$

Fig. 5 shows the weight vector component curve obtained by the PCA algorithm when the initial weight vector was $\boldsymbol{w}_0 = [-0.0316, 0.0141, 0.0308, 0.0047, 0.0885]^T$. Fig. 6 shows the weight vector component curve obtained by the MCA algorithm when the initial weight vector was $\boldsymbol{w}(0) = [0.0487, 0.0327, -0.0630, 0.0238, -0.0450]^T$.

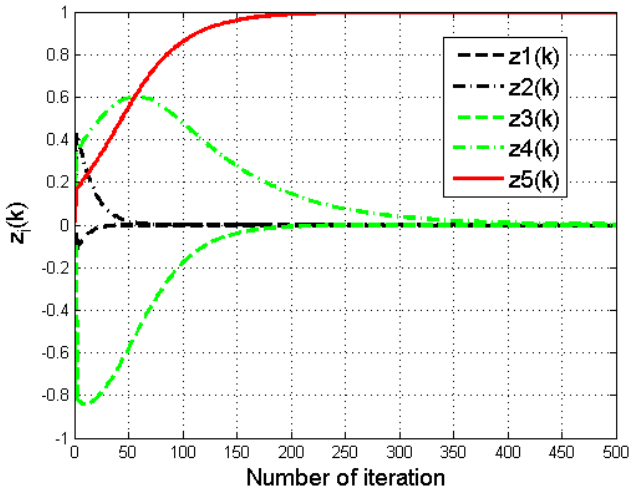

FIGURE 6. Weight vector dynamic behavior of MCA.

As shown in Fig. 5, after numerous iterations, z_1 approaches 1, while $z_i(i = 2, 3, 4, 5)$ approaches 0, which is consistent with the results of Lemmas 2 and 3. Similarly, the dynamic properties of the curve shown in Fig. 6 are consistent with the conclusions of Lemmas 5 and 6, which verifies the correctness of the dynamic behavior analysis of the algorithm.

VI. CONCLUSION

Compared with conventional PCA and MCA algorithms, the proposed dual-purpose algorithm demonstrates certain theoretical and application advantages. By studying some PCA and MCA algorithms, we propose a novel dual-purpose algorithm and analyze its dynamic properties using the DDT method. We obtained the initialization condition for ensuring the convergence of the algorithm and verified the performance and correctness of the analysis process via some numerical experiments, which lays the foundation for the subsequent application of the algorithm.

APPENDIXES

APPENDIX A

PROOF OF THEOREM 1

Proof: From (9) and (10), we have

$$\begin{aligned}
&\|\boldsymbol{w}(k+1)\|^2 \\
&= \sum_{i=1}^{n} z_i^2(k+1) \\
&= \sum_{i=1}^{n} \left\{1 + \eta\left[\lambda_i - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1\right]\right\}^2 z_i^2(k) \\
&\leq \sum_{i=1}^{n} \left\{1 + \eta\left[\lambda_1 - \lambda_n \|\boldsymbol{w}(k)\|^2 + \|\boldsymbol{w}(k)\|^4 - 1\right]\right\}^2 z_i^2(k) \\
&\leq \left\{1 + \eta\left[\lambda_1 - \lambda_n \|\boldsymbol{w}(k)\|^2 + \|\boldsymbol{w}(k)\|^4 - 1\right]\right\}^2 \|\boldsymbol{w}(k)\|^2
\end{aligned}$$

$$\leq \left\{1+\eta\left[\|\boldsymbol{w}(k)\|^4+\lambda_1-1\right]\right\}^2\|\boldsymbol{w}(k)\|^2 \tag{26}$$

We define a continuous derivable function in the interval [0, 1] as follows:

$$f(s)=\left\{1+\eta\left[s^2+\lambda_1-1\right]\right\}^2 s \tag{27}$$

The first-order subdivision of this function is

$$\dot{f}(s)=\left\{1+(\lambda_1-1)\,\eta+\eta s^2\right\}\left\{1+(\lambda_1-1)\,\eta+5\eta s^2\right\} \tag{28}$$

Based on $\eta \leq 0.45$ and $\eta\lambda_1 \leq 0.3$, if $0 < s < 1, \dot{f}(s) > 0$ is always true, that is $f(s)$ monotonically increases within the interval of (0, 1). Therefore, if $0 < s < 1$, the following equation is always true:

$$f(s) < f(1) = (1+\eta\lambda_1)^2 \tag{29}$$

Thus, if $k \geq 0$, then $\|\boldsymbol{w}(k+1)\| < 1+\eta\lambda_1$ is always true.

APPENDIX B
PROOF OF LEMMA 1

According to Theorem 1, we have $\|\boldsymbol{w}(k)\| < 1+\eta\lambda_1$. To prove this lemma, we must consider two scenarios.

Scenario 1: $0 < \|\boldsymbol{w}(k)\| \leq 1$

In this case, according to Eq. (11), we have

$$\begin{aligned}\beta &= 1+\eta\left[\lambda_i-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]\\ &> 1-\eta\lambda_1\|\boldsymbol{w}(k)\|^2+\eta\left(\|\boldsymbol{w}(k)\|^4-1\right)\\ &> 1-\eta\lambda_1-\eta > 0.6 > 0\end{aligned} \tag{30}$$

Scenario 2: $1 < \|\boldsymbol{w}(k)\| < 1+\eta\lambda_1$ in document

In this case, according to Eq. (11), we have

$$\begin{aligned}\beta &= 1+\eta\left[\lambda_i-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]\\ &> 1-\eta\lambda_1\|\boldsymbol{w}(k)\|^2+\eta\left(\|\boldsymbol{w}(k)\|^4-1\right)\\ &> 1-\eta\lambda_1(1+\eta\lambda_1)^4\\ &> 0.5853 > 0\end{aligned} \tag{31}$$

Combining these two scenarios, Lemma 1 is proved.

APPENDIX C
PROOF OF LEMMA 2

If $k \geq 0$, the following equation is always true:

$$\begin{aligned}&\frac{1+\eta\left[\lambda_i-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}\\ &\quad= 1-\frac{\eta(\lambda_1-\lambda_i)}{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}\end{aligned} \tag{32}$$

The constant θ_1 is defined as

$$\theta_1 = \frac{\eta(\lambda_1-\lambda_i)}{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]} \tag{33}$$

Based on Lemma 1 and $\lambda_1 > \lambda_i$, we have $\theta_1 > 0$. Next, we will prove that when the initialization condition is satisfied, $\theta_1 < 1$ is true.

$$\begin{aligned}\theta_1 &= \frac{\eta(\lambda_1-\lambda_i)}{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}\\ &< \frac{\eta\lambda_1}{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}\\ &< \frac{\eta\lambda_1}{1+\eta\lambda_1\left(1-\|\boldsymbol{w}(k)\|^2\right)+\eta\left(\|\boldsymbol{w}(k)\|^4-1\right)}\end{aligned} \tag{34}$$

If $0 < \|\boldsymbol{w}(k)\|^2 \leq 1$, we have

$$\begin{aligned}\theta_1 &< \frac{\eta\lambda_1}{1+\eta\lambda_1\left(1-\|\boldsymbol{w}(k)\|^2\right)+\eta\left(\|\boldsymbol{w}(k)\|^4-1\right)}\\ &< \frac{\eta\lambda_1}{1-\eta} < \frac{0.2}{1-0.2} = 0.25 < 1\end{aligned} \tag{35}$$

If $1 < \|\boldsymbol{w}(k)\|^2 < 1+\eta\lambda_1$, we have

$$\begin{aligned}\theta_1 &< \frac{\eta\lambda_1}{1+\eta\lambda_1\left(1-\|\boldsymbol{w}(k)\|^2\right)+\eta\left(\|\boldsymbol{w}(k)\|^4-1\right)}\\ &< \frac{\eta\lambda_1}{1+\eta\lambda_1\left(1-(1+\eta\lambda_1)^2\right)}\\ &< \frac{0.2}{1+0.2\times\left(1-(1+0.2)^2\right)}\\ &= 0.2193 < 1\end{aligned} \tag{36}$$

Taking Eq. (35)-(36) together, we have $0 < \theta_1 < 1$. Let $\theta = 1-\theta_1$; then, θ is a constant, and $0 < \theta < 1$ is true. Based on Eq. (10) and (33), if $k \geq 0$, the following is always true:

$$\begin{aligned}&\left[\frac{z_i(k+1)}{z_1(k+1)}\right]^2\\ &= \left\{\frac{1+\eta\left[\lambda_i-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}\right\}^2\left[\frac{z_i(k)}{z_1(k)}\right]^2\\ &\leq \theta\left[\frac{z_i(k)}{z_1(k)}\right]^2 \leq \theta^{k+1}\left[\frac{z_i(0)}{z_1(k)}\right]^2\end{aligned} \tag{37}$$

where $i = 2, 3, \cdots, n$. Because $0 < \theta < 1$, if $k \to \infty$, we have

$$\lim_{k\to\infty}\frac{z_i(k)}{z_1(k)} = 0 \tag{38}$$

where $i = 2, 3, \cdots, n$. According to Theorem 1, $z_1(k)$ is vertically bounded, and $z_1(k) \neq 0$, $\lim_{k\to\infty} z_i(k) = 0$, $(i = 2, 3, \cdots, n)$ must be true.

APPENDIX D
PROOF OF LEMMA 3

Based on Lemma 2, if $k \to \infty$, the weight vector $\boldsymbol{w}(k)$ must converge towards the direction of the PC $\boldsymbol{v}_1$. Thus, $\boldsymbol{w}(k) = z_1(k)\boldsymbol{v}_1$ is true.

Based on Eq. (9), we have

$$\begin{aligned}&z_1(k+1)\\ &= \left\{1+\eta\left[\lambda_1-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]\right\}z_1(k)\\ &= \left\{1+\eta\left[\lambda_1-z_1^2(k)\lambda_1+z_1^4(k)-1\right]\right\}z_1(k)\\ &= \left\{1+\eta\,(z_1(k)-1)\,(z_1(k)+1)\left(1+z_1^2(k)-\lambda_1\right)\right\}z_1(k)\end{aligned} \tag{39}$$

Based on Eq. (39), if $k \geq k_0$, the following expression is always true:

$$\begin{aligned} & z_1(k+1)-1 \\ & = \left\{1+\eta\left(z_1(k)-1\right)\left(z_1(k)+1\right)\left(1+z_1^2(k)-\lambda_1\right)\right\} z_1(k)-1 \\ & = z_1(k)-1+\eta\left(z_1(k)-1\right)\left(z_1(k)+1\right)\left(1+z_1^2(k)-\lambda_1\right) z_1(k) \\ & = \left[z_1(k)-1\right]\left[1+\eta\left(z_1(k)+1\right)\left(1+z_1^2(k)-\lambda_1\right) z_1(k)\right] \end{aligned} \tag{40}$$

Given that

$$\begin{aligned} & 1+\eta\left(z_1(k)+1\right)\left(1+z_1^2(k)-\lambda_1\right) z_1(k) \\ & \quad = 1+\eta\left(z_1^2(k)+z_1(k)\right)\left(1+z_1^2(k)-\lambda_1\right) \\ & \quad > 1-\eta\lambda_1\left(z_1^2(k)+z_1(k)\right) \\ & \quad \gg 1-\eta\lambda_1\left(1+\eta\lambda_1+(1+\eta\lambda_1)^2\right) \\ & \quad > 1-0.2\times(1+0.2+(1+0.2)^2) \\ & \quad = 0.4720>0 \end{aligned} \tag{41}$$

Let $\alpha = 1-\eta(\lambda_1-1)(1+z_1(k))\, z_1(k)$; then $0<\alpha<1$ is true. Based on Eqs. (40)-(41), if $k \geq k_0$, then we have

$$|z_1(k+1)-1| = \alpha\, |z_1(k)-1| \tag{42}$$

and

$$|z_1(k+1)-1| = \alpha^{k+1}|z_1(0)-1| \leq (k+1)\Phi_1 e^{-\sigma(k+1)} \tag{43}$$

where $\sigma = -\ln\alpha$ and $\Phi_1 = |z_1(0)-1|$.

For any given arbitrary small number ε, if $K \geq 1$, the following equation is always true:

$$\frac{\Phi_2 K e^{-\sigma K}}{(1-e^{-\sigma K})^2} \leq \varepsilon \tag{44}$$

where $\Phi_2 = \left|\eta(1+\eta\lambda_1)(2+\eta\lambda_1)(1+(1+\eta\lambda_1)^2-\lambda_1)\right|\Phi_1$.

When $k_1 > k_2 > K$, the following is always true:

$$\begin{aligned} & |z_1(k_1)-z_1(k_2)| \\ & = \left|\sum_{r=k_2}^{k_1}\left[z_1(r+1)-z_1(r)\right]\right| \\ & = \left|\sum_{r=k_2}^{k_1}\left\{\eta\left(z_1(k)-1\right)\left(z_1(k)+1\right)\left(1+z_1^2(k)-\lambda_1\right) z_1(r)\right\}\right| \\ & \leq \left|\eta z_1(r)\left(1+z_1(r)\right)\left(1+z_1^2(k)-\lambda_1\right)\right| \sum_{r=k_2}^{k_1}|z_1(r)-1| \\ & \leq \left|\eta(1+\eta\lambda_1)(2+\eta\lambda_1)(1+(1+\eta\lambda_1)^2-\lambda_1)\right| \\ & \quad (\sum_{r=k_2}^{k_1}|z_1(r)-1| \\ & \leq \Phi_2\sum_{r=k_2}^{k_1} r e^{-\sigma r} \leq \Phi_2\sum_{r=K}^{+\infty} r e^{-\sigma r} \\ & \leq \Phi_2 K e^{-\sigma K}\sum_{r=0}^{+\infty} r e^{-\sigma(r-1)} \\ & \leq \frac{\Phi_2 K e^{-\sigma K}}{(1-e^{-\sigma K})^2} \leq \varepsilon \end{aligned} \tag{45}$$

Based on Eq. (45), we conclude that $z_1(k)$ is a Cauchy sequence and must converge to a constant value. Let $\lim_{k\to\infty} z_1(k) = a$; then, based on Eq. (39), we have

$$a = \left\{1+\eta\left[\lambda_1 - a^2\lambda_1 + a^4 - 1\right]\right\} a \tag{46}$$

By solving the above equation, we have $a = \pm 1$; i.e.,

$$\lim_{k\to\infty} z_1(k) = \pm 1 \tag{47}$$

APPENDIX E
PROOF OF THEOREM 3

According to Eqs. (16)-(17), we have

$$\begin{aligned} & \|\boldsymbol{w}(k+1)\|^2 \\ & \leq \sum_{i=1}^{n}\left\{1-\eta\left[\lambda_n-\lambda_1\|\boldsymbol{w}(k)\|^2+\|\boldsymbol{w}(k)\|^4-1\right]\right\}^2 z_i^2(k) \\ & \leq \left\{1+\eta\left[\lambda_1\|\boldsymbol{w}(k)\|^2+1\right]\right\}^2\|\boldsymbol{w}(k)\|^2 \end{aligned} \tag{48}$$

We define a continuous derivable function $f(s) = \left\{1+\eta\left[s^2+1\right]\right\}^2 s$ within the interval of [0, 1] and can determine that $f(s)$ is monotonically increasing within that interval through derivation; i.e.,

$$f(s) < f(1) = (1+\eta\lambda_1+\eta)^2 \tag{49}$$

Thus, we conclude that if $k \geq 0$, $\|\boldsymbol{w}(k+1)\| < 1+\eta+\eta\lambda_1$ is always true.

APPENDIX F
PROOF OF LEMMA 4

From Theorem 3, we have $\|\boldsymbol{w}(k+1)\| < 1+\eta+\eta\lambda_1$. According to Eq. (18), we have

$$\begin{aligned} \beta' & = 1-\eta\left[\lambda_i-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right] \\ & > 1-\eta\lambda_1-\eta\|\boldsymbol{w}(k)\|^4 \\ & > 1-\eta\lambda_1-\eta(1+\eta+\eta\lambda_1)^4 \\ & > 1-0.2-0.2\times(1+0.2+0.2)^4 \\ & = 0.0317>0 \end{aligned} \tag{50}$$

APPENDIX G
PROOF OF LEMMA 5

If $k \geq 0$, the following equation is always true:

$$\begin{aligned} & \left[\frac{z_i(k+1)}{z_n(k+1)}\right]^2 \\ & = \left\{\frac{1-\eta\left[\lambda_i-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}{1-\eta\left[\lambda_n-\boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k)+\|\boldsymbol{w}(k)\|^4-1\right]}\right\}^2\left[\frac{z_i(k)}{z_n(k)}\right]^2 \\ & = \left\{1-\theta_1'\right\}^2\left[\frac{z_i(k)}{z_n(k)}\right]^2 \end{aligned} \tag{51}$$

where

$$\theta'_1 = \frac{\eta(\lambda_i - \lambda_n)}{1 - \eta\left[\lambda_n - \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k) + \|\boldsymbol{w}(k)\|^4 - 1\right]} \quad (52)$$

Based on the initialization condition and Theorem 3, we have

$$\theta'_1 < \frac{\eta\lambda_1}{1 + \eta\left(1 - \|\boldsymbol{w}(k)\|^4\right) - \eta\lambda_n\left(1 - \|\boldsymbol{w}(k)\|^2\right)} \quad (53)$$

When $0 < \|\boldsymbol{w}(k)\|^2 \leq 1$, we have $\theta_1 < \eta\lambda_1/(1 - \eta\lambda_1) < 1$; when $1 < \|\boldsymbol{w}(k)\|^2 < 1 + \eta + \eta\lambda_1$, we have

$$\theta_1 < \frac{\eta\lambda_1}{1 - \eta\left(\|\boldsymbol{w}(k)\|^4 - 1\right)} < 0.4633 < 1 \quad (54)$$

that is, $0 < \theta'_1 < 1$. Assuming $\theta' = 1 - \theta'_1$, then $0 < \theta' < 1$. Thus, if $k \geq 0$, the following expression is always true:

$$\left[\frac{z_i(k+1)}{z_n(k+1)}\right]^2 \leq \theta'\left[\frac{z_i(k)}{z_n(k)}\right]^2 \leq \cdots \leq (\theta')^{k+1}\left[\frac{z_i(0)}{z_n(k)}\right]^2 \quad (55)$$

where $i = 1, 2, \cdots, n-1$. Given that $0 < \theta' < 1$ and $z_n(k)$ are vertically bounded and $z_n(k) \neq 0$, $\lim_{k\to\infty} z_i(k) = 0$ $(i = 1, 2, \cdots, n-1)$ must be true.

APPENDIX H
PROOF OF LEMMA 6

Based on Lemma 5, when $k \to \infty$, we have $\boldsymbol{w}(k) = z_n(k)\boldsymbol{v}_n$. According to Eq. (17), we have

$$z_n(k+1) = \{1 - \eta\,(z_n(k) - 1)\,(z_n(k) + 1) \times \left(1 + z_n^2(k) - \lambda_n\right)\}\, z_n(k) \quad (56)$$

Based on Eq. (56), if $k \geq k_0$, the following equation is always true:

$$z_n(k+1) - 1 = (z_n(k) - 1) \times \left[1 - \eta z_n(k)\,(z_n(k) + 1)\left(1 + z_n^2(k) - \lambda_n\right)\right] \quad (57)$$

Given

$$\eta z_n(k)\,(z_n(k) + 1)\left(1 + z_n^2(k) - \lambda_n\right) < \eta\,\|\boldsymbol{w}(k)\|\,(\|\boldsymbol{w}(k)\| + 1)\left(1 + \|\boldsymbol{w}(k)\|^2\right) < 1.9891 < 2 \quad (58)$$

Assuming $\alpha' = 1 - \eta z_n(k)\,(z_n(k) + 1)\left(1 + z_n^2(k) - \lambda_n\right)$, then $0 < \left|\alpha'\right| < 1$. According to Eqs. (57)-(58), if $k \geq k_0$, the following expression is always true:

$$|z_n(k+1) - 1| < |\alpha|\,|z_n(k) - 1| < \alpha^{k+1}|z_n(0) - 1| \leq (k+1)\Phi_1 e^{-\sigma(k+1)} \quad (59)$$

where $\sigma' = -\ln\alpha'$ and $\Phi'_1 = |z_n(0) - 1|$.

For any given arbitrary small number ε', $K \geq 1$ is always true, which renders the following expression true:

$$\frac{\Phi'_2 K e^{-\sigma' K}}{(1 - e^{-\sigma' K})^2} \leq \varepsilon' \quad (60)$$

where $\Phi'_2 = \left|\eta z_n(r)\,(1 + z_n(r))\,(1 + z_n^2(k) - \lambda_n)\right|\Phi'_1$. If $k_1 > k_2 > K$, the following equation is always true:

$$\begin{aligned} &|z_n(k_1) - z_n(k_2)| \\ &= \left|\sum_{r=k_2}^{k_1}[z_n(r+1) - z_n(r)]\right| \\ &= \left|\sum_{r=k_2}^{k_1}\left\{\eta\,(z_n(r) - 1)\,(z_n(r) + 1)\left(\lambda_n - 1 - z_n^2(r)\right)z_n(r)\right\}\right| \\ &\leq \left|\eta z_n(r)\,(1 + z_n(r))\,(1 + z_n^2(k) - \lambda_n)\right|\sum_{r=k_2}^{k_1}|z_n(r) - 1| \\ &\leq \Phi'_2\sum_{r=k_2}^{k_1} r e^{-\sigma' r} \leq \frac{\Phi'_2 K e^{-\sigma' K}}{(1 - e^{-\sigma' K})^2} \\ &\leq \varepsilon' \end{aligned} \quad (61)$$

This equation indicates that $z_n(k)$ must converge to a constant (a'). By solving the equation $a' = [1 + \eta(\lambda_1 - a'^2\lambda_1 + a'^4 - 1)]a'$, we have $a' = \pm 1$. Thus, $\lim_{k\to\infty} z_n(k) = \pm 1$.

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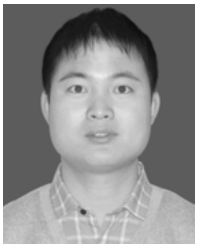
XU ZHONGYING received the B.S. and M.S. degrees from the Xi'an Research Institute of High Technology, Xi'an, China, where he is currently pursuing the Ph.D. degree in control science and engineering. His research interests include stochastic system analysis and nonlinear system modeling.



GAO YINGBIN received the bachelor's, master's, and Ph.D. degrees in control engineering from the Xi'an Research Institute of High Technology, Xi'an, Shaanxi, China, in 2009, 2011, and 2016, respectively. He is currently a Lecturer with the Xi'an Research Institute of High Technology. His current research interests include adaptive signal processing and feature extraction.

KONG XIANGYU received the bachelor's degree from the Beijing Institute of Technology, in 1990, the master's degree from the Xi'an Research Institute of High Technology, Xi'an, Shanxi, in 2000, and the Ph.D. degree in control science and engineering from Xi'an Jiaotong University, Xi'an, in 2005.

He is currently a Professor with the Xi'an Institute of High Technology. His research interests include adaptive signal processing, system modeling, and fault diagnosis.